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Instrumentation Tutorial 3 Answers

Instrumentation (Flinders University)

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INSTRUMENTATION

ENGR7732

TUTORIAL 3: ROBOT MOTION

QUESTIONS

QUESTION 1

Given a Gaussian distribution i.e. $\mathcal{N}(\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\mu^2}{2\sigma^2}}$

a) Find the support from the likelihood of an ***odometry based*** motion model, given;

- The robots state space is $[x,y,\theta]$ with its initial state, x, of $[5\text{m}, 5\text{m}, 30^\circ]$
- The command sequence of;
  1. Rotation 1 = -20°
  2. Translation = 20 m
  3. Rotation 2 = 30°
- With the error model;
  - $\sigma_{rot1} = \alpha_1 |\hat{\delta}_{rot1}| + \alpha_2 \hat{\delta}_{trans}$
  - $\sigma_{rot1} = \alpha_3 \hat{\delta}_{trans} + \alpha_4 (|\hat{\delta}_{rot1}| + |\hat{\delta}_{rot2}|)$
  - $\sigma_{rot2} = \alpha_1 |\hat{\delta}_{rot2}| + \alpha_2 \hat{\delta}_{trans}$
  - With
    - $\alpha_1 = 0.05$
    - $\alpha_2 = 0.001$
    - $\alpha_3 = 0.05$
    - $\alpha_4 = 0.01$
- With a proposed final state, x', of $[9\text{m}, 25\text{m}, 45^\circ]$

b) In this case we used a standard Gaussian distribution,

- How can these calculation be simplified? (Hint: scaling factors)
- Secondarily, what are the consequences of this simplification. (Hint: what value would be the support be if both the odometry and predicted state matched perfectly.)

S. Thrun, W. Burgard, and D. Fox, *Probabilistic robotics* vol. 1: MIT press Cambridge, 2005.

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QUESTION 2

In a 6DOF system, given the current vehicle state;

Surge	: u	= 1 m/s
Sway	: v	= 0 m/s
Heave	: w	= 0.1 m/s
North	: x	= 5 m
East	: y	= 10 m
Down	: z	= 20 m
Roll	: φ	= 6°
Pitch	: θ	= 12°
Yaw	: ψ	= 24°
Acc_x	: a_x	= -2 m/s^2
Acc_y	: a_y	= 1 m/s^2
Acc_z	: a_z	= 9.5 m/s^2
$Gyro_x$	: ω_x	= 0.1 rad/s
$Gyro_y$	: ω_y	= 0.2 rad/s
$Gyro_z$	: ω_z	= 0.3 rad/s

a) Calculate the vehicle's ***linear velocity transormation matrix***, $\boldsymbol{R}_b^n(\boldsymbol{\Theta})$
b) Calculate the vehicle's ***angular velocity transormation matrix***, $\boldsymbol{T}_\Theta(\boldsymbol{\Theta})$
c) Calculate the vehicles;
   - rate of change in velocity, $\dot{\boldsymbol{v}}_b$.
   - rate of change in position, $\dot{\boldsymbol{p}}_n$.
   - rate of change in orientation, $\dot{\boldsymbol{\Theta}}$.

d) Using Euler integration, with a time step T = 0.1 sec, calculate the vehicle pose at the next time step. (Hint: $x_t = x_{t-1} + T\dot{x}_t$)

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QUESTION 3

All robot models in this section were kinematic. In this exercise, you will consider a robot with dynamics. Consider a robot that lives in a 1-D coordinate system. Its location will be denoted by x, its velocity by $\dot{x}$, and its acceleration by $\ddot{x}$. Suppose we can only control the acceleration $\ddot{x}$.

Develop a mathematical motion model that computes the posterior over the pose x' and the velocity $\dot{x}'$ from an initial pose x and velocity $\dot{x}$, assuming that the acceleration $\ddot{x}$ is the sum of a commanded acceleration and a zero-mean Gaussian noise term with variance σ^2 (and assume that the actual acceleration remains constant in the simulation interval Δt).

Are x' and $\dot{x}'$ correlated in the posterior? Explain why/why not.

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b) In this case we used a standard Gaussian distribution,
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ANSWER

a) --

p = **motion_model_odometry**(x,x’,u)

$$\delta_{trans} = u(2)$$
$$\delta_{rot1} = u(1)$$
$$\delta_{rot2} = u(3)$$

$$\hat{\delta}_{trans} = \sqrt{(x' - x)^2 + (y' - y)^2}$$
$$\hat{\delta}_{rot1} = atan2(y' - y, x' - x) - \theta$$
$$\hat{\delta}_{rot2} = \theta' - \theta - \hat{\delta}_{rot1}$$

$$p_{trans} = pdf\left(\delta_{trans} - \hat{\delta}_{trans}, \alpha_3\hat{\delta}_{trans} + \alpha_4\left(\left|\hat{\delta}_{rot1}\right| + \left|\hat{\delta}_{rot2}\right|\right)\right)$$
$$p_{rot1} = pdf\left(\delta_{rot1} - \hat{\delta}_{rot1}, \alpha_1\left|\hat{\delta}_{rot1}\right| + \alpha_2\hat{\delta}_{trans}\right)$$
$$p_{rot2} = pdf\left(\delta_{rot2} - \hat{\delta}_{rot2}, \alpha_1\left|\hat{\delta}_{rot2}\right| + \alpha_2\hat{\delta}_{trans}\right)$$

return $p = p_{trans}p_{rot1}p_{rot2}$

----

p = **motion_model_odometry**([5,5,30],[9,25,45],[-20,20,30])

$$\delta_{trans} = u(2) = 20$$
$$\delta_{rot1} = u(1) = deg2rad(-20) = -0.3491$$
$$\delta_{rot2} = u(3) = deg2rad(30) = 0.5880$$

$$\hat{\delta}_{trans} = \sqrt{(9 - 5)^2 + (25 - 5)^2} = 20.3961$$
$$\hat{\delta}_{rot1} = atan2(25 - 5, 9 - 5) - deg2rad(30) = -0.3262$$
$$\hat{\delta}_{rot2} = deg2rad(45) - deg2rad(30) - -0.3262 = 0.5880$$

$$p_{trans} = pdf\left(20 - 20.3961, 0.05 \times 20.3961 + 0.01 \times (0.3262 + 0.5880)\right)$$
$$= 8.9522$$
$$p_{rot1} = pdf(-0.3491 + 0.3262, 0.05 \times 0.3262 + 0.001 \times 20.3961)$$
$$= 0.36$$
$$p_{rot2} = pdf(0.5880 - 0.5880, 0.05 \times 0.5880 + 0.001 \times 20.3961)$$
$$= 1.8053$$

return $p = 8.9522 \times 0.36 \times 1.8053 = 5.8176$

b) ● The scale factor infront of the exponential part can be removed

● As a consequence the value p return is between 0 and 1 with 1 meaning that there is zero error between x’ and the state produced using u.

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ANSWER

a)

$$\boldsymbol{R}_b^n(\boldsymbol{\Theta}) = \begin{bmatrix} \cos\psi\cos\theta & -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi \\ \sin\psi\cos\theta & \cos\psi\cos\theta + \sin\phi\sin\theta\sin\psi & -\cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

$$\sin\phi = \sin(0.1047) = 0.1045$$
$$\cos\phi = \cos(0.1047) = 0.9945$$
$$\sin\theta = \sin(0.2094) = 0.2079$$
$$\cos\theta = \cos(0.2094) = 0.9781$$
$$\sin\psi = \sin(0.4189) = 0.4067$$
$$\cos\psi = \cos(0.4189) = 0.9135$$

$\boldsymbol{R}_b^n(\boldsymbol{\Theta}) =$

$$\begin{bmatrix} 0.9135 \times 0.9781 & -0.4067 \times 0.9945 + 0.9135 \times 0.2079 \times 0.1045 & 0.4067 \times 0.1045 + 0.9135 \times 0.2079 \times 0.9945 \\ 0.4067 \times 0.9781 & 0.9135 \times 0.9781 + 0.1045 \times 0.2079 \times 0.4067 & -0.9135 \times 0.1045 + 0.2079 \times 0.4067 \times 0.9945 \\ -0.2079 & 0.9781 \times 0.1045 & 0.9781 \times 0.9945 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8936 & -0.3847 & 0.2314 \\ 0.3978 & 0.9174 & -0.0114 \\ -0.2079 & 0.1022 & 0.9728 \end{bmatrix}$$

b)

$$\boldsymbol{T}_\Theta(\boldsymbol{\Theta}) = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix}$$

From Part a)

$$\sin\phi = \sin(0.1047) = 0.1045$$
$$\cos\phi = \cos(0.1047) = 0.9945$$
$$\cos\theta = \cos(0.2094) = 0.9781$$
$$\tan\theta = \sin\theta\cos\theta = \frac{0.2079}{0.9781} = 0.2126$$

$$\boldsymbol{T}_\Theta(\boldsymbol{\Theta}) = \begin{bmatrix} 1 & 0.1045 \times 0.2126 & 0.9945 \times 0.2126 \\ 0 & 0.9945 & -0.1045 \\ 0 & 0.1045/0.9781 & 0.9945/0.9781 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0.0222 & 0.2114 \\ 0 & 0.9945 & -0.1045 \\ 0 & 0.1069 & 1.0167 \end{bmatrix}$$

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c) $\dot{\boldsymbol{v}}_b = \boldsymbol{a}_b - \boldsymbol{R}_b^n(\boldsymbol{\Theta})^T \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 9.5 \end{bmatrix} - \begin{bmatrix} 0.8936 & 0.3978 & -0.2079 \\ -0.3847 & 0.9174 & 0.1022 \\ 0.2314 & -0.0114 & 0.9728 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 9.81 \end{bmatrix} = \begin{bmatrix} 0.0396 \\ -0.0030 \\ -0.0431 \end{bmatrix}$

$$\dot{\boldsymbol{p}}_n = \boldsymbol{R}_b^n(\boldsymbol{\Theta})\boldsymbol{v}_b = \begin{bmatrix} 0.8936 & -0.3847 & 0.2314 \\ 0.3978 & 0.9174 & -0.0114 \\ -0.2079 & 0.1022 & 0.9728 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.9167 \\ 0.3967 \\ -0.1106 \end{bmatrix}$$

$$\dot{\boldsymbol{\Theta}} = \boldsymbol{T}_{\boldsymbol{\Theta}}(\boldsymbol{\Theta})\boldsymbol{\omega}_{\boldsymbol{nb}}^{\boldsymbol{b}} = \begin{bmatrix} 1 & 0.0222 & 0.2114 \\ 0 & 0.9945 & -0.1045 \\ 0 & 0.1069 & 1.0167 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix} = \begin{bmatrix} 0.1679 \\ 0.1675 \\ 0.3264 \end{bmatrix}$$

d)

$$x_t = x_{t-1} + T\dot{x}_t = \begin{bmatrix} 1 \\ 0 \\ 0.1 \\ 5 \\ 10 \\ 20 \\ 0.1047 \\ 0.2094 \\ 0.4189 \end{bmatrix} + 0.1 \times \begin{bmatrix} 0.0396 \\ -0.0030 \\ -0.0431 \\ 0.9167 \\ 0.3967 \\ -0.1106 \\ 0.1679 \\ 0.1675 \\ 0.3264 \end{bmatrix} = \begin{bmatrix} 1.00396 \\ -0.0003 \\ 0.0957 \\ 5.09167 \\ 10.03967 \\ 19.98894 \\ 0.1215 \\ 0.2262 \\ 0.4515 \end{bmatrix}$$

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Are x' and $\dot{x}'$ correlated in the posterior? Explain why/why not.

ANSWER

$$\dot{x}' = \dot{x} + T\ddot{x}$$

$$\mathcal{N}(\dot{x}', \sigma_v^2) = \mathcal{N}(\dot{x} + T\ddot{x}, T\sigma^2)$$

$$x' = x + T\dot{x} + \frac{T^2}{2}\ddot{x}$$

$$\mathcal{N}(x', \sigma_p^2) = \mathcal{N}\left(x + T\dot{x} + \frac{T^2}{2}\ddot{x}, \frac{T^2}{2}\sigma^2\right)$$

Yes, x' and $\dot{x}'$ are correlated as x' is dependent on $\dot{x}'$.

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