

# Calculus

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## Introduction

**Calculus** is the mathematical study of continuous change established by **Issac Newton** and **Gottfried Wilhelm Leibniz** in the 17th century. **Single variable calculus** studies **derivatives** and **integrals** of functions of one variable and their relationship stated by the **fundamental theorem of calculus**.

$$\int_a^b f(x)dx = F(b) - F(a)$$

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# **1 Functions and Limits**

## **1.1 The Limit of a Function**

## **1.2 The Precise Definition of a Limit**

## **1.3 Computing Limits**

## **1.4 Continuity**

## **1.5 Limits and Infinity**

# **2 Derivatives**

## **2.1 Derivatives**

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## **2.3 Implicit Differentiation**

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## **7 Sequences and Series**

### **7.1 Sequences**

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### **7.3 Convergence Tests**

### **7.4 Power Series**

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## **8 Parametric Equations and Polar Coordinates**

### **8.1 Calculus of Parametric Equations**

### **8.2 Calculus in Polar Coordinates**

## **9 Differential Equations**

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