Homework – Compliant Mechanism

Course	**Machine Dynamics and Vibrations**
Activity Title	Compliant Mechanism – System Identification using Free Response data
Why	System identification can be utilized to find the unknowns of a system such as its equivalent stiffness and equivalent damping even though system doesn't have an actual spring. It is important to know the natural frequency of a mechanism since mechanism undergoes large deflection if the mechanism vibrates at its natural frequency. In mechanical engineering, deriving mathematical model plays an important role to control the mechanism so it has desired output values. In this lab students will obtain the mathematical model of two translational mechanisms; one with actual 3D printed springs and the other with fixed-free flexible beams representing nonlinear springs.
Time recommended to complete this homework assignment	180 minutes
Learning objective(s)	**1.** Derive the equation of motion of a translational vibratory single degree of freedom system **2.** Obtain natural frequency both experimentally and theoretically **3.** Find the equivalent stiffness of the compliant mechanism **4.** Create Matlab Simulink model of the system and find the free-vibration response **5.** Compare the free response obtained from the simulation model and the experimental result
Performance criteria (behaviors you should follow to perform the activity well)	Confirm prerequisite knowledge needed for activity has been addressed by reviewing course notes and textbook: • Natural frequency • Modeling of SDOF free vibration translational mechanical system Use differential calculus solutions to complete your analysis of this system. Validate the measured data and modify data set appropriately for further analysis. Report outcomes using professional language that reflects deep thinking for interpretation of results.
Connections (to other topics, content areas, real-world applications)	This learning activity integrates the fundamentals of vibrations from ENGR 3125 (Machine Dynamics and Vibrations) such as natural frequency, free response of SDOF systems, data measurement from low-cost accelerometer, and system analysis using Matlab Simulink.

Instructions - after reviewing this entire activity, do these in order

1. Review the general theoretical model for SDOF systems
   Refer to your Machine Dynamics and Vibrations textbook Chapter 1 and Chapter 2 or in-class notes shared in D2L, or GitHub repository.
2. Answer the **preliminary questions** on the next page of this handout (graded only for completion).
3. Study the experimental system we are analyzing for this homework and watch the recorded video.
4. Gather the resources needed to complete the analysis
   a. Download the experimental data from GitHub
   b. Download Matlab R2020a if you haven't already. (or Matlab R2020b)
5. Answer the **procedures and calculation questions** (sections A-E below) on this handout. Please show **all** your work to receive full credit.
6. Upload your homework (your responses to the preliminary questions and sections A-E below) in pdf form in D2L before the deadline.

Preliminary Questions (Graded only for completion)

1. What are three parts of a vibratory mechanism?

2. Describe a method how to determine the damping of an underdamped system?

3. Why is it important to find the natural frequency of a system?

4. Why does the amplitude of free vibrations decrease in most practical systems?

5. List the parts of the mechanism below and explain the possible required equipment needed to collect data which could represent its free response behavior.

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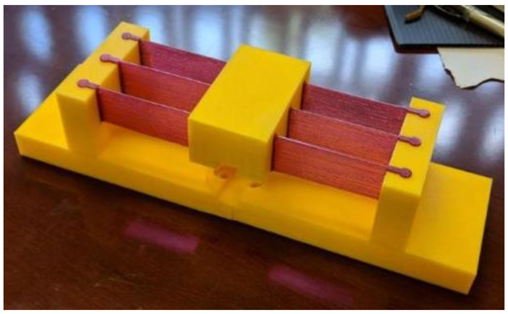

Equipment List: ___

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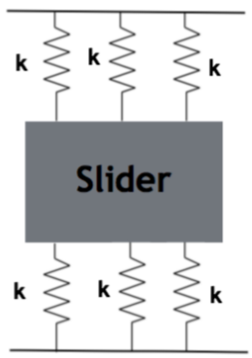
Procedures and Calculations:

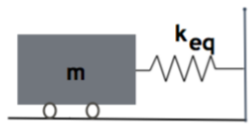
A. Theoretical Modeling using Newton's Laws of Motion

(Please refer to Chapter 2: SDOF Free Vibrations notes.)

1. Derive the equation of motion of the compliant oscillatory mechanism given in the below sketch. Show all your work including the free body diagram and the equations.

Assume there is also damping acting on the system so the system is damped!





Free Response of the Pendulum:

The slider is displaced by 0.5 cm.

Slider Mass	52 g.
Compliant Link Length	6 cm
Compliant Link Width (b)	2.5 cm
Compliant Link Thickness (h)	1 mm
E (Young/s Modulus- PLA)	3.5 GPa

2. Find the equivalent stiffness of the mechanism in terms of k.

3. If the compliant members are assumed to be fixed-free beams, what would be the theoretical equivalent stiffness of the system? ($k = \frac{3EI}{L^3}$ and $I = \frac{bh^3}{12}$)

B. Experimental Data Collection and Calculations

1. Watch the recorded video and the PowerPoint titled "Homework Compliant Mechanism".
2. Download the recorded data.
3. Import the recorded data into Matlab.
4. Plot the time vs acceleration data.
5. Calculate the damped period, damped frequency, logarithmic decrement, damping ratio, damped frequency (in rad/sec), natural frequency (in rad/sec), damping constant and equivalent stiffness of the system.

   Logarithmic decrement: $\delta = \ln\left(\frac{X_1}{X_2}\right) = \ln\left(\frac{}{}\right) =$

   Damping ratio: $\zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} = \frac{}{} =$

   Read the following from the acceleration vs time graph:

   Damped Period=T_d=

   Damped Frequency = $f_d = \frac{1}{T_d} =$

   Damped Angular Frequency= $w_d = 2\pi f_d$=

   Natural Frequency= $w_n = \frac{w_d}{\sqrt{1-\zeta^2}} = \frac{}{} =$

   Damping constant=c= $2mw_n\zeta =$

   Equivalent Stiffness=

6. Is there a significant difference between the theoretical equivalent stiffness you calculated in Section A.3 and experimental equivalent stiffness?

C. Model Validation and Simulation

1. Simulate the theoretical equation you derived in Section A.1 in Matlab Simulink.

2. Import the experimental data in Matlab Simulink and compare the experimental data with the simulation output. Include your figures in your report.

3. Is there any difference between the simulation response and experimental data? If yes, please comment on the possible reasons.

D. Analysis (post-processing) Questions

1. Rewrite the equation of motion of the compliant mechanism assuming:
   - The mechanism has no damping (i.e. no friction)
   - The mechanism has some damping (friction is not neglected). Use the damping constant you calculated in Section B.5.
2. Using the same properties and initial displacement; simulate the response of the both systems (damped and undamped) in Matlab Simulink.
3. Compare the frequency of the damped and undamped oscillations. Is there a significant difference in the frequency of oscillations?
4. Summarize in one paragraph (about ¼ of a page single spaced prose – no lists!) how to obtain the natural frequency and damping constant for any single degree of freedom system.

E. Critical Thinking Questions

1. Does damping affect the natural frequency of a system? Why or why not?
2. What are the three most important concepts that you have learned in this lab (review the learning objectives to help you decide)?
3. What are three applications (in addition to a SDOF oscillatory mechanism) where you might use these concepts?
4. What is the muddiest point remaining that you haven't yet mastered?
5. Write out one question you can ask your instructor to clear this up.