

## CHAPTER 17: INDEXING STRUCTURES FOR FILES

Consider a disk with block size  $B=512$  bytes. A block pointer is  $P=6$  bytes long, and a record pointer is  $P_R=7$  bytes long. A file has  $r=30,000$  EMPLOYEE records of fixed-length. Each record has the following fields: NAME (30 bytes), SSN (9 bytes), DEPARTMENTCODE (9 bytes), ADDRESS (40 bytes), PHONE (9 bytes), BIRTHDATE (8 bytes), SEX (1 byte), JOBCODE (4 bytes), SALARY (4 bytes, real number). An additional byte is used as a deletion marker.

- (a) Calculate the record size  $R$  in bytes.
- (b) Calculate the blocking factor  $bfr$  and the number of file blocks  $b$  assuming an unspanned organization.
- (c) Suppose the file is ordered by the key field SSN and we want to construct a primary index on SSN. Calculate (i) the index blocking factor  $bfr_i$  (which is also the index fan-out  $fo_i$ ); (ii) the number of first-level index entries and the number of first-level index blocks; (iii) the number of levels needed if we make it into a multi-level index; (iv) the total number of blocks required by the multi-level index; and (v) the number of block accesses needed to search for and retrieve a record from the file--given its SSN value--using the primary index.
- (d) Suppose the file is not ordered by the key field SSN and we want to construct a secondary index on SSN. Repeat the previous exercise (part c) for the secondary index and compare with the primary index.
- (e) Suppose the file is not ordered by the non-key field DEPARTMENTCODE and we want to construct a secondary index on SSN using Option 3 of Section 18.1.3, with an extra level of indirection that stores record pointers. Assume there are 1000 distinct values of DEPARTMENTCODE, and that the EMPLOYEE records are evenly distributed among these values. Calculate (i) the index blocking factor  $bfr_i$  (which is also the index fan-out  $fo_i$ ); (ii) the number of blocks needed by the level of indirection that stores record pointers; (iii) the number of first-level index entries and the number of first-level index blocks; (iv) the number of levels needed if we make it a multi-level index; (v) the total number of blocks required by the multi-level index and the blocks used in the extra level of indirection; and (vi) the approximate number of block accesses needed to search for and retrieve all records in the file having a specific DEPARTMENTCODE value using the index.
- (f) Suppose the file is ordered by the non-key field DEPARTMENTCODE and we want to construct a clustering index on DEPARTMENTCODE that uses block anchors (every new value of DEPARTMENTCODE starts at the beginning of a new block). Assume there are 1000 distinct values of DEPARTMENTCODE, and that the EMPLOYEE records are evenly distributed among these values. Calculate (i) the index blocking factor  $bfr_i$  (which is also the index fan-out  $fo_i$ ); (ii) the number of first-level index entries and the number of first-level index blocks; (iii) the number of levels needed if we make it a multi-level index; (iv) the total number of blocks required by the multi-level index; and (v) the number of block accesses needed to search for and retrieve all records in the file having a specific DEPARTMENTCODE value using the clustering index (assume that multiple blocks in a cluster are either contiguous or linked by pointers).

(g) Suppose the file is not ordered by the key field Ssn and we want to construct a B + -tree access structure (index) on SSN. Calculate (i) the orders p and p leaf of the B + -tree; (ii) the number of leaf-level blocks needed if blocks are approximately 69% full (rounded up for convenience); (iii) the number of levels needed if internal nodes are also 69% full (rounded up for convenience); (iv) the total number of blocks required by the B + -tree; and (v) the number of block accesses needed to search for and retrieve a record from the file--given its SSN value--using the B + -tree.

**Answer:**

(a) Record length  $R = (30 + 9 + 9 + 40 + 9 + 8 + 1 + 4 + 4) + 1 = 115$  bytes

(b) Blocking factor  $bfr = \text{floor}(B/R) = \text{floor}(512/115) = 4$  records per block  
 Number of blocks needed for file =  $\text{ceiling}(r/bfr) = \text{ceiling}(30000/4) = 7500$

(c) i. Index record size  $R_i = (V_{SSN} + P) = (9 + 6) = 15$  bytes  
 Index blocking factor  $bfr_i = fo = \text{floor}(B/R_i) = \text{floor}(512/15) = 34$   
 ii. Number of first-level index entries  $r_1 = \text{number of file blocks } b = 7500$  entries  
 Number of first-level index blocks  $b_1 = \text{ceiling}(r_1 / bfr_i) = \text{ceiling}(7500/34) = 221$  blocks  
 iii. We can calculate the number of levels as follows:  
 Number of second-level index entries  $r_2 = \text{number of first-level blocks } b_1 = 221$  entries  
 Number of second-level index blocks  $b_2 = \text{ceiling}(r_2 / bfr_i) = \text{ceiling}(221/34) = 7$  blocks  
 Number of third-level index entries  $r_3 = \text{number of second-level index blocks } b_2 = 7$  entries  
 Number of third-level index blocks  $b_3 = \text{ceiling}(r_3 / bfr_i) = \text{ceiling}(7/34) = 1$   
 Since the third level has only one block, it is the top index level.  
 Hence, the index has  $x = 3$  levels  
 iv. Total number of blocks for the index  $b_i = b_1 + b_2 + b_3 = 221 + 7 + 1 = 229$  blocks  
 v. Number of block accesses to search for a record =  $x + 1 = 3 + 1 = 4$

(d) i. Index record size  $R_i = (V_{SSN} + P) = (9 + 6) = 15$  bytes  
 Index blocking factor  $bfr_i = (\text{fan-out}) fo = \text{floor}(B/R_i) = \text{floor}(512/15) = 34$  index records per block  
 (This has not changed from part (c) above)

ii. Number of first-level index entries  $r_1 = \text{number of file records } r = 30000$   
 Number of first-level index blocks  $b_1 = \text{ceiling}(r_1 / bfr_i) = \text{ceiling}(30000/34) = 883$  blocks  
 iii. We can calculate the number of levels as follows:  
 Number of second-level index entries  $r_2 = \text{number of first-level index blocks } b_1 = 883$  entries  
 Number of second-level index blocks  $b_2 = \text{ceiling}(r_2 / bfr_i) = \text{ceiling}(883/34) = 26$  blocks  
 Number of third-level index entries  $r_3 = \text{number of second-level index blocks } b_2 = 26$  entries  
 Number of third-level index blocks  $b_3 = \text{ceiling}(r_3 / bfr_i) = \text{ceiling}(26/34) = 1$   
 Since the third level has only one block, it is the top index level.  
 Hence, the index has  $x = 3$  levels

- iv. Total number of blocks for the index  $b_i = b_1 + b_2 + b_3 = 883 + 26 + 1 = 910$   
 v. Number of block accesses to search for a record  $= x + 1 = 3 + 1 = 4$

(e) i. Index record size  $R_i = (V_{\text{DEPARTMENTCODE}} + P) = (9 + 6) = 15$  bytes  
 Index blocking factor  $bfr_i = (\text{fan-out})_{fo} = \text{floor}(B/R_i) = \text{floor}(512/15)$   
 $= 34$  index records per block

ii. There are 1000 distinct values of DEPARTMENTCODE, so the average number of records for each value is  $(r/1000) = (30000/1000) = 30$

Since a record pointer size  $P_R = 7$  bytes, the number of bytes needed at the level of indirection for each value of DEPARTMENTCODE is  $7 * 30 = 210$  bytes, which fits in one block. Hence, 1000 blocks are needed for the level of indirection.

iii. Number of first-level index entries  $r_1$

$=$  number of distinct values of DEPARTMENTCODE  $= 1000$  entries

Number of first-level index blocks  $b_1 = \text{ceiling}(r_1 / bfr_i) = \text{ceiling}(1000/34)$   
 $= 30$  blocks

iv. We can calculate the number of levels as follows:

Number of second-level index entries  $r_2 =$  number of first-level index blocks  $b_1$   
 $= 30$  entries

Number of second-level index blocks  $b_2 = \text{ceiling}(r_2 / bfr_i) = \text{ceiling}(30/34) = 1$

Hence, the index has  $x = 2$  levels

v. total number of blocks for the index  $b_i = b_1 + b_2 + b_{\text{indirection}}$   
 $= 30 + 1 + 1000 = 1031$  blocks

vi. Number of block accesses to search for and retrieve the block containing the record pointers at the level of indirection  $= x + 1 = 2 + 1 = 3$  block accesses

If we assume that the 30 records are distributed over 30 distinct blocks, we need an additional 30 block accesses to retrieve all 30 records. Hence, total block accesses needed on average to retrieve all the records with a given value for DEPARTMENTCODE  $= x + 1 + 30 = 33$

(f) i. Index record size  $R_i = (V_{\text{DEPARTMENTCODE}} + P) = (9 + 6) = 15$  bytes  
 Index blocking factor  $bfr_i = (\text{fan-out})_{fo} = \text{floor}(B/R_i) = \text{floor}(512/15)$   
 $= 34$  index records per block

ii. Number of first-level index entries  $r_1$

$=$  number of distinct DEPARTMENTCODE values  $= 1000$  entries

Number of first-level index blocks  $b_1 = \text{ceiling}(r_1 / bfr_i)$   
 $= \text{ceiling}(1000/34) = 30$  blocks

iii. We can calculate the number of levels as follows:

Number of second-level index entries  $r_2 =$  number of first-level index blocks  $b_1$   
 $= 30$  entries

Number of second-level index blocks  $b_2 = \text{ceiling}(r_2 / bfr_i) = \text{ceiling}(30/34) = 1$

Since the second level has one block, it is the top index level.

Hence, the index has  $x = 2$  levels

iv. Total number of blocks for the index  $b_i = b_1 + b_2 = 30 + 1 = 31$  blocks

v. Number of block accesses to search for the first block in the cluster of blocks  
 $= x + 1 = 2 + 1 = 3$

The 30 records are clustered in  $\text{ceiling}(30/bfr) = \text{ceiling}(30/4) = 8$  blocks.

Hence, total block accesses needed on average to retrieve all the records with a given DEPARTMENTCODE  $= x + 8 = 2 + 8 = 10$  block accesses

(g) i. For a B+ -tree of order  $p$ , the following inequality must be satisfied for each internal tree node:  $(p * P) + ((p - 1) * V_{SSN}) < B$ , or

$(p * 6) + ((p - 1) * 9) < 512$ , which gives  $15p < 521$ , so  $p=34$

For leaf nodes, assuming that record pointers are included in the leaf nodes, the following inequality must be satisfied:  $(p_{leaf} * (V_{SSN} + P_R)) + P < B$ , or

$(p_{leaf} * (9+7)) + 6 < 512$ , which gives  $16p_{leaf} < 506$ , so  $p_{leaf}=31$

ii. Assuming that nodes are 69% full on the average, the average number of key values in a leaf node is  $0.69 * p_{leaf} = 0.69 * 31 = 21.39$ . If we round this up for convenience, we get 22 key values (and 22 record pointers) per leaf node. Since the file has 30000 records and hence 30000 values of SSN, the number of leaf-level nodes (blocks) needed is  $b_1 = \text{ceiling}(30000/22) = 1364$  blocks

iii. We can calculate the number of levels as follows:

The average fan-out for the internal nodes (rounded up for convenience) is

$fo = \text{ceiling}(0.69 * p) = \text{ceiling}(0.69 * 34) = \text{ceiling}(23.46) = 24$

number of second-level tree blocks  $b_2 = \text{ceiling}(b_1 / fo) = \text{ceiling}(1364/24) = 57$  blocks

number of third-level tree blocks  $b_3 = \text{ceiling}(b_2 / fo) = \text{ceiling}(57/24) = 3$

number of fourth-level tree blocks  $b_4 = \text{ceiling}(b_3 / fo) = \text{ceiling}(3/24) = 1$

Since the fourth level has only one block, the tree has  $x = 4$  levels (counting the leaf level). Note: We could use the formula:

$x = \text{ceiling}(\log_{fo} (b_1)) + 1 = \text{ceiling}(\log_{24} 1364) + 1 = 3 + 1 = 4$  levels

iv. total number of blocks for the tree  $b_i = b_1 + b_2 + b_3 + b_4$

$= 1364 + 57 + 3 + 1 = 1425$  blocks

v. number of block accesses to search for a record  $= x + 1 = 4 + 1 = 5$