Homework3

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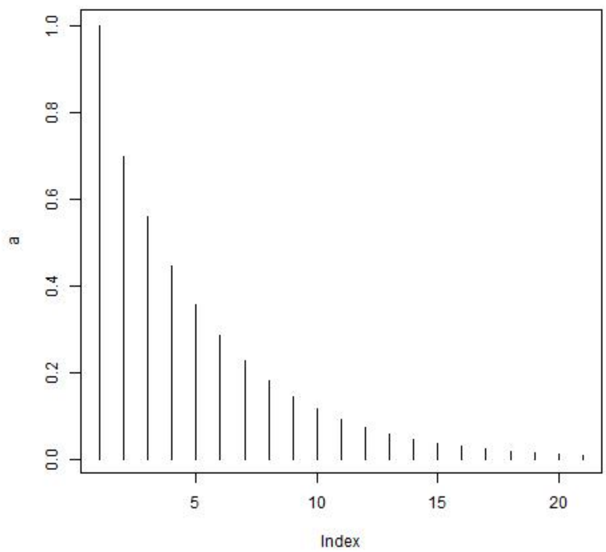
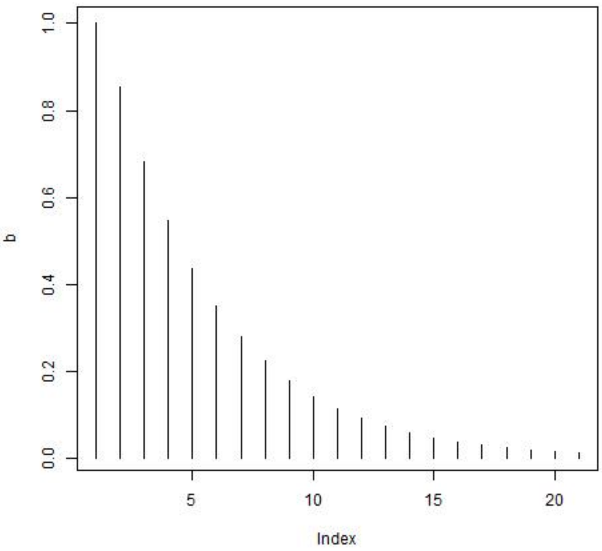
Problem 1

(a)

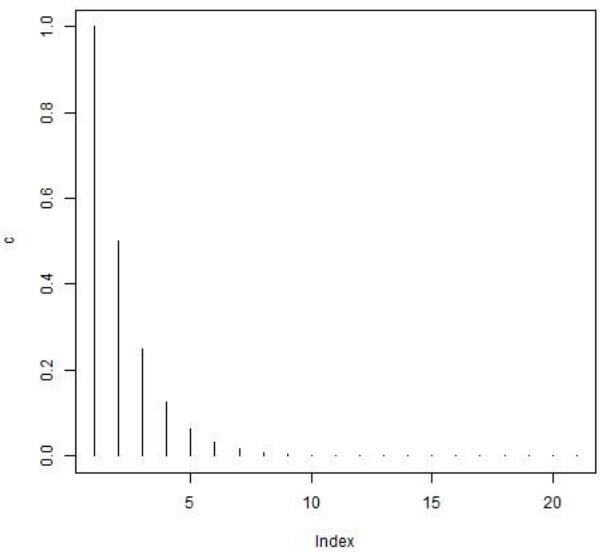


(b)


1.0
0.8
0.6
0.4
0.2
0.0
b
5
10
15
20
Index

Problem 2

(a)


1.0
0.8
0.6
0.4
0.2
0.0
c
5
10
15
20
Index

(b)

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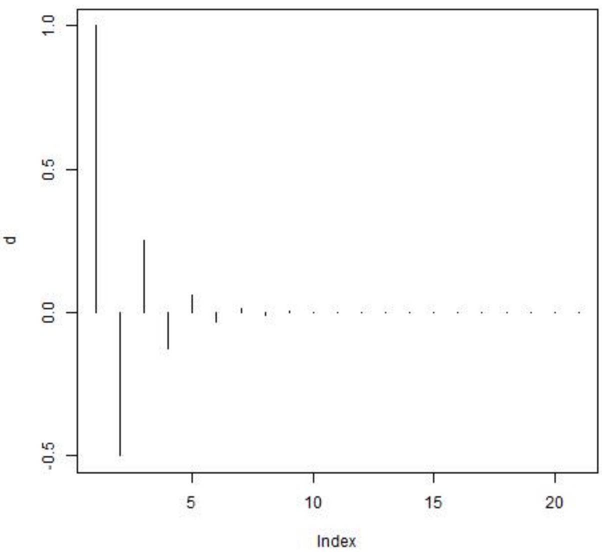
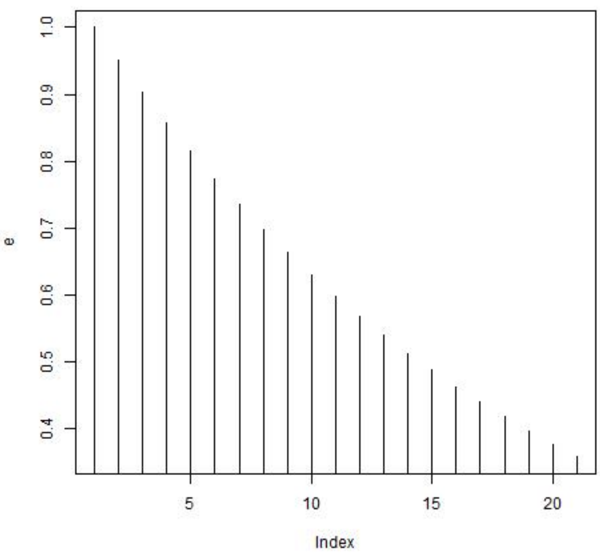
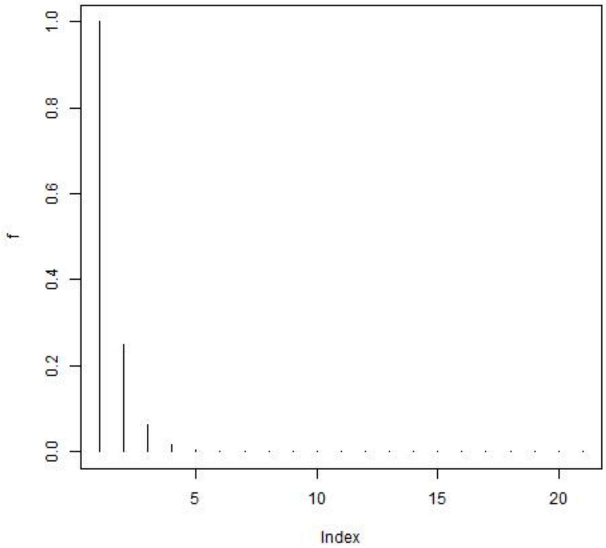


(c)



(d)



Problem 3 $b_t = Y_t - \phi Y_{t-i} + 1 = e_t + 1$

(a)

$$\begin{aligned} Cov(b_t, b_{t-k}) &= Cov(e_t + 1, e_{t-k} + 1) \\ &= 0 \end{aligned} \tag{1}$$

for all t and $k \neq 0$

(b)

$$\begin{aligned} Cov(b_t, Y_{t-k}) &= Cov(e_t + 1, \phi Y_{t-k-1} + e_{t-k}) \\ &= Cov(e_t, \phi Y_{t-k-1}) + Cov(e_t, e_{t-k}) \\ &= 0 \end{aligned} \tag{2}$$

for all t and $k > 0$

Problem 4 Since Y_t is an AR(1) process with $1 < \phi < 1$, we can rewrite W_t as

$$W_t = (\phi - 1)Y_{t-1} + e_t$$

We consider (b) first.

(b) When $k = 0$,

$$\begin{aligned} Cov(W_t, W_{t-k}) &= Var(W_t) \\ &= Var((\phi - 1)Y_{t-1} + e_t) \\ &= (\phi - 1)^2 \frac{\sigma_e^2}{1 - \phi^2} + \sigma_e^2 \\ &= \frac{2\sigma_e^2}{1 + \phi} \end{aligned} \tag{3}$$

(a) When $k > 0$

$$\begin{aligned} Cov(W_t, W_{t-k}) &= Cov((\phi - 1)Y_{t-1} + e_t, (\phi - 1)Y_{t-k-1} + e_{t-k}) \\ &= Cov((\phi - 1)Y_{t-1}, (\phi - 1)Y_{t-k-1}) + Cov((\phi - 1)Y_{t-1}, e_{t-k}) \\ &= (\phi - 1)^2 Cov(Y_{t-1}, Y_{t-k-1}) + (\phi - 1) Cov(Y_{t-1}, e_{t-k}) \\ &= (\phi - 1)^2 \phi^k \frac{\sigma_e^2}{1 - \phi^2} + (\phi - 1)\phi^{k-1}\sigma_e^2 \\ &= \frac{\phi - 1}{1 + \phi} \phi^{k-1} \sigma_e^2 \end{aligned} \tag{4}$$

Problem 5

$$\begin{aligned} Var(Y_t) &= Var(3 + e_t + 2e_{t-1} - \frac{1}{2}e_{t-2}) = [1 + 2^2 + (\frac{1}{2})^2]\sigma_e^2 \\ &= \frac{21}{4}\sigma_e^2 \\ Cov(Y_t, Y_{t-1}) &= Cov(e_t + 2e_{t-1} - \frac{1}{2}e_{t-2}, e_{t-1} + 2e_{t-2} - \frac{1}{2}e_{t-3}) \\ &= Cov(2e_{t-1}, e_{t-1}) + Cov(-\frac{1}{2}e_{t-2}, 2e_{t-2}) \\ &= \sigma_e^2 \\ Cov(Y_t, Y_{t-2}) &= Cov(e_t + 2e_{t-1} - \frac{1}{2}e_{t-2}, e_{t-2} + 2e_{t-3} - \frac{1}{2}e_{t-4}) \\ &= Cov(-\frac{1}{2}e_{t-2}, e_{t-2}) \\ &= -\frac{1}{2}\sigma_e^2 \\ Cov(Y_t, Y_{t-3}) &= Cov(e_t + 2e_{t-1} - \frac{1}{2}e_{t-2}, e_{t-3} + 2e_{t-4} - \frac{1}{2}e_{t-5}) \\ &= 0 \end{aligned} \tag{5}$$

and this persists for all lags 3 or more.

Therefore,

$$\rho_k = \begin{cases} 1, & k = 0, \\ \frac{4}{21}, & k = 1, \\ -\frac{2}{21}, & k = 2, \\ 0, & k > 2 \end{cases}$$