CS1231 TUTORIAL 4

1. Determine whether these are true or false.

(a) $\emptyset \in \{\emptyset\}$. (b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$. (c) $\{\emptyset\} \in \{\emptyset\}$. (d) $\{\emptyset\} \in \{\{\emptyset\}\}$.

(e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$. (f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$. (g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$.

2. Let $B = \{n \in \mathbb{Z} : n = 3j + 2, j \in \mathbb{Z}\}$, $D = \{n \in \mathbb{Z} : n = 3j - 1, j \in \mathbb{Z}\}$. Is $B = D$?

3. Find $|A|$ if $A = \{1, 2, \{2\}, \{4, 5\}, 5, 5\}$.

4. Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, $C = \{b, c, e\}$. Find $(A - B) - C$ and $A - (B - C)$. Are they equal?

5. Let T_P denote the truth set of the predicate $P(x)$. Prove the following:

(a) $T_{P \vee Q} = T_P \cup T_Q$, $T_{P \wedge Q} = T_P \cap T_Q$,

(b) $T_{P \to Q} = \overline{T_P} \cup T_Q$.

6. Let $A = \{1, 2, 3\}$, $B = \{u, v\}$, $C = \{m, n\}$. List the elements of $(A \times B) \times C$ and $A \times B \times C$. Are the two cartesian products equal?

7. Find the mistake in the following "proof".

Theorem: For all sets A and B, $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$.

Proof Suppose A and B are sets, and $x \in \overline{A} \cup \overline{B}$. Then $x \in \overline{A}$ or $x \in \overline{B}$. It follows that $x \notin A$ or $x \notin B$ and so $x \notin A \cup B$. Thus $x \in \overline{A \cup B}$ and hence $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$.

8. Prve that $(A - C) \cap (B - C) \cap (A - B) = \emptyset$

9. Prove that for all sets A, B, C, D,

$$\text{if} \quad A \cap C = \emptyset, \quad \text{then} \quad (A \times B) \cap (C \times D) = \emptyset.$$

10. For each of the following, either prove the statement true or find a counterexample to prove that it is false.

(a) If $B \cap C \subseteq A$, then $(A - B) \cap (A - C) = \emptyset$.

(b) $(A - B) \cap (C - B) = A - (B \cup C)$.

(c) If $\overline{A} \subseteq B$, then $A \cup B = U$.

(d) $P(A \cap B) = P(A) \cap P(B)$.

11. Define the **SYMMETRIC DIFFERENCE** as $A \oplus B = (A - B) \cup (B - A)$.

(a) Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$. Find $(A \oplus B) \oplus C$.

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.