CSE 150. Assignment 1

Out: *Tue Jan 15*
Due: *Tue Jan 22* (in class)
Reading: Russell & Norvig, Chapter 13; Korb & Nicholson, Chapter 1.

1.1 Conditional independence

Show that the following three statements about random variables X, Y, and Z are equivalent (i.e., that any one of them implies the other two):

$$\begin{aligned} P(X,Y|Z) &= P(X|Z)P(Y|Z) \\ P(X|Y,Z) &= P(X|Z) \\ P(Y|X,Z) &= P(Y|Z) \end{aligned}$$

You should become fluent with all these ways of expressing that X is conditionally independent of Y given Z.

1.2 Kullback-Leibler distance

Consider two discrete probability distributions, p_i and q_i, with $\sum p_i = \sum q_i = 1$. The Kullback-Leibler (KL) distance between these distributions is defined as:

$$\mathrm{KL}(p,q) = \sum_i p_i \log(p_i/q_i).$$

(a) By sketching graphs of $\log x$ and $x-1$, verify the inequality

$$\log x \leq x-1,$$

with equality if and only if $x=1$. Confirm this result by differentiation of $\log x - (x-1)$. (Note: all logarithms in this problem are *natural* logarithms.)

(b) Use the previous result to prove that $\mathrm{KL}(p,q) \geq 0$, with equality if and only if the two distributions p_i and q_i are equal.

(c) Provide a counterexample to show that the KL distance is not a symmetric function of its arguments:

$$\mathrm{KL}(p,q) \neq \mathrm{KL}(q,p).$$

Despite this asymmetry, it is still common to refer to $\mathrm{KL}(p,q)$ as a measure of distance. Many algorithms in machine learning are based on minimizing KL distances between probability distributions.

1.3 Creative writing

Attach events to the binary random variables X, Y, and Z that are consistent with the following patterns of commonsense reasoning. You may use different events for the different parts of the problem.

(a) Explaining away:

$$\begin{aligned} P(Y\!=\!1|X\!=\!1) &> P(Y\!=\!1), \\ P(Y\!=\!1|X\!=\!1,Z\!=\!1) &< P(Y\!=\!1|X\!=\!1) \end{aligned}$$

(b) Accumulating evidence:

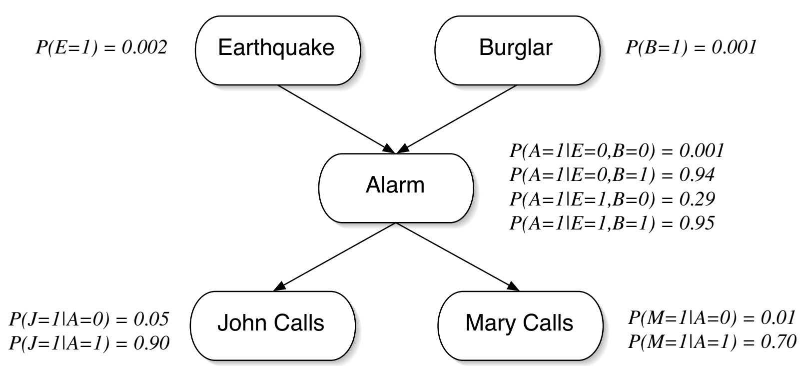
$$P(X\!=\!1) > P(X\!=\!1|Y\!=\!1) > P(X\!=\!1|Y\!=\!1,Z\!=\!1)$$

(c) Conditional independence:

$$\begin{aligned} P(X\!=\!1,Y\!=\!1) &> P(X\!=\!1)P(Y\!=\!1) \\ P(X\!=\!1,Y\!=\!1|Z\!=\!1) &= P(X\!=\!1|Z\!=\!1)P(Y\!=\!1|Z\!=\!1) \end{aligned}$$

1.4 Probabilistic inference

Recall the probabilistic model that we described in class for the binary random variables $\{E = \text{Earthquake}, B = \text{Burglary}, A = \text{Alarm}, J = \text{JohnCalls}, M = \text{MaryCalls}\}$. We also expressed this model as a belief network, with the directed acyclic graph (DAG) and conditional probability tables (CPTs) shown below:



Compute numeric values for the following probabilities, exploiting relations of conditional independence as much as possible to simplify your calculations. You may use intermediate results from lecture, but otherwise show your work.

(a) $P(E\!=\!1\|J\!=\!1)$	(c) $P(A\!=\!1\|J\!=\!0)$	(e) $P(A\!=\!1\|E\!=\!1)$
(b) $P(E\!=\!1\|J\!=\!1,B\!=\!1)$	(d) $P(A\!=\!1\|J\!=\!0,M\!=\!1)$	(f) $P(A\!=\!1\|E\!=\!1,J\!=\!0)$

Consider your results in (b) versus (a), (d) versus (c), and (f) versus (e). Do they seem consistent with commonsense patterns of reasoning?