VLBI software manual

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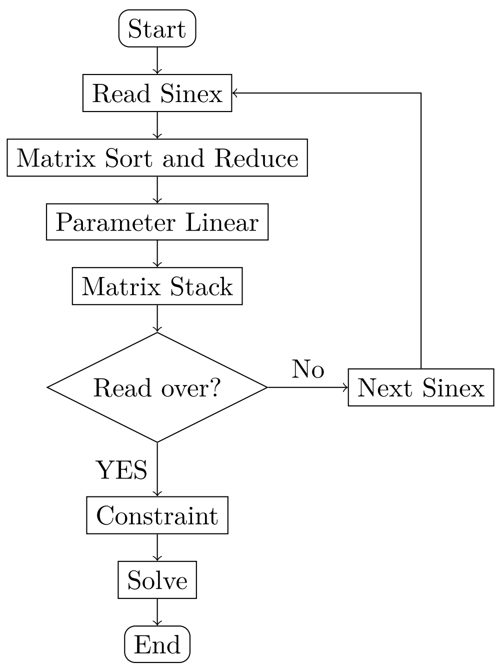
Abstract

The VLBI single solution and global solution are introduced.

1 Flow

The VLBI global solution flow is:



2 Theory

2.1 Matrix Sort and Reduce

The Normal equation is:

$$\begin{bmatrix} N_{11} & N_{12} & N_{13} & N_{14} \\ N_{21} & N_{22} & N_{23} & N_{24} \\ N_{31} & N_{32} & N_{33} & N_{34} \\ N_{41} & N_{42} & N_{43} & N_{44} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \tag{1}$$

If we want to move the parameter x_3 to first row, then need move the column firstly,

$$\begin{bmatrix} N_{13} & N_{11} & N_{12} & N_{14} \\ N_{23} & N_{21} & N_{22} & N_{24} \\ N_{33} & N_{31} & N_{32} & N_{34} \\ N_{43} & N_{41} & N_{42} & N_{44} \end{bmatrix} \cdot \begin{bmatrix} x_3 \\ x_1 \\ x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \tag{2}$$

and move row of the matrix secondly to make the matrix symmetry, like:

$$
\begin{bmatrix} N_{33} & N_{31} & N_{32} & N_{34} \\ N_{13} & N_{11} & N_{12} & N_{14} \\ N_{23} & N_{21} & N_{22} & N_{24} \\ N_{43} & N_{41} & N_{42} & N_{44} \end{bmatrix} \cdot \begin{bmatrix} x_3 \\ x_1 \\ x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_3 \\ b_1 \\ b_2 \\ b_4 \end{bmatrix} \tag{3}
$$

The estimate parameter can be divided into global and local, as:

$$
\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \cdot \begin{bmatrix} x_{glob} \\ x_{local} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{4}
$$

Then:

$$
(N_{11} - N_{12}N_{22}^{-1}N_{21}) \cdot x_{glob} = b_1 - N_{12}N_{22}^{-1}b_2 \tag{5}
$$

$$
N_{reduce} = N_{11} - N_{12}N_{22}^{-1}N_{21} \tag{6}
$$

$$
b_{reduce} = b_1 - N_{12}N_{22}^{-1}b_2 \tag{7}
$$

2.2 Parameter Linear

$$
\mathbf{x} = \mathbf{A}\mathbf{y} + \mathbf{a} \tag{8}
$$

$$
\mathbf{N}_{new} = \mathbf{C}^T \mathbf{N}_{old} \mathbf{C} \tag{9}
$$

$$
\mathbf{b}_{new} = \mathbf{C}^T (\mathbf{b}_{old} - \mathbf{N}_{old} \cdot \mathbf{a}) \tag{10}
$$

The relationship between station and velocity is:

$$
\mathbf{x} = \mathbf{C}\mathbf{y} = \begin{bmatrix} 1 & & & t_{i1} - t_0 & & & \\ & 1 & & & t_{i1} - t_0 & & \\ & & 1 & & & t_{i1} - t_0 & \\ & & & & & & ... \end{bmatrix} \cdot \begin{bmatrix} \Delta x_{i1} \\ \Delta y_{i1} \\ \Delta z_{i1} \\ \Delta v_{x_{i1}} \\ \Delta v_{y_{i1}} \\ \Delta v_{z_{i1}} \\ ... \end{bmatrix} \tag{11}
$$

where the t_i is the station epoch in sinex, Δx and Δv_x are:

$$
\Delta x = x_s - x_c, \Delta v_x = v_{sx} - v_{cx} \tag{12}
$$

where x_c and Δv_{cx} is apriori terrestrial reference.

2.3 Stack

If we get the number of all estimate parameter, for example 6, then the N_{all} can be:

$$
\mathbf{N} = \begin{bmatrix} N_{11} & ... & N_{16} \\ \vdots & \ddots & \vdots \\ N_{61} & ... & N_{66} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix} \tag{13}
$$

If the normal matrix of one session is:

$$
\mathbf{N}_i = \begin{bmatrix} N_{i11} & N_{i12} & N_{i13} \\ N_{i21} & N_{i22} & N_{i23} \\ N_{i31} & N_{i32} & N_{i33} \end{bmatrix}, \mathbf{x}_i = \begin{bmatrix} x_1 \\ x_3 \\ x_6 \end{bmatrix}, \mathbf{b}_i = \begin{bmatrix} b_{i1} \\ b_{i2} \\ b_{i3} \end{bmatrix} \tag{14}
$$

The stack N_{s1} to N_{all} is:

$$\mathbf{N} = \mathbf{N} + \begin{bmatrix} N_{i11} & 0 & N_{i12} & 0 & 0 & N_{i13} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ N_{i21} & 0 & N_{i22} & 0 & 0 & N_{i23} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ N_{i31} & 0 & N_{i32} & 0 & 0 & N_{i33} \end{bmatrix}, \mathbf{b} = \mathbf{b} + \begin{bmatrix} b_{i1} \\ 0 \\ b_{i2} \\ 0 \\ 0 \\ b_{i3} \end{bmatrix} \tag{15}$$

2.4 Constraint

For NNT/NNR/NNS on station is:

$$B_i = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \\ 0 & z_i & -y_i \\ -z_i & 0 & x_i \\ y_i & -x_i & 0 \\ x_i & y_i & z_i \end{bmatrix} \tag{16}$$

The constraint on velocity is equal to station($B_{vi} = B_i$), the final equation is:

$$\begin{bmatrix} N_{all} & B^T \\ B & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_{helmert} \end{bmatrix} = \begin{bmatrix} b_{all} \\ \mathbf{0} \end{bmatrix} \tag{17}$$

2.5 Solve

The estimate paremeter will be:

$$\mathbf{x} = \mathbf{N}^{-1}\mathbf{b} \tag{18}$$

$$\mathbf{x} = \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_{helmert} \end{bmatrix}, \mathbf{N} = \begin{bmatrix} N_{all} & B^T \\ B & \mathbf{0} \end{bmatrix}, \mathbf{b} = \begin{bmatrix} b_{all} \\ \mathbf{0} \end{bmatrix} \tag{19}$$

If $R(\mathbf{N}) < R(\mathbf{N}, \mathbf{b})$, unsolvable;
If $R(\mathbf{N}) = R(\mathbf{N}, \mathbf{b}) = n$, unique solution;
If $R(\mathbf{N}) = R(\mathbf{N}, \mathbf{b}) < n$, infinite solution;

2.6 Formal error

Usually the SINEX(Solution INdependent EXchange format) give the "WEIGHTED SQUARE SUM OF O-C" value, which is the:

$$\ell^T P \ell_{red} = (y^T {\textstyle\sum}_{yy}^{-1} y)_{red} \tag{20}$$

For single SINEX, when reduce the local parameter, 20 will be:

$$v^T P v_{red} = y^T {\textstyle\sum}_{yy}^{-1} y - b_2^T N_{22}^{-1} b_2 \tag{21}$$

If the combined parameters $\tilde{x}_1$(global parameter) are available, the squares sum of residuals of all SINEX files is:

$$v^T P v = v^T P v_{red} - \tilde{x}_1^T b_1 \tag{22}$$

Aposteriori variance of unit weight is:

$$\sigma_0^2 = \frac{v^T P v}{n - u} \tag{23}$$

Formal error of parameter will be:

$$\sigma_{x_i} = \sigma_0 N^{-1} \tag{24}$$