CS 5333.001 - Discrete Structures - F20 Course Homepage

Review Test Submission: Midterm 1 (password will be removed at 8:30 am)

Review Test Submission: Midterm 1 (password will be removed at 8:30 am)

User	Yaokun Wu
Course	CS 5333.001 - Discrete Structures - F20
Test	Midterm 1 (password will be removed at 8:30 am)
Started	9/30/20 8:35 AM
Submitted	9/30/20 9:55 AM
Status	Completed
Attempt Score	94 out of 100 points
Time Elapsed	1 hour, 19 minutes out of 1 hour and 20 minutes
Instructions	The exam will end 80 minutes after you begin. As mentioned in the email instructions, all answers can be typed directly into the exam. Only Questions IV and V can be optionally done handwritten and submitted via pdf. All other questions should be answered directly into the exam. Please submit the pdf no later than 10:00 am, if it is uploaded any time after that it is up to my discretion whether to accept it or not. Basically, you have ten more minutes (from 9:50 to 10:00) to scan and upload your pdf.
Results Displayed	All Answers, Submitted Answers, Correct Answers, Feedback, Incorrectly Answered Questions

Question 1

15 out of 15 points

Question I (Propositions)

From the following statements, select all of those that are propositions. Negative points will be given if you select a statement that is not a proposition.

Selected Answers:

a. The University of Texas at Dallas is a privately own university.

b. $\forall x(\exists y(R(x) \land Q(y) \rightarrow P(y,x)))$

Answers:

a. The University of Texas at Dallas is a privately own university.

b. $\forall x(\exists y(R(x) \land Q(y) \rightarrow P(y,x)))$

c. $\exists x(P(x) \land \exists y(Q(y))) \land Q(y)$

d. Do you have a cold?

e. Go do the dishes.

Question 2

15 out of 15 points

Question II (Logic)

Four parts. Answer all four below.

Consider the following truth table for an expression E(x,y,z)

x	y	z	E(x,y,z)
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

(i) Is E(x,y,z) a contingency, tautology, fallacy (or a combination of these)?

(ii) Write E(x,y,z) using disjunctive normal form

(iii) Consider the following expression $F(x,y,z) \equiv ((z \wedge y) \vee x) \rightarrow (y \vee z)$

Write this expression in disjunctive normal form. You can use the word "or" for disjunction and tilde ~ for negation.

(iv) Using only you answer to (ii) and (iii), can you tell me if E(x,y,z) is equivalent to F(x,y,z)?

Selected Answer:	(i). It is a contingency. (ii). (x ^ y ^ z) or (x ^ ~y ^ z) or (~x ^ y ^ z) or (~x ^ ~ y ^ ~z) (iii). (x ^ y ^ z) or (x ^ y ^~z) or (x ^ ~y ^ z) or (~x ^ y ^ z) or (~x ^ y ^ ~z) or (~x ^ ~y ^ z) or (~x ^ ~y ^ ~z) (iv). No. They are both contingency but having different truth table.
Correct Answer:	[None]
Response Feedback:	[None Given]

Question 3

7 out of 7 points

Question III (a) (Quantifiers)

From the following list of equivalences of expressions with quantifiers, select those that are correct.

Selected Answers: c. $\forall x\, \forall y (Q(x) \wedge P(y)) \equiv (\forall x\, Q(x)) \wedge (\forall y\, P(y))$

$\neg(\exists x\, \forall y\; P(x,y)) \equiv \forall x\, \exists y\; \neg P(x,y)$

e.

$\forall x\, \forall y P(x,y) \equiv \forall y\, \forall x P(x,y)$

g.

Answers:

$\exists x\, \forall y\; P(x,y) \equiv \forall y\, \exists x\; P(x,y)$

a.

$\exists x\, \forall y P(x,y) \equiv \exists y\, \forall x P(x,y)$

b.

$\forall x\, \forall y (Q(x) \wedge P(y)) \equiv (\forall x\; Q(x)) \wedge (\forall y\; P(y))$

c.

$\neg(\exists x\, \forall y\; P(x,y)) \equiv \forall y\, \exists x\; \neg P(x,y)$

d.

$\neg(\exists x\, \forall y\; P(x,y)) \equiv \forall x\, \exists y\; \neg P(x,y)$

e.

$\exists x\, \exists y (Q(x) \wedge P(y)) \equiv (\exists x\; Q(x)) \vee (\exists y\; P(y))$

f.

$\forall x\, \forall y P(x,y) \equiv \forall y\, \forall x P(x,y)$

g.

Question 4

5 out of 7 points

Question III (b) (Quantifiers)

Let the universe be the set of all cars (i.e., automobiles). Consider the following predicates:

R(x) = x has color red.
F(x) = x's owner likes to drive fast.
T(x) = x's owner has received a ticket in the last six months.

Convert the following English statements into logic. You can use the word "and" for conjunction, "or" for disjunction, tilde ~ for negation, "Exists" for existential quantification and "ForAll" for universal quantification.

(i) All cars are red and have an owner that does not drive fast.

(ii) There is a red car that has an owner that has received a ticket in the last six months.

(iii) There is one and only one car that is red and whose owner has NOT received a ticket in the last
six months. (NOTE, do NOT use unique existential, use only universal or existential quantifiers.)

Selected Answer: (i) ForAll x (R(x) and ~F(x))
(ii) Exists x (R(x) and T(x))
(iii) Exists x (R(x) and ~T(x))

Correct Answer: [None]

Response Feedback: iii wrong

Question 5

7 out of 7 points

Question IV (a) Equivalence Proof

Use a sequence of equivalence laws to prove that

$$\neg(r \vee \neg q) \vee (p \wedge q) \equiv (r \rightarrow p) \wedge q$$

you can use the word "or" for disjunction, "and" for conjunction, tilde ~ for negation and -> for implication, or you can answer in handwriting and submit pdf on the homepage

Selected Answer:	= (~r and q) or (p and q) De Morgan's law) = (~r or (p and q)) and ((p and q) or q)) Distributive law =(~r or (p and q)) and q Absorption laws =((~r or p) and (~r or q)) and q Distributive law =(~r v p) ^ q Absorption laws =(r -> p) ^ q Def of ->
Correct Answer:	[None]
Response Feedback:	[None Given]

Question 6

7 out of 7 points

Question IV (b) (Inference Proof)

Use a sequence of rules to prove the following inference

$s \wedge p$

$p \rightarrow (\neg q \vee r)$

$s \rightarrow (q \vee t)$

— — — — —

$r \vee t$

you can use the word "or" for disjunction, "and" for conjunction, tilde ~ for negation and -> for implication, or you can answer in handwriting and submit pdf on the homepage

Selected Answer:	1. s and p Premise 2. s Simplification from 1 3. s -> (q or t) Premise 4. q or t Modus ponens from 2, 3 5. p Simplification from 1 6. p -> (~q or r) Premise 7. ~q or r Modus ponens from 5, 6 8. r or t Resolution from 4, 7
Correct Answer:	[None]
Response Feedback:	[None Given]

Question 7

14 out of 14 points

Question V (Sets)

Two parts. You can write your answers here or in your handwritten pdf. If typed here, you can use Int for intersection, U for union, ' (quote) for complement

(i)

Using set identities, prove the following two sets are equal.

$$(A \cap \overline{B}) \cup (B \cap \overline{A}) = (B \cup A) \cap \overline{(A \cap B)}$$

(ii)

Consider the following two sets: A = {0, 2, 5, 9}, B = {10, 0, 9, 15}. Assume I were to ask you to calculate

$$\overline{A} \cap \overline{B}$$

What information are you missing that prevents you from computing this set expression?

Selected Answer:	(i) Prove from right to left = (B U A) Int (A' U B') Complements laws =(B Int (A' U B')) U (A Int (A' U B')) Distributive laws = ((B Int A') U (B Int B')) U ((A Int A') U (A Int B')) Distributive laws =(B int A') U (A Int B') Complements laws and Identity laws = (A Int B') U (B Int A') Commutative laws (ii) The universe is missing to find complements
Correct Answer:	[None]
Response Feedback:	[None Given]

Question 8

14 out of 14 points

Question VI (Functions)

Five parts. Let X = {a, b, c, d, e}.

(i) How many elements are in the cross product of X with (X - {e, f, g})?

(ii) Is the following a function from X to X? Briefly (one sentence) explain why yes or no.

{ (b,c), (b,c), (a,e), (e,b), (c,a), (d,e) }

(iii) Is the following a function from X to X? Briefly (one sentence) explain why yes or no.

{ (b,c), (a,b), (c,e), (d,a) }

(iv) Is the following a function from X to X? Briefly (one sentence) explain why yes or no.

{ (c,d), (a,e), (b,b), (e,a), (d, b), (c, b) }

(v) Consider the same set of pairs as in (iii) above. Show me the pairs that need to be added or removed (the least possible) to turn this set into a one-to-one and onto function from X to X.

Selected Answer:

(i)
X : 5 elements, (X - {e,f,g}) : 4 elements.
Answer: 5 * 4 = 20 elements
(ii)
Yes. All elements in the Domain is mapped to an element in CoDomain
(iii)
No. The element e is not mapped to anything
(iv)
No. The element c is mapped to two elements.
(v)
add a pair (e,d)

Correct Answer: [None]

Response Feedback: [None Given]

Question 9

10 out of 14 points

Question VII (Algorithms)

Two parts.

(i) Express a brute-force algorithm that finds the largest product of two numbers in a list a1, a2, . . . , an (n >= 2) that is less than a threshold N.

(ii) Prove that there cannot exist a procedure H'(P) that takes as input a program P and returns "halt" if P halts and "no halt" if P does not halt. <u>Note that P does not have any input.</u> Note also that programs can define constants inside their code, and constants can be of any type, such as a string of bits.

Selected Answer:

(i)
```
max:= a1 * a2
for i:=1 to n - 1
   for j: i + 1 to n
      if (ai * aj > max) and (ai * aj <  N) then max:= ai * aj
```
(ii)
Define K(P) to be
K(P) = {1. halts if H'(P) returns "no halt" and 2. loops forever if H'(P) returns "halt"}
Consider the case K(K)
K(K) = {1. halts if H'(K) returns "No halt" and 2. loops forever if H'(K) returns "halt"}
In the first case H'(K) returns "No halt" , which means K(K) return "halt", which is impossible.
In the second case H'(K) returns "halt", which mean K(K) return "no halt", which is also impossible.
So such procedure can not exist.

Correct Answer: [None]

Response Feedback: K(K), note that K has an input, H'(K) does not make sense since what should be the input to K?

Saturday, November 14, 2020 3:01:38 PM CST

← **OK**