CS1231 TUTORIAL 2

1. (a) If an animal is a rabbit, then it hops. (Alt: Every rabbit hops.)

(b) Every animal is a rabbit and hops.

(c) There is an animal such that if it is a rabbit, then it hops.

(d) There is an animal that is a rabbit and hops. (Alt: Some rabbits hop.)

2. a. Hypothesis true, conclusion false, Hypo: true, Con: true

b. $\{(1,1),(1,2),(1,-2),(2,1),(2,2),(2,-2),(-2,-2)\}$

3. (a), (b) $\forall n \in \mathbb{Z}$, if n^2 is even then n is even. (c), (d) $\forall n \in \mathbb{Z}$, if n is even then n^2 is even.

4. T, T, F(counter example $x = -3$), T

5. (a) $\forall x(D(x) \rightarrow \neg W(x))$. (b) $\forall x(O(x) \rightarrow W(x))$. (c) $\forall x(P(x) \rightarrow D(x))$. (d) $\forall x(P(x) \rightarrow \neg O(x))$. (e) Yes. Since $P(x) \rightarrow D(x) \rightarrow \neg W(x) \rightarrow \neg O(x)$.

Note: Many gave $\forall x(D(x) \wedge \neg W(x))$ as the answer for (a). This is true when there are only ducks in the domain and all of them don't waltz. But (a) is true as long as ducks don't waltz, whether or not there are other things in the domain.

6. (a) $\exists d \in \mathbb{Z}$ s.t. $\frac{6}{d} \in \mathbb{Z}$ and $d \neq 3$. (b) There is an even integer whose square is odd. Or: $\exists n \in \mathbb{Z}$ s.t. n^2 is odd and n is even.

7. (a) Some animal is able to fly but not a bird.You can also write: It is not true that an animal which is able to fly is also a bird.

(b) There is a function which is a polynomial but has no real roots.

8. $\forall x \in D$, $\exists y \in E$ such that $x + y \neq -y$. The negation is true.

9. (a) $\exists r \in \mathbb{Q}$ such that $\forall a \in \mathbb{Z}$, and $\forall b \in \mathbb{Z}$, $r \neq a/b$. (b) $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}$ such that $x + y \neq 0$. (c) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p) \equiv (\neg p \vee q) \wedge (\neg q \vee p)$. Thus its negation is $(p \wedge \neg q) \vee (q \wedge \neg p)$.

10. $S = (p \wedge \neg q) \vee (\neg p \wedge q)$ or $S = (p \vee q) \wedge (\neg p \vee \neg q)$