Calculus

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For all lovers of mathematics.

Introduction

Calculus is the study of continuous change established by **Issac Newton** (1643–1727) and **Gottfried Wilhelm Leibniz** (1646–1716) in the 17th century. **Single variable calculus** studies **derivatives** and **integrals** of functions of one variable and their relationship stated by the **fundamental theorem of calculus**.

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

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1.1 The Limit of a Function

A function $f : X \mapsto Y$ is a rule that assigns each element in set X to exactly one element $y = f(x)$ in set Y.

Definition 1.1. A **function** f is a binary relation R with domain X and codomain Y that satisfies:

- R is a subset of the Cartesian product of X and Y.

$$R \subset \{(x, y) \mid x \in X, y \in Y\}$$

- For every x in X, there exists a $y = f(x)$ such that (x, y) is in R.

$$\forall x \in X, \exists y = f(x) \in Y, (x, y) \in R$$

- If (x, y) and (x, z) are in R, then $y = z$.

$$(x, y) \in R \wedge (x, z) \in R \implies y = z$$

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