MTH 4140 Graph Theory

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Homework 4

Introduction to Graph Theory

Chapter 4 Connectivity and Paths

Section 4.1 Cuts and Connectivity

Problem 1. Excercise 4.1.1

Solution. (a) We know that a graph is k-connected if its vertex connectivity is at least k. If a graph G is 2-connected, then its vertex connectivity is at least 2. Given G has vertex connectivity 4 so we have $\kappa(G) = 4 \geq 2$ which is true. The statement "Every graph with connectivity 4 is 2-connected." is true.
(b) A 3-connected graph G has vertex connectivity at least 3 so $\kappa(G) \geq 3$. Consider the graph K_5, we know that $\kappa(G) \leq \delta(G)$ so $\kappa(K_5) \leq 4$. We know that $\kappa(K_n) = n - 1$ so we get that $\kappa(K_5) = 4 > 3$. This is a counterexample since K_5 is 3-connected but it has vertex connectivity 4. The statement "Every 3-connected graph has connectivity 3." is false.
(c) The statement "Every k-connected graph is k-edge-connected." is true.
(d) The statement "Every k-edge-connected graph is k-connected." is false. ■

Chapter 5 Coloring of Graphs

Section 5.1 Vertex Coloring and Upper Bounds

Section 5.2 Structure of k-chromatic Graphs

Section 5.3 Enumerative Aspects