Monte Carlo Simulation in Inventory Management

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***Abstract*— This project aims to utilize Monte Carlo simulation in inventory management to minimize the operating cost. In this inventory management system design projects, there are 5 individual tasks. Each task will be discussed in different sections below.**

I. A INTRODUCTION OF MONTE CARLO METHOD

Monte Carlo methods use repeated random sampling to achieve numerical results instead of performing a long complex calculation. The essential ideal of Monte Carlo method is to use randomness to approximate numerical solutions that would be difficult if not impossible to solve mathematically.

In theory, any problem that has a probabilistic interpretation can be approximated by Monte Carlo methods. According to the law of large numbers, the average of the results obtained from a huge quantity of experiments should be close to the expected value and will tend to become closer as more experiments are performed.[1]When the probability distribution of the variable is parametrized, mathematicians often use a Markov chain Monte Carlo (MCMC) sampler.[2][3][4]

II. SUMMARY OF PROJECT'S BACKGROUND

In this project, we aim to design an inventory management system that can minimize the operating cost of selling 'super apple'. The function of this inventory management system can be divided into two parts, namely paying for the operating costs and ordering from earth.The operating cost is made up of three parts, namely holding cost, shortage cost and return cost. The demand for this product is a random variable with a determined probability distribution, as shown in Table I.

TABLE I

AN EXAMPLE OF A TABLE

Demand	0	1	2	3	4	5	6
Probability	0.04	0.08	0.28	0.40	0.16	0.02	0.02

III. TASK 1

Without using simulation the reasonable range of the order number y is 3 to 4, and re-order level r is 0 to 2.

As shown in Table I, a weekly demand that is higher than 4 only has a probability of 0.04. The average weekly penalty of not satisfy this demand is 0.4 coin, compare to 9.6 coins if satisfy this demand. Hence, the most reasonable behavior is to neglect demand higher than 4.

If we choose the order number from 3 to 4, then the reasonable range of re-order stock level should be 0 to 2. The optimal combination (y,r) may be (3,1),(3,2),(4,0).

IV. TASK 2

On the basis of what we deduce from task 1, we choose order number as 3 and re-order stock level as 1. In order to generate random numbers with the specified probability distribution, we can use the rand() function in MATLAB. Rand() function can generate a uniformly distributed random number in the interval of 0 to 1. The interval between 0 to 1 can be viewed as probability space. Therefore, it can be divided into 6 different parts, each represents a possible demand. The length of each part is determined by their given probabilities. Then we can use this way to generate demands of 52 weeks and applied the given rules to calculate the total operating cost. After repeating this 500 times, a sample set can be obtained from this experiment. The histogram of this sample set is shown in Fig. 1.

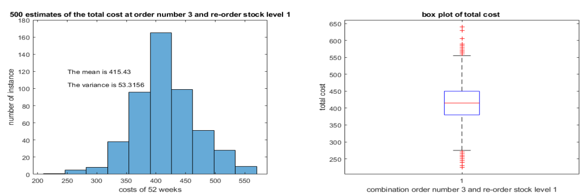

Fig. 1. Histogram and box plot of the combination (3,1)

A. *Results of Monte Carlo simulation method*

- The mean of 500 estimates of the total cost is 415.43.
- The variance of 500 estimates of the total cost is 53.31.

V. TASK 3

There are two ways of finding the optimal combination using Monte Carlo simulation method.

A. *two ways of Monte Carlo simulation method*

- Solution 1: Each simulation is a full process of 52 weeks. N simulations will be conducted for every combination and the mean of their total cost will be calculated. The combination that has the minimum mean of the total cost is the optimal combination.
- Solution 2: Each simulation is a full process of 52 weeks. For every simulation, the program will go through every combination to find combinations which lead to minimum total cost.N simulations will be conducted to produce an optimal combination set. The optimal combination is obtained by choosing the majority combination in the optimal combination set.

In this task, we choose N as 50. Both solutions suggest that the optimal order number is 3 and optimal re-order level is 1. A box plot of the mean of the total cost with the different combination is shown in the Fig. 2.The histograms of these data sets are shown in Fig. 3.

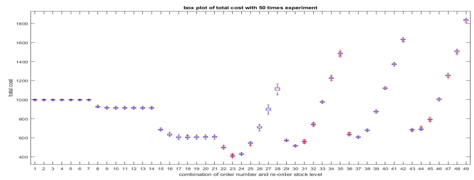

Fig. 2. Box plot of the mean of of total cost with N=50

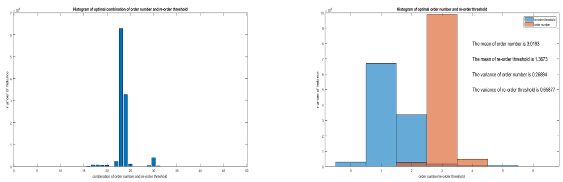

Fig. 3. Histograms of optimal combination,order number and re-order level

VI. TASK 4

We choose solution 2 in task 3 to discuss the confidence level of the optimal combination.

A. *Confidence level of optimal combination of two solutions*

- Solution 1: The optimal combination is the combination that has the minimum mean of the total cost. After repeating this process, a distribution of the mean of every combination can be obtained(see Fig.2). The combination (3,2) is the only combination that has its mean fluctuate lower than the upper boundary of combination (3,1).The fluctuation of both combination's mean with N=50 and N=100 are shown in Fig. 4. It is clear from Fig. 4 that with more simulations the area where two combination meets declined significantly. This suggests that with more simulations, the confidence level of choosing combination (3,1) increase.
- Solution 2: The optimal combination is produced by choosing the majority combination in the optimal combination set. If N=50, the possibility of combination (3,1) is the majority combination fluctuate around 98.6%. After repeating this process, a distribution of the probability of choosing every combination as optimal choice can be obtained. We can use central limit theorem to construct this distribution into a standard normal distribution and then choose confidence interval and calculate the respective confidence level.

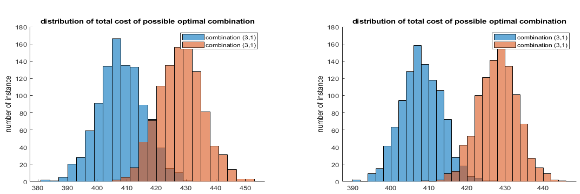

Fig. 4. Histogram of 1000 experiments of two combination's mean with N=50 and N=100

Use central limit theorem to construct the confidence intervals. As shown in equation 1.

$$\frac{\bar{\theta}-\Theta}{\delta/\sqrt{n}} \rightarrow N(0,1) \tag{1}$$

$$[L(X),U(X)] = [\bar{\theta} - Z\alpha/2\frac{\delta}{\sqrt{n}}, \bar{\theta} + Z\alpha/2\frac{\delta}{\sqrt{n}}] \tag{2}$$

According to equation 1 and 2, confidence interval of the estimates converted to N(0,1) can be calculated. The confidence interval of N=50 is [-0.61,0.61], the confidence level is 78% to 80%. The confidence interval of N=50 is [-1.287,1.287], the confidence level is 99%.

VII. TASK 5

In this projects, we discussed two ways of conducting Monte Carlo simulations. These two ways of conducting Monte Carlo simulations leads to the different thought of calculating confidence intervals and confidence levels. Both have their pros and cons.

In the first solution, we choose the confidence interval [x-a,x+a] if the mean of one combination is smaller another. However, the interval of $[-\infty, x+a]$ can satisfy decision logic. Therefore, by choosing a symmetric interval we are in fact calculating a subset of all possible solutions. In the second solution, it is difficult for us to choose a correct confidence interval.

To improve the accuracy of these models, we need to find a way to compare them. However, because during each simulation we produce a new set of random demands, we can not test our model on a standard test set. Therefore, a standard test set that contains a set of 52 weeks demand can be a good solution for this problem.

In a real-life situation, we need to consider the assumption more strictly and may request more parameters to work with. For instance, the distribution of weekly demand can only be approximate and may change over time. We need to obtain an accurate impartial observation of the real work before modeling.

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