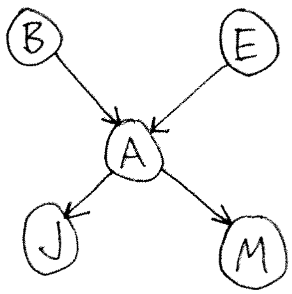
CSE 150 – 1/18/11

Review

B = burglar
E = earthquake
A = alarm
J = John Calls
M = Mary Calls



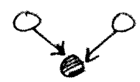
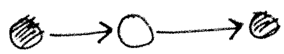
* Belief Network (BN) = DAG + CPTs

* Conditional Independence

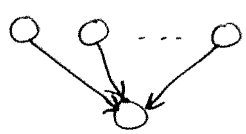
$$P(X_1, X_2, \ldots, X_n) = P(X_1)\,P(X_2 | X_1) \ldots P(X_n | X_1, \ldots, X_{n-1}) \quad \text{product rule}$$

$$= \prod_{i=1}^{n} P(X_i | X_1, X_2, \ldots, X_{i-1}) = \prod_{i=1}^{n} P(X_i | pa(X_i))$$

parents of X_i

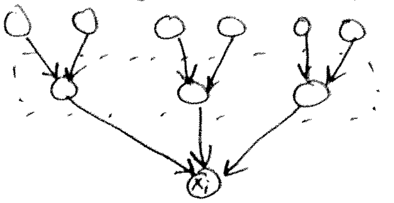
* Types of Reasoning
  1) competing explanations of observed event 
  2) multiple events with common explanation 
  3) intervening events 

* Representing CPTs $P(X_i | pa(X_i))$
  - lookup table
  - logical AND, OR operations
  - noisy OR



Conditional Independence

A node is conditionally independent of its non-parent ancestors given its parents.



$$P(x_i \mid pa(x_i)) = P(x_i \mid x_1, x_2, \dots x_{i-1})$$

* More generally:

Let X, Y, E refer to sets of nodes.

When is X <u>conditionally</u> <u>independent</u> of Y given <u>evidence</u> E?

When is $P(X \mid E, Y) = P(X \mid E)$?

$P(X, Y \mid E) = P(X \mid E)\, P(Y \mid E)$?

Above is a special case: $X = \{x_i\}$, $E = \{pa(x_i)\}$,
$Y = \{x_1, x_2, \dots, x_{i-1}\} - pa(x_i)$

* d-separation

"direction-dependent" separation

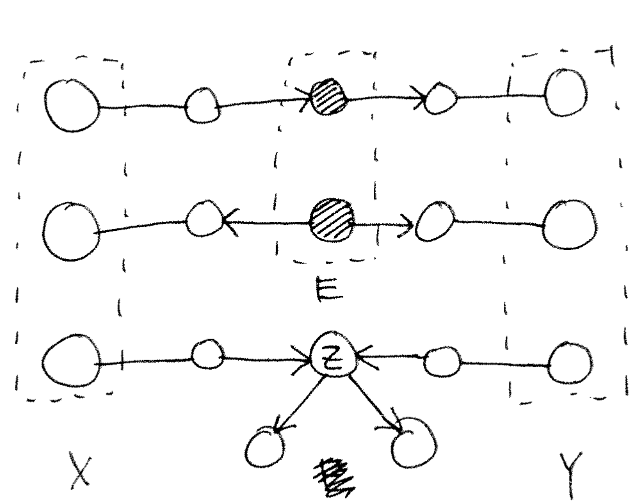
Relates conditional independence to graph-theoretic properties.

* $P(X, Y \mid E) = P(X \mid E) P(Y \mid E)$ if and only if every undirected path from a node in X to a node in Y is "d-separated" by E.

* <u>Definition:</u> a path π is <u>d-separated</u> if there exists a node $z \in \pi$ for which one of <u>3</u> conditions hold:

(I) $z \in E$ with $\rightarrow z \rightarrow$
is an "intervening" event

(II) $z \in E$ with $\leftarrow z \rightarrow$
is a common explanation



(I)

(II)

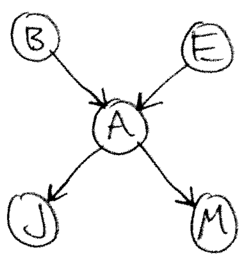
(III)

(III) $z \notin E$, $desc(z) \notin E$, with 
no observed common effects

* Proof that d-separation $\Leftrightarrow$ conditional independence is difficult, beyond course.

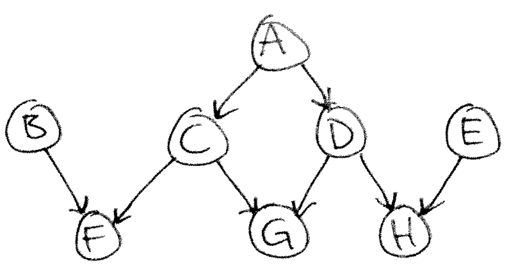
* Efficiend algorithms exist for tests of d-separation

* Alarm BN Example



1) $P(B|A,M) \overset{?}{=} P(B|A)$ true
   alarm is an intervening event
2) $P(J,M|A) \overset{?}{=} P(J|A)P(M|A)$ true
   alarm is common explanation of J, M
3) $P(B,E) \overset{?}{=} P(B)P(E)$ true, condition III
   $P(B,E|J) \overset{?}{=} P(B|J)P(E|J)$ false
   "explaining away"

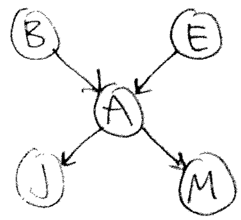
* Loopy BN Example



statement	true or false?	
$P(D\|H) = P(D\|E,H)$	false	(case III)
$P(F,H\|A) = P(F\|A)P(H\|A)$	true	top path ok by (II) zigzag path ok by (III)
$P(F,G,H\|A) = P(F\|A)P(G\|A)P(H\|A)$	false	$P(G\|A) \neq P(G\|A,F,H)$

$\hookrightarrow = P(F|A)P(\overset{H}{\cancel{G}}|A,F)P(\overset{G}{\cancel{H}}|A,F,\overset{H}{\cancel{G}})$
by product rule

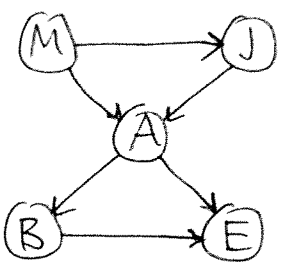
Ex: node ordering



* What is BN with node ordering $\{M, J, A, B, E\}$?

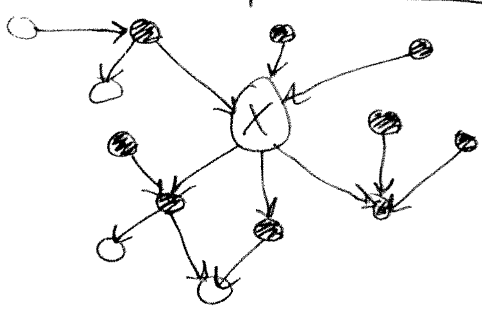
$$P(M,J,A,B,E) = P(M)\,P(J|M)\,P(A|J,M)\,P(B|J,M,A) \times P(E|M,J,A,B)$$ product rule

$$= P(M)\,P(J|M)\,P(A|J,M)\,P(B|A)\,P(E|A,B$$



graph based on node ordering $\{M, J, A, B, E\}$

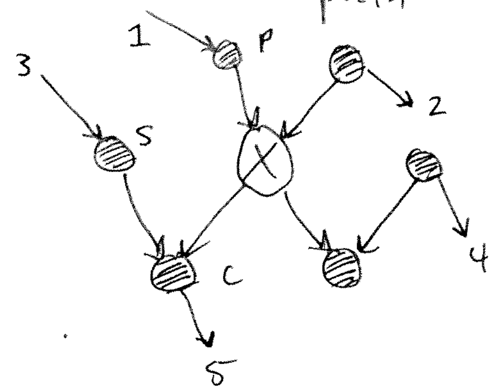
Def: Markov Blanket B_X of individual node X consists of <u>parents of X</u>, children of X, and <u>parents of children</u> of X (not including X).



Thm: A node X is conditionally independent of all nodes outside B_X given nodes inside B_X:

$$P(X|B_X, Y) = P(X|B_X) \quad \text{where } Y \notin \{B_X, X\}$$

Proof: For any node $Y \notin \{X, B_X\}$, the undirected path from Y to X must pass through B_X.



There are five cases to consider:

1) from parent of parent of X (satisfies case I)
2) from child of parent P of node X (satisfies case II)
3) from parent of spouse S of node X (satisfies case I)

4) from child of spouse S of node X
(satisfies case II)

5) from child of child C of node X
(satisfies case I)

All paths are "d-separated" from Y to X, hence $P(X \mid B_X, Y) = P(X \mid B_X)$