The 4th Assignment

March 4, 2020

Problem 1. Identify as specific **ARIMA**(p, d, q) models, that is, what are p, d, and q and what are the values of the parameters—the ϕ's and θ's?

(a) $Y_t = 0.8Y_{t-1} - 0.3Y_{t-2} + e_t - 0.1e_{t-1}$

(b) $Y_t = 1.9Y_{t-1} - 0.9Y_{t-2} + e_t$

(c) $Y_t = 0.4Y_{t-1} - 0.4Y_{t-2} + e_t - 0.1e_{t-1} + 0.05e_{t-2}$

Problem 2. Suppose $Y_t = \beta_0 + \beta_1 t + X_t$ where X_t is a random walk. First suppose β_0 and β_1 are constants.

(a) Is $\{Y_t\}$ stationary?

(b) Is $\{\nabla Y_t := Y_t - Y_{t-1}\}$ stationary?

Now suppose β_0 and β_1 are random variables that are independent of each other and the random walk $\{X_t\}$.

(c) Is $\{Y_t\}$ stationary?

(d) Is $\{\nabla Y_t\}$ stationary?

Problem 3. The data file *winnebago* contains monthly unit sales of recreational vehicles (RVs) from Winnebago, Inc. from November 1966 through February 1972.

(a) Display and interpret the time series plot for these data.

(b) Now take natural logarithms of the monthly sales figures and display the time series plot of the transformed values. Describe the effect of the logarithms on the behavior of the series.

(c) Calculate the fractional relative changes, $(Y_t - Y_{t-1})/Y_{t-1}$, and compare them to the differences of (natural) logarithms, $\nabla \log(Y_t) = \log(Y_t) - \log(Y_{t-1})$. How do they compare for smaller values and for larger values?

Problem 4. The file named *gold* contains the daily price of gold (in dollars per troy ounce) for the 252 trading days of year 2005.

(a) Display the time series plot of these data. Interpret the plot.

(b) Display the time series plot of the differences of the logarithms of these data. Interpret this plot.

(c) Calculate and display the sample ACF for the differences of the logarithms of these data and argue that the logarithms appear to follow a random walk model.

(d) Display the differences of logs in a histogram and interpret.

(e) Display the differences of logs in a quantile-quantile normal plot and interpret.

Problem 5. From a time series of 100 observations, we calculate $r_1 = -0.39$, $r_2 = 0.41$, $r_3 = -0.21$, $r_4 = 0.11$, and $|r_k| < 0.09$ for $k > 4$. On this basis alone, what **ARIMA** model would we tentatively specify for the series?

Problem 6. A stationary time series of length 121 produced sample partial autocorrelation of $\hat{\phi}_{11} = 0.7$, $\hat{\phi}_{22} = -0.74$, $\hat{\phi}_{33} = 0.075$, and $\hat{\phi}_4 = 0.00$. Based on this information alone, what model would we tentatively specify for the series?

Problem 7. From a series of length 169, we find that $r_1 = 0.44$, $r_2 = 0.36$, $r_3 = 0.28$, $r_4 = 0.22$, and $r_5| = 0.17$. What **ARIMA** model fits this pattern of autocorrelations?

Problem 8. The sample ACF for a series and its first difference are given in the following table. Here $n = 100$.

lag	1	2	3	4	5	6
ACF for Y_t	0.97	0.97	0.93	0.85	0.80	0.71
ACF for ∇Y_t	–0.42	0.18	–0.02	0.07	–0.10	–0.09

Based on this information alone, which **ARIMA** model(s) would we consider for the series?

Problem 9. Suppose the $\{X_t\}$ is a stationary $\mathbf{AR}(1)$ process with parameter ϕ but that we can only observe $Y_t = X_t + N_t$ where $\{N_t\}$ is the white noise measurement error independent of $\{X_t\}$.

(a) Find the autocorrelation function for the observed process in terms of ϕ, σ_X^2 and σ_N^2.

(b) Which **ARIMA** model might we specify for $\{Y_t\}$?