

# Calculus

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*For all lovers of mathematics.*

## Introduction

**Calculus** is the study of continuous change established by **Issac Newton** (1643–1727) and **Gottfried Wilhelm Leibniz** (1646–1716) in the 17th century. **Single variable calculus** studies **derivatives** and **integrals** of functions of one variable and their relationship stated by the **fundamental theorem of calculus**.

$$\int_a^b f(x) dx = F(b) - F(a)$$

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## **1.2 The Precise Definition of a Limit**

## **1.3 Computing Limits**

## **1.4 Continuity**

## **1.5 Limits and Infinity**

# **2 Derivatives**

## **2.1 Derivatives**

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## **9 Differential Equations**

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