NATIONAL UNIVERSITY OF SINGAPORE

CS1231 - DISCRETE STRUCTURES

(SEMESTER 2 AY 2016/2017)

Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This assessment paper contains **FIVE** questions and comprises **EIGHT** printed pages, including this page.
2. Answer **ALL** questions within the space in this booklet.
3. This is a Closed Book assessment.
4. Candidates are allowed to bring in an A4-sized help sheet, written on both sides.
5. Calculators are allowed.
6. Please write your Student Number below. Do not write your name.

Student NO: ______________________________

Question	Marks	Remarks
A(Pg 2)		
A(Pg 3)		
B		
C		
D		
E		
Total		

Question A [38 marks]. *For each of the following, just write down the answers in the spaces provided. Detailed workings are not required. Also numerical answers are to be written as integers or powers of a single integer. For example, you can write 2300 or* 3^{27} *but neither* $\binom{5}{1}\binom{3}{1}$ *nor* $3!$.

(1) Find -9724 **Mod** 19.

(2) Is 2017 a prime number? Yes/ No

(3) Find the inverse $\bar{a}$ of a modulo 251 such that $0 < \bar{a} < 251$, where $a = 149$.

(4) Find the coefficient of x^3yz^4w in the expansion of $(x - y + 2z - 2w)^9$.

(5) Find the number of integers in $\{1, 2, \ldots, 2017\}$ which are

(i) multiples of 3 or 4.

(ii) multiples of 3 or 4 but not 5.

(6) How many integer solutions are there of the equation $x+y+z+t+w = 7$ with $x \geq 5$, $y \geq -3$, $z \geq 2$, $t \geq -7$, and $w \geq 4$?

(7) A manufacturer makes **identical** transparent marbles and also makes nontransparent marbles in 20 **different colors**. In how many ways can he make up a bag of 20 marbles, given that the bag may contain up to 20 transparent marbles but not two nontransparent marbles that have the same color?

(8) Let G be a **connected** simple graph with 5 vertices.

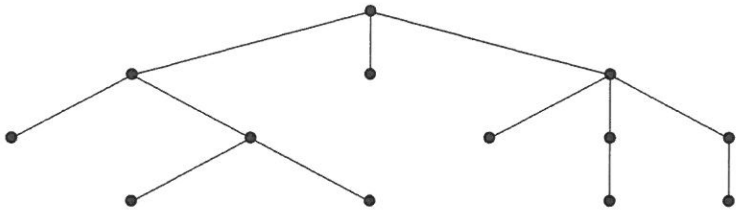
(i) Is it possible that 3 vertices are of degree 4 and 2 vertices are of degree 2?

Yes / No

(ii) If G has at least 3 vertices of degree 4, then is it true that such a G must have an Euler path? Yes / No

(9) In the hypercube Q_4, find a simple path from 1011 to 1100. (You only need to name the vertices in the path.)

(10) Label the following tree using the Universal Address system. (Write the labels beside the vertices.)



Order the vertices using the lexicographic ordering.

(11) Let T be a **full** 7-ary tree.

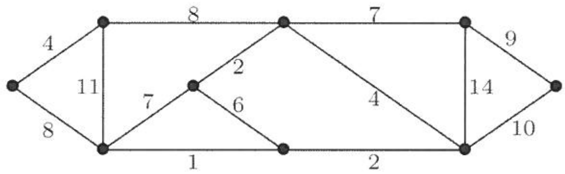
(i) How many vertices does T have if it has 12 internal vertices?

(ii) How many among the numbers 101, 222, 333, 666, can be the number of leaves of T? (Your answer ranges from 0 to 4.)

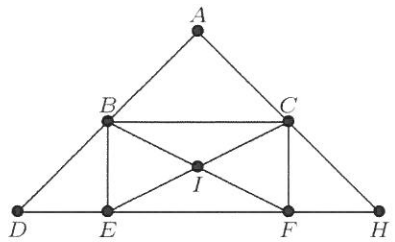
(12) (i) Find the minimum values of m if an m-**ary tree** has **at least** 120 leaves and height **exactly equal** to 5. min:

(ii) Find the maximum values of n if a **full** n-**ary tree** has **at most** 120 leaves and height **exactly equal** to 5. max:

(13) Find the **weight** of a minimum spanning tree in the following graph.



(14) Let G be the following graph. Using the alphabetical ordering, find a spanning tree by **depth first** search. Draw the tree below.



Graph G

Question B [5 marks]. Prove by using mathematical induction that for any integer $n \geq 1$,

$$\left(1 + \frac{1}{3}\right)^n \geq 1 + \frac{n}{3}.$$

Question C [5 marks]. How many ways are there to arrange 3 girls and 8 boys to dance in a circle such that there are at least two boys between any two adjacent girls? Justify your answer.

Question D [5 marks]. Prove that for any integer $n \geq 2$, a simple graph with n vertices and $\frac{1}{2}(n-1)(n-2)+1$ edges is connected.

Question E [7 marks]. (i) Fibonacci Numbers F_n is defined as follows:

$$\begin{cases} F_0 = 0,\ F_1 = 1 \\ F_n = F_{n-1} + F_{n-2} & \text{if } n \geq 2 \end{cases}$$

Prove that for any integer $n \geq 0$, F_n is even if and only if n is divisible by 3.

(ii) Now we define a new sequence of numbers. Let $A_1 = 0$ and $A_2 = 1$. For $n \geq 3$, the number A_n is defined by concatenating the decimal expansions of A_{n-1} and A_{n-2} from left to right. For example, $A_3 = (A_2$ followed by $A_1) = 10$, $A_4 = (A_3$ followed by $A_2) = 101$, $A_5 = (A_4$ followed by $A_3) = 10110$, and so forth. Prove that for all positive integer n, A_n is a multiple of 11 if and only if $n \equiv 1 \bmod 6$.

—END OF PAPER—