An Accurate Reconstruction of CMB E Mode Signal over Large Angular Scales using Prior Information of CMB Covariance Matrix in ILC Algorithm

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Polarization of CMB

- Expressed in of Stokes Q and U parameters
- Local Observables
- Transformed to E mode and B mode mathematically.

$$Q(\hat{n}) \pm iU(\hat{n}) = \sum_{\ell=2}^{\ell_{max}} a_{\pm 2,\ell m} Y_{\pm 2,\ell m}(\hat{n})$$

$$E_{\ell m} \equiv -\frac{1}{2}[a_{2,\ell m} + a_{-2,\ell m}],$$

$$B_{\ell m} \equiv -\frac{1}{2i}[a_{2,\ell m} - a_{-2,\ell m}].$$

$$C_\ell^{EE} \equiv \frac{1}{2\ell+1}\sum_m \langle E_{\ell m} E^*_{\ell m}\rangle$$

$$C_\ell^{BB} \equiv \frac{1}{2\ell+1}\sum_m \langle B_{\ell m} B^*_{\ell m}\rangle$$

E-mode

- Important cosmological probe
- Large angular scale E mode -- free electrons in space-- to tightly constrain the ionization history.
- i.g. peak height and location of EE mode are determined by the amount of the free electrons
- Accurate measures of A_s and τ of reionization epoch by breaking degeneracies between the two.
- l <15 EE mode: constrain the physics at the last scattering surface as well.

Foreground removal

- weak E mode hidden behind the weakly understood, strong polarized foreground signals
- synchrotron and thermal dust emissions that originate from the Milky Way
- Blind; Non-blind; Semi-blind

Internal Linear Combination

Full-sky observations of CMB polarization at ***n*** different frequency maps, with N pixels each map

$$X(\hat{n}) = \sum_{i=1}^{n} w_i S^i(\hat{n}) \,, \quad [N,1] \tag{4}$$

where following the usual ILC method, the weight factors w_i can be found by minimizing the variance (σ^2) of the cleaned map defined as

$$\sigma^2 = \mathbf{X}^T\mathbf{X} \,, \quad [1,N]*[N,1] = [1,1] \tag{5}$$

where, $\mathbf{X}$ represents $N \times 1$ column vector representing the cleaned map. N denotes the total number of pixels in the map. Using Eqns. 4 in Eqn. 5 we obtain,

$$\sigma^2 = \mathbf{W}\mathbf{A}\mathbf{W}^T \,, \quad [1,n]*[n,n]*[n,1] = [1,1] \tag{6}$$

where $\mathbf{W}$ is $1 \times n$ row vector containing the weights and (i,j) element of $n \times n$ matrix $\mathbf{A}$ is given by,

$$A_{ij} = \mathbf{S}_i^T \mathbf{S}_j \,. \quad [1,n]*[n,1] = [1,1] \tag{7}$$

Internal Linear Combination

Full-sky observations of CMB polarization at ***n*** different frequency maps, with N pixels each map

CMB follows black body spectrum with a very good accuracy its polarization anisotropy are independent on frequency.

$S_i = S_{CMB} + S_f^i$;
$X = \sum w_i S_i = \sum w_i S_{CMB} + \sum w_i S_f^i$

$$\sum_{i=1}^{n} w_i e_i = \mathbf{W}\mathbf{e}^T = 1$$

$$\mathbf{W} = \frac{\mathbf{e}\mathbf{A}^{\dagger}}{\mathbf{e}\mathbf{A}^{\dagger}\mathbf{e}^T}$$

ILC variants
Sudevan et al. 1612.03401

- Iterative ILC algorithm in harmonic space

- Divided in to several small regions
- ILC in each region
- Replace the region in the initial maps by the ILC-cleaned one

Sudevan & Saha 1810.08872

- Gibbs ILC method
- To estimate the joint CMB posterior density and CMB theoretical angular power spectrum
- Provide the best fit estimates of both CMB temperature map and theoretical angular power spectrum

Sudevan & Saha 1712.09804

- Global ILC method on temperature maps
- using prior information of CMB covariance matrix.
- suppressing CMB-foregrounds chance correlations at large angular scales,
- reconstructed a cleaned map and its angular power spectrum accurately without any bias.

ILC bias

- due to CMB-foreground chance correlation

- The data covariance matrix **C** follows, $\mathbf{C} = \sigma_c^2 \mathbf{e}^T\mathbf{e} + \mathbf{C}_{fc} + \mathbf{C}_f$
  where σ_c^2 represents the variance of the CMB component
- **C**fc - the chance correlation between the CMB and all foreground components for a given realization of CMB (e.g., pure CMB signal in our Universe)

$$\left\langle \hat{C}_l^{Clean} \right\rangle = \left\langle \hat{C}_l^{c} \right\rangle - n_f \frac{\left\langle \hat{C}_l^{c} \right\rangle}{2l+1}$$

Prior information

- theoretical covariance matrix C.
- minimizing CMB covariance weighted variance (σ^2_{red}) of the cleaned map instead of usual variance.

$$\sigma^2_{\mathtt{red}} = \mathbf{X}^T \mathbf{C}^\dagger \mathbf{X}$$

$$C_{ij} = \sum_{\ell=2}^{\ell_{max}} \frac{2\ell+1}{4\pi} C_\ell^E \mathcal{P}_\ell(\hat{n}_i \cdot \hat{n}_j) B_\ell^2 P_\ell^2$$

$$A_{ij} = \mathbf{S}_i^T \mathbf{C}^\dagger \mathbf{S}_j$$

Moore-Penrose generalized inverse

Simulation Data

Simulated full-sky polarization maps at COrE 15 frequencies

Dust and synchrotron with rigid frequency spectrum model of $\beta_s = -3.0$ and $\beta_d = 1.6$, fixed without any randomness.

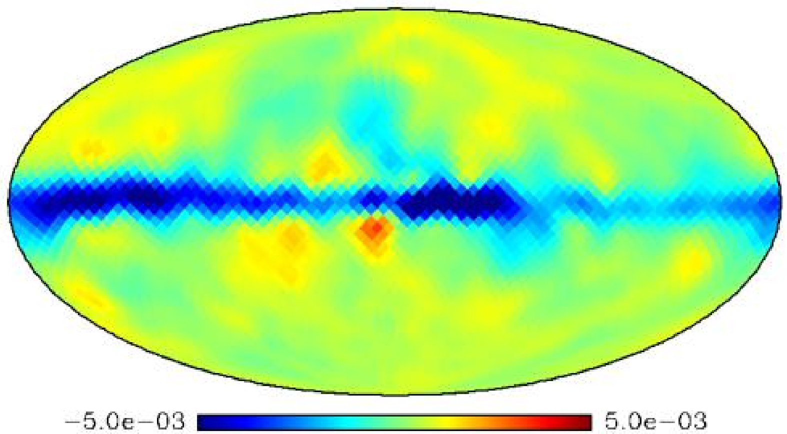
Detector noise signals are random and uncorrelated between any two of these input sets.

1000 random realizations; Nside = 16

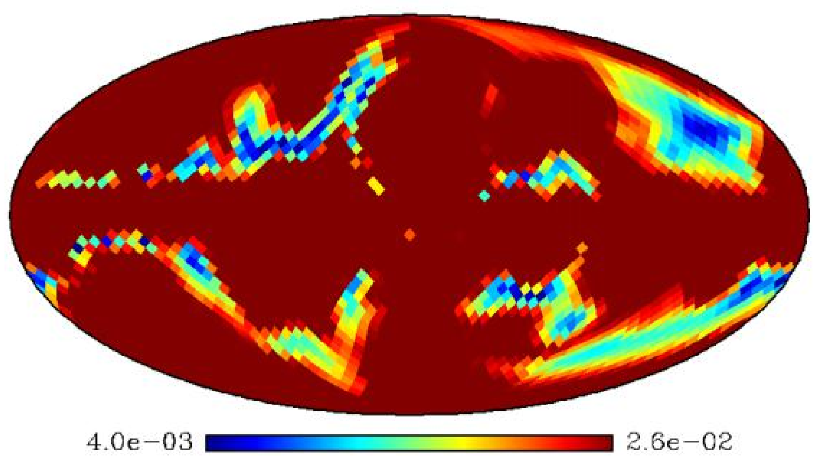
Frequency (GHz)	Beam FWHM (arcmin)	$\Delta Q = \Delta U$ (μK.arcmin)
60	17.87	7.49
70	15.39	7.07
80	13.52	6.78
90	12.08	5.16
100	10.92	5.02
115	9.56	4.95
130	8.51	3.89
145	7.68	3.61
160	7.01	3.68
175	6.45	3.61
195	5.84	3.46
220	5.23	3.81
255	4.57	5.58
295	3.99	7.42
340	3.49	11.10

Usual ILC

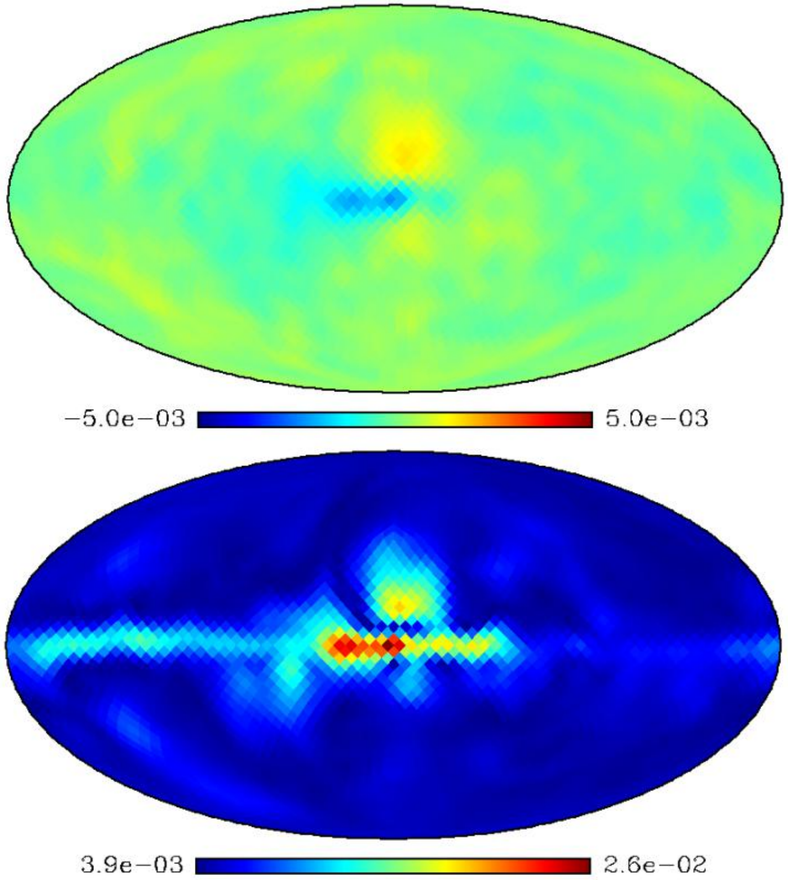
mean difference maps between input E-mode maps and ILC-output maps



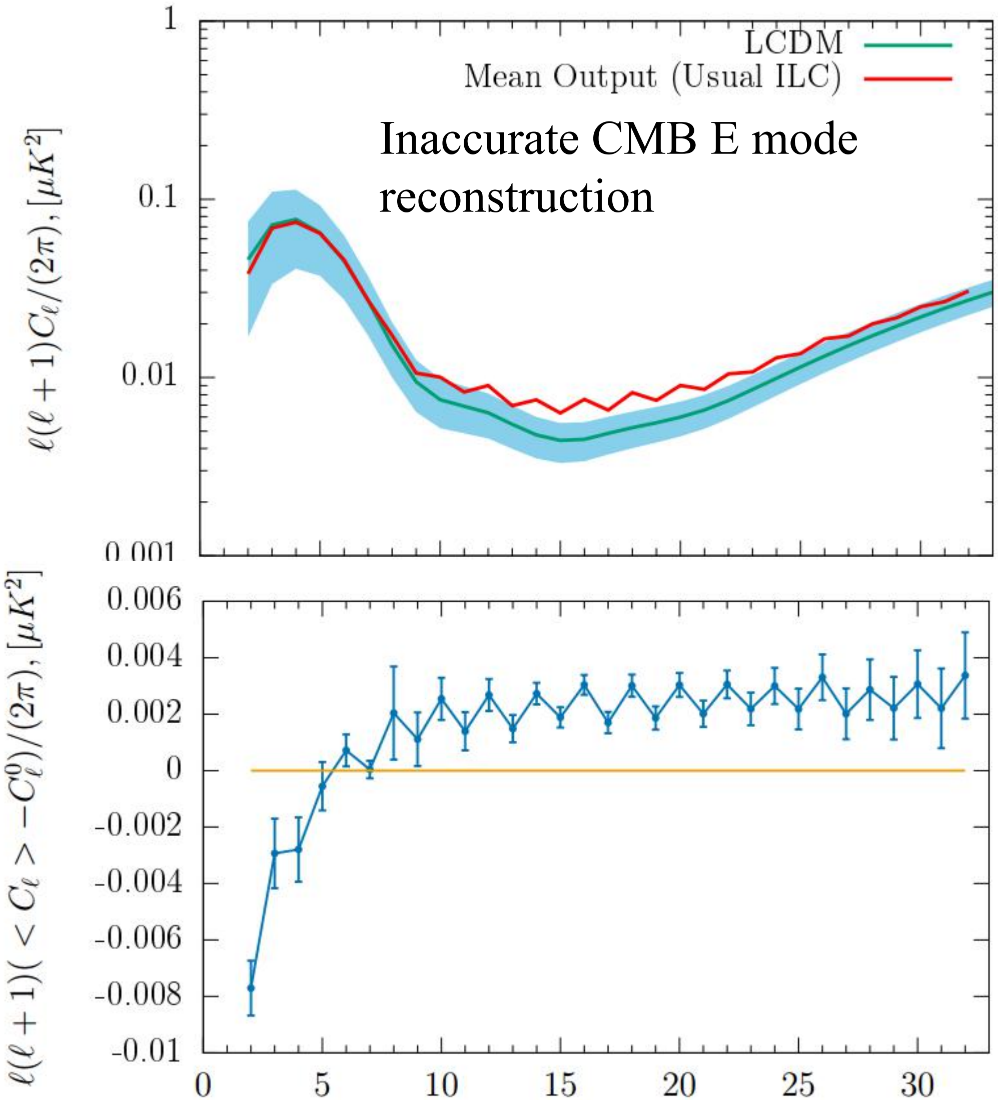
std

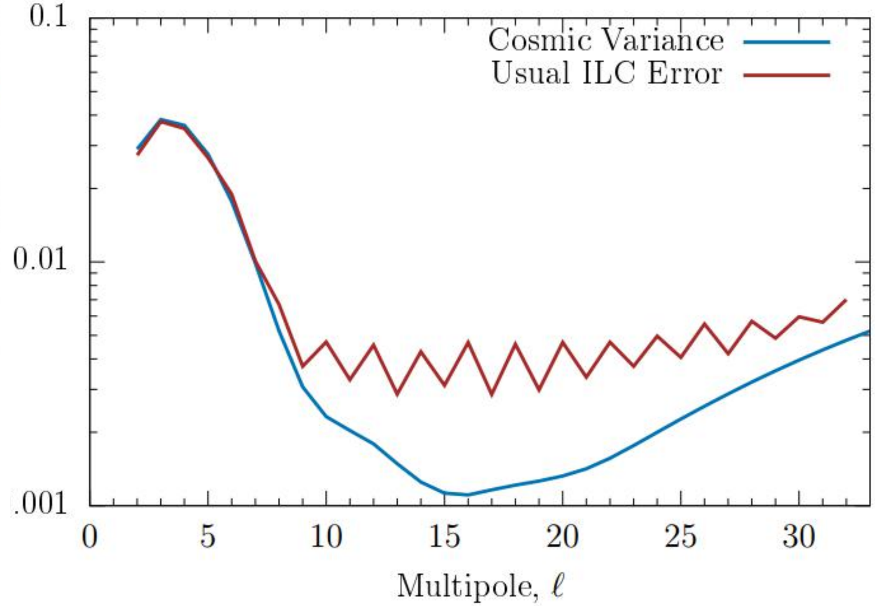


Global ILC



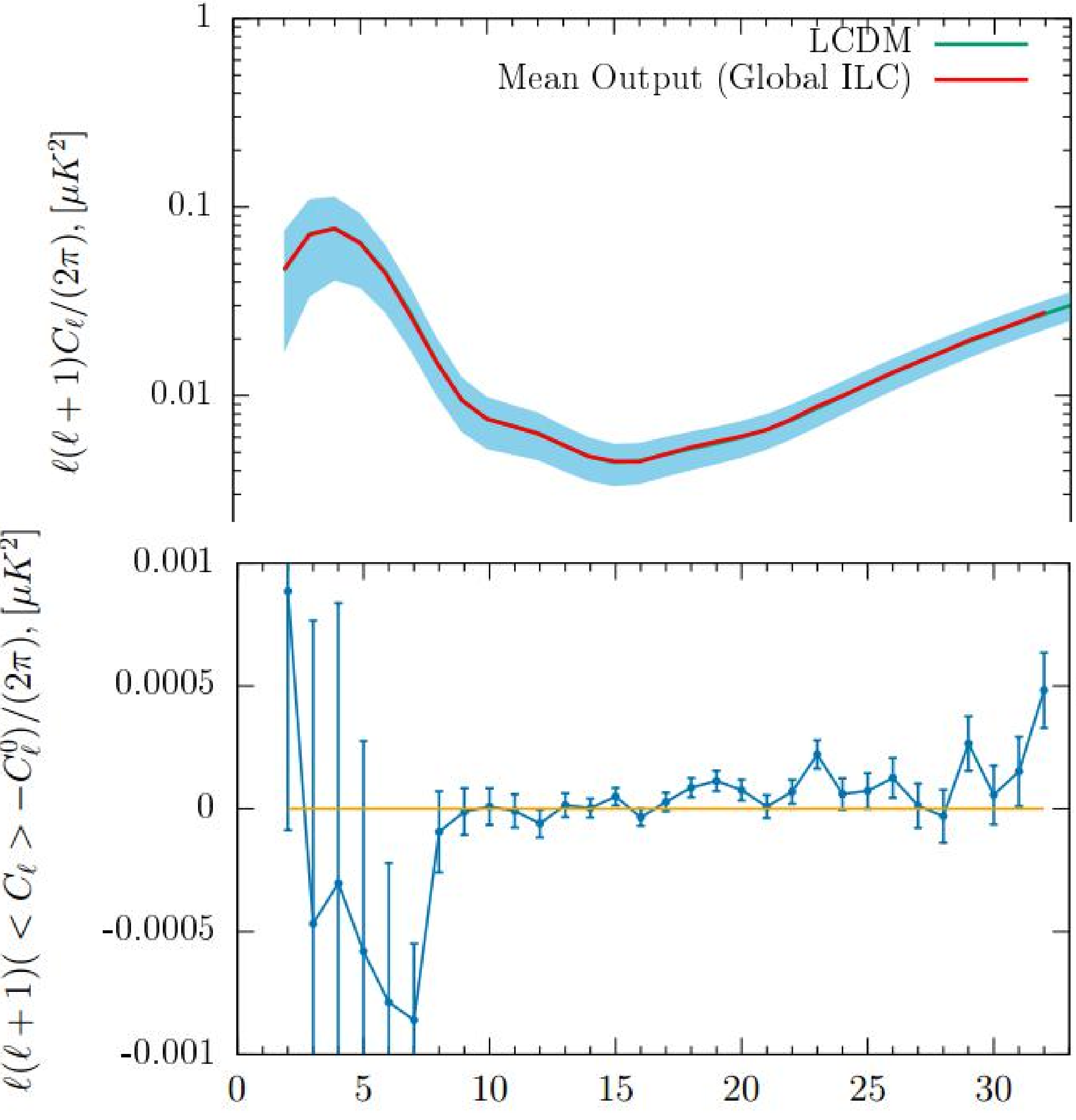
Usual ILC



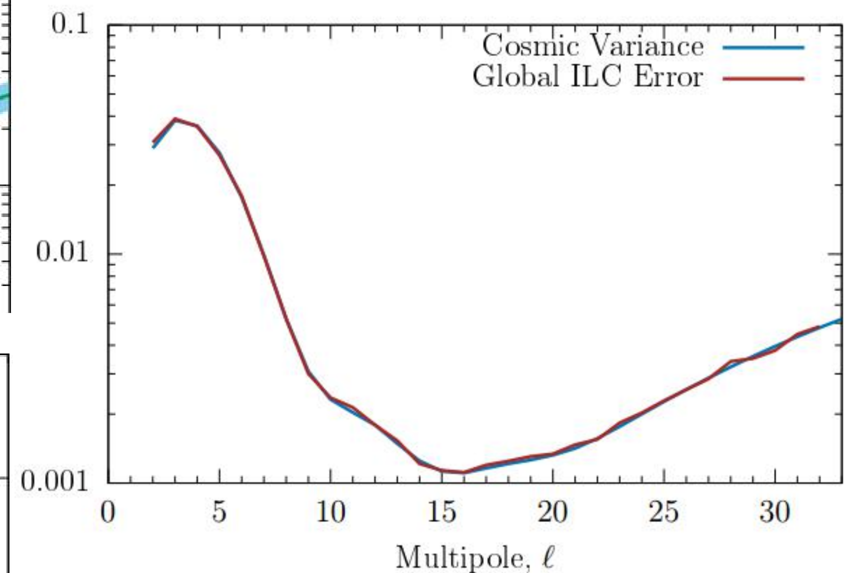


Significantly larger errors compared to the cosmic variance prediction is observed.

Global ILC



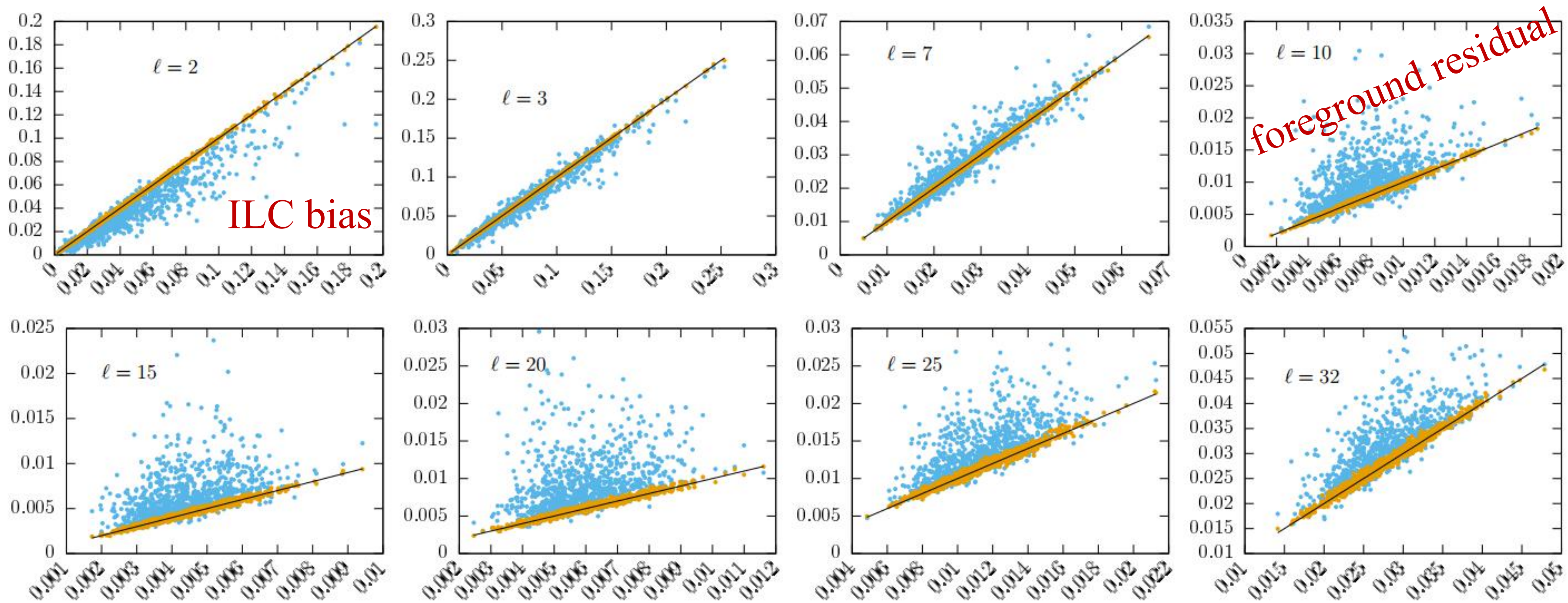
LCDM
Mean Output (Global ILC)
$\ell(\ell+1)C_\ell/(2\pi)$, $[\mu K^2]$
$\ell(\ell+1)(<C_\ell> - C_\ell^0)/(2\pi)$, $[\mu K^2]$
1
0.1
0.01
0.001
0.0005
0
-0.0005
-0.001
0
5
10
15
20
25
30



Cosmic Variance
Global ILC Error
0.1
0.01
0.001
0
5
10
15
20
25
30
Multipole, ℓ

E mode angular power spectra

blue: usual ILC; yellow: global ILC

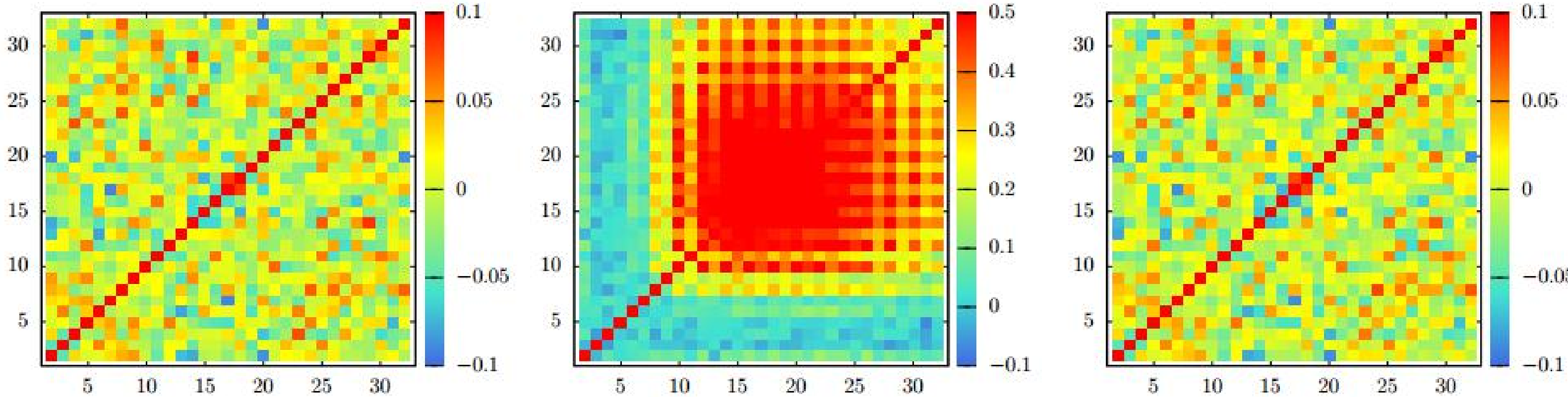


Horizontal axis of each subplot denotes pure CMB E mode power spectrum.

Vertical coordinates are given by the cleaned angular power spectra for all 1000 cases.

l-l' Correlation matrix

Estimated using the angular power spectrum from 1000 input CMB E mode maps (left panel), cleaned CMB E mode maps after removing foregrounds using usual ILC method (middle panel) and using global ILC method (right panel).



-- correlation matrix estimated from global ILC method is consistent with the input E mode CMB maps.
-- usual ILC method a strong correlation between multipoles due to residual foreground contaminations is visible.

Conclusion

- The empirical data covariance matrix in the usual ILC method contains a chance correlation power between the CMB and non-CMB components (e.g., foregrounds).
- Using the underlying covariance matrix effectively suppresses the chance correlation power and results in the accurate estimation of CMB E mode signal
- The new E mode CMB angular power spectrum contains **neither** any significant negative bias at the low multipoles **nor** any positive foreground bias at relatively higher mutlipoles.