

Angels in Space

Let's talk about angels angles in space!

Given three non-collinear points, A , B , and C , let $\angle(A,B,C)$ be the *measure* of the angle formed by the rays \overrightarrow{BA} and \overrightarrow{BC} .

Note that $0 < \angle(A,B,C) < \pi$ always holds.

Given N points, P_0, \dots, P_{N-1} , three points, (P_i,P_j,P_k) with $i < j < k$, are randomly and uniformly selected. Compute the expected value of $\angle(P_i,P_j,P_k)$. Give your answer in radians.

Note: The points are 3D!

Resources

[Dot Product](#)

[Cross Product](#)

Input Format

The first line contains a single integer, N , denoting the number of points. The next N lines describe the points.

The i th subsequent line describes a point in the form of three space-separated integers: x_i , y_i , and z_i , respectively.

Constraints

- $3 \leq N \leq 100$
- $-10^4 \leq x_i, y_i, z_i \leq 10^4$
- No three points are collinear.*

Output Format

Print a single real number: the expected value of $\angle(P_i,P_j,P_k)$ in radians. Your answer is considered to be correct if the absolute error from the correct answer is at most 10^{-8} .

Sample Input

```
4
0 0 0
2 0 0
2 2 1
0 2 1
```

Sample Output

```
1.20593249868
```

Explanation

The sample input contains $N = 4$ points which are vertices of a rectangle in space.

There are four choices for (P_i, P_j, P_k) , each equally likely:

- $(P_1, P_2, P_3): \angle = \pi/2$
- $(P_1, P_2, P_4): \angle \approx 0.841068670568$
- $(P_1, P_3, P_4): \angle \approx 0.841068670568$
- $(P_2, P_3, P_4): \angle = \pi/2$

Thus, the expected value is:

$$\approx \frac{1}{4}(\pi/2 + 0.841068670568 + 0.841068670568 + \pi/2) \\ \approx 1.20593249868$$