SUPPLEMENTARY MATERIALS

A. Definition

Denote that $a(t)$ and $b(t)$ are functions, then we have following definitions:

- Inner product:

$$a(t) \cdot b(t) = \int_{t=0}^{T} a(t)b(t)dt \tag{23}$$

- The l_2 norm

$$\|a(t)\|_2^2 = a(t) \cdot a(t) = \int_{t=0}^{T} a(t)a(t)dt \tag{24}$$

$$\|a(t)\|_2 = \sqrt{a(t) \cdot a(t)} = \sqrt{\int_{t=0}^{T} a(t)a(t)dt} \tag{25}$$

- Distance

$$d(a(t), b(t)) = \|a(t) - b(t)\|_2 \tag{26}$$

- The l_1 norm:

$$\|a(t)\|_1 = \int_{t=0}^{T} |a(t)|dt \tag{27}$$

- The ∞ norm:

$$a_{max} = \|a(t)\|_\infty = \max_{0 \le \tau \le T} a(\tau) \tag{28}$$

B. Proof of Lemma 1

For any two admissible controls $a_{i,1}(t)$ and $a_{i,2}(t)$, we can construct a new control action

$$a_{i,new}(t) = \beta a_{i,1}(t) + (1-\beta)a_{i,2}(t) \quad 0 \le \beta \le 1 \tag{29}$$

The storage dynamic is:

$$\begin{aligned} S_{i,new}(t) &= \int_{\tau=0}^{t} a_{i,new}(t)d\tau + S_i(0) \\ &= \beta \int_{\tau=0}^{t} a_{i,1}(t)d\tau + \beta S_i(0) \\ &\quad + (1-\beta) \int_{\tau=0}^{t} a_{i,2}(t)d\tau + (1-\beta)S_i(0) \\ &= \beta S_{i,1}(t) + (1-\beta)S_{i,2}(t) \end{aligned} \tag{30}$$

The key is to observe $S_{i,new}(t)$ is also a linear combination of $S_{i,1}(t)$ and $S_{i,2}(t)$:

Hence we can conclude that

$$0 \le S_{i,new}(t) \le B_i \quad S_{i,new}(T) = B_i \tag{31}$$

The constructed action $a_{i,new}$ is also in the admissible control set, yielding the fact that $\mathcal{A}_i$ is a convex set.

In addition, the set $\mathcal{C}_i$ that satisfies Eq.(2,3,4) is compact. And the set $\mathcal{U}_i$ that satisfies Eq. (1,3,4) is also compact.

Note that $\mathcal{A}_i = \mathcal{C}_i \cap \mathcal{U}_i$. So $\mathcal{A}_i$ is also compact according to [1].

C. Proof of Lemma 2

First, the unconstrained solution is defined as

$$\begin{aligned} &\hat{a}_i(z,t) = \arg\max_{a_i} J_i(a_i, z) \\ &= \int_0^T -\frac{1}{2}\gamma_i a_i^2(t) + B_i a_i(t) \\ &- f\big(D(t) + Nz(t)\big)\big(a_i(t) + \lambda a_i^2(t)\big)dt \\ &= \int_0^T -\big(\frac{1}{2}\gamma_i + \lambda f(D(t) + Nz(t))\big)a_i^2(t) \\ &\big(B_i - f(D(t) + Nz(t))\big)a_i(t)dt \end{aligned} \tag{32}$$

Given z, the corresponding unconstrained optimal solution is

$$\hat{a}_i(z,t) = h(z) = \frac{B_i - f(D + Nz)}{2\lambda f(D + Nz) + \gamma_i} \tag{33}$$

The function h is smoothed by K Lipschitz. For any z_1 and z_2, we define:

$$\begin{aligned} &\|\hat{a}_i(z_1,t) - \hat{a}_i(z_2,t)\|_2 \\ &= \left\| \frac{B_i - f(D + Nz_1)}{2\lambda f(D + Nz_1) + \gamma_i} - \frac{B_i - f(D + Nz_2)}{2\lambda f(D + Nz_2) + \gamma_i} \right\|_2 \\ &= \left\| \frac{\big(2\lambda B_i + \gamma_i\big)\big(f(D + Nz_2) - f(D + Nz_1)\big)}{(2\lambda f(D + Nz_1) + \gamma_i)(2\lambda f(D + Nz_2) + \gamma_i)} \right\|_2 \\ &\le \frac{\big(2\lambda B_i + \gamma_i\big) \|f(D + Nz_2) - f(D + Nz_1)\|_2}{(2\lambda f_{\min} + \gamma_i)^2} \\ &\le \frac{\big(2\lambda B_i + \gamma_i\big)KN \|z_1 - z_2\|_2}{(2\lambda f_{\min} + \gamma_i)^2} \end{aligned} \tag{34}$$

Where $f_{\min}$ is the minimum electricity price without EVs. Note that we define the coefficient $\frac{\left(2\lambda B_i + \gamma_i\right)KN}{(2\lambda f_{\min} + \gamma_i)^2}$ to be θ, which is frequently used in the subsequent analysis.

Since the original optimization is convex over a convex set, the optimal solution is the projection of the above unrestricted optimal solution onto the feasible set.

$$a_i^*(z,t) = Proj_{\mathcal{A}_i} \hat{a}_i(z,t) = \min_{a_i \in \mathcal{A}_i} \|a_i(z,t) - \hat{a}_i(z,t)\|_2 \tag{35}$$

where $a_i^*(z_1)$ and $a_i^*(z_2)$ are in the feasible set. According to the property of convex projection, the following hold

$$\Big(\hat{a}_i(z_1) - a_i^*(z_1)\Big) \cdot \Big(a_i^*(z_2) - a_i^*(z_1)\Big) \le 0 \tag{36}$$

$$\Big(\hat{a}_i(z_2) - a_i^*(z_2)\Big) \cdot \Big(a_i^*(z_1) - a_i^*(z_2)\Big) \le 0 \tag{37}$$

Summing up Eq.36 and 37 yields

$$(a_i^*(z_2) - a_i^*(z_1)) \cdot (a_i^*(z_2) - a_i^*(z_1)) \tag{38}$$

$$\le (\hat{a}_i(z_2) - \hat{a}_i(z_1)) \cdot (a_i^*(z_2) - a_i^*(z_1)) \tag{39}$$

$$\le \|\hat{a}_i(z_2) - \hat{a}_i(z_1)\|_2 \, \|a_i^*(z_2) - a_i^*(z_1)\|_2 \tag{40}$$

The last inequality is due to Cauchy-Swatchz inequality. Then we have

$$\begin{aligned} \|a_i^*(z_2) - a_i^*(z_1)\|_2 &\le \|\hat{a}_i(z_2) - \hat{a}_i(z_1)\|_2 \\ &\le \frac{\big(2\lambda B_i + \gamma_i\big)KN \|z_1 - z_2\|_2}{(2\lambda f_{\min} + \gamma_i)^2} \end{aligned} \tag{41}$$

which concludes the lemma.

D. Proof of Lemma 3

$$
\begin{aligned}
&\|z_1 - z_2\|_2 \\
&= \frac{1}{N}\left\|\sum\nolimits_{i=1}^{N}\left(g(a_{i,1}) - g(a_{i,2})\right)\right\|_2 \\
&= \frac{1}{N}\left\|\sum\nolimits_{i=1}^{N}\left(a_{i,1} + \lambda a_{i,1}^2 - a_{i,2} - \lambda a_{i,2}^2\right)\right\|_2 \\
&\leq \frac{1}{N}\sum\nolimits_{i=1}^{N}\left(\|a_{i,1} - a_{i,2}\|_2 + 2\lambda a_{\max}\|a_{i,1} - a_{i,2}\|_2\right) \\
&\leq (1 + 2\lambda a_{\max})\max\nolimits_i \|a_{i,1} - a_{i,2}\|_2
\end{aligned}
$$

E. Proof of Theorem 1 and 2

According to The admissible control set $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2 ... \mathcal{A}_N\}$ is convex and compact. The two lemmas indicate that the mapping from $\mathcal{A}$ to $\mathcal{Z}$ is continuous, and the mapping from $\mathcal{Z}$ to $\mathcal{A}$ is also continuous. Therefore, we can construct a continuous mapping from $\mathcal{A}$ to $\mathcal{A}$.

According to Brouwer fixed-point theorem[17], there exists at least one fix point to the system of Eq.(13) and (16), i.e., the MFG admits at least one Nash equilibrium. And when the coefficient in the mapping is less than 1, MFG admits a unique Nash equilibrium.

F. Proof Sketch of Lemma 4

Suppose that $\bar{a}_i, \bar{z} = \frac{1}{N}\sum_{i=1}^{N}\bar{a}_i$ are the optimal solution in the MFG.

$$\bar{J}_i(\bar{a}_i, \bar{z}) = \max_{a_i \in \mathcal{A}_i} J_i\Big(a_i, \frac{1}{N}\sum_{i=1}^{N}\bar{a}_j\Big) \tag{42}$$

Suppose that $\hat{a}_i$ is the optimal solution in EVCG .

$$\hat{J}_i(\hat{a}_i, \bar{a}_{-i}) = \max_{a_i \in \mathcal{A}_i} J_i\Big(a_i, \frac{1}{N}a_i + \frac{1}{N}\sum_{j\neq i}^{N}\bar{a}_j\Big) \tag{43}$$

Define the auxiliary variable $\tilde{a}_i$ and $\tilde{z}_i$ as below.

$$\tilde{J}_i(\tilde{a}_i, \tilde{z}_i) = \max_{a_i \in \mathcal{A}_i} J_i\Big(a_i, \frac{1}{N}\hat{a}_i + \frac{1}{N}\sum_{j\neq i}^{N}\bar{a}_j\Big) \tag{44}$$

Obviously, the following inequality holds

$$\tilde{J}_i(\tilde{a}_i, \tilde{z}) \geq \hat{J}_i\left(\hat{a}_i, \frac{1}{N}\hat{a}_i + \frac{1}{N}\sum_{j\neq i}^{N}\bar{a}_j\right) \tag{45}$$

$$\hat{J}_i(\hat{a}_i, \bar{a}_{-i}) \geq J_i\left(\bar{a}_i, \frac{1}{N}\bar{a}_i + \frac{1}{N}\sum_{j\neq i}^{N}\bar{a}_j\right) = \bar{J}_i(\bar{a}_i, \bar{z}) \tag{46}$$

Therefore, we can bound the error between EVCG and MFG as follows

$$
\begin{aligned}
&\left\|\bar{J}_i - \hat{J}_i\right\|_2 \leq \left\|\bar{J}_i - \tilde{J}_i\right\|_2 \\
&= \left\|\bar{J}_i(\bar{a}_i(\bar{z}_i), \bar{z}_i) - \tilde{J}_i(\tilde{a}_i(\tilde{z}_i), \tilde{z}_i)\right\|_2 \\
&\leq M\|\bar{a}_i(\bar{z}_i) - \tilde{a}_i(\tilde{z}_i)\|_2 + M\|\bar{z}_i - \tilde{z}_i\|_2 \\
&\leq M(\theta + 1)\|\bar{z}_i - \tilde{z}_i\|_2 \leq MN^{-1}(\theta + 1)\|\bar{z}_i - \tilde{z}_i\|_2
\end{aligned} \tag{47}
$$

where M is the Lipschitz coefficient of function J and θ equals to $\frac{\left(2\lambda B_i + \gamma_i\right)KN}{(2\lambda f_{\min} + \gamma_i)^2}$ according to lemma 2.

When N approaches infinity, the utility difference between EVCG and MFG approaches zero.

Each EV aims to maximize their utility, yielding the EV coordination game (EVCG). However, for each EV, it is very difficult to calculate the optimal action and arrive at the equilibrium. Then we introduce MFG to simplify the analysis and calculation of EVCG. When the optimal action is calculated by MFG and transferred to each EV, EV will follow the action. Because the profits from other actions are negligible.

REFERENCES

[1] Ciarlet P G. Linear and nonlinear functional analysis with applications[M]. Siam, 2013.