MTH 4140 Graph Theory

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Homework 4

Introduction to Graph Theory

Chapter 4 Connectivity and Paths

Section 4.1 Cuts and Connectivity

Problem 1. Excercise 4.1.1

Solution. (a) A graph is k-vertex-connected if its vertex-connectivity is at least k. If a graph G is 2-connected, then its vertex-connectivity is at least 2. Given G has vertex-connectivity 4 so we have $\kappa(G) = 4 \geq 2$ which is true. The statement "Every graph with connectivity 4 is 2-connected." is true.

(b) A 3-connected graph G has vertex-connectivity at least 3 so $\kappa(G) \geq 3$. Consider the graph K_5, we know that $\kappa(G) \leq \delta(G)$ so $\kappa(K_5) \leq 4$. We know that $\kappa(K_n) = n - 1$ so we get that $\kappa(K_5) = 4 > 3$. This is a counterexample since K_5 is 3-connected but it has vertex-connectivity 4. The statement "Every 3-connected graph has connectivity 3." is false.

(c) A graph is k-edge-connected if every disconnecting set has at least k edges. Consider a k-vertex-connected graph G, we know that $\kappa(G) \leq \kappa'(G)$ from Whitney's theorem. We can deduce that

$$k \leq \kappa(G) \leq \kappa'(G)$$

The edge-connectivity $\kappa'(G)$ is the minimum size of a disconnecting set and it is at least k so G is k-edge-connected. The statement "Every k-connected graph is k-edge-connected." is true.

(d) Consider a k-edge-connected graph G with vertex-connectivity $\kappa(G)$ and edge-connectivity $\kappa'(G)$. We have $\kappa(G) \leq \kappa'(G)$ and $k \leq \kappa'(G)$. If $k > \kappa(G)$, then we have

$$\kappa(G) < k \leq \kappa'(G)$$

and Whitney's theorem still holds so G is not k-vertex-connected.
The statement "Every k-edge-connected graph is k-connected." is false. ■

Chapter 5 Coloring of Graphs

Section 5.1 Vertex Coloring and Upper Bounds

Section 5.2 Structure of k-chromatic Graphs

Section 5.3 Enumerative Aspects