CS1231 TUTORIAL 1

1. Only c and f are propositions and both are false. The rest do not have a truth value.

2. (a) The election is not decided but the votes have been counted.

(b) If the votes have been counted, then the election is decided.

(c) The election is decided iff the votes have been counted.

(d) Either the votes have not been counted, or else the election is not decided and the votes have been counted.

3. T, T, F, T

4. (a) The above is: (get A) $\rightarrow$ (do every ex) $\vee$ (score $\geq$ 80 marks). (Note that $p \rightarrow q$ is true when p is false or p, q are both true.) Thus the proposition is true when you

(i) do not get A or

(ii) get A and either do every ex or score $\geq$80 marks.

(b) No.

5. $s =$ lhs, $t =$ rhs. From the truth tables,the answers are: (a) No (columns under s and t are not identical), (b) Yes.

p	q	r	$p \vee q$	$p \wedge r$	s	t
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F
F	0	T	F	F	F	F
F	F	F	F	F	F	F

p	q	r	$r \vee p$	$p \wedge q$	$\neg r \vee (p \wedge q)$	$r \vee q$	s	t
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	T
T	F	T	T	F	F	T	F	F
T	F	F	T	F	T	F	F	F
F	T	T	T	F	F	T	F	F
F	T	F	F	F	T	T	F	F
F	F	T	T	F	F	T	F	F
F	F	F	F	F	T	F	F	F

Alt:

$$\begin{aligned}(p \vee q) \vee (p \wedge r) &= ((p \vee q) \vee p) \wedge ((p \vee q) \vee r) \\ &= (p \vee q) \wedge ((p \vee q) \vee r) \\ &= p \vee q\end{aligned}$$

But $p \vee q \not\equiv (p \vee q) \wedge r$ as they have opposite truth values when p T and r F.

$$\begin{aligned}(r \vee p) \wedge (\neg r \vee (p \wedge q)) \wedge (r \vee q) &= (r \vee p) \wedge (r \vee q) \wedge (\neg r \vee (p \wedge q)) \\ &= (r \vee (p \wedge q)) \wedge (\neg r \vee (p \wedge q)) \\ &= (r \wedge \neg r) \vee (p \wedge q) \\ &= p \wedge q\end{aligned}$$

6.

p	q	r	$p \to q$	$\neg p \to r$	$(p \to q) \wedge (\neg p \to r)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	F
F	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	F	F

ANS: F,T,F,T,T

7. (a) The 99^{th} proposition is true and the rest are false. The reason is as follows:

Suppose there are exactly k false propositions in the list. Then only one proposition, namely, the k^{th} proposition, is true. Thus k has to 99.

(b) 1 to 50 are true and 51-100 are false.

We first suppose that there are m propositions of which k are true and $m-k$ are false. We see that proposition i is true, iff $m-k \geq i$. From this we conclude that if proposition i is true, then proposition $i-1$ is also true. By repeating this argument, we see that positions 1 to k are true and the rest are false. Since proposition k is true, we have $m-k \geq k$. Since proposition $k+1$ is false, we have $m-k \geq k+1$ is false, i.e., $m-k \leq k$. Thus $m-k=k$ or $m=2k$.

(c) From the discussion in (b), we conclude that it not possible to assign a truth value of each of the items. Thus they are not propositions.

8. It is false when p is false and q is true. (You can get this by constructing the truth table.)

9. Let each letter stand for the statement that the person who name begins with that letter is chatting. Then (i) $\neg K \rightarrow H$, (ii) $R \rightarrow \neg V$ and $\neg R \rightarrow V$, (iii) $A \rightarrow R$, (iv) $V \leftrightarrow K$, (v) $H \rightarrow A \wedge K$.

If A is chatting, then R is chatting (from iii), V is not chatting (from ii), K is not chatting (from iv), H is chatting (from i). This contradicts (v).

If A is not chatting, then H is not chatting (v), K is chatting (i), V is chatting (iv), R is not chatting (ii).

Thus only Kevin and Vijay are chatting. (You should check that this satisfies the given 5 conditions.)

Alt: Consider the bit string $KHRVA$ where the X position is 1 if X is chatting and 0 otherwise. Then (i) $KH = 10, 01$ or 11; (ii) $RV = 10, 01$; (iii) If $A = 1$, then $R = 1$; (iv) $VK = 00, 11$, (v) If $H = 1$, then $AK = 11$.

(a) $A = 1$: Then $R = 1$ (iii), $V = 0$ (ii), $K = 0$ (iv), $H = 1$ (i). This contradicts (v).

(b) $A = 0$: $H = 0$ (v), $K = 1$ (i), $V = 1$ (iv), $R = 0$ (ii).

Thus the answer is; Only Kevin and Vijay are chatting.