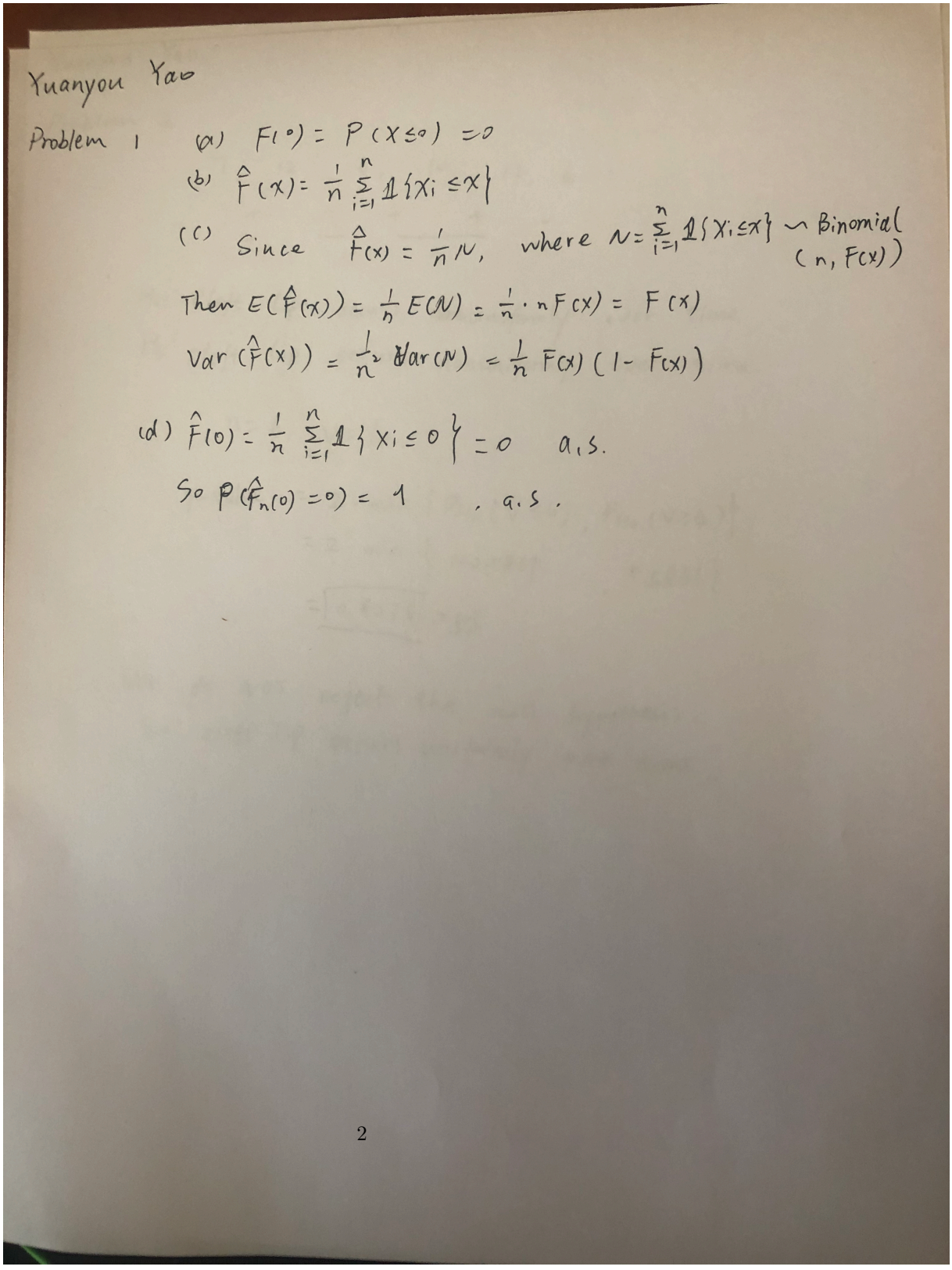
1 Problem 1



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Problem 1

(a) $F(0) = P(X \le 0) = 0$

(b) $\hat{F}(x) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}\{X_i \le x\}$

(c) Since $\hat{F}(x) = \frac{1}{n}N$, where $N = \sum_{i=1}^{n} \mathbb{1}\{X_i \le x\} \sim$ Binomial$(n, F(x))$

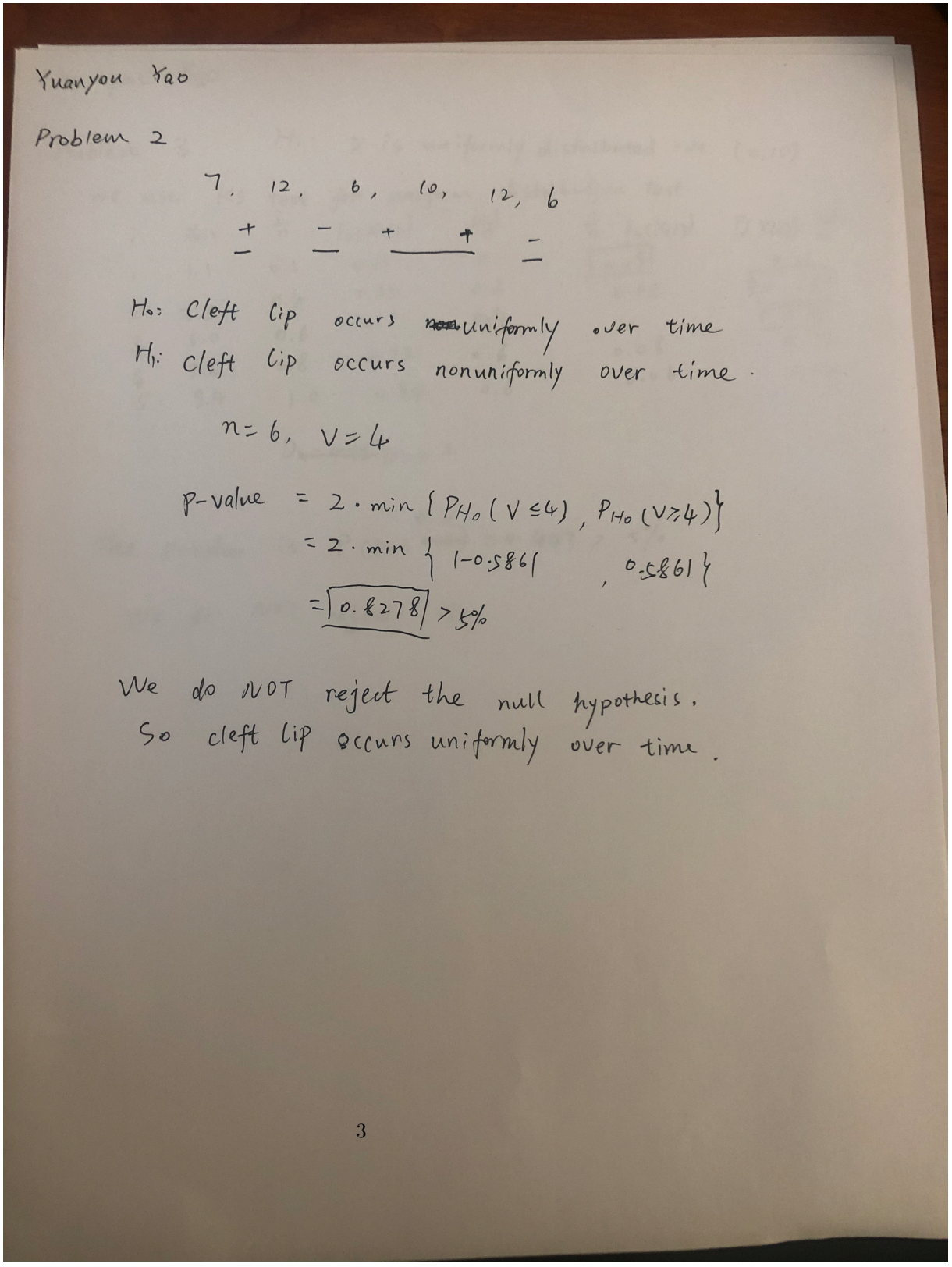
Then $E(\hat{F}(x)) = \frac{1}{n}E(N) = \frac{1}{n} \cdot nF(x) = F(x)$

$Var(\hat{F}(x)) = \frac{1}{n^2}Var(N) = \frac{1}{n}F(x)(1 - F(x))$

(d) $\hat{F}(0) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}\{X_i \le 0\} = 0$ a.s.

So $P(\hat{F}_n(0) = 0) = 1$. a.s.

2 Problem 2



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Problem 2

7, 12, 6, 10, 12, 6

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H_0: Cleft lip occurs uniformly over time

H_1: Cleft lip occurs nonuniformly over time.

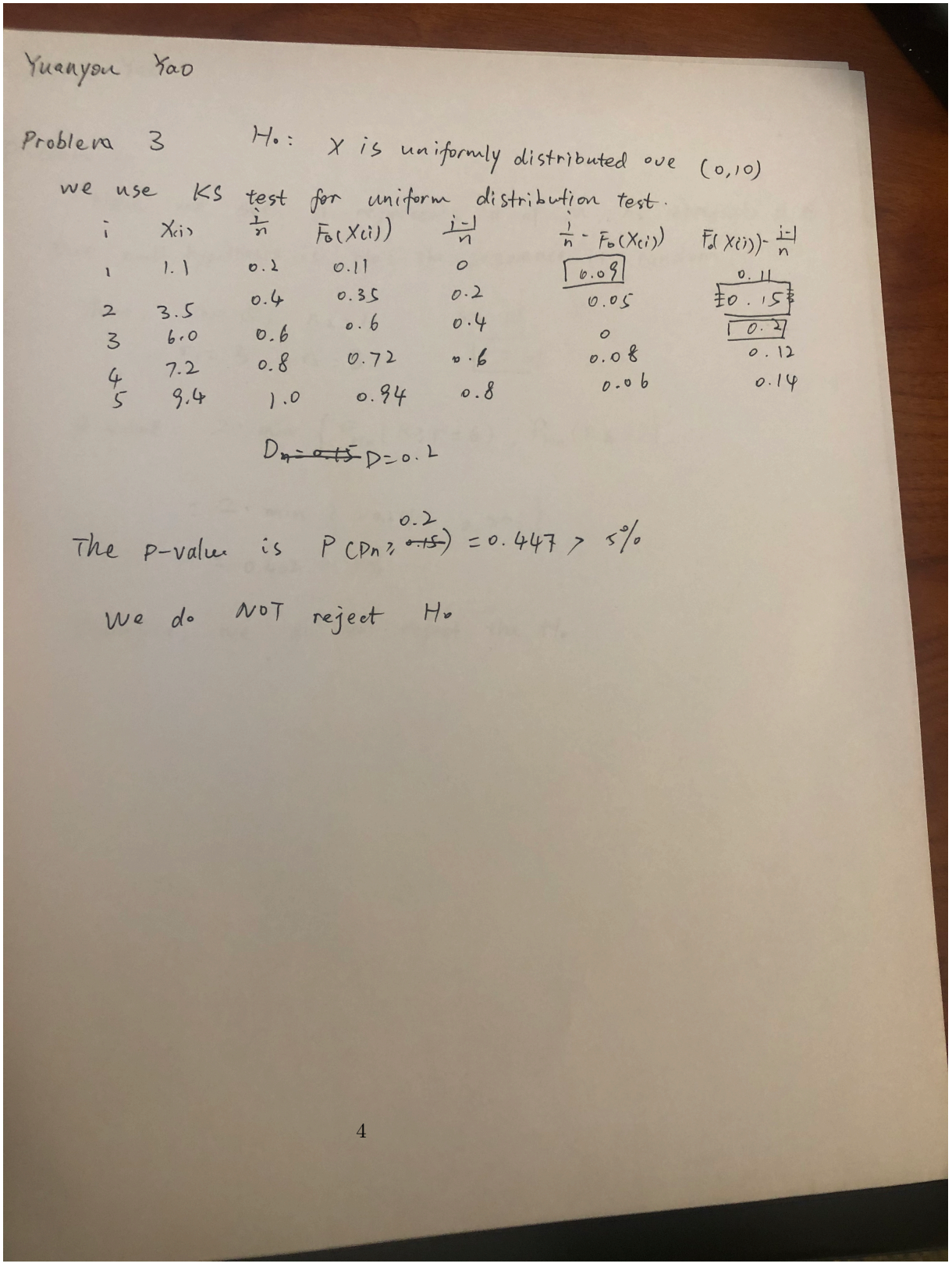
$n = 6, \quad V = 4$

$$\text{p-value} = 2 \cdot \min\{P_{H_0}(V \le 4), P_{H_0}(V \ge 4)\}$$
$$= 2 \cdot \min\{1 - 0.5861, 0.5861\}$$
$$= \boxed{0.8278} > 5\%$$

We do NOT reject the null hypothesis.

So cleft lip occurs uniformly over time.

3 Problem 3



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Problem 3 H_0: X is uniformly distributed ove (0,10)

we use KS test for uniform distribution test.

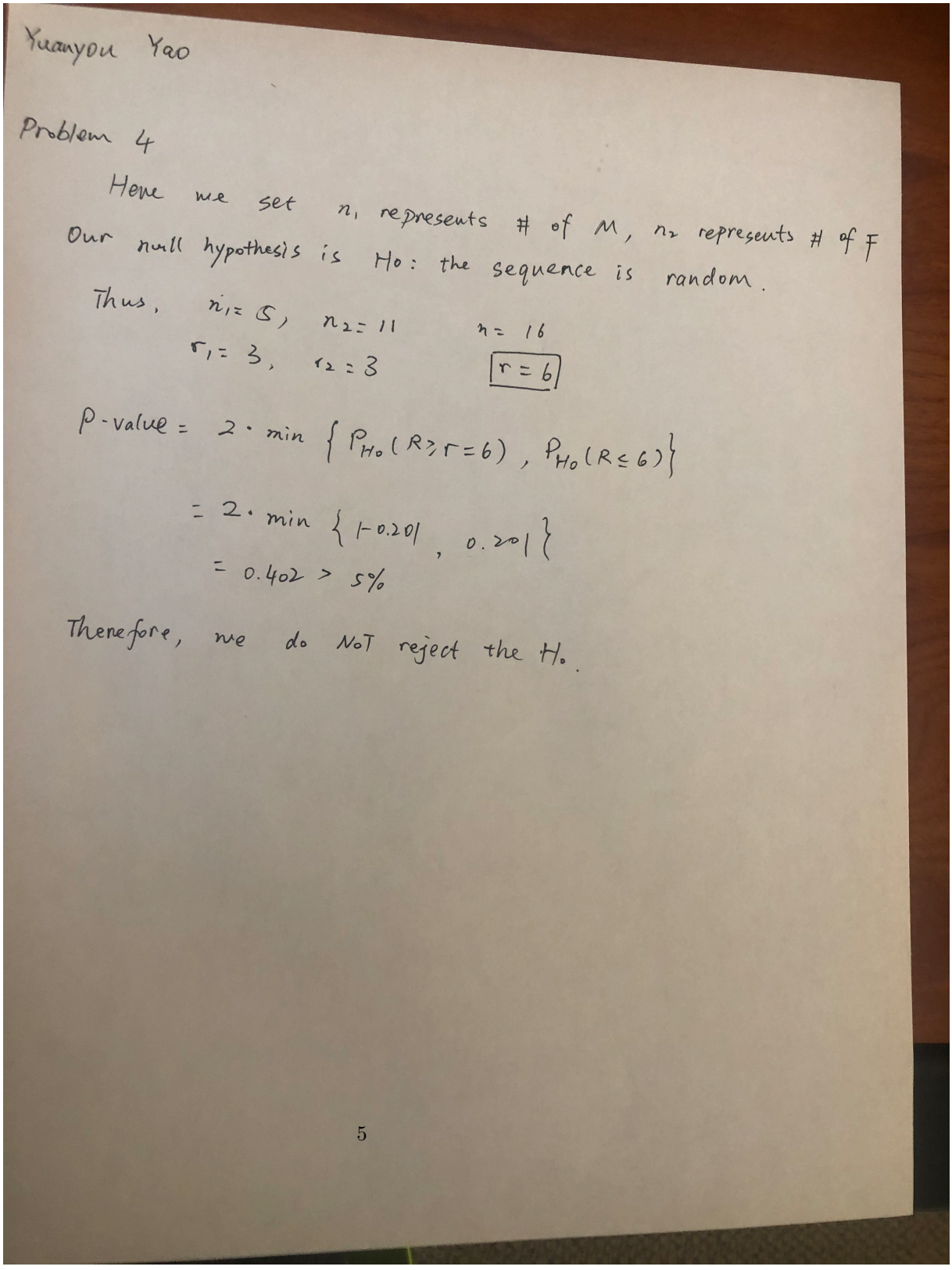
i	$X_{(i)}$	$\frac{i}{n}$	$F_0(X_{(i)})$	$\frac{i-1}{n}$	$\frac{i}{n} - F_0(X_{(i)})$	$F_0(X_{(i)}) - \frac{i-1}{n}$
1	1.1	0.2	0.11	0	0.09	0.11
2	3.5	0.4	0.35	0.2	0.05	0.15
3	6.0	0.6	0.6	0.4	0	0.2
4	7.2	0.8	0.72	0.6	0.08	0.12
5	9.4	1.0	0.94	0.8	0.06	0.14

~~$D_n = 0.15$~~ $D = 0.2$

The p-value is $P(D_n \geq 0.2) = 0.447 > 5\%$

We do NOT reject H_0

4 Problem 4



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Problem 4

Here we set n_1 represents # of M, n_2 represents # of F

Our null hypothesis is H_0: the sequence is random.

Thus, $n_1 = 5$, $n_2 = 11$ $\quad n = 16$

$r_1 = 3$, $r_2 = 3$ $\quad \boxed{r = 6}$

$$p\text{-value} = 2 \cdot \min \left\{ P_{H_0}(R \geq r = 6),\ P_{H_0}(R \leq 6) \right\}$$

$$= 2 \cdot \min \left\{ |-0.201|,\ |0.201| \right\}$$

$$= 0.402 > 5\%$$

Therefore, we do NOT reject the H_0.