MTH 4320 Homework 4

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Problem 1

Solution. The edges of the breadth-first search (BFS) tree of the graph G starting from root g are $e_1 = \{g, b\}, e_2 = \{g, i\}, e_3 = \{g, j\}, e_4 = \{b, a\}, e_5 = \{b, c\}, e_6 = \{i, d\}, e_7 = \{i, f\}, e_8 = \{j, e\}, e_9 = \{j, h\}$ in the order from first to last respectively, that which was to be demonstrated. ■

Problem 2

Solution. The algorithm is

1. Let the vertex of every squirrel be the root of a BFS tree.
2. Run a modified BFS alogrithm for every squirrel to find all vertices that each root can reach using at most 5 edges.
3. If there is a common vertex in the BFS trees of all five squirrels then return true else return false.

This is a modified BFS alogrithm so we need to visit every vertex and edge that are necessary to check. Visiting all vertices takes $O(|V|)$ time, visiting all edges takes $O(|E|)$ time, and all other operations takes $O(1)$ time. Therefore, the running time is $O(|V| + |E|)$. ■

Problem 3

Solution. We can change the graph G to G' by adding an vertex in the middle of every blue edge s.t. that G' has all uncolored edges of weight 1. Then we can apply the BFS algorithm on G' starting from root s to find the shortest paths tree in G'. The shortest paths tree from the vertex s in G' will be the same as the shortest paths tree in G after we remove the extra vertices. The number of vertices we can add in G' is at most $|E(G)|$ so G' has $|V(G)| + |E(G)|$ vertices. Hence the number of edges in G' is at most $2 \cdot |E(G)|$. Since we are using the BFS algorithm on G',the running time is

$$\begin{aligned} O(|V(G')| + |E(G')|) &= O(|V(G)| + |E(G)| + 2 \cdot |E(G)|) \\ &= O(|V(G)| + |E(G)|) \\ &= O(|V(G) + E(G)|) \end{aligned}$$

Therefore, the time complexity of the algorithm is $O(|V + E|)$. ■

Problem 4

Solution. The algorithm is

1. Start from any vertex v and run the BFS algorithm on graph G.
2. If G is connected then the set of all of the vertices in G is the largest commune.
3. If G is not connected then we run BFS for each component of G. We update the largest commune with the set of all vertices in the component that has the largest number of vertices.

The BFS algorithm visits all of the vertices and edges in the graph so the time complexity is $O(|V| + |E|)$. ■