# 6th Assignment

Zhengjun Zhang
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**Problem 1.** From a series of length 100, we have computed $r_1 = 0.8$, $r_2 = 0.5$, $r_3 = 0.4$, $\bar{Y} = 2$, and a sample variance of 5. If we assume that an **AR**(2) model with a constant term is appropriate, how can we get (simple) estimates of $\phi_1$, $\phi_2$, $\beta_0$, and $\sigma_e^2$?

**Problem 2.** Consider an **ARMA**$(1,1)$ model with $\phi = 0.5$ and $\theta = 0.45$. Recall that for maimum likelihood estimators of **ARMA**$(1,1)$

$$Var(\hat{\phi}) \approx \Big[\frac{1-\phi^2}{n}\Big]\Big[\frac{1-\phi\theta}{\phi-\theta}\Big]^2,$$
$$Var(\hat{\theta}) \approx \Big[\frac{1-\theta^2}{n}\Big]\Big[\frac{1-\phi\theta}{\phi-\theta}\Big]^2,$$
$$Corr(\hat{\phi},\hat{\theta}) \approx \frac{\sqrt{(1-\phi^2)(1-\theta^2)}}{1-\phi\theta}.$$

(a) For n = 48, evaluate the variances and correlation of the maximum likelihood estimators of $\phi$ and $\theta$ using the above equations and comment on the results.

(b) Repeat part (a) but now with $n = 120$. Comment on the new results

**Problem 3.** Simulate an **AR**(1) model with $n = 30$ and $\phi = 0.5$.

(a) Fit the correctly specified **AR**(1) model and look at a time series plot of the residuals. Does the plot support the **AR**(1) specification?

(b) Display a normal quantile-quantile plot of the standardized residuals. Does the plot support the **AR**(1) specification?

(c) Display the sample ACF of the residuals. Does the plot support the **AR**(1) specification?

(d) Calculate the Ljung-Box statistic summing to $K = 8$. Does this statistic support the **AR**(1) specification?

**Problem 4.** Fit an **AR**(3) model by maximum likelihood to the square root of the hare abundance series (filename *hare*).

(a) Plot the sample ACF of the residuals. Comment on the size of the correlations.

(b) Calculate the Ljung-Box statistic summing to $K = 9$. Does this statistic support the **AR**(3) specification?

(c) Perform a runs test on the residuals and comment on the results.

(d) Display the quantile-quantile normal plot of the residuals. Comment on the plot.

(e) Perform the Shapiro-Wilk test of normality on the residuals.