# Assignment #1

10 points
1) Suppose your calculator only did base 10 logarithms.

Write an expression to compute log base 2 of 2048 using only log base 10.

$$\log_A B = \log_C B / \log_C A$$

$$\Rightarrow \quad \log_2 2048 = \frac{\log_{10} 2048}{\log_{10} 2}$$

10 points
2) Express the following summation in closed form (an expression that can be directly computed from k). (Refer to slide 11)

3 + 5 + 7 + 9 + ... + 2k+1

Writen as: $2(1+2+3+\cdots+k)+(1+1+1+\cdots+1)$

$$= 2\left(\frac{(k+1)k}{2}\right) + k$$

$$= k(k+2)$$

10 points
3) Proof by counterexample

Prove that the following statement is false: n^3 > 2^n for any n >= 1

for $n=1$, $n^3 = 1$, $2^n = 2$, $n^3 < 2^n$

Conclude: the statement is false.

10 points
4) Proof by contradiction

Prove that the following statement is true: the square of an even number is also even

Assume the statement is false: given even $x$, then $x^2$ is odd.

If $x^2$ is odd, then $x^2 = 2b+1$

But $x = 2a$ means $x^2 = (2a)^2 = 4a^2 = 2 \cdot (2a^2)$

So $2(2c^2) = 2b+1$, but this says an even number equals and odd number, which is impossible.

Therefore, the square of an even number is also even.

30 points

5) Induction proofs.

a. Prove by induction:

$$\sum_{i=1}^{n} i^3 = [n^2][(n+1)^2]/4$$

Note: "sum" is the summation symbol, and ^ is for an exponent

b. Prove by induction:

$n^2 - n$ is even for any $n >= 1$

a. Basic step: $n=1$, the sum is 1, and $1^2 \cdot 2^2/4 = 1$, so it is true for $n=1$.

Inductive step:

Assume true for $k$.

$$\sum_{i=1}^{k} i^3 = k^2 \cdot (k+1)^2/4$$

Show true for $k+1$:

$$\sum_{i=1}^{k+1} i^3 = (k+1)^2 \cdot (k+2)^2/4$$

$$\sum_{i=1}^{k+1} = \sum_{i=1}^{k} + (k+1)^3 = k^2 \cdot (k+1)^2/4 + (k+1)^3$$

$$= k^2(k+1)^2/4 + 4(k+1)^3/4$$

$$= (k+1)^2(k^2 + 4(k+1))/4$$

$= (k+1)^2 (k^2+4k+2^2)/4$

$= (k+1)^2 \cdot (k+2)^2/4$

Conclusion: by induction. the statement holds true for all $n \geq 1$

b. Basic step: $n=1$, $1^2-1=0$ which is even, so it is true for $n=1$.

Induction step.

Assume true for $k$:

$$k^2-k = 2a$$

show true for $k+1$:

$$(k+1)^2-(k+1) = 2b$$

$$(k+1)^2-(k+1) = k^2+2k+1-k-1$$

$$= k^2-k+2k$$

$$= 2a+2k$$

$$= 2(a+k)$$

$$= 2b$$

Conclusion: by induction. the statement holds true for all $n \geq 1$

20 points
6) Recursion.

Note: You can use Java or pseudocode for these. If pseudocode then the logic must be complete and easy to understand.

a. Write a recursive function that when passed a value n displays

n (n-1) (n-2) (n-3) ... 0 ... (n-3) (n-2) (n-1) n

for example, if passed 5 displays

5 4 3 2 1 0 1 2 3 4 5

```
class Solution {

    public static void display(int n) {
        if (n == 0) {
            System.out.print(n + " ");
            return;
        }
        System.out.print(n + " ");
        display(n - 1);
        System.out.print(n + " ");
    }

}
```

b. Write a recursive function that receives an array of integers and a position as parameters and returns the count of odd numbers in the array. Let each recursive call consider the next integer in the array.

```
class Solution {

    public static int countOdd(int[] arr) {
        return helper(arr, 0);
    }

    private static int helper(int[] arr, int pos) {
        if (pos == arr.length) {
            return 0;
        }
        int count = helper(arr, pos + 1);
        if ((arr[pos] & 1) == 1) {
            return count + 1;
        }
        return count;
    }

}
```

10 points

7) Suppose there exists a generic Java class named Pair with type parameter T that stores two objects with get and set methods for each. Write the statements necessary to create an object of type Pair with String as its type parameter, and use the set methods to set the two strings, then the get methods to retrieve them for printing. Note that you do not need to write the Pair class itself.

```
public class Pair<T> {

    private T obj1;
    private T obj2;

    public T getObj1() {
        return obj1;
    }
    public void setObj1(T obj1) {
        this.obj1 = obj1;
    }
    public T getObj2() {
        return obj2;
    }
    public void setObj2(T obj2) {
        this.obj2 = obj2;
    }

    public static void main(String[] args) {
        Pair<String> pair = new Pair<>();
        pair.setObj1("123"); // Arbitrary String
        pair.setObj2("456"); // Arbitrary String
        System.out.println(pair.getObj1());
        System.out.println(pair.getObj2());
    }

}
```