

Given:

DeMorgan's Law:

$$\overline{A_1 \cap A_2} = \overline{A_1} \cup \overline{A_2}$$

Prove by induction:

$$\overline{\bigcap_{j=1}^n A_j} = \bigcup_{j=1}^n \overline{A_j}$$

where $n \geq 2$.

Base case:

for $n=2$, true because of DeMorgan's Law

Inductive step:

Assume true for k :

$$\overline{\bigcap_{j=1}^k A_j} = \bigcup_{j=1}^k \overline{A_j}$$

where $k \geq 2$.

Show true for $k+1$:

$$\overline{\bigcap_{j=1}^{k+1} A_j} = \bigcup_{j=1}^{k+1} \overline{A_j}$$

For $k+1$:

$$\overline{\bigcap_{j=1}^{k+1} A_j} = \overline{\bigcap_{j=1}^k A_j \cap A_{k+1}}$$

$$\overline{\bigcap_{j=1}^k A_j} \cup \overline{A_{k+1}}$$

$$\bigcup_{j=1}^k \overline{A_j} \cup \overline{A_{k+1}}$$

$$\bigcup_{j=1}^{k+1} \overline{A_j}$$

Comments:

unroll last term

split apart using DeMorgan's law

use the inductive hypothesis

combine

Conclusion:

Since the base case was true and the inductive step was shown to be true, by induction it is true for all $n \geq 2$.