University of Waterloo | ACTSC974/STAT974 - Financial Econometrics
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Problem Set #3 | Posted Date: Oct 23, 2022 at 9:59 am
Topic: Statistics of (G)ARCH | Due Date: Nov 6 , 2022 at 11:59 pm

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Instruction: (i) Please read slide **2022_974_S0.pdf** for information related to the homework report and the homework submission process*before* you begin working on it; and (ii) submit your completed homework by uploading it to a dropbox folder called **H3** on the course's LEARN site to be created by the TA soon. The dropbox will be closed for submission at the specified due date's time for this assignment.

Use functions in the **ruGARCH** package for this homework. Be sure to read the package documentation before you start implementing the procedures in the package.
Download adjusted closing daily prices (P_t) on Microsoft's stock (with a ticker symbol of MSFT) dated from Jan 3, 2000 to Oct 16, 2015 from:

$$https://finance.yahoo.com/quote/MSFT/history?p = MSFT$$

See **R3.pdf** in the **R Instructional Materials** module on the course's LEARN site on how to download daily adjusted-closing price data from the yahoo website.
Calculate the daily log (or continuously compounded) return as $X_t = ln\left(\frac{P_t}{P_{t-1}}\right)$. Henceforth, I simply call it return.
Let a sequence of return random variables be denoted as $\{X_t, t \in \mathbb{Z}\}$, such that the conditional mean equation is given by:

$$X_t = \mathbb{E}(X_t|\mathcal{F}_{t-1}) + \epsilon_t$$

and the conditionally heteroskedastic process is

$$\epsilon_t = \sigma_t \eta_t,\ \eta_t \sim i.i.d(0, 1)$$

For ae GARCH(p,q) model, the conditional variance equation is

$$\sigma_t^2 = \omega + \alpha(B)\epsilon_t^2 + \beta(B)\sigma_t^2,\ \omega > 0$$

where $\alpha(B) = \sum_{i=1}^{q} \alpha_i B^i$, $B^i\epsilon_t^2 = \epsilon_{t-i}^2$, $\alpha_i \geq 0$, $\beta(B) = \sum_{j=1}^{p} \beta_j B^j$, $\beta_j \geq 0$, and, for a ARCH(q) model, the conditional variance equation is

$$\sigma_t^2 = \omega + \alpha(B)\epsilon_t^2$$

where $\omega > 0$, $\alpha_i \geq 0, i = 1, \cdots, q-1$, and $\alpha_q > 0$.

Question 1. Identification Tests (20%)

(a) **Assessing Distributional Poperties of Asset Returns**
(i) Create time(-series) plots of daily returns and squared returns on Microsoft. Comment

on the empirical properties of this asset return.
(ii) Create histograms and normal QQ plots for the daily return on Microsoft. Briefly comment on each of these plots.
(iii) Compute sample statistics (i.e., sample mean, sample standard deviation, sample skewness, and sample kurtosis) for the daily returns on Microsoft. Test the null hypothesis of normality of this return by using a Jarque-Bera (JB) statistic. Interpret the test results by using a p-value approach.
(b) **Assessing Serial Correlation Poperties of Asset Returns**
(i) Plot the sample ACF of the daily return on Microsoft. Briefly comment on the plots.
(ii) Compute the Ljung-Box Q statistic for the daily returns on Microsoft for lags 1-10 and use these statistics to test the null hypothesis of no serial correlation in the daily return on Microsoft. Interpret the test results using a p-value approach.
(iii) Repeat excercise in part (i) for the daily squared return on Microsoft. Briefly comment on the plots.
(iv) Repeat excercise in part (ii) for the daily squared return on Microsoft. Interpret the test results by using a p-value approach.
(v) Is the Q statistic computed in part (ii) and (iv) valid for testing for weak white noises in the return series? Explain. Note: You can manually compute tests that are *robust* to the presence of conditional heteroskedasticity in the return process.
(c) **Testing for an ARCH Effect in Asset Returns**
(i) For the daily return on Microsoft, compute a Lagrange Multiplier test for an ARCH effect by using 5 and 10 lags. Interpret the resulting test result by using a p-value approach.
(ii) Repeat the excercise in part (i) for the daily squared return on Microsoft. Interpret the resulting test result by using a p-value approach and link your answer in this part to your anwer in (iv) of (b).
(d) **Summary Remarks**
(i) Based on the results of the identification tests conducted in this question, what is a "good" candidate in terms of a statistical model for the daily return on Microsoft over the sample period under consideration for the purpose of *model estimation*? Explain. Hint: Write down your candidate model explicitly and commen on it.
Notes. In conducting a hypothesis test, be sure to state the null and alternative hypotheses, the test statistic and its (asymptotic) distribution under H_0.

Question 2. Model Estimation (20%)

Set $\mathbb{E}(X_t|\mathcal{F}_{t-1}) = 0$ and assume $\eta_t \sim i.i.d\mathcal{N}(0,1)$. Then Consider the estimation of an ARCH(p) model and a GARCH(p,q) models for the daily returns on Microsoft by the conditional maximum likelihood (CML) approach.
(a) **Estimation of an ARCH(5) model**
(i) Report the estimated value of $\alpha_1 + \cdots + \alpha_5$ and interpret the estimation result.
(ii) Comment on the sign and size of the estimated parameters: $\hat{\omega}$, $\hat{\alpha}_1, \cdots, \hat{\alpha}_5$.
(iii) Comment on the CML standard error estimates of the estimated parameters of the model and contrast them with the corresponding QML estimates, which are reported in the R output as "Robust Standard Errors".
(iv) Plot the *in-sample* estimates of the conditional volatility of the daily return, i.e., $\hat{\sigma}_t$, and compare it with its absolute return, $|X_t|$.
(v) What is the estimate of the unconditional volatility (i.e., standard deviation) of the

daily MSFT returns, $\sigma = \sqrt{Var(X_t)}$? Compare this with the sample standard deviation of the daily MSFT returns.

(b) **Estimation of a GARCH(1,1) model**

(i) Report the estimated value of $\alpha + \beta$ and interpret the result.

(ii) Plot the in-sample estimates of the conditional volatility of the return and compare it with its absolute return. Do the conditional volatility estimates of the returns look similar to the ARCH(5) estimates? Explain. (iii) What is the estimate of the unconditional volatility (i.e., the standard deviation) of the return? Compare this with the sample standard deviation of the daily MSFT returns.

Question 3. Model Validation/Model Diagnostic Check (20%)

(a) **Fitted ARCH(5)**

(i) Is the standardized residuals of the fitted model, $\hat{\eta}_t := \frac{\hat{\epsilon}_t}{\hat{\sigma}_t}$, serially correlated? What is the objective of this test? Interpret the test result reported for the Weighted Ljung-Box Test on Standardized Residuals in the R output.

(ii) Is the squared standardized residuals of the fitted model, $\hat{\eta}_t^2$, serially correlated? What is the objective of this test? Interpret the test result reported for the Weighted Ljung-Box Test on Standardized Squared Residuals.

(iii) Is there any ARCH effect left in the residuals after it has been fitted to an ARCH(5) model? Interpret the test result reported for the Weighted ARCH LM Tests. noindent (iv) What is the connection, if any, between the test carried out in part (iii) and the test carried out in part (ii)? Explain.

(b) **Fitted GARCH(1,1)**

(i) Repeat excercise in (i) of part (a).

(ii) Repeat excercise in (ii) of part (a).

(iii) Repeat excercise in (iii) of part (a).

(c) **Summary Remarks**

(i) Based on your analysis in Question 2 and your analysis in parts (a) and (b) of Question 3, which model will you recommend to be used for the purpose of *prediction*? Explain.

Question 4. Model Prediction/Forecast (20%)

(a) **Fitted ARCH(5)**

(i) Using the fitted ARCH(5) model, compute h-step ahead volatility forecasts of the daily return on Microsoft for $h = 1, \cdots, 100$ days.

(ii) Plot these along with the estimate of unconditional volatility of the daily return on Microsoft. From the plot, do the forecasts appear to converge fast to the unconditional volatility of the returns? Explain.

(b) **Fitted GARCH(1,1)**

(i) Repeat exercise in (i) of part (a).

(ii) Repeat exercise in (ii) of part (a).

Question 5. Rolling Estimation, Rolling Forecasts, and VaR Measures for Fitted GARCH(1,1) (20%)

The *ugarchroll()* function in the ruGARCH package allows you to perform a rolling estimation and forecasting of a model. Once you specify a model with *ugarchspec()*, you can

apply *ugarchroll()* for rolling estimation. Take a look at the examples in the **ruGARCH** vignette and read the help file for *ugarchroll()*, so you understand how it works.
(a) Assume that the daily return follows a normal GARCH(1,1) process. Write down the formulas for 1-day conditional Value at Risk (VaR) and h-day conditional VaR for the daily return on Microsoft.
(b) Compute a thousand 1-period ahead rolling forecasts of a normal GARCH(1,1) model for Microsoft, refitting the GARCH(1,1) model every 20 observations (this is a monthly rolling window). In *ugarchroll(), n:ahead* =1 means compute 1-step ahead forecasts, *forecast:length* =1000 means set aside 1000 observations for the forecast, *refit:every* = 20 means you will refit our model every 20 days for calculation speed, and *refit:window* = *moving* is the rolling forecast. Make sure that you calculate 1% VaR when you perform the rolling estimation.
(c) What is the Mean Squares Errors and Mean Absolute Errors of the forecast?
(d) Plot the forecast of conditional volatility and discuss the result.

References

[1] Brockwell, P. J. and R. A. Davis. *Time Series: Theory and Methods*. Second Edition. Springer-Verlag, New York, 2006 (BD).

[2] Wirjanto, T. S., *ACTSC974/STAT974, Fall 2022*, Slides: 2022_974_S1 - 2022_974_S14 and R-dedicated Slides: 2022_974_R1 - 2022_974_R4.