**Homework 5**
**Statistics 201B**
**Due Dec 4th 9:30am**

1. The owner of a ski shop must order skis for the upcoming season. Orders must be placed in quantities of 25 pairs of skis. The cost per pair of skis is \$50 if 25 are ordered, \$45 if 50 are ordered, and \$40 if 75 are ordered. The skis will be sold at \$75 per pair. Any skis left over at the end of the year can be sold (for sure) at \$25 per pair. If the owner runs out of skis during the season, she will suffer a loss of "goodwill" among unsatisfied customers. She rates this loss at \$5 per unsatisfied customer. For simplicity, suppose the owner feels that demand for the skis will be 30, 40, 50, or 60 pairs of skis, with probabilities 0.2, 0.4, 0.2, and 0.2, respectively.

   (a) Describe the parameter space $\Theta$ and the space of possible actions $\mathcal{A}$.

   (b) What is the prior distribution?

   (c) For each possible $\theta \in \Theta$ and $a \in \mathcal{A}$, compute the loss. Display these possibilities in a matrix.

   (d) What is the Bayes rule? That is, what action minimizes the Bayes risk? Note that in this example, there is no data, so the frequentist risk is the same as the loss.

2. Suppose $X|p \sim Binomial(n, p)$ and $p \sim Beta(\alpha, \beta)$. Suppose also that the loss function $L(p, \hat{p}(x)) = (p - \hat{p}(x))^2$.

   (a) Calculate the posterior risk $r(\hat{p}|x)$ for an arbitrary estimator $\hat{p}$.

   (b) For a given $x$, what value of $\hat{p}(x)$ minimizes the posterior risk? Use this to construct a Bayes estimator.

   (c) What is the posterior risk for the Bayes estimator you found in (b)? How does it relate to the posterior distribution?

3. Let $\Theta = \{\theta_1, \ldots, \theta_k\}$ be a finite parameter space. Prove that the posterior mode is the Bayes estimator under zero-one loss.

4. Consider a decision problem with possible states of nature $\theta_1$ and $\theta_2$ . Let X be a random variable with probability function $p(x|\theta)$:

$P(X = 0|\theta_1) = 0.2, P(X = 1|\theta_1) = 0.8; P(X = 0|\theta_2) = 0.4, P(X = 1|\theta_2) = 0.6.$

Two non-randomized actions $a_1$ and $a_2$ are considered with the following loss function:

$$L(\theta_1, a_1(0)) = 1, L(\theta_1, a_1(1)) = 2, L(\theta_1, a_2(0)) = 4, L(\theta_1, a_2(1)) = 0;$$

$$L(\theta_2, a_1(0)) = 3, L(\theta_2, a_1(1)) = 1, L(\theta_2, a_2(0)) = 1, L(\theta_2, a_2(1)) = 4.$$

(a) Give and plot the risk set $S = \{(r_1, r_2) : r_1 = \lambda R(\theta_1, a_1) + (1-\lambda)R(\theta_1, a_2), r_2 = \lambda R(\theta_2, a_1) + (1 - \lambda)R(\theta_2, a_2), \lambda \in [0, 1]\}$.

(b) Suppose $\theta$ has the prior distribution $\Lambda(\theta)$ defined by $P(\theta = \theta_1) = 0.9, P(\theta = \theta_2) = 0.1$. What is the Bayes rule with respect to $\Lambda(\theta)$?

(c) Find the minimax rule(s).