6.4 Grade distribution An instructor always assigns final grades such that 20% are A, 40% are B, 30% are C, and 10% are D. The grade point scores are 4 for A, 3 for B, 2 for C, and 1 for D.

a. Specify the probability distribution for the grade point score of a randomly selected student of this instructor.

b. Find the mean of this probability distribution. Interpret it.

a. Let X = grade point score.

X	P(X)	X•P(X)
4	0.20	0.80
3	0.40	1.20
2	0.30	0.60
1	0.10	0.10

b. Sum of X•P(X) is the mean. It is 2.70.
The average grade point score for the class is 2.70.

6.38 Passing by guessing A quiz in a statistics course has four multiple-choice questions, each with five possible answers. A passing grade is three or more correct answers to the four questions. Allison has not studied for the quiz. She has no idea of the correct answer to any of the questions and decides to guess at random for each.

a. Find the probability she lucks out and answers all four questions correctly.

b. Find the probability that she passes the quiz.

a. Let X = number of questions answered correctly.
$X\sim$ Binomial (n = 4, p = 0.20)
$P(X=4) = (0.2)^4 = 0.0016$

b. P(Pass the quiz) $= P(X\geq 3)$
$= P(X=3) + P(X=4)$
$= 4(0.2)^3(0.8)^1 + 0.0016$
$= 0.0272$

6.42 Exit poll An exit poll is taken of 3000 voters in a statewide election. Let X denote the number who voted in favor of a special proposition designed to lower property taxes and raise the sales tax. Suppose that in the population, exactly 50% voted for it.

TRY

a. Explain why this scenario would seem to satisfy the three conditions needed to use the binomial distribution. Identify n and p for the binomial.

b. Find the mean and standard deviation of the probability distribution of X.

c. Using the normal distribution approximation, give an interval in which you would expect X almost certainly to fall, if truly $p = 0.50$. (*Hint*: You can follow the reasoning of Example 14 on racial profiling.)

d. Now, suppose that the exit poll had $x = 1706$. What would this suggest to you about the actual value of p?

a. Let X = number of people who voted in favor of the special proposition.
(1) Data is binary (voted or not).
(2) Same probability of success (p =0.50)
(3) Independent trials

n = 3000, p =0.50 ➔ $X\sim$ Binomial (n = 3000, p = 0.50)

b. Mean of X = np = (3000)(0.5) = 1500;
Standard deviation of $X = \sqrt{[np(1-p)]} = \sqrt{(3000)(0.5)(0.5)} = \sqrt{750} = 27.39$

c. The interval of three standard deviations of the mean
= 1500 ± 27.39 = (1418, 1582).

d. The actual value of p is <u>higher</u> than 0.50.

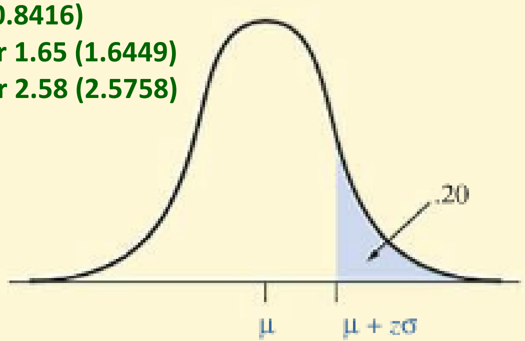
6.20 ***z*-score for right-tail probability**

a. For the normal distribution shown below, find the z-score.

b. Find the value of z (rounding to two decimal places) for right-tail probabilities of (i) 0.05 and (ii) 0.005.

a. z = 0.84 (0.8416)

b. (i) 1.64 or 1.65 (1.6449)
(ii) 2.57 or 2.58 (2.5758)



6.27 MDI TRY The Mental Development Index (MDI) of the Bayley Scales of Infant Development is a standardized measure used in observing infants over time. It is approximately normal with a mean of 100 and a standard deviation of 16.

a. What proportion of children has an MDI of (i) at least 120? (ii) at least 80?

b. Find the MDI score that is the 99th percentile.

c. Find the MDI score such that only 1% of the population has MDI below it.

6.28 Quartiles and outliers Refer to the previous exercise.

a. Find the z-score corresponding to the lower quartile (Q1) of a normal distribution.

b. Find and interpret the lower quartile and upper quartile of the MDI.

c. Find the interquartile range (IQR) of MDI scores.

d. Section 2.5 defined an observation to be a potential outlier if it is more than 1.5 × IQR below Q1 or above Q3. Find the intervals of MDI scores that would be considered potential outliers.

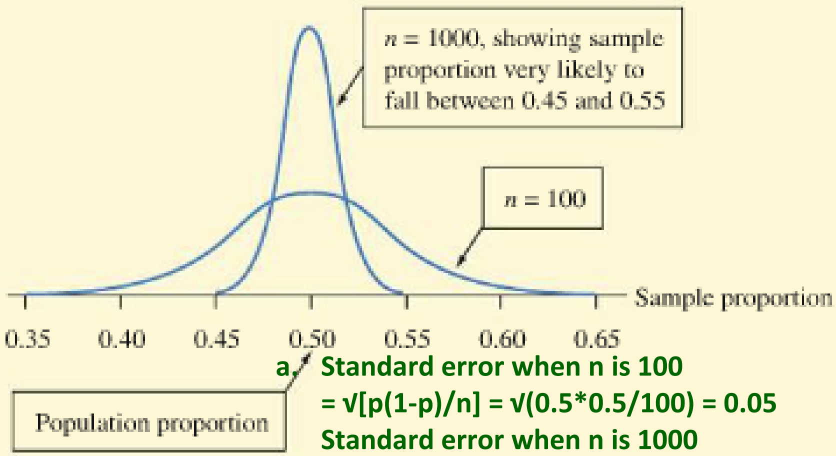
a. Q1 for Z is -0.6745.

b. Q1 = μ +Zσ = 100 + (-0.6745)(16) = 89.2
Q1 = μ +Zσ = 100 + (0.6745)(16) = 110.8
25% of MDI is below 89.2 while 25% of MDI is above 110.8

c. IQR = 110.8 – 89.2 = 21.6

d. Q1-1.5IQR = 89.2 – 1.5(21.6) = 56.8
Q3+1.5IQR = 110.8 + 1.5(21.6) = 143.2
MDI scores lower than 56.8 or higher than 143.2 would be considered as outliers.

7.10 Effect of n on sample proportion The figure illustrates two sampling distributions for sample proportions when the population proportion $p = 0.50$.



a. Standard error when n is 100
$= \sqrt{[p(1-p)/n]} = \sqrt{(0.5*0.5/100)} = 0.05$
Standard error when n is 1000
$= \sqrt{[p(1-p)/n]} = \sqrt{(0.5*0.5/1000)} = 0.016$

a. Find the standard deviation for the sampling distribution of the sample proportion with (i) $n = 100$ and (ii) $n = 1000$.

b. Explain why the sample proportion would be very likely (as the figure suggests) to fall (i) between 0.35 and 0.65 when $n = 100$, and (ii) between 0.45 and 0.55 when $n = 1000$. (*Hint:* Recall that for an approximately normal distribution, nearly the entire distribution is within 3 standard deviations of the mean.)

c. Explain how the results in part b indicate that the sample proportion tends to more precisely estimate the population proportion when the sample size is larger.

b. The sample proportion is likely to fall within 3 standard errors of the mean. In (i) mean = 0.5, SE= 0.05, this would be between 0.35 to 0.65. In (ii) mean = 0.5, SE = 0.016, this would be between 0.45 to 0.55.

c. SE is smaller when n is larger (A larger sample is more accurate than a smaller sample). Thus the interval of 3 SE of the mean from a larger sample will be shorter (more precise estimate) than a smaller sample.

7.12 Gender distributions At a university, 60% of the 7,400 students are female. The student newspaper reports results of a survey of a random sample of 50 students about various topics involving alcohol abuse, such as participation in binge drinking. They report that their sample contained 26 females.

a. Explain how you can set up a binary random variable X to represent gender.

b. Identify the population distribution of gender at this university. Sketch a graph.

c. Identify the data distribution of gender for this sample. Sketch a graph.

d. Identify the sampling distribution of the sample proportion of females in the sample. State its mean and standard deviation for a random sample of size 50. Sketch a graph.

a. Let X = 1 if a female is selected, X = 0 if a male is selected.
b. Population distribution: P(female) = P(1) = 0.60, P(male) = P(0) = 0.40
c. Sample Data distribution: P(1) = 26/50 = 0.52, P(0) = 0.48
d. Sampling distribution of the sample proportion of females, P(1) is an approximately normal distribution. The mean = P(1) = 0.60, the standard deviation$=\sqrt{[p(1-p)/n]}=\sqrt{(0.60*0.40/50)} = \sqrt{0.0048} = 0.06928$