Section 1.1

6. a) True, because $288 > 256$ and $288 > 128$.

b) True, because C has 5 MP resolution compared to B's 4 MP resolution. Note that only one of these conditions needs to be met because of the word *or*.

c) False, because its resolution is not higher (all of the statements would have to be true for the conjunction to be true).

d) False, because the hypothesis of this conditional statement is true and the conclusion is false.

e) False, because the first part of this biconditional statement is false and the second part is true.

14. a) $r \wedge \neg q$ b) $p \wedge q \wedge r$ c) $r \to p$ d) $p \wedge \neg q \wedge r$ e) $(p \wedge q) \to r$ f) $r \leftrightarrow (q \vee p)$

Section 1.3

14. This is not a tautology. It is saying that knowing that the hypothesis of an conditional statement is false allows us to conclude that the conclusion is also false, and we know that this is not valid reasoning. To show that it is not a tautology, we need to find truth assignments for p and q that make the entire proposition false. Since this is possible only if the conclusion if false, we want to let q be true; and since we want the hypothesis to be true, we must also let p be false. It is easy to check that if, indeed, p is false and q is true, then the conditional statement is false. Therefore it is not a tautology.

20.

Truth table for ~(p XOR q)

p	q	p xor q	~(p XOR q)
T	**T**	**F**	**T**
T	F	T	F
F	T	T	F
F	**F**	**F**	**T**

DNF of the above: (p and q) or (~p and ~q)

Given (by definition) p <-> q equivalent (p ->q) and (q -> p)

We transform one into the other:

(p and q) or (~p and ~q)

equiv {distribution of or over and}

((p and q) or ~p) and ((p and q) or ~q)

equiv {distribution or over and}

((p or ~p) and (~p or q)) and ((p or ~q) and (~q or q))

equiv {identity laws and associativity of and}

T and (~p or q) and (p or ~q) and (T)

equiv {identity laws}

(~p or q) and (p or ~q)

eqiv { p -> q equiv ~p or q, also commutativity of or}

(p -> q) and (~q or p)

equiv {~q or p equiv q -> p}

(p -> q) and (q -> p)

equiv {by definition}

p <-> q

Section 1.5

48. We need to show that each of these propositions implies the other. Suppose that $\forall x P(x) \vee \forall x Q(x)$ is true. We want to show that $\forall x \forall y (P(x) \vee Q(y))$ is true. By our hypothesis, one of two things must be true. Either P is universally true, or Q is universally true. In the first case, $\forall x \forall y (P(x) \vee Q(y))$ is true, since the first expression in the disjunction is true, no matter what x and y are; and in the second case, $\forall x \forall y (P(x) \vee Q(y))$ is also true, since now the second expression in the disjunction is true, no matter what x and y are. Next we need to prove the converse. So suppose that $\forall x \forall y (P(x) \vee Q(y))$ is true. We want to show that $\forall x P(x) \vee \forall x Q(x)$ is true. If $\forall x P(x)$ is true, then we are done. Otherwise, $P(x_0)$ must be false for some x_0 in the domain of discourse. For this x_0, then, the hypothesis tells us that $P(x_0) \vee Q(y)$ is true, no matter what y is. Since $P(x_0)$ is false, it must be the case that $Q(y)$ is true for each y. In other words, $\forall y Q(y)$ is true, or, to change the name of the meaningless quantified variable, $\forall x Q(x)$ is true. This certainly implies that $\forall x P(x) \vee \forall x Q(x)$ is true, as desired.

Instead of the above, which is fine, I would prefer a proof where you present a small universe, e.g., U = {1, 2, 3}, and argue by expanding the qualifiers that the statement is true, and then arguing that increasing the size of the universe will result in the same type of proof.

In principle, a proof by induction would be best but we have not gotten to induction yet.

Section 1.6

12. Applying Exercise 11, we want to show that the conclusion r follows from the five premises $(p \wedge t) \to (r \vee s)$, $q \to (u \wedge t)$, $u \to p$, $\neg s$, and q. From q and $q \to (u \wedge t)$ we get $u \wedge t$ by modus ponens. From there we get both u and t by simplification (and the commutative law). From u and $u \to p$ we get p by modus ponens. From p and t we get $p \wedge t$ by conjunction. From that and $(p \wedge t) \to (r \vee s)$ we get $r \vee s$ by modus ponens. From that and $\neg s$ we finally get r by disjunctive syllogism.

28. We want to show that the conditional statement $\neg R(a) \to P(a)$ is true for all a in the domain; the desired conclusion then follows by universal generalization. Thus we want to show that if $\neg R(a)$ is true for a particular a, then $P(a)$ is also true. For such an a, universal modus tollens applied to the second premise gives us $\neg(\neg P(a) \wedge Q(a))$. By rules from propositional logic, this gives us $P(a) \vee \neg Q(a)$. By universal generalization from the first premise, we have $P(a) \vee Q(a)$. Now by resolution we can conclude $P(a) \vee P(a)$, which is logically equivalent to $P(a)$, as desired.

Better written (my way)

(1) ForAll x (P(x) or Q(x))	{premise}
(2) ForAll x ((~P(x) and Q(x)) -> R(x))	{premise}
(3) P(a) or Q(a)	{universal generalization on (1) }
(4) (~P(a) and Q(a)) -> R(a)	{universal generalization on (2), we pick the same value as in (3)}
(5) ~(~P(a) and Q(a) or R(a)	{a -> b equiv ~a or b}
(6) (P(a) or ~Q(a)) or R(a)	{DeMorgan's or LHS of 5}
(7) (P(a) or R(a)) or ~Q(a)	{associativity and commutativity of or}
(8) (P(a) or R(a)) or P(a)	{resolution on (3) and (7)}
(9) P(a) or R(a)	{associativity of or and idempotent property of or}
(10) ~R(a) -> P(a)	{a -> b equiv ~a or b}
(11) Forall x (~R(a) -> P(a))	{universal generalization, notice that we made no specific assumptions about the value of a}

Section 1.7

32. We give direct proofs that (i) implies (ii), that (ii) implies (iii), and that (iii) implies (i). That will suffice. For the first, suppose that $x = p/q$ where p and q are integers with $q \neq 0$. Then $x/2 = p/(2q)$, and this is rational, since p and $2q$ are integers with $2q \neq 0$. For the second, suppose that $x/2 = p/q$ where p and q are integers with $q \neq 0$. Then $x = (2p)/q$, so $3x - 1 = (6p)/q - 1 = (6p - q)/q$ and this is rational, since $6p - q$ and q are integers with $q \neq 0$. For the last, suppose that $3x - 1 = p/q$ where p and q are integers with $q \neq 0$. Then $x = (p/q + 1)/3 = (p + q)/(3q)$, and this is rational, since $p + q$ and $3q$ are integers with $3q \neq 0$.

Section 1.8

8. The number 1 has this property, since the only positive integer not exceeding 1 is 1 itself, and therefore the sum is 1. This is a constructive proof.

16. We know from algebra that the following equations are equivalent: $ax + b = c$, $ax = c - b$. $x = (c - b)/a$. This shows, constructively, what the unique solution of the given equation is.

Section 2.1

10. a) true b) true c) false—see part (a) d) true
e) true—the one element in the set on the left is an element of the set on the right, and the sets are not equal
f) true—similar to part (e) g) false—the two sets are equal

Section 2.2

18. a) Suppose that $x \in A \cup B$. Then either $x \in A$ or $x \in B$. In either case, certainly $x \in A \cup B \cup C$. This establishes the desired inclusion.
b) Suppose that $x \in A \cap B \cap C$. Then x is in all three of these sets. In particular, it is in both A and B and therefore in $A \cap B$, as desired.
c) Suppose that $x \in (A - B) - C$. Then x is in $A - B$ but not in C. Since $x \in A - B$, we know that $x \in A$ (we also know that $x \notin B$, but that won't be used here). Since we have established that $x \in A$ but $x \notin C$, we have proved that $x \in A - C$.
d) To show that the set given on the left-hand side is empty, it suffices to assume that x is some element in that set and derive a contradiction, thereby showing that no such x exists. So suppose that $x \in (A-C)\cap(C-B)$. Then $x \in A - C$ and $x \in C - B$. The first of these statements implies by definition that $x \notin C$, while the second implies that $x \in C$. This is impossible, so our proof by contradiction is complete.
e) To establish the equality, we need to prove inclusion in both directions. To prove that $(B-A)\cup(C-A) \subseteq (B \cup C) - A$, suppose that $x \in (B-A)\cup(C-A)$. Then either $x \in (B-A)$ or $x \in (C-A)$. Without loss of generality, assume the former (the proof in the latter case is exactly parallel.) Then $x \in B$ and $x \notin A$. From the first of these assertions, it follows that $x \in B \cup C$. Thus we can conclude that $x \in (B \cup C) - A$, as desired. For the converse, that is, to show that $(B \cup C) - A \subseteq (B-A)\cup(C-A)$, suppose that $x \in (B \cup C) - A$. This means that $x \in (B \cup C)$ and $x \notin A$. The first of these assertions tells us that either $x \in B$ or $x \in C$. Thus either $x \in B - A$ or $x \in C - A$. In either case, $x \in (B-A)\cup(C-A)$. (An alternative proof could be given by using Venn diagrams, showing that both sides represent the same region.)

Section 2.3

12. a) This is one-to-one, since if $n_1 - 1 = n_2 - 1$, then $n_1 = n_2$.
b) This is not one-to-one, since, for example, $f(3) = f(-3) = 10$.
c) This is one-to-one, since if $n_1^3 = n_2^3$, then $n_1 = n_2$ (take the cube root of each side).
d) This is not one-to-one, since, for example, $f(3) = f(4) = 2$.

72. If f is one-to-one, then every element of A gets sent to a different element of B. If in addition to the range of A there were another element in B, then $|B|$ would be at least one greater than $|A|$. This cannot happen, so we conclude that f is onto. Conversely, suppose that f is onto, so that every element of B is the image of some element of A. In particular, there is an element of A for each element of B. If two or more elements of A were sent to the same element of B, then $|A|$ would be at least one greater than the $|B|$. This cannot happen, so we conclude that f is one-to-one.