

# MTH 4320 Homework 7

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## Problem 1

*Solution.* We run Prim's algorithm to find a minimum spanning tree (MST) of the graph with root  $d$  then the edges added to the MST are  $\{d, e\}, \{e, g\}, \{e, h\}, \{b, e\}, \{a, b\}, \{a, c\}, \{c, f\}$  from first to last respectively. ■

## Problem 2

*Solution.* We can use a modified Prim's algorithm to find the maximum spanning tree of  $G$ . We can find the greatest edge  $e$  instead of the lightest edge while keeping everything else in the algorithm the same. We can multiply the weight of every edge by  $-1$  and we can use  $-\infty$  instead of  $\infty$  in the priority queue. Therefore, when we remove a vertex with the highest priority in the queue we will choose the edge with the highest weight in the original graph. The time complexity of the modified Prim's algorithm is  $O(|V| \log |V| + |E|)$ . ■

## Problem 3

*Solution.* Since the weight of each edge is either 0 or 5 thus we can directly choose the lightest edge incidental to each vertex. Let  $u$  be an arbitrary vertex in  $G$ , we check all of its incidental edges. If an edge has weight 0 then we add that edge to connect the adjacent vertex  $v$ . If all the edges have weight 5 then we can add any edge. Hence, we can make sure that we select the lightest edge to connect each vertex in  $G$ . We run operations with  $O(1)$  time for  $|V|$  vertices and  $|E|$  edges so the time complexity of the algorithm is  $O(|V| + |E|)$ . ■

## Problem 4

*Solution.* The time complexity of the algorithm is  $O(|V| + |E|)$ . ■