CSE 150 Lecture 3 1/11/10

HW1 due next Tues

Review

* Probabilities

  Unconditional $P(x)$

  Conditional $P(y|x)$

  Joint $P(x,y)$

* Conditional independence

$$\left.\begin{aligned} P(x|y) &= P(x) \\ P(y|x) &= P(y) \\ P(x,y) &= P(x)P(y) \end{aligned}\right\} \text{equivalent}$$

* Rules

$P(A,B,C,\ldots) = P(A)P(B|A)P(C|A,B)\ldots$ product rule

$P(X|Y) = P(Y|X)P(X)/P(Y)$ Bayes rule

$P(x) = \sum_y P(X, Y=y)$ marginalization

Probabilistic inference

Today: do probabilities capture patterns of commonsense reasoning?

Examples — reasoning about:

1) multiple explanations of a single event
2) multiple events with a single explanation
3) intervening events

* Binary random variables

B = burglary?

E = earthquake?

A = alarm?

* Joint distribution

$$P(B,E,A) = P(B)\,P(E|B)\,P(A|B,E)$$

* Domain knowledge

$P(B=1) = 0.001$

$P(E=1|B=0) = 0.002$

$P(E=1|B=1) = 0.002$

$P(E=1|B) = P(E=1) = 0.002$

} Conditional independence.

B	E	$P(A=1\|B,E)$
0	0	0.001
0	1	0.29
1	0	0.94
1	1	0.95

1) reasoning about multiple explanations.

Compare: $P(B=1) = 0.001$

$P(B=1|A=1) = ?$

$P(B=1|A=1, E=1) = ?$

· Bayes rule

$$P(B=1|A=1) = \frac{P(A=1|B=1)\,\overbrace{P(B=1)}^{(0.001)}}{\underbrace{P(A=1)}_{\boxed{0.00252}}}$$

Term in denominator:

$$P(A=1) = \sum_{e,b\in\{0,1\}} P(B=b, E=e, A=1) \quad \text{marginalization}$$

$$= \sum_{e,b} P(B=b)\,P(E=e|B=b)\,P(A=1|B=b, E=e) \quad \text{product rule}$$

$$P(A=1) = \sum_{e,b} P(B=b)\,P(E=e)\,P(A=1 \mid B=b, E=e) \quad \text{Conditional independence}$$

$$= P(B=0)P(E=0)P(A=1 \mid B=E=0) + P(B=1)P(E=0)P(A=1 \mid B=1, E=0) +$$
$$P(B=0)P(E=1)P(A=1 \mid B=0, E=1) + P(B=1)P(E=1)P(A=1 \mid B=E=1)$$
$$= (1-0.001)(1-0.002)(0.001) + (0.001)(1-0.002)(0.94) +$$
$$(1-0.001)(0.002)(0.29) + (0.001)(0.002)(0.95)$$
$$= \boxed{0.00252}$$

* Term in numerator

$$P(A=1 \mid B=1) = \sum_{e \in \{0,1\}} P(A=1, E=e \mid B=1) \quad \text{"Conditionalized" marginalization}$$

$$= \sum_e P(A=1 \mid E=e, B=1)\,\underbrace{P(E=e \mid B=1)}_{P(E=e)} \quad \text{"Conditionalized" product rule; Conditional independence}$$

$$= P(A=1 \mid E=0, B=1)P(E=0) + P(A=1 \mid E=1, B=1)P(E=1)$$
$$= (0.94)(1-0.002) + (0.95)(0.002)$$
$$= 0.94002$$

So: $$P(B=1 \mid A=1) = \frac{P(A=1 \mid B=1)P(B=1)}{P(A=1)} = \frac{(0.94002)(0.001)}{(0.00252)} = 0.37$$

Compare to $P(B=1) = 0.001$

Now Compare to $P(B=1 \mid A=1, E=1) = ?$

* Conditionalized Bayes rule:

$$P(B=1 \mid A=1, E=1) = \frac{\overset{(0.95)}{P(A=1 \mid B=1, E=1)}\;\overset{P(B=1)=0.001}{P(B=1 \mid E=1)}}{P(A=1 \mid E=1)} \quad \text{Conditional independence}$$

Denominator: $$P(A=1 \mid E=1) = \sum_b P(A=1, B=b \mid E=1) \quad \text{conditional marginalization}$$
$$= \sum_b P(A=1 \mid B=b, E=1)\,P(B=b \mid E=1) \quad \text{cond. product rule}$$

⋮ Same steps as for computing $P(A=1 \mid B=1)$

$$\cong 0.29$$

$$P(B=1 \mid A=1, E=1) = \frac{(0.95)(0.001)}{0.29} = 0.0033$$

Summary : $P(B=1) = 0.001$
$P(B=1 \mid A=1) = 0.37 \uparrow$ non-monotonic
$P(B=1 \mid A=1, E=1) = 0.0033 \downarrow$

$\Rightarrow$ earthquake "explained away" the alarm, weakening our belief in burglary.
Arises from multiple (causal) explanations of an observed event.

2) Multiple events with a common explanation

* Two more binary random variables :
  J = John calls?
  M = Mary calls?
* Conditional independence assumptions
  Already : $P(B \mid E) = P(B)$
  Also assume : $P(J \mid A) = P(J \mid A, B, E)$
  $P(M \mid A) = P(M \mid A, B, E, J)$
* Joint distribution

$$P(B, E, A, J, M) = P(B)\,P(E \mid B)\,P(A \mid B, E)\,P(J \mid B, E, A)\,P(M \mid B, E, A, J)$$
$$= P(B)\,P(E)\,P(A \mid B, E)\,P(J \mid A)\,P(M \mid A) \quad \text{Conditional independence}$$

* Conditional probabilities
  $P(J=1 \mid A=0) = 0.05$
  $P(J=1 \mid A=1) = 0.9$
  $P(M=1 \mid A=0) = 0.01$
  $P(M=1 \mid A=1) = 0.7$.

2) multiple events with a common explanation

Compare $P(A=1) = 0.00252$ (from example #1)

$P(A=1|J=1) = \;?$ $\uparrow$ 0.0435 another case of

$P(A=1|J=1, M=0) = \;?$ $\downarrow$ 0.0136 non-monotonic reasoning

· Bayes rule:

$$P(A=1|J=1) = \frac{\overset{(0.9)}{P(J=1|A=1)}\;\overset{(0.00252)}{P(A=1)}}{P(J=1)}$$

Denominator: $P(J=1) = \sum_a P(A=a, J=1)$ marginalization

inefficient!

$$= \sum_{a,b,e,m} P(A=a, B=b, E=e, M=m, J=1)$$

$$= \sum_a P(J=1|A=a)P(A=a) \quad \text{product rule}$$

$$= P(J=1|A=0)P(A=0) + P(J=1|A=1)P(A=1)$$

$$= (0.05)(1-0.00252) + (0.9)(0.00252)$$

$$= 0.0521$$

So $P(A=1|J=1) = \dfrac{(0.9)(0.00252)}{0.0521} = 0.0435$

Bayes rule with multiple pieces of evidence:

$$P(A=1|J=1, M=0) = \frac{P(J=1, M=0|A=1)\,P(A=1)}{P(J=1, M=0)}$$

$$= \frac{\overset{(0.9)}{P(J=1|A=1)}\;\overset{(1-0.7)}{P(M=0|A=1)}\;\overset{(0.00252)}{P(A=1)}}{\underset{(0.05)}{P(J=1, M=0)}}$$

$$= 0.0136$$

Denominator:

$$P(J=1, M=0) = \sum_{a\in\{0,1\}} P(A=a, J=1, M=0) \quad \text{marginalization}$$

$$= \sum_a P(A=a)P(J=1|A=a)P(M=0|J=1, A=a) \quad \text{product rule}$$

$$= \sum_a P(A=a)P(J=1|A=a)P(M=0|A=a) \quad \text{conditional independence}$$

$\vdots$ write out terms $a \in \{0,1\}$; plug in #s

$$= 0.05$$

Conditional independence

$P(J=1|A=1)=0.9$

3) Reasoning about intervening events:

from example #1

Compare: $P(A=1) = 0.00252$

(0.94002)

$P(A=1|J=1) = 0.0435 \uparrow$

$$P(A=1|J=1,B=1) = \frac{P(J=1|A=1,B=1)\,P(A=1|B=1)}{P(J=1|B=1)}$$ Conditionalized Bayes rule

$\leftarrow 0.849$

$= 0.9965 \uparrow$ ~~0.0435~~

Denominator:

$P(J=1|B=1) = \sum_a P(J=1, A=a|B=1)$ Conditional marginalization

$= \sum_a P(A=a|B=1)P(J=1|A=a,B=1)$ product rule

$= \sum_a P(A=a|B=1)P(J=1|A=a)$ Conditional independence

Computed in example #1

given from model

$= 0.849$