Instrumentation Tutorial 4 Answers

Instrumentation (Flinders University)

INSTRUMENTATION

ENGR4732, SEMESTER 2 2014

TUTORIAL 4: KALMAN FILTERS

QUESTIONS

QUESTION 1

Suppose a robot is equipped with a sensor for measuring range and bearing to a landmark; and for simplicity suppose that the robot can also sense the landmark identity (the identity sensor is noise-free). We want to perform global localization with EKFs. When seeing a single landmark, the posterior is usually poorly approximated by a Gaussian. However, when sensing two or more landmarks at the same time, the posterior is often well-approximated with a Gaussian.

(a) Explain why.

(b) Given k simultaneous measurements of ranges and bearings of k identifiably landmarks, devise a procedure for calculating a Gaussian pose estimate for the robot under uniform initial prior. You should start with the range/bearing measurement model provided in the lectures on "Perception Models".

S. Thrun, W. Burgard, and D. Fox, *Probabilistic robotics* vol. 1: MIT press Cambridge, 2005.

QUESTION 2

Consider robot with one dimensional motion with which we want to estimate its velocity and position. The robot has a ranging sensor that provides an integer measurement of range to a fixed land mark at 1 m, with a 1σ uncertainty of 5. The integer value is related to its resolution of 5mm, i.e. a value of $30 = 15cm$ and an accelerometer that returns a value equal to the sensed acceleration. The initial position and velocity are 0 m and 0 m/s respectively, with a 1σ uncertainty of 1 m and 0.1 m/s, respectively. Given the Kalman Filter structure;

$$\hat{\boldsymbol{x}}_{k|k-1} = \boldsymbol{F}_{k-1}\hat{\boldsymbol{x}}_{k-1|k-1} + \boldsymbol{G}_{k-1}\boldsymbol{u}_{k-1}$$
$$\boldsymbol{P}_{k|k-1} = \boldsymbol{Q}_{k-1} + \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1|k-1}\boldsymbol{F}_{k-1}^T$$
$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k\left(\boldsymbol{z}_k - \boldsymbol{H}_k\hat{\boldsymbol{x}}_{k|k-1}\right)$$
$$\boldsymbol{P}_{k|k} = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_k\boldsymbol{S}_k\boldsymbol{K}_k^T$$

where

$$\boldsymbol{S}_k = \boldsymbol{R}_k + \boldsymbol{H}_k\boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^T$$
$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^T\boldsymbol{S}_k^{-1}$$

(a) Define the required state vector, $\boldsymbol{x}_k$.

(b) Define the motion model, $\boldsymbol{F}_{k-1}$.

(c) Define the control model, $\boldsymbol{G}_{k-1}$.

(d) Define the measurement model, $\boldsymbol{H}_k$.

(e) Define the measurement uncertainty, $\boldsymbol{R}_k$.

(f) Define the initial state estimate, $\langle\hat{\boldsymbol{x}}_0, \boldsymbol{P}_0\rangle$

S. Thrun, W. Burgard, and D. Fox, *Probabilistic robotics* vol. 1: MIT press Cambridge, 2005.

QUESTION 3

Given robot in *Question 2*, with an update period of T=0.1 sec, $\boldsymbol{Q}_{k-1} = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix}$, the first accelerometer measurement of 1 m/s^2 and the position measurement is 5, find;

(a) The predicted state mean, $\hat{\boldsymbol{x}}_{k|k-1}$.

(b) The predicted state covariance, $\boldsymbol{P}_{k|k-1}$.

(c) The Kalman gain, $\boldsymbol{K}_k$.

(d) The corrected state mean, $\hat{\boldsymbol{x}}_{k|k}$.

(e) The corrected state covariance, $\boldsymbol{P}_{k|k}$.

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(a) Explain why.

(b) Given k simultaneous measurements of ranges and bearings of k identifiably landmarks, devise a procedure for calculating a Gaussian pose estimate for the robot under uniform initial prior. You should start with the range/bearing measurement model provided in the lectures on “Perception Models”.

ANSWERS

(a) For one range-bearing measurement the uncertainty of the measurement has an arc “banana” shape. This banana shape is poorly approximated by Gaussian uncertainty which has the shape of an ellipse.

When more than one measurement is available multiple banana shapes are correlated together, exploiting the “central limit theorem”, generates an uncertainty shape like an ellipse thus better approximated by the Gaussian uncertainty. This is dependent on the configuration of the landmarks, i.e. if the landmarks are very close together the resulting distribution would be very similar to the distribution of the individual measurements, conversely if the landmarks are orthogonal, as perceived by the robot, the distribution is Gaussian.

S. Thrun, W. Burgard, and D. Fox, *Probabilistic robotics* vol. 1: MIT press Cambridge, 2005.

(b)

```
process
    for k measuements
```

$$x_r(k) = x_k - d_k \sin\theta_k$$

$$y_r(k) = y_k - d_k \cos\theta_k$$

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    end for
```

$$x_r = \frac{1}{k}\sum_{i=1}^{k} x_r(i)$$

$$y_r = \frac{1}{k}\sum_{i=1}^{k} y_r(i)$$

```
end process
```

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$$\boldsymbol{P}_{k|k-1} = \boldsymbol{Q}_{k-1} + \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1|k-1}\boldsymbol{F}_{k-1}^{T}$$

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_{k}\left(\boldsymbol{z}_{k} - \boldsymbol{H}_{k}\hat{\boldsymbol{x}}_{k|k-1}\right)$$

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where

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(e) Define the measurement uncertainty, $\boldsymbol{R}_{k}$.

(f) Define the initial state estimate, $\langle\hat{\boldsymbol{x}}_{0}, \boldsymbol{P}_{0}\rangle$

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Answers

(a) $\boldsymbol{x}_k = \begin{bmatrix} p_k \\ v_k \end{bmatrix}$

(b) $\boldsymbol{F}_{k-1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$

(c) $\boldsymbol{G}_{k-1} = \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix}$

(d) $\boldsymbol{H}_k = [200 \quad 0]$

(e) $\boldsymbol{R}_k = [25]$

(f) $\langle \hat{\boldsymbol{x}}_0, \boldsymbol{P}_0 \rangle = \langle \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0.01 \end{bmatrix} \rangle$

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(c) The Kalman gain, $\boldsymbol{K}_k$.

(d) The corrected state mean, $\hat{\boldsymbol{x}}_{k|k}$.

(e) The corrected state covariance, $\boldsymbol{P}_{k|k}$.

ANSWERS

(a) $\hat{\boldsymbol{x}}_{k|k-1} = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{0.1^2}{2} \\ 0.1 \end{bmatrix} [1] = \begin{bmatrix} 0.005 \\ 0.1 \end{bmatrix}$

(b) $\boldsymbol{P}_{k|k-1} = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix} + \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0.01 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix} = \begin{bmatrix} 1.0011 & 0.001 \\ 0.001 & 0.011 \end{bmatrix}$

(c) $\boldsymbol{S}_k = [25] + [200 \quad 0] \begin{bmatrix} 1.0011 & 0.001 \\ 0.001 & 0.011 \end{bmatrix} \begin{bmatrix} 200 \\ 0 \end{bmatrix} = [40069]$

$\boldsymbol{K}_k = \begin{bmatrix} 1.0011 & 0.001 \\ 0.001 & 0.011 \end{bmatrix} \begin{bmatrix} 200 \\ 0 \end{bmatrix} \left[\frac{1}{40069} \right] = \begin{bmatrix} 0.005 \\ 0.000005 \end{bmatrix}$

(d) $\hat{\boldsymbol{x}}_{k|k} = \begin{bmatrix} 0.005 \\ 0.1 \end{bmatrix} + \begin{bmatrix} 0.005 \\ 0.000005 \end{bmatrix} \left(5 - [200 \quad 0] \begin{bmatrix} 0.005 \\ 0.1 \end{bmatrix} \right) = \begin{bmatrix} 0.025 \\ 0.10002 \end{bmatrix}$

(e) $\boldsymbol{P}_{k|k} = \begin{bmatrix} 1.0011 & 0.001 \\ 0.001 & 0.011 \end{bmatrix} - \begin{bmatrix} 0.005 \\ 0.000005 \end{bmatrix} [40069][0.005 \quad 0.000005] =$

$\begin{bmatrix} 0.0006 & 6e-7 \\ 6e-7 & 0.011 \end{bmatrix}$

S. Thrun, W. Burgard, and D. Fox, *Probabilistic robotics* vol. 1: MIT press Cambridge, 2005.