

# Period of a Pendulum (Part 2)

## Post-Lab Assignment

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## Goals/RQs(?)

- How is the period of oscillation of a pendulum related to its angle? How does it compare with theoretical models describing its motion?
- (More broadly): How can experiment design and biases influence experimental results? How do these impacts affect the evaluation of experimental results in relation to their theoretical values?

## Procedure & Setup (incl. Modifications)



(Setup image by Ray)

This procedure is generally the same as the one used last week (the procedure is also reworded from what was originally included in the notebook for clarity):

1. Move the string from its initial position, at 0 degrees, upwards to its starting position such that the angle between the string and the protractor affixed onto the retort stand is at 10 degrees and that the string is tensioned.
2. While recording the pendulum for one period, release the pendulum from its starting position so that the pendulum starts to swing/oscillate.
3. Using a video playback feature on a video editing software or otherwise, find and record the difference in time from when the pendulum was released to when it completes a period of oscillation.
4. Repeat steps 1-3 for two more trials.
5. Repeat steps 1-4 for the angles of 15, 30, 60 and 90.

The main modification we have made to our setup is to affix the protractor to the pendulum and to use the protractor to measure the angle. For the experiment last week, we did not account for potential parallax errors from placing the protractor onto the side of the retort stand whenever we needed to measure an angle, allowing for random uncertainties in the angle measurement, so we affixed the protractor so that there is no random error in the measurement of the angle as the angle is measured the same constantly between trials and changes in the dependent variable.

To minimize the parallax errors from perceiving the protractor, we measured the angle from the protractor by viewing the protractor directly, level to its position and ensuring that the string is at the “90 degrees” position on the protractor before making any adjustments to the string position while maintaining the same position/view of the protractor.

## Data

(Modified from original data table in lab notebook):

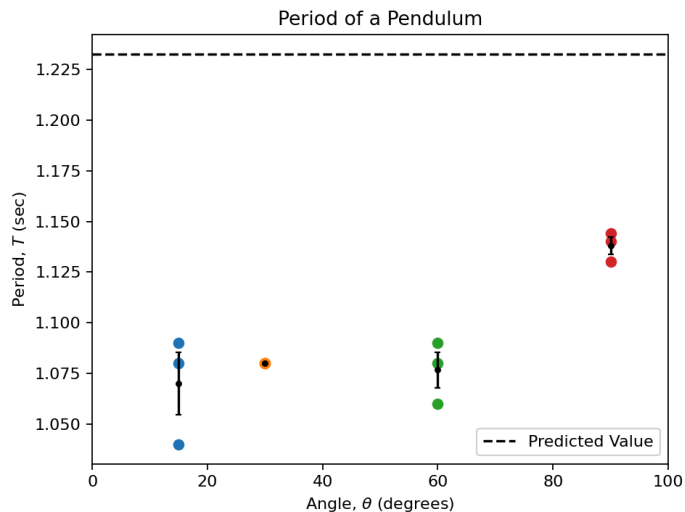
Table 1: Data table for 10 degrees (verifying agreement with past lab results)

Angle ( $^{\circ} \pm 0.5^{\circ}$ )	Measurements ( $s \pm 0.01s$ )			Means ( $s \pm 0.01s$ )
	Trial 1	Trial 2	Trial 3	
10.0	1.06	1.08	1.08	1.07

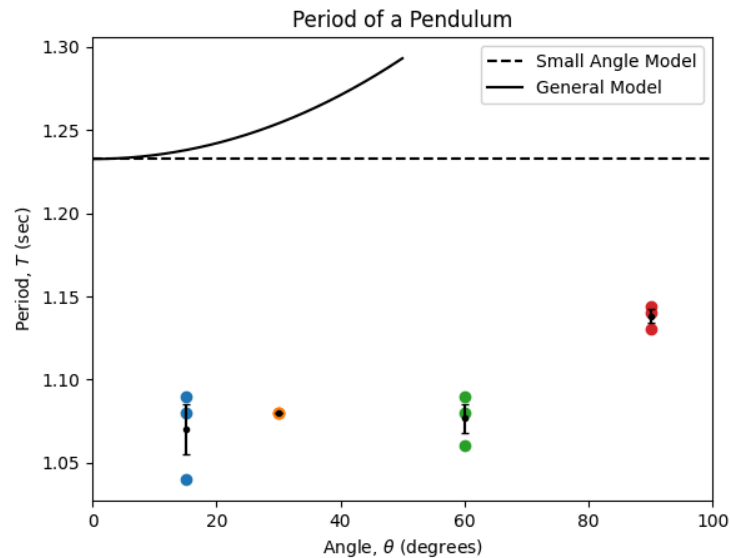
Angle ( $^{\circ} \pm 0.5^{\circ}$ )	Measurements ( $s \pm 0.01s$ )			Means ( $s \pm 0.01s$ )
	Trial 1	Trial 2	Trial 3	
15.0	1.08	1.04	1.09	1.07
30.0	1.08	1.08	1.08	1.08
60.0	1.09	1.08	1.06	1.08
90.0	1.14	1.13	1.14	1.14
			Overall Mean	$1.09 \pm 0.04$

The uncertainty of the overall mean is derived from the formula  $\frac{\max - \min}{2}$  where  $\frac{1.14 - 1.07}{2} \approx 0.04$ . I believe that the uncertainty here is more representative of the potential random uncertainty compared to the value of  $\pm 0.01$  derived from our measurement “tool” (method).

When plotted, the data points are as follows:



And when evaluated in consideration with the improved model, the data plotted is as follows:



When calculating the t-test in comparison with the values of last week's experiment (with smaller angles) to determine the level of agreement with the lower angles, we obtain the following value:

$$t' = \frac{A-B}{\sqrt{(\delta A)^2 + (\delta B)^2}} = \frac{1.04-1.09}{\sqrt{0.02^2 + 0.04^2}} = -1.11$$
, implying that the relationship is inconclusive between the values of this experiment and the previous. This entails that there isn't a clear relationship between the small-angle experiment and our current experiment.

The t-tests for each angle in relation to the predicted values of the small-angle model are calculated as follows:

```
for n, theta in enumerate(angles):
    print("For", theta, "degrees, t' =", t_prime(avg[n], davg[n], T_predicted))

For 15 degrees, t' = -10.641264733900355
For 30 degrees, t' = -inf
For 60 degrees, t' = -17.675282229887877
For 90 degrees, t' = -22.70969593620121
/tmp/ipython-input-4162965860.py:23: RuntimeWarning: divide by zero encountered in scalar divide
    return((A-B)/np.sqrt(dA**2 + dB**2)) # If only 3 arguments are given, assumes dB = 0 (e.g. a literature value)
```

The t-tests for each angle in relation to the predicted values of the general model is as follows:

Finally, we can perform the  $t'$  test on each point.

```
for n, theta in enumerate(angles):
    print("For", theta, "degrees, t' =", t_prime(avg[n], davg[n],
        T_predicted_expanded(L,theta)))

For 15 degrees, t' = -10.988235576381971
For 30 degrees, t' = -inf
For 60 degrees, t' = -27.638485268819878
For 90 degrees, t' = -70.12370622425146
/tmp/ipython-input-4162965860.py:23: RuntimeWarning: divide by zero encountered in scalar divide
    return((A-B)/np.sqrt(dA**2 + dB**2)) # If only 3 arguments are given, assumes dB = 0 (e.g. a literature value)
```

The reason that for  $\theta = 30$  degrees, the  $t$ -value is negative infinity is that the uncertainty derived in the calculations of the Google Collab notebook is dependent on the experimental uncertainty between each trial for a given angle. Out of the three trials we took for the angle of 30 degrees, all three trials happened to have the same value of 1.08 seconds. To calculate a more accurate  $t'$  value, we can recalculate  $t'$  using the measurement uncertainty:

For the small-angle model, we get  $t' = -22.7$ :

```
[30]
✓ Os  #for n, theta in enumerate(angles):
#    print("For", theta, "degrees, t' =", t_prime(avg[n], davg[n], T_predicted))

print("For", 30, "degrees, t' =", t_prime(average(T90), standard_error(T90), T_predicted))

 For 30 degrees, t' = -22.70969593620121
```

For the general model, we get  $t' = -27.9$ :

```
print (""")
print ("For", 30, "degrees, t' =", t_prime(average(T90), standard_error(T90),
    T_predicted_expanded(L,30)))



For 30 degrees, t' = -27.85438181466429
```

## Conclusions

The conclusion is your *interpretation* and *discussion* of your data.

- What do your data tell you?
- How do your data match the model (or models) you were comparing against, or to your expectations in general? (Sometimes this means using the  $t'$  test, but other times it means making qualitative comparisons.)

- Were you able to estimate uncertainties well, or do you see room to make changes or improvements in the technique?
- Do your results lead to new questions?
- Can you think of other ways to extend or improve the experiment?

In about one or two paragraphs, draw conclusions from the pendulum data you collected today. Address both the qualitative and quantitative aspects of the experiment and feel free to use plots, tables or anything else from your notebook to support your words. Don't include throw-away statements like "Looks good" or "Agrees pretty well"; instead, try to be precise.

Remember... your goal is not to discover some "correct" answer. In fact, approaching any experiment with that mind set is the wrong thing to do. You must always strive to reach conclusions which are supported by your data, regardless of what you think the "right" answer should be. Never should you state a conclusion which is contradicted by the data. Stating that the results of your experiment are inconclusive, or do not agree with theoretical predictions is completely acceptable if that is what your data indicate. Trying to shoehorn your data into agreement with some preconceived expectation when you cannot support that claim is fraudulent.

Statistically, there appears to be disagreement between the values we obtained from our experiment and the theoretical values in both the general and small-angle models. This is evidenced by t-test values of  $\sim 10$  or higher, indicating that the experimental values we have obtained differ from the theoretical value to a significant extent. By observation of the graph plots, it seems that the behavior of our measurements somewhat appears to match the behavior of the general model, as when the angle increases significantly, it also results in an increase in the time for the pendulum to oscillate, which is particularly noticeable for  $\theta = 90$  degrees. The fact that our values seem to match the behavior of the general model yet differ substantially from the values it predicts suggests that our values are low-accuracy, likely due to some systematic error in our measurement, but have relatively high precision.

I believe we eliminated a few significant sources of random error in our methodology, such as using the video to measure the period to eliminate human error in recording, which could arise from manually timing the experiment. However, there are systematic errors that influence the accuracy of the value. We speculated that this might be due to an applied force from when the pendulum was released, but while it is possible for other systematic errors to come into play, the ones that we have concluded (including air resistance or a resistive force caused by the string) would likely cause the duration of the period to shorten, not lengthen.

# Questions

Consider the following questions:

- Early in the lab, you had a discussion with a neighboring group (or groups) to compare your pendulums and measurement technique(s). What did you learn from the other group(s)? Did any of your measurement techniques or ideas change after speaking with the other group(s)? If so, how and why?
- At the start of Part 1, we provided you the model against which you were going to compare your data ( $T = \sqrt{L/g}$ ) and you therefore were collecting data with an *expected* outcome in mind.
  - How might having an expected outcome in mind before starting an experiment be *useful* or *helpful*?
  - How might having an expected outcome in mind before starting an experiment be *unproductive* or *unhelpful*?
  - Assuming we can never remove all expectations or predictions, what are some strategies we can take in the lab to mitigate potential issues? Provide *actionable* strategies (not just “don't be biased”).

Our main modification to the setup (by affixing the podtractor to the retort stand) was inspired by another group doing the same thing, as it should eliminate a source of random error from imprecision in measuring the starting angle. We also attempted to release the pendulum in a different way than our previous experiment by “pinching” the bob and releasing it (see image at start of report) after class discussions related to the release method (such as the suggested “hot wheels method” which we have found to not be replicable in our experiment), since we believed that the force applied upon the pendulum would be minimized or at the very least be kept constant if we use the same release method.

Having a “theoretical value” in mind while experimenting somewhat shoehorns me into making early assumptions about my results, such as whether data points are “outliers” and potentially excluding useful experimental data as a result, but it is still pretty useful, as it gives us considerable room to consider the efficacy and accuracy of our experiment via the comparison of data/theoretical models and doing statistical tests (i.e. t-test) to see the accuracy of our results.

I believe that issues could be mitigated by greater consideration of our experimental design and by minimizing potential sources of error through the formalization and inclusion of a wide range of control variables. Even if some of these variables may be redundant or have a negligible effect on the results (for example, we decided to record the mass of the bob this week and attempted to keep it constant with the mass of the bob from last week, even though the mass

theoretically has no influence on the period at all), I think it's worth it to be more careful and try and try to make the experiment as accurate as possible.