AI6123 Time Series Analysis Project 2

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Introduction

The aim of this project is to fit an appropriate ARIMA model in real situations. ARIMA models provide an approach to time series forecasting in which they describe the autocorrelations in the data. The data is for monthly anti-diabetic drug sales in Australia from 1992 to 2008. Total monthly scripts for pharmaceutical products falling under ATC code A10, as recorded by the Australian Health Insurance Commission. Therefore, in this project, we will explore different ARIMA models to find the most appropriate model to explore the correlations in data for monthly anti-diabetic drug sales in Australia, and forecast the time series for three years after 2008.

Original Data Analysis

First, we load the data and get the original data plot, which is shown in Figure 1.

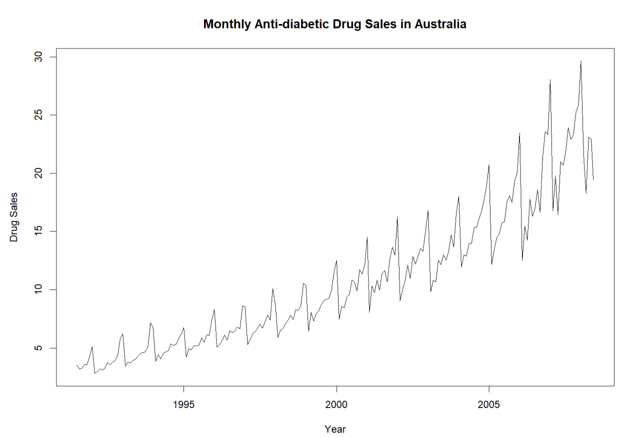


Figure 1: Original data plot

We can see that the minimum value of the data is 2.81, the maximum value is 29.67, and the average value is 10.7. Furthermore, it can be seen that there is an increasing trend in the original data. Besides that, there is a certain seasonal trend in the data. We then use the *stl()* function on the original data to do the seasonal decomposition. We can see a more obvious trend in Figure 2. The results show a clear increasing trend and seasonal trend in the data.

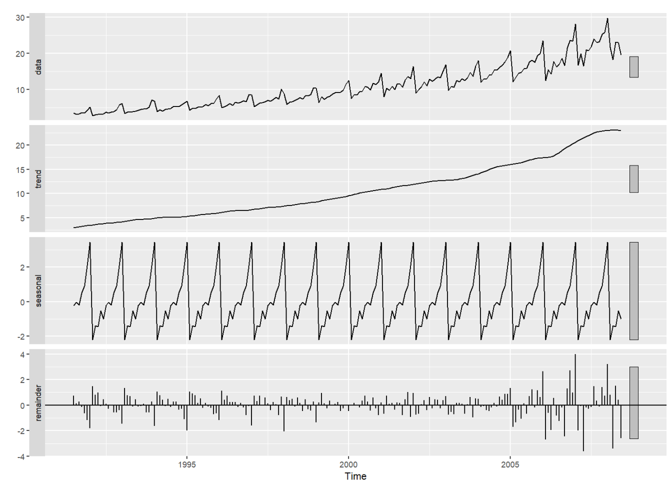


Figure 2: Result of seasonal decomposition

Then we generate ACF and PACF of the original data. The results can be seen in Figure 3. We know that stationary series usually have short-term correlation. This property can be explained by the fact that the autocorrelation coefficient of a stationary series decays quickly to zero as lag increases. Conversely, the autocorrelation coefficient of a non-stationary series usually decays to zero more slowly. However, we can find in the plot that the autocorrelation coefficients of the series decay to zero slowly, and the data remained positive for a long period. Therefore, we can conclude that the original data is non-stationary.

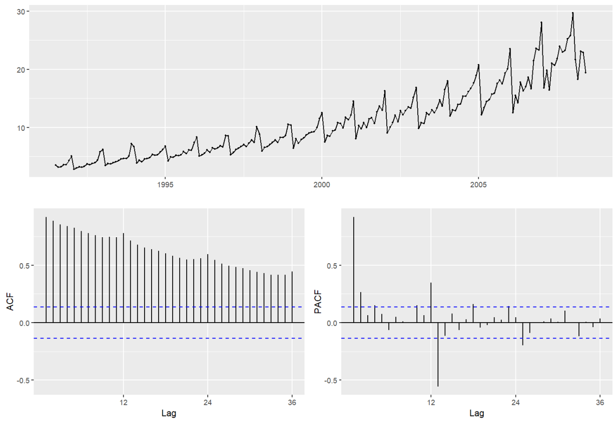


Figure 3: ACF and PACF of the oringinal data

We divided the data into training and validation sets in a 4:1 ratio. After the division, the training set has 163 data points and the validation set has 41 data points.

All the above analysis indicates that the series is non-stationary and has seasonal trend. Therefore, Box-Cox transformation needs to be done, and then we should do data differencing.

From ARIMA to SARIMA

Just as we learned from our lectures, ARIMA models are primarily used for non-seasonal time series data. It may not be reasonable to assume that the seasonal component repeats itself precisely in the same way cycle after cycle. On the other hand, SARIMA models are used for data that exhibits a seasonal pattern. In addition to the AR, I, and MA components, SARIMA also includes seasonal autoregression (SAR) and seasonal moving average (SMA) components. SARIMA can identify and model seasonal factors in the data and make predictions accordingly. Our monthly anti-diabetic drug sales data in Australia appears to be seasonal data, and using the SARIMA model may be appropriate. Therefore, we will primarily use the 'auto.arima()' function to prove our assumption, as shown in the Figure 4 below. :

```
> auto_model = auto.arima(data_all)
> auto_model
Series: data_all
ARIMA(1,1,1)(0,1,1)[12]

Coefficients:
          ar1      ma1     sma1
      -0.2504  -0.6674  -0.4725
s.e.   0.1007   0.0870   0.0641

sigma^2 = 0.8756:  log likelihood = -258.82
AIC=525.63   AICc=525.85   BIC=538.64
```

Figure 4: auto.ARIMA

Based on Figure 4, we can observe that the output parameters obtained using the 'auto.arima()' function include seasonal components, which aligns with the assumption we made earlier. Therefore, it is appropriate to use a SARIMA model to fit the drug data.

Transformation

Box-Cox

As seen on the seasonal decomposition of the original data, there is an increasing trend in its variance. Hence, it is necessary to transform the data using a Box-Cox transformation to stabilize the variance. The Box-Cox transformation is a statistical method to transform the response variable, so the data follows a normal distribution. In the Box-Cox transformation, λ is a hyper-parameter which has to be tuned according to the data set. The first option is to set $\lambda = 0$, where the natural log of the data is used. The second option is to use the function BoxCox.lambda to find the non-zero λ. In this case, the non-zero $\lambda = 0.1313326$. We choose the value of λ that provides the best approximation for the normal distribution of our response variable. After plotting the seasonal decomposition of the data after the Box-Cox transformation, it is found that $\lambda = 0$ yields better results. Hence, we will be using the data after the Box-Cox transformation with hyper-parameter $\lambda = 0$. The data after the Box-Cox transformation with hyper-parameter $\lambda = 0$ is shown in Figure 5.

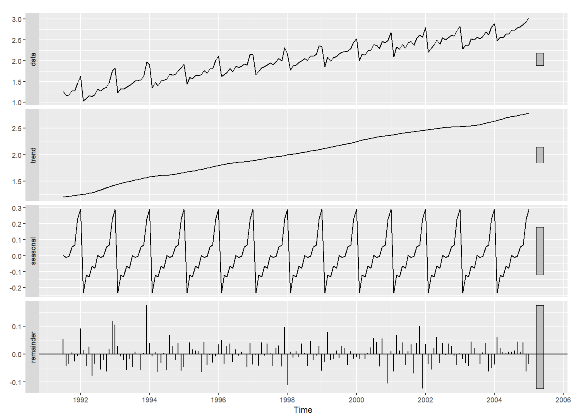


Figure 5: Seasonal decomposition after Box-Cox transformation

Differencing

As mentioned earlier, according to the original data's ACF and PACF plots, the data is non-stationary. Since the data is non-stationary, it is necessary to take first and second differences of the data until the data is stationary. Consequently, we do a lag-12 differencing to remove the seasonal component of the data. The data after differencing is shown in Figure 6.

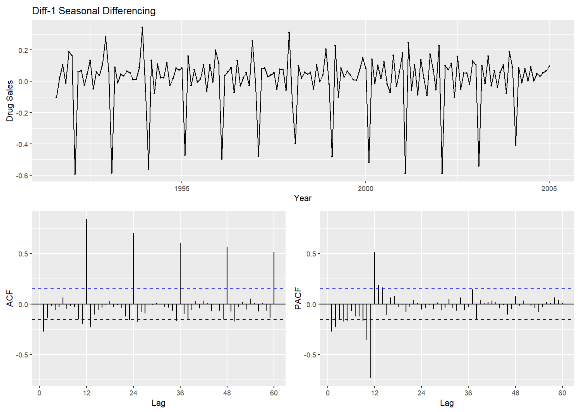


Figure 6: Data after Differencing

Model Parameter Chosen

When using the SARIMA model, the most important thing is to select the appropriate parameter pairs including the difference order d for eliminating the trending; D for eliminating the seasonal components; q and Q associated with the autocorrelation information; p and P dealing with partial autocorrelation terms.

Selecting appropriate parameter pairs is crucial when using a SARIMA model. These pairs include the difference order d for eliminating trending, D for eliminating seasonal components, q and Q associated with autocorrelation information, and p and P dealing with partial autocorrelation terms.

As seen in Figure 6, the cutoff point at stage 1 (12-months for one stage) near lag 7 is evident in the ACF figure, suggesting that we can set Q to 1 and q to 7. Additionally, in the PACF figure, the curves cut off at the first stage with lag 5, indicating that we can set P to 1 and p to 5.

Based on these findings, we can derive various hyperparameter pairs for our SARIMA model, as discussed in our lectures:

1. SARIMA(0, 1, 7)(0, 1, 1)
2. SARIMA(0, 1, 7)(1, 1, 0)
3. SARIMA(5, 1, 0)(0, 1, 1)
4. SARIMA(5, 1, 0)(1, 1, 0)
5. SARIMA(5, 1, 7)(1, 1, 1)

After building these models, we continue to evaluate their performance.

SARIMA Model Fitting

We can use the *tsdiag()* function and *checkresiduals()* function to implement the diagnostic check smoothly. Furthermore, we can plot the Normal Q-Q Plot of residuals. Figure 7, 8, 9, 10 and 11 shows the results.

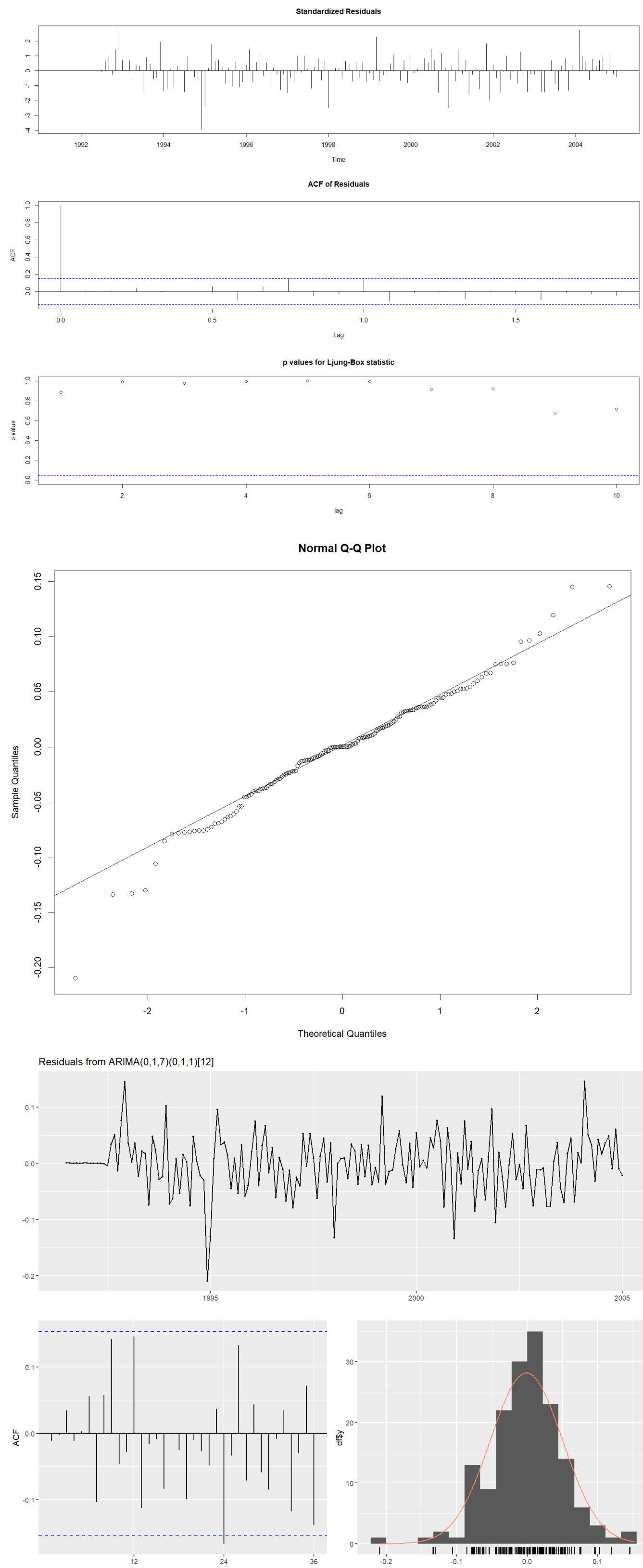


Figure 7: Diagnostic Check of SARIMA(0, 1, 7)(0, 1, 1)

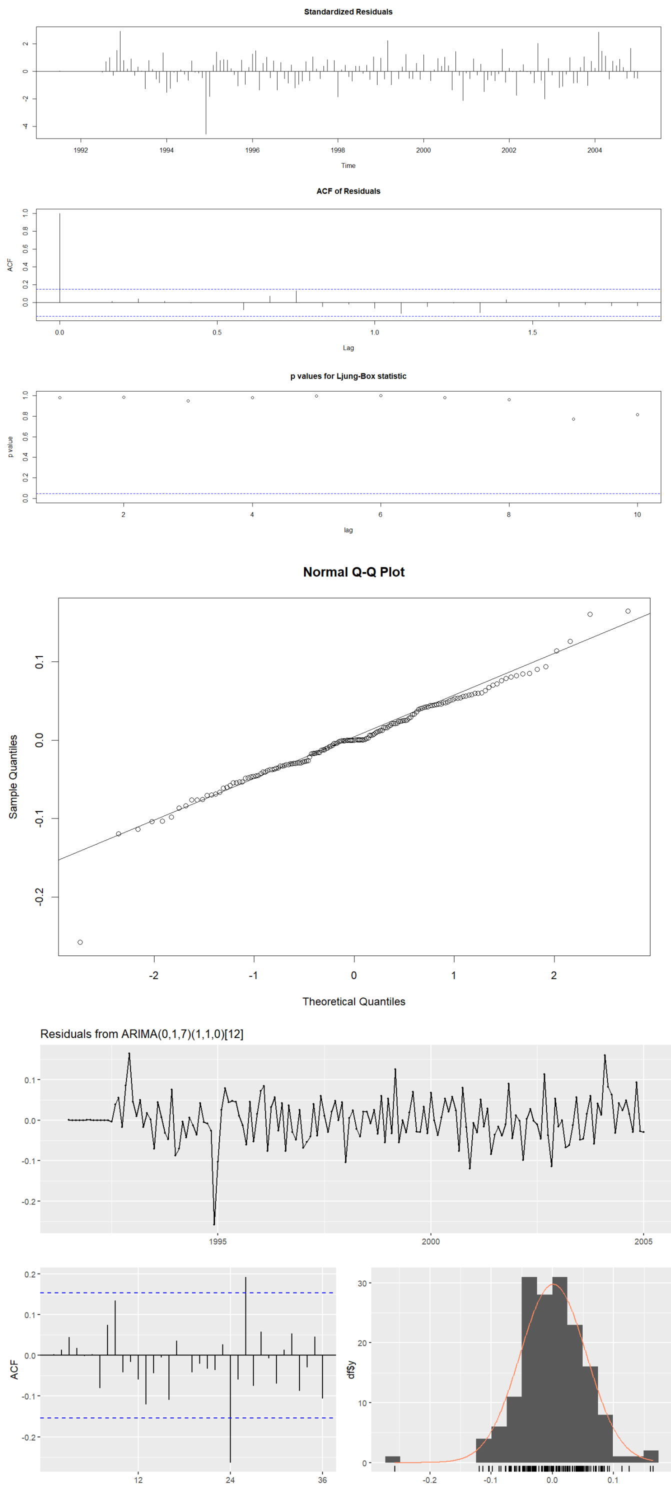


Figure 8: Diagnostic Check of SARIMA(0, 1, 7)(1, 1, 0)

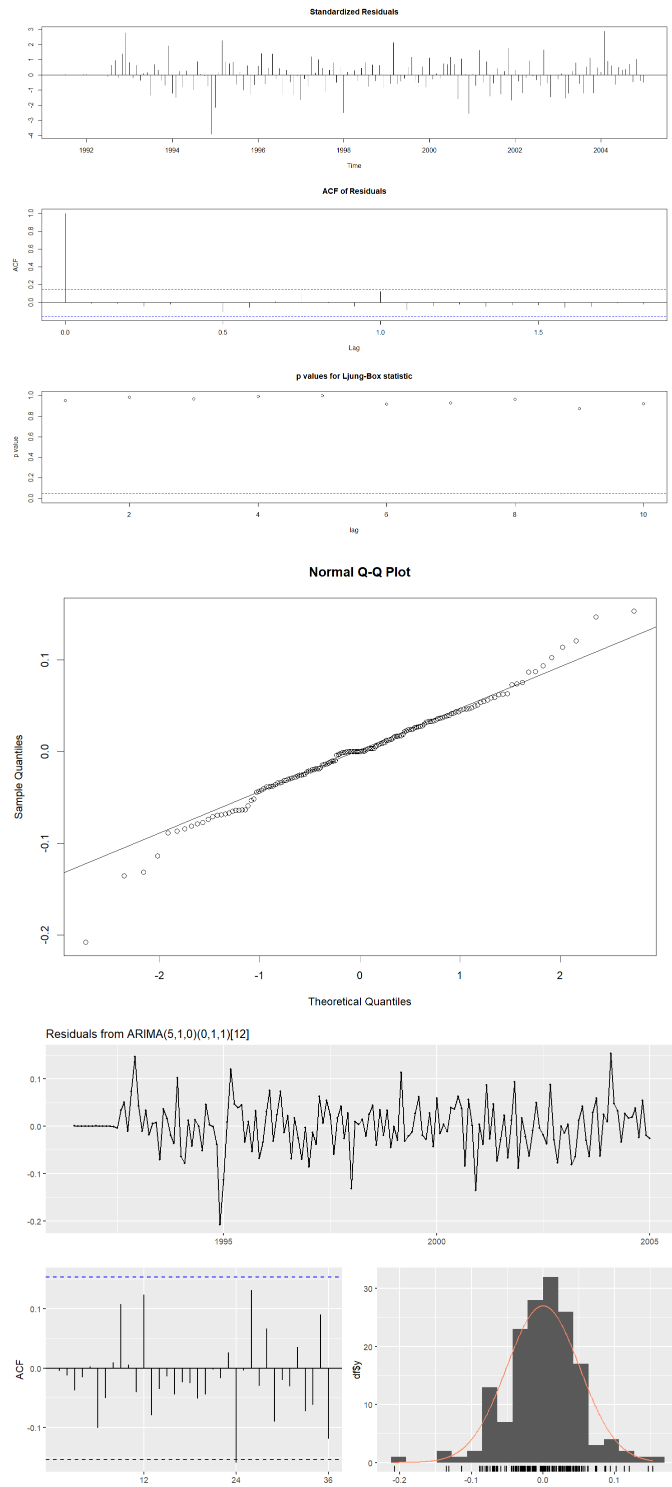


Figure 9: Diagnostic Check of SARIMA(5, 1, 0)(0, 1, 1)

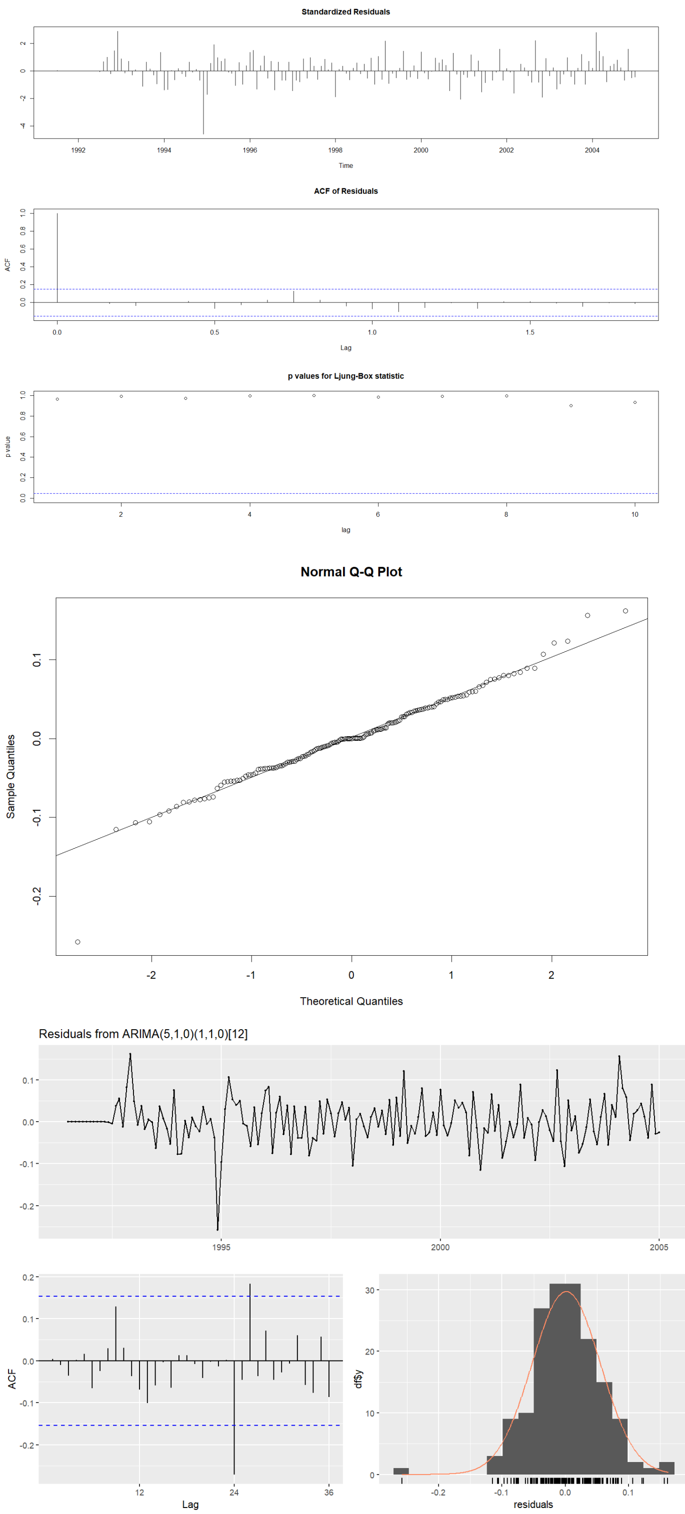


Figure 10: Diagnostic Check of SARIMA(5, 1, 0)(1, 1, 0)

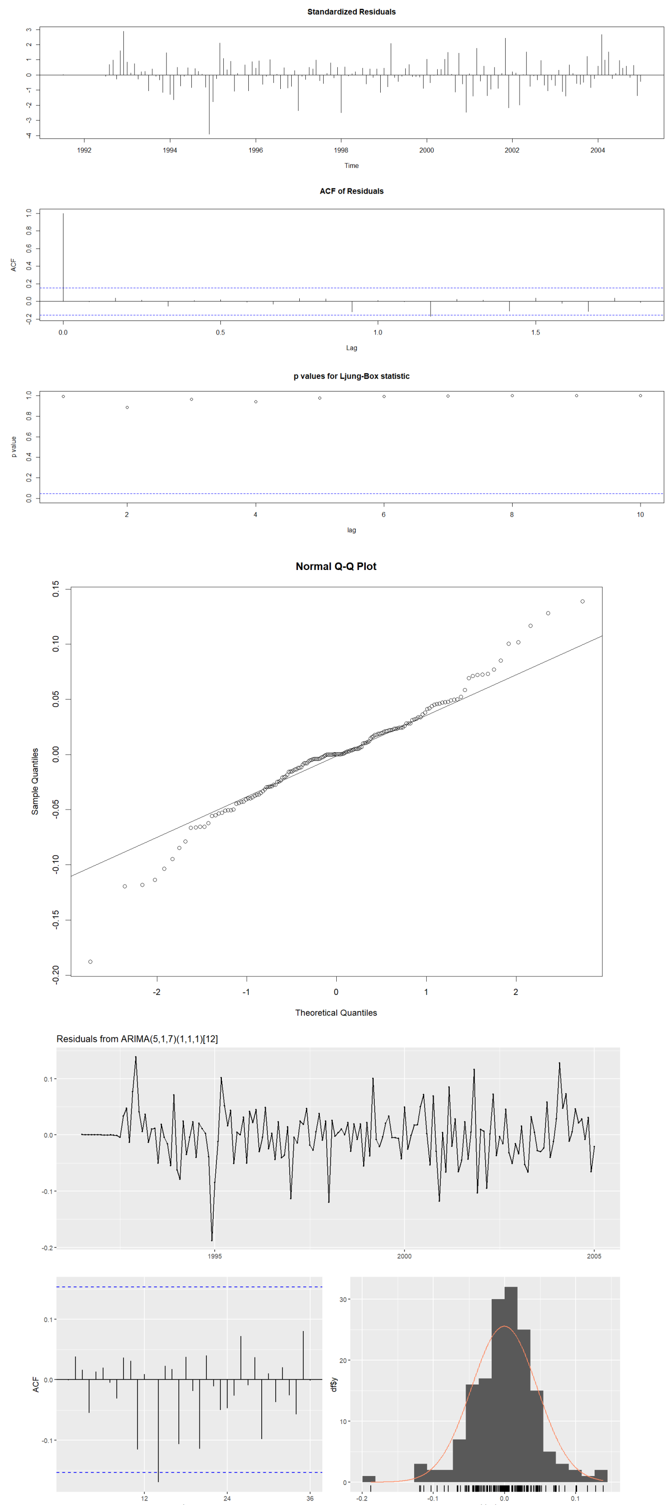


Figure 11: Diagnostic Check of SARIMA(5, 1, 7)(1, 1, 1)

For these plots, we can implement a series of analyses similar to what we have done in the project I:

1. The residuals (standardized residuals) look random enough.

2. The histogram of residuals and Q-Q plot of residuals show that residuals are enough to be regarded as white noise.

3. The ARMA and MA model's ACF of the residuals cut off after lag 0. However, the AR model's ACF does not cut off after lag 0. Combined with the residuals checking, the AR model may not be a satisfactory model to fit the data.

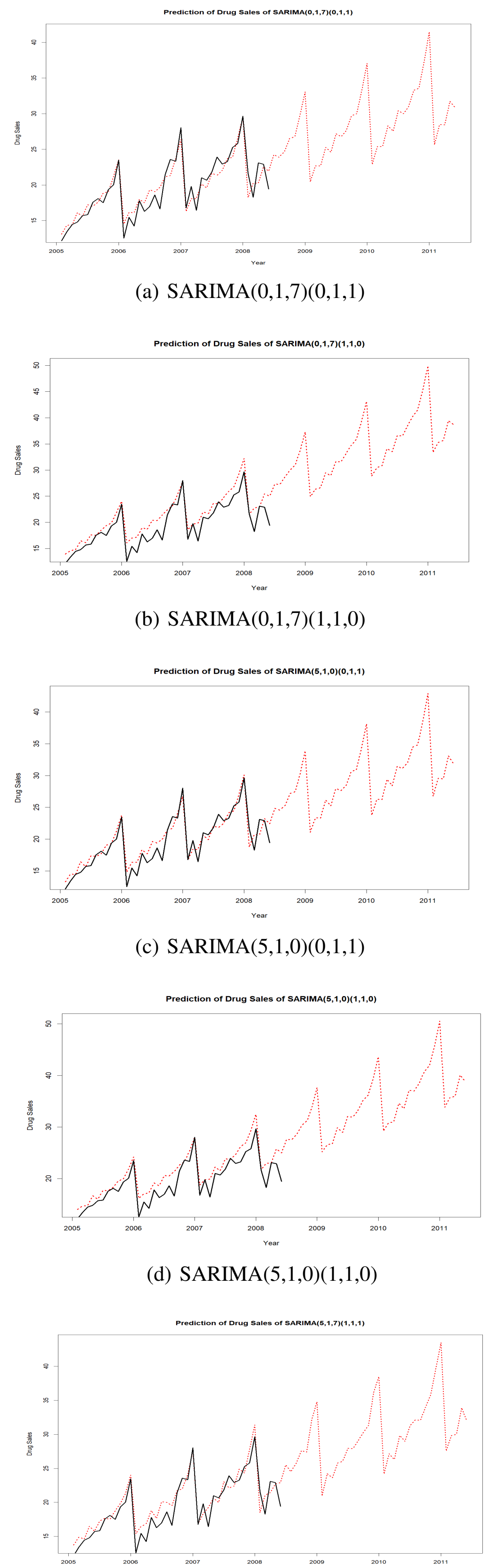
4. The p-value for the Ljung-Box statistic is high enough, which is beyond 0.05, based on the Box.test() function. The p-value of the MA model is apparently lower than the ARIMA model, even though still higher than 0.05. This may indicate that the MA model's performance will be lower than the ARIMA model.

We also calculate the AIC and BIC of these models, shown in Table I. Having concluded that those models may provide an adequate fit, we implement the forecasting to the following 20 data points and validation with the model. We can easily use the *forecast()* function to create the forecasting data, as shown in Figure 12. The black curve is the forecasting result of the corresponding model, while the blue curve is the validation data (Ground truth). The validation length is 41, and the forecasting length is 20.

With the predicting result, we can calculate the RMSE and MAE of both the training set and validation set as two criteria to evaluate model fitting performance. The results are also shown in Table 1. From the result, we can conclude that the ARMA model has the best performance in forecasting, while the AIC and BIC are only a little bit higher than the MA model. The forecasting results of the AR model are super bad, which coincides with our analysis above.

set	Model	AIC	BIC	RMSE	MAE
	SARIMA(0, 1, 7)(1, 1, 0)	-416.971	-389.8753	0.05409898	0.04039567
	SARIMA(0, 1, 7)(0, 1, 1)	-428.6995	-401.6038	0.05118094	0.03782738
Training	SARIMA(5, 1, 0)(1, 1, 0)	-422.1871	-401.1126	0.053885	0.04006019
	SARIMA(5, 1, 0)(0, 1, 1)	-434.6062	-413.5318	0.05098949	0.03778493
	SARIMA(5, 1, 0)(1, 1, 1)	-439.346	-394.1865	0.04605911	0.03299559
	SARIMA(0, 1, 7)(1, 1, 0)	-	-	2.229626	1.766395
	SARIMA(0, 1, 7)(0, 1, 1)	-	-	**1.483472**	**1.179295**
Testing	SARIMA(5, 1, 0)(1, 1, 0)	-	-	2.289985	1.812177
	SARIMA(5, 1, 0)(0, 1, 1)	-	-	1.507028	1.507028
	SARIMA(5, 1, 0)(1, 1, 1)	-	-	1.674455	1.376838

Table 1: ACCURACY



(a) SARIMA(0,1,7)(0,1,1)

(b) SARIMA(0,1,7)(1,1,0)

(c) SARIMA(5,1,0)(0,1,1)

(d) SARIMA(5,1,0)(1,1,0)

(e) SARIMA(5,1,7)(1,1,1)

Figure 12: Predictions for five SARIMA models

Thus, in the five models with zero lambda, the ARMA model SARIMA(0, 1, 7)(0, 1, 1)[12] has the best forecasting performance.

Sequential Search

On top of the 4 previous models we have proposed, we are also interested to see if there is a better model. We will use sequential search to fit different parameters. By using this method, we can record AIC, BIC, RSME, and MAE. As per the previous explanation, the data is stationary after applying one time seasonal differencing and one time differencing. Since each s has 12 lags, we are tuning small p and q values within the range [0,13] for each seasonal parameter (0,1,1), (1,1,0) and (1,1,1). (Check from Appendix for detailed data). By comparing all AIC and BIC values, we get 3 models with minimum values, and use RMSE and MAE to decide which model is the best(shown in appendix:tables).

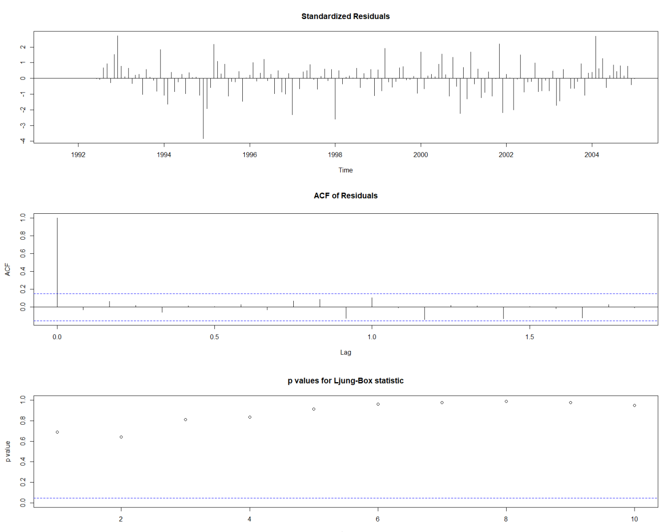


Figure 13: Diagnostic Check of SARIMA(2, 1, 3)(0, 1 ,1)

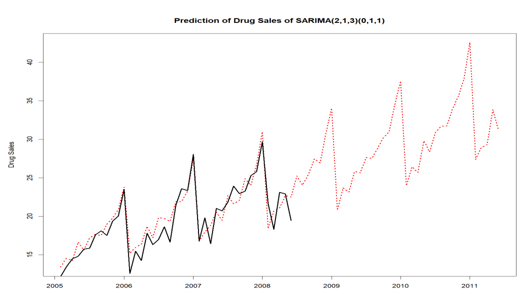


Figure 14: Prediction SARIMA(2, 1, 3)(0, 1 ,1)

From the result, SARIMA(2,1,3)(0,1,1) model is one of the 3 best AIC and BIC, and it gives lowest mean absolute error.

Holt-Winters

Another common time series model is the Holt-Winters method. The Holt-Winters method is a time series method that uses triple exponential smoothing. Hence, a time series must have trend and seasonality for the Holt-Winters method to work. Since our dataset has shown a clear seasonal trend, it is also worth considering the performance of the Holt-Winters method on this dataset. In the following section, the Holt-Winters method will be used and compared with the ARIMA method result obtained in the section above.

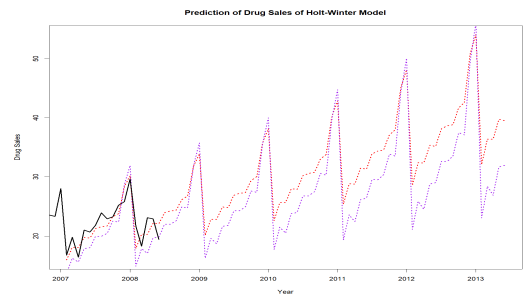


Figure 15: The prediction of holt-winter

Figure 15 shows the prediction of the Holt-Winters model obtained from hw() function. With red curve showing the additive Holt-Winters model, and purple curve showing the multiplicative Holt-Winters model.

Conclusion

Table 2 presents the root mean squared error (RMSE) and mean absolute error (MAE) of the Holt-Winters and ARIMA methods analyzed previously. For the two types of Holt-Winters models, the additive model exhibited greater accuracy than the multiplicative model, with much smaller RMSE and MAE indicators. Moreover, the red curve predictions of the additive model were much closer to the validation GT curve. However, when comparing these models to the best SARIMA model we found through our search, both types of Holt-Winters models had noticeably lower prediction accuracy. The **SARIMA(2,1,3)(0,1,1) model** remained the optimal choice for predicting the monthly anti-diabetic drug sales data, with the lowest RMSE and MAE values and relatively low model complexity.

model	RMSE	MAE
multiplicative Holt	3.258563	2.777955
addictive Holt	1.857903	1.593702
SARIMA	1.552908	1.269335

Table 2: comparison of RMSE and MAE of SARIMA and Holt-winter model

Appendix: tables

-	0	1	2	3	4	5	6	7	8	9	10	11	12
0	NA	-431.1108	-431.0102	-429.3551	-431.1846	-432.1427	-430.2072	-428.6995	-429.4884	-429.0842	-427.2597	-425.8231	-425.2574
1	-398.6925	-431.0193	-429.0625	-428.1484	-430.6700	-430.1688	-428.2580	-426.2358	-428.1695	-429.9025	-427.9940	-423.9520	-427.8535
2	-428.7299	-429.2737	-430.0777	**-446.6317**	-439.3665	-438.8606	-438.0492	-436.0781	-435.3244	-438.1261	-436.7132	-436.8528	-427.2694
3	-426.8400	-433.3559	-432.2394	-439.1022	-438.6519	-437.3153	-440.5049	-439.7806	-433.3244	-431.3415	-436.1917	-436.1773	-434.1221
4	-427.8700	-433.5360	-432.0383	-438.6038	-437.6565	-438.2019	**-445.7893**	-434.1266	-443.2794	-429.3347	-430.4537	-428.2518	-428.5570
5	-434.6062	-432.8345	-431.6301	-438.6667	-436.6880	-440.2225	-433.5546	-437.5164	-431.0424	-432.2981	-430.3461	-433.8518	-423.9219
6	-432.6670	-433.5181	-431.5621	-441.8127	-439.3838	-440.7318	-430.0599	**-444.8412**	-430.2660	-429.2714	-428.3362	-430.6985	-431.8370
7	-430.9045	-430.5968	-429.4641	-440.0072	-439.2770	-437.3619	-438.3418	-426.4327	-428.3938	-427.5544	-432.8759	-430.6438	-430.5193
8	-433.5001	-431.5002	-434.4370	-434.0114	-437.8681	-437.7082	-436.5290	-428.4162	-433.0823	-430.3590	-427.5987	-435.6535	-420.8704
9	-431.5003	-429.5184	-436.6081	-432.8083	-431.1365	-430.6633	-430.9058	-426.7600	-427.0084	-428.8355	-426.7155	-434.7746	-432.8493
10	-430.0623	-430.5945	-436.6535	-435.8627	-432.8272	-430.8419	-428.3550	-431.1786	-433.6581	-433.7117	-433.3338	-432.7460	-421.9710
11	-431.5788	-431.0119	-428.2959	-434.1298	-432.2480	-430.2684	-429.2767	-431.3594	-429.4619	-433.4616	-428.8401	-428.3538	-425.9791
12	-431.5809	-427.8515	-427.3674	-425.5194	-424.6617	-425.6913	-420.7841	NA	-429.5248	-429.5131	-431.8348	-425.8116	-417.9422

Table 3: the table of AIC $\mathrm{D}_S(011)$

-	0	1	2	3	4	5	6	7	8	9	10	11	12
0	NA	-417.9456	-417.2751	-415.3231	-416.8528	-419.9487	-418.0667	-416.9710	-416.2172	-415.3071	-418.8935	-418.5209	-419.8460
1	-387.3152	-417.2323	-415.2815	-415.2971	-416.8353	-418.0123	-416.2779	-415.6420	-414.3786	-413.8635	-419.3338	-416.3906	-421.5840
2	-414.3632	-415.5126	-416.7522	-424.4755	-422.7780	-422.8520	-421.7724	-420.2614	-419.4072	-417.6256	-415.6289	-419.4284	-423.9992
3	-412.3915	-420.7882	-419.9396	**-433.0092**	**-432.2934**	-419.1575	-430.2999	-418.6092	-417.5541	-420.7369	-419.9966	-421.7563	-423.7882
4	-415.5847	-420.8151	-421.8412	-421.0574	**-432.1570**	-420.9085	-420.5132	-418.5820	-416.4288	-419.3746	-420.0106	-420.1438	-423.3997
5	-422.1871	-420.3724	-418.9212	-429.4023	-428.2267	-431.8995	-431.0164	-428.7303	NA	-423.6219	-416.5510	-420.9167	-424.6145
6	-420.2372	-422.2358	-420.3429	-419.2510	NA	-430.4879	NA	-427.9724	-425.6449	NA	-415.4956	-425.2255	-422.6680
7	-418.6729	-417.7143	-423.7617	-428.9388	NA	-429.8843	-430.0615	NA	-418.9392	-412.7972	-416.4484	NA	-420.6794
8	-420.6729	-418.7418	-417.2174	-420.7727	NA	-430.1058	NA	NA	-428.2820	-426.3516	-417.0809	NA	-418.8693
9	-418.8114	-418.8364	-417.0444	-416.2721	-417.5481	NA	-411.6345	-417.8928	-418.1236	-426.9307	-413.8219	-424.6440	NA
10	-417.8476	-416.2103	NA	-424.0924	NA	-425.3837	-422.9447	-421.0084	-427.6324	-425.5981	-423.0245	-424.9033	-417.1298
11	-416.0747	-414.2637	-419.6351	-418.1843	-417.8106	NA	-425.4146	-424.6932	NA	-424.2073	-421.5464	NA	-414.1816
12	-414.8719	-417.8302	-418.0607	NA	-415.8384	-413.8402	-413.6600	-421.2722	-424.2568	-420.1179	NA	-425.2957	NA

Table 4: the table of AIC $\mathrm{D}_S(110)$

-	0	1	2	3	4	5	6	7	8	9	10	11	12
0	NA	-440.9182	-439.3247	-437.6533	-439.0311	-439.1194	-437.4696	-436.1717	-436.0174	-435.0666	-434.6034	-432.6138	-433.6592
1	-412.5249	-439.3531	-437.1373	-436.2086	-438.2430	-437.2327	-435.4175	-435.1702	-434.3212	-432.0370	-432.8487	-430.8487	-433.2617
2	-434.5480	-437.3667	-437.3483	**-444.7899**	-443.5757	-441.8599	-441.6356	-439.6358	-438.3491	-436.5799	-434.5799	-432.6223	-431.9751
3	-432.6499	-440.2073	-439.3375	-443.2936	**-448.9244**	-440.6101	-439.3449	-438.5226	-436.7410	-436.5053	NA	-439.6603	NA
4	-435.1203	-440.3868	-438.5154	-441.5708	-440.3381	**-446.7794**	-438.1561	-436.5600	-433.8757	-436.1605	NA	-432.0517	-427.5967
5	-440.4262	-438.7769	-437.6906	-439.5144	-440.1797	-443.4640	-441.2254	-439.3460	-432.7987	-434.3015	-432.3129	-430.3504	-429.2738
6	-438.4769	-439.7928	-438.0769	NA	-438.8768	-437.0906	NA	-438.7588	-436.8137	NA	NA	-434.9191	-425.2305
7	-437.3753	-437.2054	-435.6028	-438.9417	-437.1274	-435.3004	-433.4544	NA	-430.9568	NA	-428.8703	-432.8938	NA
8	-439.1786	-437.1963	-435.3617	-431.5020	NA	-432.9666	-435.3930	NA	-426.5441	-429.6287	-429.0460	-433.8548	NA
9	-437.2030	-435.2172	-434.7680	-436.2394	NA	-440.4582	-433.6132	-431.5556	-430.2697	NA	NA	-435.4868	NA
10	-435.3148	-434.9088	NA	-439.4200	-436.5787	-436.5401	-436.6108	-434.5518	-432.4326	-432.9947	-428.6270	-422.2226	-430.1087
11	-434.0782	-433.5561	-431.6794	NA	-438.1839	-435.2151	-434.6734	-422.2626	-430.7448	-421.2925	NA	-424.3220	-427.5144
12	-432.7293	-430.9493	-429.1496	-427.1664	-429.4983	-432.9304	-426.8513	-428.4001	-431.2754	-429.2609	-429.6514	NA	-424.6237

Table 5: the table of AIC $\mathrm{D}_S(111)$

-	0	1	2	3	4	5	6	7	8	9	10	11	12
0	NA	**-422.0789**	-418.9676	-414.3019	-413.1208	-411.0682	-406.1221	-401.6038	-399.3821	-395.9672	-391.1321	-386.6849	-383.1085
1	-389.6606	**-418.9767**	-414.0093	-410.0845	-409.5956	-406.0837	-401.1623	-396.1295	-395.0525	-393.7749	-388.8558	-381.8031	-382.6939
2	-416.6874	-414.2205	-412.0139	**-425.5573**	-415.2814	-411.7649	-407.9428	-402.9611	-399.1968	-398.9879	-394.5643	-391.6932	-379.0993
3	-411.7868	-415.2921	-411.1650	-415.0171	-411.5562	-407.2089	-407.3879	-403.6530	-394.1862	-389.1926	-391.0322	-388.0071	-382.9413
4	-409.8062	-412.4616	-407.9533	-411.5081	-407.5502	-405.0849	-409.6617	-394.9883	-401.1305	-384.1752	-382.2835	-377.0710	-374.3655
5	-413.5318	-408.7495	-404.5344	-408.5604	-403.5710	-404.0948	-394.4163	-395.3675	-385.8829	-384.1280	-379.1653	-379.6604	-366.7198
6	-408.5819	-406.4223	-401.4558	-408.6958	-403.2561	-401.5936	-387.9110	-399.6817	-382.0958	-378.0906	-374.1448	-373.4964	-371.6243
7	-403.8088	-400.4905	-396.3472	-403.8795	-400.1387	-395.2130	-393.1822	-378.2625	-377.2130	-373.3629	-375.6739	-370.4311	-367.2960
8	-403.3938	-398.3832	-398.3094	-394.8731	-395.7192	-392.5486	-388.3589	-377.2354	-378.8908	-373.1569	-367.3860	-372.4302	-354.6364
9	-398.3833	-393.3908	-397.4698	-390.6594	-385.9770	-382.4932	-379.7250	-372.5686	-369.8064	-368.6228	-363.4922	-368.5406	-363.6047
10	-393.9347	-391.4562	-394.5046	-390.7032	-384.6570	-379.6611	-374.1635	-373.9766	-373.4454	-370.4884	-367.0998	-363.5014	-349.7158
11	-392.4405	-388.8630	-383.1364	-385.9596	-381.0672	-376.0770	-372.0747	-371.1467	-366.2386	-367.2276	-359.5954	-356.0985	-350.7132
12	-389.4320	-382.6920	-379.1973	-374.3386	-370.4703	-368.4892	-360.5714	NA	-363.2909	-360.2685	-359.5796	-350.5457	-339.6657

Table 6: the table of BIC D$_S(011)$

-	0	1	2	3	4	5	6	7	8	9	10	11	12
0	NA	**-408.9137**	**-405.2325**	-400.2699	-398.7890	-398.8742	-393.9817	-389.8753	-386.1109	-382.1901	-382.7658	-379.3826	-377.6971
1	-378.2833	-405.1898	-400.2284	-397.2333	-395.7608	-393.9272	-389.1822	-385.5357	-381.2616	-377.7359	-380.1956	-374.2417	-376.4245
2	-402.3206	-400.4594	-398.6883	-403.4011	-398.6929	-395.7563	-391.6661	-387.1444	-383.2796	-378.4873	-373.4800	-374.2689	-375.8290
3	-397.3383	-402.7244	-398.8652	**-408.9241**	-405.1977	-389.0512	-397.1829	-382.4816	-378.4159	-378.5880	-374.8371	-373.5862	-372.6074
4	-397.5208	-399.7407	-397.7562	-393.9617	-402.0506	-387.7915	-384.3856	-379.4438	-374.2799	-374.2151	-371.8405	-368.9630	-369.2083
5	-401.1126	-396.2873	-391.8255	-399.2960	-395.1097	-395.7719	-391.8782	-386.5814	NA	-375.4517	-365.3702	-366.7252	-367.4124
6	-396.1521	-395.1401	-390.2366	-386.1341	NA	-391.3497	NA	-382.8129	-377.4748	NA	-361.3042	-368.0234	-362.4553
7	-391.5771	-387.6080	-390.6448	-392.8112	NA	-387.7354	-384.9019	NA	-367.7584	-358.6058	-359.2463	NA	-357.4560
8	-390.5665	-385.6248	-381.0898	-381.6344	NA	-384.9463	NA	NA	-374.0906	-369.1495	-356.8682	NA	-352.6353
9	-385.6944	-382.7088	-377.9061	-374.1232	-372.3886	NA	-360.4537	-363.7014	-360.9215	-366.7180	-350.5986	-358.4100	NA
10	-381.7199	-377.0720	NA	-378.9328	NA	-374.2029	-368.7533	-363.8063	-367.4197	-362.3748	-356.7905	-355.6587	-344.8746
11	-376.9364	-372.1148	-374.4755	-370.0141	-366.6298	NA	-368.2126	-364.4805	NA	-357.9733	-352.3018	NA	-338.9157
12	-372.7230	-372.6706	-369.8905	NA	-361.6470	-356.6381	-353.4473	-358.0489	-358.0229	-350.8733	NA	-350.0298	NA

Table 7: the table of BIC D$_S(110)$

-	0	1	2	3	4	5	6	7	8	9	10	11	12
0	NA	**-428.8757**	**-424.2715**	-419.5895	-417.9566	-415.0343	-410.3739	-406.0653	-402.9004	-398.9390	-395.4651	-390.4649	-388.4997
1	-400.4824	**-424.2999**	-419.0735	-415.1342	-414.1579	-410.1370	-405.3112	-402.0532	-398.1936	-392.8987	-390.6998	-385.6892	-385.0916
2	-419.4948	-419.3028	-416.2739	-420.7049	-416.4800	-411.7536	-408.5186	-403.5081	-399.2108	-394.4310	-389.4203	-384.4521	-380.7943
3	-414.5861	-419.1328	-415.2524	-416.1979	-418.8180	-407.4931	-403.2173	-399.3844	-394.5921	-391.3457	NA	-388.4795	NA
4	-414.0459	-416.3018	-411.4196	-411.4645	-407.2211	-410.6517	-399.0178	-394.4111	-388.7161	-387.9903	NA	-377.8602	-370.3947
5	-416.3411	-411.6812	-407.5843	-406.3974	-404.0521	-404.3258	-399.0765	-394.1865	-384.6285	-383.1207	-378.1214	-373.1483	-369.0611
6	-411.3812	-409.6864	-404.9599	NA	-399.7386	-394.9417	NA	-390.5886	-385.6329	NA	NA	-374.7064	-362.0072
7	-407.2690	-404.0884	-399.4752	-399.8034	-394.9785	-390.1409	-385.2843	NA	-376.7654	NA	-368.6576	-369.6705	NA
8	-406.0616	-401.0687	-396.2234	-389.3531	NA	-384.7964	-384.2122	NA	-369.3420	-369.4160	-365.8227	-367.6208	NA
9	-401.0753	-396.0789	-392.6191	-391.0798	NA	-389.2774	-379.4217	-374.3535	-370.0570	NA	NA	-366.2422	NA
10	-396.1766	-392.7599	NA	-391.2499	-385.3979	-382.3486	-379.4087	-374.3391	-369.2093	-366.7608	-359.3824	-349.9673	-354.8429
11	-391.9293	-388.3966	-383.5093	NA	-383.9925	-378.0131	-374.4607	-359.0393	-364.5109	-352.0479	NA	-349.0561	-349.2378
12	-387.5698	-382.7791	-377.9688	-372.9750	-372.2962	-372.7177	-363.6280	-362.1661	-362.0308	-357.0056	-354.3855	NA	-343.3366

Table 8: the table of BIC D$_S(111)$

-	0	1	2	3	4	5	6	7	8	9	10	11	12
0	NA	1.190636	1.181774	1.181495	1.203319	1.190437	1.179560	1.179295	1.255756	1.184769	1.194196	1.242667	1.235442
1	1.264970	1.181255	1.181450	1.185191	1.190034	1.186180	1.174107	1.172418	1.242245	1.177767	1.184974	1.230251	1.211054
2	1.175361	1.184237	1.187258	1.269335	1.180662	1.176344	1.185316	1.190701	1.177409	1.172639	1.209748	1.156071	1.235382
3	1.175873	1.227364	1.202311	1.182207	1.184699	1.172614	1.241797	1.212351	1.177358	1.180359	1.195917	1.180982	1.168443
4	1.188193	1.178809	1.190425	1.176485	1.171200	**1.149103**	1.186765	1.175003	1.224216	1.172997	1.200768	1.189059	1.253471
5	1.198604	1.196697	1.189485	1.187995	1.185120	1.301320	1.229436	1.238929	1.213062	1.215376	1.211063	1.182855	1.270347
6	1.199110	1.168931	1.171505	1.207264	1.289438	1.231584	1.232315	1.267993	1.237185	1.248944	1.211966	1.237476	1.274825
7	1.202449	1.182614	1.184083	1.175663	1.216663	1.206339	1.223195	1.164108	1.237727	1.231406	1.220491	1.279121	1.246110
8	1.165244	1.165285	1.164266	1.162005	1.200600	1.253111	1.216489	1.224091	1.288024	1.228696	1.192558	1.207395	1.306116
9	1.165329	1.167392	1.187459	1.170287	1.161935	1.216847	1.227376	1.223644	1.220419	1.199170	1.210845	1.243219	1.249365
10	1.166790	1.155907	1.213546	1.212013	1.214089	1.215527	1.198531	1.207240	1.261238	1.211287	1.240140	1.228609	1.426516
11	1.162682	1.163491	**1.143896**	1.214450	1.207508	1.206093	1.187972	1.220429	1.221555	1.175270	1.239342	1.249468	1.455213
12	1.163242	1.173490	1.171549	1.168908	1.182419	1.256283	1.180454	NA	**1.155088**	1.196682	1.196422	1.218843	1.189769

Table 9: the table of MAE $\mathrm{D}_S(011)$

-	0	1	2	3	4	5	6	7	8	9	10	11	12
0	NA	1.687874	1.765894	1.736360	1.349959	2.017362	1.970552	1.766395	2.111272	1.450796	1.150185	1.137998	1.142732
1	1.354834	1.751622	1.762369	1.800044	1.474236	1.998764	2.018608	1.814788	2.065372	2.183902	1.146501	1.161871	1.137362
2	1.680022	1.789698	1.780785	1.520591	1.527001	1.579005	1.413003	1.655686	1.798468	1.733540	1.713419	1.141886	**1.102417**
3	1.683849	1.167854	1.169409	1.721536	1.742118	1.711284	1.312643	1.195794	1.782239	1.198677	1.170700	**1.113738**	1.172677
4	1.784469	1.774708	1.779101	1.593779	1.308418	1.208903	1.197754	1.195505	1.192995	1.188392	1.156451	1.152180	1.197716
5	1.812177	1.825882	1.810617	1.613177	1.933094	1.772806	1.822051	1.798101	NA	2.006064	1.192629	1.171453	1.396179
6	1.816380	1.237491	1.225108	1.154716	NA	1.237245	NA	1.807217	1.263255	NA	1.208634	1.313556	1.184163
7	1.840743	1.791546	1.720173	1.312050	NA	1.329114	1.309444	NA	1.163482	1.190359	1.173115	NA	1.181402
8	1.689885	1.695656	1.694304	1.726324	NA	1.304480	NA	NA	1.623738	1.639918	**1.136365**	NA	1.229191
9	1.702429	1.272964	1.262065	1.693814	1.725241	NA	1.651652	1.257515	1.221289	1.288322	1.167011	1.272658	NA
10	1.704191	1.759370	NA	1.340854	NA	1.219899	1.356885	1.380370	1.287398	1.284997	1.292457	1.288696	1.213357
11	1.754614	1.775488	1.684814	1.182236	1.159428	NA	1.332375	1.441984	NA	1.301065	1.302842	NA	1.151484
12	1.727539	1.145426	1.221227	NA	1.177831	1.174490	1.248205	1.331746	1.300308	1.309370	NA	1.264794	NA

Table 10: the table of MAE $\mathrm{D}_S(110)$

-	0	1	2	3	4	5	6	7	8	9	10	11	12
0	NA	1.333306	1.330542	1.309529	1.264830	1.346798	1.296620	1.258691	1.347646	1.276215	1.236764	1.239221	1.239000
1	1.317235	1.329023	1.340708	1.345473	1.276554	1.336884	1.350326	1.262559	1.332743	1.354050	1.247657	1.247453	1.264326
2	1.240186	1.326355	1.293426	1.282028	1.258345	1.269061	1.250700	1.251365	1.293256	1.265280	1.265111	1.275657	1.205365
3	1.243026	1.234615	1.239355	1.256765	1.374934	1.254875	1.252437	1.260273	1.276548	1.278219	NA	1.344189	NA
4	1.297161	1.336539	1.350579	1.266672	1.258590	1.381747	1.265205	1.260958	1.281275	1.236885	NA	1.237164	1.271881
5	1.340264	1.354693	1.350075	1.261733	1.231498	1.372869	1.375863	1.376838	1.299961	1.237504	1.237785	1.239060	1.243948
6	1.338083	1.280044	1.273532	NA	1.249701	1.259566	NA	1.391872	1.316772	NA	NA	1.290862	1.244109
7	1.359696	1.349146	1.306059	1.288805	1.284842	1.268436	1.266575	NA	1.257683	NA	1.247567	1.306835	NA
8	1.340340	1.341751	1.335313	1.311851	NA	1.252009	1.252702	NA	1.207554	1.224299	**1.188922**	1.222268	NA
9	1.342102	1.342095	1.298338	1.242698	NA	1.346326	1.247330	1.246977	1.227724	NA	NA	1.303500	NA
10	1.335786	1.293136	NA	1.376813	1.321320	1.388392	1.338234	1.337759	1.350986	1.252866	1.314592	1.233091	1.267254
11	1.297069	1.259637	1.262287	NA	1.376784	1.205582	1.319218	1.243837	1.313588	**1.133414**	NA	1.226796	1.421340
12	1.274578	1.280885	1.277187	1.274370	1.266663	1.332733	1.282914	**1.203080**	1.347252	1.238687	1.217785	NA	1.241406

Table 11: the table of MAE $\mathrm{D}_S(111)$

-	0	1	2	3	4	5	6	7	8	9	10	11	12
0	NA	1.493438	1.492674	1.489465	1.506380	1.501873	1.492748	1.483472	1.569050	1.489658	1.494706	1.539861	1.543773
1	1.583746	1.491531	1.492119	1.496067	1.492201	1.498591	1.487637	1.485483	1.549511	1.490997	1.496491	1.527320	1.496204
2	1.483873	1.495266	1.500175	1.552908	1.473512	1.484432	1.477924	1.480329	1.485912	1.465663	1.497041	1.457781	1.510621
3	1.484790	1.536873	1.503134	1.474920	1.483088	1.481449	1.532016	1.506853	1.485865	1.488430	1.494719	1.472847	1.474522
4	1.498049	1.488619	1.500966	1.484944	1.479457	**1.452829**	1.485691	1.482144	1.524342	1.481172	1.492645	1.495003	1.538027
5	1.507028	1.506543	1.498296	1.476669	1.474899	1.593040	1.514117	1.525499	1.507780	1.518952	1.515133	1.485777	1.558490
6	1.508555	1.484595	1.483035	1.502308	1.587643	1.564725	1.510697	1.551763	1.518778	1.528277	1.516561	1.549994	1.534913
7	1.509494	1.488884	1.491423	1.473252	1.513654	1.503379	1.564312	1.467463	1.518881	1.509446	1.531483	1.585980	1.564979
8	1.475824	1.475838	1.468473	1.465484	1.493011	1.576626	1.543002	1.508276	1.583232	1.509957	1.486312	1.515330	1.586582
9	1.475869	1.477580	1.475808	1.465516	1.465993	1.502611	1.505410	1.505164	1.502033	1.502675	1.491953	1.541338	1.546391
10	1.478619	1.462629	1.497795	1.499921	1.500062	1.501105	1.505488	1.513139	1.563249	1.535896	1.536265	1.525193	1.741996
11	1.460328	1.459188	**1.455624**	1.500230	1.493434	1.491668	1.474847	1.512203	1.512730	1.503517	1.516613	1.555117	1.765075
12	1.459001	1.469645	1.467806	1.465019	1.468007	1.525350	1.465773	NA	**1.456906**	1.489197	1.481809	1.535780	1.461752

Table 12: the table of RMSE $\mathrm{D}_S(011)$

-	0	1	2	3	4	5	6	7	8	9	10	11	12
0	NA	2.132970	2.232992	2.195522	1.717882	2.506176	2.461152	2.229626	2.591614	1.818990	1.455926	1.466745	1.447230
1	1.721538	2.214802	2.228518	2.281344	1.859566	2.488517	2.506740	2.293566	2.548085	2.659892	1.461363	1.461982	**1.436002**
2	2.119439	2.263814	2.259491	1.918036	1.914405	1.993876	1.780969	2.087728	2.265200	2.185944	2.161708	1.481611	**1.411715**
3	2.125107	1.501421	1.485767	2.172278	2.196256	2.158288	1.611202	1.520615	2.246788	1.500134	1.492885	1.436709	1.444121
4	2.262705	2.246958	2.251909	1.999942	1.605079	1.551246	1.526468	1.524347	1.520001	1.470738	1.485518	1.458183	1.502468
5	2.289985	2.308970	2.290354	2.042368	2.412453	2.243618	2.294321	2.269519	NA	2.488871	1.475807	1.473161	1.734143
6	2.295245	1.573026	1.558775	1.472886	NA	1.530596	NA	2.281120	1.550750	NA	1.507068	1.617646	1.476028
7	2.322462	2.265471	2.137362	1.607154	NA	1.628428	1.600343	NA	1.471202	1.508484	1.508657	NA	1.472073
8	2.123924	2.132573	2.129120	2.146424	NA	1.596414	NA	NA	2.079795	2.096828	**1.419180**	NA	1.521780
9	2.142626	1.608880	1.596880	2.129506	2.151992	NA	2.071843	1.591241	1.547308	1.587417	1.495377	1.552780	NA
10	2.143793	2.218356	NA	1.661949	NA	1.496416	1.685632	1.716419	1.602606	1.577392	1.599624	1.574086	1.487470
11	2.210758	2.237558	2.100469	1.468983	1.446450	NA	1.656735	1.810663	NA	1.613544	1.604696	NA	1.468896
12	2.180522	1.456761	1.508178	NA	1.463890	1.460638	1.568651	1.640435	1.615740	1.613530	NA	1.578948	NA

Table 13: the table of RMSE $\mathrm{D}_S(110)$

-	0	1	2	3	4	5	6	7	8	9	10	11	12
0	NA	1.690317	1.679409	1.653088	1.569951	1.695553	1.632450	1.559703	1.701944	1.576156	1.514285	1.516586	1.511886
1	1.589414	1.677233	1.701878	1.697630	1.592646	1.682537	1.697639	1.561449	1.677844	1.710809	1.524819	1.524625	1.541372
2	1.534711	1.674809	1.606621	1.600801	1.559265	1.572014	1.551097	1.552247	1.605298	1.564078	1.563856	1.577524	**1.482012**
3	1.538541	1.519662	1.522254	1.556942	1.655373	1.554488	1.546583	1.563740	1.587928	1.575622	NA	1.629228	NA
4	1.621915	1.678045	1.694882	1.572524	1.559376	1.687674	1.569390	1.565386	1.594380	1.505224	NA	1.526845	1.556736
5	1.668787	1.696801	1.687966	1.564038	1.519413	1.657873	1.673600	1.674455	1.600038	1.504045	1.505463	1.506846	1.507892
6	1.665446	1.591752	1.574729	NA	1.543860	1.558698	NA	1.688646	1.614076	NA	NA	1.579021	1.509280
7	1.697365	1.689202	1.607789	1.593617	1.586587	1.551971	1.549518	NA	1.570254	NA	1.515219	1.592594	NA
8	1.661663	1.662608	1.649202	1.636581	NA	1.534237	1.529149	NA	1.482584	1.528072	**1.459491**	1.551222	NA
9	1.662674	1.662274	1.605342	1.527055	NA	1.628414	1.518667	1.521251	1.495277	NA	NA	1.599326	NA
10	1.653564	1.597587	NA	1.678452	1.610154	1.686522	1.620319	1.619716	1.635745	1.558111	1.603456	1.537325	1.584172
11	1.602998	1.558536	1.560534	NA	1.696500	1.505195	1.601521	1.542913	1.602652	**1.461922**	NA	1.495504	1.734419
12	1.573806	1.579368	1.575661	1.573716	1.559399	1.649865	1.583421	1.497872	1.633088	1.518665	1.495554	NA	1.520343

Table 14: the table of RMSE $\mathrm{D}_S(111)$