Section 1.1

6. a. true

b. true

c. false

d. false

e. false

14. a. $r \wedge \neg q$

b. $p \wedge q \wedge r$

c. $r \rightarrow p$

d. $p \wedge \neg q \wedge r$

e. $(p \wedge q) \rightarrow r$

f. $r \leftrightarrow (p \vee q)$

Section 1.3.

14. $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ ($\rightarrow$ defined from $\vee$ and $\neg$)

$= (\neg p \wedge (\neg p \vee q)) \rightarrow \neg q$ (contrapositive)

$= q \rightarrow (\neg (\neg p \wedge (\neg p \vee q)))$ (Double negation and De Morgan's laws)

$= q \rightarrow (p \vee \neg(\neg p \vee q))$ (Double negation and De Morgan's laws)

$= q \rightarrow (p \vee (p \wedge q))$ (Absorption laws)

$= q \rightarrow p$

if q is true and p is false, then $q \rightarrow p$ is false. So the original one is not a tautology.

20. $\neg(p \oplus q) \equiv p \leftrightarrow q$

p	q	$p \oplus q$	$\neg(p \oplus q)$
T	T	F	T
T	F	T	F
F	T	T	F
F	F	F	T

$\neg(p \oplus q)$	(DNF)
$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$	(distribution)
$\equiv ((p \wedge q) \vee \neg p) \wedge ((p \wedge q) \vee \neg q))$	(distribution)
$\equiv ((p \vee \neg p) \wedge (q \vee \neg p)) \wedge ((p \vee \neg q) \wedge (q \vee \neg q))$	(Negation law)
$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$	(Def of $\leftrightarrow$)
$\equiv p \leftrightarrow q$	

Section 1.5

4f. $\forall x \forall y (P(x) \vee Q(y))$

$\equiv \forall x (\forall y (P(x) \vee Q(y)))$

$\equiv \forall x (P(x) \vee \forall y (Q(y)))$

$\equiv \forall x P(x) \vee \forall y Q(y)$

$\equiv \forall x P(x) \vee \forall x Q(x)$

Section 1.6.

12. From exercise 11,

We can conclude $q \to r$ is valid with premises $P_1, P_2 \cdots P_n$

if we can prove $P_1, P_2, \cdots P_n, q \to r$ is valid

Thus we need to prove:

$$\begin{array}{l} (p \wedge t) \to (r \vee s) \\ q \to (u \wedge t) \\ u \to p \\ \neg s \\ q \\ \hline r \end{array}$$

(1) q premise

(2) $q \to (u \wedge t)$ premise

(3) $u \wedge t$ Modus ponens from (1).(2)

(4) u Simplification from (3)

(5) $u \to p$ premise

(6) p Modus ponens from (4).(5)

(7) t Simplification from (3)

(8) $p \wedge t$ Conjuction from (6),(7)

(9) $(p \wedge t) \to (r \vee s)$ premise

(10). $r \vee s$ Modus ponens from (8).(9).

(11). $\neg s$ premise.

(12). r Disjunctive syllogism.

28. To proof

$$\begin{array}{l} \forall x (P(x) \vee Q(x)) \\ \underline{\forall x ((\neg P(x) \wedge Q(x)) \to R(x))} \\ \forall x (\neg R(x) \to P(x)) \end{array}$$

(1) $\forall x (P(x) \vee Q(x))$ Premise

(2) $P(c_0) \vee Q(c_0)$ Universal Instantiation from (1)

(3) $\forall x ((\neg P(x) \wedge Q(x)) \to R(x))$ premise

(4) $(\neg P(c_0) \wedge Q(c_0)) \to R(c_0)$ Universal Instantiation from (3)

Let p: $P(c_0)$. q: $Q(c_0)$ r: $R(c_0)$

Convert the proof (using exec. 11)

$$\begin{array}{l} p \vee q \\ \neg p \wedge q \to r. \\ \hline \neg r \to p. \end{array} \quad \text{to} \quad \begin{array}{l} p \vee q \\ \neg p \wedge q \to r \\ \neg r \\ \hline p \end{array}$$

(1)	$\neg p \wedge q \to r$	premise.
(2)	$\neg r \to \neg(\neg p \wedge q)$	Contrapositive from (1)
(3)	$\neg r$	premise
(4)	$\neg(\neg p \wedge q)$	Modus ponens from (3)(2)
(5)	$p \vee \neg q$	Double negation from (4)
(6)	$p \vee q$	premise
(7)	$p \vee p$	Resolution
(8)	p	idempotent law

Thus. $\neg R(c_0) \to P(c_0)$

$\forall x(\neg R(x) \to P(x))$ Universal generalization

Section 1.7

32. x is rational

Let $x = a/b$ where $b \neq 0$.

then $x/2 = a/2b$ where $2b \neq 0$

So that $x/2$ is also rational.

thus x is rational $\rightarrow$ $x/2$ is rational

Let $x/2 = a/b$ where $b \neq 0$.
then $x = 2a/b$
so that $3x-1 = (6a-b)/b$ is also a rational

thus. $x/2$ is rational $\rightarrow 3x-1$ is rational

Let $3x-1 = a/b$, where $b \neq 0$.
then $x = (a+b)/3b$ is also a rational
thus $3x-1$ is rational $\rightarrow$ x is rational.

therefore

x is rational $\leftrightarrow$ $x/2$ is rational $\leftrightarrow$ $3x-1$ is rational
these statements are equavalent.

Section 1-8

8. positive integer 1 has this property. the number 1 is the only positive integers not exceeding it. So the sum is 1 which equals 1 itself. This is a constructive proof.

16. Assume there are two solutions x_1 and x_2 where $x_1 \neq x_2$.

then $ax_1 + b = c$ ① subtract ② from ①

$ax_2 + b = c$ ② $a(x_1 - x_2) = 0$ ③

Since $a \neq 0$. So $x_1 - x_2 = 0$ which means $x_1 = x_2$

that contradict with $x_1 \neq x_2$.

So there is a unique solution to $ax + b = c$

Section 2.1

10.

a). true

b). true

c). false

d). true

e). true

f). true

g). false

Section 2.2.

18. a). $x \in A \cup B$

$\equiv x \in A \vee x \in B$

$\equiv x \in A \vee x \in B$

$\equiv x \in A \vee x \in B \vee x \in C$

$\equiv x \in A \cup B \cup C$

b). $x \in A \cap B \cap C$

$\equiv x \in A \wedge x \in B \wedge x \in C$

$\equiv x \in A \wedge x \in B$

$\equiv x \in A \cap B$

c). $(A-B)-C \subseteq A-C$

$(A \cap \overline{B})-C \subseteq A-C$

$(A \cap \overline{B}) \cap \overline{C} \subseteq A \cap \overline{C}$

$x \in A \cap \overline{B} \cap \overline{C}$

$\equiv x \in A \wedge x \in \overline{B} \wedge x \in \overline{C}$

$\equiv x \in A \wedge x \in \overline{C}$

$\equiv x \in A \cap \overline{C}$

d). $(A-C) \cap (C-B)$ def of difference

$\equiv (A \cap \overline{C}) \cap (C \cap \overline{B})$ Commutative law

$\equiv \overline{C} \cap C \cap A \cap \overline{B}$ Complement law

$\equiv \phi \cap A \cap \overline{B}$ Domination law

$\equiv \phi$

e). $(B \cup C) - A$ def of difference

$\equiv (B \cup C) \cap \bar{A}$ Distribution law

$\equiv (B \cap \bar{A}) \cup (C \cap \bar{A})$ def of difference

$\equiv (B - A) \cup (C - A)$

Section 2.3.

12.

a. Suppose that $x, y \in \mathbb{Z}$ with $f(x) = f(y)$

so that $x - 1 = y - 1$. This means $x = y$.

Hence, $f(n) = n - 1$ is one-to-one function.

b. This function is not one-to-one because

$f(-1) = f(1) = 2$, but $-1 \neq 1$

c. Suppose that $x, y \in \mathbb{Z}$ with $f(x) = f(y)$

so that $x^3 = y^3$ This means $x = y$.

Hence, $f(n) = n^3$ is one-to-one function.

d. This function is not one-to-one because

$f(3) = f(4) = 2$, but $3 \neq 4$

72. If f is one-to-one, then every element in A will be mapped to a different element of B. Assume f is not onto, which means that $|B| \geq |A|+1$, which contradicts $|B|=|A|$, so f must be onto.

On the other hand, if f is onto, then every element in B is the image of some element in A. Assume f is not one-to-one, which means $|A| \geq |B|+1$, which contradicts $|B|=|A|$, so f must be one-to-one.

Therefore, if $|A|=|B|$,

$$f \text{ is one-to-one} \Leftrightarrow f \text{ is onto.}$$