CS1231–Midterm 1, 2015

Name: **Matric No:**

1.

Yes, it's a tautology.

p	q	r	$p \to q$	$q \to r$	$p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	T	T
F	T	T	T	T	T	T
T	F	F	F	T	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

2. $(p \to (p \land q)) = \neg p \lor (p \land q) = (\neg p \lor p) \land (\neg p \lor q) = \mathbf{T} \land (\neg p \lor q) = \neg p \lor q$

Thus the given expression is

$$\neg(\neg p \lor q) \lor q = (p \land \neg q) \lor q = (p \lor q) \land (\neg q \lor q) = p \lor q$$

3. $\neg(p \to q) = \neg(\neg p \lor q) = p \land \neg q$. So the ans is d

4. (a) $\exists x D(x) \land \neg P(x)$.

(b) $\forall x D(x) \to P(x)$.

5. (a) $\forall x \neg(D(x) \land \neg P(x)) = \forall x P(x) \lor \neg D(x) = \forall x D(x) \to P(x)$

English: Every old dog can learn new tricks

(b) $\exists x \neg(D(x) \to P(x)) = \exists x \neg(\neg D(x) \lor P(x)) = \exists x D(x) \land \neg P(x)$.

English: Some old dogs cannot learn new tricks.

6. $\exists x \in P\ \forall y \in Q, W(x, y) \to \neg T(y)$.

7. Hypotheses :

(i) $a \land w \to p$, (ii) $\neg a \to i$, (iii) $\neg w \to m$, (iv) $\neg p$, (v) $e \to (\neg i \land \neg m) = e \to \neg(i \lor m)$

Proof:

1. $\neg p$, (iv)
2. $a \land w \to p$, (i)
3. $\neg(a \land w) = \neg a \lor \neg w$, from 1,2 (modus tollen)
4. $\neg a \to i$, (ii)
5. $\neg w \to m$, (iii)
6. $i \lor m$, from 3, 4, 5. (modus ponen)
7. $e \to \neg(i \lor m)$, (iv)
8. $\neg e$, from 6, 7 (modus tollen)

Comments:

Q6: Many wrote $\exists x \in P \ \forall y \in Q, W(x, y) \wedge T(y)$.

Note that $W(x, y) \wedge T(y)$ is false when x does not write y. Thus the universal quantification $\forall y \ W(x, y) \wedge T(y)$ is false since there certainly will be programmes not written by x. Thus we see that $\exists x \in P \ \forall y \in Q, W(x, y) \wedge T(y)$ is false even if there is a programmer who writes only programmes that don't terminate.

Q7. You have to note that "able to prevent evil" and "prevent evil" are different attributes. For example having the ability to climb Mt Everest and the act of climbing Mt Everest are different. Thus the 1st hypothesis is $a \wedge w \to p$ and not $a \wedge w$.