Problem 1 Using $\hat{\phi}_1 = \frac{r_1(1-r_2)}{1-r_1^2}$ and $\hat{\phi}_2 = \frac{r_2-r_1^2}{1-r_1^2}$, we have $\hat{\phi}_1 = 1.11$ and $\hat{\phi}_2 = -0.389$.

Using $\theta_0 = \mu(1-\phi_1-\phi_2)$, we have $\hat{\theta_0} = 0.558$.

Using $\hat{\sigma_e^2} = (1-\hat{\phi}_1 r_1 - \hat{\phi}_2 r_2)s^2$, we have $\hat{\sigma_e^2} = 1.5325$.

Problem 2

(a)

$$Var(\hat{\phi}) \approx [\frac{1-\phi^2}{n}][\frac{1-\phi\theta}{\phi-\theta}]^2 = 3.75$$

$$Var(\hat{\theta}) \approx [\frac{1-\theta^2}{n}][\frac{1-\phi\theta}{\phi-\theta}]^2 = 3.99$$

$$Corr(\hat{\phi},\hat{\theta}) \approx \frac{\sqrt{(1-\phi^2)(1-\theta^2)}}{1-\phi\theta} = 0.998$$

The estimates are highly correlated. The estimates are not significantly different from 0.

(b)

$$Var(\hat{\phi}) \approx [\frac{1-\phi^2}{n}][\frac{1-\phi\theta}{\phi-\theta}]^2 = 1.50$$

$$Var(\hat{\theta}) \approx [\frac{1-\theta^2}{n}][\frac{1-\phi\theta}{\phi-\theta}]^2 = 1.60$$

$$Corr(\hat{\phi},\hat{\theta}) \approx \frac{\sqrt{(1-\phi^2)(1-\theta^2)}}{1-\phi\theta} = 0.998$$

The situation did not inprove. The estimates are still highly correlated.

Problem 3

(a)

```
> arima(series,order = c(1,0,0))

Call:
arima(x = series, order = c(1, 0, 0))
```

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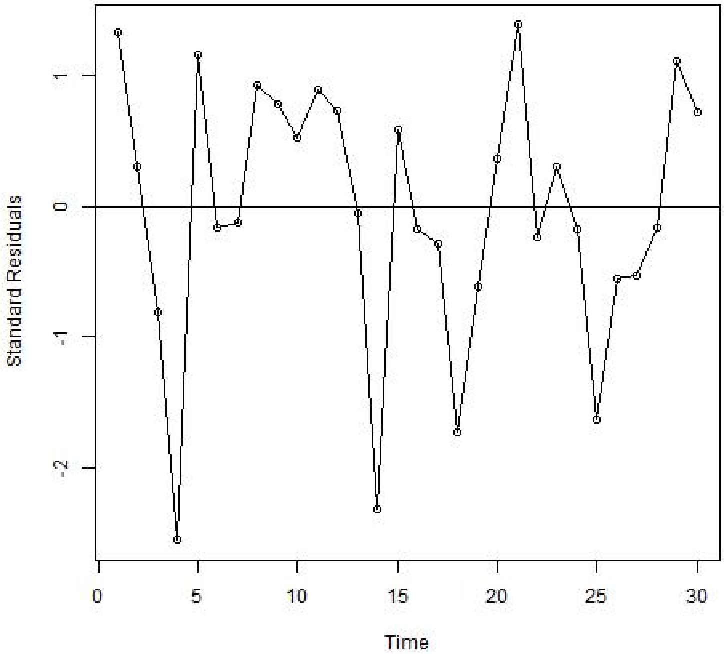
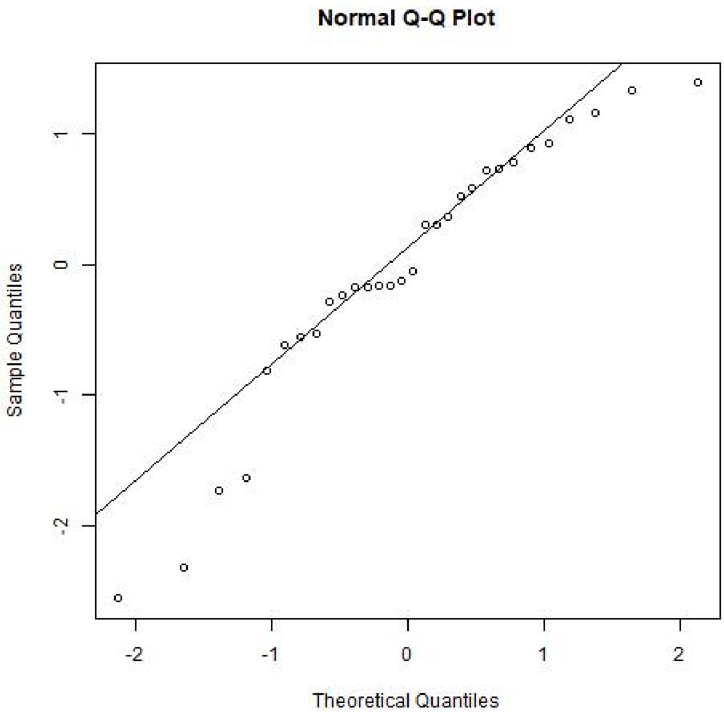
```
Coefficients:
         ar1   intercept
      0.5110      0.2076
s.e.  0.1619      0.3311

sigma^2 estimated as 0.822:  log likelihood = -39.78,  aic = 83.5
```



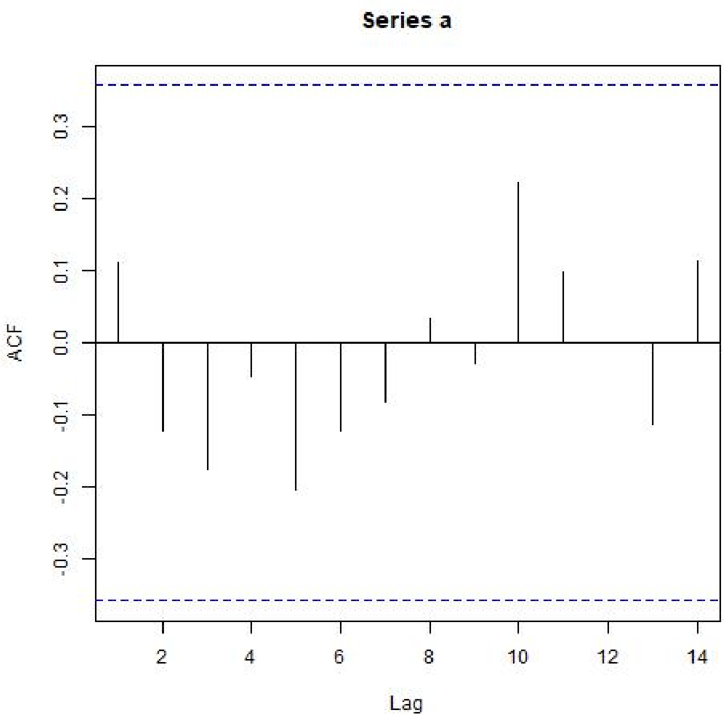
These standardized residuals look fairly ”random” with no particular patterns.

(b)



With a few outliers, the qq plot of the standardized resiuals looks reasonably "normal".

(c)



The graph shows no signnificant correlation in residuals.

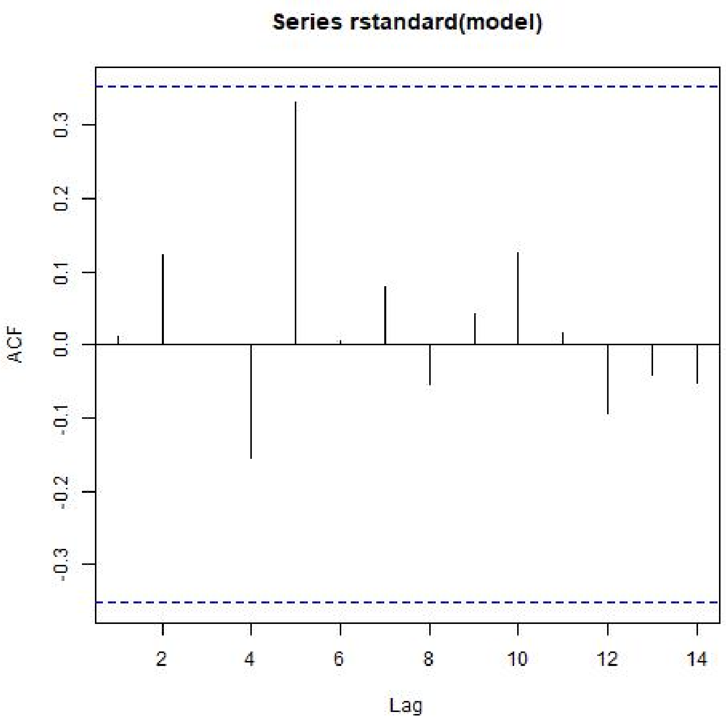
(d)

```
> LB.test(model, lag = 8)

        Box-Ljung test

data:  residuals from  model
X-squared = 4.621, df = 7, p-value = 0.7061
```

The test does not reject randomness of the error terms based on the first eight autocorrelations of the residuals.

Problem 4

(a)



The graph shows no signnificant correlation in residuals.

(b)

```
> LB.test(model, lag = 9)

        Box-Ljung test

data:  residuals from model
X-squared = 6.2475, df = 6, p-value = 0.396
```

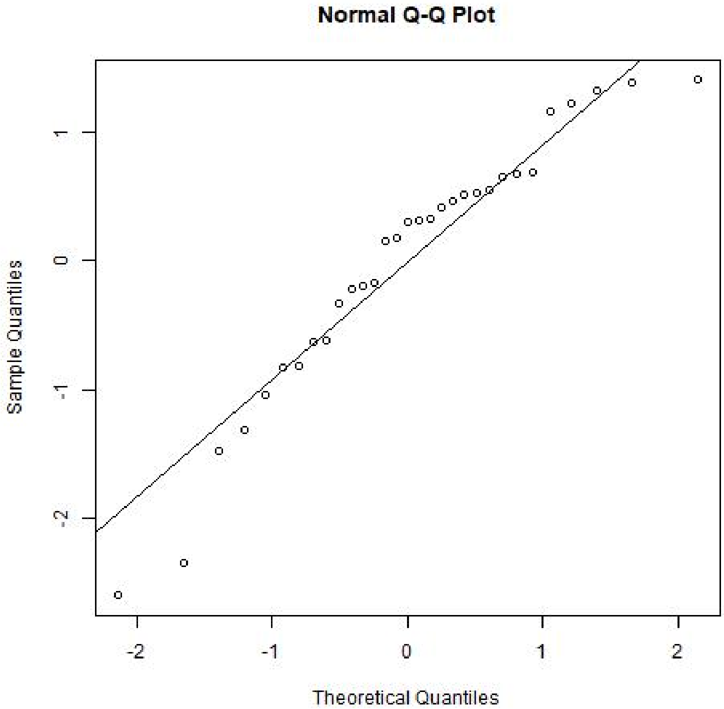
The test does not reject independence of the error terms.

(c)

```
> runs(rstandard(model))
$pvalue
[1] 0.602

$observed.runs
[1] 18

$expected.runs
[1] 16.09677

$n1
[1] 13

$n2
[1] 18

$k
[1] 0
```

The test does not reject independence of the error terms.

(d)



With a few outliers, the qq plot of the standardized resiuals looks reasonably "normal".

(e)

```
> shapiro.test(residuals(model))

        Shapiro-Wilk normality test

data:  residuals(model)
W = 0.93509, p-value = 0.06043
```

We will not reject the normality of the error terms.