Most questions had two versions, you received one the two versions randomly.

QUESTION I

Details: Multiple Answer Question

Question Title	Question I (Propositions)
Question	**Question I (Propositions)** From the following statements, select all of those that are propositions. Negative points will be given if you select a statement that is not a proposition.
Answer	a. Go do the dishes. ✓ b. The University of Texas at Dallas is a privately own university. c. Do you have a cold? d. $\exists x(P(x) \wedge \exists y(Q(y))) \wedge Q(y)$ ✓ e. $\forall x(\exists y(R(x) \wedge Q(y) \rightarrow P(y,x)))$

Details: Multiple Answer Question

Question Title	Question I (Proposition)
Question	**Question I (Propositions)** From the following statements, select all of those that are propositions. Negative points will be given if you select a statement that is not a proposition
Answer	✓ a. The half-life of caffeine in the human body is one hour. b. $\forall y(P(x) \wedge R(y) \leftrightarrow Z(y))$ c. Do not return this item to the store. ✓ d. $\exists y(\forall x(Z(x,y) \rightarrow \neg(P(x) \wedge P(y))))$ e. Is gravity constant everywhere on the surface of the earth?

QUESTION II

Details: Essay Question

Question Title	Question II (Logic)
Question	**Question II (Logic)** Four parts. Answer all four below. Consider the following truth table for an expression E(x,y,z)

x	y	z	E(x,y,z)
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	T

(i) Is E(x,y,z) a contingency, tautology, fallacy (or a combination of these)?

(ii) Write E(x,y,z) using disjunctive normal form

(iii) Consider the following expression $F(x,y,z) \equiv ((x \vee y) \wedge x) \rightarrow (x \wedge z)$

Write this expression in disjunctive normal form. You can use the word "or" for disjunction and tilde ~ for negation.

(iv) Using only you answer to (ii) and (iii), can you tell me if E(x,y,z) is equivalent to F(x,y,z)?

Answer

Below, I am using space for "and", sorry for my laziness, but I had to prepare answers for two exams not just one (looks cleaner too!).

(i)	Contingency	(2 points)
(ii)	(x y z) or (x ~y ~z) or (~x ~y z) or (~x ~y ~z)	(5 points)
(iii)	(x y z) or (~x y z) or (~x y ~z) or (x ~y z) or (~x ~y z) or (~x ~y ~z)	(6 points)
(iv)	<u>Different DNF</u> so not same	(2 points)

Details: Essay Question

Question Title	Question II (Logic)
Question	**Question II (Logic)**

Four parts. Answer all four below.

Consider the following truth table for an expression E(x,y,z)

x	y	z	E(x,y,z)
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

(i) Is E(x,y,z) a contingency, tautology, fallacy (or a combination of these)?

(ii) Write E(x,y,z) using disjunctive normal form

(iii) Consider the following expression $F(x,y,z) \equiv ((z \wedge y) \vee x) \rightarrow (y \vee z)$

Write this expression in disjunctive normal form. You can use the word "or" for disjunction and tilde ~ for negation.

(iv) Using only you answer to (ii) and (iii), can you tell me if E(x,y,z) is equivalent to F(x,y,z)?

Answer

(v)	Contingency	(2 points)
(vi)	(x y z) or (x ~y z) or (~x y z) or (~x ~y ~z)	(5 points)
(vii)	All 8 values except (x and ~y and ~z)	(6 points)
(viii)	<u>Different DNF</u> so not same	(2 points)

QUESTION III(A)

Details: Multiple Answer Question

Question Title	Question III (a) (Quantifiers)
Question	**Question III (a) (Quantifiers)** From the following list of equivalences of expressions with quantifiers, select those that are correct.
Answer	a. $\neg(\exists x\, \forall y\; P(x,y)) \equiv \forall y\, \exists x\; \neg P(x,y)$
	b. $\exists x\, \forall y\; P(x,y) \equiv \forall y\, \exists x\; P(x,y)$
	✔ c. $\forall x\, \forall y (Q(x) \wedge P(y)) \equiv (\forall x\; Q(x)) \wedge (\forall y\; P(y))$
	d. $\exists x\, \forall y P(x,y) \equiv \exists y\, \forall x P(x,y)$
	✔ e. $\neg(\exists x\, \forall y\; P(x,y)) \equiv \forall x\, \exists y\; \neg P(x,y)$

QUESTION III(b) (first version)

Question Title	Question III (b) (Quantifiers)
Question	**Question III (b) (Quantifiers)** Let the universe correspond to the set of all students at UTD. Consider the following predicates: CS(x) = x is a computer science student. G(x) = x is a graduate student. UG(x) = x is an undergraduate student. L(x) = x has checked out a book from the library. Convert the following English statements into logic. (to save time, you can use "or" for disjunction, "and" for conjunction, tilde ~ for negation, "ForAll" for universal quantification, and "Exists" for existential quantification) (i) There is a computer science student that is either a graduate or undergraduate student. (ii) All undergraduate students have checked out a book from the library. (iii) There is one and only one computer science student that is both an undergraduate and a graduate student. (NOTE: do NOT use unique existential quantifier, use only universal or existential quantifiers)
Answer	

(i) [E x CS(X) and (UG(x) + G(X))] 3 points

(ii) [A x UG(X) → L(X)] 2 points

(iii) [E x CS(X) and UG(X) and G(X) and [A y CS(Y) → (X = Y) OR ~(UG(Y) and G(Y))]] 2 points

Question Title	Question III (b) (Quantifiers)
Question	**Question III (b) (Quantifiers)** Let the universe be the set of all cars (i.e., automobiles). Consider the following predicates: R(x) = x has color red. F(x) = x's owner likes to drive fast. T(x) = x's owner has received a ticket in the last six months. Convert the following English statements into logic. You can use the word "and" for conjunction, "or" for disjunction, tilde ~ for negation, "Exists" for existential quantification and "ForAll" for universal quantification. (i) All cars are red and have an owner that does not drive fast. (ii) There is a red car that has an owner that has received a ticket in the last six months. (iii) There is one and only one car that is red and whose owner has NOT received a ticket in the last six months. (NOTE, do NOT use unique existential, use only universal or existential quantifiers.)

(i) [A x R(X) and ~F(X)] 3 points

(ii) [E x R(X) and T(X)] 2 points

(iii) [E x R(X) and ~T(X) and [A y (X = Y) OR ~(R(X) and ~T(X))]] 2 points

QUESTION IV (a)

(first version)

Question IV (a) Equivalence Proof

Question IV (a) Equivalence Proof

Use a sequence of equivalence laws to prove that

$$p \to (\neg q \wedge r) \equiv \neg p \vee \neg(r \to q)$$

you can use the word "or" for disjunction, "and" for conjunction, tilde ~ for negation and -> for implication, or you can answer in handwriting and submit pdf on the homepage

$p \to (\neg q \wedge r) \equiv \neg p \vee \neg(r \to q)$

$p \to (\neg q \wedge r)$

$\equiv \neg p \vee (\neg q \wedge r)$ {def of $\to$}

$\equiv \neg p \vee \neg(\neg(\neg q \wedge r))$ {double negation}

$\equiv \neg p \vee \neg(q \vee \neg r)$ {de Morgan's}

$\equiv \neg p \vee \neg(r \to q)$ {def of implication}

(second version)

Question IV (a) Equivalence Proof

Question IV (a) Equivalence Proof

Use a sequence of equivalence laws to prove that

$$\neg(r \vee \neg q) \vee (p \wedge q) \equiv (r \to p) \wedge q$$

you can use the word "or" for disjunction, "and" for conjunction, tilde ~ for negation and -> for implication, or you can answer in handwriting and submit pdf on the homepage

$\neg(r \vee \neg q) \vee (p \wedge q) \equiv (r \to p) \wedge q$

$\equiv (\neg r \wedge q) \vee (p \wedge q)$ {de Morgan}

$\equiv (\neg r \vee p) \wedge q$ {distribution}

$\equiv (r \to p) \wedge q$ {def of $\to$}

$\equiv (\neg r \vee p) \wedge q$ {def of $\to$}

$\equiv (\neg r \wedge q) \vee (p \wedge q)$ {dist}

$\equiv \neg(r \vee \neg q) \vee (p \wedge q)$ {de Morgan}

Two different ways just for fun

QUESTION IV (b)

First version

Question IV (b) Inference Proof

Question IV (b) (Inference Proof)

Use a sequence of rules to prove the following inference

$s \wedge p$

$p \rightarrow (\neg q \vee r)$

$s \rightarrow (q \vee t)$

$-----$

$r \vee t$

you can use the word "or" for disjunction, "and" for conjunction, tilde ~ for negation and -> for implication, or you can answer in handwriting and submit pdf on the homepage

(1) p (Simplification on hypothesis 1)
(2) $(\neg q \vee r)$ (Modus ponens on (1) and hypothesis 2)
(3) s (Simplification on hypothesis 1)
(4) $(q \vee t)$ (Modus ponens ~~on~~ (3) and hypothesis 3)
(5) $r \vee t$ (resolution on (2) and (4))

Second version

Question IV (b) Inference Proof

Question IV (b) (Inference Proof)

Use a sequence of rules to prove the following inference

$s \to r$

$r \to (p \wedge x)$

$p \to (q \wedge r)$

— — — — —

$s \to q$

you can use the word "or" for disjunction, "and" for conjunction, tilde ~ for negation and -> for implication, or you can answer in handwriting and submit pdf on the homepage

a) $S \to p \wedge x$ (hyp. Syll. on hyp 1 and hyp 2)

b) $p \wedge x \to p$ (Simplification rule, thus it is a tautology)

c) $S \to p$ (hyp Syll on a) and c))

d) $S \to q \wedge r$ (hyp syll on c) and hyp 3)

e) $q \wedge r \to q$ (simplification rule, hence a tautology)

f) $S \to q$ (hyp syll on d) and e))

You could have also used that s -> (p and x) equivalent (s -> p) and (p -> x), and then use simplification to choose s -> p

Question VI

First version

Question V (Sets)

Question V (Sets)

Two parts. You can write your answers here or in your handwritten pdf. If typed here, you can use Int for intersection, U for union, ' (quote) for complement

(i)

Using set identities, prove the following two sets are equal.

$$\overline{((A-B)\cup(A-C))}=\overline{(\overline{A}\cup B\cup C)}$$

(ii)

Consider the following two sets: A = {0, 2, 5, 9}, B = {10, 0, 9, 15}. Assume I were to ask you to calculate

$$\overline{A}\cap\overline{B}$$

What information are you missing that prevents you from computing this set expression?

7 points

(i)
On this one, don't hate me ☺, but there was a typo in the question, and it turns out that they are not equal to each other.

However, using your knowledge of sets you should be quite capable to determine that they are not the same. So I get a B+ instead of an A as a professor, but you should be able to determine the issue since, after all, you are graduate students. If your boss at work asks you to do something that can't be done, shouldn't you be able to tell that this is the case and then tell your boss? Think about it.

I made this part 7 points rather than 10 to mitigate the issue (the other version of the question I have 10 points for part (i) and 4 points for part (ii)). Yes, some people did tell me that they were not equal, and of course they got full credit.

```
     LHS
  ((A - B)  U  (A - C))'
= (A - B)' Int (A - C)'
= (A Int B')' Int (A Int C')'
= (A' U B) Int (A' U C)
=  A' U (B Int C)

     RHS
  (A' U B U C)'
= (A I B' I C')

If B = {} and C = {}, then LHS = A', and RHS = A
```

ii. The Universe set

7 points

second version

Question V (Sets)

Two parts. You can write your answers here or in your handwritten pdf. If typed here, you can use Int for intersection, U for union, ' (quote) for complement

(i)

Using set identities, prove the following two sets are equal.

$$(A \cap \overline{B}) \cup (B \cap \overline{A}) = (B \cup A) \cap \overline{(A \cap B)}$$

(ii)

Consider the following two sets: A = {0, 2, 5, 9}, B = {10, 0, 9, 15}. Assume I were to ask you to calculate

$$\overline{A} \cap \overline{B}$$

What information are you missing that prevents you from computing this set expression?

```
(i) 10 points

(B U A) Int (A Int B)’                    {RHS}
=
(B U A) Int (A’ U B’)                           {De Morgan's}
=
[(B U A) Int A’] U [(B U A) Int B’]       {Distribution}
=
[(A’ Int A) U (A’ Int B)] U [(B’ Int B) U (B’ Int A)] {Distribution}
=
[{} U (A’ Int B)] U [{} U (B’ Int A)]           {Complement}
=
(A’ Int B) U (B’ Int A)                   {Identity}
=
(A Int B’) U (B Int A’)                   {Commutative}

ii) We are missing the universal set.

4 points
```

Question VI

Question Title	Question VI (Functions)
Question	**Question VI (Functions)** Five parts. Let X = {1, 2, 3, 4, 5}. (i) How many elements are in the cross product of the set X - {1} with the set X - {3, 5}? (ii) Is the following a function from X to X? Briefly (one sentence) explain why yes or no. { (2,4), (1,5), (3,2), (5,1) } (iii) Is the following a function from X to X? Briefly (one sentence) explain why yes or no. { (2,4), (2,4), (1,5), (3,2), (4, 2), (5,1) } (iv) Is the following a function from X to X? Briefly (one sentence) explain why yes or no. { (2,4), (1,5), (3,2), (5,1), (4, 2), (2, 1) } (v) Consider the same set of pairs as in (ii) above. Show me the pairs that need to be added or removed (the least possible) to turn this set into a one-to-one and onto function from X to X.
Answer	

3 points each maximum of 14

- (i) 12
- (ii) NO missing 4
- (iii) YES (it is a set)
- (iv) NO (2 has two images)
- (v) ADD (4, 3)

Question Title	Question VI (Functions)
Question	**Question VI (Functions)** Five parts. Let X = {a, b, c, d, e}. (i) How many elements are in the cross product of X with (X - {e, f, g})? (ii) Is the following a function from X to X? Briefly (one sentence) explain why yes or no. { (b,c), (b,c), (a,e), (e,b), (c,a), (d,e) } (iii) Is the following a function from X to X? Briefly (one sentence) explain why yes or no. { (b,c), (a,b), (c,e), (d,a) } (iv) Is the following a function from X to X? Briefly (one sentence) explain why yes or no. { (c,d), (a,e), (b,b), (e,a), (d, b), (c, b) } (v) Consider the same set of pairs as in (iii) above. Show me the pairs that need to be added or removed (the least possible) to turn this set into a one-to-one and onto function from X to X.
Answer	

3 each, max 14

(i) 20
(ii) YES
(iii) NO (missing e)
(iv) NO (double image for c)
(v) (e, d) add

QUESTION VII

Details: Essay Question

Question Title	Question VII (Algorithms)
Question	**Question VII (Algorithms)** Two parts. (i) Express a brute-force algorithm that finds the largest product of two numbers in a list a1, a2, . . . , an (n >= 2) that is less than a threshold N. (ii) Prove that there cannot exist a procedure H'(P) that takes as input a program P and returns "halt" if P halts and "no halt" if P does not halt. <u>Note that P does not have any input.</u> Note also that programs can define constants inside their code, and constants can be of any type, such as a string of bits.
Answer	

(i)

```
procedure LargestProduct(a1,…,an)

maxProduct := 0

for i:=1 to n - 1

  for j:=i+1 up to n

        newProduct:= ai*aj

        if newProduct > maxProduct and newProduct < N then

                maxProduct := newProduct
```

(ii)

For every program P and every possible input I of P, create a new program Q that has P and I defined as constants inside of it (Q thus has no input). Q is simple, it simply runs P on input I.

If H’ could exist, it would correctly identify if Q halts or not (for every possible Q, i.e., for every P and I), and we showed in class that is not possible.