NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
MA1101R Linear Algebra I

2018-2019 (Semester 1) **Tutorial 3**

1. For each of the following matrix $\boldsymbol{A}$ below, use elementary row operations to determine if $\boldsymbol{A}$ is invertible. If it is, find $\boldsymbol{A}^{-1}$.

(a) $\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{pmatrix}$; (b) $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{pmatrix}$; (c) $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

2. Suppose $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{X}$ are square matrices of order n such that $\boldsymbol{A}$, $\boldsymbol{X}$ and $\boldsymbol{A} - \boldsymbol{AX}$ are invertible. Suppose

$$(\boldsymbol{A} - \boldsymbol{AX})^{-1} = \boldsymbol{X}^{-1}\boldsymbol{B} \tag{*}$$

(i) Is $\boldsymbol{B}$ invertible? Justify your answer.

(ii) Solve $(*)$ for $\boldsymbol{X}$. In your working, if you need to invert a matrix, make sure the matrix is invertible.

3. For each of the following, solve for $\boldsymbol{X}$.

(a)

$$\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 3 & 2 \end{pmatrix} \boldsymbol{X} = \begin{pmatrix} 2 & 3 & 4 & 1 \\ 1 & 0 & 3 & 7 \\ 2 & 1 & 1 & 2 \end{pmatrix}.$$

(b)

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{pmatrix} \boldsymbol{X} = \begin{pmatrix} 6 & 1 \\ 10 & 1 \\ 12 & 1 \end{pmatrix}.$$

4. Let $\boldsymbol{A} = \begin{pmatrix} 0 & 1 & -3 \\ -2 & 5 & 4 \\ -1 & 2 & 3 \end{pmatrix}$.

(i) Find a sequence of elementary matrices $\boldsymbol{E_1}, \boldsymbol{E_2}, \cdots, \boldsymbol{E_k}$ such that $\boldsymbol{E_k}\boldsymbol{E_{k-1}} \cdots \boldsymbol{E_1}\boldsymbol{A}$ is the reduced row-echelon form of $\boldsymbol{A}$.

(ii) Is $\boldsymbol{A}$ invertible? If so, express $\boldsymbol{A}$ as a product of elementary matrices. If not, explain why.

5. Compute the determinant of the following matrices by cofactor expansion.

(a) $\begin{pmatrix} 5 & 2 & 0 \\ 0 & -2 & 5 \\ 0 & 0 & 4 \end{pmatrix}$; (b) $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 3 & 1 \\ 4 & 5 & -2 \end{pmatrix}$; (c) $\begin{pmatrix} 4 & 2 & 2 & 0 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 2 \\ 0 & 1 & 1 & 4 \end{pmatrix}$.