

Homework 4

Yuanyou Yao

April 22, 2021

Problem 1

Firstly, we need to sort the data in ascending order, thus we have:

Verbal	Student	Socre	D_i^2
1	9	2	1
2	5	1	1
3	7	3	0
4	2	4	0
5	6	7.5	6.25
6	11	7.5	2.25
7	12	10	9
8	14	5	9
9	4	6	9
10	1	12.5	6.25
11	8	9	4
12	13	11	1
13	3	14	1
14	15	12.5	2.25
15	10	15	0

where the null hypothesis H_0 is No association between verbal and general scores.

Now after some simple calculations, we have $\sum D_i^2 = 52, n = 15$ and Spearman rank correlation is $R = 1 - \frac{6 \times 52}{15 \times (225 - 1)} = 0.91$.

The p-value therefore is smaller than 0.001. So it is safe for us to reject the null hypothesis.

Problem 2

Firstly, we need to sort the data according to students' points, thus we have:

Test	1	2	3	4	5	6	7	8
Quantitative	1	4	7	6	5	3	8	2
Vocabulary	8	2	3	4	7	5	1	6
Reading	8	2	4	5	3	7	1	6
Total	17	9	13	15	15	15	10	14

Using concordance test, we have $\sum R^2 = 1510, S = 52, W = 0.138$

The test statistic is

$$Q = 2.89,$$

where degrees of freedom is 7.

The corresponding p-value is greater than 0.5, so we reject the null hypothesis. In other words, there is no association between scores.

To check if there is an indirect relationship between quantitative and verbal abilities, we can sum up the scores of vocabulary and reading. Below is the sum and the rank.

Scores	122	172	177	157	160	151	188	152
Ranks	8	3	2	5	4	7	1	6

A Spearman rank correlation coefficient can be applied:

$$R = -0.88, p = 0.004.$$

Therefore a strong negative relationship is found between verbal ability and quantitative ability.

Problem 3

Since $\sum \omega_j(t) = 1$, we have:

$$Y_{(1)} = \sum \omega_j(t) Y_{(1)} \leq \sum \omega_j(t) Y_{(j)} = \hat{m}(t) \leq \sum \omega_j(t) Y_{(n)} = Y_{(n)}.$$