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# Generative Adversarial Networks

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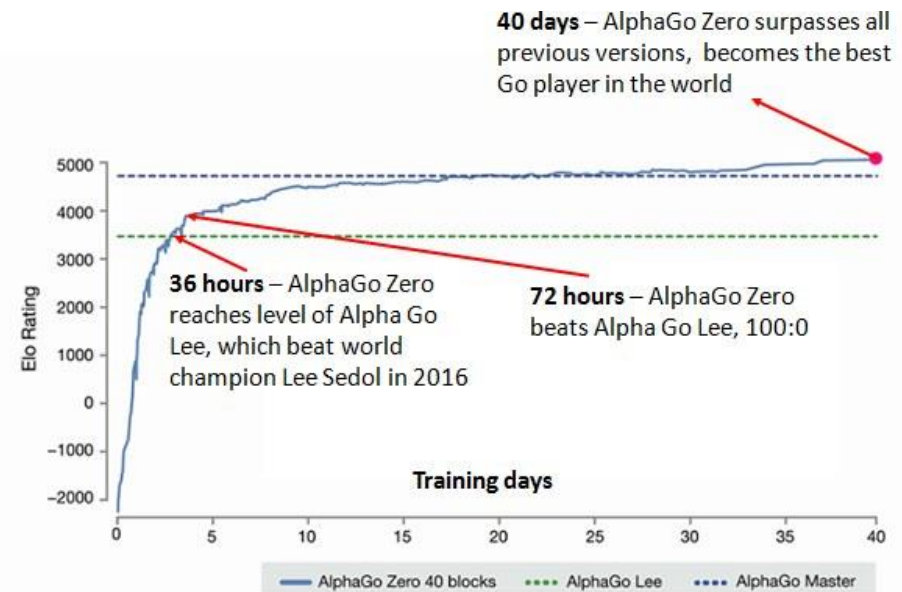
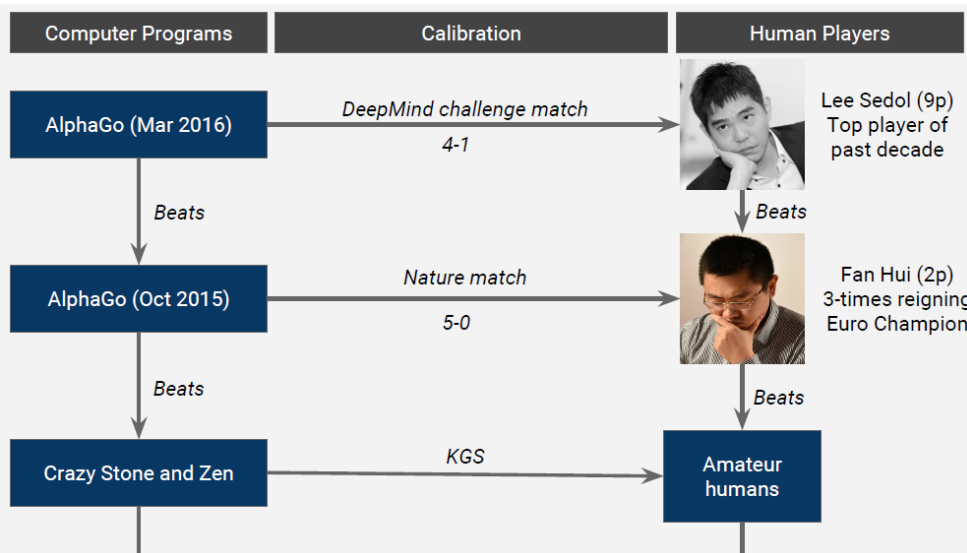
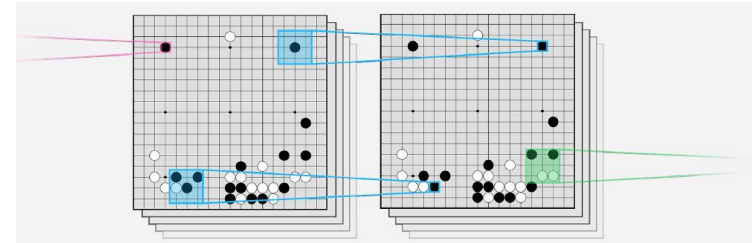
# Generative Adversarial Network (GAN)

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- **Adversarial Learning**
- **GAN Architecture**
- **DCGAN**
- **GAN Applications**
- **The Fundamentals of GAN**
- **FAT-GAN**

# Adversarial Learning

- **AlphaGo**
  - Convolutional Neural Network (CNN)
- **AlphaGo Zero**
  - Adversarial Network



# Generative Adversarial Network (GAN)

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- **Generative Adversarial Networks (GAN)**

- Introduced by Ian Goodfellow et al. in 2014
- Deep neural network architectures comprised of two nets
  - A Generator
  - A Discriminator
- Both nets are trying to optimize a different and opposing loss function in a zero-sum game

- **Potential of GAN**

- Can be trained to mimic any distribution of data
- Create worlds eerily similar to our own in any domain

“The most interesting idea in the last 10 years in machine learning” – Yann LeCun

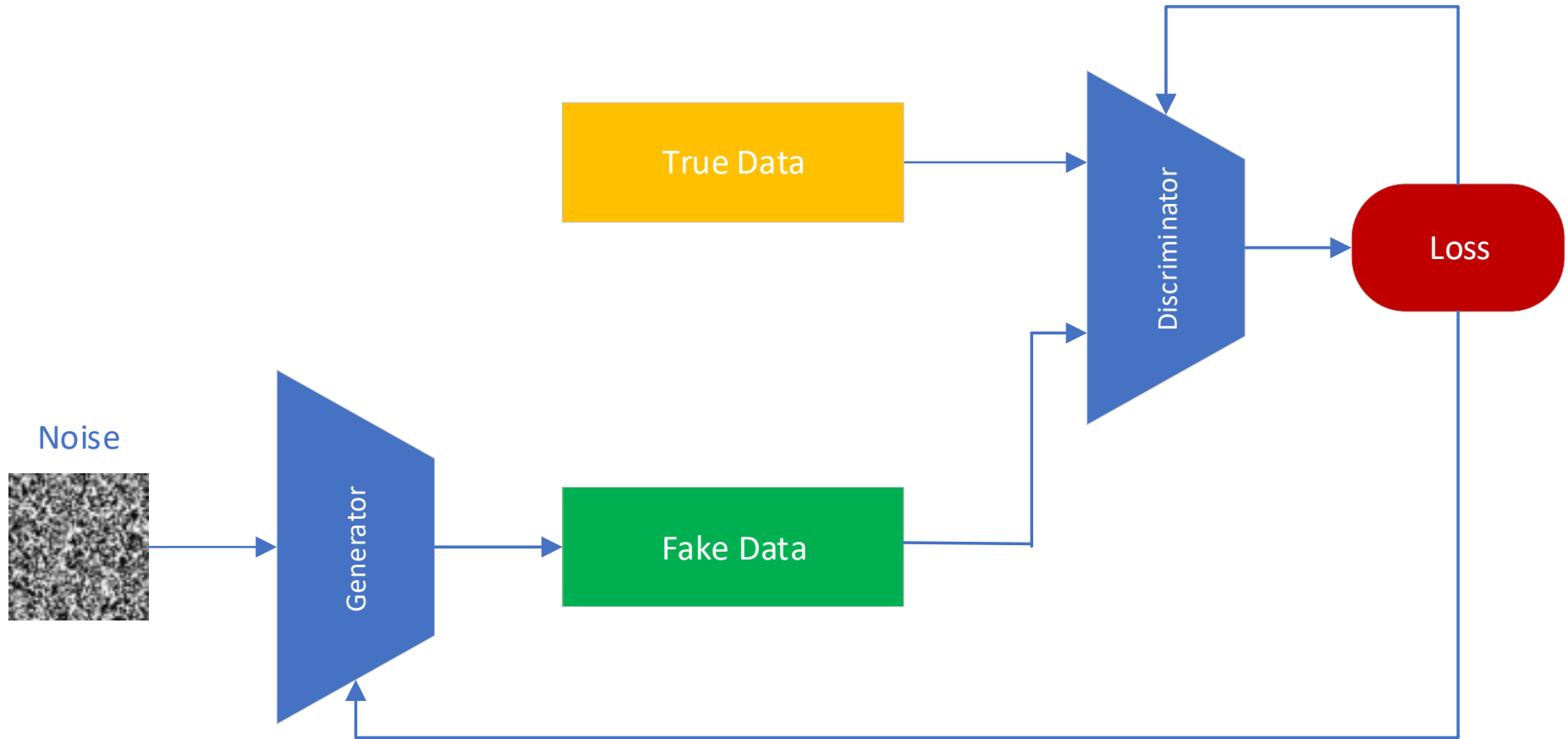
# Generative Adversarial Network (GAN)

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# General Architecture of GAN

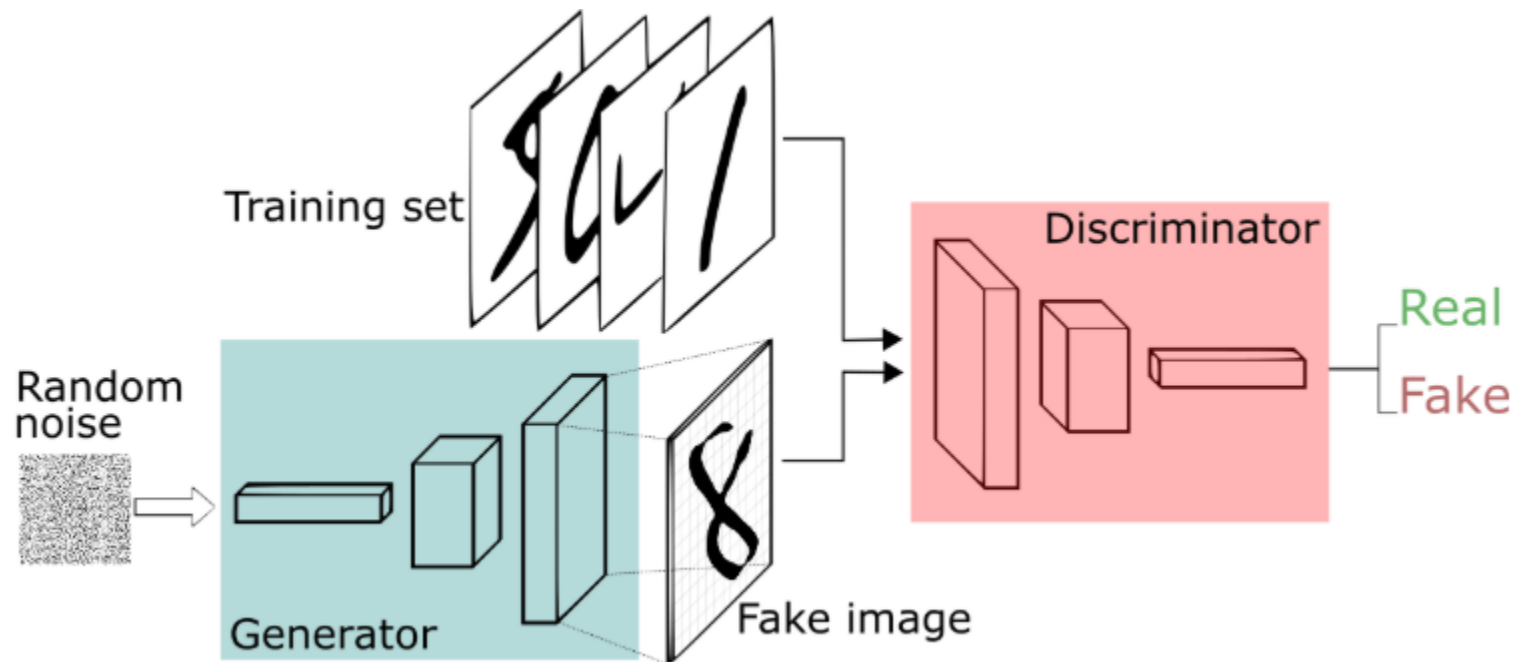
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# A GAN Example

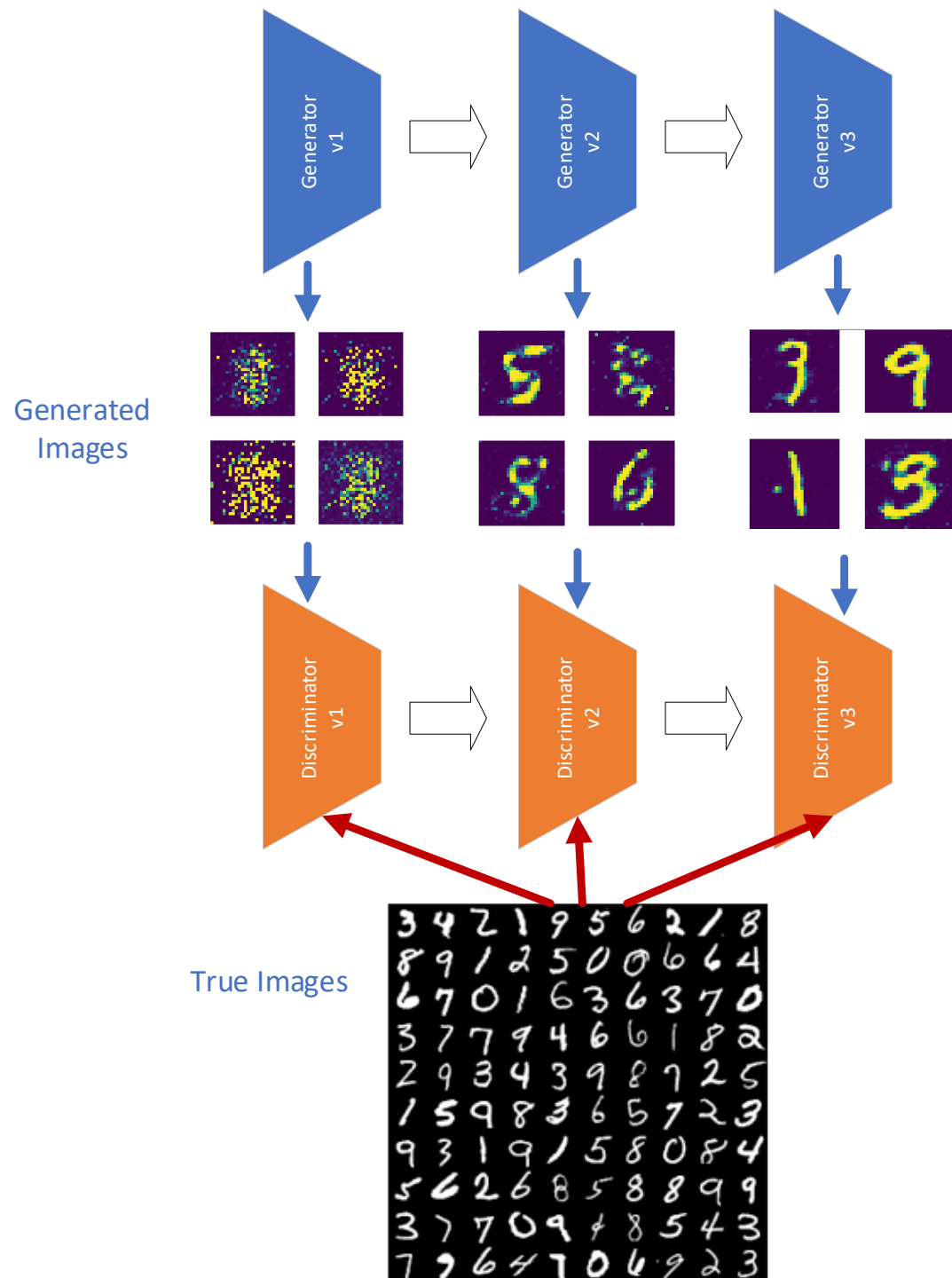
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- Generating Hand-writing Digits



# Step by Step GAN

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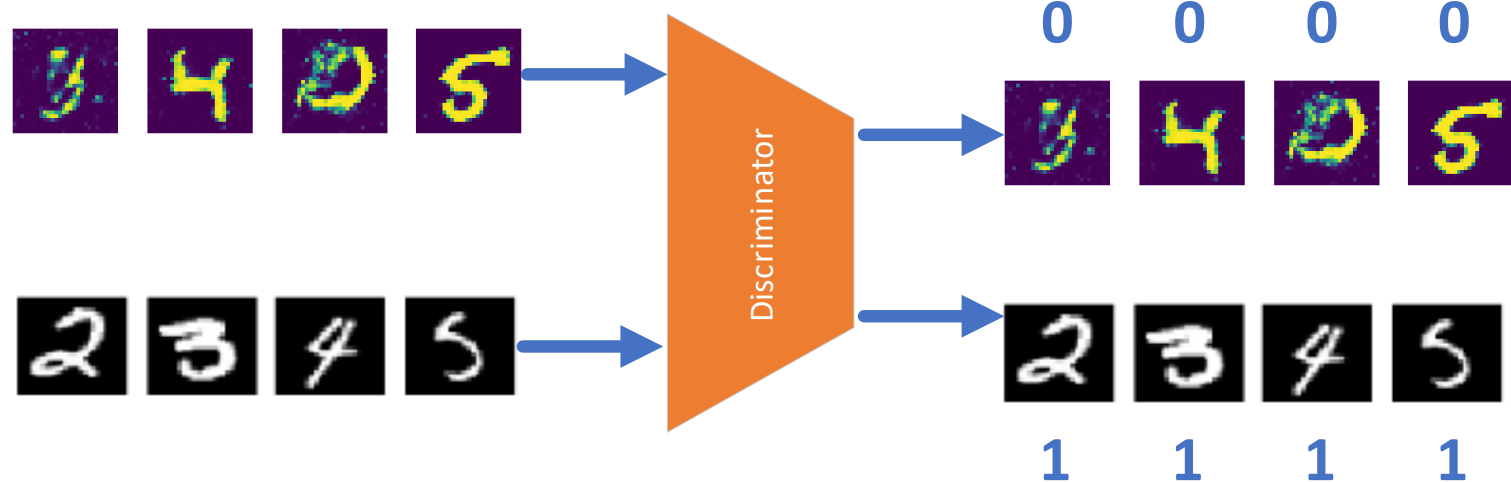




# Training a Discriminator

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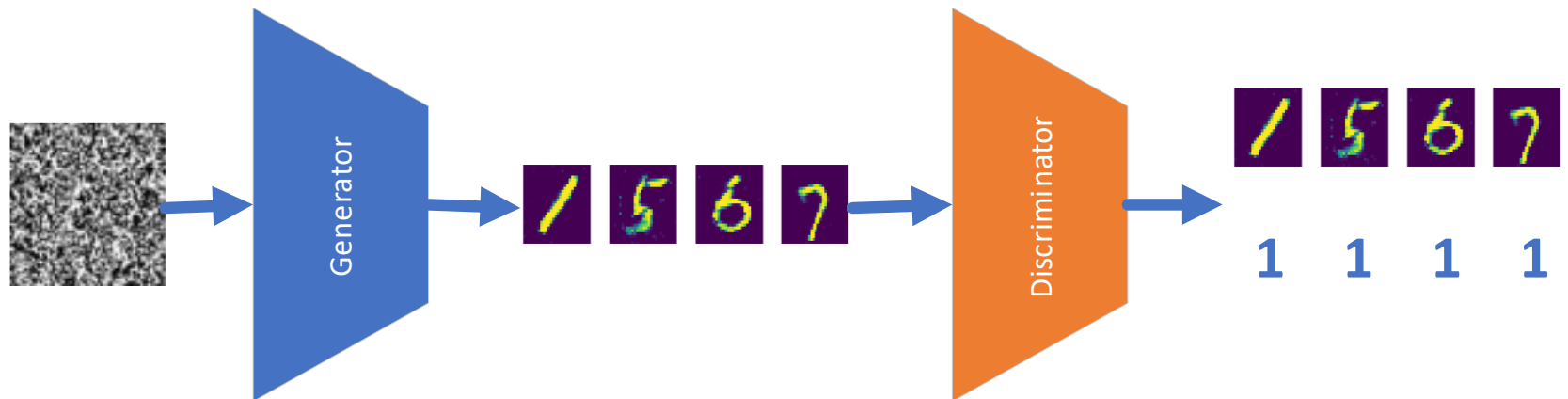
- Binary Discriminator



# Training the Generator

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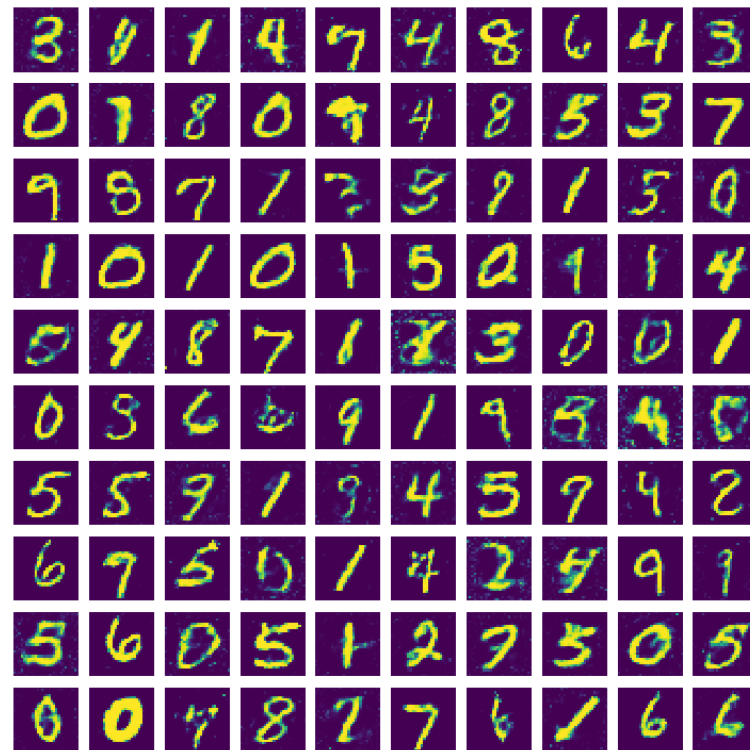
- Try to Fool the Discriminator



## Demo: MNIST GAN

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- Jupyter notebook `GAN_MNIST.ipynb`



# Generative Adversarial Network (GAN)

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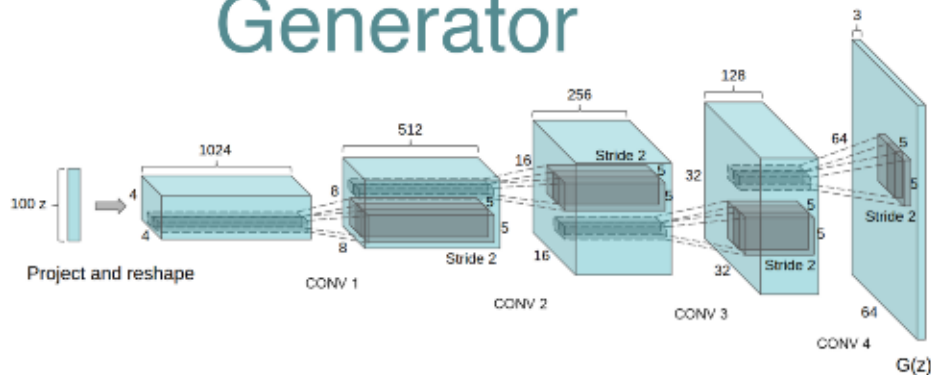
- Adversarial Learning
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# Deep Convolutional GAN (DCGAN)

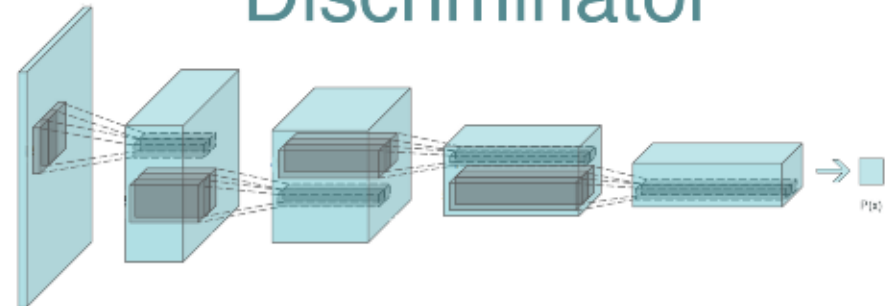
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- Improve the sensitivity of Discriminator
- Improve the approximation of Generator

## Generator



## Discriminator



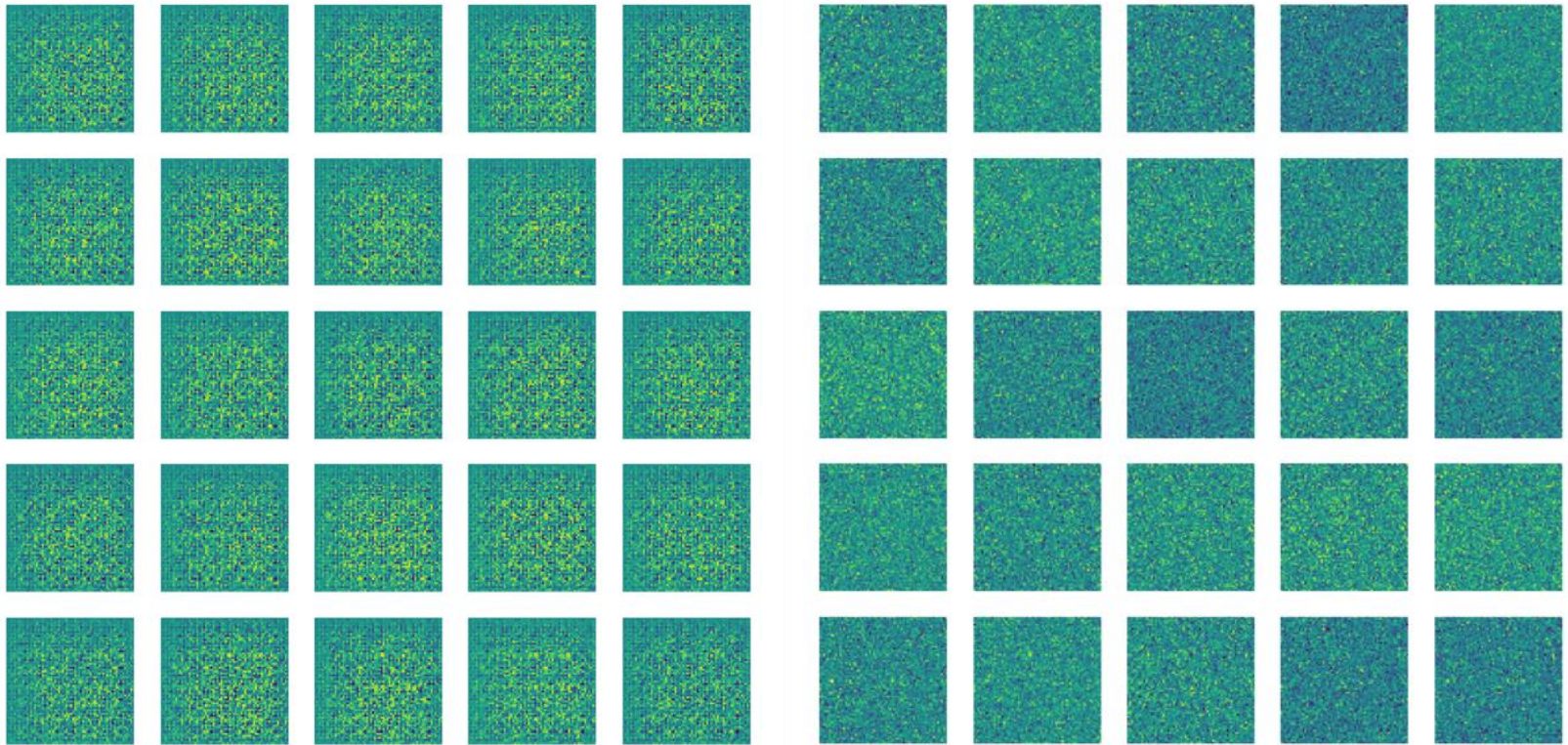
# DCGAN Architecture

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- **Convolution Layer in both Discriminator and Generator**
  - Strided Convolution (Discriminator)
  - Fractional-strided Convolution (Generator)
- **Batch Normalization**
- **Remove Fully-Connected Hidden Layers for Deeper Architecture**
- **ReLU in Generator except for output (Tanh)**
- **LeakyReLU for Discriminator**

# DCGAN on MNIST

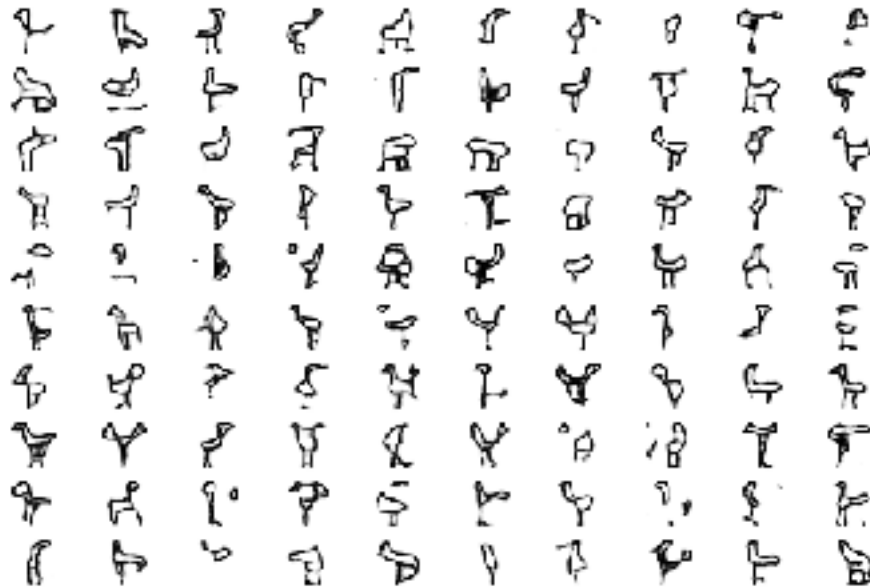
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# Demo: Using GAN to draw your own flamingo

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Jupyter notebook [GAN\\_flamingo.ipynb](#)





# Generative Adversarial Network (GAN)

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- Adversarial Learning
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- **GAN Applications**
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# The Power of GAN

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- **Can be trained to mimic any distribution of data**
- **Applications**
  - Artificial Arts
  - Virtual Reality
  - New Characters
  - Artificial Music

# Generating Virtual Arts

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# AI Arts

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- AI arts sold for \$432,500 in Christie's

CHRISTIE'S

AUCTIONS

PRIVATE SALES

LOCATIONS

DEPARTMENTS

STORIES

SERVICES



Is artificial intelligence set to become art's next medium?

12 December 2018  
PHOTOGRAPHS & PRINTS |

AI artwork sells for \$432,500 — nearly 45 times its high estimate — as Christie's becomes the first auction house to offer

# Virtual Reality

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**Source: Metz and Collins, 2018**

## 4.5 Years of GAN Progress on Face Generation

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 **Ian Goodfellow**  
@goodfellow\_ian

4.5 years of GAN progress on face generation.  
[arxiv.org/abs/1406.2661](https://arxiv.org/abs/1406.2661) [arxiv.org/abs/1511.06434](https://arxiv.org/abs/1511.06434)  
[arxiv.org/abs/1606.07536](https://arxiv.org/abs/1606.07536) [arxiv.org/abs/1710.10196](https://arxiv.org/abs/1710.10196)  
[arxiv.org/abs/1812.04948](https://arxiv.org/abs/1812.04948)

♡ 3,754 8:40 PM - Jan 14, 2019





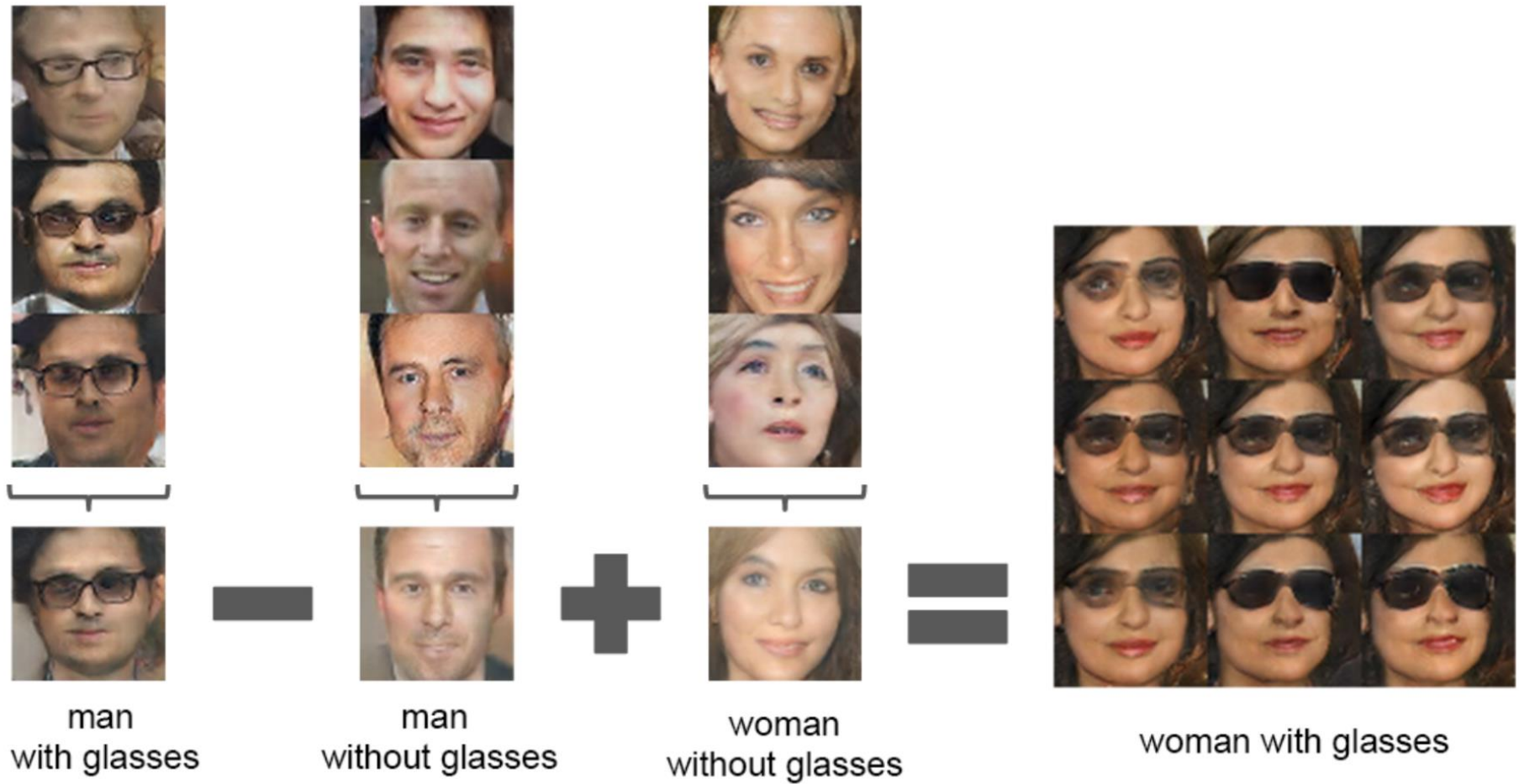
# GAN Face Generation Today

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# Latent Vectors Generated by GAN

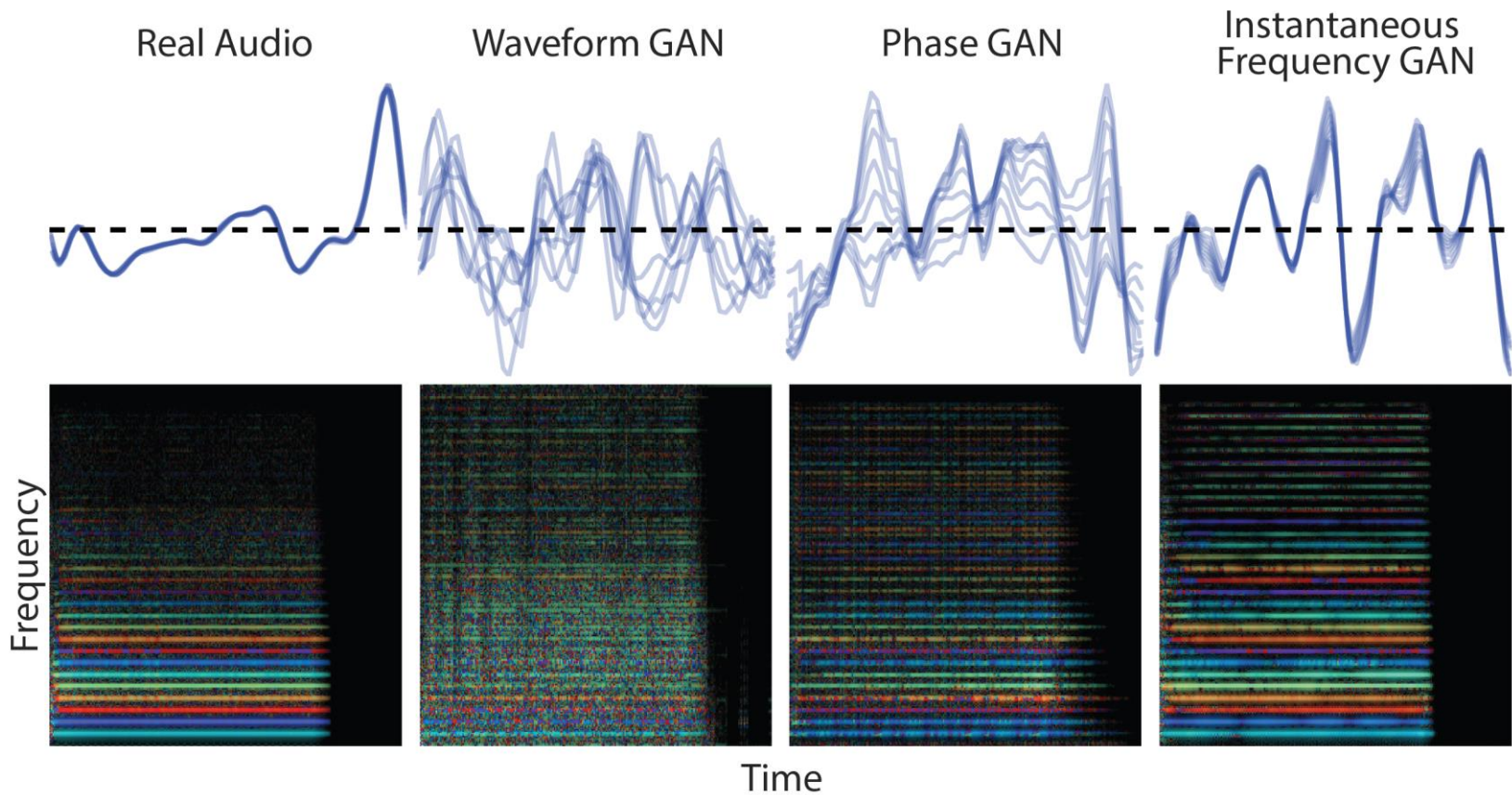
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# GAN Music

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# Generation of Poems

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I snatched a letter from my  
mother's hand  
And ran into the hope of  
turning back.  
She adored me

Ours learned the wisdom of  
ancient groups  
Of ancient kings.  
I remember the wild useless war

Your help has given  
countless people around the  
world a voice - the one thing  
they need most.

Oh! horrid Night! Melody release  
thee from my aching Heart, And  
Fate copied on my Mind the  
wandering Misfortunes?

Home, home!  
Home, sweet home!  
Home, sweet!  
Home! home! home! home!

# Generation of Cartoon Figures

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Source of images: <https://zhuanlan.zhihu.com/p/24767059>  
From Dr. HY Lee's notes.

## Natural Language to Images

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### PPGN for caption to image



oranges on a table next to a liquor bottle

(Nguyen et al 2016)



# Image Beautification

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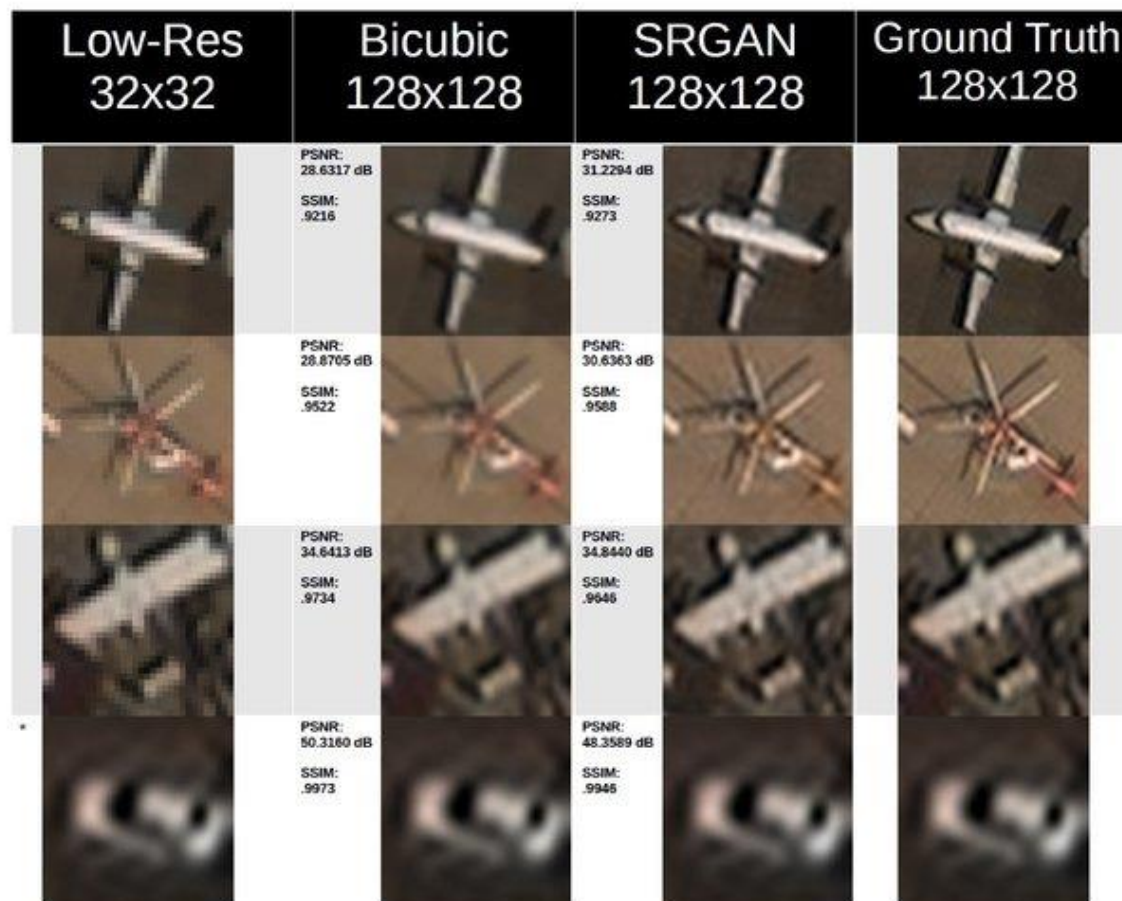
# Image Super-Resolution

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- Convert Low Resolution Images into High Resolution Images



# Image Super-Resolution in Military Applications



\* Vehicle low res is 64x64 and upsampled by 2.

# Generative Adversarial Network (GAN)

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# Fundamentals of GAN

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- **Generator G**

- A Function: Input  $z$ , Output  $x$
- Given a prior distribution  $P_{\text{prior}}(z)$ , a probability distribution  $P_G(x)$  is defined by function  $G$

- **Discriminator D**

- A Function: Input  $x$ , Output a scalar
- Evaluate the difference between  $P_G(x)$  and  $P_{\text{data}}(x)$

# Kullback–Leibler Divergence

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- **Kullback–Leibler divergence (Relative Entropy)**

- measures how one probability distribution is different from a reference probability distribution
- Given probability distributions  $P$  and  $Q$

- Discrete version

$$D_{\text{KL}}(P||Q) = - \sum_x P(x) \log \left( \frac{Q(x)}{P(x)} \right)$$

- Continuous version

$$D_{\text{KL}}(P||Q) = - \int P(x) \log \left( \frac{Q(x)}{P(x)} \right) dx$$

# Properties of Kullback–Leibler Divergence

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- Explanation of KL divergence

**Cross Entropy of  $P$  and  $Q$**

$$D_{\text{KL}}(P||Q) = - \sum_x P(x) \log \left( \frac{Q(x)}{P(x)} \right)$$
$$= - \sum_x P(x) \log Q(x) - \left( - \sum_x P(x) \log P(x) \right)$$

**Entropy of  $P$**

- Properties of KL divergence

- Non-symmetric
- Non-negative

# Jensen-Shannon Divergence

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- **Jensen-Shannon Divergence**

- Measures the similarity between two probability distributions
- A symmetrized and smoothed version of the Kullback–Leibler divergence
- Definition

$$JSD(P||Q) = \frac{1}{2}D_{\text{KL}}(P||M) + \frac{1}{2}D_{\text{KL}}(M||Q)$$

where

$$M = \frac{1}{2}(P + Q)$$

- Bounds

$$0 \leq JSD(P||Q) \leq \log(2)$$

# GAN Cost Function

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- **An optimization problem**

- Find an optimal generator  $G^*$  such that

$$G^* = \arg \min_G \max_D V(G, D)$$

- A minimax algorithm

- **Jensen-Shannon Divergence (Information Radius)**

- $V = E_{x \sim P_{\text{data}}} [\log D(x)] + E_{x \sim P_G} [\log(1-D(x))]$

- Measures the similarity between two distributions

$$\max_D V(G, D)$$

---

- $\max_D V(G, D)$ 
  - Given a generator  $G$
  - $\max_D V(G, D)$  evaluates the “difference” between  $P_G$  and  $P_{data}$
- **What is the optimal  $D^*$  that maximize  $V(G, D)$ ?**

$$\begin{aligned} V &= E_{x \sim P_{data}} [\log D(x)] + E_{x \sim P_G} [\log(1 - D(x))] \\ &= \sum_x P_{data}(x) \log D(x) + \sum_x P_G(x) \log(1 - D(x)) \end{aligned}$$

**Then**

$$D^* = P_{data}(x) / (P_{data}(x) + P_G(x))$$

$$\min_G \max_D V(G, D)$$


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$$\begin{aligned} & \max_D V(G, D) \\ &= V(G, D^*) \quad \text{where } D^* = P_{data}(x)/(P_{data}(x) + P_G(x)) \end{aligned}$$

$$\begin{aligned} &= E_{x \sim P_{data}} [\log D^*(x)] + E_{x \sim P_G} [\log(1 - D^*(x))] \\ &= \sum_x P_{data}(x) \log D^*(x) + \sum_x P_G(x) \log(1 - D^*(x)) \\ &= -2 \log 2 + 2 JSD(P_{data} || P_G) \end{aligned}$$

**What is  $G^*$  with  $\min_G \max_D V(G, D)$ ?**

$$JSD(P_{data} || P_G) = 0$$

**i.e.,  $P_{data} = P_G$**

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# Problems in Training a GAN

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- **Training a GAN is notoriously difficult**
  - Perfect Discriminator
  - Mode Collapse
  - Non-convergence
  - Imbalance Generator and Discriminator Training
  - Model parameter oscillation
  - Destabilization
  - Vanishing gradient



# Training GAN in Practice

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## ■ Discriminator (Binary Classifier)

- Given G
  - Sample  $x_{\text{true}}$  from  $P_{\text{data}}$
  - Sample  $x_G$  from generator  $P_G$
- Maximize

$$\sum_{x_{\text{true}}} \log D(x_{\text{true}}) + \sum_{x_G} \log(1 - D(x_G))$$

**If x is true**

**If x is fake**

- Minimize cross-entropy
  - Positive sample: minimize  $-\log D(x)$
  - Negative sample: minimize  $-\log(1 - D(x))$

# Too Perfect Discriminator

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## ■ Generator

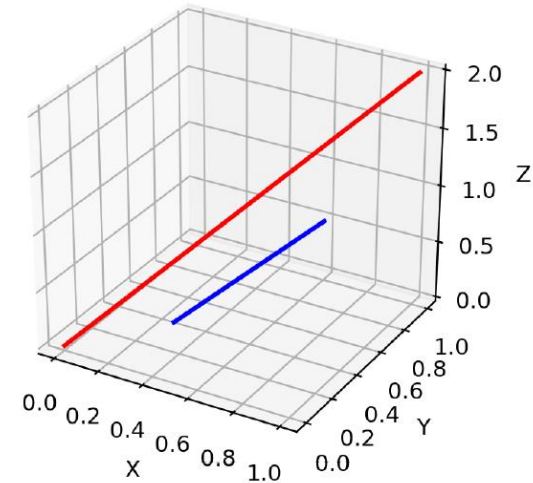
– Given D

■ Minimize

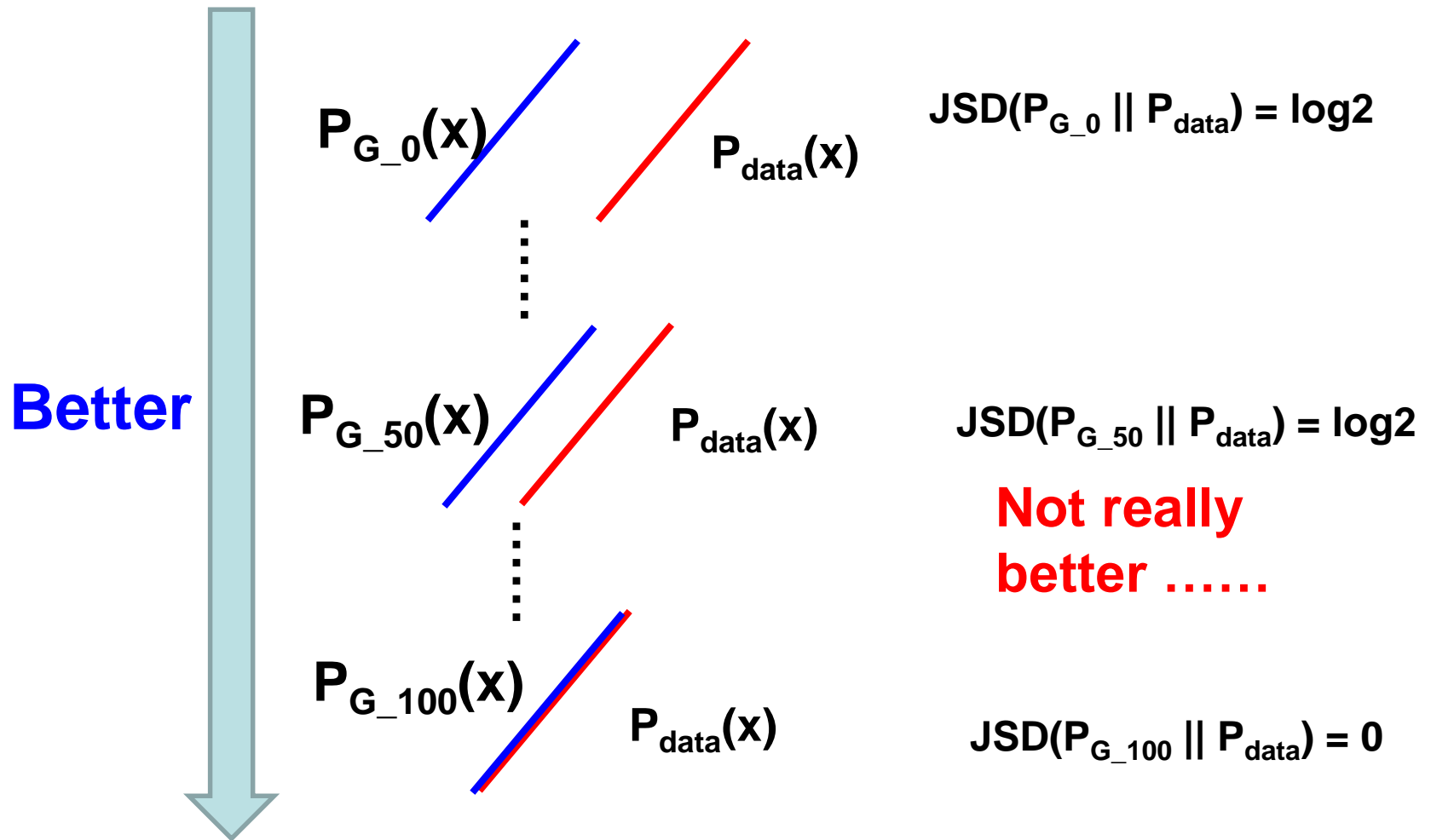
$$\cancel{\sum_{x_{true}} \log D(x_{true})} + \sum_{x_G} \log(1 - D(x_G))$$

## ■ Problem

- A nearly perfect discriminator
- No guideline to generate  $x_G$  close to  $x_{true}$
- Low dimensional manifolds in high-dimensional space barely have overlaps

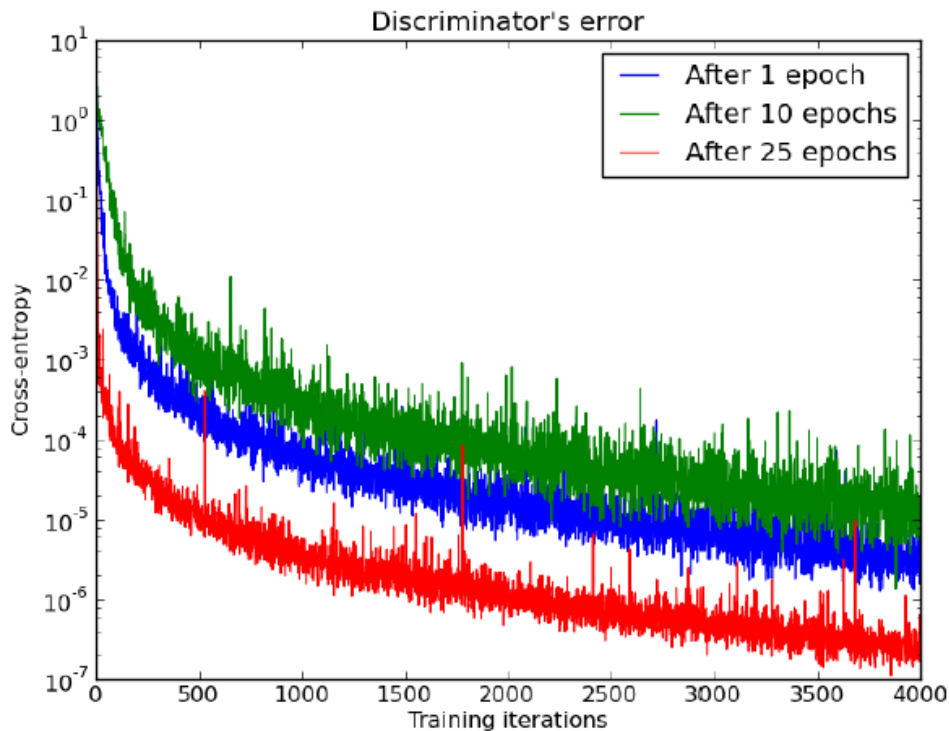


## Too Perfect Discriminator

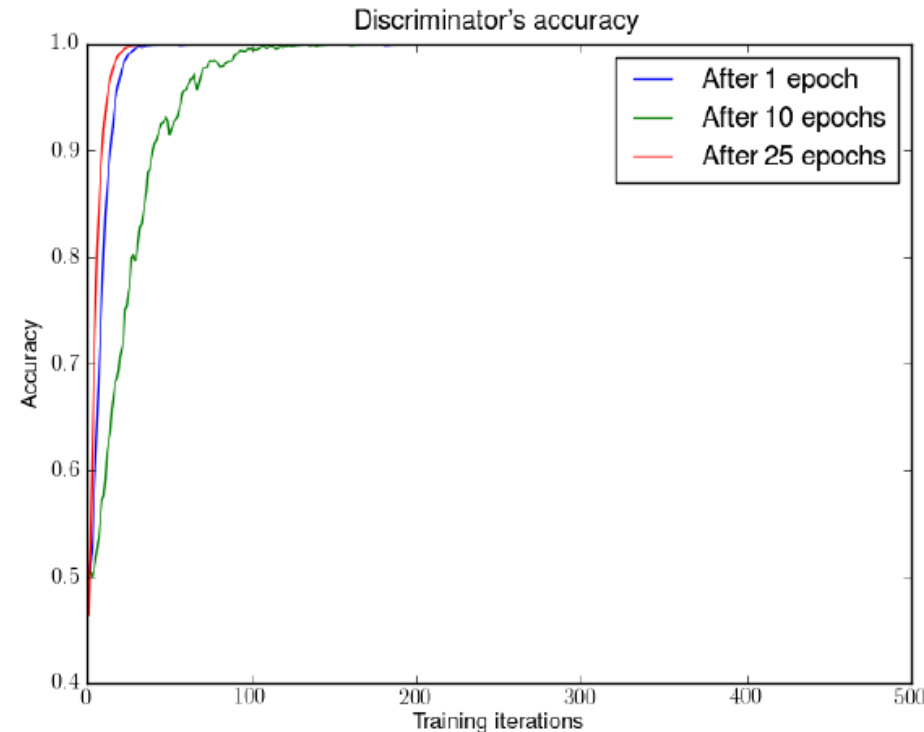


# Discriminator too perfect

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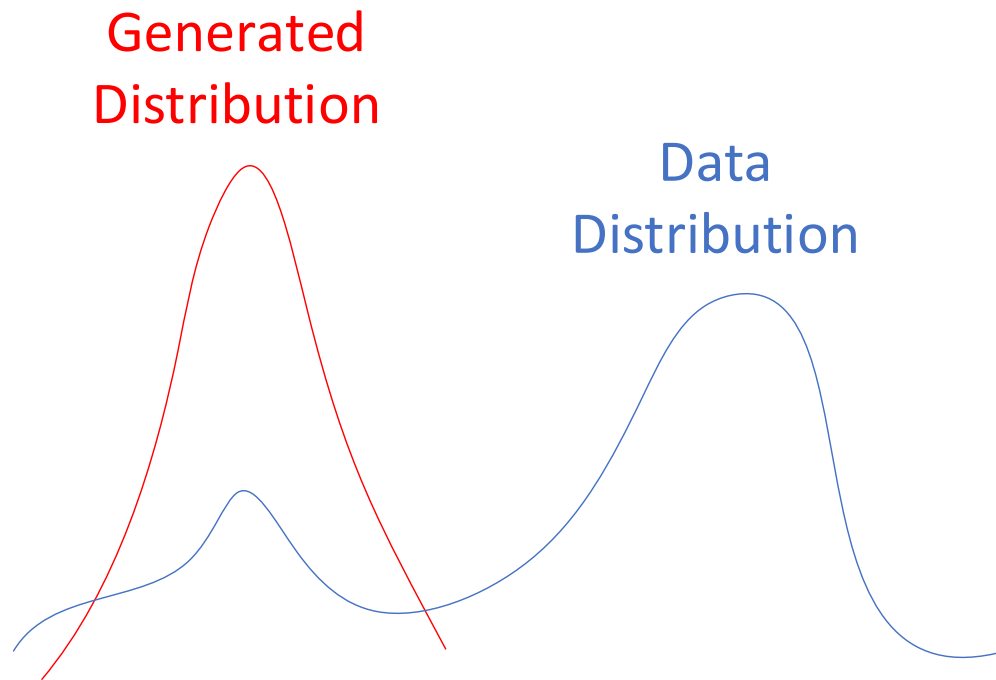
**Discriminator is too strong: for all three Generators, JSD = 0**



# Mode Collapse

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- Generating the same sample



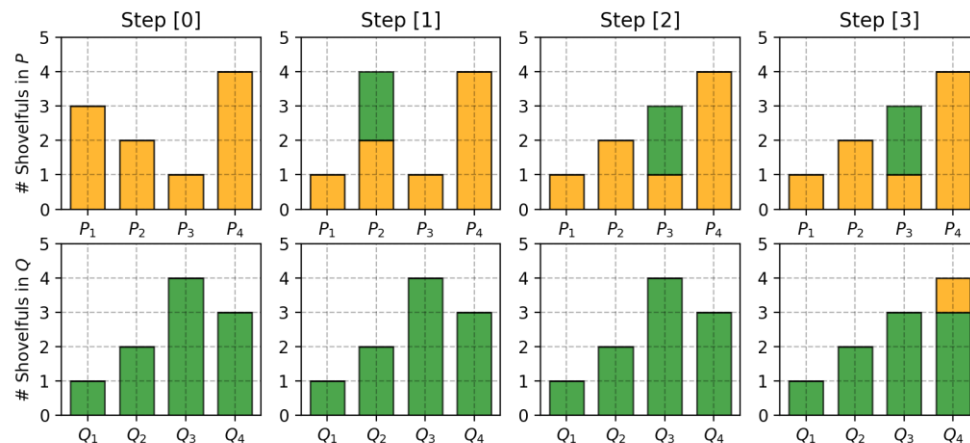
# Wasserstein Distance

## ■ Wasserstein Distance (Earth Mover's Distance)

- Measure of the distance between two probability distributions
- Interpreted as the minimum energy cost of moving and transforming a pile of dirt in the shape of one probability distribution to that of the other
- Toy Example: Matching Distributions P and Q

$P_1=3, P_2=2, P_3=1, P_4=4$

$Q_1=1, Q_2=2, Q_3=4, Q_4=3$



# Wasserstein GAN

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- **Discriminator**

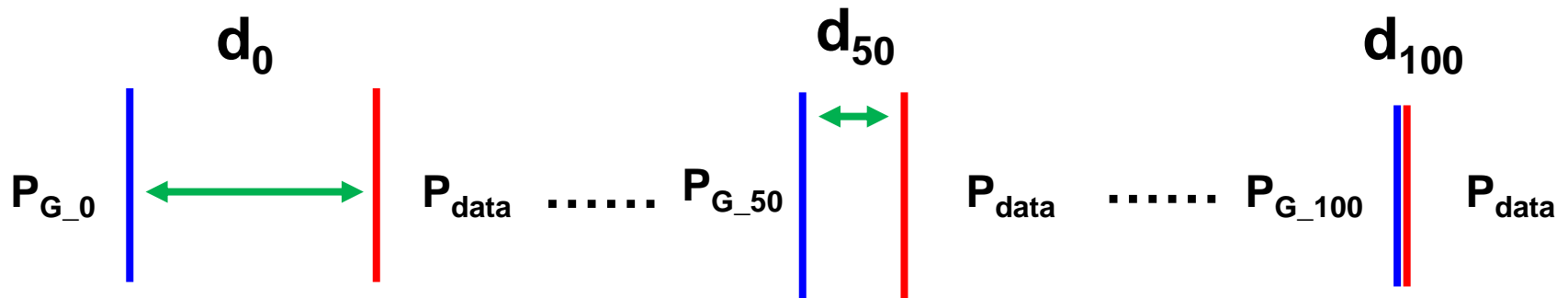
- No longer a direct critic of telling the fake samples apart from the real ones
- Tries to bring  $x_G$  distribution closer to  $x_{true}$  distribution

- **Wasserstein Loss**

$$D_{loss} = E(D(x_{true})) - E(D(x_G))$$
$$G_{loss} = -E(D(x_G))$$

# Wasserstein Distance vs. JSD

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$$JS(P_{G_0}, P_{data}) = \log 2$$

$$JS(P_{G_{50}}, P_{data}) = \log 2$$

$$JS(P_{G_{100}}, P_{data}) = 0$$

$$W(P_{G_0}, P_{data}) = d_0$$

$$W(P_{G_{50}}, P_{data}) = d_{50}$$

$$W(P_{G_{100}}, P_{data}) = 0$$



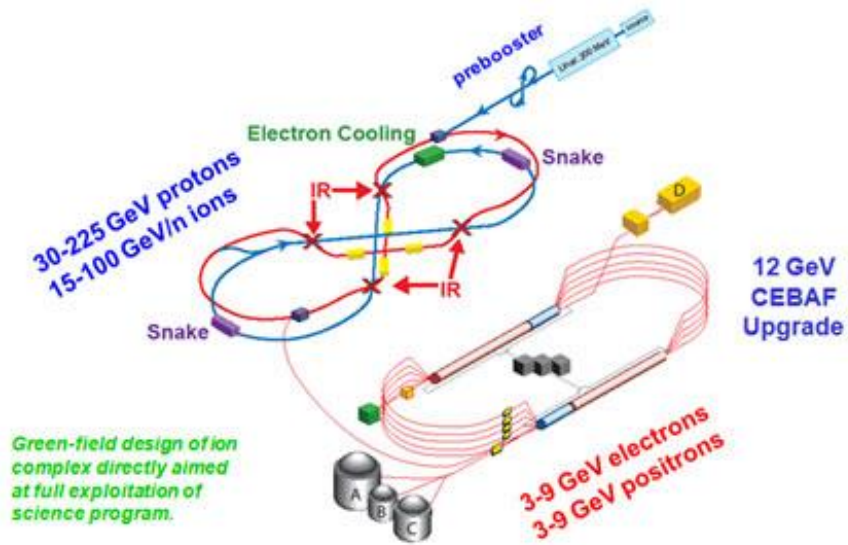
# Generative Adversarial Network (GAN)

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# Simulation of Physics Events

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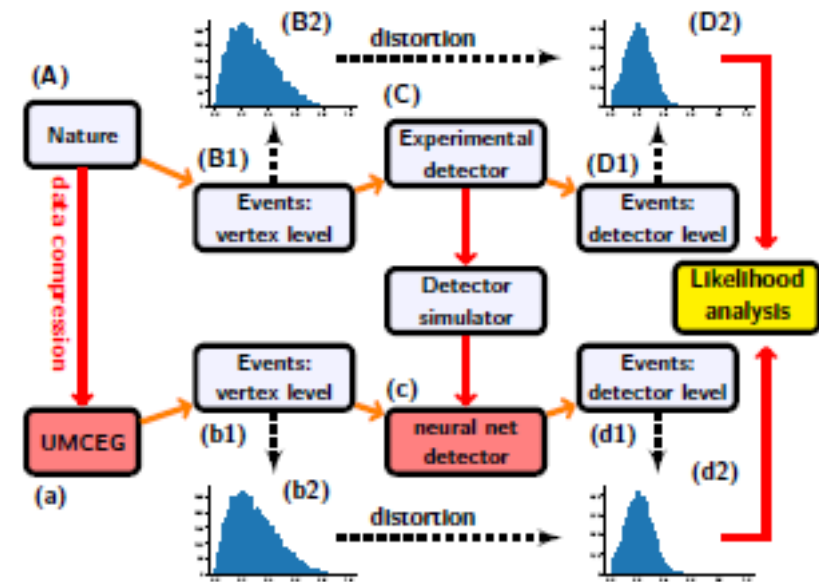


## Jefferson Lab Electron-Proton Collider

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# GAN-based Event Generators

- **Learning from real electron-proton scattering data**
  - Capture rich underlying distributions over data
    - Difficult to model using explicit parameters
- **Faithfully reproducing particle reaction events**
  - No assumptions on femtometer-scale physics theory
- **Overcome the limitations of MCEGs**
- **Proof-of-concept on inclusive electrons**



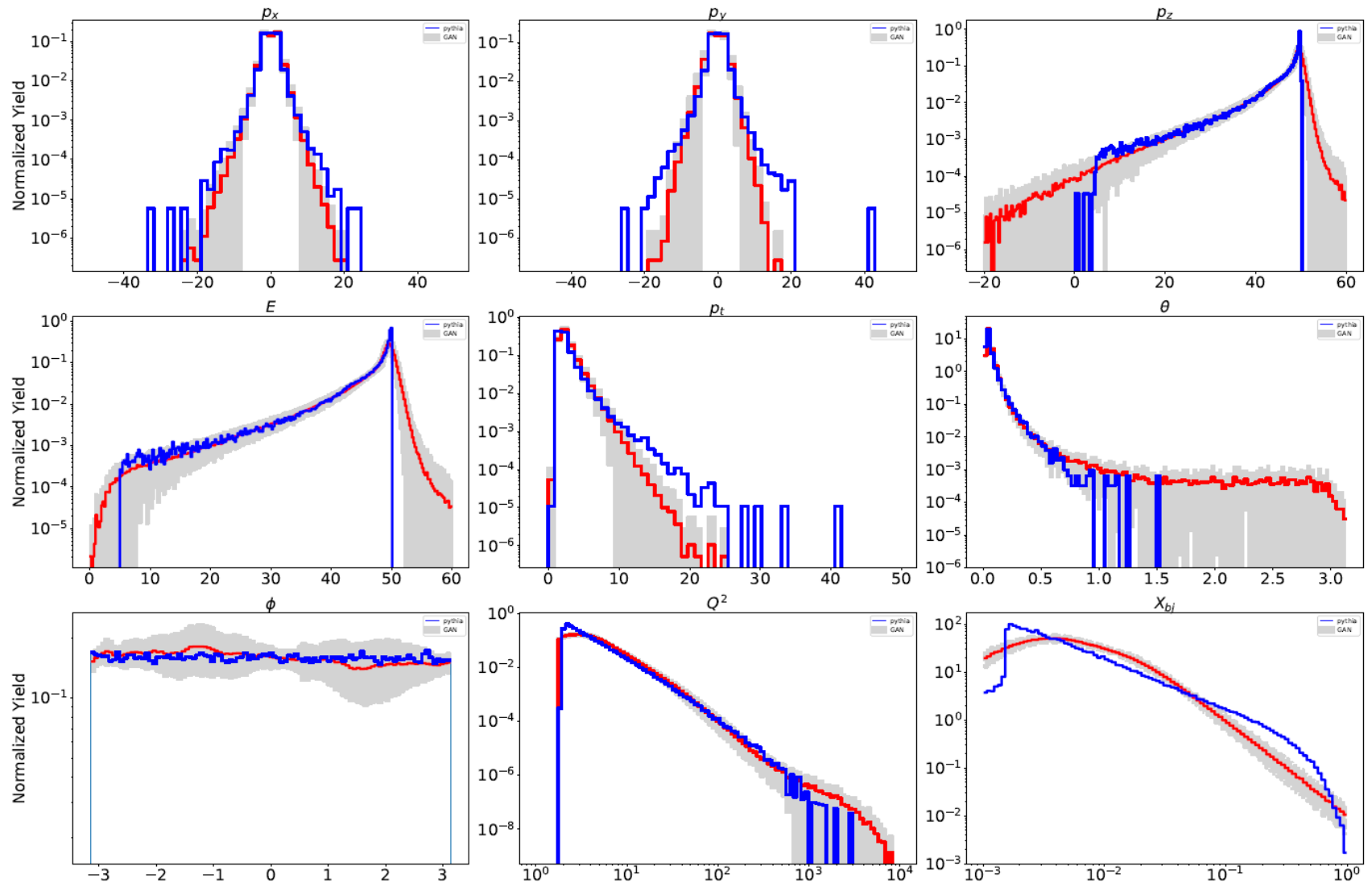
# Additional (GAN) Challenges

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- Precise Event Feature Distributions
  - Replicate the nature of particle reactions faithfully
- Obeying the fundamental Physics Laws
  - Energy Conservation
  - Momentum Conservation

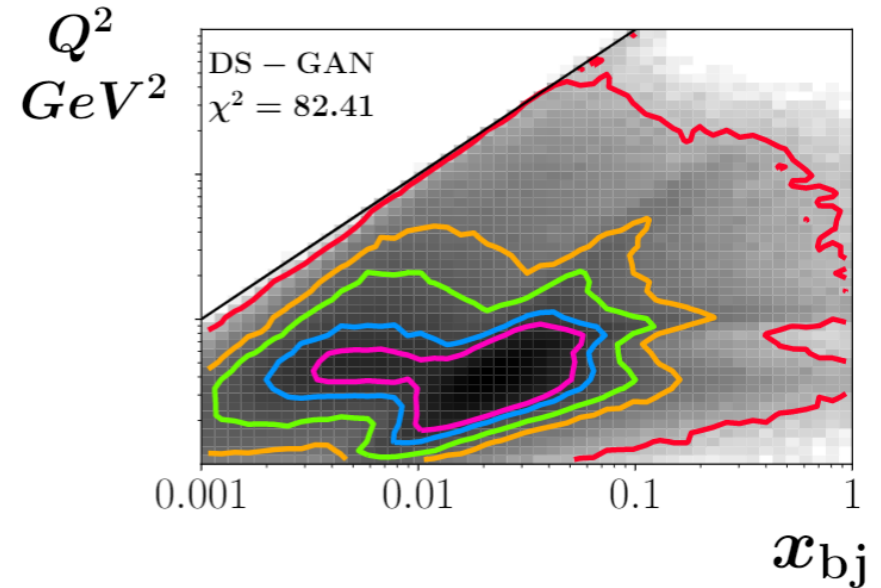
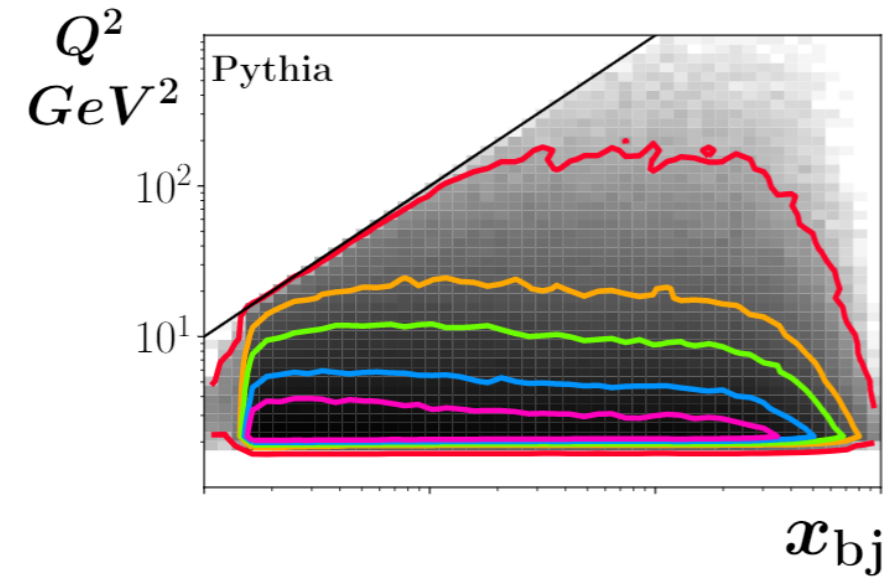
# Direct Simulation GAN

- Directly learn from electron three-momentum vector ( $p_x, p_y, p_z$ )



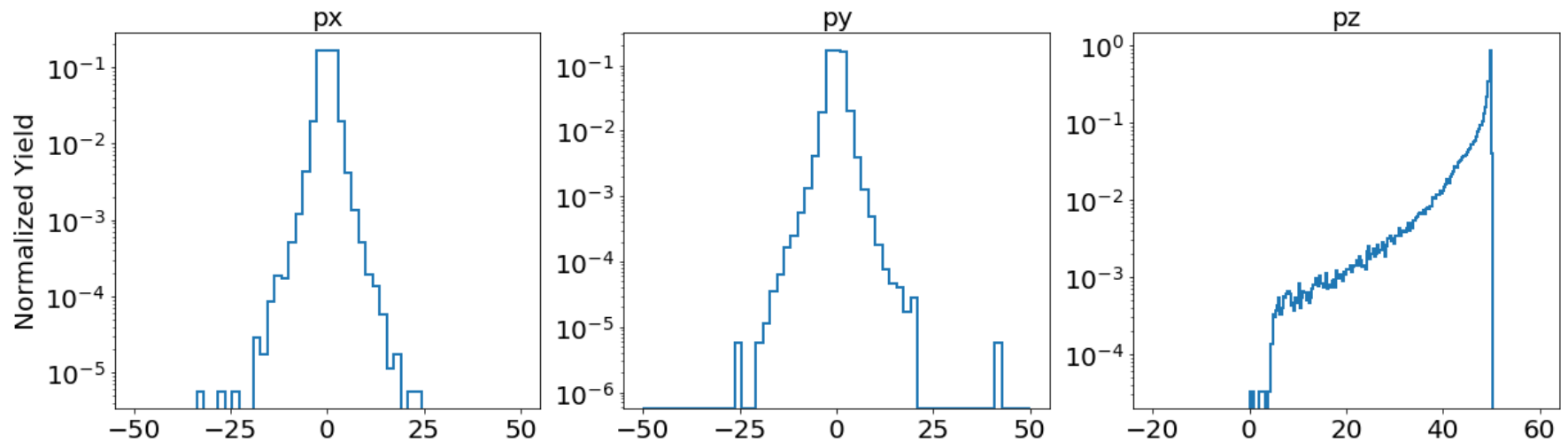
## Direct Simulation GAN (cont.)

- Inter-correlation between physical quantities



# Momenta Distributions of Electrons

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# Feature-Augmented Transformed GAN (FAT-GAN)

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- **FAT-GAN**

- Features Transformation
  - Select generated features
    - Not necessary meaningful physics properties
  - Easier to be generated by the generator
- Features Augmentation
  - Expand feature space
  - Improve sensitivity of the discriminator
- Maximum Mean Discrepancy (MMD)
  - Improve Distribution Match
- Wasserstein Loss
  - Reduce the chance of mode collapse
  - Enhance GAN convergence

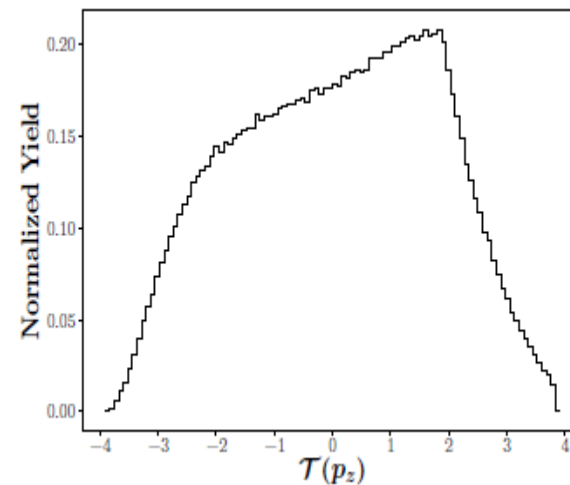
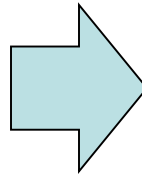
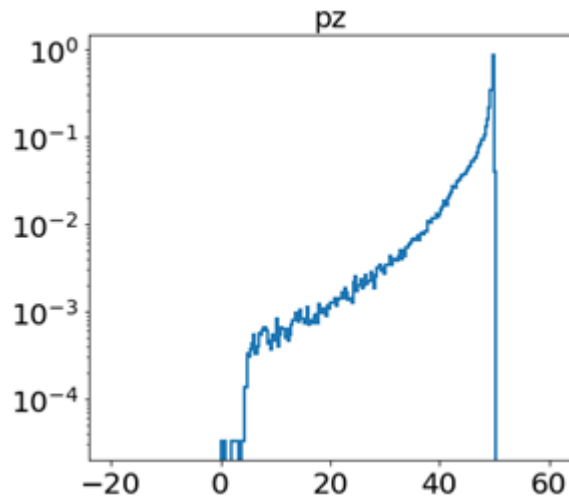


# Features Transformation

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$$\mathcal{T}(p_z) = \log(E_b - p_z)$$

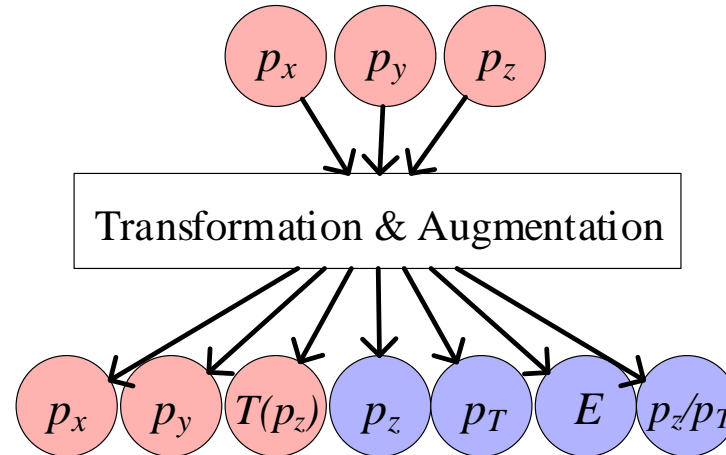
- Conversion to eliminate sharp edges
- Guarantee no generation of non-physical electrons



# Features Augmentation

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- Augment the Feature Space to improve the Sensitivity of Discriminator



# Maximum Mean Discrepancy (MMD)

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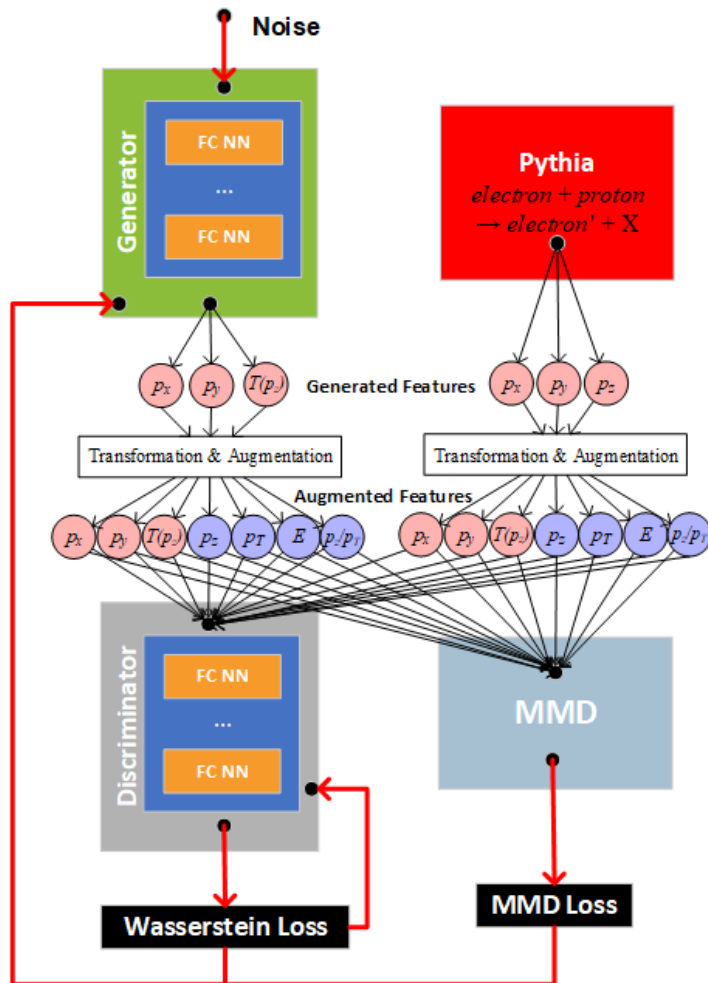
- MMD

- A kernel-based two-sample test to compare two distributions
- Determine if the two sets of samples are drawn from different distributions

$$\begin{aligned}\text{MMD}^2(\mathbf{p}, \tilde{\mathbf{p}}) &= \mathbb{E}_{\mathbf{p}_a, \mathbf{p}_{a'} \sim P_{\mathbf{p}}} [k(\mathbf{p}_a, \mathbf{p}_{a'})] \\ &\quad + \mathbb{E}_{\mathbf{p}_b, \mathbf{p}_{b'} \sim P_{\tilde{\mathbf{p}}}} [k(\mathbf{p}_b, \mathbf{p}_{b'})] \\ &\quad - 2 \mathbb{E}_{\mathbf{p}_a \sim P_{\mathbf{p}}, \mathbf{p}_b \sim P_{\tilde{\mathbf{p}}}} [k(\mathbf{p}_a, \mathbf{p}_b)]\end{aligned}$$

- $k(\mathbf{p}_a, \mathbf{p}_b)$  is a Gaussian kernel

# FAT-GAN Architecture



- Discriminator Loss

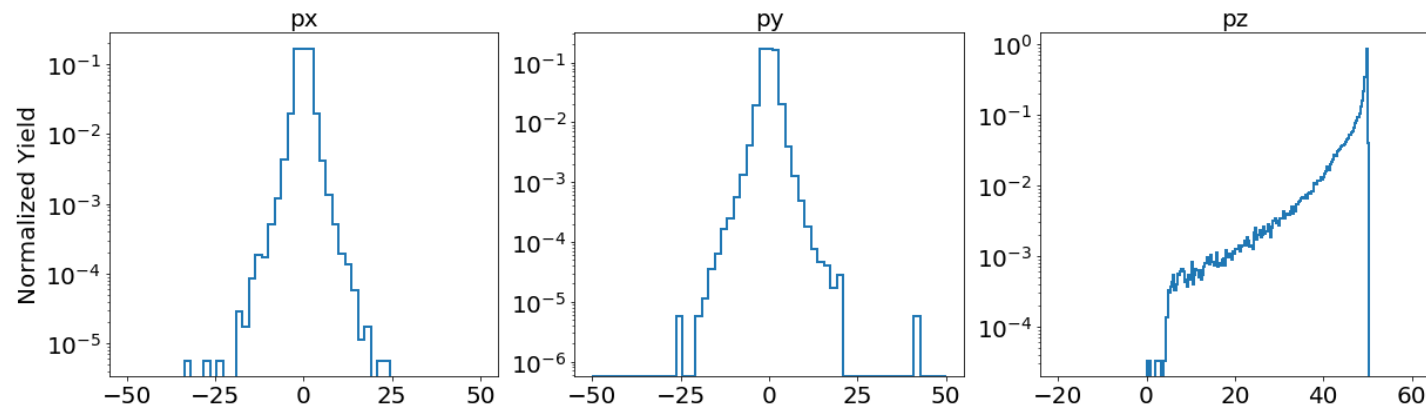
$$L_D = (\mathbb{E}[D(\tilde{p})]) - \mathbb{E}[D(p)] + \lambda \mathbb{E}_{\hat{p} \sim P_{\hat{p}}} [(\|\nabla_{\hat{p}} D(\hat{p})\|_2 - 1)^2]$$

- Generator Loss

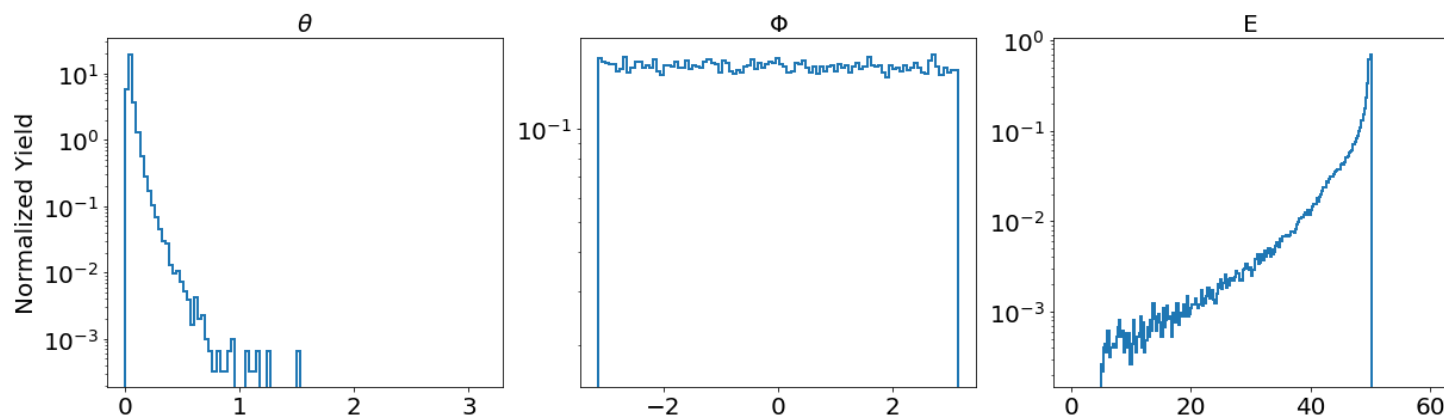
$$L_G = -\mathbb{E}[D(\tilde{p})] + \eta \text{MMD}^2(p, \tilde{p})$$

# Comparison between Representations in Cartesian Coordinates and Spherical Coordinates

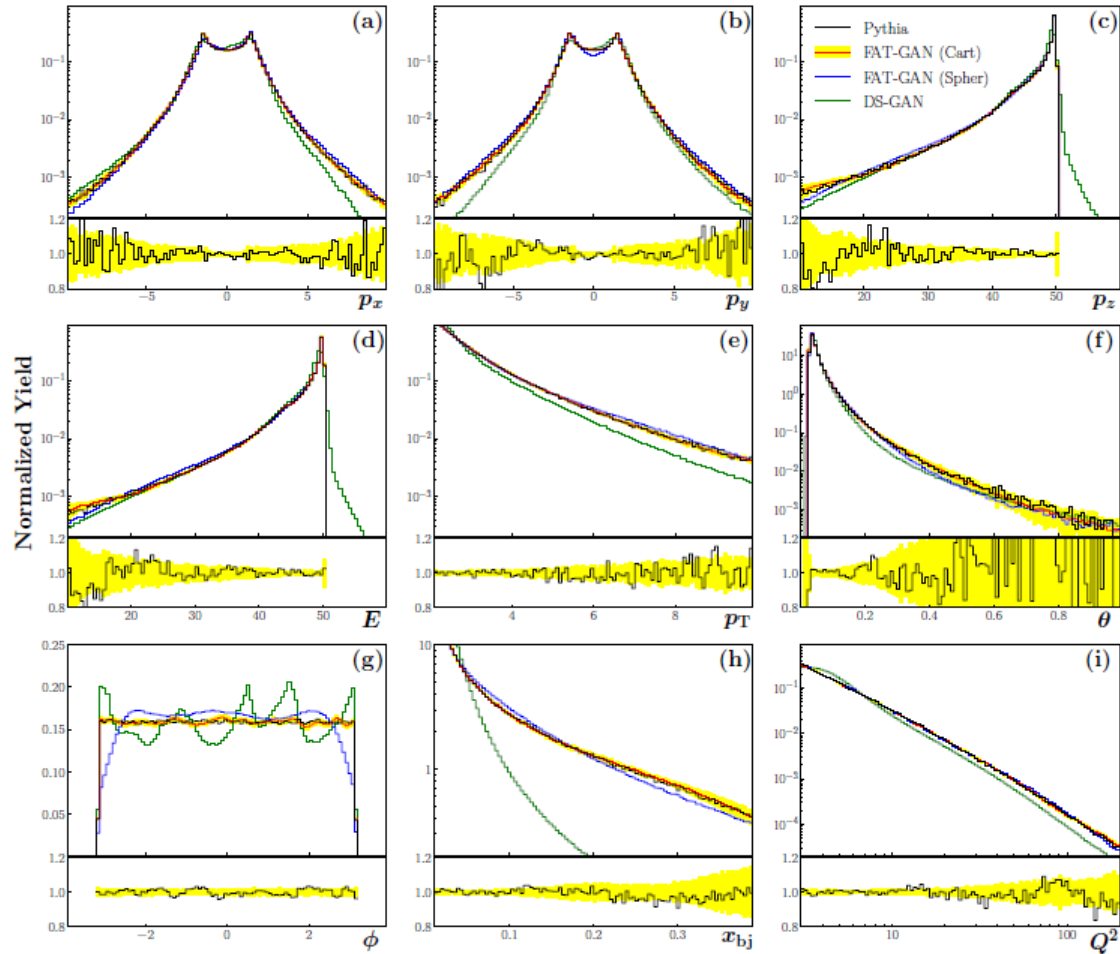
## ■ Representation in Cartesian Coordinates [FAT-GAN (Cart)]



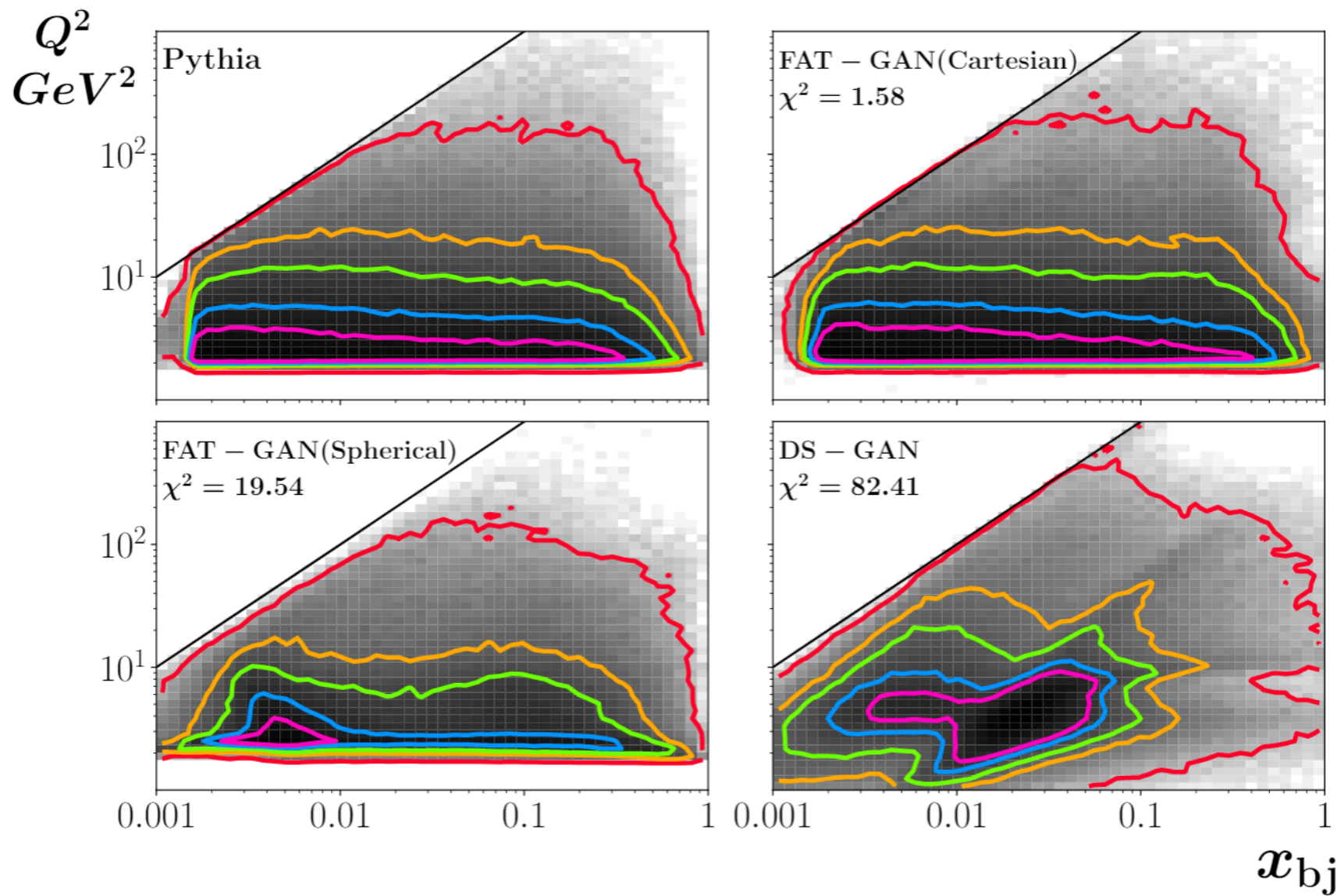
## ■ Representation in Spherical Coordinates [FAT-GAN (Spher)]



# Distributions of Generated Physical Properties

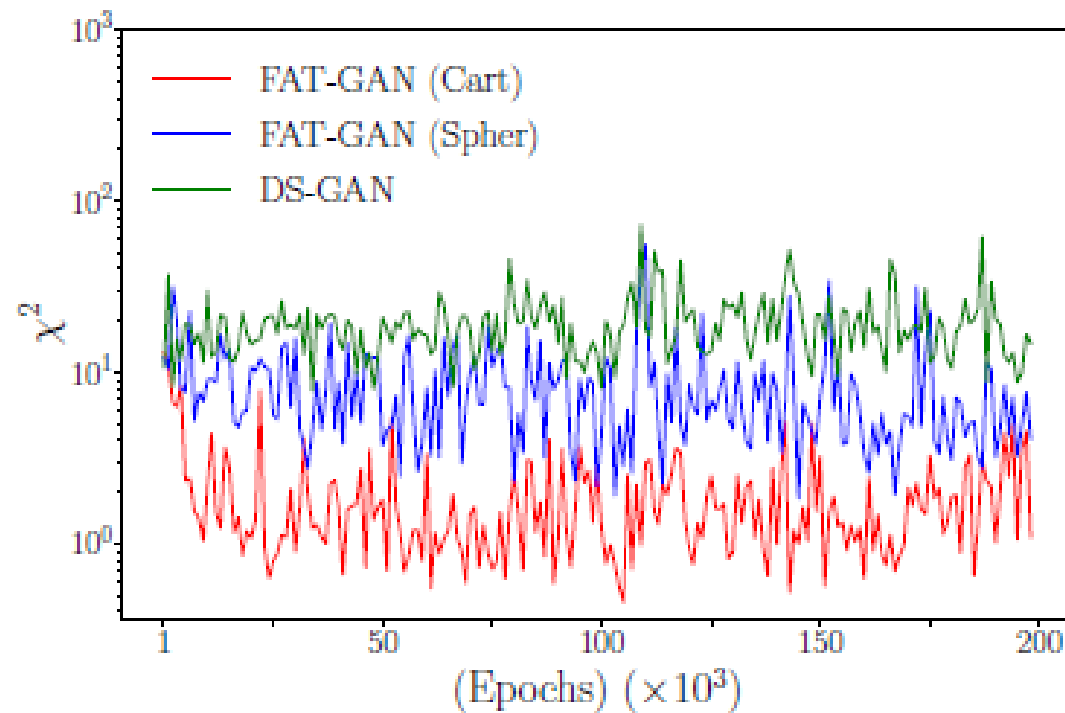


# Features Inter-correlations



# Convergence Comparison

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# Summary

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- **GAN**
  - Adversarial Training
  - Can mimic any data distributions
- **Mathematics under a GAN**
- **Problems of Training a GAN**
  - Attractive but Difficult to Train
- **FAT-GAN**
- **DCGAN**