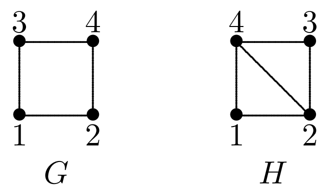
CS1231 TUTORIAL 11(Last Tutorial)

1. Let $n(a)$ denote the number of '1' bits in a. If a and b are adjacent, then a is obtained from by changing a '1' bit to '0' or a '0' bit to '1'. Thus $n(a) - n(b) = \pm 1$. Therefore $n(a)$ and $n(b)$ are of opposite parity., i.e., one a even and the other odd. Let V_i be the set of vertices with $n(a)$ even and V_2 be the set of vertices with $n(a)$ odd. Then any two vertices in V_1 are nonadjacent. The same goes for V_2. Thus V_1, V_2 form a bipartition and therefore Q_n is bipartite.

2. Two vertices are adjacent if their bitstrings differ in exactly one position. Since two bitstrings can differ in at most n positions, the simple connecting them is of length $\leq n$.

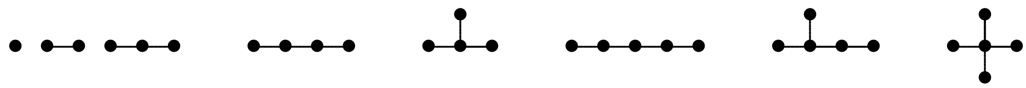
3.



4. 1. Just add a new edge joining 2 to 4.

5. Only a is a tree.

6.



7. No. A tree with 10 vertices has 9 edges and thus total degree is 18.

8. (a) a, (b) a, b, d, e, g, h, i, o (c) $c, f, j, k, l, m, n, p, q, r$ (d) e, f, g, (e) e, (f) i, d, a (g) h, i, n, o, p, q, r (h) 4.

9. If the height is h, then there are at most 2^h leaves. Hence the answers are: (a) ≥ 5, (b) ≥ 6, (c) ≥ 6

10. Basis step: $i = 0$. This is the tree with a single vertex which is a leave.

Inductive step: Suppose that a full binary tree with j internal vertices has $j+1$ leaves, for $j = 0, \ldots, k$ where $k \geq 0$. Consider a full binary tree T with $k+1$ internal vertices and ℓ leaves. We want to prove that $\ell = k+2$.

The root a of T is an internal vertex. Consider its children b, c. Let T_b be the subtree at b. Denote the number of internal vertices and leaves of T_b by i_b and ℓ_b, respectively. Note that the internal vertices and leaves of T_b are still internal vertices and leaves of T, respectively. Likewise for T_c. Thus $k+1 = i_b + i_c + 1$ and $ib \leq k$ and $i_c \leq k$.

By the induction hypothesis, $i_b+1 = \ell_b$, $i_c+1 = \ell_c$. Thus, $k+1 = i_b+i_c+1 = \ell_b+\ell_c-1 = \ell - 1$ or $\ell = k+2$.

Alternative proof of the inductive step: Let T be as above. Suppose the height of T is h. Consider a leave u at level h. Then its sibling v is also a leave. Let w be their parent. Let T' be the tree obtained by deleting u, v. Then T' is still a full binary tree but w is now a leave. Thus T' has k internal vertices and $\ell - 1$ leaves. By the induction hypothesis, T' has $k+1$ leaves. Thus $\ell - 1 = k+1$ or $\ell = k+2$ as required.