MTH 4140 Homework 1

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Problem 1

Section 1.1 Problem 9

Solution. Let the graph on the left be G and the graph on the right be H. It is equivalent to show that $G \cong \overline{H}$. If we label every vertex in G then two vertices are adjacent if and only if they are adjacent in $\overline{H}$. Therefore, it is proved that $G \cong \overline{H}$ so $\overline{G} \cong H$. ■

Problem 2

Section 1.1 Problem 17

Solution. We know that $G \cong H \iff \overline{G} \cong \overline{H}$ so it is equivalent to count all 2-regular graphs on 7 vertices. Only the graph C_7 and the graph with only C_3, C_4 satisfy this. Therefore, the number of isomorphic classes of simple 7-vertex 4-regular graphs is 2. ■

Problem 3

Section 1.1 Problem 25

Solution. Suppose the Peterson graph has C_7 then there are 3 remaining vertices outside of C_7. The Peterson graph is 3-regular so each of the 7 vertices on C_7 has one extra edge outside of C_7. The Peterson graph has girth 5 so the 7 extra edges have to have one of the 3 remaining vertices as an endpoint. By the pigeonhole principle, at least 3 of the extra edges end on the same vertex. WLOG, we will have a C_3 but the Peterson graph has girth 5 so there is a contradiction. Therefore, it is proved that the Peterson graph does not have a cycle of length 7. ■