Assignment #2

Section 3.2.

#2.

a. Yes, $|17x+11| \le |17x+11x| = 28|x| \le 28|x^2|$ for all $(x>1)$ $(C=28, k=1)$

b. Yes, $|x^2+1000| \le |x^2+1000x^2| = 1001|x^2|$ for all $x>1$ $(C=1001, k=1)$

c. Yes. $|x\log x| \le |x^2|$ for all $x>0$ $(C=1, k=0)$

d. No, to satisfy $|x^4/2| \le C|x^2| \Rightarrow |x^2/2| \le C$, but there is no such constant C than can not be exceeded as x is growing.

e. No, $|2^x| \le C|x^2| \Rightarrow \left|\frac{x}{\log x}\right| \le 2\log C = C_2$, but $\frac{x}{\log x}$ is monotonically increasing when $x > 2^{\frac{1}{\ln 2}}$, thus there is no such constant C that can not be exceeded as x is growing.

f. Yes, $|\lfloor x\rfloor\lceil x\rceil| \le |x\cdot(x+1)| \le 2|x^2|$ for all $x>1$ $(C=2, k=1)$

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Use $k=1$

$$|x^3| \le |x^3|\cdot|x| = |x^4| \quad \forall x \mid x>1$$

Hence, $k=1$, $C=1$ will do. x^3 is $O(x^4)$

Assume x^4 is $O(x^3)$. Thus, there is a C and k such that (1) $|x^4| \le C\cdot|x^3|$ for all x, $x>k$

Choose $m=\max(C,k,1)$ and $x'=m+1$

(2) $|x'^4| = |(m+1)\cdot x'^3| = (m+1)\times|x'^3| > C\cdot|x'^3|$

which contradicts (1). Thus, x^4 is not $O(x^3)$

Section 3.3.

#38

The sorting requires $O(n\log n)$ $\Rightarrow$ $f_1(n) = O(n\log n)$

The selecting requires $O(n)$ $\Rightarrow$ $f_2(n) = O(n)$

$f(n) = (f_1 + f_2)(n) = O(\max(|n\log n|, |n|)) = O(n\log n)$

Section 5.1.

#10.

a).

$\frac{1}{1\times 2} = \frac{1}{2}$ for $n=1$

$\frac{1}{1\times 2} + \frac{1}{2\times 3} = \frac{2}{3}$ for $n=2$

$\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} = \frac{3}{4}$ for $n=3$

The formula would be: $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

b). $P(n) \equiv \left(\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}\right)$

$c=1$. $\frac{1}{1\cdot 2} = \frac{1}{2}$, so $P(1)$ is true.

Inductive: let $k \geq 1$. We show that

$\left(\sum_{j=1}^{k} \frac{1}{j\cdot(j+1)} = \frac{k}{k+1}\right) \rightarrow \left(\sum_{j=1}^{k+1} \frac{1}{j(j+1)} = \frac{k+1}{k+2}\right)$

$$\sum_{j=1}^{k+1} \frac{1}{j(j+1)}$$

$$= \frac{1}{(k+1)\cdot(k+2)} + \sum_{j=1}^{k} \frac{1}{j(j+1)}$$

$$= \frac{1}{(k+1)(k+2)} + \frac{k}{k+1}$$

$$= \frac{k(k+2)+1}{(k+1)(k+2)}$$

$$= \frac{k^2+2k+1}{(k+1)(k+2)}$$

$$= \frac{(k+1)^2}{(k+1)(k+2)}$$

$$= \frac{k+1}{k+2} \quad \text{as desired}$$

Section 5.2.

#4.

a). $P(18) \equiv (18 = 4+7+7)$ true

$P(19) \equiv (19 = 4+4+4+7)$ true

$P(20) \equiv (20 = 4+4+4+4+4)$ true

$P(21) \equiv (21 = 7+7+7)$ true

b). Inductive Hypothesis:

For $k \geq 21$, $P(k)$ is true

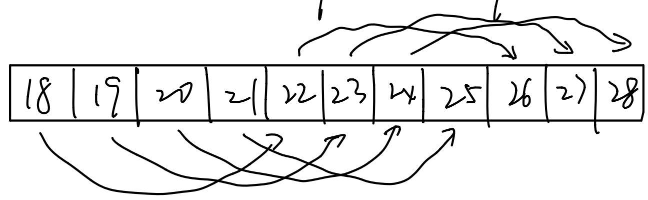
c). Inductive: Let $k \geq 21$. We need to prove that

$(P(18) \wedge P(19) \wedge P(20) \wedge P(21) \wedge \cdots \wedge P(k)) \rightarrow P(k+1)$

d). $k+1$ can be done from the stamps chosen for $k-3$ plus a 4 cent stamp.

Since $p(k-3)$ is true for $k \geq 21$ from inductive hypothesis, $p(k+1)$ is true as desired.

e). Begining from 18, we simply add 4 to the current amount and repeat the process, the sequence is Continuous.



Section 6.1.

#26.

a). $10 \times 9 \times 8 \times 7 = 5040$

b). $10 \times 10 \times 10 \times 5 = 5000$

c). Case 1: 999x, 9 ways

Case 2: 99x9, 9 ways

Case 3: 9x99, 9 ways

Case 4: x999, 9 ways

$9+9+9+9 = 36$ ways

Section 6.2.

#14.

a). There are 5 pairs in total that can form 11.

So the number of pigeonhole would be 5.

Fill seven pigeons into pigeonholes at least two of which must find their partner with sum 11.

b). It is not true for six integer because it can have only one pair of them with sum 11.

e.g. 1, 2, 3, 4, 5, 6. in this case, only 1 pair is found.

Section 6.5.

#14.

$$| \quad | \quad |$$
$$x_1 \quad x_2 \quad x_3 \quad x_4$$

$$C(4+17-1, 17) = C(20, 3) = \frac{20\times19\times18}{3\times2\times1} = 60\times19 = 1140$$

#44.

a. $C(4+12-1, 12) = C(15, 3) = 455$

b. $C(15, 3) \times P(12, 12) = \frac{15!}{12! \cdot 3!} \times 12! = \frac{15!}{3!}$