

Introduction to Machine Learning

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Slides link:

<https://yaoxiangding.github.io/introML-2023/lec7-RL.pdf>



Summer 2023
Week 16

Announcements

- **考查方法 Evaluation Method**

平时成绩 : 20分 4次 小测 (5分/次)

报告 : 70分

墙报展示 : 10分

1. 报告内容(70分)

Report Types

论文综述 Paper review

算法实践 Case study

二选一 (select one from the above two)

Slides link:



Reinforcement Learning

- Basics
 - Markov decision process
 - RL with known model
 - RL with unknown model
 - Policy gradient & actor-critic methods
- Deep reinforcement learning
- Integrating learning and planning
- RL from human preference
- Take-home messages

Slides link:



Reinforcement Learning

- **Basics**

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Decision Making



Decision Making



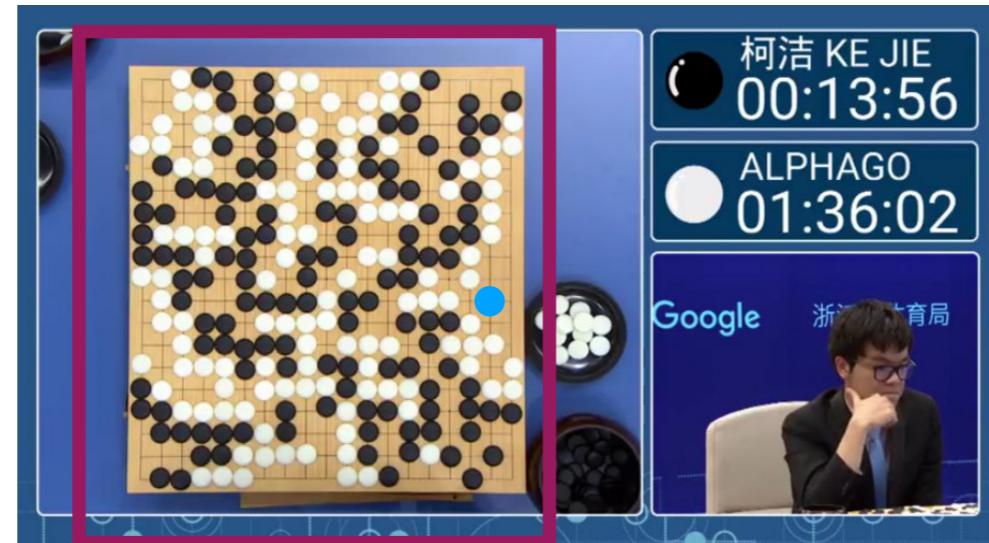
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Decision Making



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- Need to choose the corresponding “actions”.

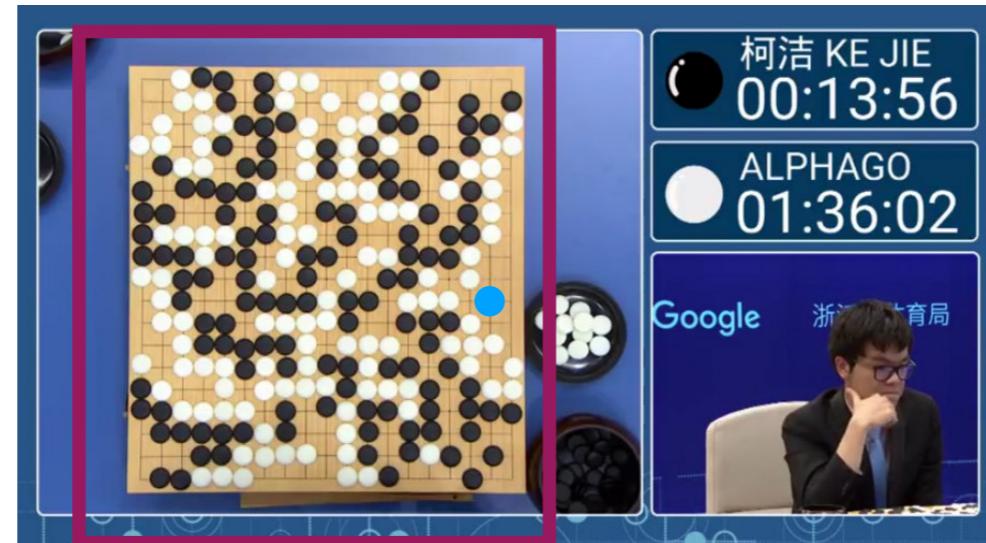
Decision Making



- The agent faces with a series of “states”.
- Need to choose the corresponding “actions”.
- Each action has a utility/cost.

In this lecture,
we given utility a name: reward.

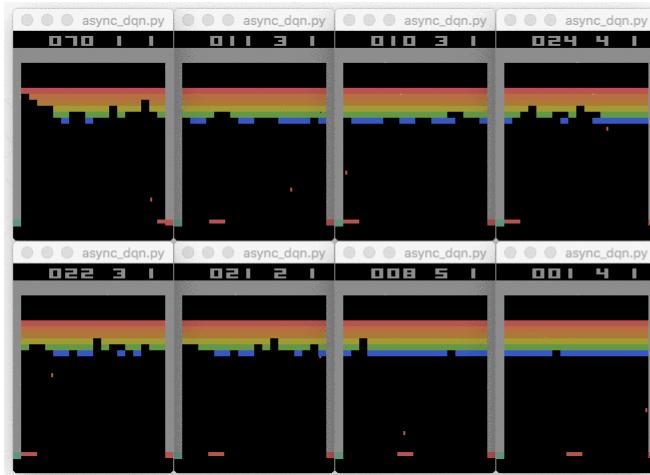
Decision Making



- The agent faces with a series of “states”.
- Need to choose the corresponding “actions”.
- Each action has a utility/cost.
- Target: maximize the total reward in a decision sequence by always choosing the right action.

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Decision Making

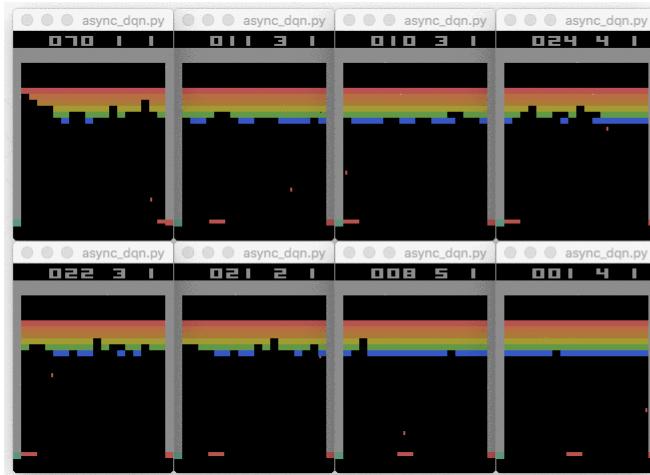


- Conduct **action** in any **state** of an **environment**.



In most problems, the agent needs to do a sequence of actions w.r.t. a sequence of states.

Decision Making

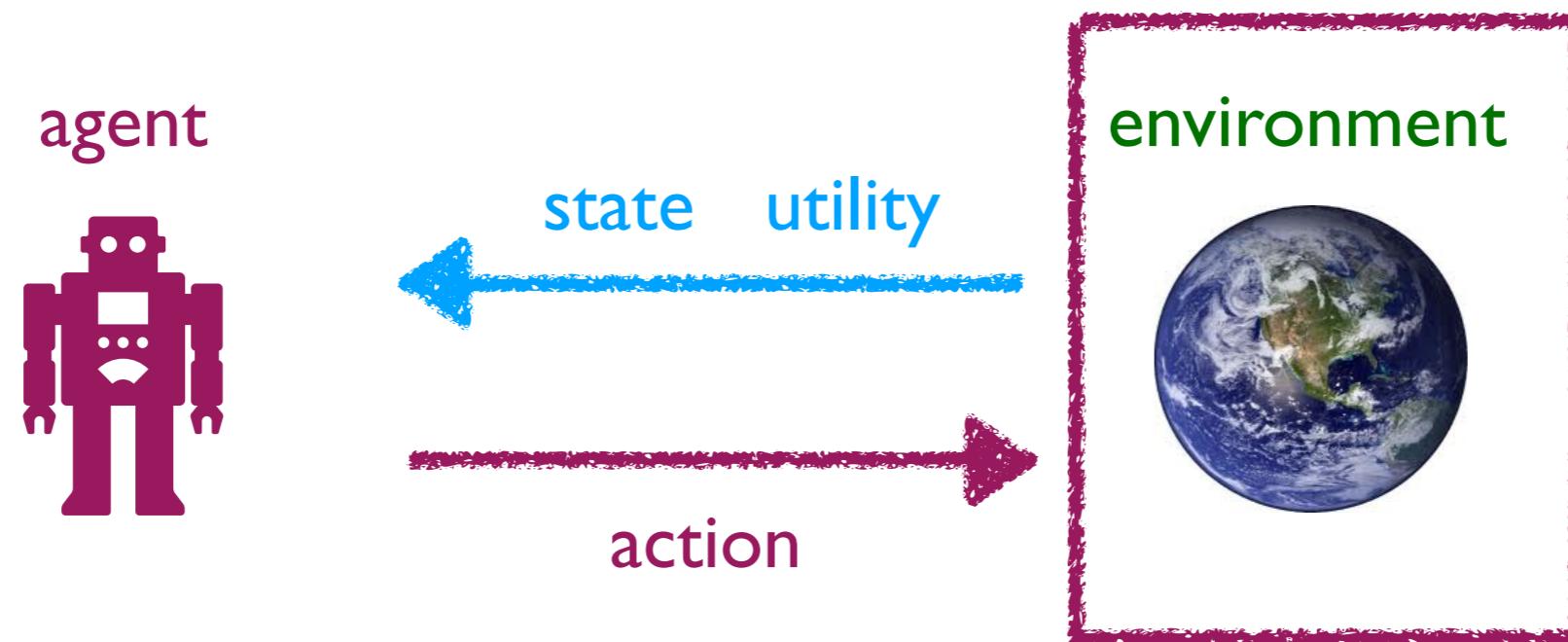


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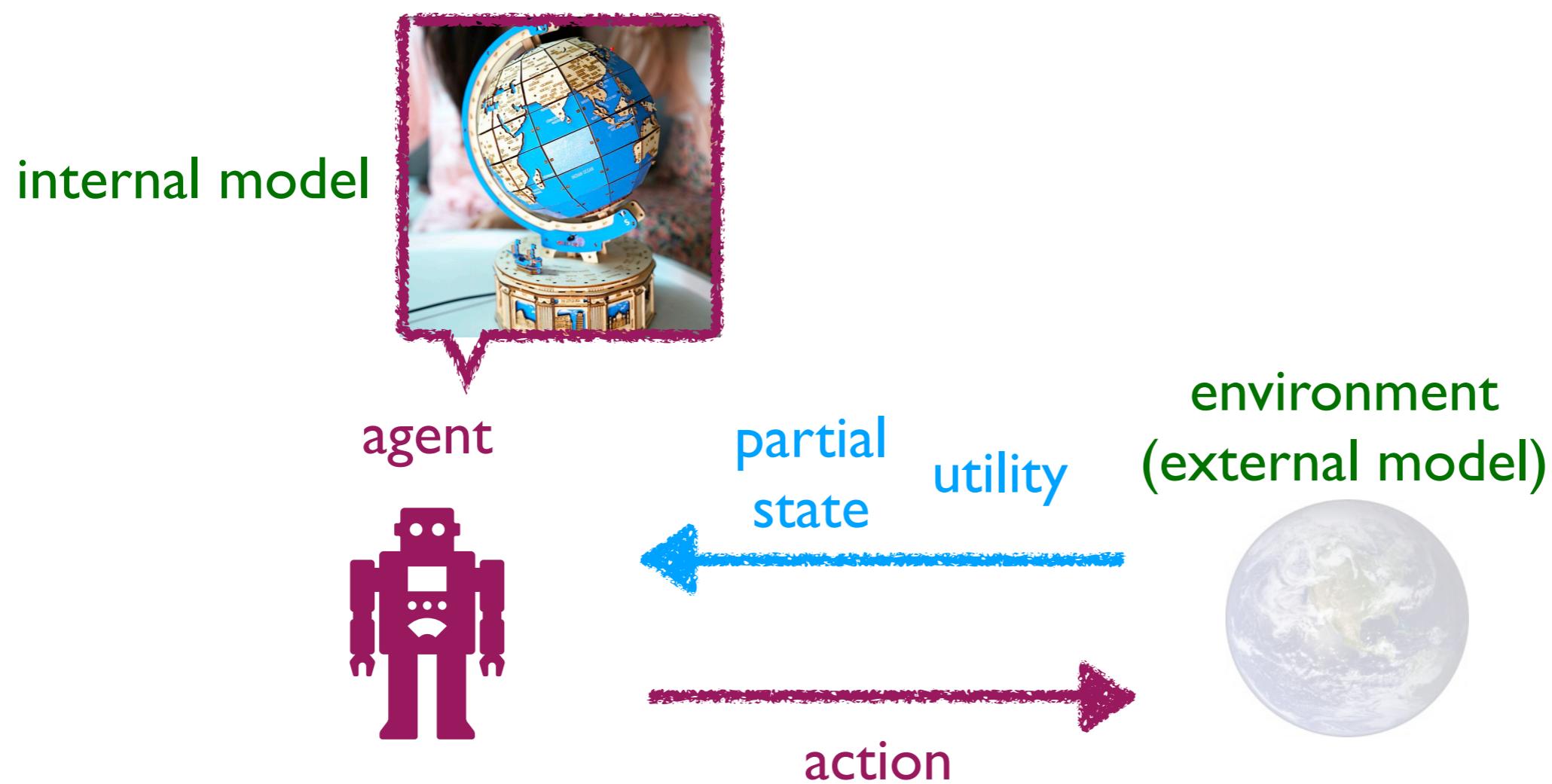
Model of the Environment



To make decisions in the environment, the agent usually needs a model of the environment to know how the things go on.
Where does this model come from?
Given by the problem (external) or built by the agent? (internal)

Internal vs. External Model

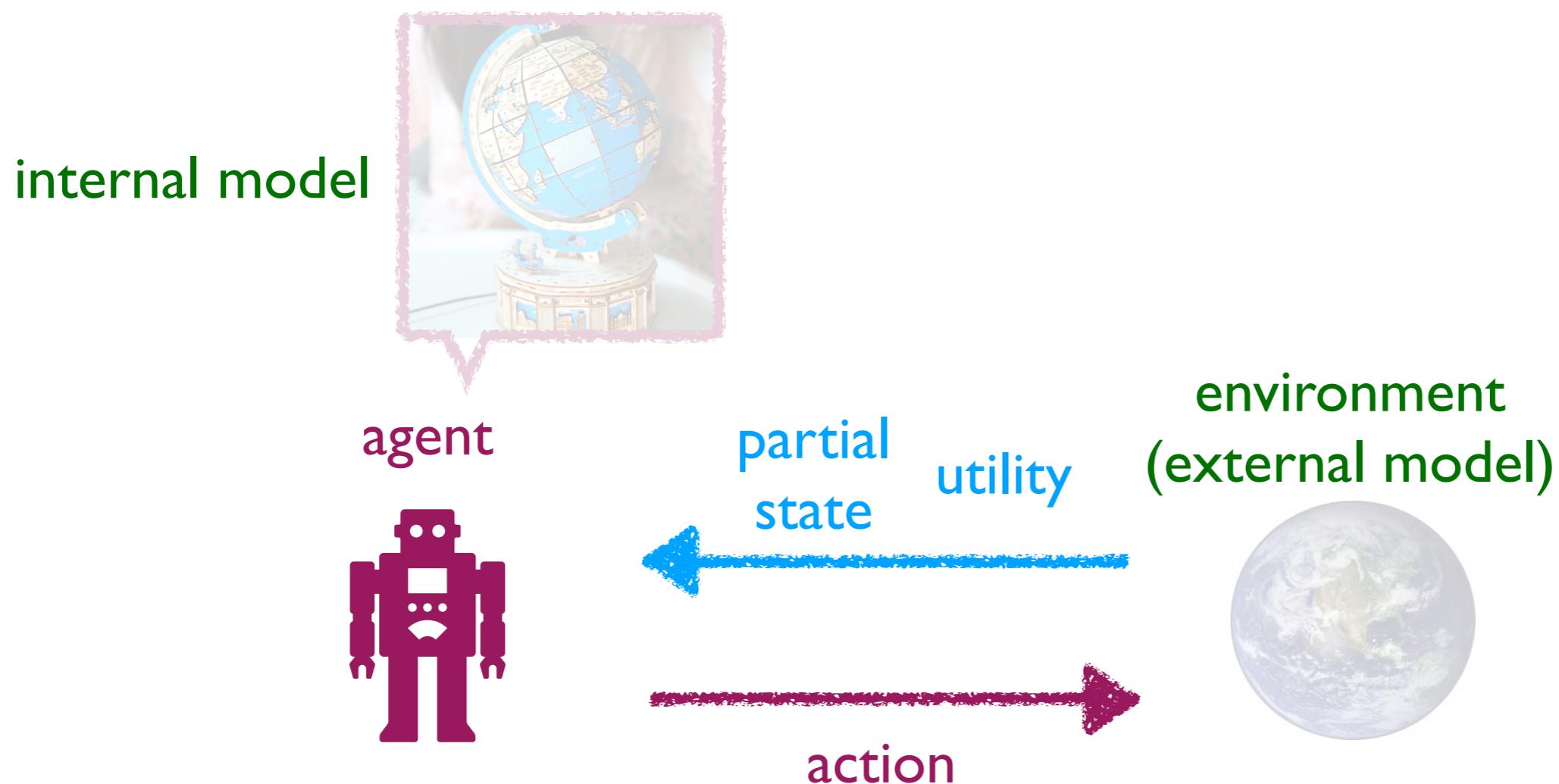
A decision-making agent can make use of external model when available, or build its own internal model when unavailable.



Internal vs. External Model

A decision-making agent can make use of external model when available, or build its own internal model when unavailable.

But what if both the external and internal models can not be used?



Reinforcement Learning

- Decision making is to find the optimal **policy**:
 - Decide best actions on all states.
 - No labeled <state, action> data, only receive reward.
 - The target is to maximize the long term total reward.

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- Search-based decision making:
 - When the model is known, and the search cost is reasonable.
- Reinforcement learning:
 - Decision making in **unknown model** or search cost is too high.

Reinforcement Learning

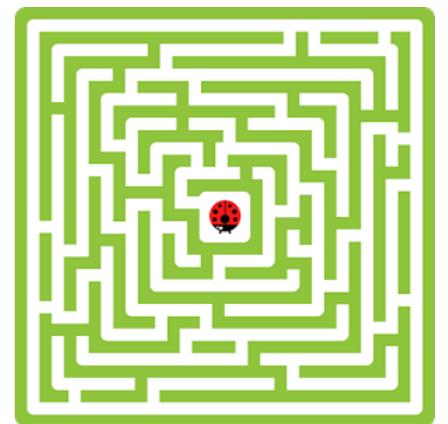
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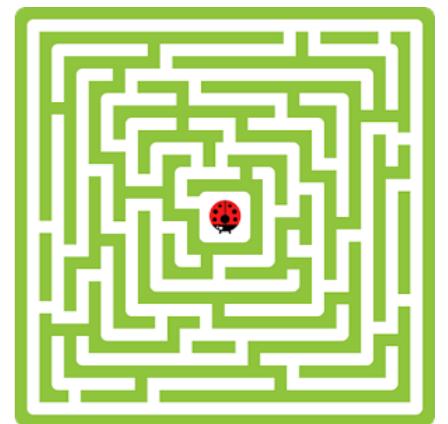
Markov Decision Process

- How to model a maze problem?



Markov Decision Process

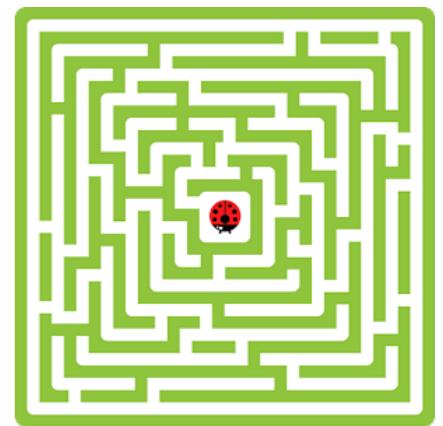
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 - **State:** the current position.



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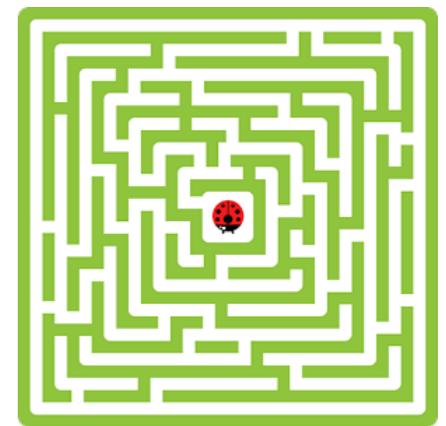
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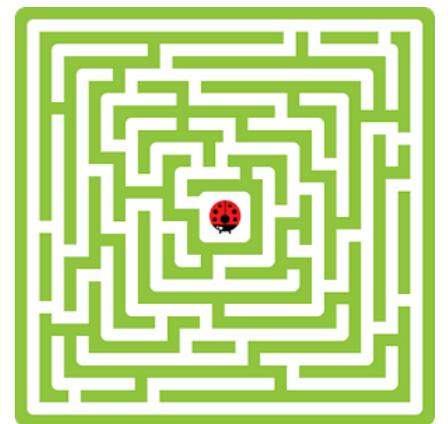
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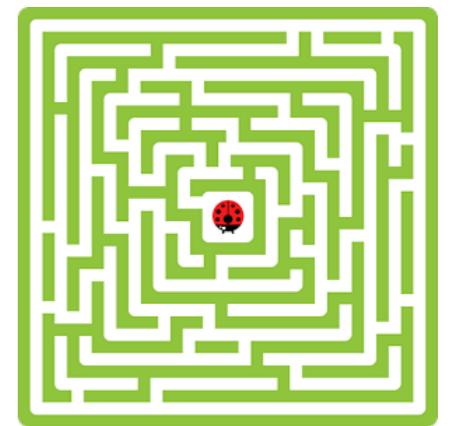
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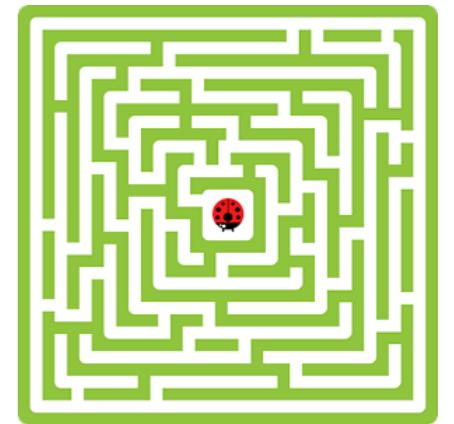
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Markov decision process (MDP) is the decision making model in RL with specific assumptions.

Mathematical Formulation

- A Markov Decision Process (MDP) is a five-tuple

$$< \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma >$$



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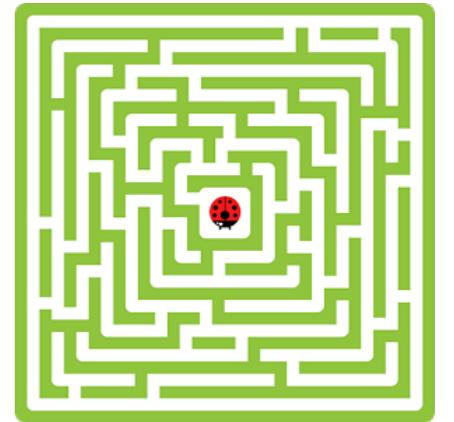


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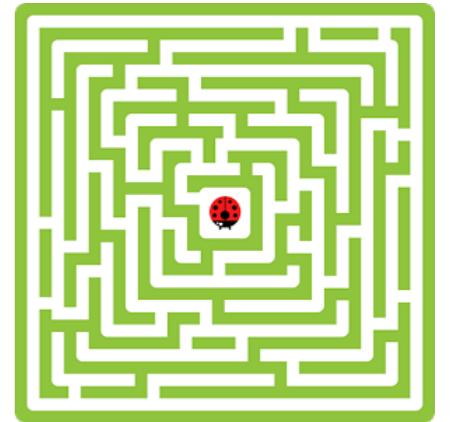


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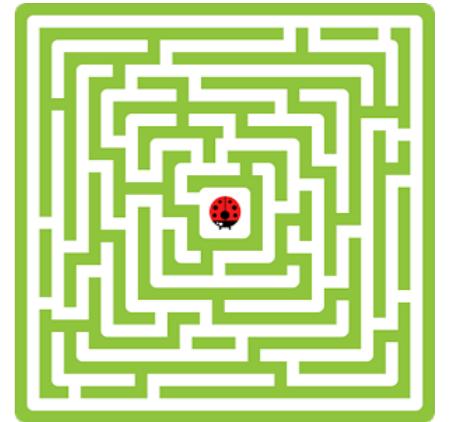


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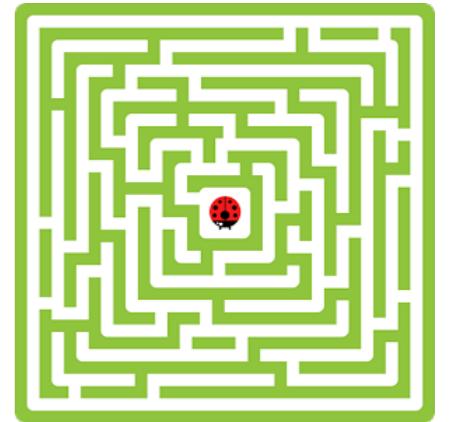


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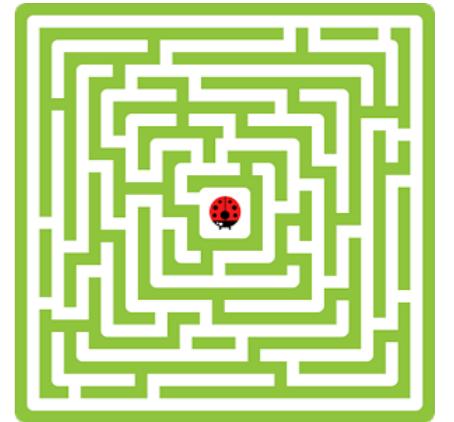


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The transition and reward functions can be stochastic!

The Markov Property

“The future is independent of the past given the present”

- The Markov property:

$$p(s_{t+1}|s_t, a_t) = p(s_{t+1}|s_1, s_2, \dots, s_t, a_t)$$



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- The current state is a sufficient statistic.
- Non-Markovian decision problem:

小张出生于中国，2016年来到美国留学。小张的母语是(?)



The Learning Agent

- The agent takes a series of actions, experiences a series of states, and receives a series of rewards:
 $\{s_1, a_1, r_1\}, \{s_2, a_2, r_2\}, \{s_3, a_3, r_3\} \dots$
- **policy**: function $p(a|s)$ used to select actions on any states.

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- **policy**: function $p(a|s)$ used to select actions on any states.
- The target is to find the optimal policy to maximize the discounted total reward along the timeline:
$$r_1 + \gamma r_2 + \gamma^2 r_3 + \dots = \sum_{t=1}^{\infty} \gamma^{t-1} r_t$$
- The discount factor measures how much the current action cares about the long term effect.

Value Functions: State Value Function V

- The state value function of a given policy is the expected total reward start from **a given state**, then follow the policy:

$$V_\pi(s) = \mathbb{E}_{\pi, p(s|s,a)} \left[\sum_{t=0}^{\infty} \gamma^t r_t | s_0 = s \right]$$

- The optimal policy have the optimal value function:

$$\forall s, \quad V_{\pi^*}(s) = \max_{\pi} V_{\pi}(s)$$

Value Functions: Action-State Value Function Q

- The action-state value function of a given policy is the expected total reward start from a given state, execute a given action, then follow the policy:

$$Q_\pi(s, a) = \mathbb{E}_{\pi, p(s|s, a)} \left[\sum_{t=0}^{\infty} \gamma^t r_t | s_0 = s, a_0 = a \right]$$

- The optimal deterministic policy chooses the optimal action:

$$\pi^*(s) = \arg \max_a Q_{\pi^*}(s, a)$$

If the optimal action-state value function is known, so is the optimal policy!

Bellman Equation

- For the state value function,

$$\begin{aligned} V_\pi(s) &= \mathbb{E}_{\pi(s), p(s|s,a)} \left[\sum_{t=0}^{\infty} \gamma^t r_t | s_0 = s \right] \\ &= \mathbb{E}_{\pi(s_0), p(s_1|s_0,a_0)} \left[r_0 + \gamma \mathbb{E}_{\pi(s), p(s|s,a)} \left[\sum_{t=1}^{\infty} \gamma^{t-1} r_t | s_1 \right] | s_0 = s \right] \\ &= \mathbb{E}_{\pi(s_0), p(s_1|s_0,a_0)} \left[r_0 + \gamma \underline{V_\pi(s_1)} | s_0 = s \right] \end{aligned}$$

Recursive Definition

- For discrete state and action, and deterministic policy,

$$V_\pi(s) = \sum_{s'} p(s'|s, \pi(s)) \left[r(s, \pi(s), s') + \gamma V_\pi(s') \right]$$

Bellman Equation (Cont.)

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Bellman Equation (Cont.)

- For optimal deterministic policy π^* ,

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- Due to the recursive structure, the optimal value functions can be solved by **dynamical programming**. This assumes that **the full information of the MDP is known!**

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Value Iteration

- Initialize value function V_0
- For $i=1,2,3\dots$ until convergence
 - Update V_i for each state

$$V_i(s) = \max_a \left[\sum_{s'} p(s'|s, a)[r(s, a, s') + \gamma V_{i-1}(s')] \right]$$

- Theoretical convergence guarantee to V^* and π^*

Value Iteration

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Why iterative update?
Loop exists in the MDP!

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Policy Iteration

- Initialize value function V_0 and policy π_0
- For $i=1,2,3\dots$ until convergence
 - Policy evaluation step: update V_i for each state **until converge**

$$V_i(s) = r + \gamma V_{i-1}^{\pi_{i-1}}(s')$$

- Policy improvement step: update π_i for each s-a pair.

$$\pi(s) \leftarrow \arg \max_a \sum_{s',r'} p(s',r|s,a)[r + \gamma V(s')]$$

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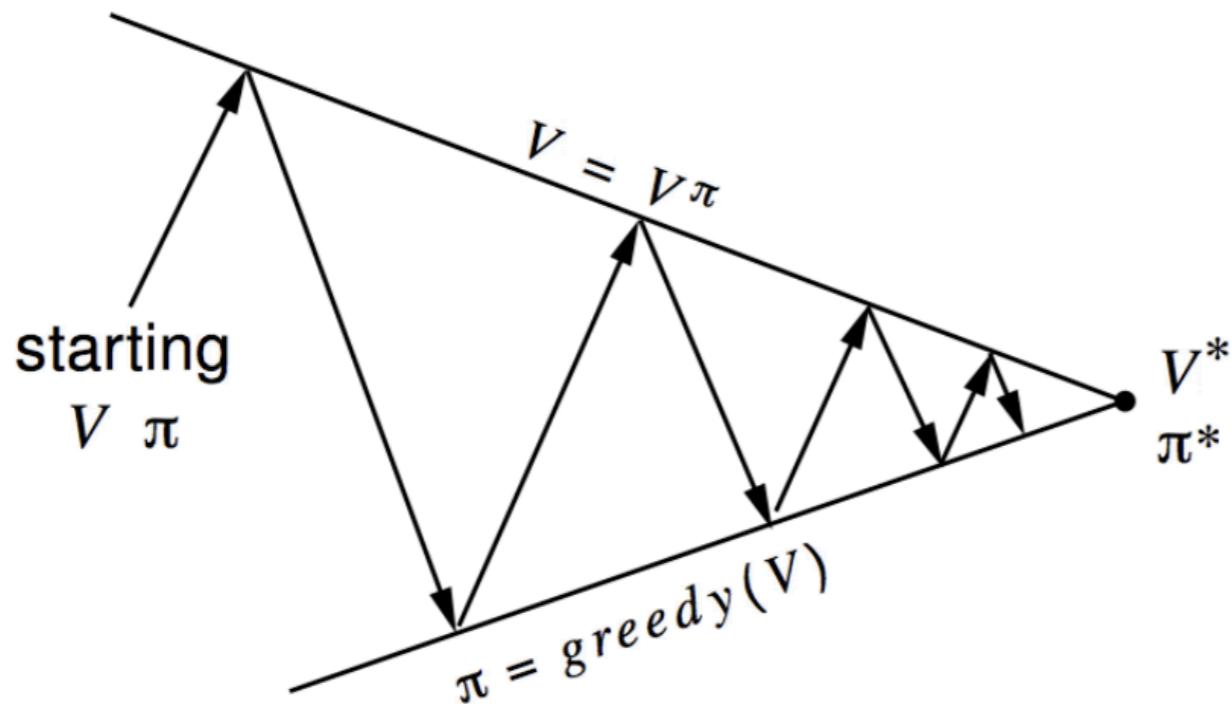
Calculated based on π_{i-1}
Actually an inner loop to do
iterative update until convergence

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Policy Iteration (Cont.)

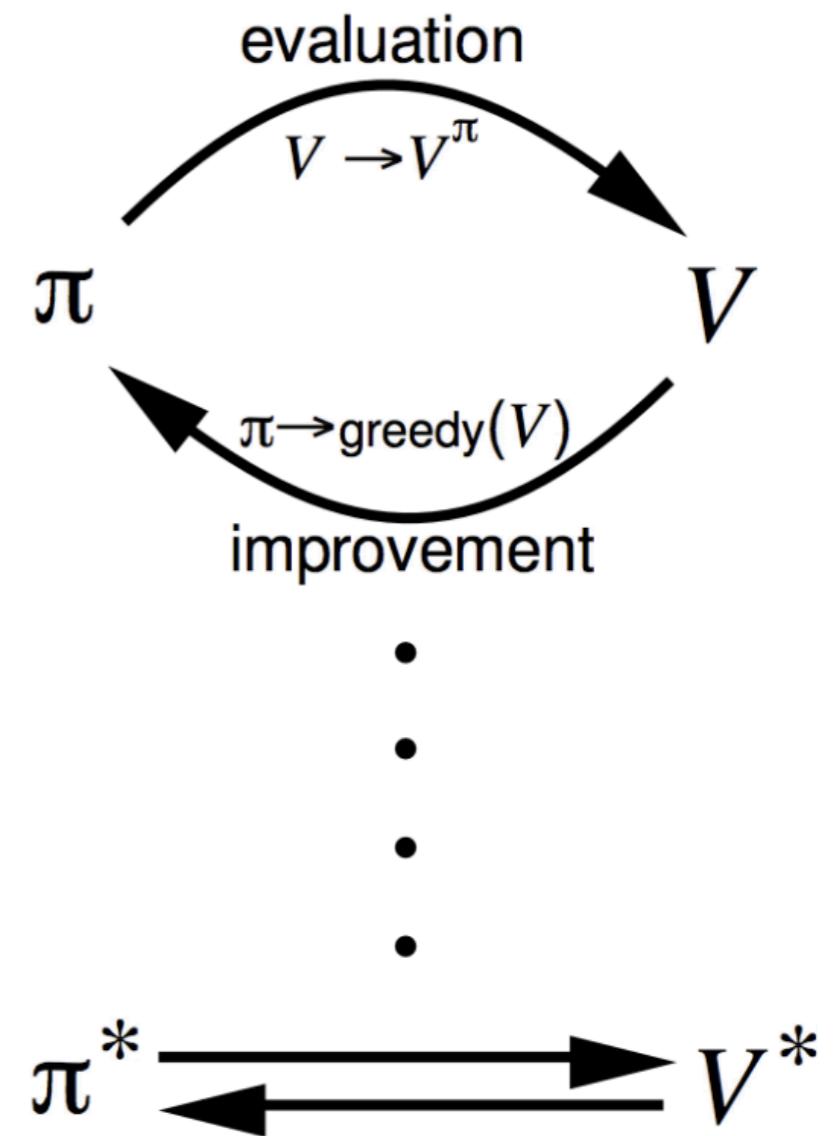


Policy evaluation Estimate v_π

Iterative policy evaluation

Policy improvement Generate $\pi' \geq \pi$

Greedy policy improvement



Slide courtesy: David Silver

Learning with Unknown Model

- When full information of the MDP is known, the value function can be solved by planning.
- But how to solve when not fully known? $\langle \mathcal{S}, \mathcal{A}, \underline{\mathcal{P}}, \mathcal{R}, \gamma \rangle$
- In RL, usually the state transition \mathcal{P} and reward function \mathcal{R} are not known.
- The agent has to learn by trial and error, facing with the exploration and exploitation problem.

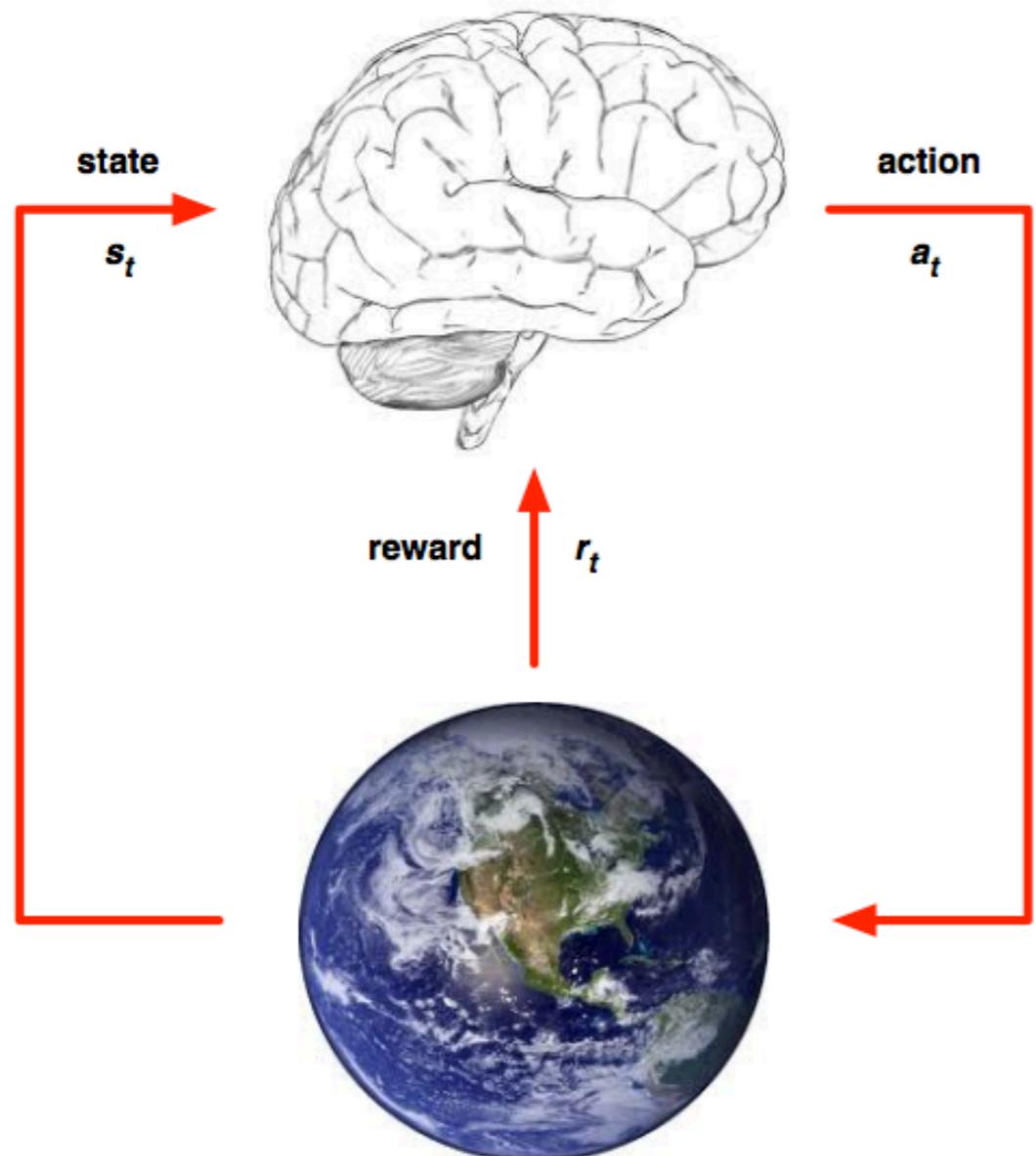
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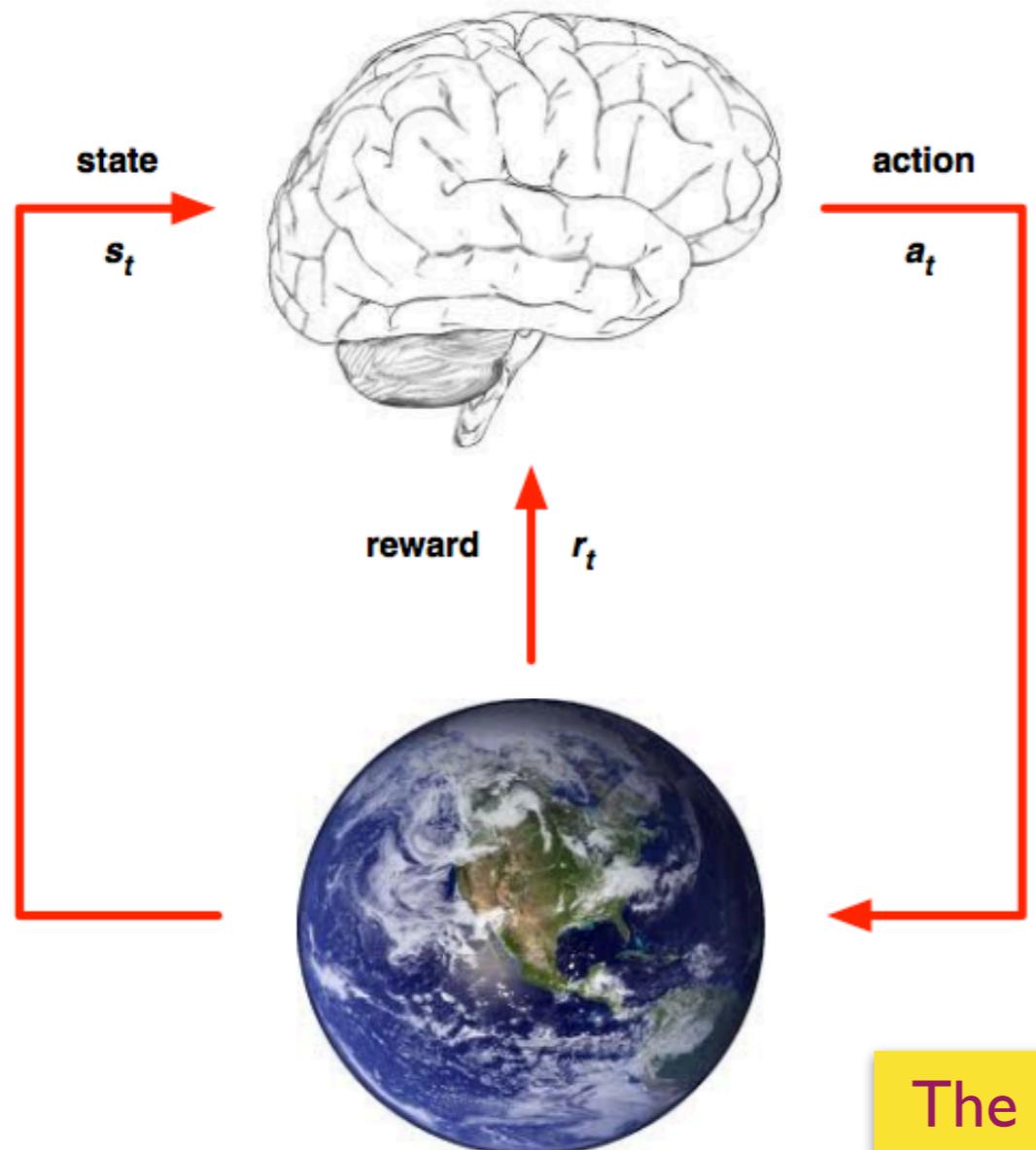
Agent and Environment



- At each step t the agent:
 - ▶ Receives state s_t
 - ▶ Receives scalar reward r_t
 - ▶ Executes action a_t
- ▶ The environment:
 - ▶ Receives action a_t
 - ▶ Emits state s_t
 - ▶ Emits scalar reward r_t
- The target is still to learn the optimal value function.

Slide courtesy: David Silver

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The agent can only interact with true environment.
Can not use model for search or planning.

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- Value function update: update using new estimation

$$Q_{i+1}(s, a) = (1 - \alpha)Q_i(s, a) + \alpha\hat{Q}_i(s, a)$$

- Monte-Carlo RL — Estimate by sampled trajectories
- Temporal difference Learning — SARSA and Q-learning.

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- Temporal difference Learning — SARSA and Q-learning.
- Policy Improvement:
 - Based on new value function, with ϵ - greedy.

Monte-Carlo RL

- Given policy π_i , we can sample trajectories:

$$\{s_1, a_1, r_1\}, \{s_2, a_2, r_2\}, \{s_3, a_3, r_3\} \dots$$

- Then we can get empirical estimate:

$$\hat{Q}_i(s_1, a_1) = r_1 + \gamma r_2 + \gamma^2 r_3 + \dots$$

- Update value function:

$$Q_{i+1}(s, a) = (1 - \alpha)Q_i(s, a) + \alpha \hat{Q}_i(s, a)$$

- Follow the spirit of policy iteration, update $\pi_i \rightarrow \pi_{i+1}$

Monte-Carlo RL

- Given policy π_i , we can sample trajectories:

$$\{s_1, a_1, r_1\}, \{s_2, a_2, r_2\}, \{s_3, a_3, r_3\} \dots$$

- Then we can get empirical estimate:

$$\hat{Q}_i(s_1, a_1) = r_1 + \gamma r_2 + \gamma^2 r_3 + \dots$$

Can we still
update the
policy greedily?

- Update value function:

$$Q_{i+1}(s, a) = (1 - \alpha)Q_i(s, a) + \alpha \hat{Q}_i(s, a)$$

No!

- Follow the spirit of policy iteration, update $\pi_i \rightarrow \pi_{i+1}$

Exploration vs. Exploitation



"Behind one door is tenure - behind the other is flipping burgers at McDonald's."

- There are two doors in front of you.
 - You open the left door and get reward 0
 $V(\text{left}) = 0$
 - You open the right door and get reward +1
 $V(\text{right}) = +1$
 - You open the right door and get reward +3
 $V(\text{right}) = +2$
 - You open the right door and get reward +2
 $V(\text{right}) = +2$
 - Are you sure you've chosen the best door?
- ⋮

Slide courtesy: David Silver

Exploration vs. Exploitation (Cont.)

- In policy iteration, the policy improvement step is greedy:

$$\pi_i(s) = \arg \max_a Q_i(s, a)$$

- But for RL, since the environment is not fully known, greedy update may perform arbitrarily bad — need to allow some **exploration**.
- Common choice: use ϵ -greedy policy:
 - with prob. $1 - \epsilon$, execute as greedy
 - with prob. ϵ , execute randomly
- Theoretical guarantee: If the exploration vanishes, we can ensure convergence.

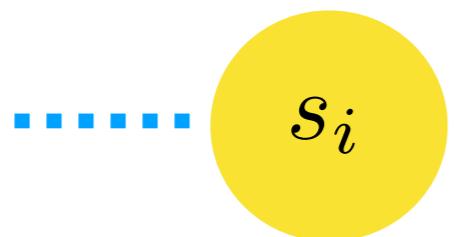
TD vs. MC

- Temporal-difference (TD) learning has several advantages over Monte-Carlo (MC)
 - Lower variance
 - Online
 - Incomplete sequences
- Natural idea: use TD instead of MC in our control loop
 - Apply TD to $Q(S, A)$
 - Use ϵ -greedy policy improvement
 - Update every time-step

Slide courtesy: David Silver

SARSA

- “State-Action-Reward-State-Action” — SARSA



Run the current
best ϵ -greedy policy

SARSA

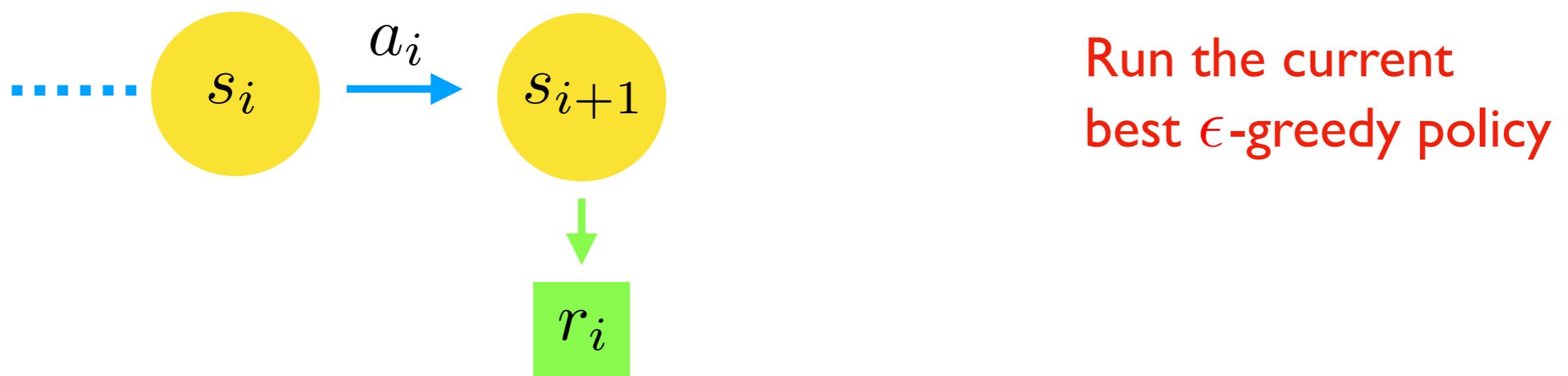
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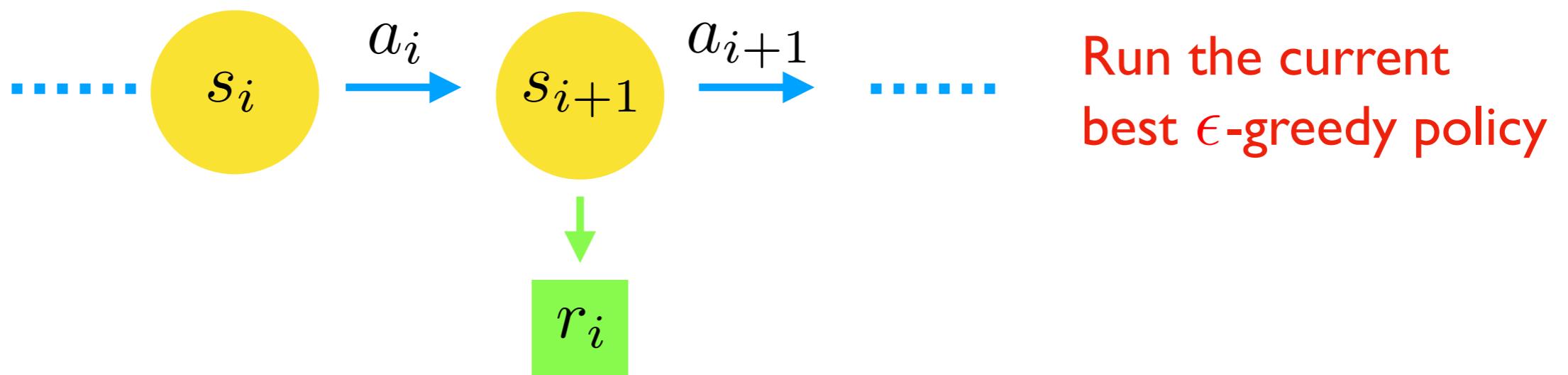
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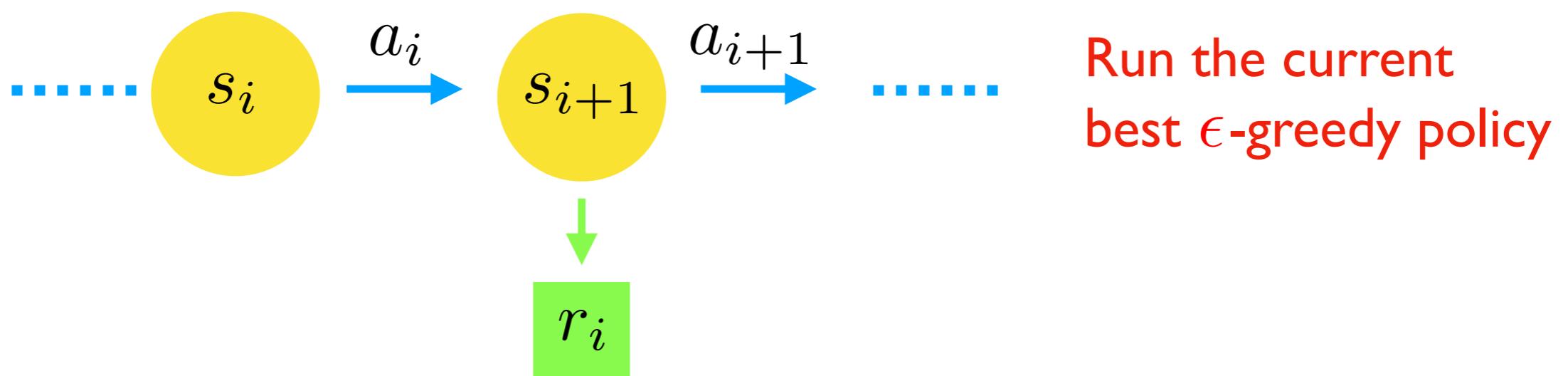
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SARSA

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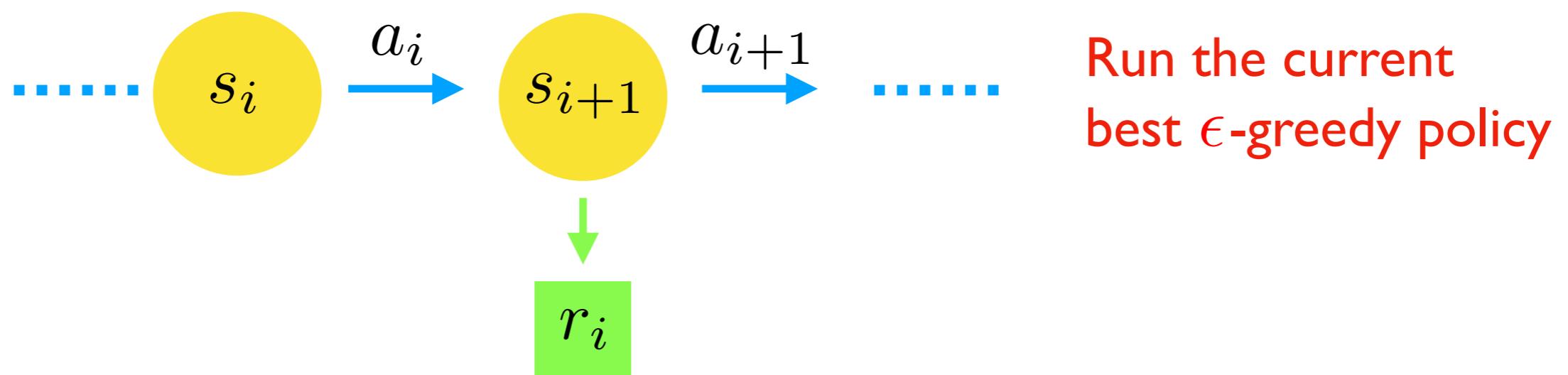


- Once collect s-a-r-s-a sample, do value function update:

$$Q(s_i, a_i) \leftarrow Q(s_i, a_i) + \alpha [r_i + \gamma Q(s_{i+1}, a_{i+1}) - Q(s_i, a_i)]$$

SARSA

- “State-Action-Reward-State-Action” — SARSA

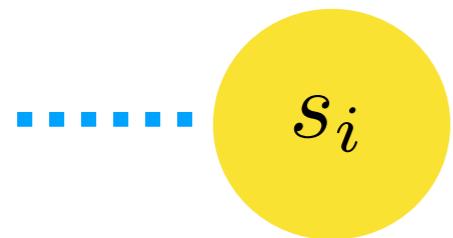


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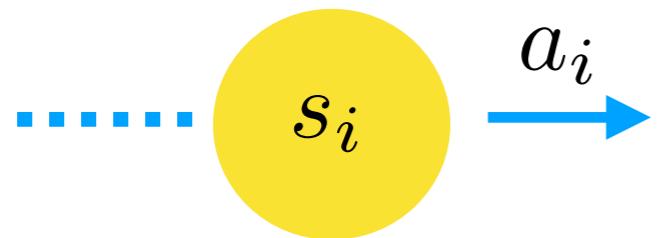
Always use the policy on-the-run, called “on-policy”

Q-Learning



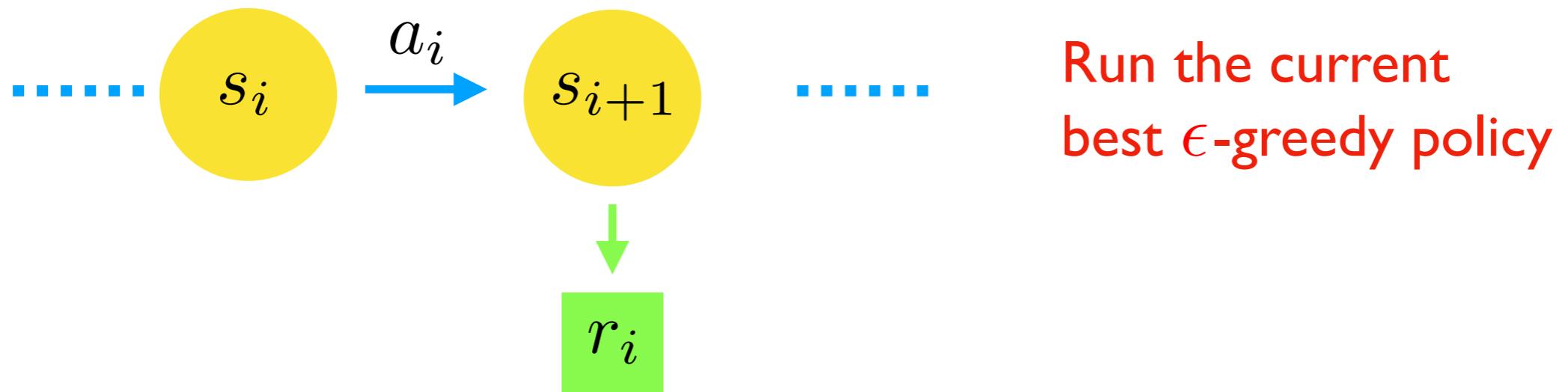
Run the current
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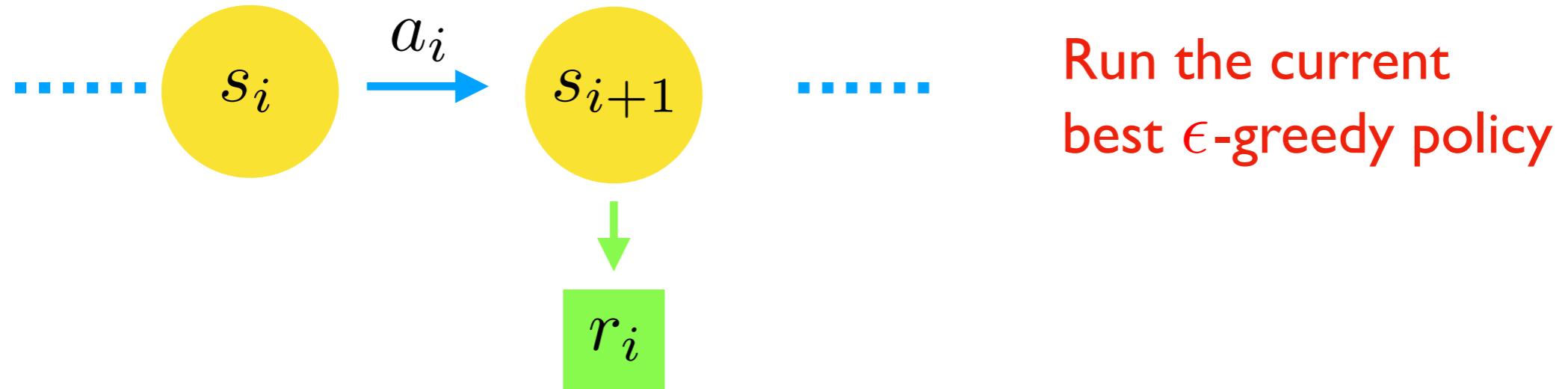


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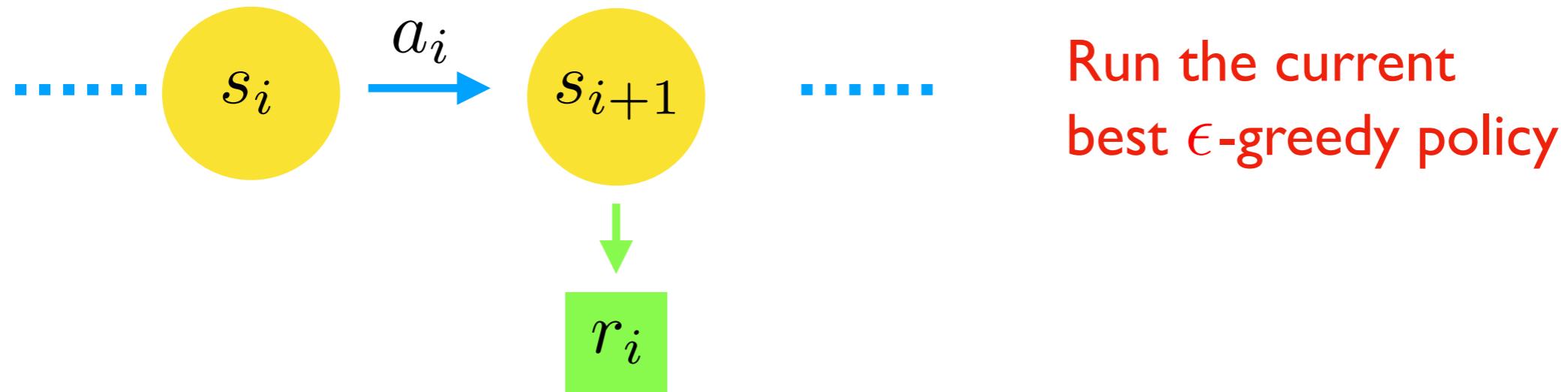
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$$Q(s_i, a_i) \leftarrow Q(s_i, a_i) + \alpha \left[r_i + \gamma Q(s_{i+1}, \hat{\pi}^*(s_{i+1})) - Q(s_i, a_i) \right]$$

Q-Learning



- Once collect s-a-r-s sample, do value function update:

$$Q(s_i, a_i) \leftarrow Q(s_i, a_i) + \alpha \left[r_i + \gamma \underline{Q(s_{i+1}, \hat{\pi}^*(s_{i+1}))} - Q(s_i, a_i) \right]$$

The current best policy

The policy on the run can be different from
the current best policy in the update, called “off-policy”

Off-Policy Learning

- Evaluate target policy $\pi(a|s)$ to compute $v_\pi(s)$ or $q_\pi(s, a)$
- While following behaviour policy $\mu(a|s)$

$$\{S_1, A_1, R_2, \dots, S_T\} \sim \mu$$

- Why is this important?
- Learn from observing humans or other agents
- Re-use experience generated from old policies $\pi_1, \pi_2, \dots, \pi_{t-1}$
- Learn about *optimal* policy while following *exploratory* policy
- Learn about *multiple* policies while following *one* policy

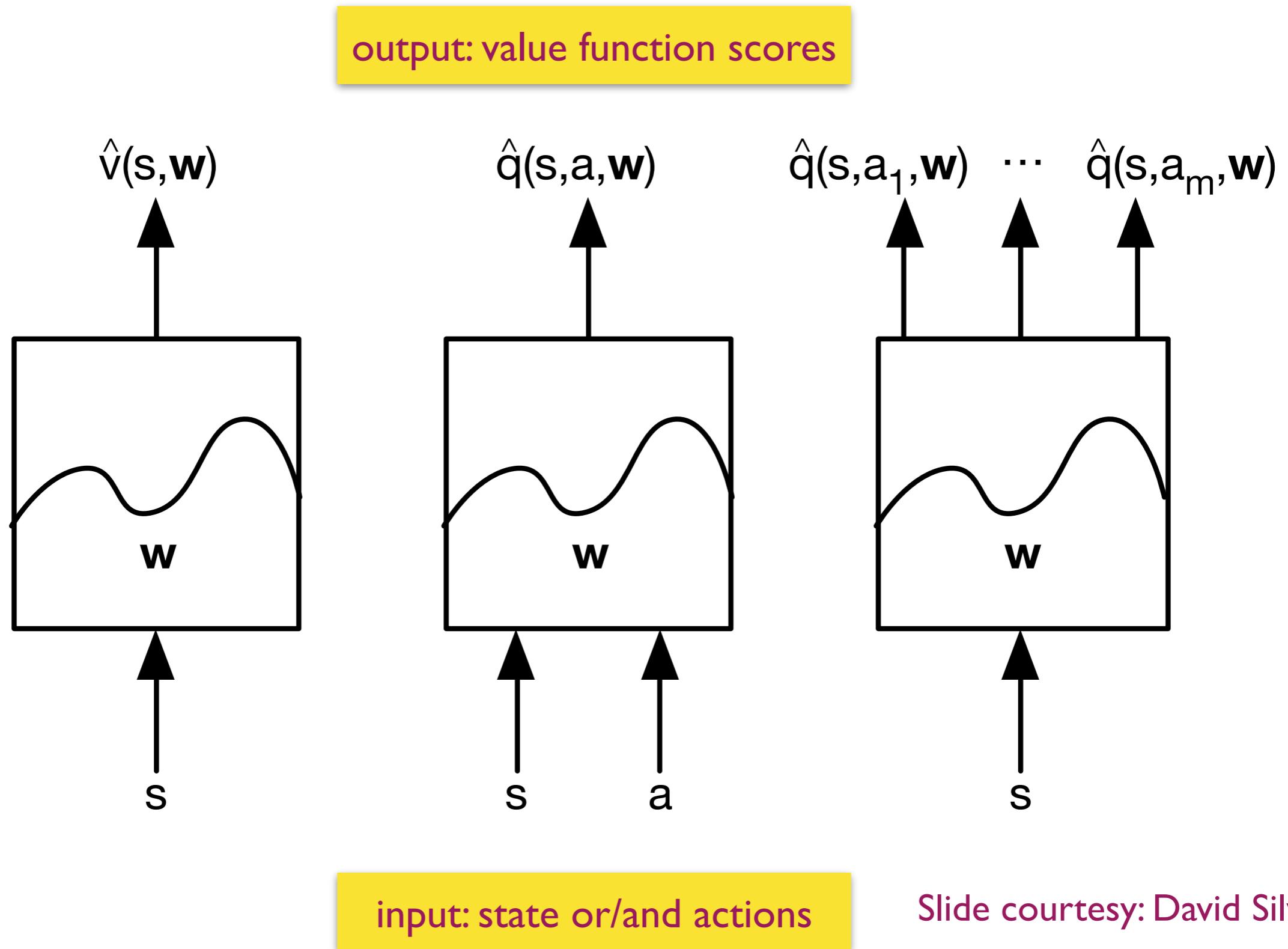
Slide courtesy: David Silver

Large-Scale RL

- Large decision-making problems:
 - Backgammon: 10^{20} states
 - Go: 10^{170} states
 - Robot control: continuous state space

Classic value function methods rely on tabular representation of value functions.
Obviously needing more compact representations.

Types of Value Function Approximation



Slide courtesy: David Silver

Feature Vectors

- Represent state by a *feature vector*

$$\mathbf{x}(S) = \begin{pmatrix} \mathbf{x}_1(S) \\ \vdots \\ \mathbf{x}_n(S) \end{pmatrix}$$

- For example:
 - Distance of robot from landmarks
 - Trends in the stock market
 - Piece and pawn configurations in chess

Slide courtesy: David Silver

Linear Value Function Approximation

- Represent value function by a linear combination of features

$$\hat{v}(S, \mathbf{w}) = \mathbf{x}(S)^\top \mathbf{w} = \sum_{j=1}^n x_j(S) w_j$$

- Objective function is quadratic in parameters \mathbf{w}

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[(v_\pi(S) - \mathbf{x}(S)^\top \mathbf{w})^2 \right]$$

- Stochastic gradient descent converges on *global* optimum
- Update rule is particularly simple

$$\nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) = \mathbf{x}(S)$$

$$\Delta \mathbf{w} = \alpha (v_\pi(S) - \hat{v}(S, \mathbf{w})) \mathbf{x}(S)$$

Update = step-size \times prediction error \times feature value

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unknown true value.
need to estimate during learning!

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Update = step-size \times prediction error \times feature value

beyond simple linear regression

Function Approximators

There are many function approximators, e.g.

- Linear combinations of features
- Neural network
- Decision tree
- Nearest neighbour
- Fourier / wavelet bases
- ...

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more commonly used

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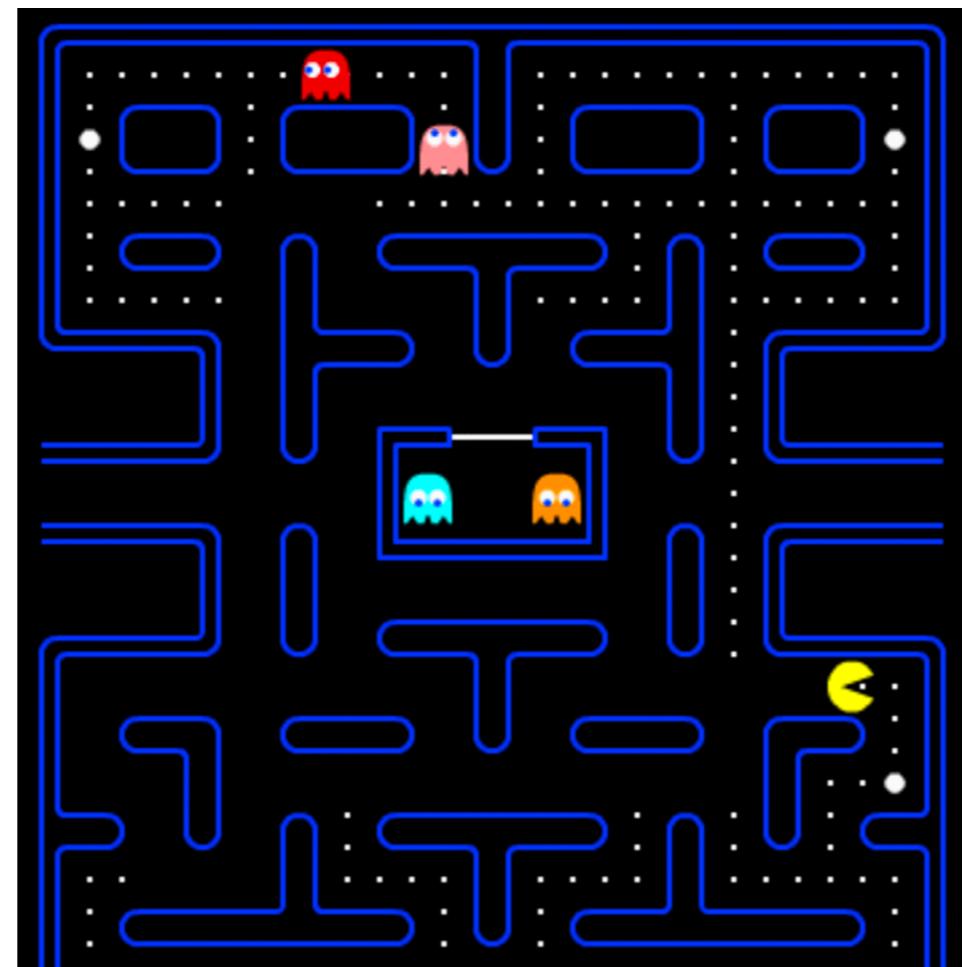
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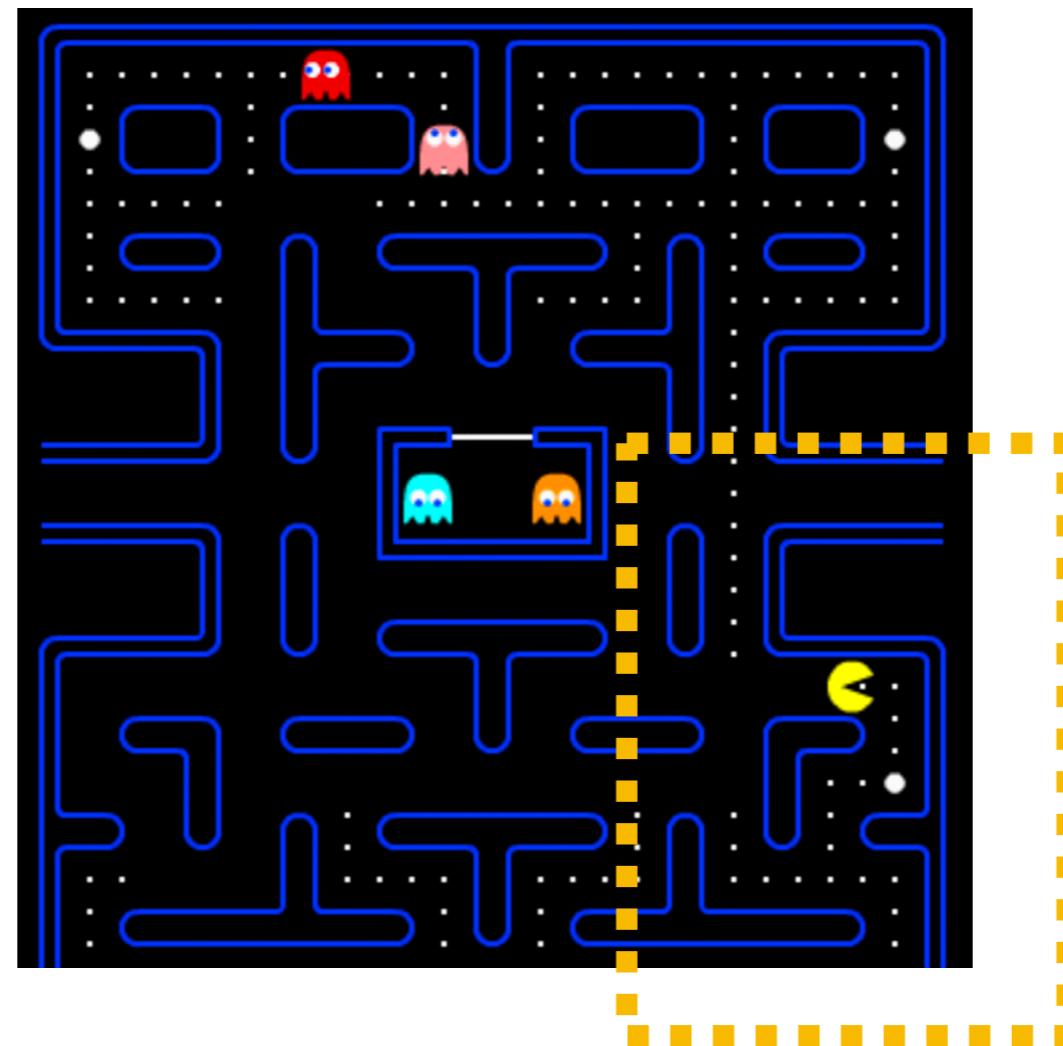
Different from traditional supervised learning,
we need learning algorithms that can handle data collected by the learner online:
biased and unstable.

Slide courtesy: David Silver

Bias and Instability



Bias and Instability



The learner collects
data by itself.
“Only see
what it want to see”

The issue of bias and instability for data collection lies at the heart of RL.
This is also why we need exploration.

Approximate Targets via Bellman Equation

- Have assumed true value function $v_\pi(s)$ given by supervisor
- But in RL there is no supervisor, only rewards
- In practice, we substitute a *target* for $v_\pi(s)$
 - For MC, the target is the return G_t

$$\Delta \mathbf{w} = \alpha(\textcolor{red}{G_t} - \hat{v}(S_t, \mathbf{w}))\nabla_{\mathbf{w}}\hat{v}(S_t, \mathbf{w})$$

- For TD(0), the target is the TD target $R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})$

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- For TD(λ), the target is the λ -return G_t^λ

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Slide courtesy: David Silver

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Temporal-difference estimation

Slide courtesy: David Silver

Action Value Function Approximation

- Approximate the action-value function

$$\hat{q}(S, A, \mathbf{w}) \approx q_\pi(S, A)$$

- Minimise mean-squared error between approximate action-value fn $\hat{q}(S, A, \mathbf{w})$ and true action-value fn $q_\pi(S, A)$

$$J(\mathbf{w}) = \mathbb{E}_\pi [(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))^2]$$

- Use stochastic gradient descent to find a local minimum

$$-\frac{1}{2} \nabla_{\mathbf{w}} J(\mathbf{w}) = (q_\pi(S, A) - \hat{q}(S, A, \mathbf{w})) \nabla_{\mathbf{w}} \hat{q}(S, A, \mathbf{w})$$

$$\Delta \mathbf{w} = \alpha (q_\pi(S, A) - \hat{q}(S, A, \mathbf{w})) \nabla_{\mathbf{w}} \hat{q}(S, A, \mathbf{w})$$

Linear Action-Value Function Approximation

- Represent state *and* action by a *feature vector*

$$\mathbf{x}(S, A) = \begin{pmatrix} \mathbf{x}_1(S, A) \\ \vdots \\ \mathbf{x}_n(S, A) \end{pmatrix}$$

- Represent action-value fn by linear combination of features

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Linear Action-Value Function Approximation

- Like prediction, we must substitute a *target* for $q_\pi(S, A)$

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- For backward-view TD(λ), equivalent update is

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SARSA here.
Can also do
Q-learning
(more later)

Reinforcement Learning

- Basics
 - Markov decision process
 - RL with known model
 - RL with unknown model
 - **Policy gradient & actor-critic methods**
- Deep reinforcement learning
- Integrating learning and planning
- RL from human preference
- Take-home messages

Slides link:



Direct Policy Learning

- For value function based RL, policy is not directly optimized.
 - Not capable to learn in continuous state and action space.
 - Not capable to learn stochastic policy.
 - May not learn fast.

Direct Policy Learning

- For value function based RL, policy is not directly optimized.
 - Not capable to learn in continuous state and action space.
 - Not capable to learn stochastic policy.
 - May not learn fast.
- Can learn **stochastic policy** directly:
 - Parametrize policy $\pi_\theta(a|s)$ with parameter θ
 - For discrete action: softmax $\pi_\theta(s, a) \propto e^{\phi(s, a)^\top \theta}$
 - For continuous action: Gaussian $a \sim \mathcal{N}(\mu(s), \sigma^2)$

Direct Policy Learning

Advantages:

- Better convergence properties
- Effective in high-dimensional or continuous action spaces
- Can learn stochastic policies

Disadvantages:

- Typically converge to a local rather than global optimum
- Evaluating a policy is typically inefficient and high variance

Objective Function

- Goal: given policy $\pi_\theta(s, a)$ with parameters θ , find best θ
- But how do we measure the quality of a policy π_θ ?
- In episodic environments we can use the **start value**

$$J_1(\theta) = V^{\pi_\theta}(s_1) = \mathbb{E}_{\pi_\theta} [v_1]$$

- In continuing environments we can use the **average value**

$$J_{avV}(\theta) = \sum_s d^{\pi_\theta}(s) V^{\pi_\theta}(s)$$

- Or the **average reward per time-step**

$$J_{avR}(\theta) = \sum_s d^{\pi_\theta}(s) \sum_a \pi_\theta(s, a) \mathcal{R}_s^a$$

- where $d^{\pi_\theta}(s)$ is **stationary distribution** of Markov chain for π_θ

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The challenge is that the distribution can only be estimated when the agent itself samples data.

Policy Gradient

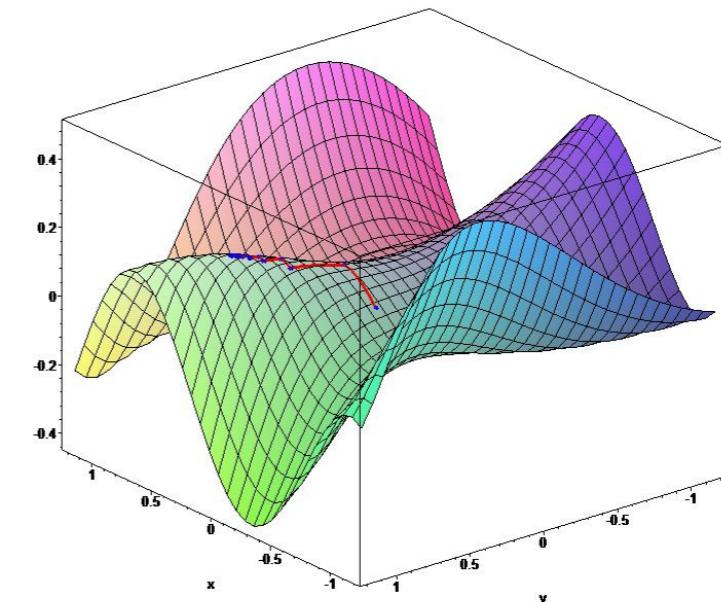
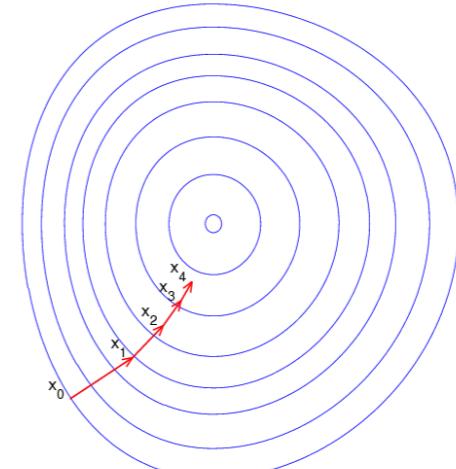
- Let $J(\theta)$ be any policy objective function
- Policy gradient algorithms search for a *local* maximum in $J(\theta)$ by ascending the gradient of the policy, w.r.t. parameters θ

$$\Delta\theta = \alpha \nabla_{\theta} J(\theta)$$

- Where $\nabla_{\theta} J(\theta)$ is the **policy gradient**

$$\nabla_{\theta} J(\theta) = \begin{pmatrix} \frac{\partial J(\theta)}{\partial \theta_1} \\ \vdots \\ \frac{\partial J(\theta)}{\partial \theta_n} \end{pmatrix}$$

- and α is a step-size parameter



Slide courtesy: David Silver

Policy Gradient Theorem

Policy Gradient Methods for Reinforcement Learning with Function Approximation

Richard S. Sutton, David McAllester, Satinder Singh, Yishay Mansour
AT&T Labs – Research, 180 Park Avenue, Florham Park, NJ 07932

Theorem

*For any differentiable policy $\pi_\theta(s, a)$,
for any of the policy objective functions $J = J_1, J_{avR}$, or $\frac{1}{1-\gamma} J_{avV}$,
the policy gradient is*

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) Q^{\pi_\theta}(s, a)]$$

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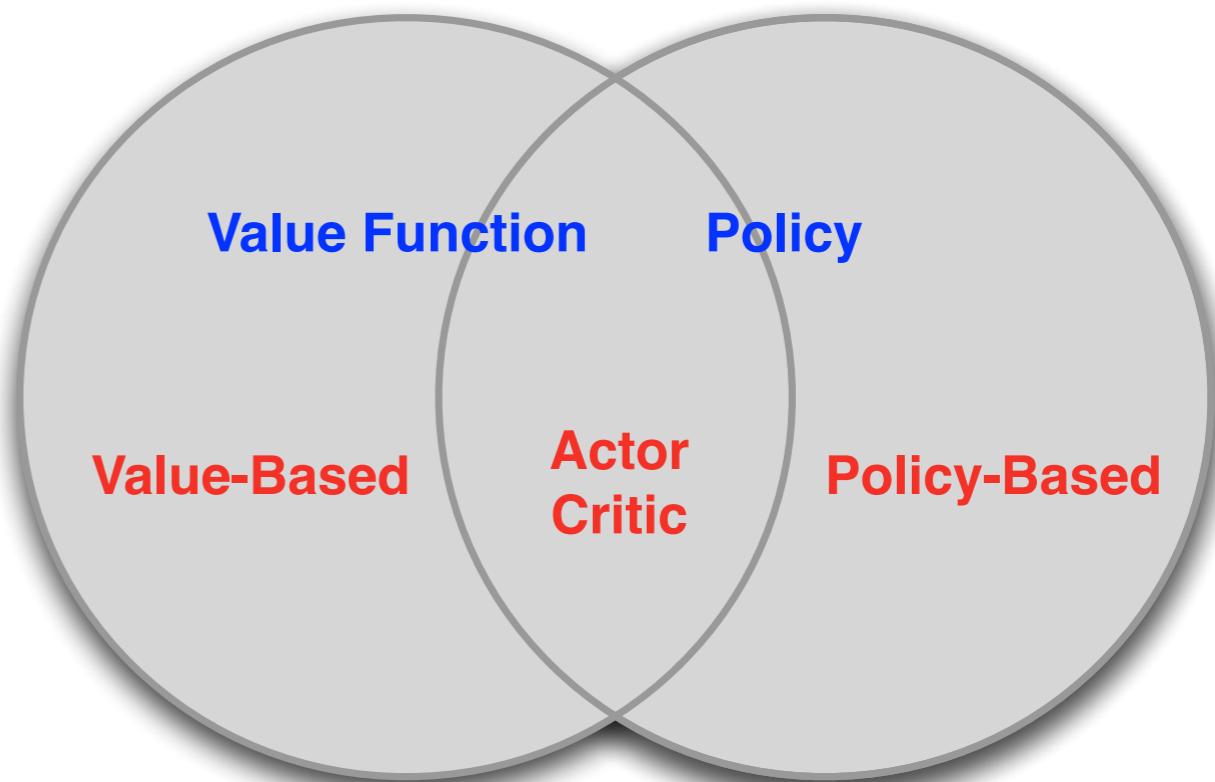
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Need to estimate value function

Monte-Carlo or temporal difference

Actor-Critic

- Value Based
 - Learnt Value Function
 - Implicit policy
(e.g. ϵ -greedy)
- Policy Based
 - No Value Function
 - Learnt Policy
- Actor-Critic
 - Learnt Value Function
 - Learnt Policy



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Slides link:



Practical Issues for RL

- Reward design: an art
- State feature design: also an art
- The environment is too complicated to model.
 - ▶ Can we apply deep learning to RL?
 - ▶ Use deep network to represent value function / policy / model
 - ▶ Optimise value function / policy /model **end-to-end**
 - ▶ Using stochastic gradient descent

Deep Q-Network

- ▶ Represent value function by deep **Q-network** with weights w

$$Q(s, a, w) \approx Q^\pi(s, a)$$

- ▶ Define objective function by mean-squared error in Q-values

$$\mathcal{L}(w) = \mathbb{E} \left[\left(\underbrace{r + \gamma \max_{a'} Q(s', a', w)}_{\text{target}} - Q(s, a, w) \right)^2 \right]$$

- ▶ Leading to the following **Q-learning** gradient

$$\frac{\partial \mathcal{L}(w)}{\partial w} = \mathbb{E} \left[\left(r + \gamma \max_{a'} Q(s', a', w) - Q(s, a, w) \right) \frac{\partial Q(s, a, w)}{\partial w} \right]$$

- ▶ Optimise objective end-to-end by SGD, using $\frac{\partial \mathcal{L}(w)}{\partial w}$

Recall function approximation in P. 42.

Stability Issues for Deep RL

Naive Q-learning **oscillates** or **diverges** with neural nets

1. Data is sequential
 - ▶ Successive samples are correlated, non-iid
2. Policy changes rapidly with slight changes to Q-values
 - ▶ Policy may oscillate
 - ▶ Distribution of data can swing from one extreme to another
3. Scale of rewards and Q-values is unknown
 - ▶ Naive Q-learning gradients can be large
unstable when backpropagated

The bias and instability issue we have discussed.
More severe for NNs.

DQN Techs

DQN provides a stable solution to deep value-based RL

1. Use **experience replay**
 - ▶ Break correlations in data, bring us back to iid setting
 - ▶ Learn from all past policies
2. Freeze **target Q-network**
 - ▶ Avoid oscillations
 - ▶ Break correlations between Q-network and target
3. **Clip** rewards or **normalize** network adaptively to sensible range
 - ▶ Robust gradients

Experience Replay

To remove correlations, build data-set from agent's own experience

- ▶ Take action a_t according to ϵ -greedy policy
- ▶ Store transition $(s_t, a_t, r_{t+1}, s_{t+1})$ in replay memory \mathcal{D}
- ▶ Sample random mini-batch of transitions (s, a, r, s') from \mathcal{D}
- ▶ Optimise MSE between Q-network and Q-learning targets, e.g.

$$\mathcal{L}(w) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}} \left[\left(r + \gamma \max_{a'} Q(s', a', w) - Q(s, a, w) \right)^2 \right]$$

off-policy!

Fixed Network

To avoid oscillations, fix parameters used in Q-learning target

- ▶ Compute Q-learning targets w.r.t. old, fixed parameters w^-

$$r + \gamma \max_{a'} Q(s', a', w^-)$$

- ▶ Optimise MSE between Q-network and Q-learning targets

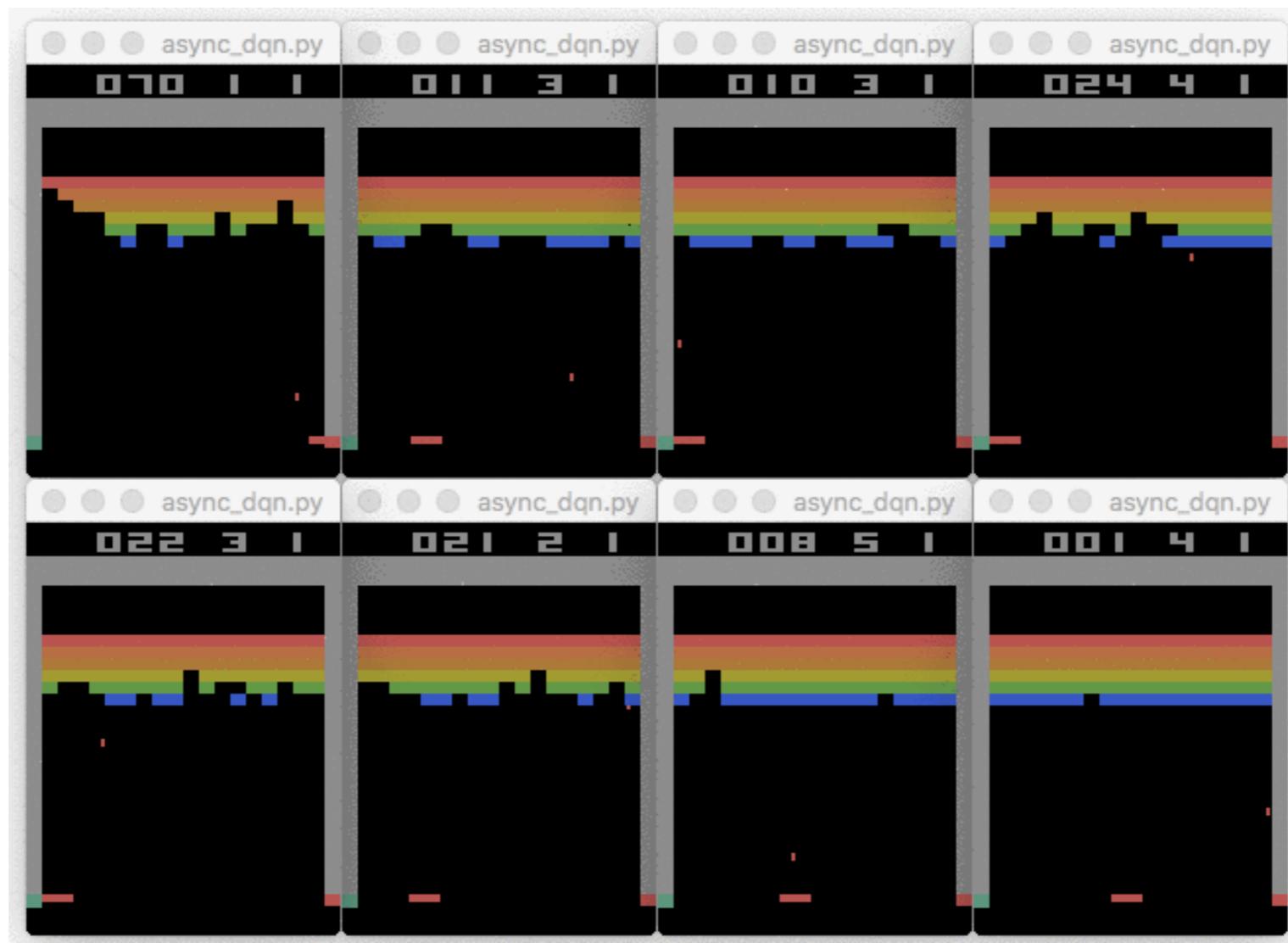
$$\mathcal{L}(w) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}} \left[\left(r + \gamma \max_{a'} Q(s', a', w^-) - Q(s, a, w) \right)^2 \right]$$

- ▶ Periodically update fixed parameters $w^- \leftarrow w$

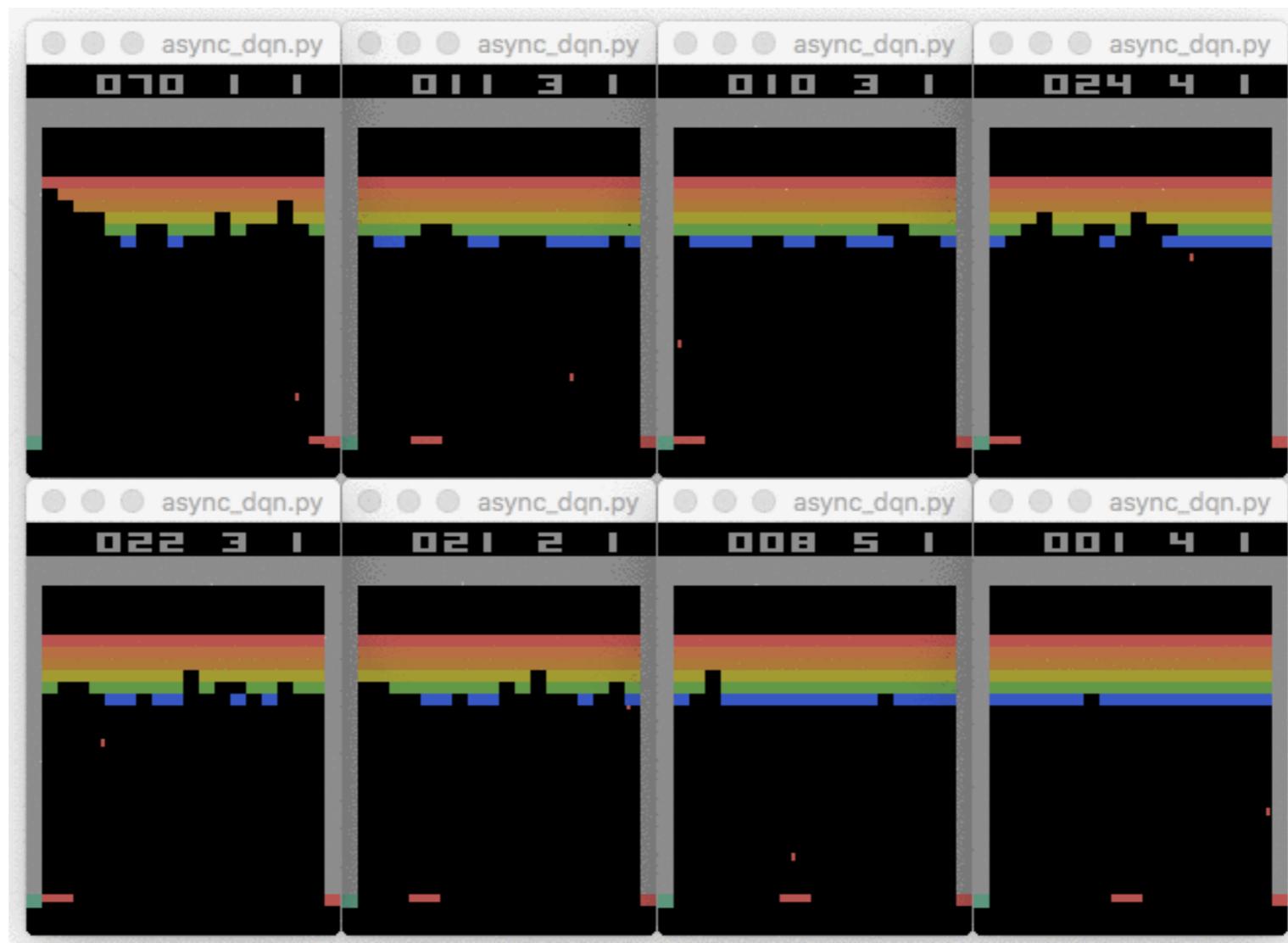
Clipped Rewards

- ▶ DQN clips the rewards to $[-1, +1]$
- ▶ This prevents Q-values from becoming too large
- ▶ Ensures gradients are well-conditioned
- ▶ Can't tell difference between small and large rewards

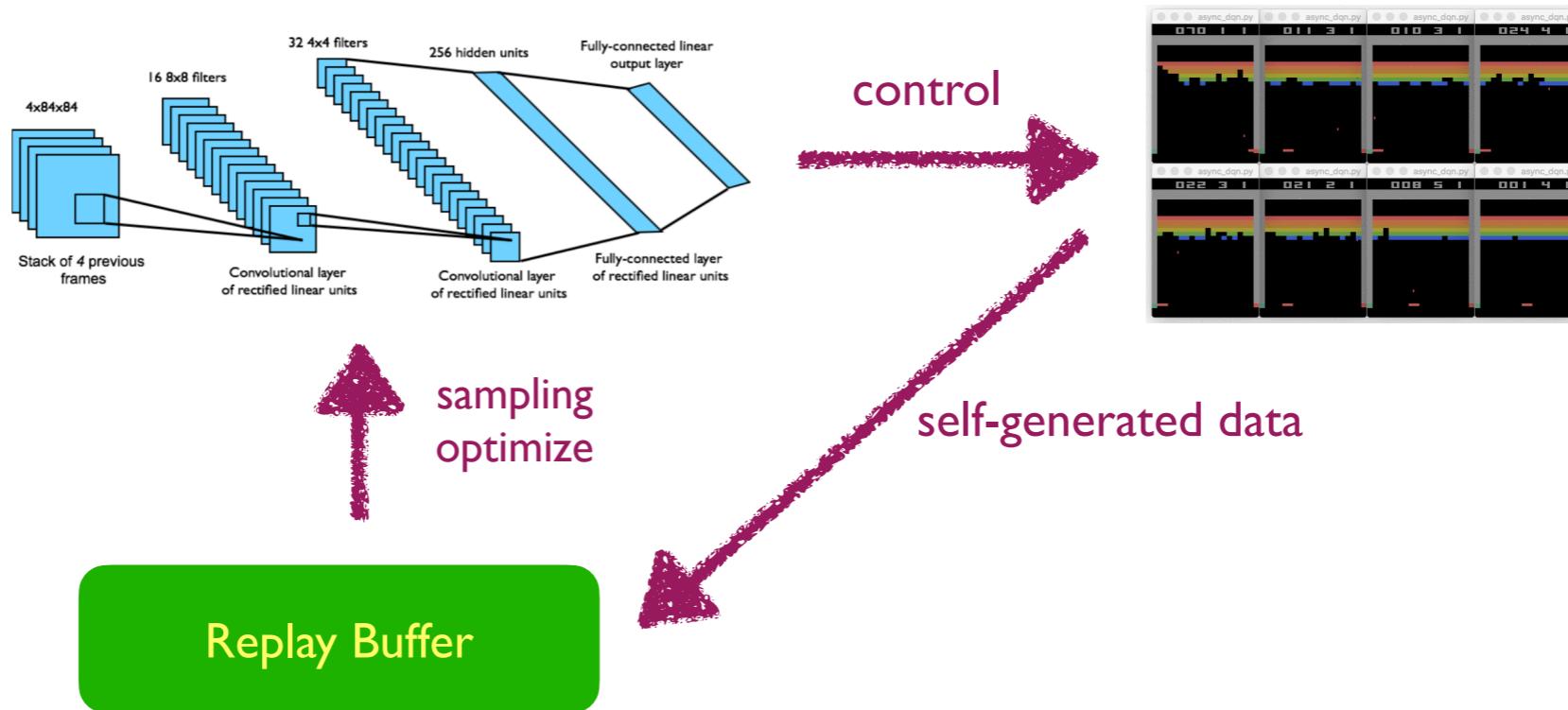
DQN for Atari Games



DQN for Atari Games



DQN for Atari Games



Published: 25 February 2015

Human-level control through deep reinforcement learning

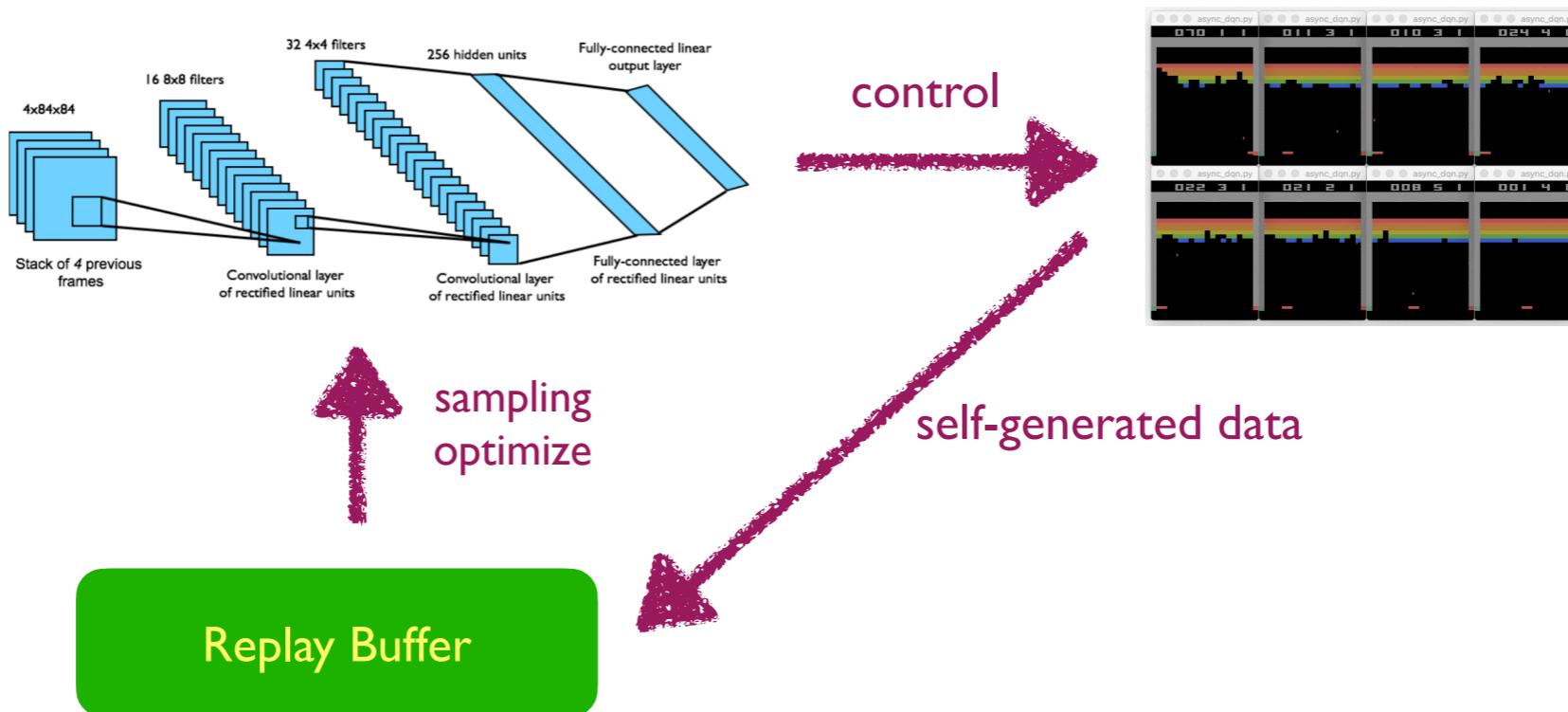
[Volodymyr Mnih](#), [Koray Kavukcuoglu](#)✉, [David Silver](#), [Andrei A. Rusu](#), [Joel Veness](#), [Marc G. Bellemare](#), [Alex Graves](#), [Martin Riedmiller](#), [Andreas K. Fidjeland](#), [Georg Ostrovski](#), [Stig Petersen](#), [Charles Beattie](#), [Amir Sadik](#), [Ioannis Antonoglou](#), [Helen King](#), [Dharshan Kumaran](#), [Daan Wierstra](#), [Shane Legg](#) & [Demis Hassabis](#)✉

[Nature](#) **518**, 529–533 (2015) | [Cite this article](#)

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First break through of deep RL.

DQN for Atari Games



Published: 25 February 2015

Human-level control through deep reinforcement learning

[Volodymyr Mnih](#), [Koray Kavukcuoglu](#)✉, [David Silver](#), [Andrei A. Rusu](#), [Joel Veness](#), [Marc G. Bellemare](#), [Alex Graves](#), [Martin Riedmiller](#), [Andreas K. Fidjeland](#), [Georg Ostrovski](#), [Stig Petersen](#), [Charles Beattie](#), [Amir Sadik](#), [Ioannis Antonoglou](#), [Helen King](#), [Dharshan Kumaran](#), [Daan Wierstra](#), [Shane Legg](#) & [Demis Hassabis](#)✉

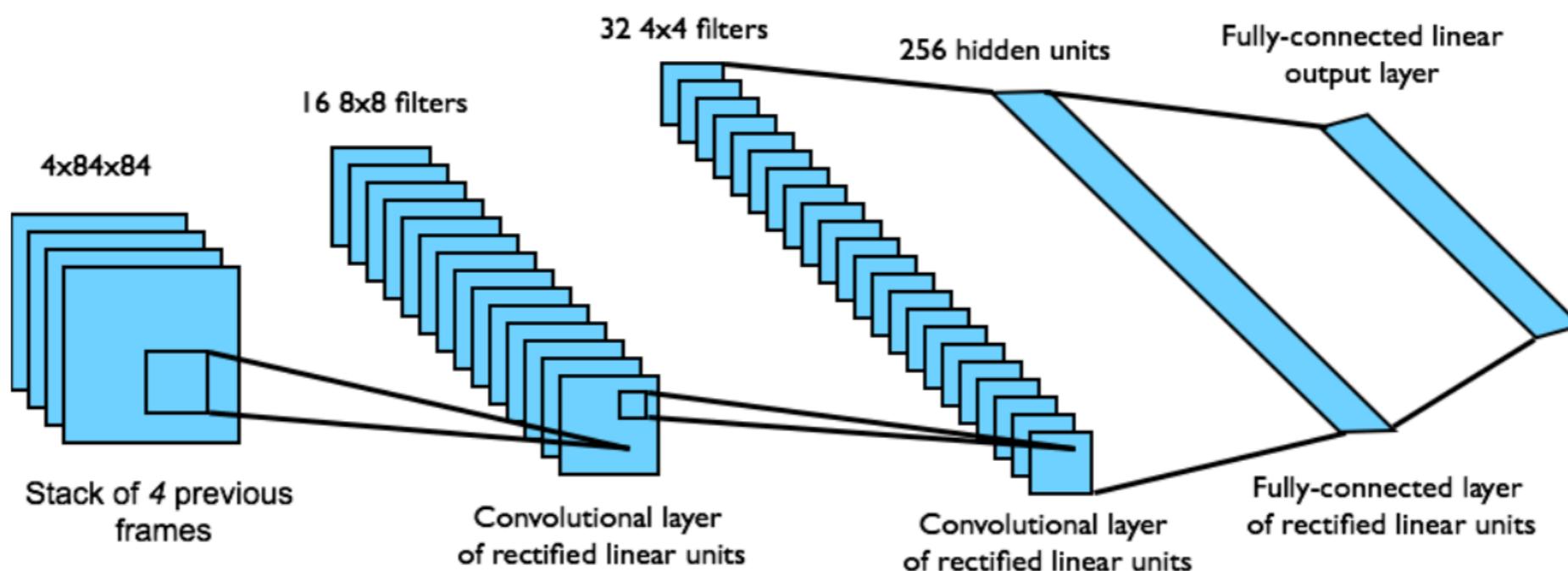
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First break through of deep RL.

DQN for Atari Games

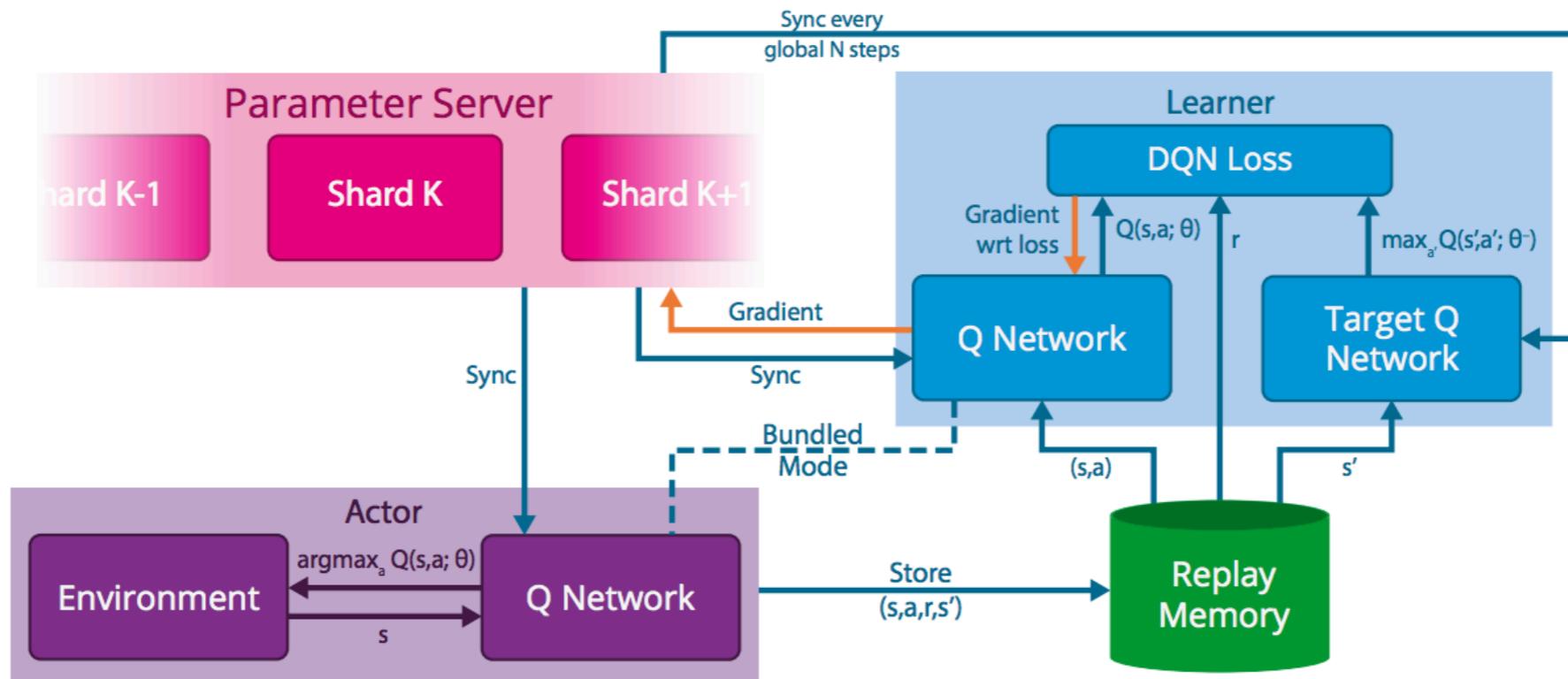
- ▶ End-to-end learning of values $Q(s, a)$ from pixels s
- ▶ Input state s is stack of raw pixels from last 4 frames
- ▶ Output is $Q(s, a)$ for 18 joystick/button positions
- ▶ Reward is change in score for that step



Network architecture and hyperparameters fixed across all games
[Mnih et al.]

Deep RL Architecture

Gorila (GOogle ReInforcement Learning Architecture)



- ▶ **Parallel acting:** generate new interactions
- ▶ **Distributed replay memory:** save interactions
- ▶ **Parallel learning:** compute gradients from replayed interactions
- ▶ **Distributed neural network:** update network from gradients

Slide courtesy: David Silver

Reinforcement Learning

- Basics
 - Markov decision process
 - RL with known model
 - RL with unknown model
 - Policy gradient & actor-critic methods
- Deep reinforcement learning
- Integrating learning and planning
- RL from human preference
- Take-home messages

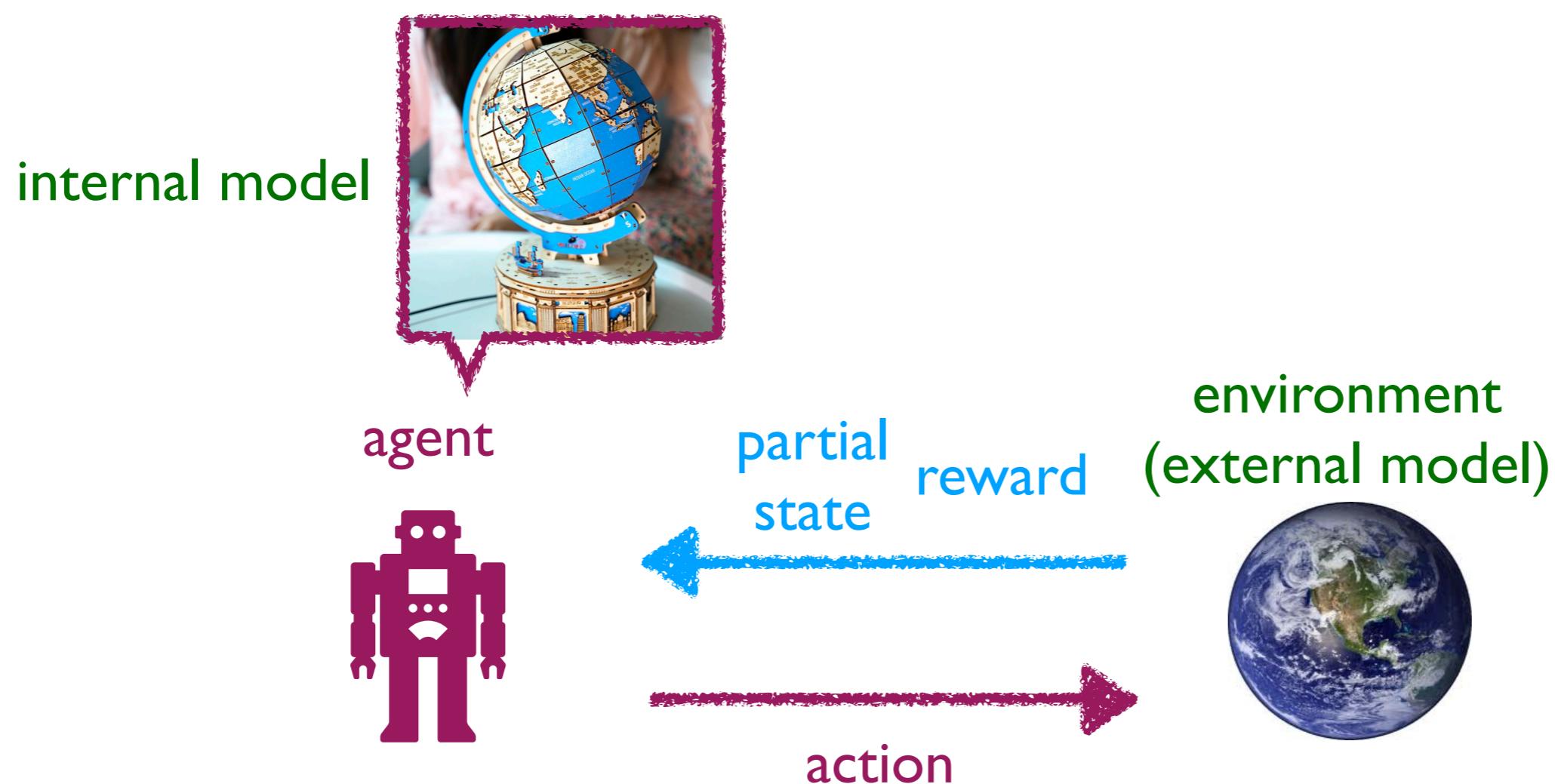
Slides link:



Internal and External Model

Models are crucial in decision making problems.

Whenever we have the external model or can obtain the internal model, we can combine the power of learning and planning (e.g. search, dynamic programming).



Integrating Learning and Planning

- Model-Free RL
 - No model
 - **Learn** value function (and/or policy) from real experience
- Model-Based RL (using Sample-Based Planning)
 - Learn a model from real experience
 - **Plan** value function (and/or policy) from simulated experience
- Dyna
 - Learn a model from real experience
 - **Learn and plan** value function (and/or policy) from real and simulated experience



Machine Learning Proceedings 1990
Proceedings of the Seventh International Conference, Austin, Texas, June 21–23, 1990
1990, Pages 216-224



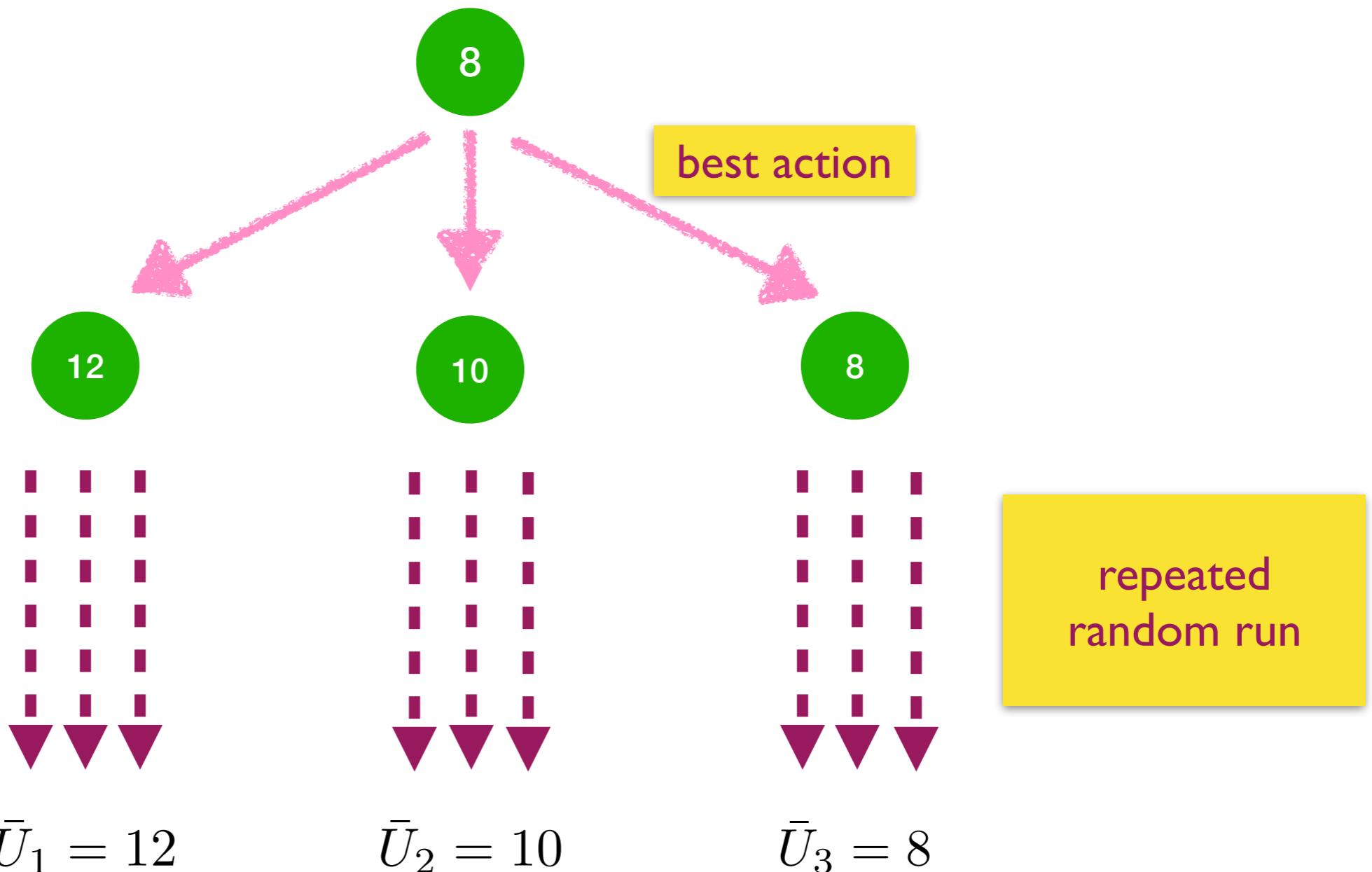
Integrated Architectures for Learning, Planning,
and Reacting Based on Approximating Dynamic
Programming

Planning by DP

Problem	Bellman Equation	Algorithm
Prediction	Bellman Expectation Equation	Iterative Policy Evaluation
Control	Bellman Expectation Equation + Greedy Policy Improvement	Policy Iteration
Control	Bellman Optimality Equation	Value Iteration

- Algorithms are based on state-value function $v_\pi(s)$ or $v_*(s)$
- Complexity $O(mn^2)$ per iteration, for m actions and n states
- Could also apply to action-value function $q_\pi(s, a)$ or $q_*(s, a)$
- Complexity $O(m^2n^2)$ per iteration

Monte-Carlo Simulation

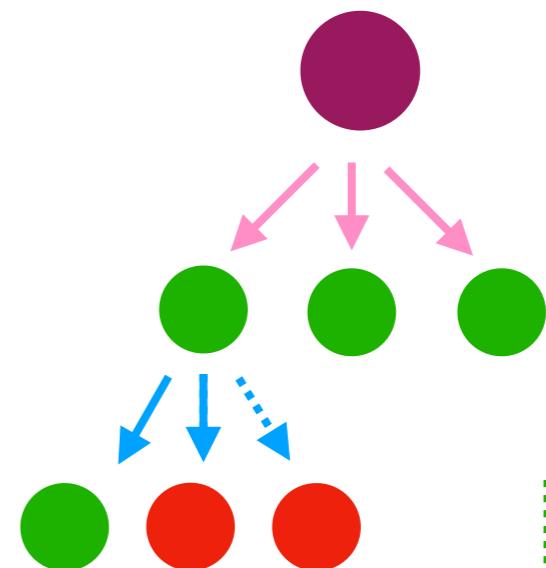


Converge to true utility when the #run is sufficient!

But in real game playing, the time and space for simulation is limited.
We need a smart strategy to decide the order of simulation.

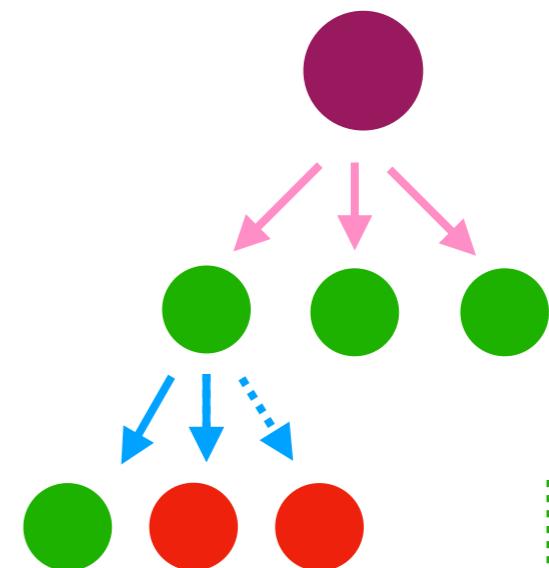
Monte-Carlo Tree Search (MCTS)

- Nodes of two kinds:  visited node with unexpanded child, and  unvisited node.



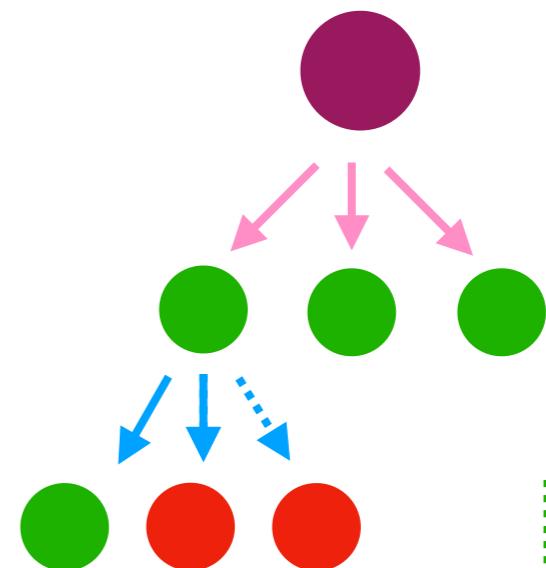
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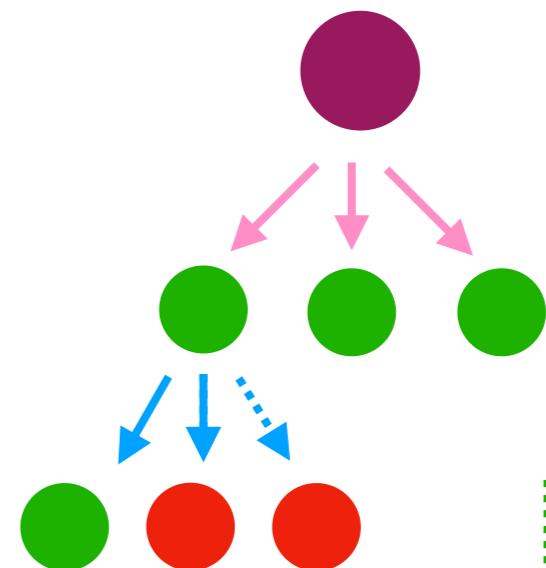
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- Algorithm: repeat until time or space limit:



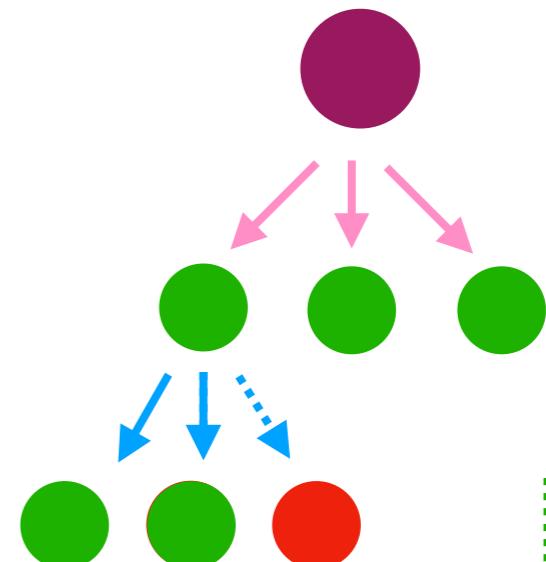
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- Algorithm: repeat until time or space limit:
 - Selection: choose one node among  using **tree strategy**



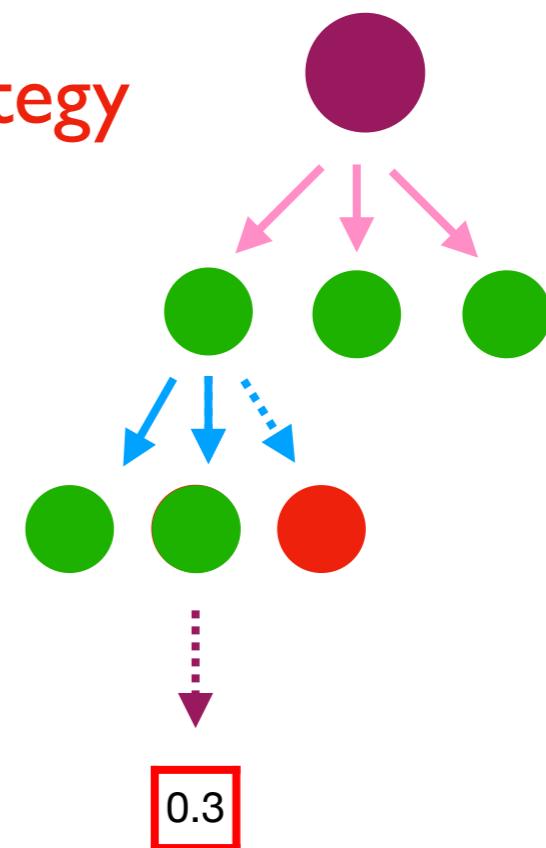
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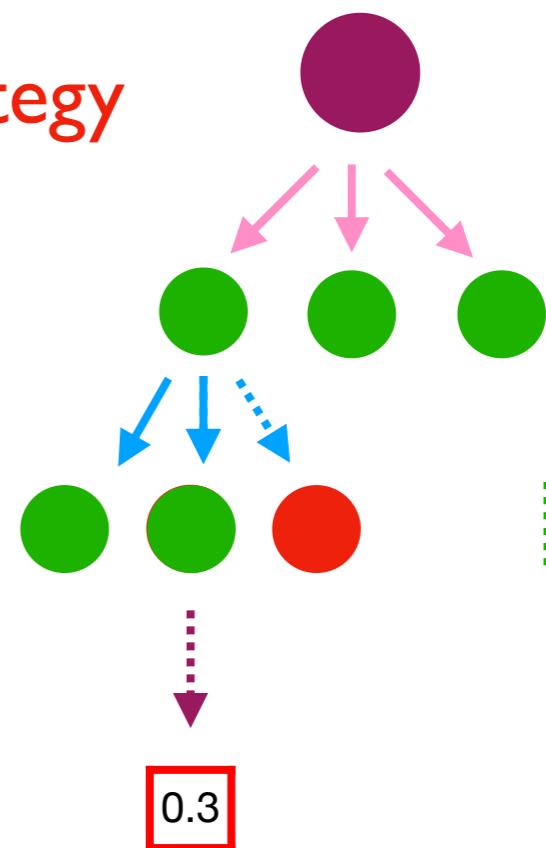
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- Algorithm: repeat until time or space limit:
 - Selection: choose one node among  using **tree strategy**
 - Expansion: expand an unvisited child and put into 
 - Simulation: simulate down using **default strategy**



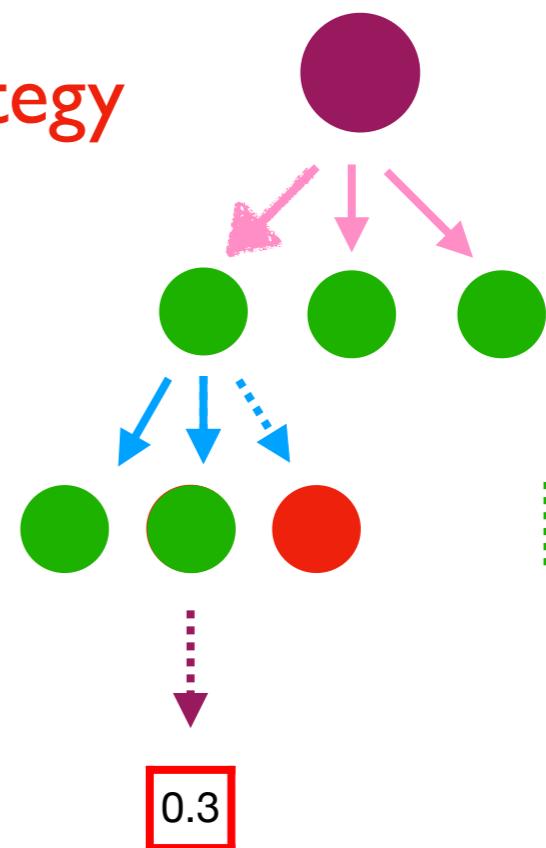
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 - Expansion: expand an unvisited child and put into 
 - Simulation: simulate down using **default strategy**
 - Update: update MC estimation through path



Monte-Carlo Tree Search (MCTS)

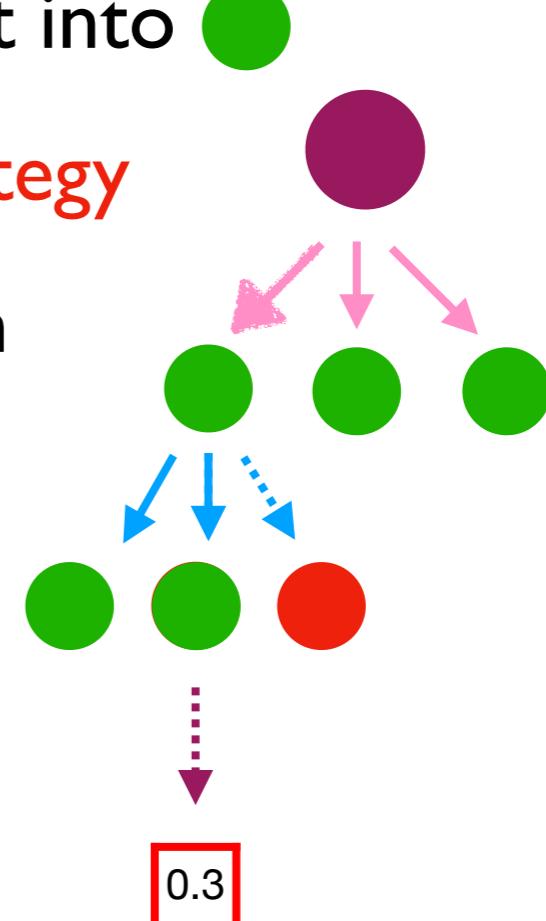
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- Algorithm: repeat until time or space limit:
 - Selection: choose one node among  using **tree strategy**
 - Expansion: expand an unvisited child and put into 
 - Simulation: simulate down using **default strategy**
 - Update: update MC estimation through path
- Output the best action to play



Monte-Carlo Tree Search (MCTS)

- Nodes of two kinds:  visited node with unexpanded child, and  unvisited node.
- During MCTS, we use two strategies: **tree** and **default**
- Algorithm: repeat until time or space limit:
 - Selection: choose one node among  using **tree strategy**
 - Expansion: expand an unvisited child and put into 
 - Simulation: simulate down using **default strategy**
 - Update: update MC estimation through path
- Output the best action to play

The default strategy is usually random play
The tree strategy is essential:
Deciding the order of search



AlphaGo: Integration of Learning & Planning

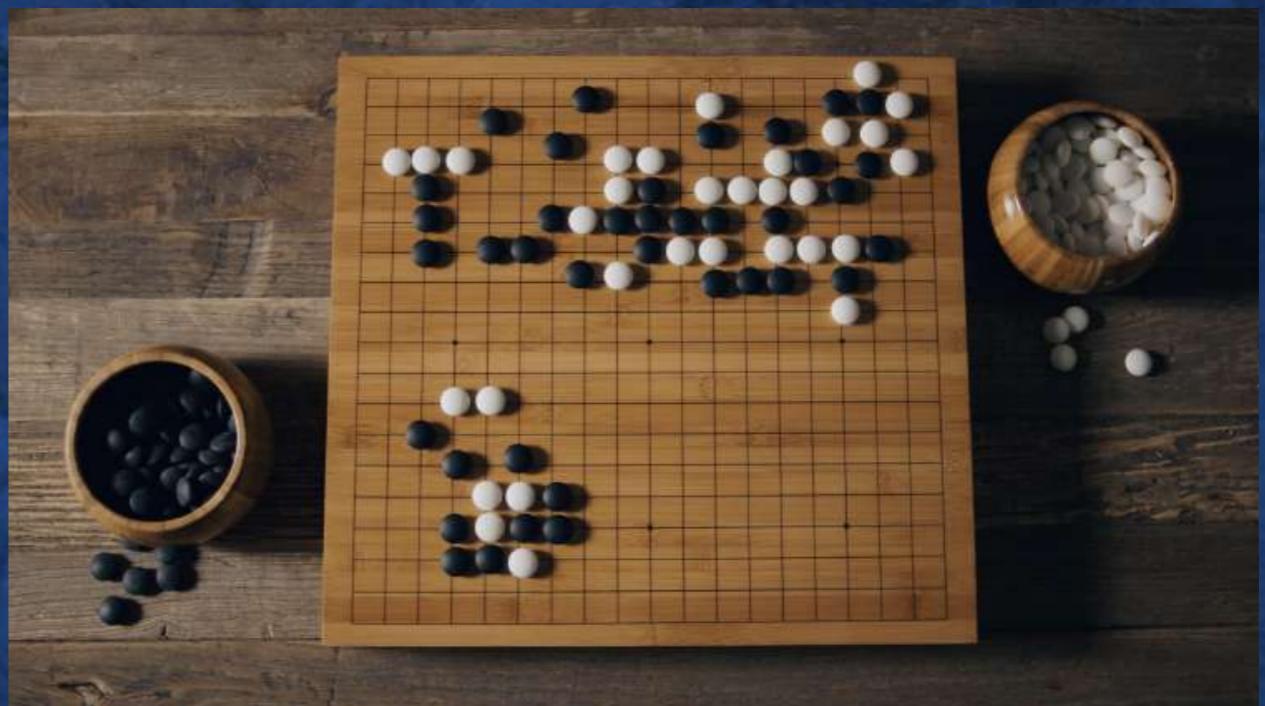


Why is Go hard for computers to play?

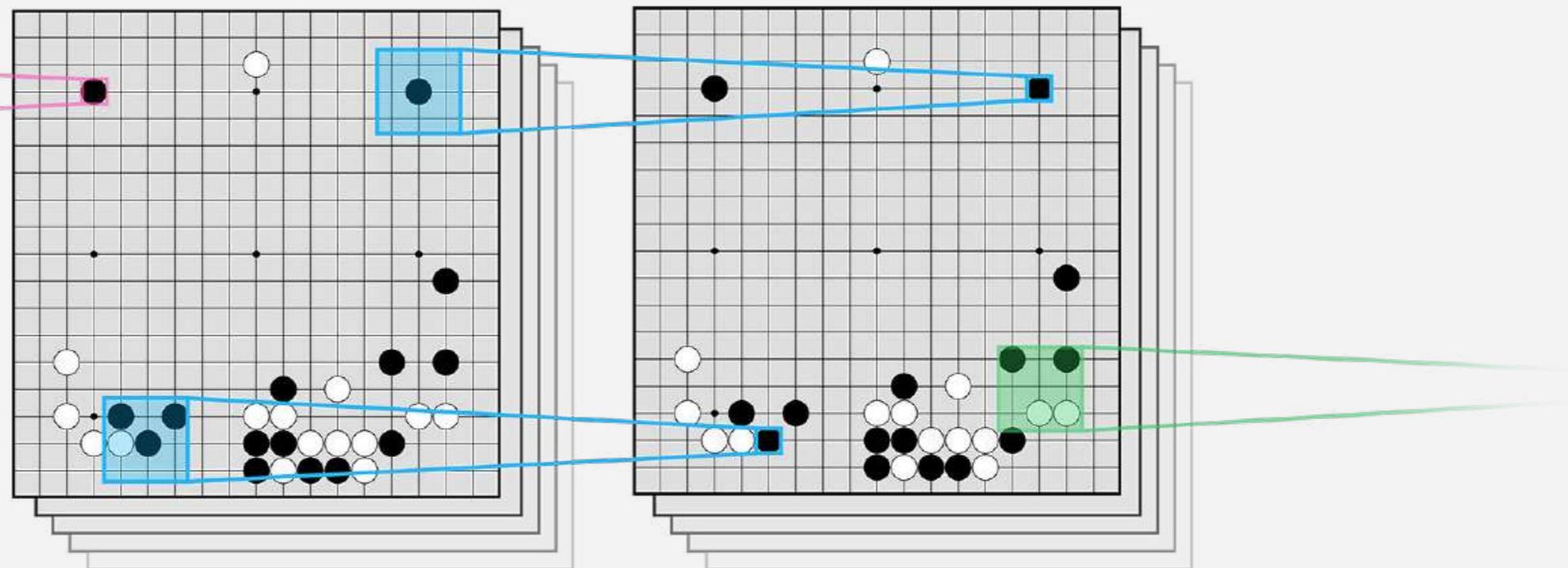
Game tree complexity = b^d

Brute force search intractable:

1. Search space is huge
2. “Impossible” for computers to evaluate who is winning



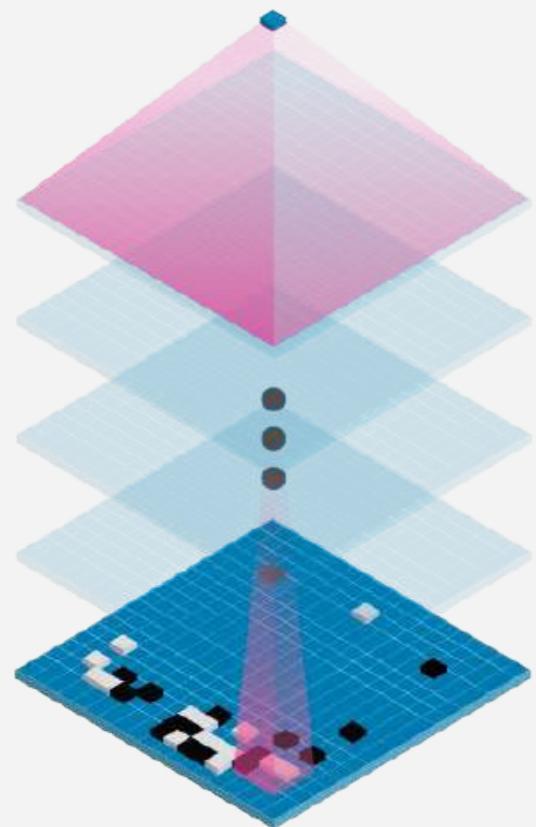
Convolutional neural network



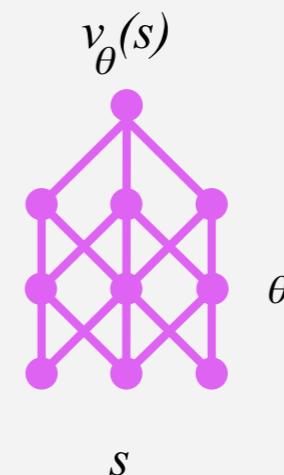
AlphaGo introduces three conv. nets for learning.
Use images of board as state inputs.

Value network

Evaluation



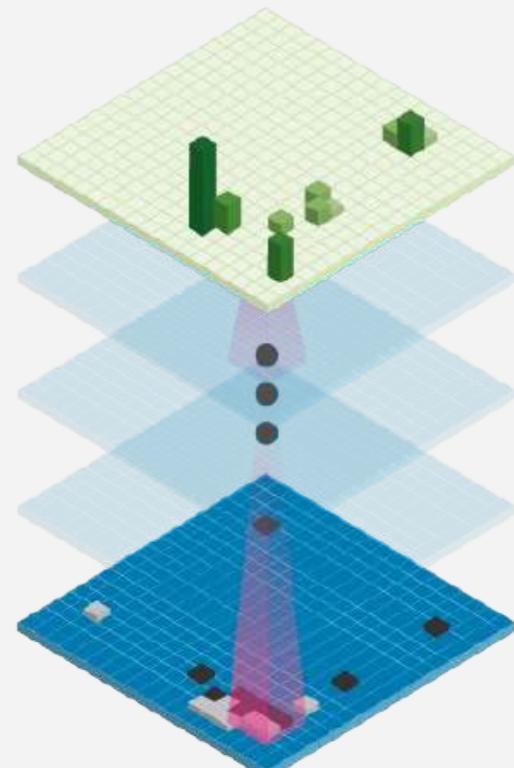
Position



The value function of RL.

Policy network

Move probabilities



Position

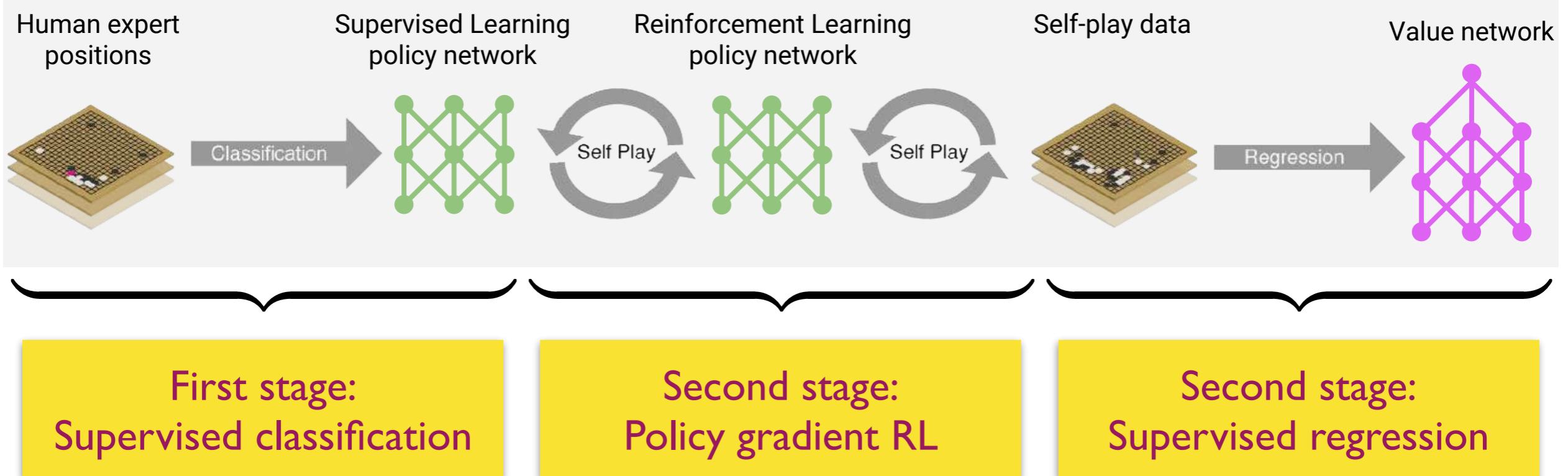
$$p_{\sigma}(a|s)$$

A diagram of a neural network structure. It consists of three layers of nodes. The bottom layer is labeled s , the middle layer is labeled σ , and the top layer is labeled $p_{\sigma}(a|s)$. The nodes are represented by green circles, and the connections between them are shown as green lines.



Policy network of RL.

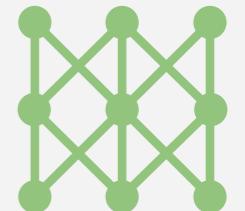
Neural network training pipeline



Learning: First Stage

Supervised learning of policy networks

Policy network: 12 layer convolutional neural network



Training data: 30M positions from human expert games (KGS 5+ dan)

Training algorithm: maximise likelihood by stochastic gradient descent

$$\Delta\sigma \propto \frac{\partial \log p_\sigma(a|s)}{\partial \sigma}$$

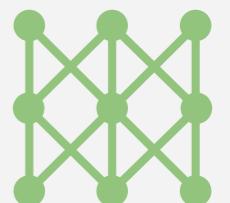
Training time: 4 weeks on 50 GPUs using Google Cloud

Results: 57% accuracy on held out test data (state-of-the art was 44%)

Learning: Second Stage

Reinforcement learning of policy networks

Policy network: 12 layer convolutional neural network



Training data: games of self-play between policy network

Training algorithm: maximise wins z by policy gradient reinforcement learning

$$\Delta\sigma \propto \frac{\partial \log p_\sigma(a|s)}{\partial \sigma} z$$

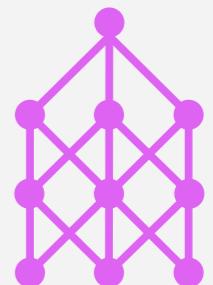
Training time: 1 week on 50 GPUs using Google Cloud

Results: 80% vs supervised learning. Raw network ~3 amateur dan.

Learning: Third Stage

Reinforcement learning of value networks

Value network: 12 layer convolutional neural network



Training data: 30 million games of self-play

Training algorithm: minimise MSE by stochastic gradient descent

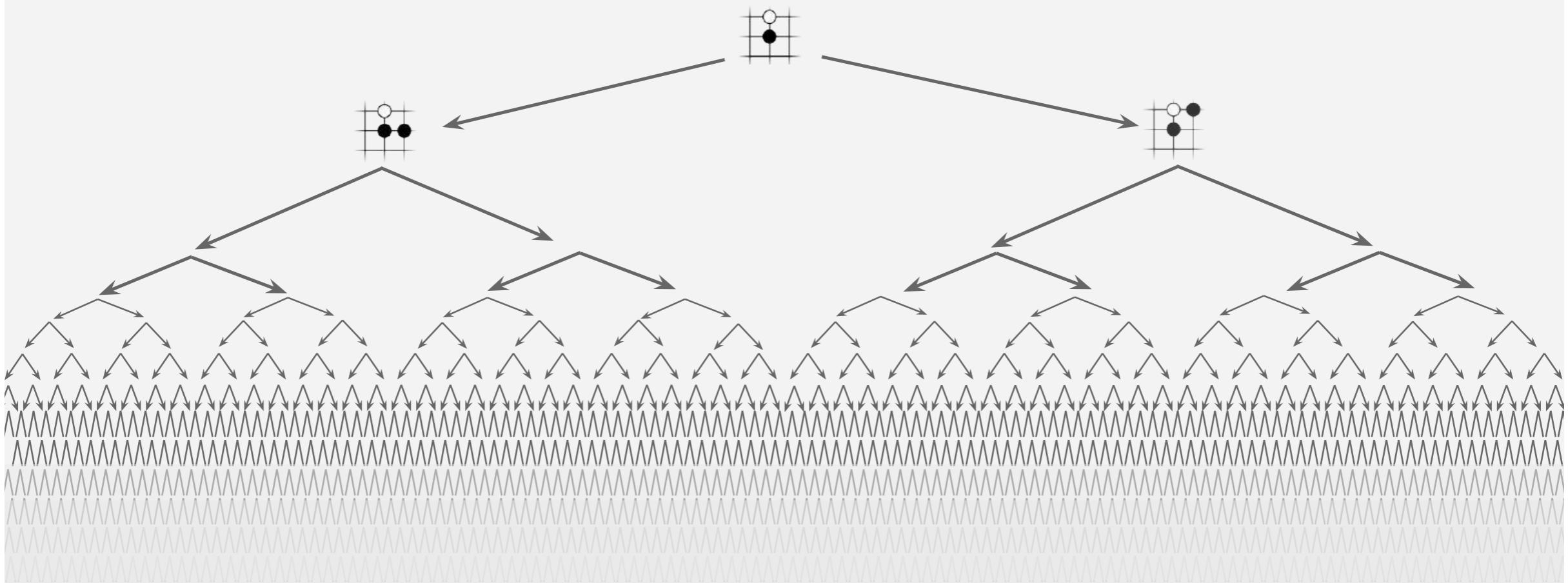
$$\Delta\theta \propto \frac{\partial v_\theta(s)}{\partial \theta} (z - v_\theta(s))$$

Training time: 1 week on 50 GPUs using Google Cloud

Results: First strong position evaluation function - previously thought impossible

Real Play: MCTS

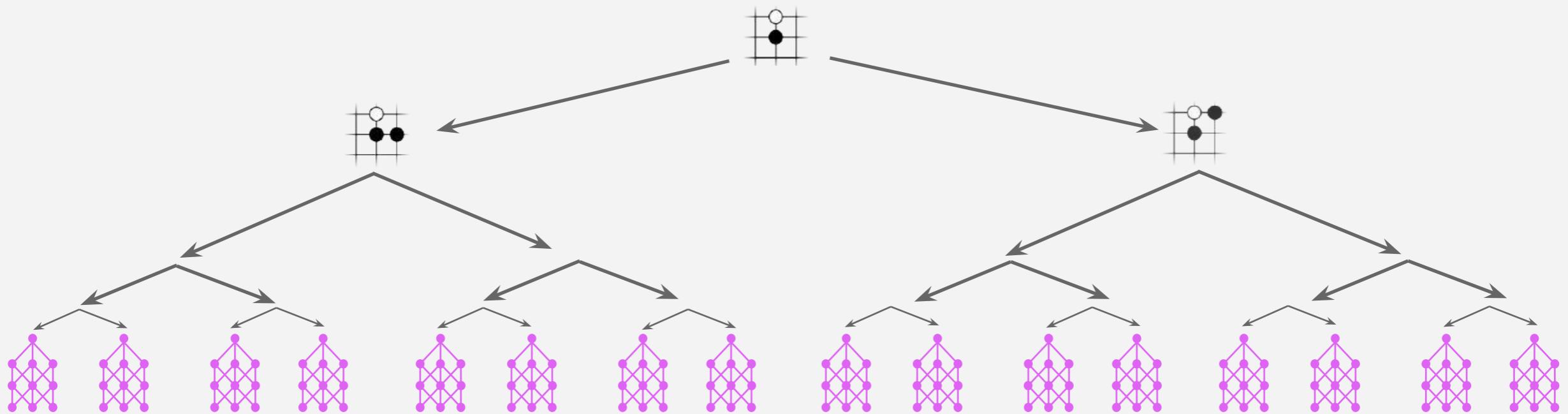
Exhaustive search



Two key steps:
node expansion and repeated random simulation

Real Play: MCTS

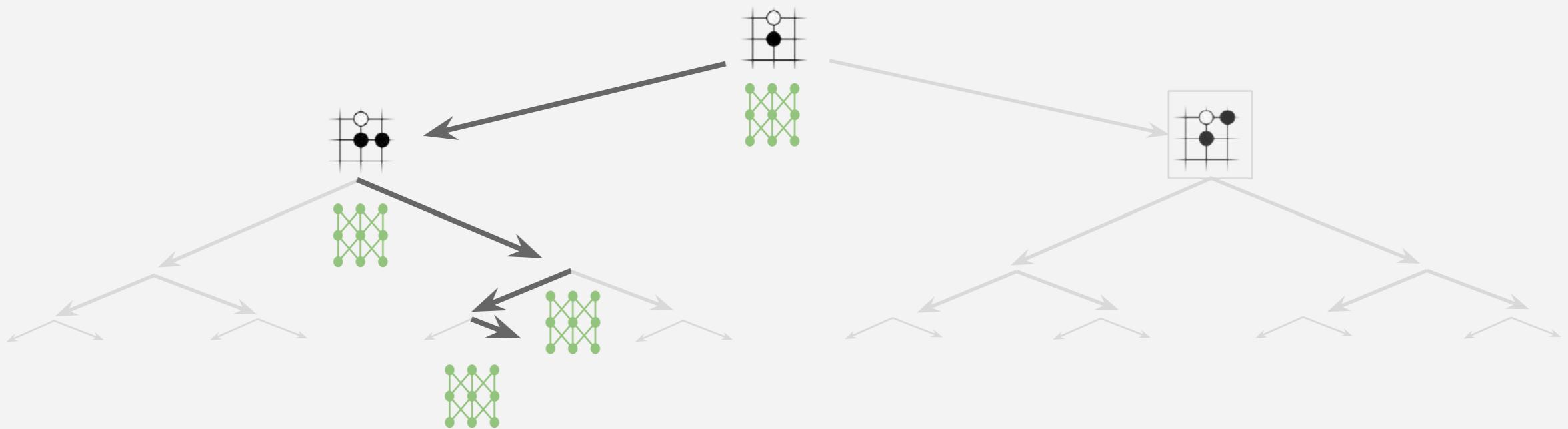
Reducing depth with value network



With value network, we can expand fewer depth
since the value of nodes can also be obtained from the value network.

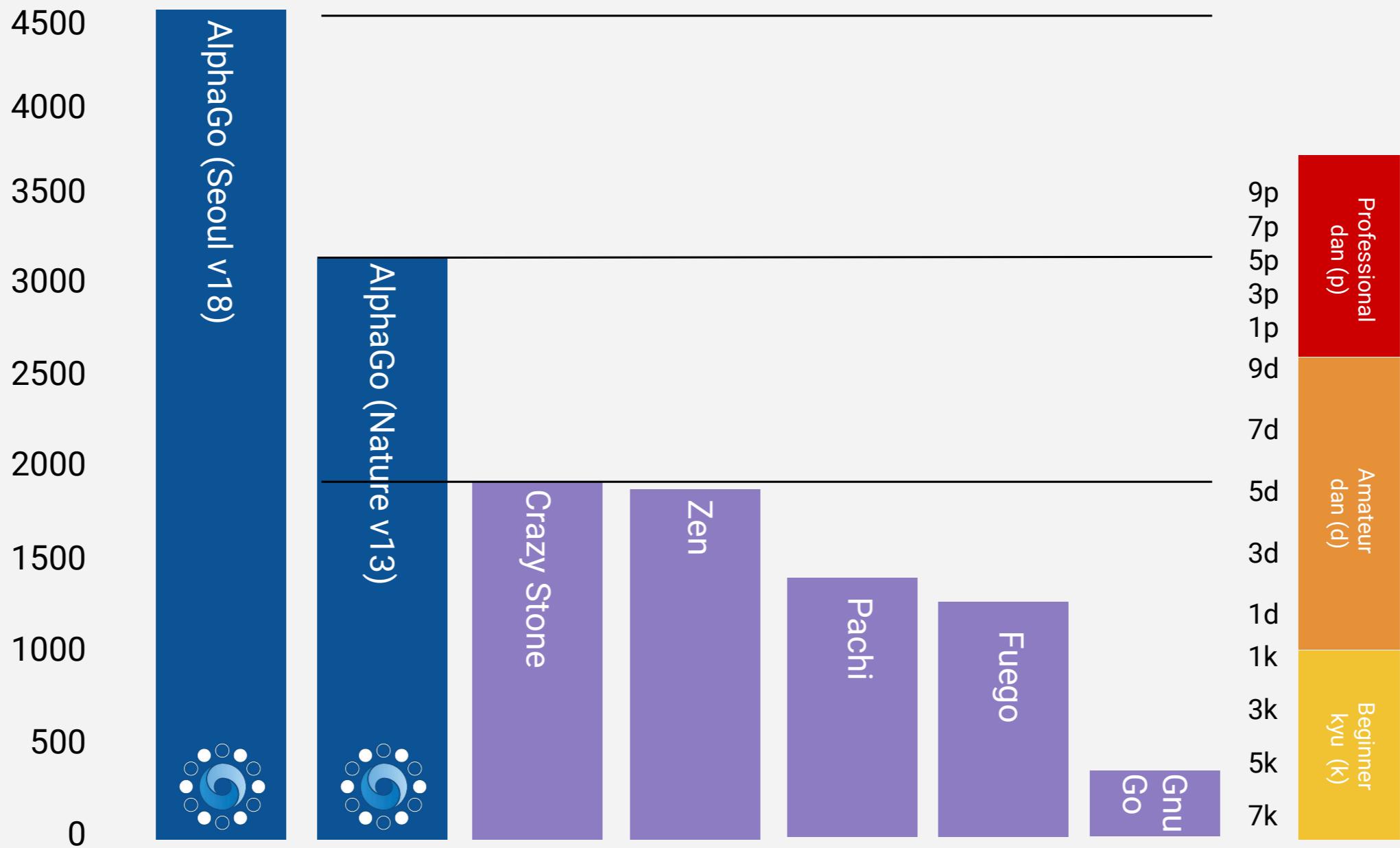
Real Play: MCTS

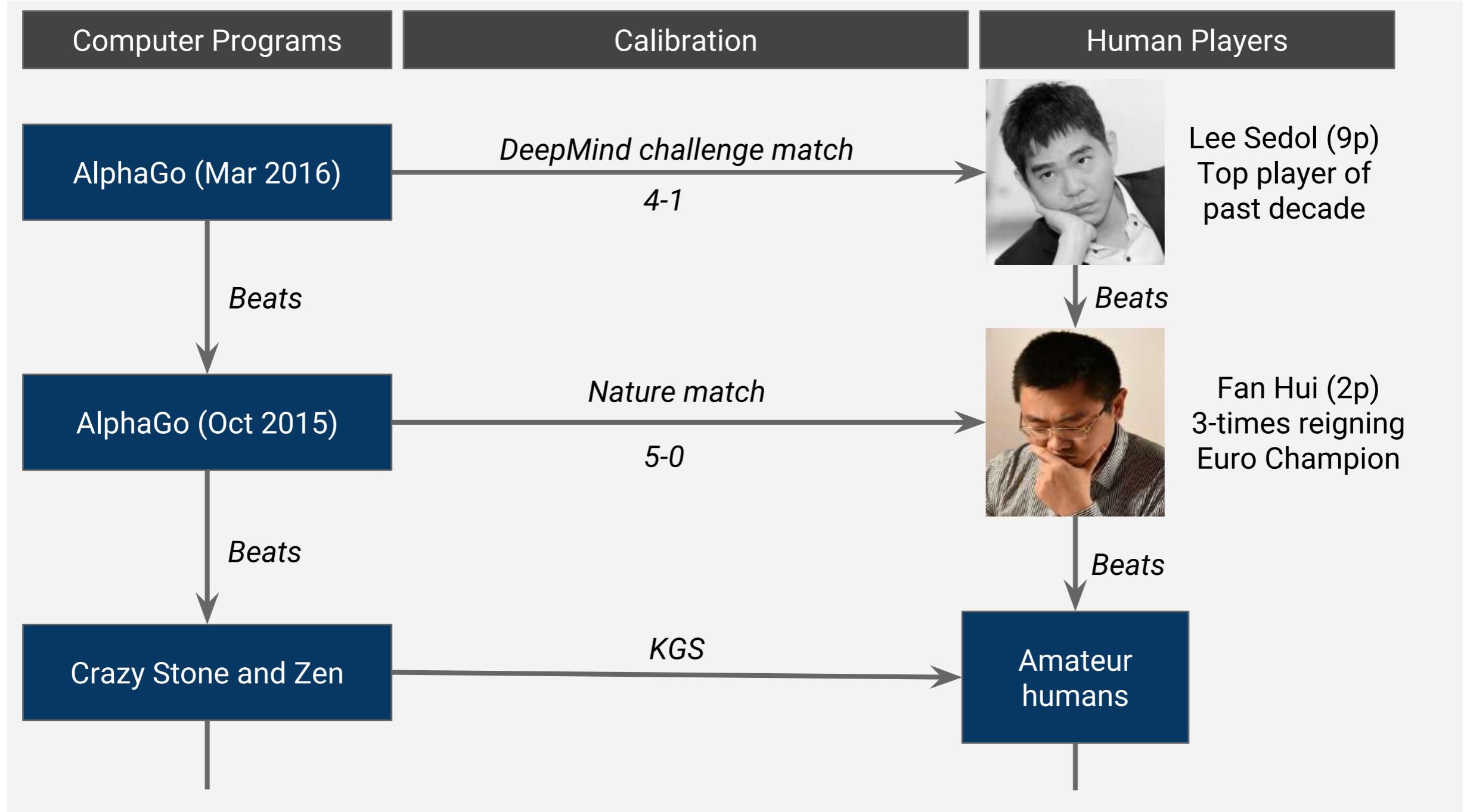
Reducing breadth with policy network



With policy network, we can simulate with fewer times
since the simulation can be guided by policy network over random play.

Evaluating AlphaGo against computers





Reinforcement Learning

- Basics
 - Markov decision process
 - RL with known model
 - RL with unknown model
 - Policy gradient & actor-critic methods
- Deep reinforcement learning
- Integrating learning and planning
- RL from human preference
- Take-home messages

Slides link:



Aligning Language Models to Follow Instructions

Prompt *Explain the moon landing to a 6 year old in a few sentences.*

Completion GPT-3

Explain the theory of gravity to a 6 year old.

Explain the theory of relativity to a 6 year old in a few sentences.

Explain the big bang theory to a 6 year old.

Explain evolution to a 6 year old.

InstructGPT

People went to the moon, and they took pictures of what they saw, and sent them back to the earth so we could all see them.

Fine-Tune GPT by RL

Step 1

**Collect demonstration data,
and train a supervised policy.**

A prompt is
sampled from our
prompt dataset.

Explain the moon
landing to a 6 year old

A labeler
demonstrates the
desired output
behavior.

Some people went
to the moon...

This data is used
to fine-tune GPT-3
with supervised
learning.

SFT

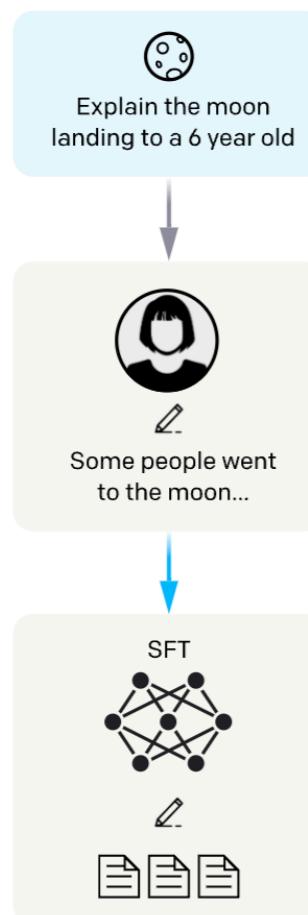
Some people went
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Fine-Tune GPT by RL

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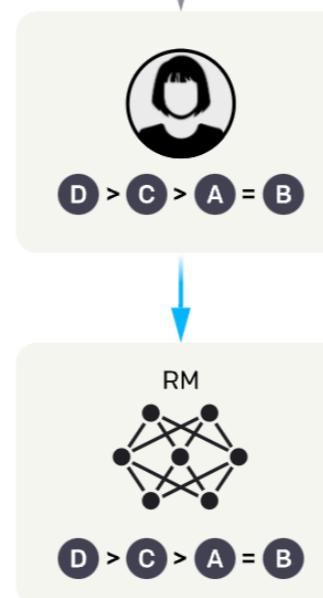
Step 2

**Collect comparison data,
and train a reward model.**

A prompt and several model outputs are sampled.



A labeler ranks the outputs from best to worst.



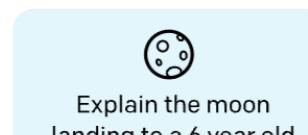
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Fine-Tune GPT by RL

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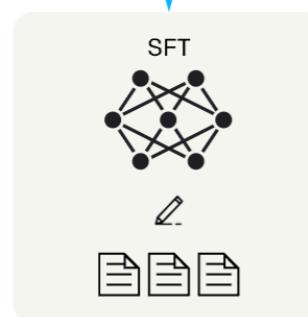
A prompt is sampled from our prompt dataset.



A labeler demonstrates the desired output behavior.



This data is used to fine-tune GPT-3 with supervised learning.



Step 2

Collect comparison data, and train a reward model.

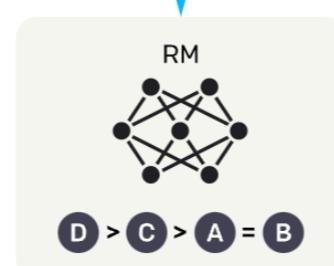
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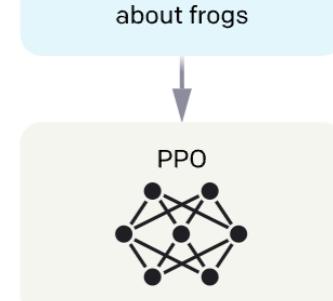
Step 3

Optimize a policy against the reward model using reinforcement learning.

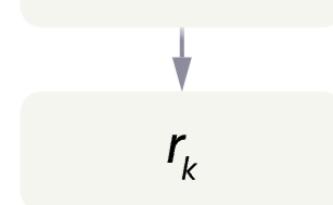
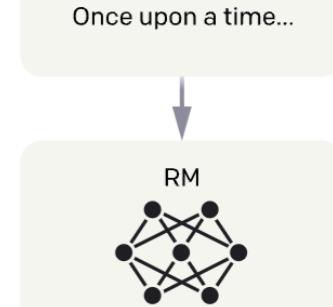
A new prompt is sampled from the dataset.



The policy generates an output.



The reward model calculates a reward for the output.



The reward is used to update the policy using PPO.

Learning Reward Function from Human Preference

- Reward function for evaluating behavior segments: $r(o, a)$

Learning Reward Function from Human Preference

- Reward function for evaluating behavior segments: $r(o, \underline{a})^\sigma$

Learning Reward Function from Human Preference

- Reward function for evaluating behavior segments: $r(o, \underline{a})^\sigma$
- Given two segments, human expert labels preference: $\sigma_1 \succ \sigma_2$

Learning Reward Function from Human Preference

- Reward function for evaluating behavior segments: $r(o, \underline{a})^\sigma$
- Given two segments, human expert labels preference: $\sigma_1 \succ \sigma_2$
- Turn reward function into classifier to estimate the preference:

$$\hat{P}[\sigma^1 \succ \sigma^2] = \frac{\exp \sum \hat{r}(o_t^1, a_t^1)}{\exp \sum \hat{r}(o_t^1, a_t^1) + \exp \sum \hat{r}(o_t^2, a_t^2)}. \quad \text{The Bradley–Terry model [1952]}$$

Learning Reward Function from Human Preference

- Reward function for evaluating behavior segments: $r(o, \underline{a})^\sigma$
- Given two segments, human expert labels preference: $\sigma_1 \succ \sigma_2$
- Turn reward function into classifier to estimate the preference:
$$\hat{P}[\sigma^1 \succ \sigma^2] = \frac{\exp \sum \hat{r}(o_t^1, a_t^1)}{\exp \sum \hat{r}(o_t^1, a_t^1) + \exp \sum \hat{r}(o_t^2, a_t^2)}. \quad \text{The Bradley–Terry model [1952]}$$
- Learn the reward function (classifier) with cross-entropy loss.

Learning Reward Function from Human Preference

- Reward function for evaluating behavior segments: $r(o, \underline{a})^\sigma$
- Given two segments, human expert labels preference: $\sigma_1 \succ \sigma_2$
- Turn reward function into classifier to estimate the preference:
$$\hat{P}[\sigma^1 \succ \sigma^2] = \frac{\exp \sum \hat{r}(o_t^1, a_t^1)}{\exp \sum \hat{r}(o_t^1, a_t^1) + \exp \sum \hat{r}(o_t^2, a_t^2)}. \quad \text{The Bradley–Terry model [1952]}$$
- Learn the reward function (classifier) with cross-entropy loss.

Why not let human directly label single segment?

Reinforcement Learning

- Basics
 - Markov decision process
 - RL with known model
 - RL with unknown model
 - Policy gradient & actor-critic methods
- Deep reinforcement learning
- Integrating learning and planning
- RL from human preference
- Take-home messages

Slides link:



Take-Home Messages

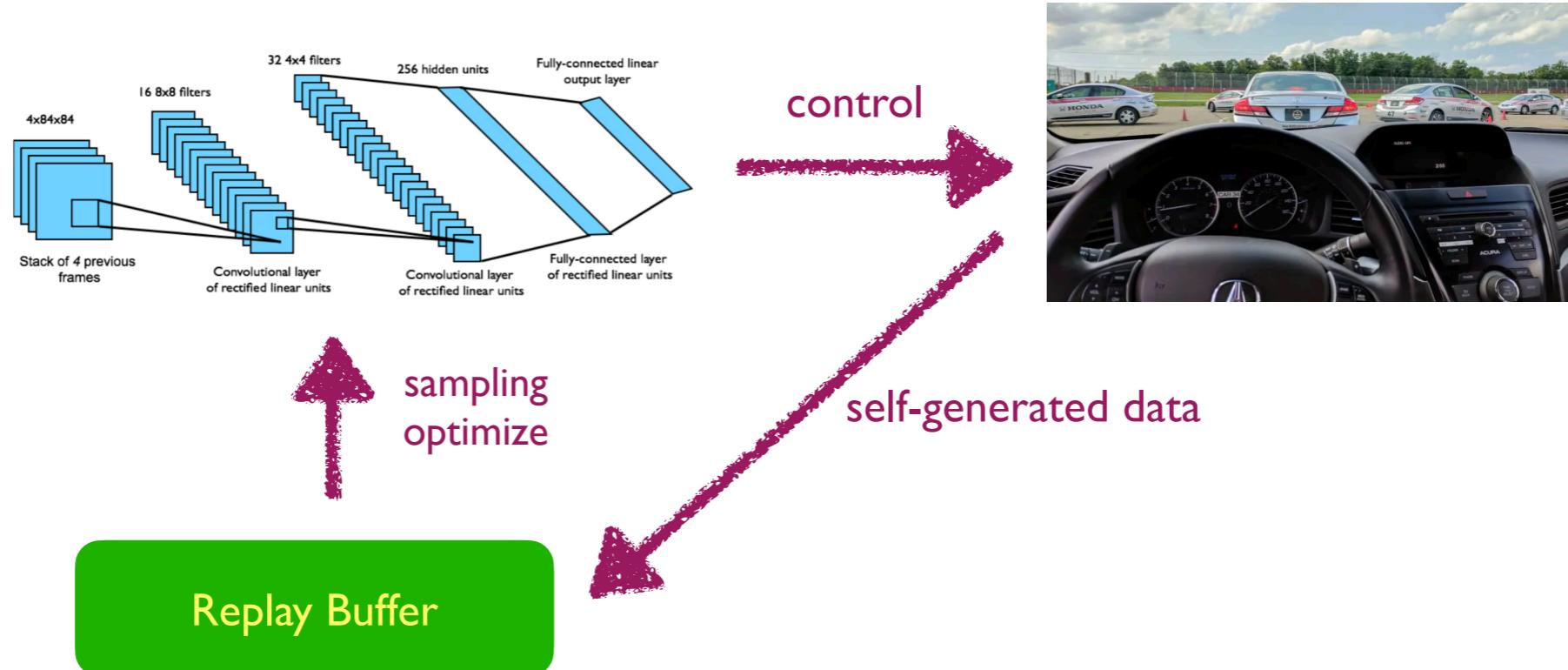
- Reinforcement learning solves decision problems by interaction with the environment.
- Markov decision process models the decision problem, when full information is known, dynamical programming can be used.
- Value function based RL utilizes MC or TD estimate of the value function.

Take-Home Messages

- To solve large-scale RL problems, functional approximation of value functions or policies is essential.
- Policy gradient & actor-critic: direct learning of policies. Usually more efficient for deep RL.
- Large-scale RL systems: Integrating learning and planning & RL from human preference.
- Next-steps of RL?

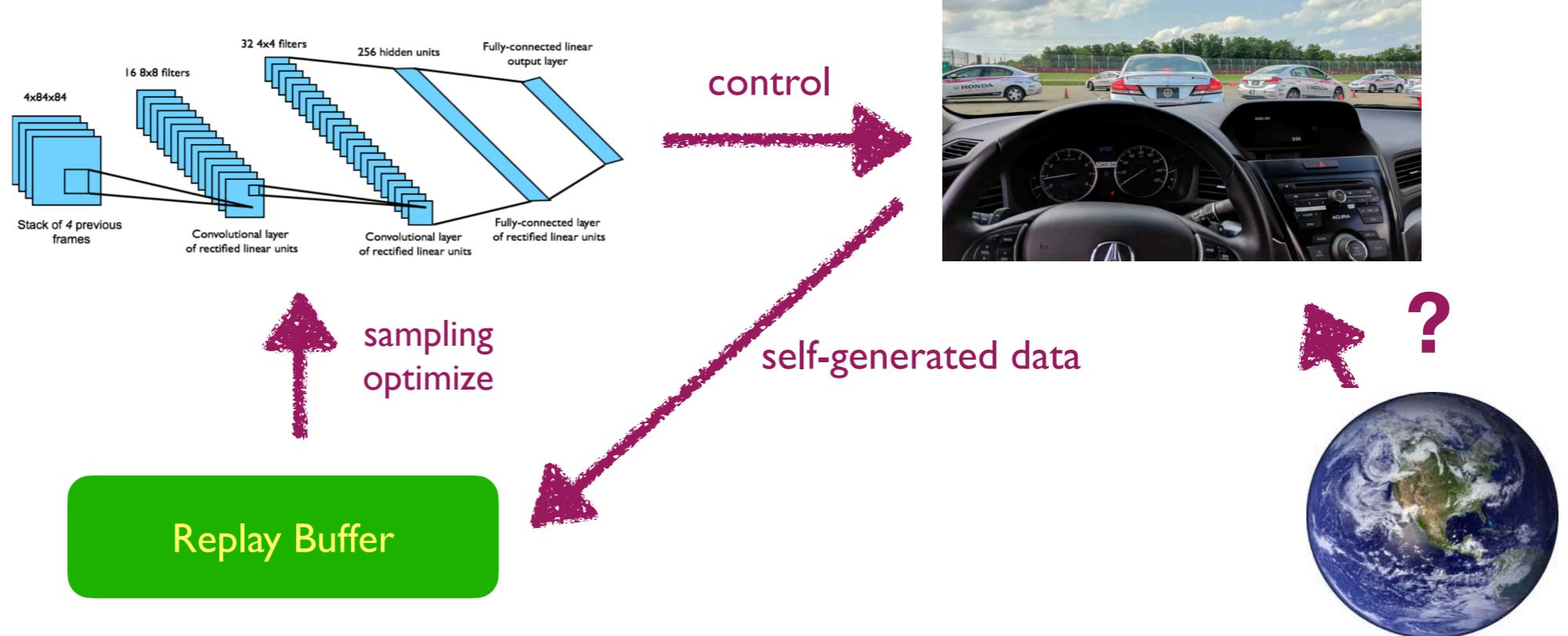
Why hasn't DRL solved many real-world problems?

- Deep RL usually does not use (learn) a world model.



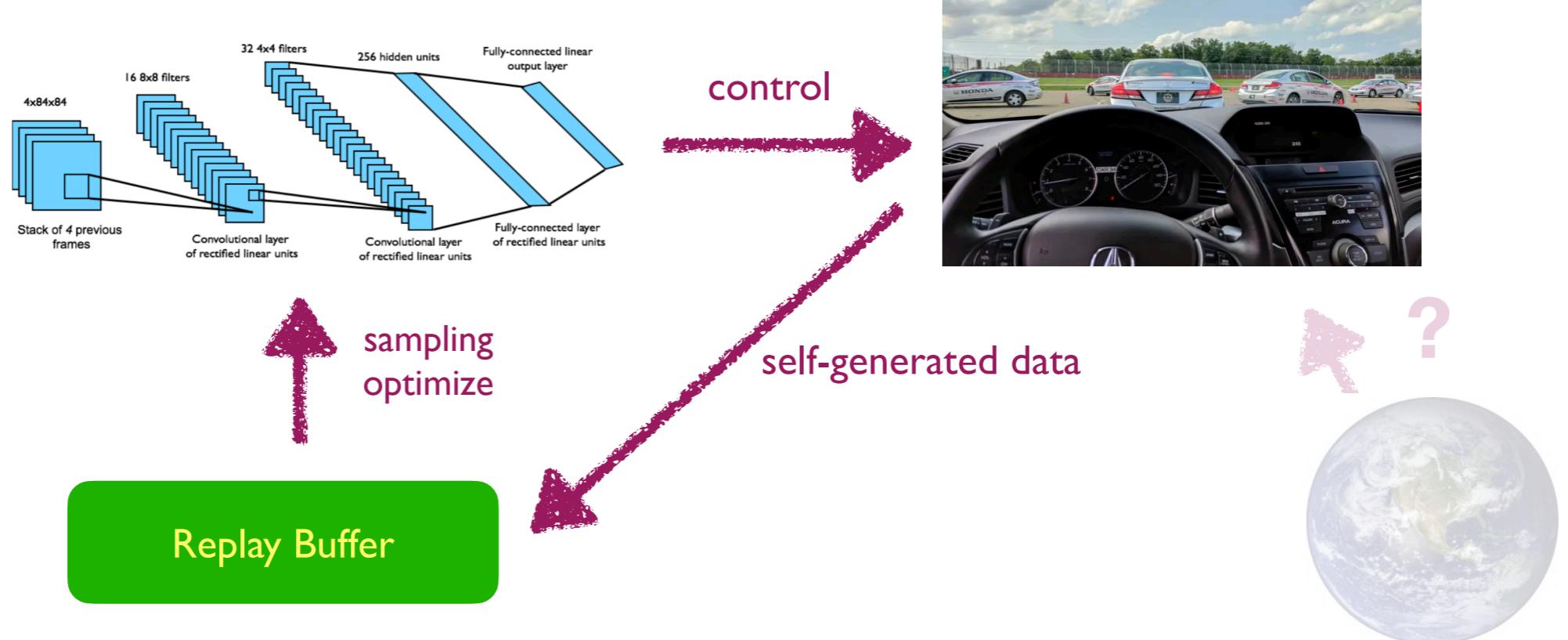
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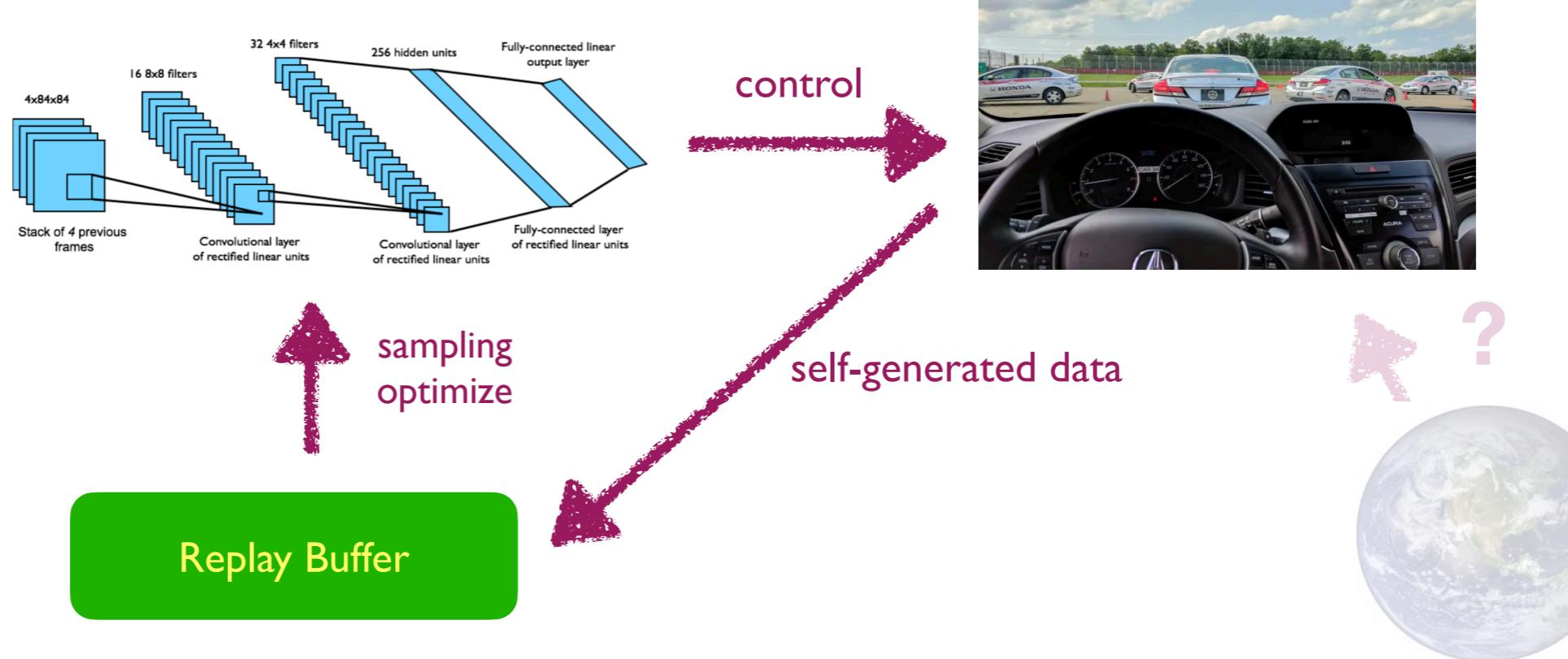
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Without a world model, learning from self-generated data requires many trial-and-errors in the real world.
Large cost. Bad generalization to new task.

Model-Based RL

Scenarios of decision making:

Model-Based RL

Scenarios of decision making:

- Planning: directly solve the optimal policy when the world model is known (dynamic programming, Monte-Carlo tree search...)

Model-Based RL

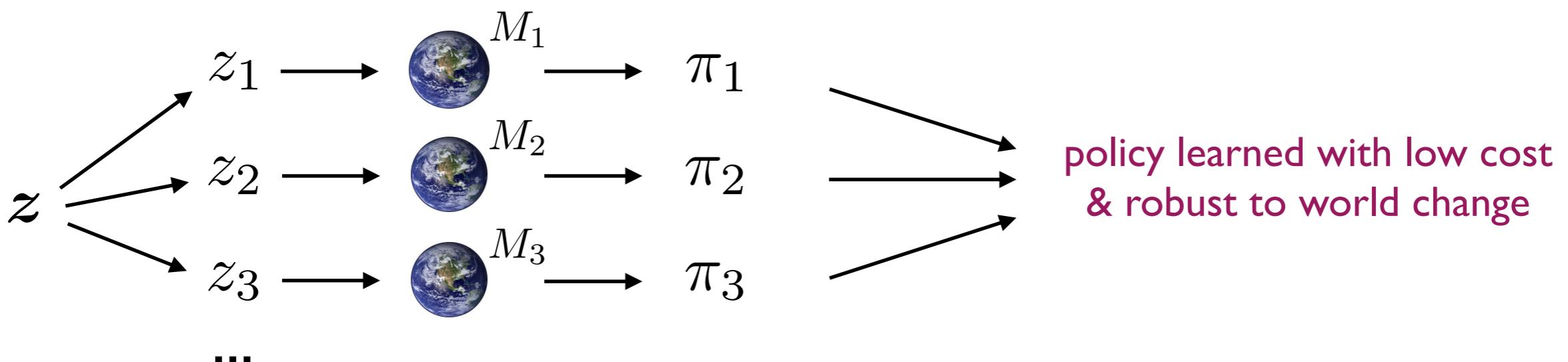
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- Model-free RL: learn by trial-and-error (Q-learning, policy gradient, actor-critic...)

Model-Based RL

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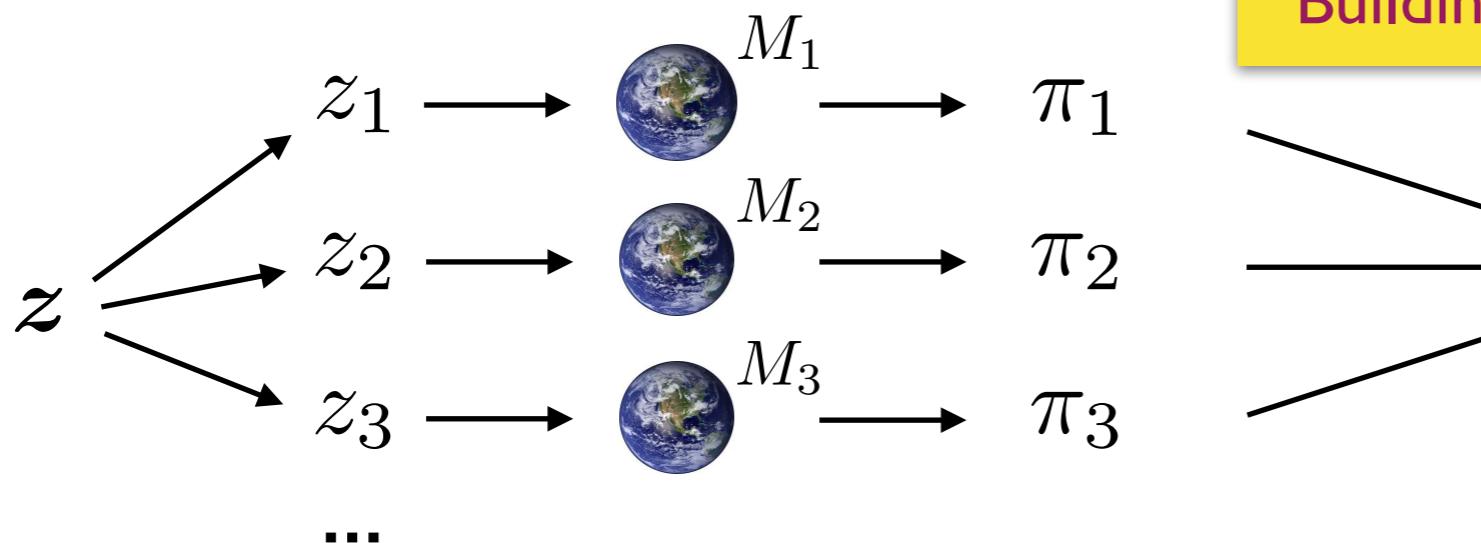
- Planning: directly solve the optimal policy when the world model is known (dynamic programming, Monte-Carlo tree search...)
- Model-free RL: learn by trial-and-error (Q-learning, policy gradient, actor-critic...)
- Model-based RL: learn the world model during learning, do RL or planning using the model.



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Building the internal model is essential.

policy learned with low cost
& robust to world change

Not Covered...

- Search, planning and game theory
- Bandit learning
- Imitation learning and Inverse RL
- RL Theory
- ...

Further Reading

- David Silver's RL Course:
<https://www.davidsilver.uk/teaching/>
- Deep RL course @ Berkeley:
<https://rail.eecs.berkeley.edu/deeprlcourse/>
- Sutton and Barto book:
<http://incompleteideas.net/book/the-book-2nd.html>
- OpenAI Gym platform:
<https://github.com/Farama-Foundation/Gymnasium>

Thanks for your attention! Discussions?

Acknowledgement: Many materials in this lecture are taken from
<https://www.davidsilver.uk/teaching/>

https://www.davidsilver.uk/wp-content/uploads/2020/03/AlphaGo-tutorial-slides_compressed.pdf