Instrumentation Tutorial 2 Answers

Instrumentation (Flinders University)

INSTRUMENTATION

ENGR7732

TUTORIAL 2: ROBOT PERCEPTION

QUESTIONS

QUESTION 1

In beam based models;

(a) What are the four sources of error modelled?
(b) In what way does each of these errors manifest themselves in the likelihood? (Include mathematical descriptions)
(c) What are the drawbacks of the beam based model?

S. Thrun, W. Burgard, and D. Fox, *Probabilistic robotics* vol. 1: MIT press Cambridge, 2005.

QUESTION 2

In scan based models;

(a) What are the three sources of error modelled?
(b) In what way does each of these errors manifest themselves in the likelihood?
(c) Sketch the likelihood for the scan (dotted red line) in the likelihood field in Figure 1. Include all sources of error.
(d) What are the drawbacks of the scan based model?
(e) What are the advantages of scan based model over a beam based model?
(f) Define an algorithm that would calculate the support (i.e. the value of the likelihood) at a given point defined by a measurement z_t^k.

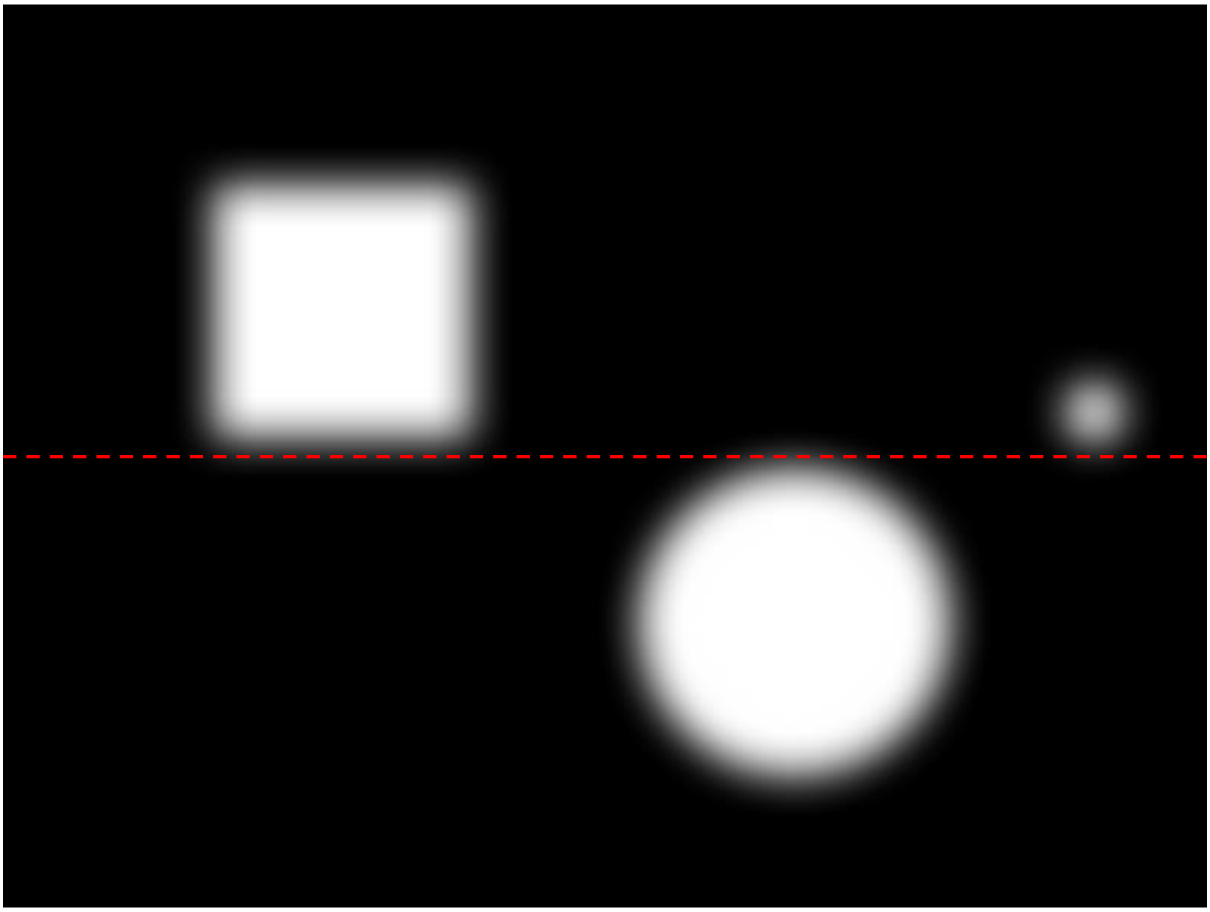

FIGURE 1 - LIKELIHOOD FIELD

S. Thrun, W. Burgard, and D. Fox, *Probabilistic robotics* vol. 1: MIT press Cambridge, 2005.

QUESTION 3

Many early robots navigating using features used artificial landmarks in the environment that were easy to recognize. A good place to mount such markers is a ceiling **(why?)**. A classical example is a visual marker: Suppose we attach the following marker to the ceiling:

Let the world coordinates of the marker be x_m and y_m and its orientation relative to the global coordinate system θ_m We will denote the robot's pose by x_r , y_r and θ_r.

Now assume that we are given a routine that can detect the marker in the image plane of a perspective camera. Let x_i and y_i denote the coordinates of the marker in the image plane, and θ_i its angular orientation. The camera has a focal length of f. From projective geometry, we know that each displacement d in x-y-space gets projected to a proportional displacement of $d\frac{f}{h}$ in the image plane. (You have to make some choices on your coordinate systems; Make these choices explicit).

Your questions:

(a) Describe mathematically where to expect the marker (in global coordinates x_m, y_m, θ_m) when its image coordinates are x_i, y_i, θ_i, and the robot is at x_r , y_r, θ_r.

(b) Provide a mathematical equation for computing the image coordinates x_i, y_i, θ_i, from the robot pose x_r , y_r, θ_r and the marker coordinates x_m, y_m, θ_m.

(c) Now give a mathematical equation for determining the robot coordinates x_r , y_r, θ_r assuming we know the true marker coordinates x_m, y_m, θ_m and the image coordinates x_i, y_i, θ_i.

(d) So far we assumed there is only a single marker. Now suppose there are multiple (indistinguishable) markers of the type shown above. How many such markers must a robot be able to see to uniquely identify its pose? Draw such a configuration, and argue why it is sufficient.
Hint: You don't need to consider the uncertainty in the measurement for answering this question. Also, note that the marker is symmetrical. This has an impact on the answer of these questions!

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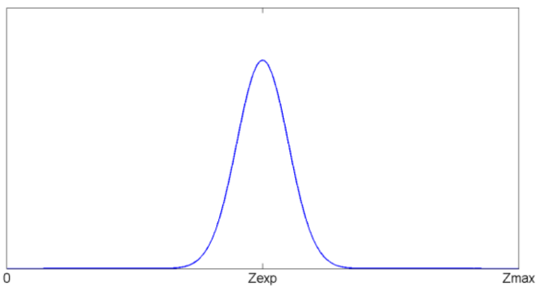
Answer

(a)
- Measurement Noise
- Unexpected Obstacles
- Random Measurement
- Max Range

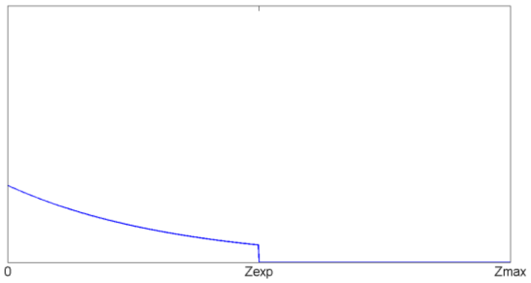
(b)
- Measurement Noise



$$P_{hit}(z|x,m) = \eta \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-z_{exp})^2}{2\sigma^2}}$$

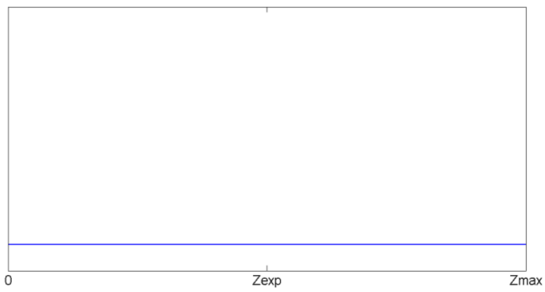
S. Thrun, W. Burgard, and D. Fox, *Probabilistic robotics* vol. 1: MIT press Cambridge, 2005.

- Unexpected Obstacles



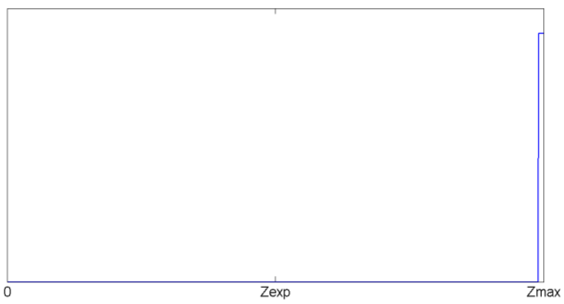
$$P_{unexp}(z|x,m) = \begin{cases} \eta\lambda e^{-\lambda z} & z < z_{exp} \\ 0 & otherwise \end{cases}$$

- Random Measurement



$$P_{rand}(z|x,m) = \eta \frac{1}{z_{max}}$$

- Max Range



$$P_{max}(z|x,m) = \begin{cases} \eta \frac{1}{z_{max}} & z = z_{max} \\ 0 & otherwise \end{cases}$$

(c)
- not smooth for small obstacles and at edges.
- not very efficient.

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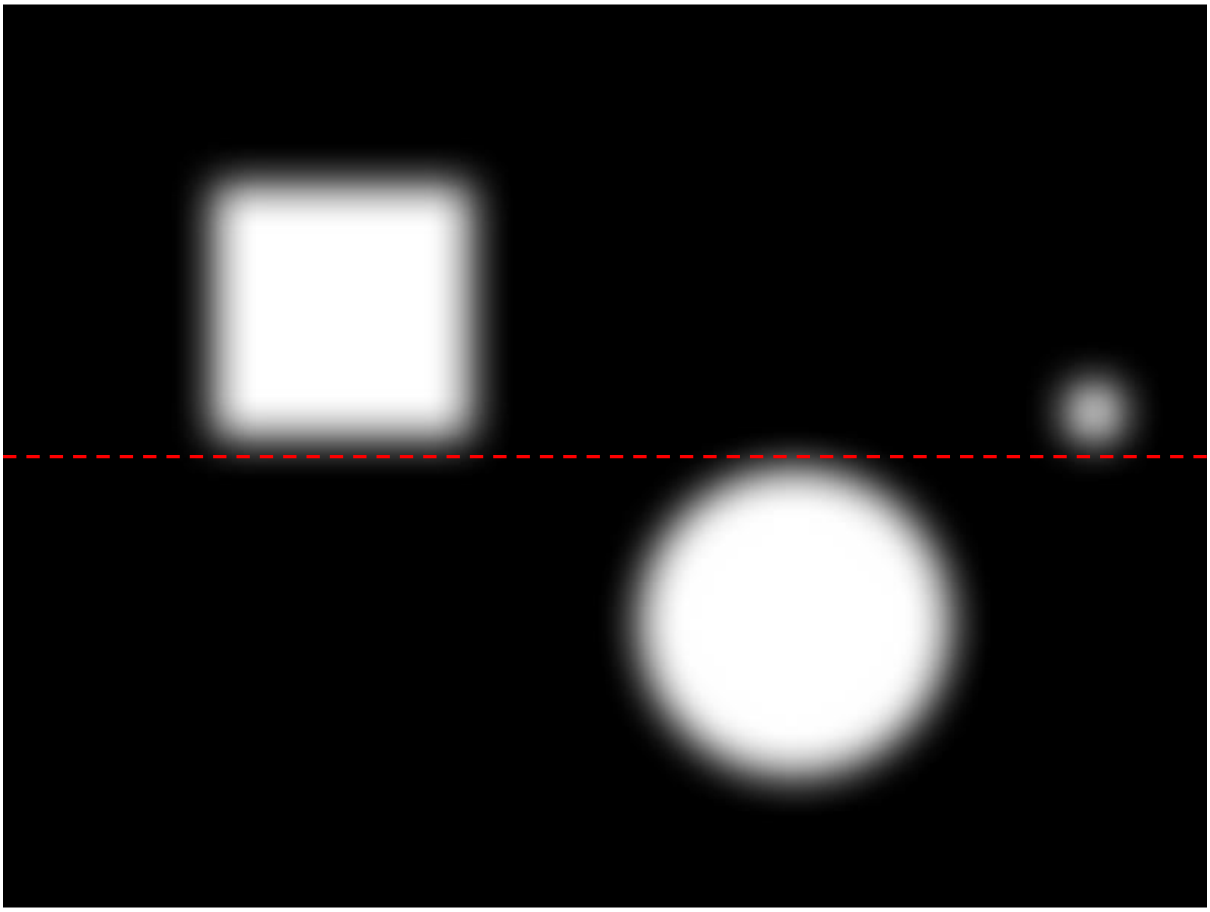


FIGURE 1 - LIKELIHOOD FIELD

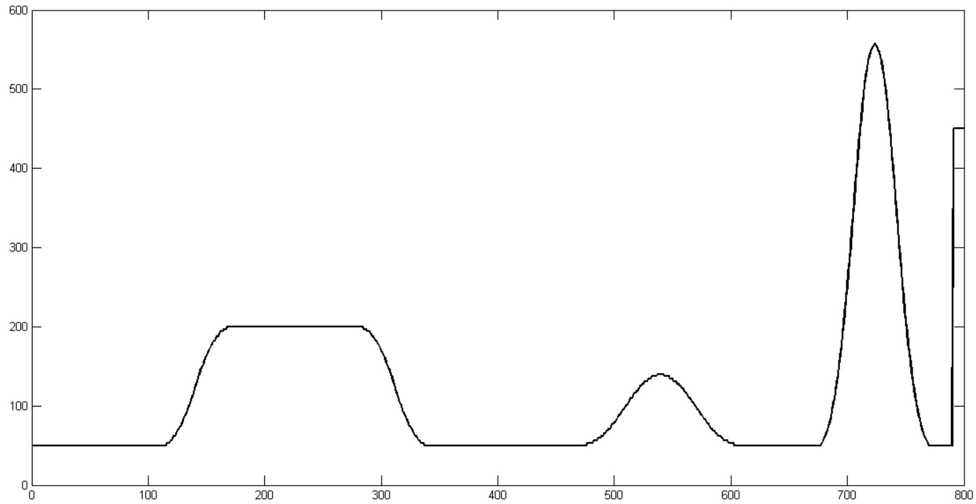
ANSWERS

(a) • Measurement Noise
• Random Measurement
• Max Range

(b) See Quesiton 1 (b)

S. Thrun, W. Burgard, and D. Fox, *Probabilistic robotics* vol. 1: MIT press Cambridge, 2005.

(c)



(d) ● Reliance on the accuracy of the map

● Computational load exponentially increases when extend to a 3D map.

(e) See Question 1 (c)

(f)

p = **landmark_detection_model**(z,x,m)

$z_{detected} = \langle i, d, \alpha \rangle, \quad x = \langle x, y, \theta \rangle$

$\hat{d} = \sqrt{(m_x(i) - x)^2 + \left(m_y(i) - y\right)^2}$

$\hat{\alpha} = atan2\left(m_y(i) - y, m_x(i) - x\right) - \theta$

$p_{det} = pdf\left(\hat{d} - d, \varepsilon_d\right) pdf\left(\hat{\alpha} - \alpha, \varepsilon_\alpha\right)$

return $p = z_{detected} p_{detected} + z_{false\ positive} P_{uniform}(z|x, m)$

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Answers

(a)

$$\theta_m = \theta_r + \theta_i$$
$$x_m = x_r + x_i \cos\theta_r - y_i \sin\theta_r$$
$$y_m = y_r + x_i \sin\theta_r + y_i \cos\theta_r$$

(b)

$$\theta_i = \theta_m - \theta_r$$
$$x_i = \cos\theta_r\,(x_m - x_r) + \sin\theta_r\,(y_m - y_r)$$
$$y_i = -\sin\theta_r\,(x_m - x_r) + \cos\theta_r\,(y_m - y_r)$$

(c)

$$\theta_r = \theta_m - \theta_i$$
$$x_r = x_m - x_i \cos\theta_r + y_i \sin\theta_r$$
$$y_r = y_m - x_i \sin\theta_r - y_i \cos\theta_r$$

(d) 3 marker that DO NOT lie in a straight line.

If the markers were distinguishable only two would be required, as which marker was to the left/right of the other.

Without distinguishing features a third is required due to the guaranteed symmetry of two markers.

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