Instrumentation Tutorial 7 Answers

Instrumentation (Flinders University)

INSTRUMENTATION

ENGR4732

TUTORIAL 7: FILTER REVIEW

QUESTIONS

QUESTION 1 – BAYESIAN FILTER

What are the two significant stages of the Bayesian filter?

QUESTION 2 – KALMAN FILTER

What assumptions does the Kalman Filter make on the characteristics on the plant and the uncertainties?

How is this uncertainty represented?

What are the equations are utilised to implement the first stage of the Bayesian filter?

What are the equations are utilised to implement the second stage of the Bayesian filter? (Excluding the calculation of the Kalman gain).

QUESTION 3 – EXTENDED KALMAN FILTER

Conceptually what additions, to the algorithm defined in Question 2, are required to implement non-linear plant and observation model producing and Extended Kalman Filter?

How are these additions implemented mathematically? Give two alternatives.

QUESTION 4 – PARTICLE FILTER

In the particle filter, how is the uncertainty represented?

What are the equations are utilised to implement the first stage of the Bayesian filter?

What are the equations are utilised to implement the second stage of the Bayesian filter?

What additional stage is required?

What does this stage attempt to do?

Define a process to perform this stage?

INSTRUMENTATION

ENGR4732

TUTORIAL 7: FILTER REVIEW

ANSWERS

QUESTION 1 – BAYESIAN FILTER

What are the two significant stages of the Bayesian filter?

ANSWER

- Prediction Stage
- Correction Stage

QUESTION 2 – KALMAN FILTER

What assumptions does the Kalman Filter make on the characteristics on the plant and the uncertainties?

How is this uncertainty represented?

What are the equations are utilised to implement the first stage of the Bayesian filter?

What are the equations are utilised to implement the second stage of the Bayesian filter? (Excluding the calculation of the Kalman gain).

ANSWER

- Linear State Transition and Observation relationships
- Gaussian

Gaussian Uncertainties can be uniquely defined by its mean, μ, and standard deviation, σ.

First stage (prediction stage)

$$\boldsymbol{x}_{k|k-1} = \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1|k-1} + \boldsymbol{G}_{k-1}\boldsymbol{u}_{k-1}$$

$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1|k-1}\boldsymbol{F}_{k-1}^{T} + \boldsymbol{G}_{k-1}\boldsymbol{Q}_{k-1}\boldsymbol{G}_{k-1}^{T}$$

Second stage (correction stage)

$$\boldsymbol{x}_{k|k} = \boldsymbol{x}_{k|k-1} + \boldsymbol{K}_{k}\left(\boldsymbol{z}_{k} - \boldsymbol{H}_{k}\boldsymbol{x}_{k|k-1}\right)$$

And either

$$\boldsymbol{P}_{k|k} = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_{k}\boldsymbol{S}_{k}\boldsymbol{K}_{k}^{T}$$

Or

$$\boldsymbol{P}_{k|k} = (I - \boldsymbol{K}_{k}\boldsymbol{H}_{k})\boldsymbol{P}_{k|k-1}$$

There are a few formulations for the last equation but they are all mathematically identical.

QUESTION 3 – EXTENDED KALMAN FILTER

Conceptually what additions, to the algorithm defined in Question 2, are required to implement non-linear plant and observation model producing and Extended Kalman Filter?

How are these additions implemented mathematically? Give two alternatives.

Replace

$$\boldsymbol{x}_{k|k-1} = \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1|k-1} + \boldsymbol{G}_{k-1}\boldsymbol{u}_{k-1}$$

$$\boldsymbol{H}_k\boldsymbol{x}_{k|k-1}$$

With

$$\boldsymbol{x}_{k|k-1} = f_{k-1}\left(\boldsymbol{x}_{k-1|k-1},\ \boldsymbol{u}_{k-1}\right)$$

$$h_k\left(\boldsymbol{x}_{k|k-1}\right)$$

And

$$\boldsymbol{F}_{k-1}, \boldsymbol{G}_{k-1}, \boldsymbol{H}_k$$

With

$$\widehat{\boldsymbol{F}}_{k-1}, \widehat{\boldsymbol{G}}_{k-1}, \widehat{\boldsymbol{H}}_k$$

where

$$\widehat{\boldsymbol{F}}_{k-1} = \frac{\partial f_{k-1}\left(\boldsymbol{x}_{k-1|k-1},\ \boldsymbol{u}_{k-1}\right)}{\partial \boldsymbol{x}_{k-1|k-1}}$$

$$\widehat{\boldsymbol{G}}_{k-1} = \frac{\partial f_{k-1}\left(\boldsymbol{x}_{k-1|k-1},\ \boldsymbol{u}_{k-1}\right)}{\partial \boldsymbol{u}_{k-1|k-1}}$$

$$\widehat{\boldsymbol{H}}_{k-1} = \frac{\partial h_k\left(\boldsymbol{x}_{k|k-1}\right)}{\partial \boldsymbol{x}_{k|k-1}}$$

QUESTION 4 – PARTICLE FILTER

In the particle filter, how is the uncertainty represented?

What are the equations are utilised to implement the first stage of the Bayesian filter?

What are the equations are utilised to implement the second stage of the Bayesian filter?

What additional stage is required?

What does this stage attempt to do?

Define a process to perform this stage?

Answers

The uncertainty in the estimate is represented by a set of weighted points, a multinomial distribution approximation to the true uncertainty.

$$\boldsymbol{x}_k^i \sim p(\boldsymbol{x}_k | \boldsymbol{x}_{k-1}^i, \boldsymbol{u}_k)|_{i=1,2,\ldots,N}$$

$$w_k^i \propto w_{k-1}^i p(\boldsymbol{z}_k | \boldsymbol{x}_k^i)|_{i=1,2,\ldots,N}$$

The additional stage required is the resampling stage where the particle population is redistributed to better represent the current estimate of the uncertainty. I.e. Particles with low weights are removed and particles with large weights are duplicated.

This pseudo code performs systematic resampling.

```
[w[], x[]] = systematic_resampling(w[], x[], N)

c[1] = w[1]
for i = 2..N
      c[i] = c[i-1] + w[i]
end

u ~ U[0,1/N]

for j = 1..N
      while u[j] > c[i]
            i = i + 1
      end
      x[j] = x[i]
      w[j] = 1/N
      u = u + 1/N
end
```