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Simultaneous sensor selection and routing of unmanned aerial vehicles for complex mission plans

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ABSTRACT

Military reconnaissance missions often employ a set of unmanned aerial vehicles (UAVs) equipped with sensors to gather intelligence information from a set of known targets. UAVs are limited by the number of sensors they can hold; also attaching a sensor adds weight to the aircraft which in turn reduces the flight time available during a mission. The task of optimally assigning sensors to UAVs and routing them through a target field to maximize intelligence gain is a generalization of the team orienteering problem studied in the vehicle routing literature. This work presents a mathematical programming model for simultaneous sensor selection and routing of UAVs, which solves optimally using CPLEX for simple missions. Larger missions required the development of three heuristics, which were augmented by Column Generation. Results from a performance study indicated that the heuristics quickly found good solutions. Column Generation improved the solution in many instances, with minimal impact on overall solution time. The rapid nature of the overall solution approach allows it to be used in other mission planning tasks. A fleet sizing application is discussed as an example of its flexible usage.

1. Introduction

1.1. Problem description

In unmanned aerial vehicle (UAV) mission planning, there exists a set of predetermined targets that require surveillance. The surveillance required at each target is unique and can only be satisfied with a specific sensor, or set of sensors. Surveillance benefit is obtained when UAVs visit targets with the appropriate sensors attached. The goal of mission planning is to route the UAV fleet through the target field in an effort to maximize surveillance benefit. For simplicity, mission planners often assume that the sensors attached to each UAV in the fleet are fixed. Modern UAVs have the ability to interchange sensors, providing greater flexibility in mission planning. The cost/time associated with a sensor change is usually small, so swapping sensors between missions is practical. Assuming fixed sensor attachments simplifies the complexity associated with the planning phase, but also hinders the effectiveness of the mission. The consideration of interchangeable sensors adds complexity for the following reasons:

1. *Increased combinations*: When sensors are fixed on UAVs, each target has a unique benefit when it is visited. Consideration of interchangeable sensors adds a great deal of complexity because the benefit of visiting a target is no longer static. Thus, the increase in complexity is dependent on the quantity of sensors considered and the sensor capacity of the UAV.

2. *Travel time variability*: UAVs with predefined sensor attachments have a fixed travel range. When interchangeable sensors are considered the sensor payload weight assigned to each UAV will vary. As the payload weight increases, the travel time available for UAVs to visit targets decreases. This variation in travel time adds additional complexity to the problem.

Furthermore, each sensor may not be compatible with every type of UAV. In Section 1.2, an example case is presented to illustrate the problem. This case will be referenced in subsequent sections to aid in the explanation of the solution approach.

1.2. Example case

Consider six targets spread over a terrain of 100 by 80 units. Intelligence is gained when these targets are surveyed with a particular sensor. Some targets may only benefit from a single sensor, while others may benefit from a combination of sensors. The quantitative benefit of surveillance is based on priority and importance, as determined by a mission planner. Typically, these data would be derived from a prior reconnaissance mission or existing intelligence. Fig. 1 details the spatial layout of targets.

The Cartesian coordinates of each target are listed in parenthesis. A single base is considered and located at the origin. All UAVs must depart from the base, survey a set of targets, and return to the base

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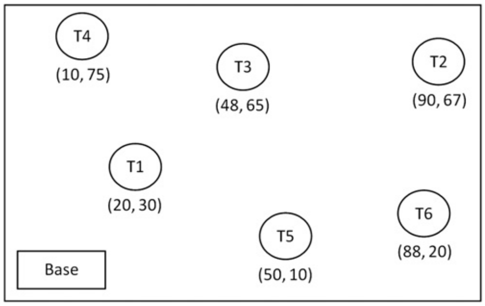

Fig. 1. Spatial arrangement of targets and base.

Table 1
Sensor target benefit matrix.

	S1	S2	S3	S4
T1	100	0	130	135
T2	145	120	100	75
T3	80	0	0	120
T4	160	80	50	25
T5	0	0	0	300
T6	50	45	110	0

Table 2
UAV attributes.

UAV	Sensor capacity	Unloaded range	Load limit
1	2	300	125
2	2	350	140

before their available travel time is depleted. The requirement for each UAV to return to the base is realistic, as many of the UAVs and sensors are relatively expensive. However, relaxation of this requirement can be incorporated for disposable UAVs. Euclidean distance is assumed as the travel time between targets.

Four sensors are considered in this example. While not explicitly defined, the sensor set could include electro-optical/infrared cameras, video recording devices, and radiation detectors. The sensor-target benefit matrix (STBM) is shown in Table 1.

Here, if a UAV visits target T1 with sensor S1, a benefit of 100 units is obtained. Additionally, if sensor S3 was also attached, the surveillance benefit of visiting target T1 would increase to 230. Target T3 receives no benefit if it is visited by sensor S2 or S3. The characteristics of the resources available to survey the targets will now be detailed.

Two UAVs are considered, each UAV with the ability to carry two sensors. Additional attributes are detailed in Table 2.

The unloaded range represents the time a UAV can travel without any sensor attachments while the load limit indicates the maximum sensor payload weight. The attributes of each sensor is displayed in Table 3.

If sensor S1 is attached to UAV 1, the travel time would reduce to 200 units. Also, sensors S1 and S2 cannot simultaneously be attached to UAV 1 because doing so would exceed the load limit. The weight of a sensor is strongly correlated to its hindrance on travel time [15]. Thus, in this example, the travel time reduction is equivalent to the weight of a sensor. It should be noted that the mathematical models and algorithms developed do not require this equivalence.

If one were to consider the sensor selection and routing aspects of the problem independently, the solution procedure may elect to assign sensors to UAVs first and then route them through the target field. Alternatively, one could create a route for each UAV first and subsequently assign sensors. Here, the former procedure will be investigated.

Table 3
Sensor attributes.

Sensor	Sensor weight	Travel time reduction
S1	100	100
S2	75	75
S3	125	125
S4	40	40

In a two step method that assigns sensors first and routes UAVs second, two logical approaches are suggested for the sensor assignment step. These methods are not intended to be exact procedures. They are presented to illustrate general approaches a mission planner might take if other tools were unavailable.

1. *Highest potential benefit*: Using this approach, one would assign sensors based on their potential benefit. It is assumed that assignment precedence is based on greatest unloaded travel range. The potential benefit of a sensor is the benefit it would obtain if it visited all of the targets. For the example problem, the potential benefit of sensors S1, S2, S3, and S4 is 535, 245, 390, and 655, respectively. Using this approach, one would assign sensor S3 to UAV 1 and sensors S1 and S4 to UAV 2. Note that we cannot assign both sensors S2 and S3 to UAV 1 due to payload limitations. S3 was chosen over S2 because it has a higher potential benefit.

2. *Lowest travel time reduction*: The logic behind this approach is to load each UAV with sensors that minimize travel time reduction. Hence, UAVs will be able to visit more targets, even though the benefit per visit may be less than the previously stated method. Once again, assignment precedence is based on unloaded travel range, with the lightest sensor being assigned to the UAV with the lowest range. Here, sensor S4 would be assigned to UAV 1 and sensor S2 would be assigned to UAV 2. Alternatively, one could assign sensor S4 to both UAVs.

After assigning sensors using either of these methods, the UAVs can be optimally routed through the targets. Given a small problem, the optimal route can be obtained using a straightforward mathematical model. Fig. 2 shows the optimal routes when the above approaches are used. The solid and dashed lines represent UAV 1 and UAV 2, respectively. Table 4 summarizes the results.

The optimal sensor and route assignment is shown in Fig. 3. Clearly, the sensor selection utilized for the optimal solution is some combination of the selection procedures above. The optimal solution was acquired using the Integrated Sensor Selection and Routing Model (ISSRM). The ISSRM is a mixed integer linear programming formulation that will be detailed in Section 2. For this example, a 9% improvement is obtained due to a combined consideration of sensor assignment and routing of the aircraft.

Several other problem features are considered in this work, but are omitted from the sample problem. The sample problem assumes instantaneous sensor visitations. This assumption is valid for surveillance tasks including the capture of still pictures, but may need to be relaxed in some instances. For example, a video surveillance task may require a UAV to stay at a target for a duration of time. Additionally, targets may be assigned a time window, or a specified period of time they may be visited. The mathematical models and algorithms presented in this work accommodate each feature differently, with some approaches supporting a subset of the problem features.

1.3. Connection to team orienteering problem

The UAV sensor selection and routing problem is a generalization of the team orienteering problem (TOP) [4]. To establish

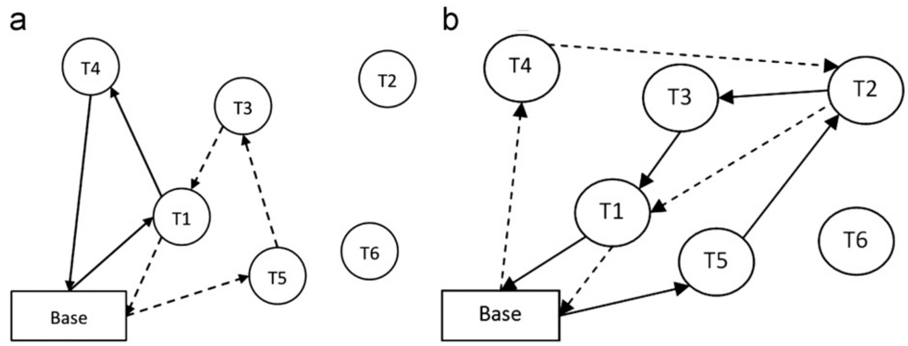

Fig. 2. Route assignments for two-step heuristics. (a) Highest potential benefit and (b) lowest travel time reduction.

Table 4
Summary of results for each sensor selection procedure.

Sensor selection procedure	UAV 1 sensors	UAV 2 sensors	Benefit
Highest potential benefit	S3	S1,S4	915
Lowest travel time reduction	S4	S2	830

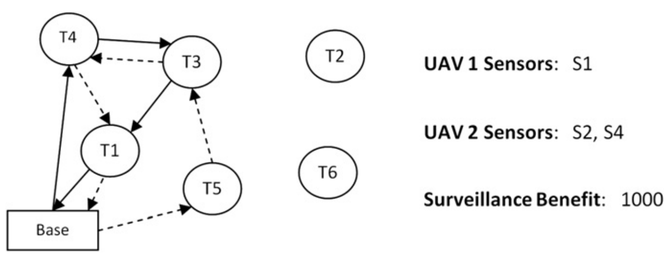

Fig. 3. Optimal solution using integrated sensor and routing model.

this connection with the TOP, we first discuss the orienteering problem (OP) which was initially investigated by Tsiligirides [18]. In the OP, a single vehicle begins at a starting location and must reach a designated destination prior to time T_{max}. Along with the starting and ending points, a set of locations exists with an associated benefit that may be collected by the vehicle. The objective of the OP is to route the vehicle through a subset of the locations in a way which maximizes the collected benefit. Of course, the vehicle must reach the end point by time T_{max}. The problem gets its name from the sport of orienteering. In this navigation based game, players begin at a central control location and must visit other control locations where they accumulate points. The quantity of points received is typically based on how difficult it is to reach the control location. Players are disqualified if they fail to reach a predefined destination before the time limit, and the winner is the individual who accumulates the greatest number of points. Clearly, the optimal route is the solution to the OP. The TOP is identical to the OP, but multiple vehicles may be used to visit the control points.

Two papers have been published which present exact algorithms for the TOP: Boussier et al. [3] present a branch and price algorithm while Butt and Ryan [5] use Column Generation. Unfortunately, each of these approaches can only solve problems of limited size in a reasonable amount of time. Butt and Ryan find optimal solutions to problems with 100 nodes, but the vehicle travel time is limited so the average tour size contains fewer than four nodes. These results are not surprising, however, as the OP was shown by Golden et al. [7] to be NP-hard. For this reason, most of the literature has focused on the presentation of heuristic approaches.

Chao et al. [6] developed a heuristic to solve the TOP. They compare their heuristic with an extension of a stochastic heuristic originally designed by Tsiligirides [18] to solve the OP. In our paper, a variation of Tsiligirides' method is used to develop a heuristic we will present in Section 4.3. Tang and Miller-Hooks [17] used a tabu search heuristic to solve the TOP. Their procedure included an adaptive memory procedure to store and update solutions. Archetti et al. [1] developed two additional tabu search heuristics along with a variable neighborhood search, which provided results superior to the above mentioned. An ant colony optimization approach was used by Ke et al. [9] which produced results on par or better than Archetti et al. [1] with quicker solution times. In a recent paper by Vansteenwegen et al. [20] a guided local search framework was implemented to rapidly find good solutions to the TOP. For a thorough overview of the OP and TOP, see Vansteenwegen et al. [19].

In the TOP, split deliveries are not allowed and a customer may not be visited by more than one vehicle. If the sensor assignment and routing problem in this work considered a homogeneous UAV fleet with fixed sensors, it would be equivalent to the TOP. In this special case, T_{max} would be equivalent to the travel range of each UAV in the fleet. Also, the surveillance benefit gained for visiting a target would be identical for each UAV. Specifically, the benefit would correspond to the fixed sensor assignment.

Generally speaking, however, T_{max} will be different for each UAV in the fleet for two reasons. Primarily, the fleet may be heterogeneous so the unloaded travel time for each UAV, T_{max}, need not be equivalent. Moreover, each sensor attachment impacts travel time differently. Since each UAV can be equipped with a unique set of sensors, impact on travel distance will not be consistent for each UAV in the fleet. Additionally, the benefit value associated with each target will be different for each vehicle as it also corresponds to the attached sensors. Butt and Cavalier [4] refer to the TOP as the Multiple Tour Maximum Collection Problem (MTMCP) and assume the start and end point of each vehicle to be the same. Similarly, the work presented here assumes that each UAV takes off and lands at the same location. Table 5 compares the sensor selection and routing problem to the team orienteering problem.

Additionally, the mathematical model developed in this work allows for the inclusion of time windows. We note that Kantor and Rosenwein [8] initially investigated the orienteering problem with time windows and developed a tree-based heuristic. The tree heuristic is compared to an insertion procedure centered on an extension of a heuristic developed by Laporte and Martello [11]. For problems with few nodes and small T_{max} values, the tree-based heuristic outperforms the insertion heuristic in terms of solution quality. As the quantity of nodes, T_{max}, and time window width increase, the tree heuristic is unable to find a solution in reasonable time.

1.4. Relationship with UAV routing and planning literature

In this section, we briefly review contributions that have been made in UAV routing and scheduling. There has been significant work

Table 5
Comparison of team orienteering problem and sensor selection and routing problem.

Modeling feature	Traditional TOP	SSRP
Vehicle travel time	Fixed across all vehicles	Variable across all vehicles
Customer benefit	Fixed across all vehicles	Variable across all vehicles
Start and end nodes	Different	Same
Split deliveries	Prohibited	Allowed

in the area of path planning for UAVs. See, for example, the papers by Kim et al. [10], Nikolos et al. [13], Qu et al. [14], and Yang and Kapila [23]. The focus of path planning is primarily on satisfying vehicle dynamic requirements as well as avoiding obstacles. Mission planning can be viewed as a complex version of path planning where the objective is to visit a sequence of targets to achieve the objectives of the mission. For example, in a recent paper by Wu et al. [22] the mission scenario involves the delivery of a medical package to a remote location using a small UAV. The goal of our paper is related to a much more complex mission. We are dealing with targets in an area of interest that can be visited by UAVs to gain information about the targets (i.e., reconnaissance). The additional complications include the facts that the UAVs can be equipped with several alternative sensor assignments, each with its own gain values; the weight of sensors reduces the UAV range; there are time windows for the targets to be visited; and a fuel constraint exists. Thus, our contribution to the UAV routing and planning literature is in the domain of planning a complex mission, not in the domain of satisfying realistic vehicle dynamic requirements or obstacle avoidance.

1.5. Organization of paper

Section 2 contains the formulation for the Integrated Sensor Selection and Routing Model (ISSRM), which is useful for finding a direct solution using CPLEX when the mission contains few targets and UAVs. Section 3 details a Column Generation heuristic, which includes a presentation of the master and subproblem formulations, along with a description of the algorithm used to solve the subproblem. Three additional heuristics are included in Section 4. Our computational experience is detailed in Section 5 where experimental conditions are defined, small/medium sized problems and large problems are separately considered, and a fleet sizing application is discussed. The paper ends with a set of conclusions and future work directions in Section 6.

2. Integrated sensor selection and routing model

The Integrated Sensor Selection and Routing Model (ISSRM) is defined as a mixed integer linear programming formulation. The formulation is based on the single commodity problem proposed by Warrier [21]. The inputs, decision variables, and formulation are detailed below:

Indices:

- i,j indices for targets ($i, j=0$ represents base location)
- h index for UAVs
- s index for sensors

Inputs:

- N number of targets
- O number of UAVs
- S number of sensors
- τ_h maximum number of sensors that can be attached to UAV h
- Q_s quantity of sensor s available at base
- V_{js} demand of sensor s at target j
- R_{js} benefit obtained when sensor s visits target j
- D_{ij} travel time from target i to target j
- λ_h unloaded travel time of UAV h
- C_s travel time reduction when sensor s is attached
- δ fuel minimization weight factor
- A_i earliest time target i can be visited
- B_i latest time target i can be visited
- W_{js} time required to deliver a single surveillance unit of sensor s to target j

Decision variables:

- f_{hs} 1 if UAV h is equipped with sensor s, 0 otherwise
- y_{ijh} 1 if UAV h travels from target i to target j
- x^s_{ijh} flow of sensor s from i to j using UAV h
- z_{jsh} delivery amount of sensor s to target j using UAV h
- t_{jh} arrival time of UAV h at target $j (j \geq 1)$
- t_{0h} return time of UAV h to the base

ISSRM formulation:

$$\text{maximize} \quad \sum_{h=1}^{O}\sum_{j=1}^{N}\sum_{s=1}^{S} R_{js} z_{jsh} - \delta \sum_{i=0}^{N}\sum_{j:j\neq i}^{N}\sum_{h=1}^{O} D_{ij} y_{ijh} \tag{1}$$

$$\text{subject to} \quad \sum_{h=1}^{O} z_{jsh} \leq V_{js} \quad \forall j,s \tag{2}$$

$$\sum_{h=1}^{O} f_{hs} \leq Q_s \quad \forall s \tag{3}$$

$$\sum_{\substack{i=0\\ i\neq j}}^{N} x^s_{ijh} - \sum_{\substack{i=0\\ i\neq j}}^{N} x^s_{jih} = z_{jsh} \quad \forall j,h,s \tag{4}$$

$$x^s_{ijh} \leq \sum_{j=1, j\neq i}^{N} V_{js} f_{hs} \quad \forall i,h,s \tag{5}$$

$$x^s_{ijh} \leq \sum_{j=1, j\neq i}^{N} V_{js} y_{ijh} \quad \forall i,h,s \tag{6}$$

$$z_{jsh} \leq V_{js} f_{hs} \quad \forall j,s,h \tag{7}$$

$$z_{jsh} \leq V_{js} \sum_{i=0, i\neq j}^{N} y_{ijh} \quad \forall j,s,h \tag{8}$$

$$\sum_{s=1}^{S} f_{hs} \leq \tau_h \quad \forall h \tag{9}$$

$$\sum_{\substack{j=0\\ j\neq i}}^{N} y_{ijh} - \sum_{\substack{j=0\\ j\neq i}}^{N} y_{jih} = 0 \quad \forall i,h \tag{10}$$

$$\sum_{j=0, i\neq j}^{N} y_{ijh} \leq 1 \quad \forall i,h \tag{11}$$

$$t_{0h} \leq \lambda_h - \sum_{s=1}^{S} C_s f_{hs} \quad \forall h \tag{12}$$

$$\sum_{i=0}^{N}\sum_{j=0, j\neq i}^{N} D_{ij}y_{ijh} + \sum_{s=1}^{S} C_s f_{hs} \leq \lambda_h \quad \forall h \tag{13}$$

$$\sum_{j=0}^{N} y_{0jh} = 1 \quad \forall h \tag{14}$$

$$\sum_{j=0}^{N} y_{j0h} = 1 \quad \forall h \tag{15}$$

$$t_{jh} \geq D_{ij}y_{ijh} + t_{ih} - M(1-y_{ijh}) + W_{is}z_{ish} \quad \forall i,j,h,s \tag{16}$$

$$A_i \leq t_{ih} \leq B_i \quad \forall i,h \tag{17}$$

$$f_{hs}, y_{ijh} \in \{0,1\} \tag{18}$$

$$x^s_{ijh} \geq 0 \quad \forall i,j,h,s \tag{19}$$

$$z_{jsh} \quad \forall j,s,h \tag{20}$$

To recap, the ISSRM solves the simultaneous sensor selection and routing problem. In a traditional team orienteering problem, nodes must simply be visited by a vehicle (regardless of which sensors it has attached to it) to instantaneously collect a reward. The most novel aspect of this work is the requirement of a vehicle sensor assignment to collect the reward. Subsequent novelties include vehicle sensor assignment limitations and the travel time penalty associated with sensor attachments. Additionally, the consideration of sensors provides the possibility of split deliveries at targets. Split deliveries are always possible between targets. As discussed in Section 1.2, a sensor may be required to stay at a target for some duration of time to collect all of the surveillance benefit. The amount of benefit is linearly correlated to the length of stay. When durations are present, the possibility of split deliveries among sensors at a given target exists. Fuel conservation is also considered in the model and will be discussed shortly. It is important to understand that the ISSRM is the only solution approach presented that considers all of the problem's intricacies. The fundamental novelties associated with sensor assignments, however, are considered in all of the approaches. Following the discussion of the solution methods, a summary of the problem characteristics supported by each approach is provided in Section 4.5.

The first term in the objective function (1) attempts to maximize the surveillance benefit for the entire fleet of UAVs, while the second term is included to minimize fuel cost when there are alternative optimal target sets and sensor assignments. If δ is selected properly, the ISSRM will select the optimal solution alternative that minimizes fuel consumption. Selection of δ is important because if the selection is too large, fuel minimization will take precedence over maximizing surveillance benefit. When δ is set according to Eq. (21) fuel minimization will never take precedence over surveillance benefit

$$\delta \leq \frac{\min R_{js}}{O\sum_{i,j} D_{ij}} \tag{21}$$

The mission planner may prefer to use a larger value for δ if he or she anticipates pop-up targets to appear after the mission begins. This will allow the UAVs to have adequate fuel available to potentially visit these additional targets. Constraint (2) ensures that the total sensor surveillance at a target does not exceed the demand of the target. For example, if target 6 required three photographs from a camera sensor S2, $V_{62}=3$. Constraint (3) makes sure that the total number of sensors assigned among all UAVs does not exceed the number of sensors available at the base. Constraints (4)–(6) are included to preserve sensor delivery. In order for a UAV to provide surveillance on a target, the UAV must have the appropriate sensor equipped and the UAV must visit the target. These requirements are satisfied in constraints (7) and (8), respectively. UAV sensor capacity is represented in constraint (9). Route continuity is ensured by the inclusion of constraint (10), and constraint (11) does not allow a single UAV to visit a target more than once. Each UAV has a maximum flight time when its sensor payload is empty. The weight associated with sensor attachments reduces flight time. Constraints (12) and (13) ensure that the route assignments for each UAV do not exceed the total adjusted flight time. This is performed by restricting the time at which a UAV returns to the base, t_{0h}, to be less than or equal to the adjusted flight time. Constraints (14) and (15) require that all UAVs begin and end their route at the base. Constraint (16) keeps track of the time each UAV visits a target. This constraint works in conjunction with constraint (12), but also is necessary for the inclusion of time windows which are represented in constraint (17). Constraints (18)–(20) are binary and non-negativity restrictions.

The ISSRM works well for simple missions containing few targets and UAVs. The x^s_{ijh} variables are flow variables which are not required to validate the formulation. A small study indicated these variables improved computation time, which justifies their inclusion. The example case in Section 1.2 with six targets, two UAVs, and four sensors was optimally solved in 15 s using CPLEX. However, for complex missions an optimal solution cannot be found within an acceptable time limit. Thus, a Column Generation heuristic was developed to quickly provide good solutions. For a survey of recent contributions in Column Generation, see Lubbecke and Desrosiers [12].

3. Column Generation heuristic

3.1. Master problem

The Column Generation approach considered in this work decomposes the problem by UAV. Thus, the number of subproblems in the procedure is equivalent to the number of UAVs considered in the mission. The subproblems are solved at each iteration of Column Generation, providing new sensor and route combinations for the master problem. The inputs, decision variables, and formulation for master problem are defined as follows:

Indices:

- i,j indices for targets
- k index for route/sensor combination
- h index for UAV
- s index for sensors

Inputs:

- K_h set of routes/sensor combinations for UAV h
- R_{js} benefit of delivering one unit of sensor s to target j
- $F_{(kh)s}$ 1 if sensor s is included in (kh), 0 otherwise
- $U_{(kh)j}$ 1 if target j is included in (kh), 0 otherwise
- $P_{(kh)ji}$ 1 if UAV h travels from j to i in (kh)
- V_{js} maximum demand for sensor s at target j
- Q_s quantity of sensor s available at base
- $E_{(kh)}$ additional travel time remaining after flight path is executed for (kh)
- D_{ij} travel time from target i to target j
- W_{js} time required to deliver a single unit of sensor s to target j
- A_i earliest time UAV may arrive at location i
- B_i latest time UAV may arrive at location i

Decision variables:

- $x_{(kh)}$ 1 if UAV h selects route/sensor combination k, 0 otherwise

g_{hjs} units of sensor s delivered to target j using UAV h
t_{jh} arrival time of UAV h at location j

The objective function of the master problem is equivalent to that of the ISSRM, with the exception of fuel minimization. Unlike the ISSRM, the master problem does not generate routes and sensor combinations. It simply selects from those that it currently has available. The selection of a single route/sensor combination for each UAV in the fleet is reflected in constraint (23). Constraint (24) states that the sensors included in the selected combinations cannot exceed the number available at the base. The cumulative sensor delivery at a target among the selected combinations cannot exceed the demand at a target. This is satisfied in constraint (25).

CG master problem formulation:

$$\text{maximize} \quad \sum_h \sum_j \sum_s R_{js} g_{hjs} \tag{22}$$

$$\text{subject to} \quad \sum_{(kh) \in K_h} x_{(kh)} = 1 \quad \forall h \ \text{Dual cost} : \alpha \tag{23}$$

$$\sum_h \sum_{(kh) \in K_h} x_{(kh)} F_{(kh)s} \le Q_s \quad \forall s \ \text{Dual cost} : \beta_s \tag{24}$$

$$\sum_h g_{hjs} \le V_{js} \quad \forall j,s \ \text{Dual cost} : v_{js} \tag{25}$$

$$g_{hjs} \le V_{js} \sum_k F_{(kh)s} x_{(kh)} \quad \forall j,s,h \ \text{Dual cost} : \psi_{js} \tag{26}$$

$$g_{hjs} \le V_{js} \sum_k U_{(kh)s} x_{(kh)} \quad \forall j,s,h \ \text{Dual cost} : \xi_{js} \tag{27}$$

$$\sum_j \sum_s W_{js} g_{hjs} \le \sum_k E_{(kh)} x_{(kh)} \quad \forall h \tag{28}$$

$$t_{jh} \ge D_{ij} \sum_k P_{(kh)ij} x_{(kh)} + t_{ih} + W_{is} g_{his} \quad \forall (kh),i,j,s \quad \text{if } P_{(kh)ij} \text{ is in } (kh) \tag{29}$$

$$A_i \le t_{ih} \le B_i \quad \forall i,h \tag{30}$$

$$x_{(kh)} \in \{0,1\} \quad \forall (kh) \tag{31}$$

$$g_{hjs} \quad \forall h,j,s \tag{32}$$

The constraints mentioned thus far tie in closely to those included in the ISSRM. Since the subproblem is generating solutions for each UAV independently, there is a chance that the selected master problem may select combinations that collectively could deliver more of a sensor than demanded at a target. Constraint (25) prevents excess delivery. Additionally, a combination cannot survey a target unless the combination includes the appropriate sensor and visits the correct target. These requirements are satisfied in constraints (26) and (27), respectively. Constraints (28) and (29) allow the UAVs to optimally allocate the remaining travel time on their route for surveillance tasks that have duration $W_{js} > 0$. $E_{(kh)}$ is the travel time remaining after the route is executed. Constraint (30) specifies the time windows for each target. Constraints (31) and (32) are binary and non-negativity restrictions. Next, the subproblem will be discussed.

3.2. Subproblem

The LP relaxation of the master problem is solved to obtain dual costs which are passed to the objective function of the subproblem. A separate subproblem is solved for each UAV, and thus, the number of subproblems is equivalent to the number of UAVs. The subproblem for UAV h is as follows:

Indices:

i,j indices for targets
h index for UAV
s index for sensors

Inputs:

C_s travel time reduction when sensor s is attached
τ_h quantity of sensors UAV h can carry
λ_h unloaded range of UAV h
W_{js} time required to deliver one unit of sensor s to target j
D_{ij} travel time from target i to target j
R_{js} benefit of delivering sensor s to target j
V_{js} demand of sensor s at target j
A_i earliest arrival time for target i
B_i latest arrival time for target i

Decision variables:

f_s 1 if sensor s is selected, 0 otherwise
y_{ij} 1 if UAV travels from target i to target j, 0 otherwise
z_{js} delivery amount of sensor s to target j
t_j arrival time of UAV at target j
t_0 return time of UAV to the base
x_{ij}^s flow variable from i to j with sensor s

CG subproblem formulation for UAV h:

$$\text{maximize} \quad \sum_{j,s} R_{js} z_{js} - \alpha - \sum_s \beta_s f_s - \sum_{j,s} v_{js} z_{js} - \sum_{j,s} \psi_{js} (z_{js} - V_{js} f_s) - \sum_{j,s} \xi_{js} \left(z_{js} - V_{js} \sum_{\substack{i \\ i \ne j}} y_{ij} \right) \tag{33}$$

$$\text{subject to} \quad z_{js} \le V_{js} f_s \quad \forall j,s \tag{34}$$

$$z_{js} \le V_{js} \sum_i y_{ij} \quad \forall j,s \tag{35}$$

$$\sum_{\substack{i=0 \\ i \ne j}} x_{ij}^s - \sum_{\substack{i=0 \\ i \ne j}}^{N} x_{ji}^s = z_{js} \quad \forall j,s \tag{36}$$

$$x_{ij}^s \le \sum_{j=1, j \ne i}^{N} V_{js} f_s \quad \forall i,s \tag{37}$$

$$x_{ij}^s \le \sum_{j=1, j \ne i}^{N} V_{js} y_{ij} \quad \forall i,s \tag{38}$$

$$\sum_s f_s \le \tau_h \tag{39}$$

$$\sum_{\substack{j \\ j \ne i}} y_{ij} - \sum_{\substack{j \\ j \ne i}} y_{ji} = 0 \quad \forall i \tag{40}$$

$$\sum_{\substack{j \\ i \ne j}} y_{ij} \le 1 \quad \forall i \tag{41}$$

$$t_0 \le \lambda_h - \sum_s f_s C_s \tag{42}$$

$$\sum_i \sum_{\substack{j \\ j \neq i}} D_{ij} y_{ij} + \sum_s C_s f_s \le \lambda_h \tag{43}$$

$$\sum_j y_{0j} = 1 \tag{44}$$

$$\sum_j y_{j0} = 1 \tag{45}$$

$$t_j \ge D_{ij} y_{ij} + t_i - M(1-y_{ij}) + W_{is} z_{is} \quad \forall i,j,s \tag{46}$$

$$A_i \le t_i \le B_i \quad \forall i \tag{47}$$

$$f_s, y_{ij} \in \{0,1\} \tag{48}$$

$$z_{js}, x_{ij}^s \ge 0 \tag{49}$$

As mentioned earlier, the goal of the subproblem is to generate a beneficial route/sensor combination to pass back into the master problem. A beneficial solution is one with a positive reduced cost solution, which is represented by the objective function. Thus, if the objective function assumes a positive value, then adding the corresponding route/sensor combination into the master problem will improve the master problem's objective provided that this column can be brought into the current basis at a non-zero level. Similar to those found in the ISSRM, constraints (34) and (35) ensure that a UAV cannot collect surveillance benefit at a target unless the target is visited and the appropriate sensor is attached. The flow variables considered in constraints (36)–(38) improve the computation time. The maximum sensor attachments allowed per UAV are modeled in constraint (39). Route continuity is preserved in constraint (40), while constraint (41) does not allow the same target to be visited multiple times. Constraints (42) and (43) guarantee that the UAV's return time to the base does not exceed its total travel time adjusted for sensor attachments. Constraints (44) and (45) force the UAV to start and end at the base location. Constraint (46) is used to preserve the cumulative travel time during the course of the route. This is used in conjunction with constraint (42) and also with constraint (47) which specifies the time windows.

As the number of sensors and targets increases, the problem becomes difficult to solve optimally. However, if a quick solution can be found with a positive reduced cost, the procedure may terminate and the corresponding solution can be added back to the master problem. Extensive testing revealed that the solution time to obtain a beneficial reduced cost for mid to large size problems was exorbitant. The goal of the subproblem is to determine the sensor/route combination for a UAV that yields the maximum benefit in reduced cost. However, any sensor/route combination with a positive reduced cost suffices in terms of its potential to improve the objective function of the master problem. With this realization we focus on an efficient heuristic to solve the subproblem in Section 3.3. Section 4 presents several heuristics which provide fast and effective stand alone solutions. In an effort to improve these solutions, they will also serve as initial columns for the Column Generation procedure.

3.3. Two-phase sensor selection and routing heuristic

As mentioned in Section 3.2, a heuristic was necessary to solve the subproblem. The Two Phase Sensor Selection and Routing Heuristic (TPSSRH) assigns sensors in the first phase and selects routes in the second phase. All of the constraints found in the subproblem are satisfied, and the heuristic makes decisions using principles of the objective function. Phase I begins by calculating the potential sensor benefit of each sensor included in the mission as shown in Eq. (50). This value is calculated assuming that each sensor can visit all of the targets

$$\text{Potential benefit}_s = \sum_j R_{js} + \sum_j \psi_{js} V_{js} - \beta_s \tag{50}$$

Next, the weighted potential benefit of each sensor is calculated according to Eq. (51) and one is stochastically selected using a procedure that mimics the route selection process (see Section 4.3)

$$\text{Weighted potential benefit of sensor}_s = \frac{\text{Potential benefit}_s}{\sum_s \text{Potential benefit}_s} \tag{51}$$

Subsequent iterations are performed with probabilities $\{P_2, P_3, \ldots, P_{\tau_h}\}$ to potentially include additional sensors. Only feasible sensors are considered at each iteration. The sensor selection phase concludes if an iteration fails to include a sensor, τ_h sensors have been assigned, or no additional sensors can feasibly be added. The visitation benefit of each target with respect to the selected sensor set, S, is computed using Eq. (52). This concludes the first phase of the procedure

$$\text{Benefit of visiting target}_j = \sum_{s \in S} R_{js} - \sum_{s \in S} v_{js} - \sum_{s \in S} \psi_{js} \tag{52}$$

Phase II (Table 6) utilizes a technique presented by Bodin et al. [2] for routing the vehicle. The selected sensor set and associated target visitation benefits are required as an input from phase I. The iteration number is defined as l, while P and T correspond to the cumulative benefit and cumulative travel time, respectively. R_l is defined as the value of a time unit at iteration l. This value is a ratio of the cumulative surveillance benefit to the cumulative travel time. Thus, the value of R_l will be high when the cumulative sensor benefit is large relative to the cumulative travel time. The parameter α is used to prevent rapid fluctuations in R_l and introduce randomness to the procedure. τ is set to the travel time adjusted for the attached sensors passed in from phase I.

Step 1 assumes that each UAV begins at the base location. The changes in time, Δt, and benefit, ΔP, are computed for each target i with an associated benefit >0 as follows: $\Delta t = t(0,i) + t(i,0)$, and $\Delta P = P(i)$. The target i which maximizes ΔP does not violate $\Delta t \le \tau$, and satisfies the appropriate time window is selected. Once selected, target i is removed from future consideration.

Step 2 updates all of the variables. The iteration, cumulative benefit, cumulative travel time, and remaining travel time are updated according to: $l = l+1$, $P = P_{l-1} + \Delta P$, $T = T_{l-1} + \Delta t$, and $\tau = \tau_{l-1} - \Delta t$. The value of a time unit is evaluated by $R_l = (\alpha P/T) + (1-\alpha)(R_{l-1})$. As mentioned earlier, α is a smoothing constant used to prevent extreme changes in R_l. Further, if the

Table 6
Two-phase sensor selection and routing heuristic—phase II.

Step 0	Set l, P, T, and $R_l = 0$ Choose smoothing constant α, set $\tau =$ adjusted travel time
Step 1	Compute Δt and ΔP for all targets i with an associated benefit >0. This is based on the sensor assignment passed in from Phase I Select i that maximizes ΔP and satisfies time constraints Insert target i in the current route and remove target i from consideration
Step 2	Update l, P, T, τ, and R_l
Step 3	Compute Δt and ΔP for remaining targets with an associated benefit >0 Select target from list that maximizes $\Delta P - R_0\ \Delta t$ and satisfies time constraints If such a target exists, insert it in the route and remove target from consideration, and proceed to **Step 2**. If such a target does not exist, go to **Step 4**
Step 4	End procedure and check reduced cost of the solution

procedure stalls by generating identical columns, α can be altered to introduce randomization.

Step 3 adds subsequent targets to the route. For each target k in the target list, Δt and $\Delta P - R_l(\Delta t)$ are computed for insertion between all existing targets i and j currently in the route. We have $\Delta t = t(i,k)+t(k,j)-t(i,j)$ and $\Delta t - R_{l-1}\Delta t = P(k) - R_l\Delta t$. The triplet that maximizes $\Delta P - R_l(\Delta t)$ and satisfies $T+\Delta t \le \tau$ as well as time window constraints for k and all existing targets in the route is selected. If such a triplet exists, target k is inserted into the current route and removed from the target list. The process then moves to *Step* 2. If no additional targets can feasibly be added to the route and the process advances to *Step* 4.

Step 4 checks the reduced cost of the discovered solution. If it is favorable, the route and sensor combination is added to the master problem as a new column. If the reduced cost is not favorable the current sensor combination is removed from consideration. A new combination is randomly selected from phase I, and phase II is restarted. If all sensor combinations are exhausted and a favorable solution is not found, the entire process may be repeated with a different α value. If a favorable column is still not found, no column is added for the UAV at the current CG iteration. However, a favorable column may still be discovered for the UAV in a future CG iteration. Column Generation ends when an iteration is unable to find a favorable column for all UAVs, or a predefined maximum number of columns are generated. An integer feasible solution is obtained by solving the final master problem using CPLEX with the integrality constraints enforced.

4. Additional heuristics

Three heuristic are presented that provide effective standalone solutions. The first two heuristics employ a generative approach that assigns sensors to the UAV fleet first and subsequently routes them through the targets. They share a common approach for sensor selection, but perform routing decisions differently. Heuristic I deterministically makes routing decisions while Heuristic II utilizes a stochastic approach. The third heuristic is based on a local search and is the focus of Section 4.4.

4.1. Sensor selection for Heuristic I and II

Sensor selection is based on the premise that the most favorable sensors will be those which have the greatest surveillance potential and the lowest travel time degradation. The sensor potential to weight ratio (SP/W) shown in Eq. (53) gives insight on which sensors to select

$$\mathrm{SP/W} = \frac{\text{Potential sensor benefit}}{\text{Travel distance reduction}} \tag{53}$$

The sensor, or sensor combination, with the highest SP/W ratio is assigned to the UAV with the longest travel distance and is removed from the assignable sensor pool. Sensors are assigned to the remaining UAVs in the same manner. The SP/W calculations for the example case presented in Section 1.2 are shown in Table 7. We use the column sum of Table 1 to find the Potential Sensor Benefit (PSB) for the sensors as follows: S1 = 535, S2 = 245, S3 = 390, and S4 = 655.

Sensor S4 has the highest SP/W Ratio so it is assigned to UAV 2, which has the greatest unloaded travel range. The remaining sensors and sensor combinations excluding S4 are examined, and S1 is selected for UAV 2. The travel ranges of UAV 1 and UAV 2 after sensor attachment are 200 and 310, respectively. Some combinations may not be compatible with a UAV as a result of load limitations. For example, the S3,S4 combination could not be assigned. The selectable sensor pool is reset if all sensors have been considered and additional UAVs are awaiting sensor assignment.

Table 7
SP/W ratios.

Sensor(s)	SP/W ratio	Sensors	SP/W ratio
S1	5.35	S1,S3	4.11
S2	3.27	S1,S4	8.5
S3	3.12	S2,S3	3.18
S4	16.375	S2,S4	7.83
S1,S2	4.46	S3,S4	6.33

4.2. Heuristic I route selection

The principle behind the routing portion of both heuristics is to visit targets that provide the greatest surveillance benefit and require the least amount of fuel. The process is performed iteratively among each UAV in the mission and continuously accesses and updates three matrices. The remaining benefit matrix (RBM) stores the surveillance benefit for each sensor remaining at all targets as UAVs visit targets. The distance benefit matrix (DBM) combines the information from the RBM with the current UAV location to create a numerical value which identifies the benefit to distance ratio of traveling to a target. Finally, the remaining travel time for each UAV is stored in the remaining travel distance matrix (RTDM). See Table 8 for Heuristic I details.

Continuing with the example case presented in Section 1.2, this algorithm can be applied to determine routes for both of the UAVs in the fleet. The DBM for UAV 1 and UAV 2 are shown in Table 9 and are used to select the first targets for their respective routes.

The value of visiting a target is the result when the remaining benefit for a target is divided by the travel distance to the target from the current location. For example, the value of target 1 with UAV 1 is obtained by dividing 100 (remaining benefit of visiting target 1 with UAV 1) by 46.06 (travel distance from the base to target 1). The target with the highest value is selected as it provides the highest surveillance benefit per unit of travel. In this

Table 8
Heuristic I routing algorithm.

Step 0	Initialize RBM, DBM, and RTDM Set current UAV target to 0 (location of base); Set current UAV $n=0$
Step 1	Set current UAV to $n+1$
Step 2	Scan DBM and select target with highest value If no feasible targets exist, proceed to **Step 5**
Step 3	Check feasibility of target selection; if feasible, proceed to **Step 4** If infeasible, remove target as an option and return to **Step 2**
Step 4	Update RBM, DBM, and RTDM
Step 5	Return current UAV to base If all UAVs have returned to base, **end** procedure Otherwise, proceed to **Step 1**

Table 9
Distance benefit matrix For UAV 1 and UAV 2.

UAV 1		UAV 2	
Target	Values	Target	Value
1	**2.17**	1	2.93
2	2.13	2	1.10
3	1.04	3	1.57
4	1.89	4	0.30
5	0	**5**	**4.96**
6	0.57	6	0

case, targets 1 and 5 are selected for UAV 1 and UAV 2, respectively. Next, a feasibility check is performed. This check ensures that a UAV will have enough fuel remaining to return to the base after it visits the selected target.

For UAV 1, the sum of the distance to target 1 and the distance from target 1 back to the base is less than the remaining travel distance indicating that this target selection is feasible. The decision to visit target 1 is finalized and the RBM and RTDM are updated. The process is repeated for UAV 2. If the target with the highest DBM value was an infeasible option, the next best target would be selected and checked for feasibility. The routing procedure ends when all of the targets are either infeasible or have a value of zero. At this point, the UAV is routed back to the base and the heuristic concludes. If the procedure were run to completion, the routes in Fig. 4 would be obtained. Recall (Section 4.1) UAV 1 has sensor S1 and UAV 2 has sensor S4. The heuristic provides excellent results for the example case for a total surveillance benefit of 995 and an optimality gap of only 0.5%. While this level of solution quality is not always obtained, the heuristic provides good solutions that may be used as initial columns in the Column Generation procedure of Section 3.

4.3. Heuristic II route selection

The routing procedure developed for Heuristic II is similar to the construction procedure used for Heuristic I. The key difference is the inclusion of a stochastic decision for each iteration of the procedure. In Heuristic I, the target with most attractive DBM value was selected. Here, the top four feasible targets ($j=1$–4) are considered and one is stochastically selected. When fewer than four feasible targets with positive DBM values remain, the procedure is adjusted to select among them. Assuming that four targets are among those to be selected, a weight is assigned to them as $W_i = B_i/(\sum_{j=1}^{4} B_j)$. W_i is the weight assigned to target i, while B_i and B_j are the benefits of visiting targets i and j, respectively. Each target i is assigned a W_i section of a roulette wheel and a random selection is made. After selection, the matrices are updated as in Heuristic I and the process continues until all feasible targets are exhausted for every UAV.

Heuristic II has two advantages over Heuristic I. Primarily, due to its stochastic nature, it allows for the solution to avoid getting stuck at a local minimum. Furthermore, it allows for multiple initial columns to be generated for Column Generation. The computational results shown in Section 5 indicate a significant advantage of using Heuristic II over Heuristic I.

4.4. Local search heuristic

A third heuristic based on local search is outlined in Table 10. The basis of this heuristic relies on the sensor benefit similarities between targets. Intuitively, targets that have similar sensor benefit requirements should be included on the same route for several reasons. Primarily, a sensor or small subset of sensors can provide similar surveillance for every target that is visited. Eq. (54) is used to determine the similarity, χ_{ab}, between a pair of targets

$$\chi_{ab} = \sum_{s} |R_{as} - R_{bs}| \tag{54}$$

Here, R_{as} and R_{bs} indicate the benefit obtained when the first and second members of the target pair are visited by sensor s, respectively. A value of zero indicates that the targets have identical benefit requirements. As the value increases, the sensor benefits of the target pair become dissimilar.

In Table 10, *Step* 0 initializes counters and establishes routes for each UAV considered in the problem. To establish these initial routes, the similarity rating is computed for each target pair. The pair of targets A,B that are most dissimilar (have the highest similarity rating) are selected. The process continues by selecting targets that are most dissimilar to previously selected targets, until the number of targets selected is equivalent to the number of UAVs in the mission. These targets will be the first added to each of the UAV routes, with the target farthest away from the base being assigned to the UAV with the greatest range. After the first target is assigned, additional targets are selected which are most similar in terms of sensor requirements to those existing on the route. The idea is to diversify the type of surveillance performed by each UAV route, while maximizing the surveillance each UAV performs with a subset of sensors. The latter is accomplished by assigning targets to routes that have low

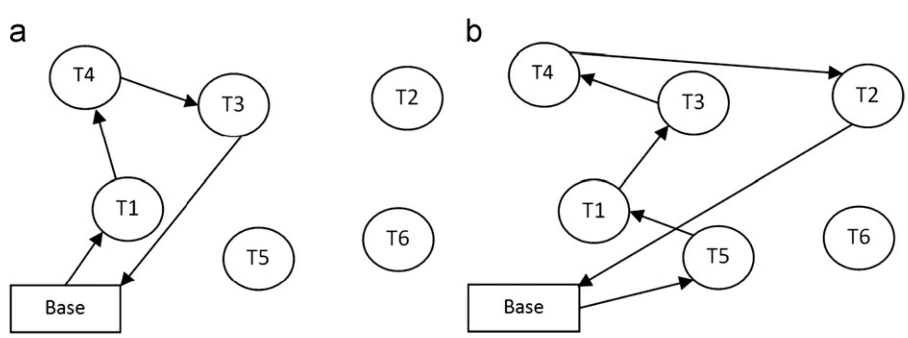

Fig. 4. Route assignments for two step heuristics. (a) Final route For UAV 1 and (b) final route For UAV 2.

Table 10
Local search heuristic.

Step 0	Set iteration counter to 0; Set no improvement iteration counter to 0
	Set iteration limit; Set no improvement iteration limit
	Create initial routes for each vehicle
Step 1	Remove targets from routes until sensors can feasibly be added for all UAVs and solve the sensor assignment problem
Step 2	Add most beneficial targets to UAV routes until no additional targets can feasibly be added
	Compare mission effectiveness with current best and update if necessary
	If an improved solution was not found, increase no improvement iteration counter by 1; Increase iteration counter
Step 3	If iteration counter or no improvement counter reach their respective limits, stop. Otherwise, go to **Step 1**

similarity scores $\chi_{a,b}$ with each other. The addition of targets to each route ceases when no targets can be added without exceeding the travel time of the UAV. Next, *Step*1 assigns sensors to each UAV by solving the Sensor Selection Model.

Sensor selection model:

$$\text{maximize} \quad \sum_{j,s,h} R_{js} z_{jsh} \tag{55}$$

$$\text{subject to} \quad \sum_{h} f_{hs} \le Q_s \quad \forall s \tag{56}$$

$$\sum_{h} z_{jsh} \le V_{js} \quad \forall j,s \tag{57}$$

$$z_{jsh} \le V_{js} f_{sh} \quad \forall j,s,h \tag{58}$$

$$z_{jsh} \le V_{js} Y_{jh} \quad \forall j,s,h \tag{59}$$

$$\sum_{s} f_{hs} \le \tau_h \quad \forall h \tag{60}$$

$$\sum_{s} C_s f_{hs} \le \lambda_h \quad \forall h \tag{61}$$

$$\sum_{s} f_{hs} \ge 1 \quad \forall h \tag{62}$$

$$f_{hs} \in \{0,1\} \quad \forall h,s \tag{63}$$

The formulation follows ISSRM, with the objective of maximizing surveillance benefit, but is only concerned with the selection of sensors for fixed routes determined from other steps in the heuristic. The input routes are represented by Y_{jh}, where a value of 1 indicates that UAV h visited target j and a value of 0 indicates no visitation took place. Additionally, Constraint (61) ensures that the sensor attachments do not violate the remaining travel time of a UAV.

If the remaining travel time of the UAV is not great enough to assign the lightest sensor, the model will return with infeasibility due to Constraint (62). Additionally, the remaining travel time directly limits the sensor(s) that can feasibly be assigned. A key part of *Step*1 involves removing targets to free up travel time to accommodate a variety sensor attachment opportunities. Prior to solving the model, a set of sensors is selected at random for each UAV. The size of this sensor set, X, for each UAV h follows $1 \le X_h \le \tau_h$. Targets are randomly removed from each route in an iterative manner until the randomly selected sensor set, X_h, can feasibly be assigned and the model is solved. The resulting sensor set assigned by the model for UAV h is defined as S_h.

After the model is solved and sensors are assigned to the UAVs, there is a reasonable possibility that additional travel time will remain and thus, opportunity for additional target insertions. *Step*2 evaluates Eq. (64) for each target not currently included in a given route

$$\frac{\sum_{s \in S_h} R_{js} + \sum_{s \notin S_h} \dfrac{R_{js}}{C_s}}{(Cost_{Insertion})(1+\sum_h I_{jh}) + \sum_{b \in r_{current}} \chi_{jb}} \tag{64}$$

Targets included in other routes are not omitted from consideration. The first term of the numerator evaluates how well target j benefits the mission with the current set of sensors, while the second term evaluates the target's benefit potential if included with additional sensors. The second term also considers the travel time hindrance incurred when a sensor is attached. The first term of the denominator considers the product of insertion cost and the number of times the considered target appears in the routes of other UAVs. This is denoted with variable I_{jh}, while a cheapest insertion policy with respect to the existing route is used to determine the insertion cost. The similarity of the entering target with existing targets in the current route, $r_{current}$, is evaluated in the second term. Looking at the numerator and denominator independently, it is clear that a desirable target would have a large numerator and small denominator. Therefore, targets with the highest overall value are considered first for entry.

Table 11
Summary of model and algorithm features.

Feature	ISSRM	CG	CG TPSSRH	Heuristic I, II	Local search
Fuel conservation	•				
Target time windows	•	•	•	•	
Surveillance duration	•	•			
Sensor travel reduction	•	•	•	•	•
UAV sensor capacity	•	•	•	•	•
UAV load capacity	•	•	•	•	•
Split delivery between sensors	•	•	•	•	•
Split delivery among sensors	•	•			

Targets are ranked for each route and sequentially added based on feasibility, until all feasible target entries are exhausted. After this step is completed for all routes, the mission effectiveness is evaluated, and compared to the best discovered thus far, which is updated accordingly. The heuristic updates the iteration counter and concludes if the maximum number of iterations has been reached, or a predefined number of iterations are subsequently performed with no solution improvement. Otherwise, the procedure returns to *Step* 1.

4.5. Summary of model and algorithm features

Each method presented thus far supports the overall problem slightly differently. Table 11 summarizes the features that each solution approach supports. For clarification, split delivery among sensors indicates that various UAVs can collect a portion of reward using the same sensor at a given target. For example, if a target benefited from 20 min of video collection, two UAVs may each collect 10 min. Split delivery between sensors specifies that the multiple UAVs can collect benefit at the same target using different sensors.

5. Computational experience

5.1. Experimental conditions

The computational experiments were carried out using software developed in Java. CPLEX 12.2 was used to solve the ISSRM, the sensor selection phase of the local search heuristic, and the master problem for the Column Generation heuristic. All testing was carried out on a PC with a 3.00 GHz Intel Core 2 Duo processor and 4 GB of RAM. Since the authors were unaware of any existing instances of this problem, 14 scenarios were developed. Ten replications of each scenario were run, for a total of 140 test cases. Table 12 summarizes the test cases.

Target locations were uniformly distributed within the target field and three sensors were considered for all test cases. The benefit assigned for each sensor/target pair was generated independently as follows. The probability that a target had benefit from the first sensor was 1, while the probabilities for sensor 2 and sensor 3 were 0.8 and 0.64, respectively. The actual benefit value was assigned using the distribution, benefit range 1–2 with probability (w.p.) 0.1, 3–5 w.p. 0.1, 6–8 w.p. 0.3, and 9–12 w.p. 0.5. Demand values, V_{js}, were set to 1 for all test cases.

Table 12
Summary of test cases.

Number of targets	Fleet size	Dimensions of target field
15	2	100×100
30	2,3,4	100×100
50	3,4,5,6	150×150
100	3,4,5,6,7,8	200×200

Table 13
Results for 15 targets and two UAVs.

Instance #	ISSRM 120 min	Heuristic I	Heuristic II	Local search	CG	Percent increase (%)
1	113	69	74	113	113	0.0
2	97	65	74	83	97	16.9
3	148	118	132	138	145	5.1
4	116	95	104	104	116	11.5
5	130	105	127	101	130	2.4
6	120	92	131	51	142	8.4
7	93	58	88	84	88	0.0
8	74	74	82	82	82	0.0
9	114	90	103	93	119	15.5
10	104	105	110	115	115	0.0

The unloaded range of each UAV in the fleet is dependent on the number of UAVs considered in the problem. The first UAV had an unloaded range of 250 units and each subsequent UAV inclusion added an additional 50 units. The distance reduction imposed for sensor 1, sensor 2, and sensor 3 was 125, 40, and 100, respectively. Column Generation was run for a maximum of 1000 iterations in all test cases. While we generated and tested problems with time windows, they tend to make the problem easier to solve. For this reason, the numerical tests do not include time windows as our principle interest is in determining the largest problem instances which can be solved in a worst-case computational time setting.

5.2. Small and medium sized problems

CPLEX was given 2 h to solve the 10 test instances presented in Table 13. These CPLEX results are reported in the ISSRM column. For all instances, CPLEX was set to emphasize feasibility and the memory emphasis setting was enabled. The next four columns are for the solution methods, and the last column is the level of improvement of CG over the starting heuristic solution.

The solution times for Heuristic I, Heuristic II, and Local Search were essentially instantaneous and Column Generation solutions were obtained within 50 s for all cases. The maximum number of iterations for Local Search was set to 50,000 and terminated when no improvement was found in 1000 subsequent iterations. Local Search and Heuristic II independently found the best solution in 40% of the cases. In the remaining 20% of the cases, Local Search and Heuristic II both found the best solution. Heuristic I was outperformed by either Heuristic II or Local Search in each test case. Column Generation was started with the heuristic solution that provided the best result. In the event of a tie, both solutions were used as initial columns. In 60% of the cases, Column Generation improved the solution of the other heuristic solutions. Of these cases, the average improvement in solution quality was 9.97%. The use of Column Generation is justified as it significantly improved the solution in several instances with minimal increase in solution time.

The results for moderately sized test cases containing 30 targets are shown in Tables 14–16. A comparison is made between the ISSRM's progress after predefined time intervals, each of the three heuristics, and Column Generation. For instance 33, CPLEX was not able to find either a feasible solution within 120 min and is reported in the table as NS. Local Search was the most favorable heuristic solution and was improved by Column Generation in many of the test instances.

Table 14
Results for 30 targets and two UAVs.

Instance #	ISSRM 30 min	ISSRM 60 min	ISSRM 120 min	Heuristic I	Heuristic II	Local search	CG
11	160	160	160	127	161	215	215
12	108	114	118	124	142	192	196
13	137	137	137	129	153	188	188
14	156	161	165	149	157	199	211
15	109	118	118	100	103	151	151
16	91	91	91	113	124	145	145
17	209	209	209	173	174	225	225
18	101	101	101	86	91	144	144
19	154	154	154	102	109	184	184
20	139	139	139	123	136	169	176

Table 15
Results for 30 targets and three UAVs.

Instance #	ISSRM 30 min	ISSRM 60 min	ISSRM 120 min	Heuristic I	Heuristic II	Local search	CG
21	207	211	211	127	160	255	258
22	231	247	247	183	219	264	283
23	139	150	150	140	163	240	241
24	236	236	236	151	176	232	245
25	226	226	226	186	206	262	301
26	166	178	188	140	163	237	250
27	204	204	215	164	177	269	269
28	174	174	174	143	164	216	216
29	256	256	256	194	213	295	300
30	264	264	284	175	192	311	323

Table 16
Results for 30 targets and 4 UAVs.

Instance #	ISSRM 30 min	ISSRM 60 min	ISSRM 120 min	Heuristic I	Heuristic II	Local search	CG
31	231	248	268	229	250	369	395
32	279	307	315	250	294	361	377
33	NS	NS	NS	211	251	333	351
34	278	293	313	263	305	384	393
35	226	269	282	231	271	348	367
36	225	281	292	273	275	376	416
37	268	270	299	252	262	373	413
38	216	216	221	212	252	321	327
39	288	288	299	283	317	432	432
40	266	303	314	252	273	350	358

The results presented in Tables 14–16 represent moderately sized problems. CPLEX was given 120 min to solve the ISSRM and was outperformed by a heuristic solution in all 30 runs. Local Search solved quickly with solution times ranging from 70.8 to 356.2 s. The average solution time for Local Search was 130.3 s. Column Generation solutions were obtained within 18 min for all runs. The average time was 7 min and 19 s. In 20 of the 30 instances Column Generation found an improved solution. Of the cases where an improved solution was found, the minimum and maximum improvement was 0.4% and 14.9%, respectively. The average improvement was 5.4%. When a solution is needed quickly for problems of this size, it is evident that the three heuristics augmented by Column Generation provide results that are more favorable than those produced by directly solving the ISSRM.

Table 17
Results for large problems.

Target quantity	Fleet size	Heuristic I	Heuristic II	Local search	CG	% Improvement Min	Avg	Max
50	3	125	140.3	209.6	214.8	0.0	2.4	11.7
50	4	198.7	231.4	296.4	319.1	0.0	8.5	30.5
50	5	253.6	297.2	384.8	417.6	2.4	8.7	14.6
50	6	305.6	366.1	453.2	510.2	6.8	12.6	21.1
100	3	152.7	173.8	289.1	303.7	0.0	3.2	14.0
100	4	236.1	266.5	327.1	344.3	0.3	5.9	18.0
100	5	279.4	311.7	410.2	426.6	0.0	4.1	9.9
100	6	395.6	434.8	551.6	585.8	1.5	6.2	23.3
100	7	436.5	481.4	650.5	687.2	1.9	5.7	10.9
100	8	548.1	583.8	724.7	779.5	1.5	8.1	15.2

5.3. Large size problems

The results for larger problems are displayed in Table 17. Average results for 10 replications of each scenario are provided. A 10 min time limit was imposed for solving the final integer master problem, as it was unable to converge within reasonable time for several of the test instances.

Local Search was the superior algorithm for each test instance, and Heuristic II once again consistently outperformed Heuristic I. Column Generation was able to improve the initial solution in 86 out of the 100 test cases. The right most column in Table 17 represents the average overall percent improvement and includes the 14 cases where CG did not improve the solution. For the 86 cases where Column Generation did improve the solution, the average improvement was 7.6%, with a maximum improvement of 30.5% and a minimum improvement of 0.3%. Once again, the use of Column Generation is justified as it provides noticeable improvement while adding relatively little to the overall computation time.

5.4. Other applications

Fleet sizing: As evident from Sections 5.2 and 5.3, the three heuristics augmented by Column Generation quickly provide a good solution to the sensor selection and routing problem. This behavior allows the solution approach to be used as a fleet sizing tool. A mission planner may be unsure as to how many UAVs he or she should allocate to a mission. Alternative solutions for a mission could be obtained quickly for a varying number of UAVs. The mission planner could then evaluate the surveillance benefit for each of the alternatives and make a decision. A 15 target case with three sensors is presented for the sake of illustration. The case was generated using the approach described in Section 5.1, with unloaded range of each UAV set to 350 units.

As seen in Fig. 5, the surveillance benefit jumps significantly when the fleet size increases from one to two UAVs. As the fleet size rises from 2 to 6, the surveillance benefit experiences a linear increase. The addition of 7 or 8 UAVs has a marginal impact on surveillance benefit.

Combat missions: The problem and solution approaches presented in this paper are extendable to combat missions where the UAV can be replaced with an Unmanned Combat Aerial Vehicle (UCAV), similar to [16]. The targets could have effectiveness by certain type of weaponry (e.g., conventional undirected munition, or directed Joint Direct Attack Munition (JDAM), and Laser Guided Bombs), and the mission planning problem is to decide the allocation of munitions to UCAVs and routing them to maximize combat mission effectiveness.

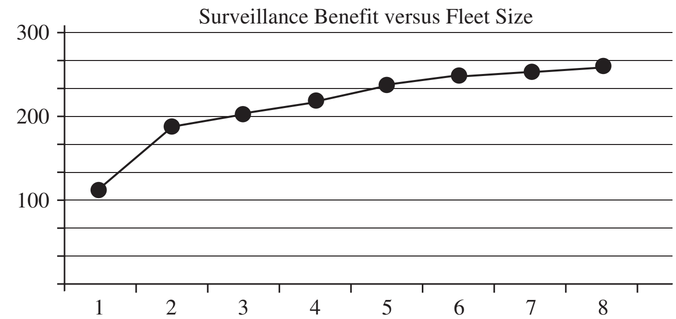

Fig. 5. Fleet sizing example.

6. Conclusions and future work

The sensor selection and routing of unmanned aerial vehicles can be modeled as a generalization of the team orienteering problem. While the ISSRM developed in this work solves well using CPLEX for small problems, larger problems required the use of heuristics augmented with Column Generation. All three heuristics solved quickly, with local search and Heuristic II consistently outperforming Heuristic I. For small problems, Local Search and Heuristic II shared the ability to provide the best solution. As the problem size grows, Local Search consistently provides superior results. Column Generation improved the solution in many of the test cases, but also did not significantly increase the overall solution time. Also, since the solution approach quickly finds good solutions, it provides a foundation for applications beyond sensor and route selection. This work demonstrated the solution approach functioning as a fleet sizing tool for a single mission. This idea can be extended to multiple missions and will be briefly discussed.

Consider the role of a mission planner with a set of target clusters requiring surveillance. The target clusters are not in close proximity, so each will require a separate mission plan. If the clusters were close to one another, only one mission plan would be required which could be obtained using the procedures presented in this work. Also, it is assumed that the surveillance of each target cluster is simultaneous, so UAVs and sensors may not be shared among mission plans. Thus, the resources allocated to each target cluster will play a significant role in the success across all missions. Since the solution procedure presented solves quickly, multiple allocation scenarios could be evaluated in a short amount of time. Furthermore, the solution procedure could be implemented to optimize UAV and sensor allocations across multiple simultaneous missions.

Lastly, the ISSRM could not prove optimality for relatively small test cases. The authors are currently investigating a branch and cut approach as a more effective method to prove optimality for simultaneous sensor selection and routing problems.

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