24 Hour Project

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Contents

1 Introduction

The purpose of the article is to model the mortality associated with two pollution variables and three climate and socioeconomic variables. The data is collected from five Standard Metropolitan Statistical Areas (SMSA). We treat mortality, deaths per 100,000 popylation, as the response variable. NOX, SO2 associated with three climate and socioeconomic factors are the explanatory variables.

We decide to use *Multi Linear Regression* to solve this problem. The problem states that two cities—Lancaster and York—have lower years of education because of their religion.Thus,we delete the data from Lancaster and York to make sure no other variables like religion will affect the analysis.

After data cleaning, we run R to get two best fitted models. To make the result more persuasive, we do other statistical tests to support our conclusions. Accordingly, all R codes are put in the appendix.

2 Methods and Models

2.1 Preparation

We firstly delete Lancaster and York, and directly run a multiple linear regression and get the summary which is listed below:

```
> summary(modelf)

Call:
lm(formula = Mort ~ Precip + Educ + NonWhite + NOX + SO2, data = dat1)

Residuals:
    Min      1Q  Median      3Q     Max 
-89.941 -16.161  -2.946  16.326  84.869 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1111.29221   93.32589  11.908  < 2e-16 ***
Precip         1.39448    0.64863   2.150 0.036237 *  
Educ         -24.36985    7.24155  -3.365 0.001443 ** 
NonWhite       2.69812    0.59901   4.504  3.8e-05 ***
NOX           -0.06373    0.12650  -0.504 0.616545    
SO2            0.31216    0.08569   3.643 0.000622 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 34.58 on 52 degrees of freedom
Multiple R-squared:  0.7146,    Adjusted R-squared:  0.6872 
F-statistic: 26.04 on 5 and 52 DF,  p-value: 4.584e-13
```

Then, we consider the multicollinearity. To diagnose, we perform Variance Inflation Factor(VIF), the result is listed below:

```
> vif(modelf)
   Precip      Educ  NonWhite       NOX       SO2
 2.049842  1.573586  1.368329  1.688255  1.452572
```

The largest VIF values of VIF_K is considerably much smaller than 10, we can assert that multicollinearity problem does not affect inference. In addition, the correlation matrix support our assertion.

```
> cor(dat1[,c(-1,-2)])
```

	Precip	Educ	NonWhite	NOX	SO2
Precip	1.00	−0.49	0.43	−0.48	−0.10
Educ	−0.49	1.00	−0.29	0.22	−0.27
NonWhite	0.43	−0.29	1.00	0.01	0.15
NOX	−0.48	0.22	0.01	1.00	0.41
SO2	−0.10	−0.27	0.15	0.41	1.00

2.2 ANOVA

We conduct an ANOVA table to see if the pollution variables have an effect on our response.

```
> anova(lm(Mort~Precip+Educ+NonWhite,data = dat1),lm(Mort~Precip+Educ+NonWhite+NOX+SO2,data = dat1))
Analysis of Variance Table

Model 1: Mort ~ Precip + Educ + NonWhite
Model 2: Mort ~ Precip + Educ + NonWhite + NOX + SO2
  Res.Df   RSS Df Sum of Sq      F   Pr(>F)   
1     54 79354                                
2     52 62190  2     17163 7.1755 0.00177 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

The result shows they are significant.

2.3 All Possible Subsets Methods

Because the number of variable is only 5, we can search all possible subset to choose the predictors. All possible subsets methods is performed and smallest BIC is the critrerion to pick the best model.

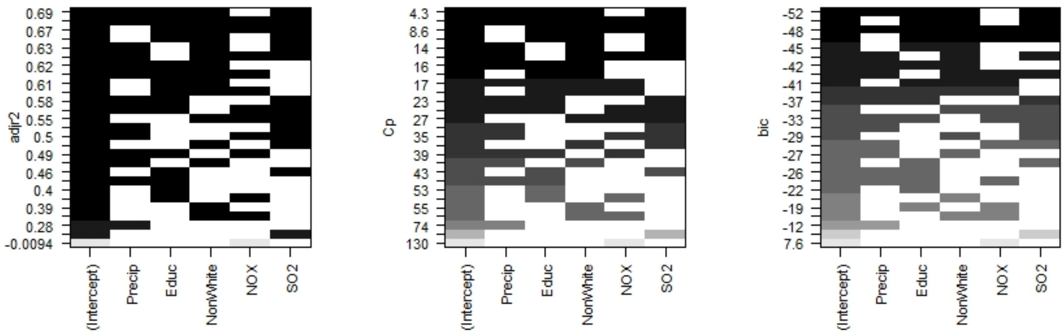
All the output is put into the appendix B. And we get our model:

```
> summary(model)

Call:
lm(formula = Mort ~ Precip + Educ + NonWhite + SO2, data = dat1)

Residuals:
    Min      1Q  Median      3Q     Max 
-92.188 -21.337  -2.957  16.031  85.155 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1112.77945   92.62020  12.014  < 2e-16 ***
Precip         1.53022    0.58586   2.612 0.011687 *  
Educ         -24.93874    7.10243  -3.511 0.000920 ***
NonWhite       2.63232    0.58047   4.535 3.33e-05 ***
SO2            0.29490    0.07799   3.781 0.000398 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 34.34 on 53 degrees of freedom
Multiple R-squared:  0.7132,    Adjusted R-squared:  0.6916 
F-statistic: 32.95 on 4 and 53 DF,  p-value: 8.391e-14
```

$$\text{Mort} \sim \text{Precip+Educ+NonWhite+SO2}$$

With the coefficients:

$$Mort = 1112.77945 + 1.53022 Precip - 24.93874 Educ + 2.63232 NonWhite + 0.29490 SO2$$



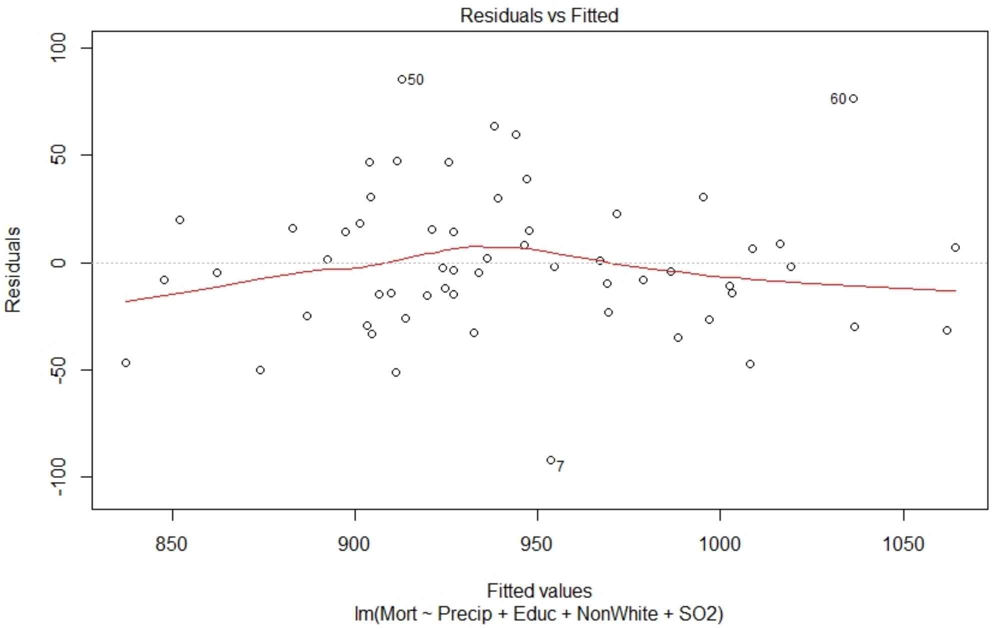
We get adjusted R^2=0.6916.

The image above shows the values of adjust R^2, C_p Criterion and BIC when different variables are chosen. It satisfied the model we proposed before.

2.4 Assumptions Check

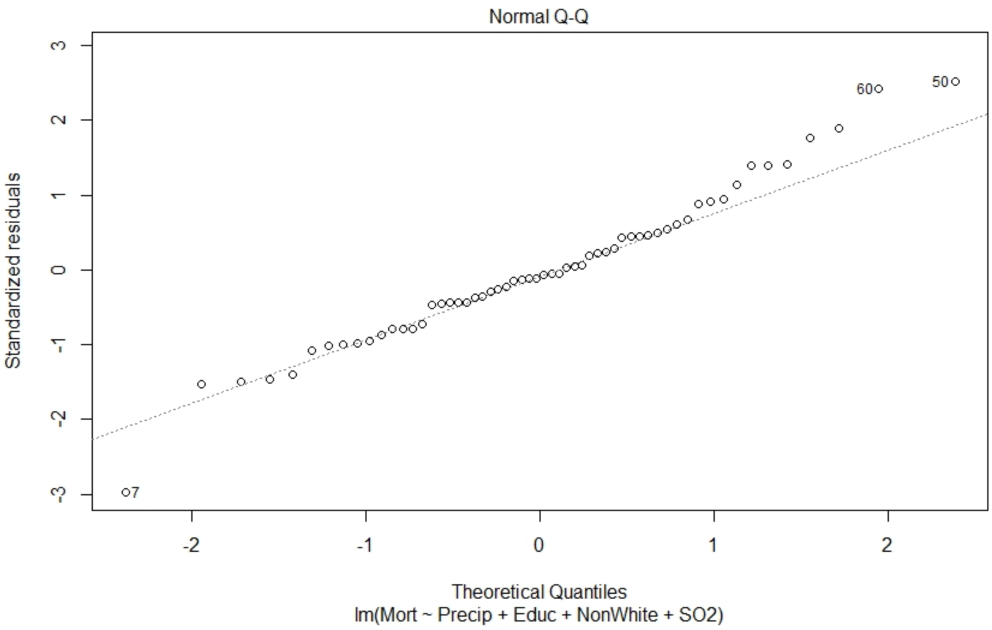
To use multi linear regression, we need to check several assumptions:

1. Equal Variance



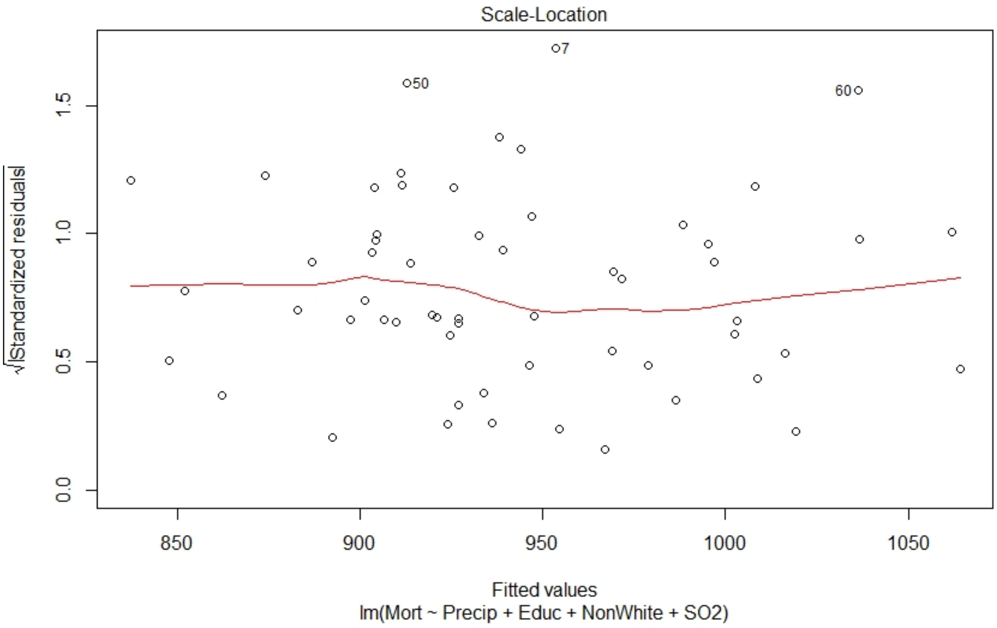
From the residuals vs fitted plot, we don't see the residuals getting larger or smaller. We may draw the conclusion that they are of equal variance.

2. Normality



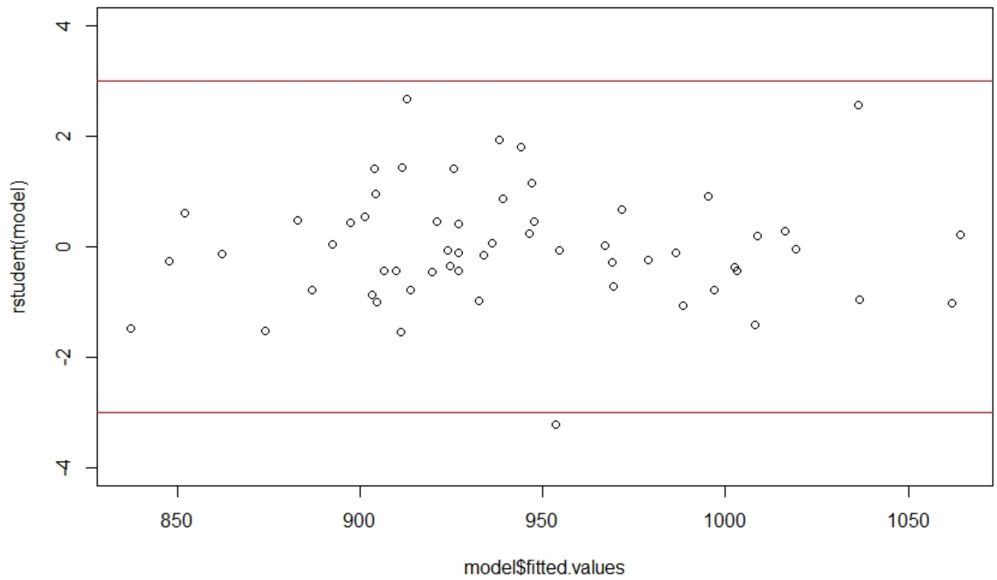
From the qq plot, we can see that normality assumption holds.

3. Outliers

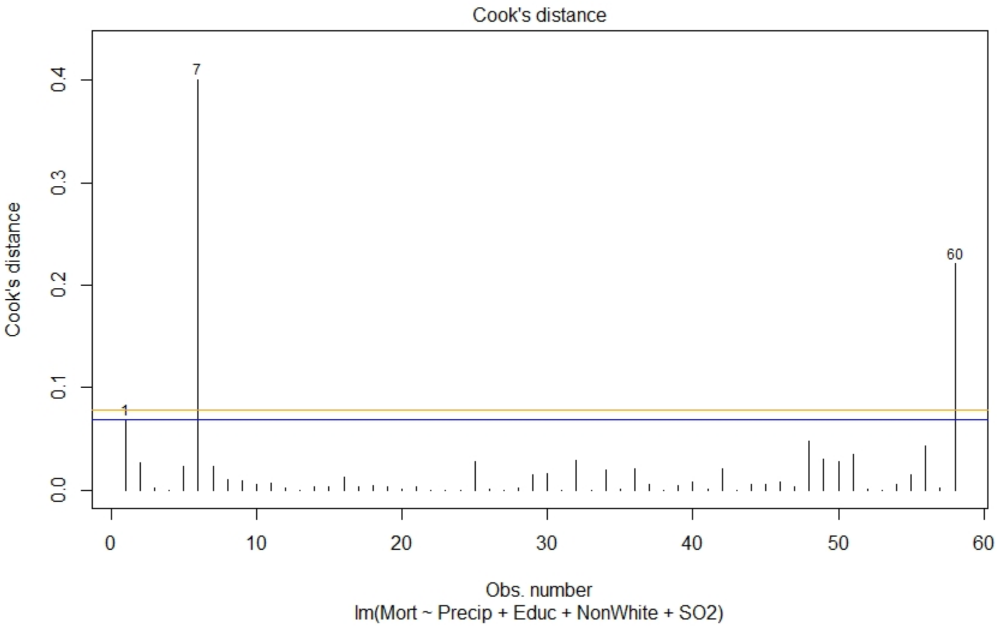


We can see from the standardized residual plot that 7(Miami) is an outlier because of the rule of thumb that $|r| > 3$

Also, we draw the studentized residual plot and see the same outlier.



4. Strong Influence Point
   We use the Cook's distance to identify Strong Influence Point.We use two rule of thumb $d > \frac{4}{n}$(the blue line) and $d > \frac{4}{n-p-2}$(the orange line). They all show 7(Miami) and 60(New orleans) are influential points.



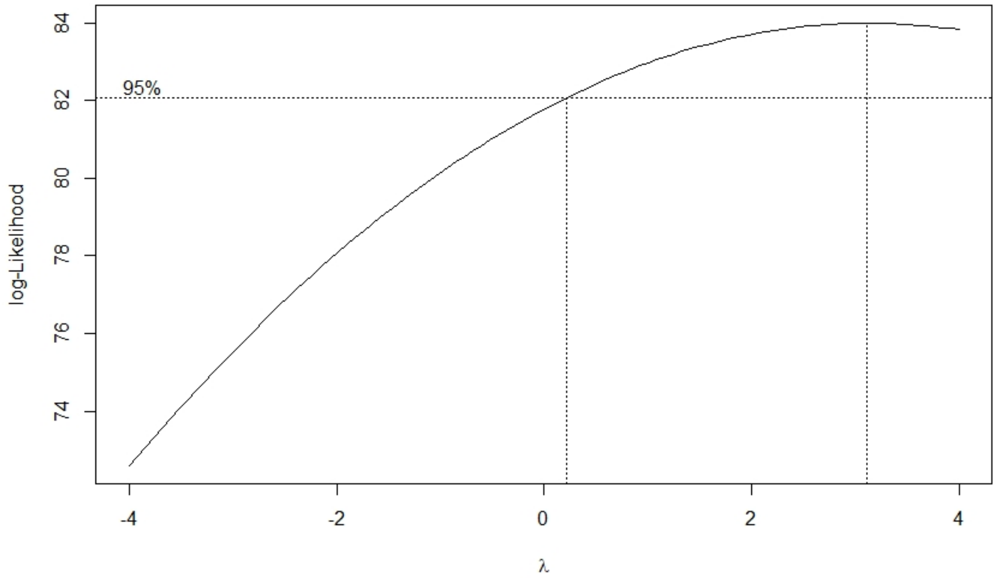
2.5 Remove the Outliers

We remove the outliers and strong influence point(7 and 60) and refit the model. From the summary, we can see that the R square and Adjusted R square have increased(from 0.71 to 0.75, 0.69 to 0.73 seperately), which show our linear model working better.

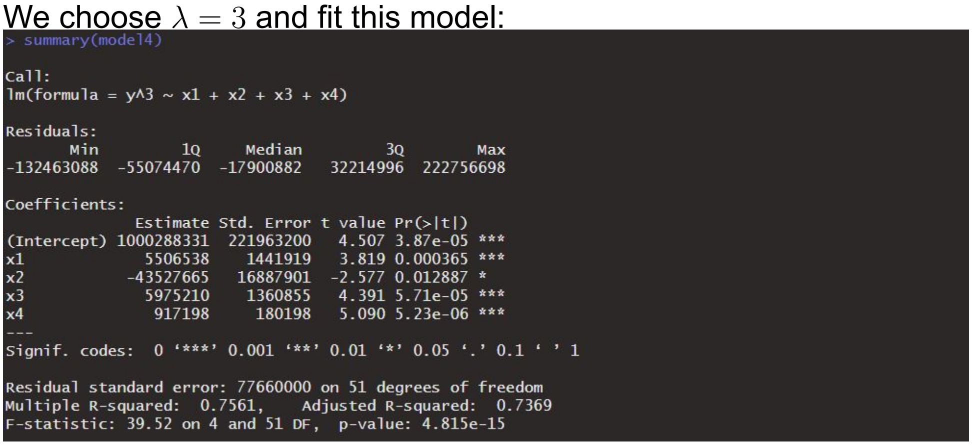
```
> summary(modelnew)

Call:
lm(formula = Mort ~ Precip + Educ + NonWhite + SO2, data = dat2)

Residuals:
    Min      1Q  Median      3Q     Max 
-52.836 -20.592  -4.882  14.660  82.805 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 992.98222   85.42710  11.624 5.98e-16 ***
Precip        2.28573    0.55495   4.119 0.000140 ***
Educ        -16.23669    6.49966  -2.498 0.015752 *  
NonWhite      2.09800    0.52375   4.006 0.000201 ***
SO2           0.34237    0.06935   4.937 8.91e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 29.89 on 51 degrees of freedom
Multiple R-squared:  0.7498,	Adjusted R-squared:  0.7302 
F-statistic: 38.22 on 4 and 51 DF,  p-value: 9.092e-15
```

2.6 Box-cox transformation

We use the boxcox transformation to fit another model. We can calculate that when $\lambda = 3.11$ the likelihood function gets its maximum.



We choose $\lambda = 3$ and fit this model:



```
> summary(model4)

Call:
lm(formula = y^3 ~ x1 + x2 + x3 + x4)

Residuals:
       Min         1Q     Median         3Q        Max 
-132463088  -55074470  -17900882   32214996  222756698 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1000288331  221963200   4.507 3.87e-05 ***
x1             5506538    1441919   3.819 0.000365 ***
x2           -43527665   16887901  -2.577 0.012887 *  
x3             5975210    1360855   4.391 5.71e-05 ***
x4              917198     180198   5.090 5.23e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 77660000 on 51 degrees of freedom
Multiple R-squared:  0.7561,	Adjusted R-squared:  0.7369 
F-statistic: 39.52 on 4 and 51 DF,  p-value: 4.815e-15
```

$$\text{Mort}^3 \sim \text{Precip+Educ+NonWhite+SO2}$$

With the coefficients:

$$Mort^3 = 1000288331 + 1000288331 Precip - 43527665 Educ + 5975210 NonWhite + 917198 SO2$$

We get adjusted R^2=0.7369, showing the model works well.

3 Results

Two best fitted models are

$$Mort = 1112.77945 + 1.53022 Precip - 24.93874 Educ + 2.63232 NonWhite + 0.29490 SO2$$

and

$$Mort^3 = 1000288331 + 1000288331 Precip - 43527665 Educ + 5975210 NonWhite + 917198 SO2$$

4 Limitations

1. The model is too simple and cannot reflect the reality very well.
2. The boxcox transformation doesn't produce a statisfying result.
3. While processing, we drop York and Lancaster at the beginning. If a proper method is used in our modeling without dropping these two observations, the result can be more accurate

5 Conclusions

We firstly perform an ANOVA table to show that pollution variables are significant in our model. Secondly, we search through all possible subset to get the predictors. Then, we check the assumptions and delete the outliers and strong influential point. Lastly, we make a boxcox transformation to get another model.

A Appendix R Code

```
dat=read.csv("Data_24h_project.csv")
dat
#delete 4 and 20 line due to the question(Lancaster
#and York)
dat1=dat[c(1:3,5:19,21:60), ]
modelf=lm(Mort~Precip+Educ+NonWhite+NOX+SO2
,data=dat1)
summary(modelf)
anova(lm(Mort~Precip+Educ+NonWhite,data = dat1),
lm(Mort~Precip+Educ+NonWhite+NOX+SO2,data = dat1))
#check for multicollinearity
require(car)
vif(modelf)
# result shows no ...

#model selection.
#due to there is only 5 varibles, we can get through
 #all subsets to find the best model

if (!require("leaps")) {
  install.packages("leaps")
  stopifnot(require("leaps"))
}

myleaps <- regsubsets(Mort~Precip+Educ+NonWhite+NOX+SO2
,data=dat1,nbest=8)
(myleaps.summary <- summary(myleaps)) # hard to interpret

# A better view:
bettertable <- cbind(myleaps.summary$which,
                     myleaps.summary$rsq,
                     myleaps.summary$rss,
                     myleaps.summary$adjr2,
                     myleaps.summary$cp,
                     myleaps.summary$bic)
dimnames(bettertable)[[2]] <- c(dimnames(
myleaps.summary$which)[[2]],"rsq", "rss", "adjr2",
 "cp", "bic")
show(bettertable)
#we use the smallest BIC to pick the best model:Mort~
#Precip+Educ+NonWhite+SO2

par(mfrow=c(1,3), pty="s")
plot(myleaps, scale = "adjr2")
plot(myleaps, scale = "Cp")
plot(myleaps, scale = "bic")
```

```
#all shows the same result.

#outliers ,normality ,equal variance
model=lm(Mort~Precip+Educ+NonWhite+SO2,data = dat1)
summary(model)
par(mfrow=c(2,2))
plot(model,which = 1:4)

#the first plot shows equal variance.
#the second plot shows normality assumption holds
#the third plot shows an outlier 7(Miami)(because r>3)
# check studentized residuals
plot(model$fitted.values, rstudent(model))
plot(model$fitted.values, rstudent(model), ylim=c(-4,4))
abline(h=c(-3,3), col="red") # rule of thumb

p=5
n=58
plot(model, which = 4)
abline(h=qf(0.5, p, n-p), col="green")
abline(h=4/n, col="blue")
abline(h=4/(n-p-1-1), col="orange")
#7(Miami) and 60(New orleans) are influential points.

dat2=dat1[dat1$City!="Miami, FL" & dat1$City!=
"New Orleans, LA", ]
modelnew=lm(Mort~Precip+Educ+NonWhite+SO2,
data = dat2)
summary(modelnew)
summary(model)

library(MASS)
boxcox(modelnew,seq(-4,4,1/10))
y=dat2$Mort
x1=dat2$Precip
x2=dat2$Educ
x3=dat2$NonWhite
x4=dat2$SO2
bc=boxcox(y~x1+x2+x3+x4,lambda = seq(-4,4,1/10))
(lambda <- bc$x[which.max(bc$y)])

model4=lm(y^3~x1+x2+x3+x4)
summary(model4)
```

B Appendix Output of R

```
> myleaps <- regsubsets(Mort~Precip+Educ+NonWhite+
NOX+SO2, data=dat1,nbest=8)
```

```
>(myleaps.summary <- summary(myleaps)) # hard to interpret
Subset selection object
Call: regsubsets.formula(Mort ~ Precip + Educ + NonWhite +
NOX + SO2, data = dat1, nbest = 8)
5 Variables  (and intercept)
         Forced in Forced out
Precip       FALSE      FALSE
Educ         FALSE      FALSE
NonWhite     FALSE      FALSE
NOX          FALSE      FALSE
SO2          FALSE      FALSE
8 subsets of each size up to 5
Selection Algorithm: exhaustive
         Precip Educ NonWhite NOX SO2
1  ( 1 ) " "    "*"  " "      " " " "
1  ( 2 ) " "    " "  "*"      " " " "
1  ( 3 ) "*"    " "  " "      " " " "
1  ( 4 ) " "    " "  " "      " " "*"
1  ( 5 ) " "    " "  " "      "*" " "
2  ( 1 ) " "    "*"  "*"      " " " "
2  ( 2 ) "*"    " "  " "      " " "*"
2  ( 3 ) " "    " "  "*"      " " "*"
2  ( 4 ) "*"    " "  "*"      " " " "
2  ( 5 ) " "    "*"  " "      " " "*"
2  ( 6 ) "*"    "*"  " "      " " " "
2  ( 7 ) " "    "*"  " "      "*" " "
2  ( 8 ) " "    " "  "*"      "*" " "
3  ( 1 ) " "    "*"  "*"      " " "*"
3  ( 2 ) "*"    " "  "*"      " " "*"
3  ( 3 ) "*"    "*"  "*"      " " " "
3  ( 4 ) " "    "*"  "*"      "*" " "
3  ( 5 ) "*"    "*"  " "      " " "*"
3  ( 6 ) " "    " "  "*"      "*" "*"
3  ( 7 ) "*"    " "  " "      "*" "*"
3  ( 8 ) "*"    "*"  " "      "*" " "
4  ( 1 ) "*"    "*"  "*"      " " "*"
4  ( 2 ) " "    "*"  "*"      "*" "*"
4  ( 3 ) "*"    " "  "*"      "*" "*"
4  ( 4 ) "*"    "*"  "*"      "*" " "
4  ( 5 ) "*"    "*"  " "      "*" "*"
5  ( 1 ) "*"    "*"  "*"      "*" "*"
> # A better view:
>bettertable <- cbind(myleaps.summary$which,
                      myleaps.summary$rsq,
                      myleaps.summary$rss,
                      myleaps.summary$adjr2,
                      myleaps.summary$cp,
                      myleaps.summary$bic)
```

```
>dimnames(bettertable)[[2]] <- c(dimnames(
myleaps.summary$which)[[2]],"rsq", "rss", "adjr2",
 "cp", "bic")
> show(bettertable)
  (Intercept) Precip Educ NonWhite          bic
1           1      0    1        0  0  0 -22.487738
1           1      0    0        1  0  0 -21.650853
1           1      1    0        0  0  0 -12.183221
1           1      0    0        0  0  1  -3.514529
1           1      0    0        0  1  0   7.634815
2           1      0    1        1  0  0 -44.662119
2           1      1    0        0  0  1 -31.707652
2           1      0    0        1  0  1 -29.425281
2           1      1    0        1  0  0 -26.760868
2           1      0    1        0  0  1 -25.706699
2           1      1    1        0  0  0 -25.539000
2           1      0    1        0  1  0 -18.670697
2           1      0    0        1  1  0 -18.500585
3           1      0    1        1  0  1 -49.180295
3           1      1    0        1  0  1 -44.072533
3           1      1    1        1  0  0 -42.349958
3           1      0    1        1  1  0 -40.628517
3           1      1    1        0  0  1 -37.186604
3           1      0    0        1  1  1 -33.739374
3           1      1    0        0  1  1 -27.648555
3           1      1    1        0  1  0 -25.402038
4           1      1    1        1  0  1 -52.142636
4           1      0    1        1  1  1 -47.486132
4           1      1    0        1  1  1 -40.996805
4           1      1    1        1  1  0 -39.242149
4           1      1    1        0  1  1 -33.318620
5           1      1    1        1  1  1 -48.364581
```