Calculus

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Introduction

Calculus is the mathematical study of continuous change established by **Issac Newton** and **Gottfried Wilhelm Leibniz** in the 17th century. **Single variable calculus** studies **derivatives** and **integrals** of functions of one variable and their relationship stated by the **fundamental theorem of calculus**.

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

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