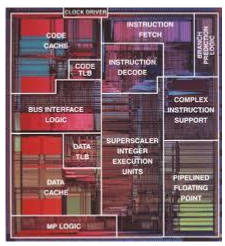
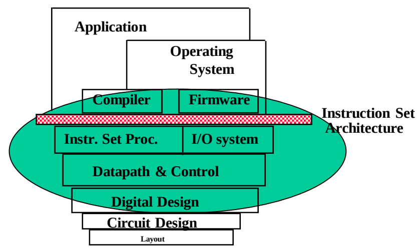
CS/SE 3340
Computer Architecture





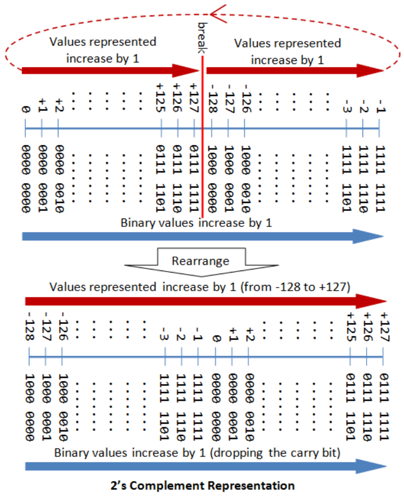
Other Number Representations

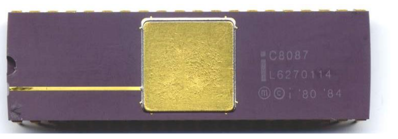
Adapted from "Computer Organization and Design, 4th Ed." by D. Patterson and J. Hennessy

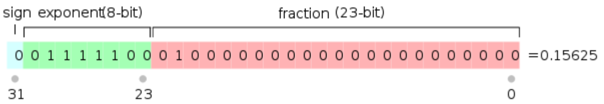
Questions

- How to represent negative integers?
- What is sign-magnitude, 1-complement and 2-complement representations and which on we should use?
- What is sign-extension for integers and how to do it?
- How to represent floating-point numbers?
- How to convert floating-point numbers represented in binary to decimal and vice-versa?

2







Negative Integers

- Natural (positive) integer numbers can be represented easily in memory of a computer in bits (binary format)
- But what about negative integers?
  - How do we handle negative integer numbers in a computer?
- How do we encode negative integers in binary format ?
  - *Can we just think of negative numbers as a positive one with the '-' sign in front of it?*

4

Representing Negative Integers

- Two common methods: *sign-magnitude representation* and ***complementary representation***
- Sign-magnitude representation
  - Encode sign (+/-) information separately
- Complementary representation
  - Do not encode sign information separately, use a complementary number
  - The complementary has a special relationship (complementary) to the original number and can be derived from it easily

5

Sign-Magnitude Representation

- The MSb (Most Significant bit) is used to encode the sign
  - '1' means negative, '0' means positive
  - The rest is used to encode the magnitude
- Thus an n-bit word can be used to represent $-(2^{(N-1)} - 1)$ to $+(2^{(N-1)} - 1)$
  - For example an 8-bit word: -127 to +127
  - *Why only 255 possibilities with 8-bit?*

6

Sign-Magnitude Representation Issues

- Zero (0) is represented twice!
  - +0 and -0
  - No such thing as two zeros in math!
- Not intuitive
  - Direct adding a number I and its negative (-I) in binary does not yield 0!
  - Have to treat the sign bit separately
- Not efficient to implement sign handling in hardware!

1-complement Representation

- Recall that in complementary representation N (positive) and –N (negative) has a special relationship
  - Must be easy to derive –N from N
- How about reversing all bits of N to get –N?
  - Replacing '1' with '0' and '0' with '1' in N to get -N
  - E.g., in 8-bit words: `9 = 00001001, -9 = 11110110`
- This is 1-complement representation
  - Efficient hardware implementation using a NOT (inverse) gate for each bit

1-Complement Representation – cont'd

- An n-bit word can be used to represent

  from $-(2^{(N-1)} - 1)$ to $+(2^{(N-1)} - 1)$

  255 possibilities for 8-bit words, still missing one possibility!
- For 8-bit words

  `+127 = 01111111, -127 = 10000000`

  `+0   = 00000000, -0   = 11111111`
- Still having the problem of two '0's!
- But at least now I – I = -0!
- How about I + 0 ?
  - Only works with +0!!!

2-complement Representation

- Let tweak the relationship in 1-complement scheme a bit: get –N by inversing all bits and then add 1
  - That is, get 1-complement of N and then add 1 to it
  - For example: with 8-bit words, **9 = 00001001,**

    -9 = 11110111 (11110110 + 1)
- This is 2-complement representation
  - Can be implemented easily and efficiently in hardware (inverse and then +1)
- An n-bit word can be used to represent numbers from $-2^{(N-1)}$ to $+(2^{(N-1)} - 1)$

2-complement Representation – cont'd

```
+10     =   00001010
-10     =   11110110
+127    =   01111111
-127    =   10000001
+0      =   00000000
-0      =   00000000
```

- No more two '0's problem!!!
- And I – I = 0!
- Complementary representation's added benefit: *the MSb can be used to determine whether a number is positive or negative!*

Integer Representations

Example: *Using 4 bit numbers*

unsigned	sign/magnitude	1's complement	2's complement
1111 [15]	0111 [7]	0111 [7]	0111 [7]
1110	0110	0110	0110 [6]
1101	0101	0101	0101 [5]
1100	0100	0100	0100 [4]
1011	0011	0011	0011 [3]
1010	0010	0010	0010 [2]
1001	0001	0001	0001 [1]
1000	0000, **1**000 [-0]	0000, **1**111 [-0]	0000 [0]
0111	**1**001	**1**110	**1**111 [-1]
0110	**1**010	**1**101	**1**110 [-2]
0101	**1**011	**1**100	**1**101 [-3]
0100	**1**100	**1**011	**1**100 [-4]
0011	**1**101	**1**010	**1**011 [-5]
0010	**1**110	**1**001	**1**010 [-6]
0001	**1**111 [-7]	**1**000 [-7]	**1**001 [-7]
0000 [0]			**1**000 [-8]

Sign Extension

- Representing a number using more bits
  - Preserve the numeric value
- *Replicate the sign bit to the left*
  - c.f. unsigned values: extend with 0s
- Examples: 8-bit to 16-bit
  - +2: 0000 0010 => 0000 0000 0000 0010
  - –2: 1111 1110 => 1111 1111 1111 1110
- In MIPS instruction set
  - `addi`: extend immediate value
  - beq, bne: extend the displacement

Floating Point

- Representation for non-integral numbers
  - Including very small and very large numbers
- Like scientific notation (in base 10)
  - -2.34×10^{56} ← normalized
  - $+0.002 \times 10^{-4}$ ← not normalized
  - $+987.02 \times 10^{9}$ ← not normalized
- In binary (base 2)
  - $\pm 1.xxxxxxx_2 \times 2^{yyyy}$
- Types **`float`** and **`double`** in C

Floating Point Standard

- Defined by IEEE Std 754-1985
- Developed in response to divergence of representations
  - Portability issues for scientific code
- Now almost universally adopted
- Two representations
  - *Single precision* (32-bit)
  - *Double precision* (64-bit)

15

IEEE Floating-Point Format

	single: 8 bits double: 11 bits	single: 23 bits double: 52 bits
S	Exponent	Fraction

$$x = (-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$$

- S: sign bit ($0 \Rightarrow$ non-negative, $1 \Rightarrow$ negative)
- Normalize significand: 1.0 ≤ |significand| < 2.0
  - Always has a leading pre-binary-point 1 bit, so no need to represent it explicitly (hidden bit)
  - Significand is Fraction with the “1.” restored
- Exponent: excess representation: actual exponent + Bias
  - Ensures exponent is unsigned
  - Single: Bias = 127; Double: Bias = 1023

16

Single-Precision Range

- Exponents 00000000 and 11111111 reserved
- Smallest value
  - Exponent: 00000001
    $\Rightarrow$ actual exponent = 1 – 127 = –126
  - Fraction: 000…00 $\Rightarrow$ significand = 1.0
  - $\pm 1.0 \times 2^{-126} \approx \pm 1.2 \times 10^{-38}$
- Largest value
  - exponent: 11111110
    $\Rightarrow$ actual exponent = 254 – 127 = +127
  - Fraction: 111…11 $\Rightarrow$ significand ≈ 2.0
  - $\pm 2.0 \times 2^{+127} \approx \pm 3.4 \times 10^{+38}$

17

Double-Precision Range

- Exponents 0000…00 and 1111…11 reserved
- Smallest value
  - Exponent: 00000000001
    $\Rightarrow$ actual exponent = 1 – 1023 = –1022
  - Fraction: 000…00 $\Rightarrow$ significand = 1.0
  - $\pm 1.0 \times 2^{-1022} \approx \pm 2.2 \times 10^{-308}$
- Largest value
  - Exponent: 11111111110
    $\Rightarrow$ actual exponent = 2046 – 1023 = +1023
  - Fraction: 111…11 $\Rightarrow$ significand ≈ 2.0
  - $\pm 2.0 \times 2^{+1023} \approx \pm 1.8 \times 10^{+308}$

18

Floating-Point Precision

- Relative precision
  - all fraction bits are significant
  - Single: approx 2^{-23}
    - Equivalent to $23 \times \log_{10}2 \approx 23 \times 0.3 \approx 6$ decimal digits of precision
  - Double: approx 2^{-52}
    - Equivalent to $52 \times \log_{10}2 \approx 52 \times 0.3 \approx 16$ decimal digits of precision

19

Floating-Point Example

- Represent –0.625 in floating point binary

  $-0.625 = -5/8 = -1 \times 101_2 \times 2^{-3} = \mathbf{-1 \times 1.01_2 \times 2^{-1}}$

  normalized

  - S = 1
  - Fraction = $01000\ldots00_2 = (0/2^1 + 1/2^2 + 0/2^3 + \ldots)$
  - Exponent = –1 + Bias
    - Single: $-1 + 127 = 126 = 01111110_2$
    - Double: $-1 + 1023 = 1022 = 01111111110_2$
- Single: 10111111001000…00
- Double: 1011111111101000…00

20

Floating-Point Example – cont'd

- What number is represented by the single-precision float below?

  11000000101000…00

  - S = 1
  - Fraction = $01000…00_2$
  - Exponent = 10000001_2 = 129
- $x = (-1)^1 \times (\mathbf{1} + 0.01_2) \times 2^{(129-127)}$

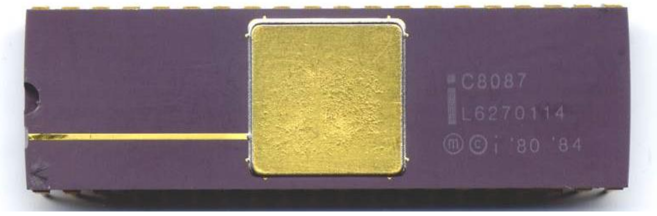
  $= (-1) \times 1.25 \times 2^2$

  $= -5.0$

Was removed when encoded

21

Intel C8087 FPU

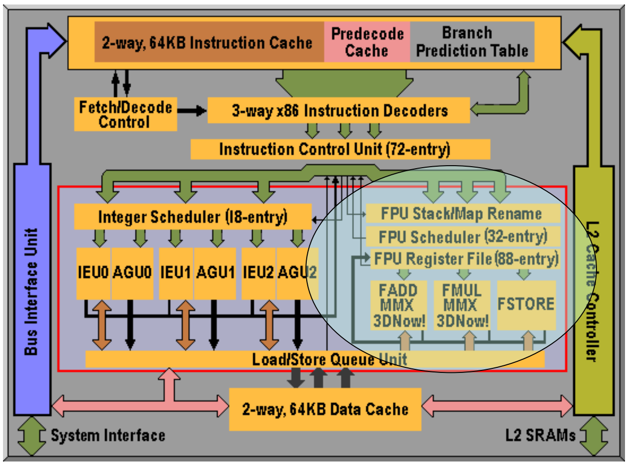


- Introduced by Intel in 1980, cost ~$150
- Runs at 4MHz ~ 10MHz
- Can perform 50,000 FLOPS using around 2.4 watts
- Boost application performance by %20 to %500

http://en.wikipedia.org/wiki/Intel_8087

22

AMD Athlon



2.4 GFLOPS at 600MHz!

http://www.pctechguide.com/amd-technology/amd-athlon

23