MTH 4320 Homework 9

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Problem 1

Solution. Let $P[i]$ be the length of the longest primish subsequence $S[i]$ that ends with element $L[i]$ for $i = 1, 2, \ldots, n$. We have

$$P[1] = 1$$

$$P[2] = \begin{cases} 1 + P[1], & \text{if } S[1] + L[2] \text{ is prime} \\ 1, & \text{otherwise} \end{cases}$$

$$P[3] = \begin{cases} 1 + \max_{1 \le j < 3} P[j], & \text{if } S[j] + L[3] \text{ is prime} \\ 1, & \text{otherwise} \end{cases}$$

In general, we can deduce that

$$P[i] = \begin{cases} 1 + \max_{1 \le j < i} P[j], & \text{if } S[j] + L[i] \text{ is prime} \\ 1, & \text{otherwise} \end{cases}$$

The algorithm returns the length of the longest primish subsequence in L by computing

$$P = \max_{1 \le i \le n} P[i]$$

Assuming we can check if a number is prime in $O(1)$ time, computing $P[i]$ takes $O(i)$ time so computing P takes $O(1) + O(2) + \cdots + O(n) = O(n^2)$ time. The time complexity of the algorithm is $O(n^2)$. ■

Problem 2

Solution. Let $P(i, j)$ be the length of the longest palindrome in substring $S[i, j]$ that starts with the ith letter and ends with jth letter. We have

$$P(i, j) = \begin{cases} 1, & \text{if } i = j \\ 2 + P(i+1, j-1), & \text{if } S[i] = S[j] \\ \max\{P(i, j-1), P(i+1, j)\}, & \text{otherwise} \end{cases}$$

For $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, n$, we compute each $P(i, j)$ in $O(1)$ time so the time complexity of the algorithm is $O(n^2)$. ■

Problem 3

Solution. Let $F(i,j)$ be the minimum damage for traveling a list of oases that starts at $L[i]$ and ends at $L[j]$. We have

$$F(i,j) = \begin{cases} 0, & \text{if } i = j \\ (10-x)^2 \text{ where } x \text{ is the distance between } L[i] \text{ and } L[j], & \text{if } i = j-1 \\ (10-x)^2 + \min_{1 \le k < i} F(i,k) + \min_{i < k \le j} F(k,j), & \text{otherwise} \end{cases}$$

For $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, n$, we compute each $F(i,j)$ in $O(1)$ time so the time complexity of the algorithm is $O(n^2)$. ■