

Sequences

CS4248 Natural Language Processing

Week 07

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Recap of Week 06

Move from term–document matrix to a term–context matrix

- Solves semantically relatedness

Embed the resultant term–context vectors into a denser space

- The side effect is the objective!
- Solves sparsity problem

Vectorial differences yields semantics relationships

Many extensions, we'll see some later

Week 07 Agenda

Sequences

Parts of Speech (POS)

POS Tagging

Hidden Markov Model (HMM)

Forward Computation

Viterbi Algorithm

Sequences

Matters Ordering

Bag of words loses semantic information

Bob kills mosquitos using the book of Hamlet

vs

Hamlet kills Bob using the book of mosquitos

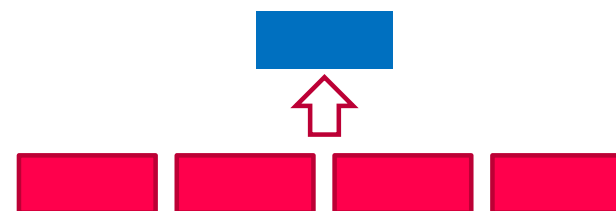
The food tastes good and does not look bad

vs

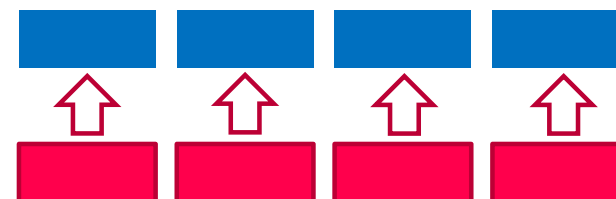
The food tastes bad and does not look good

Type of Sequence Tasks

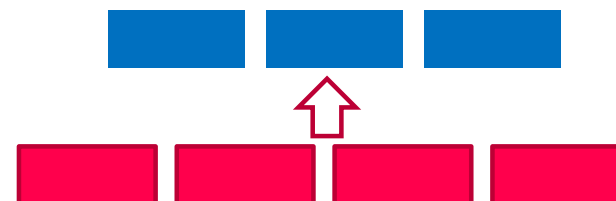
Sequence classification ($N \rightarrow 1$)



Sequence labelling ($N \rightarrow N$)



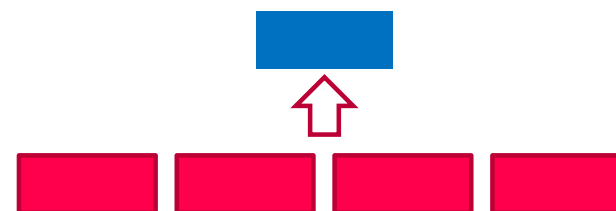
Sequence to sequence ($N \rightarrow M$)



Type of Sequence Tasks

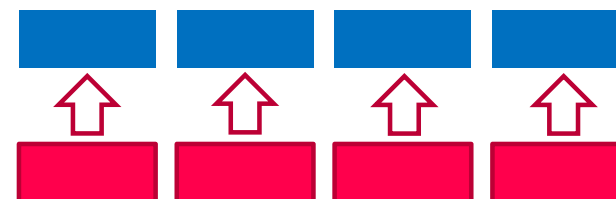
Sequence classification ($N \rightarrow 1$)

- Sentiment analysis
- Sentence factual checking



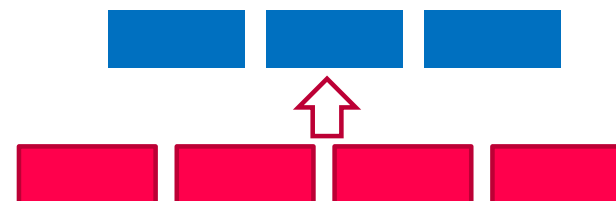
Sequence labelling ($N \rightarrow N$)

- Named-entity recognition
- Part-of-speech tagging



Sequence to sequence ($N \rightarrow M$)

- Machine translation



Parts of Speech

A case study of sequence tagging

Parts of Speech

Also called word classes or syntactic categories

Every word in the vocabulary belongs to one or more of these classes

~ 8 of them (in English)

Parts of Speech	
NOUN <i>Name of a person, place, thing or idea.</i> Examples: Daniel, London, table, hope - Mary uses a blue pen for her notes.	PRONOUN <i>A pronoun is used in place of a noun or noun phrase to avoid repetition.</i> Examples: I, you, it, we, us, them, those - I want her to dance with me.
ADJECTIVE <i>Describes, modifies or gives more information about a noun or pronoun.</i> Examples: cold, happy, young, two, fun - The little girl has a pink hat.	VERB <i>Shows an action or a state of being.</i> Examples: go, speak, eat, live, are, is - I listen to the word and then repeat it.
ADVERB <i>Modifies a verb, an adjective or another adverb. It tells how (often), where, when.</i> Examples: slowly, very, always, well, too - Yesterday, I ate my lunch quickly.	PREPOSITION <i>Shows the relationship of a noun or pronoun to another word.</i> Examples: at, on, in, from, with, about - I left my keys on the table for you.
CONJUNCTION <i>Joins two words, ideas, phrases together and shows how they are connected.</i> Examples: and, or, but, because, yet, so - I was hot and tired but still finished it.	INTERJECTION <i>A word or phrase that expresses a strong emotion. It is a short exclamation.</i> Examples: Ouch! Hey! Oh! Watch out! - Wow! I passed my English exam.
www.grammar.cl www.woodwardenglish.com www.vocabulary.cl	

POS – Two broad categories

Closed class

- Small fixed membership
- Usually function words
(words which play a grammatical role)
- Example:
 - Prepositions: on, in, of
 - Pronoun: she, him, them
 - Particles: up, down, on, off
 - Determiners: a, an, the
 - Conjunctions: and, but, or
 - Numerals: one, two, three
 - Auxiliary verbs: was, should

Open class

- New vocabulary items can be created
- Most languages have four:
 - Noun: Singapore, boy/boys
 - Verb: eat/eats/eaten
 - Adjective: good, bad, worse
 - Adverb: quickly, extremely

POS Tagging

The process of assigning a part of speech to words in a text

Input: a sequence of tokenized words and a tagset

Output: a sequence of tags, one per token

- A word can belong to more than POS.

For example, the word *back*

- The *back* door (adjective)
- On my *back* (noun)
- Win the voters *back* (adverb)
- Promised to *back* the bill (verb)

POS Tagging

Why is it useful?

First step for a vast number of practical tasks

- Named Entity Recognition (**Trump** = proper or common noun?)
- Information Extraction (finding relations, names, etc.)
- Parsing (need to know if word is noun or verb before we create dependency tree)
- Speech synthesis/recognition (DIScount-disCOUNT, how do we pronounce *lead*)

POS Tagging

Words are ambiguous, tagging is a **disambiguation** task

How common is this ambiguity?

Types:		WSJ	Brown
Unambiguous	(1 tag)	44,432 (86%)	45,799 (85%)
Ambiguous	(2+ tags)	7,025 (14%)	8,050 (15%)
Tokens:			
Unambiguous	(1 tag)	577,421 (45%)	384,349 (33%)
Ambiguous	(2+ tags)	711,780 (55%)	786,646 (67%)

Goal: find the correct tag for each particular token.

POS Tagging

A simple way to deal with ambiguity

Most frequent class baseline: Always compare a classifier against a **baseline** at least as good as the most frequent class baseline:

- assign each word to the class it occurred in most often in the training set
- assign unknown words as nouns

POS Tagging

How good is this baseline: Accuracy for POS taggers is measured as the percent of tags that are correctly labeled as compared to human labels on a test set.

Model	Accuracy (on a news corpus: <i>Wall Street Journal</i>)
Most frequent class baseline	92.34%
Flair (SOTA)	97.85%

Baseline already has >90% accuracy

- many words are unambiguous
- you get points for them (*the*, *a*) and for punctuation marks

Penn Treebank Tagset

45 possible tags

Used to label many corpora

Example:

Preliminary/JJ findings/NNS were/VBD reported/VBN in/IN
today/NN 's/POS New/NNP England/NNP Journal/NNP of/IN
Medicine/NNP ./.

Penn Treebank Tagset

Tag	Description	Example	Tag	Description	Example	Tag	Description	Example
CC	coordinating conjunction	<i>and, but, or</i>	PDT	predeterminer	<i>all, both</i>	VBP	verb non-3sg present	<i>eat</i>
CD	cardinal number	<i>one, two</i>	POS	possessive ending	<i>'s</i>	VBZ	verb 3sg pres	<i>eats</i>
DT	determiner	<i>a, the</i>	PRP	personal pronoun	<i>I, you, he</i>	WDT	wh-determ.	<i>which, that</i>
EX	existential 'there'	<i>there</i>	PRP\$	possess. pronoun	<i>your, one's</i>	WP	wh-pronoun	<i>what, who</i>
FW	foreign word	<i>mea culpa</i>	RB	adverb	<i>quickly</i>	WP\$	wh-possess.	<i>whose</i>
IN	preposition/ subordin-conj	<i>of, in, by</i>	RBR	comparative adverb	<i>faster</i>	WRB	wh-adverb	<i>how, where</i>
JJ	adjective	<i>yellow</i>	RBS	superlatv. adverb	<i>fastest</i>	\$	dollar sign	<i>\$</i>
JJR	comparative adj	<i>bigger</i>	RP	particle	<i>up, off</i>	#	pound sign	<i>#</i>
JJS	superlative adj	<i>wildest</i>	SYM	symbol	<i>+, %, &</i>	“	left quote	<i>‘ or “</i>
LS	list item marker	<i>1, 2, One</i>	TO	“to”	<i>to</i>	”	right quote	<i>’ or ”</i>
MD	modal	<i>can, should</i>	UH	interjection	<i>ah, oops</i>	(left paren	<i>[, (, {, <</i>
NN	sing or mass noun	<i>llama</i>	VB	verb base form	<i>eat</i>)	right paren	<i>],), }, ></i>
NNS	noun, plural	<i>llamas</i>	VBD	verb past tense	<i>ate</i>	,	comma	<i>,</i>
NNP	proper noun, sing.	<i>IBM</i>	VBG	verb gerund	<i>eating</i>	.	sent-end punc	<i>. ! ?</i>
NNPS	proper noun, plu.	<i>Carolinas</i>	VBN	verb past part.	<i>eaten</i>	:	sent-mid punc	<i>: ; ... --</i>

Penn Treebank Tagset – More examples

The/DT grand/JJ jury/NN commented/VBD on/IN a/DT number/NN of/IN
other/JJ topics/NNS .

Mrs/NNP Shaefer/NNP never/RB got/VBD around/RP
to/TO joining/VBG

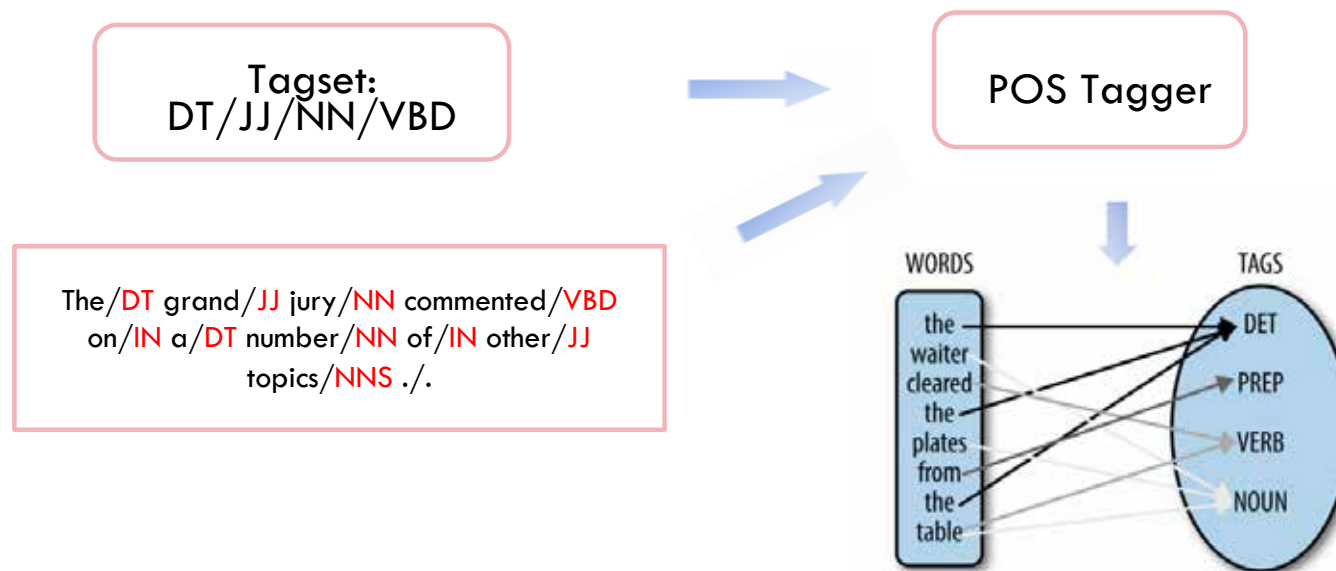
All/DT we/PRP goka/VBN do/VB is/VBZ go/VB around/IN
the/DT corner/NN

Chateau/NNP Petrus/NNP costs/VBZ around/RB 250/CD

POS Tagging via Supervised Learning

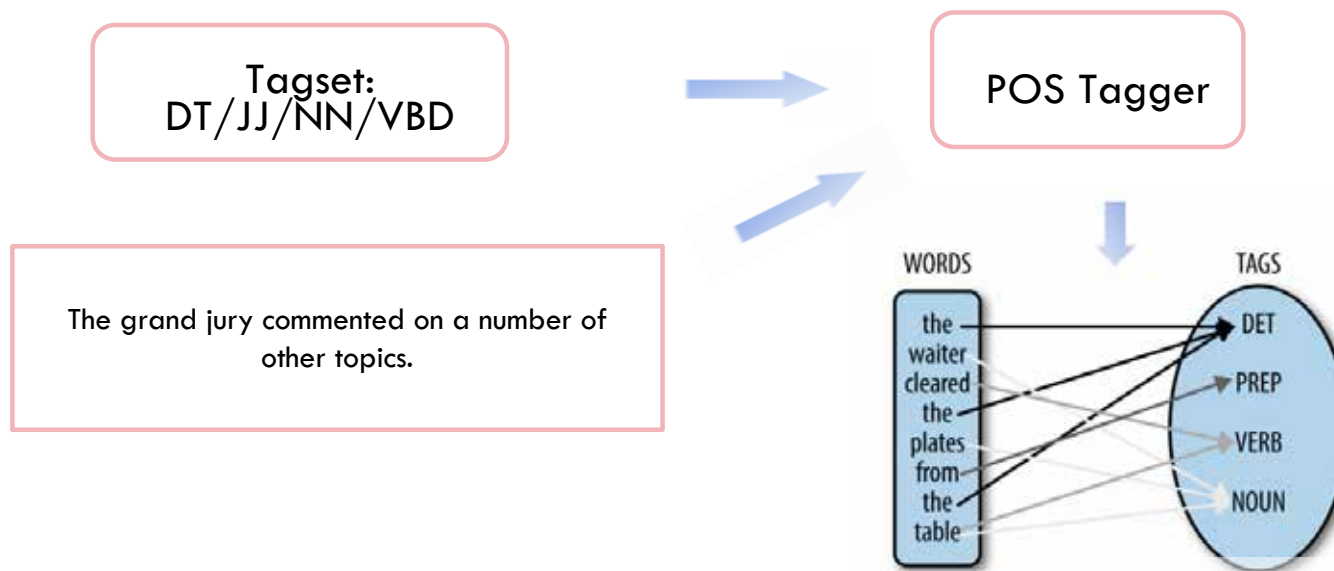
Linguists annotate input sentences with their POS.

We use this as input training data to build a supervised model.



POS Induction via Unsupervised Learning

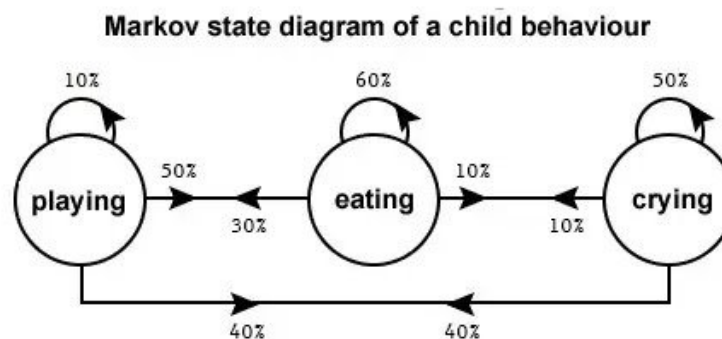
We assume we only have unannotated corpora:



Hidden Markov Model (HMM)

What is an HMM

Based on the Markov Chain



Embodies the Markov assumption

“when predicting the future, the past doesn’t matter, only the present” – Markov

“hidden” → contains unobserved event

Image Credit: <https://www.quora.com/What-is-a-state-of-Markov-chain>

Component of HMM

- Set of states (Q)
- Transition probability (a_{ij})
the probability of moving from state i to state j
- Sequence of observations (o_t)
observation drawn for each time-step t from vocabulary V
- Emission probability ($b_i(o_t)$)
the probability of observation o generated from a state i
- Initial probability distribution (π)
the probability for the Markov chain to start at state i

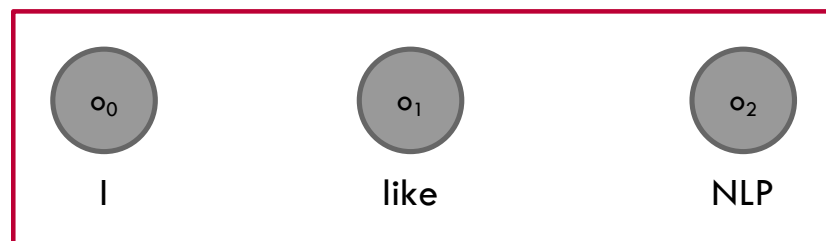
HMM Components: Observations

I

like

NLP

HMM Components: Emission Probabilities

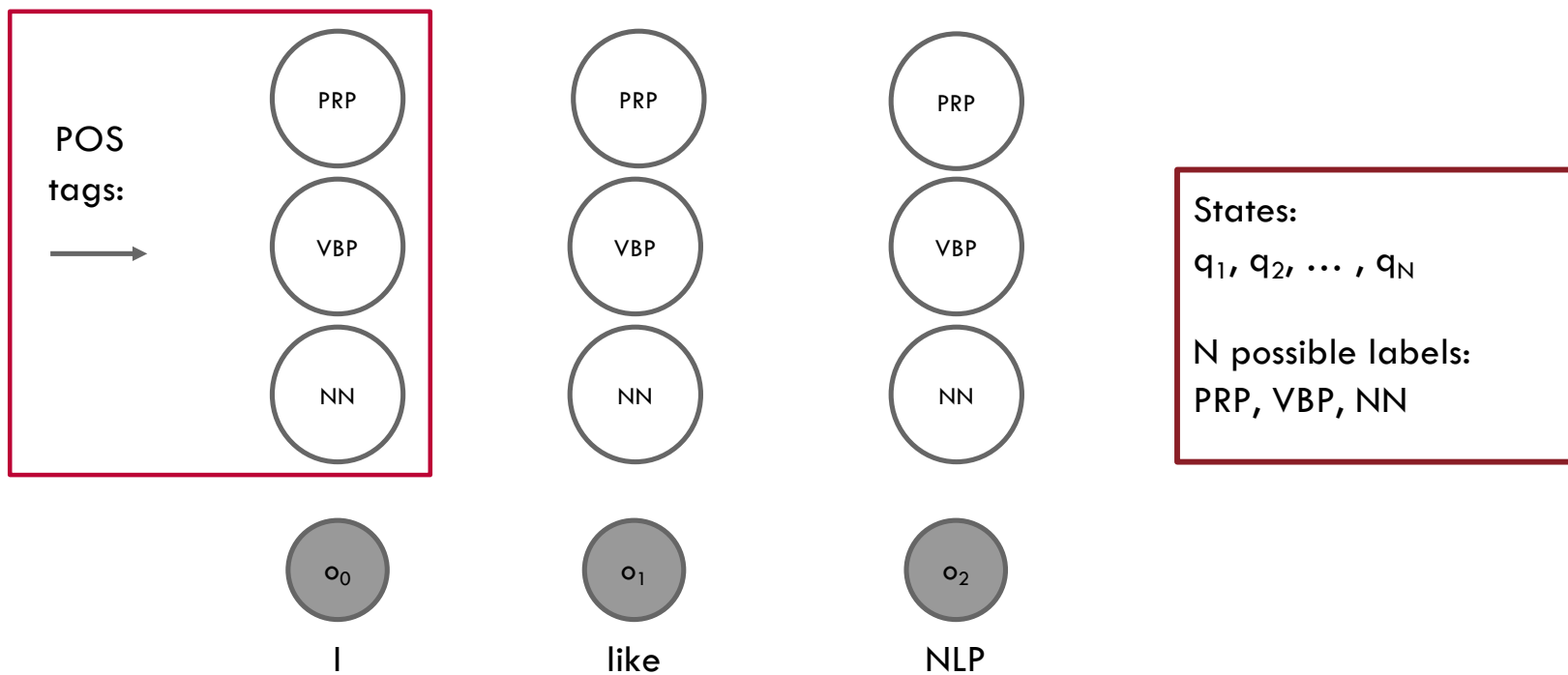


Emission prob.:

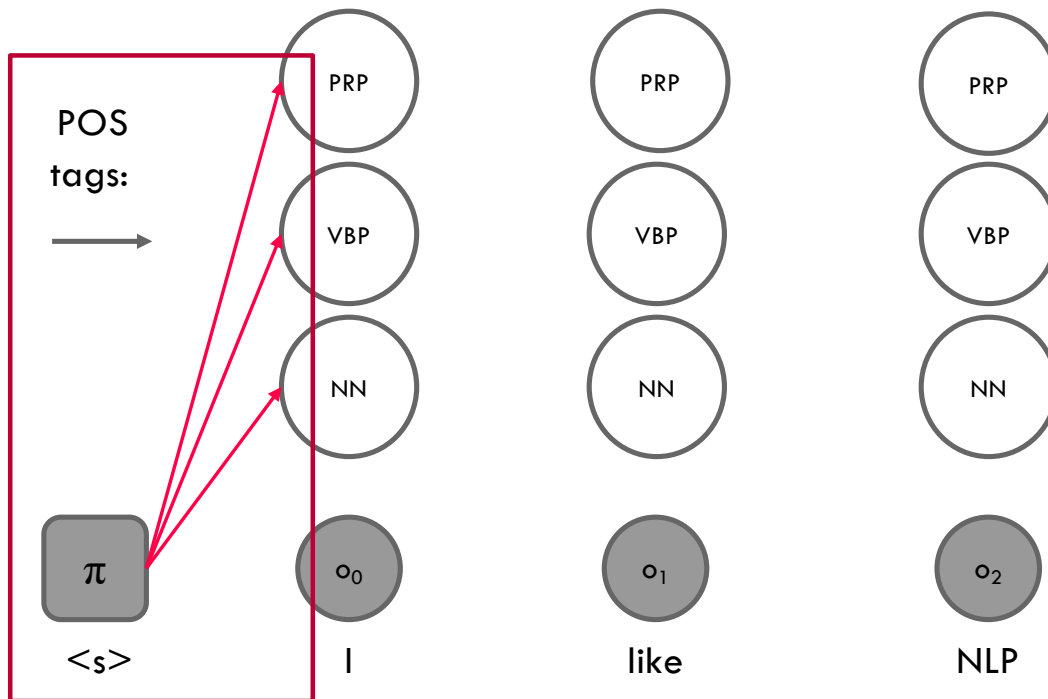
$$b_i(o_t) = p(o_t \mid q_i)$$

Given a state q_i , what's the probability that it emits o_t ?

HMM Components: States



HMM Component: Initial Start Probability



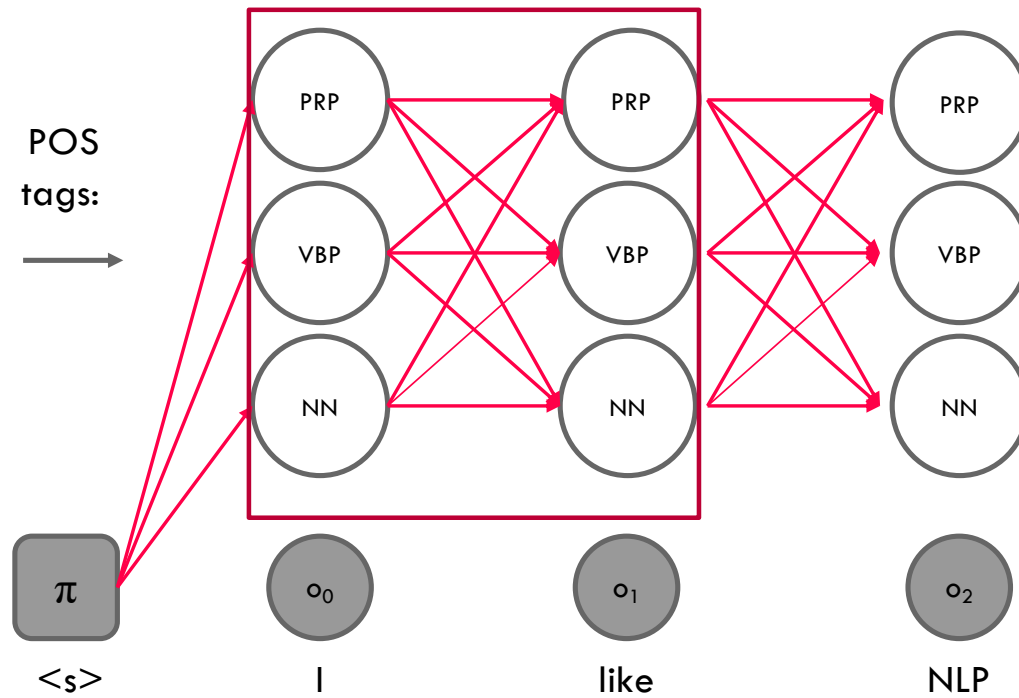
Start probability:

$$\pi_i = p(q_i | <s>)$$

What's the probability that a sequence starts with label q_i ?

Component of HMM:

Initial Transition Distribution



Transition prob.:

$$a_{ij} = p(q_i | q_j)$$

Given a label q_j ,
what's the probability
that it is followed by
label q_i ?

Getting the probabilities

Maximum Likelihood Estimates:

$$\pi_i = p(q_i | \langle s \rangle) = \frac{c(\langle s \rangle q_i)}{c(\langle s \rangle)}$$

$$a_{ij} = p(q_i | q_j) = \frac{c(q_j q_i)}{c(q_j)}$$

$$b_i(o_t) = p(o_t | q_i) = \frac{c(o_t, q_i)}{c(q_i)}$$

Getting the probabilities

Maximum Likelihood Estimates!

$$\pi_i = p(q_i | \langle s \rangle) = \frac{c(\langle s \rangle q_i)}{c(\langle s \rangle)}$$

Number of sequences that starts with label q_i

Number of sequences

$$a_{ij} = p(q_j | q_i) = \frac{c(q_i q_j)}{c(q_i)}$$

Occurrences of label q_i followed by label q_j

Occurrences of label q_i

$$b_i(o_t) = p(o_t | q_i) = \frac{c(o_t, q_i)}{c(q_i)}$$

Occurrences of word o_t labeled as q_i

Occurrences of label q_i

Forward Computation

What's the likelihood of a sequence?

Forward computation

Calculate the probabilities, given the HMM & sentence length T

$O = o_1 o_2 o_3 \cdots o_T \rightarrow$ observations, words from 1 to T

$Q = q_1 q_2 q_3 \cdots q_T \rightarrow$ order of the labels for each word, $q_0 = \langle s \rangle$

$$P(O, Q) = P(O|Q) \times P(Q) = \prod_{i=1}^T P(o_i|q_i) \times P(q_i|q_{i-1})$$

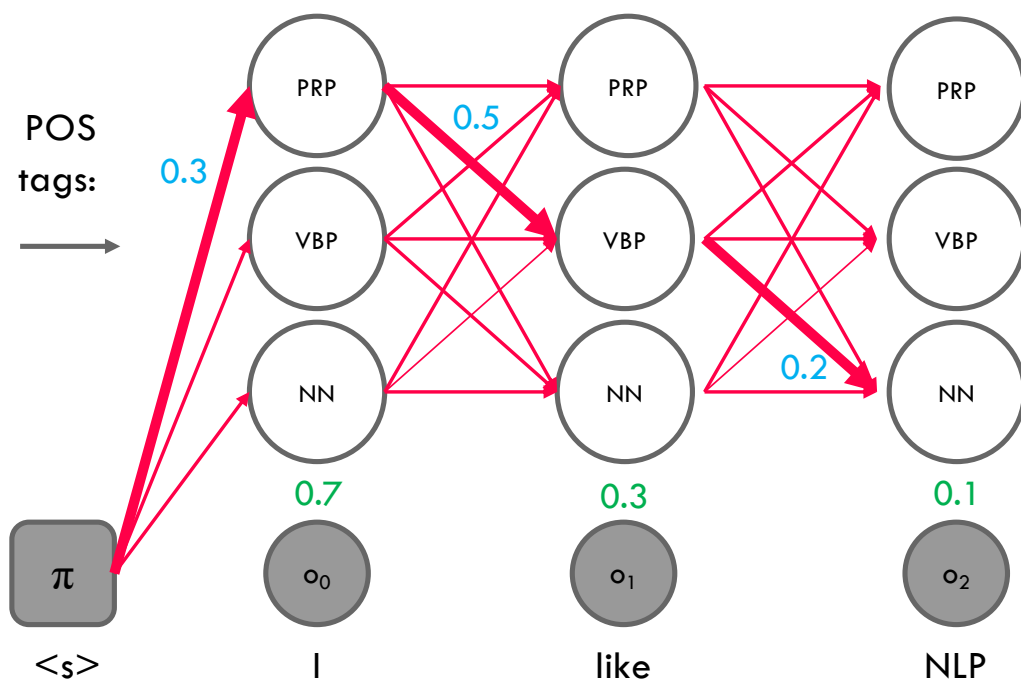
$$P(O, Q) = P(o_1|q_1) \times P(q_1|q_0) \times \cdots \times P(o_T|q_T) \times P(q_T|q_{T-1})$$

Forward computation

$$P(O, Q) = P(O|Q) \times P(Q) = \prod_{i=1}^T P(o_i|q_i) \times P(q_i|q_{i-1})$$

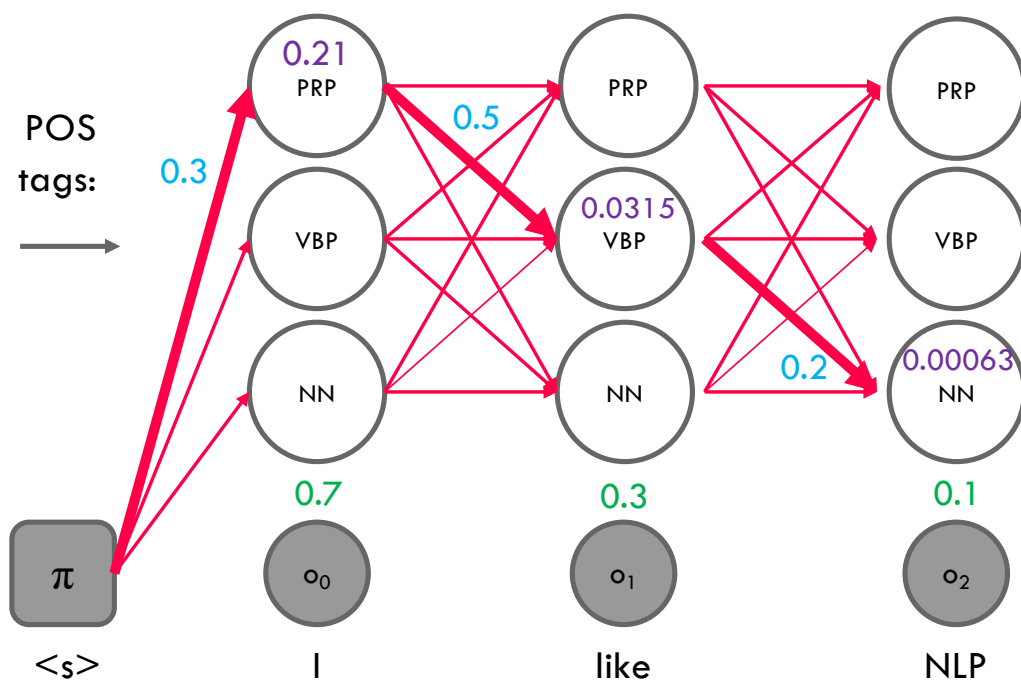
$$\begin{aligned} P(\text{"I like NLP"}, PRP - VBP - NN) = \\ P(I|PRP) \times P(PRP|\langle s \rangle) \times \\ P(Like|VBP) \times P(VBP|PRP) \times \\ P(NLP|NN) \times P(NN|VBP) \end{aligned}$$

Forward Computation



$$\begin{aligned}
 &P\left(\text{"I like NLP"}, \right. \\
 &\quad \left. PRP - VBP - NN\right) \\
 &= 0.3 \times 0.7 \times \\
 &\quad 0.5 \times 0.3 \times \\
 &\quad 0.2 \times 0.1
 \end{aligned}$$

Forward Computation



$$P\left(\text{"I like NLP", }_{PRP - VBP - NN}\right)$$

$$= 0.3 \times 0.7 \times 0.5 \times 0.3 \times 0.2 \times 0.1$$

$$= 0.21 \times 0.5 \times 0.3 \times 0.2 \times 0.1$$

$$= 0.0315 \times 0.2 \times 0.1$$

$$= 0.00063$$

Viterbi Decoding

What's the most likely (hidden state) path?

Decoding task

Determine the underlying sequence of variables that generate the observation

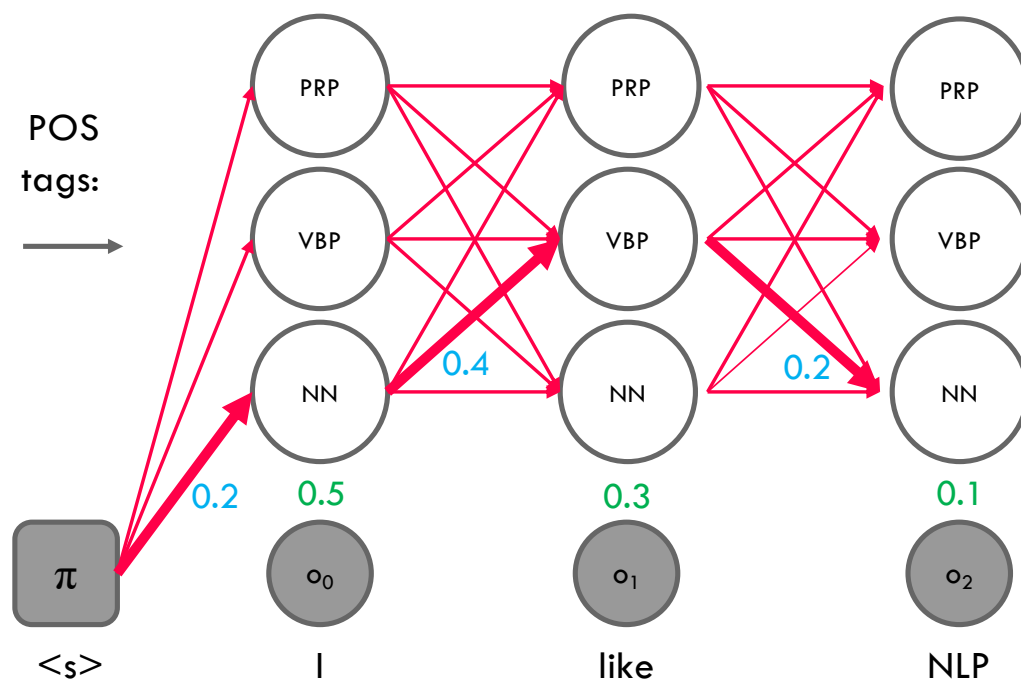
- What is the most probable labels for the sentence “I like NLP”?

$$Q = \operatorname{argmax}_{q_1 \dots q_T} \prod_{i=1}^T P(o_i | q_i) \times P(q_i | q_{i-1})$$

Try all combinations?

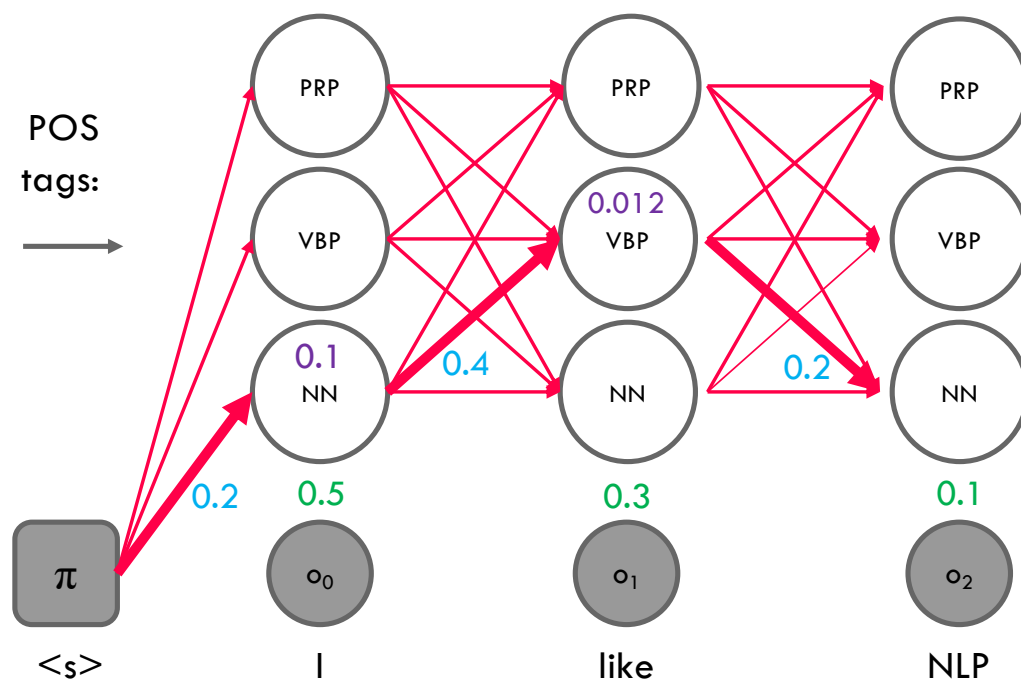
We can save the states and use **dynamic programming**

Forward Computation



$$P \left(\begin{array}{c} \text{"I like NLP",} \\ NN - VBP - NN \end{array} \right) \\
 = 0.2 \times 0.5 \times \\
 0.4 \times 0.3 \times \\
 0.2 \times 0.1$$

Forward Computation



$$P\left(\text{"I like NLP",}\right. \\ \left. NN - VBP - NN\right)$$

$$= 0.2 \times 0.5 \times \\ 0.4 \times 0.3 \times \\ 0.2 \times 0.1$$

$$= 0.1 \times \\ 0.4 \times 0.3 \times \\ 0.2 \times 0.1$$

$$= 0.012 \times \\ 0.2 \times 0.1$$

$$= 0.00024$$

Save the state

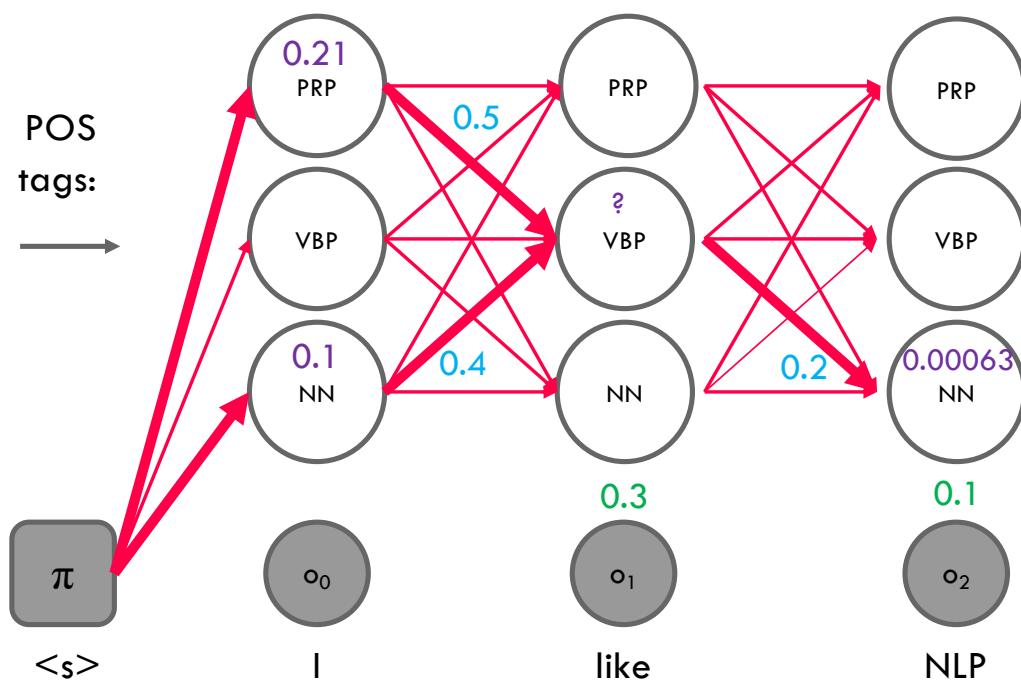
$$P(\text{"I like NLP"}, PRP - VBP - NN) = \\ P(I|PRP) \times P(PR|\langle s \rangle) \times \\ P(Like|VBP) \times P(VBP|PRP) \times \\ P(NLP|NN) \times P(NN|VBP)$$

$$= v_1(PR) \times \\ P(Like|VBP) \times P(VBP|PR) \times \\ P(NLP|NN) \times P(NN|VBP)$$

$$= v_2(VBP) \times \\ P(NLP|NN) \times P(NN|VBP)$$

$$P(\text{"I like NLP"}, PRP - VBP - NN) \\ = 0.3 \times 0.7 \times \\ 0.5 \times 0.3 \times \\ 0.2 \times 0.1 \\ = 0.21 \times \\ 0.5 \times 0.3 \times \\ 0.2 \times 0.1 \\ = 0.0315 \times \\ 0.2 \times 0.1 \\ = 0.00063$$

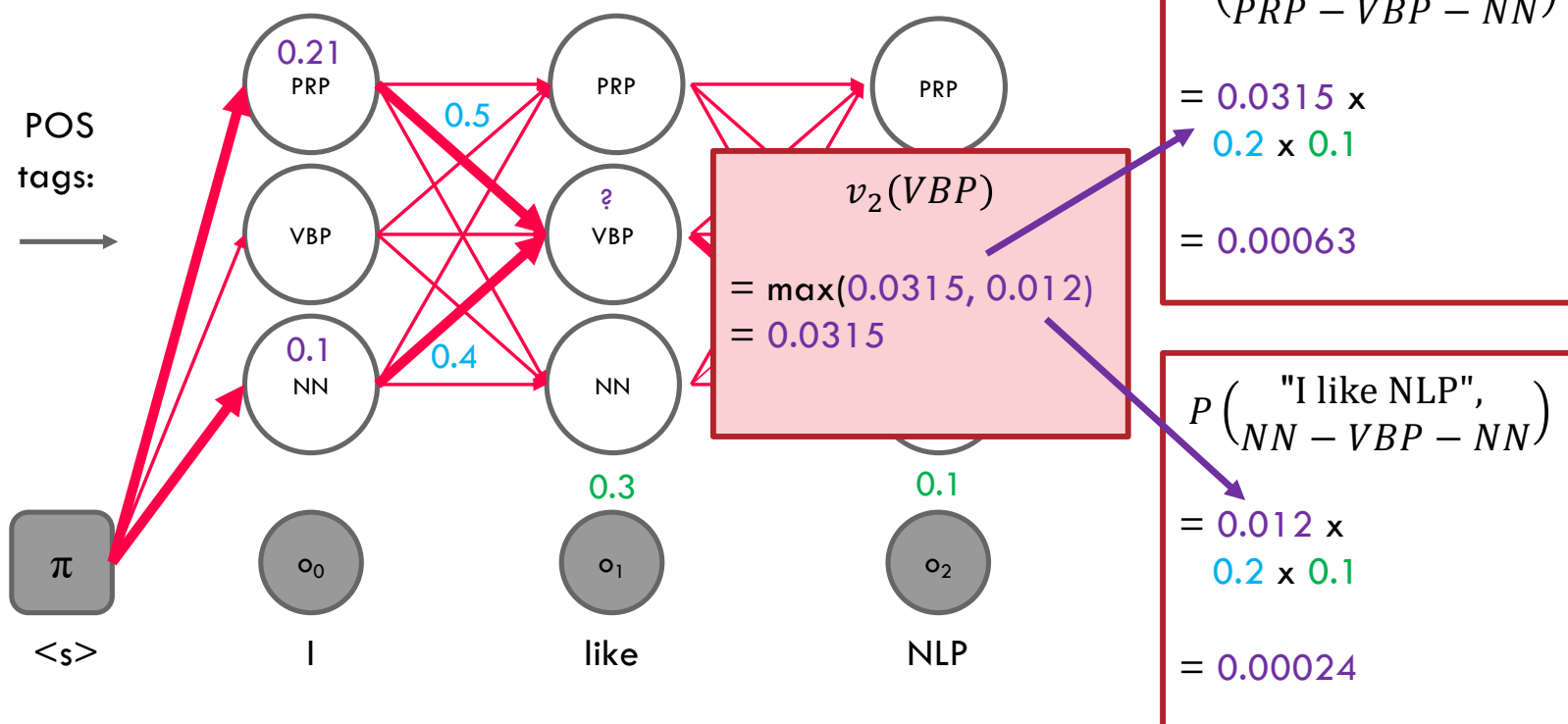
Which path to choose?



$$\begin{aligned}
 &P\left(\text{"I like NLP",}\right. \\
 &\quad \left.PRP - VBP - NN\right) \\
 &= 0.0315 \times \\
 &\quad 0.2 \times 0.1 \\
 &= 0.00063
 \end{aligned}$$

$$\begin{aligned}
 &P\left(\text{"I like NLP",}\right. \\
 &\quad \left.NN - VBP - NN\right) \\
 &= 0.012 \times \\
 &\quad 0.2 \times 0.1 \\
 &= 0.00024
 \end{aligned}$$

Which path to choose?

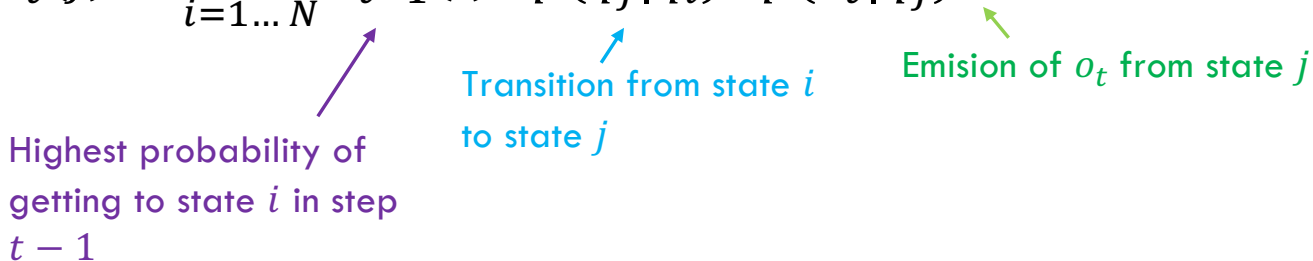


Formal definition

Save the probability of best path to state j in step $t \rightarrow v_t(j)$

$$v_t(j) = \max_{q_1 \dots q_{t-1}} P(q_1 \dots q_{t-1}, o_1, o_2 \dots o_t, q_t = j | \lambda)$$

$$v_t(j) = \max_{i=1 \dots N} v_{t-1}(i) \times p(q_j | q_i) \times p(o_t | q_j)$$



Highest probability of getting to state i in step $t - 1$

Transition from state i to state j

Emission of o_t from state j

Formal definition

Save the probability of best path to state j in step $t \rightarrow v_t(j)$

$$v_t(j) = \max_{q_1 \dots q_{t-1}} P(q_1 \dots q_{t-1}, o_1, o_2 \dots o_t, q_t = j | \lambda)$$

$$v_t(j) = \max_{i=1 \dots N} v_{t-1}(i) \times p(q_j | q_i) \times p(o_t | q_j)$$

$$v_t(j) = \max_{i=1 \dots N} v_{t-1}(i) a_{ij} b_j(o_t)$$

Remember,

$$a_{ij} = p(q_j | q_i)$$

$$b_i(o_t) = p(o_t | q_i)$$

1. Initialization:

$$\begin{aligned}v_1(j) &= \pi_j b_j(o_1) & 1 \leq j \leq N \\bt_1(j) &= 0 & 1 \leq j \leq N\end{aligned}$$

2. Recursion

$$\begin{aligned}v_t(j) &= \max_{i=1}^N v_{t-1}(i) a_{ij} b_j(o_t); & 1 \leq j \leq N, 1 < t \leq T \\bt_t(j) &= \operatorname{argmax}_{i=1}^N v_{t-1}(i) a_{ij} b_j(o_t); & 1 \leq j \leq N, 1 < t \leq T\end{aligned}$$

3. Termination:

$$\text{The best score: } P^* = \max_{i=1}^N v_T(i)$$

$$\text{The start of backtrace: } q_T^* = \operatorname{argmax}_{i=1}^N v_T(i)$$

function VITERBI(*observations* of len T , *state-graph* of len N) **returns** *best-path*, *path-prob*

create a path probability matrix $viterbi[N, T]$

for each state s **from** 1 **to** N **do** ; initialization step

$viterbi[s, 1] \leftarrow \pi_s * b_s(o_1)$

$backpointer[s, 1] \leftarrow 0$

for each time step t **from** 2 **to** T **do** ; recursion step

for each state s **from** 1 **to** N **do**

$viterbi[s, t] \leftarrow \max_{s'=1}^N viterbi[s', t-1] * a_{s', s} * b_s(o_t)$

$backpointer[s, t] \leftarrow \operatorname{argmax}_{s'=1}^N viterbi[s', t-1] * a_{s', s} * b_s(o_t)$

$bestpathprob \leftarrow \max_{s=1}^N viterbi[s, T]$; termination step

$bestpathpointer \leftarrow \operatorname{argmax}_{s=1}^N viterbi[s, T]$; termination step

$bestpath \leftarrow$ the path starting at state $bestpathpointer$, that follows $backpointer[]$ to states back in time

return $bestpath$, $bestpathprob$

Example

Fruit flies like bananas

Tags: NN, VBZ, IN

	¹ Fruit	² Flies	³ Like	⁴ Bananas	</s>
NN					
VBZ					
IN					

Example probabilities

$p(NN \langle s \rangle) = 0.7$	$p(NN NN) = 0.4$	$p(NN VBZ) = 0.5$	$p(NN IN) = 0.7$
$p(VBZ \langle s \rangle) = 0.2$	$p(VBZ NN) = 0.3$	$p(VBZ VBZ) = 0.1$	$p(VBZ IN) = 0.1$
$p(IN \langle s \rangle) = 0.1$	$p(IN NN) = 0.1$	$p(IN VBZ) = 0.2$	$p(IN IN) = 0.1$
$p(\langle /s \rangle \langle s \rangle) = 0.0$	$p(\langle /s \rangle NN) = 0.2$	$p(\langle /s \rangle VBZ) = 0.2$	$p(\langle /s \rangle IN) = 0.0$

$p(fruit NN) = 0.4$	$p(fruit VBZ) = 0.1$	$p(fruit IN) = 0.0$
$p(flies NN) = 0.2$	$p(flies VBZ) = 0.4$	$p(flies IN) = 0.0$
$p(like NN) = 0.1$	$p(like VBZ) = 0.4$	$p(like IN) = 0.3$
$p(bananas NN) = 0.1$	$p(bananas VBZ) = 0.1$	$p(bananas IN) = 0.7$

Example

Fruit flies like bananas

Tags: NN, VBZ, IN

	¹ Fruit	² Flies	³ Like	⁴ Bananas	</s>
NN	$p(NN \langle s \rangle)$ $\times p(\text{fruit} NN)$				
VBZ	$p(VBZ \langle s \rangle)$ $\times p(\text{fruit} VBZ)$				
IN	$p(IN \langle s \rangle)$ $\times p(\text{fruit} IN)$				

Example

Fruit flies like bananas

Tags: NN, VBZ, IN

	¹ Fruit	² Flies	³ Like	⁴ Bananas	</s>
NN	$= 0.7 \times 0.4$ $= 0.28$				
VBZ	$= 0.2 \times 0.1$ $= 0.02$				
IN	$= 0.1 \times 0.0$ $= 0.0$				

Example

Fruit flies like bananas

Tags: NN, VBZ, IN

	¹ Fruit	² Flies	³ Like	⁴ Bananas	</s>
NN	0.28	0.0224			
VBZ	0.02				
IN	0.0				

$$\begin{aligned}
 v_2(NN) &= \max_{i=1 \dots N} v_1(i) \times p(NN|q_i) \times p(Flies|NN) \\
 &= \max\{0.28 \times p(NN|NN), 0.02 \times p(NN|VBZ), 0.0\} \times p(Flies|NN) \\
 &= \max\{0.28 \times 0.4, 0.02 \times 0.5, 0.0\} \times 0.2 \\
 &= \max\{0.112, 0.01, 0.0\} \times 0.2 = 0.0224
 \end{aligned}$$

Example

Fruit flies like bananas

Tags: NN, VBZ, IN

	¹ Fruit	² Flies	³ Like	⁴ Bananas	</s>
NN	0.28	0.0224			
VBZ	0.02	0.0336			
IN	0.0				

$$\begin{aligned}
 v_2(VBZ) &= \max_{i=1 \dots N} v_1(i) \times p(VBZ|q_i) \times p(Flies|VBZ) \\
 &= \max\{0.28 \times p(VBZ|NN), 0.02 \times p(VBZ|VBZ), 0.0\} \times p(Flies|VBZ) \\
 &= \max\{0.28 \times 0.3, 0.02 \times 0.1, 0.0\} \times 0.4 \\
 &= \max\{0.084, 0.02, 0.0\} \times 0.4 = 0.0336
 \end{aligned}$$

Example

Fruit flies like bananas

Tags: NN, VBZ, IN

	¹ Fruit	² Flies	³ Like	⁴ Bananas	</s>
NN	0.28	0.0224			
VBZ	0.02	0.0336			
IN	0.0	0.0			

$$\begin{aligned}
 v_2(IN) &= \max_{i=1 \dots N} v_1(i) \times p(IN|q_i) \times p(Flies|IN) \\
 &= \max\{0.28 \times p(IN|NN), 0.02 \times p(IN|VBZ), 0.0\} \times p(Flies|IN) \\
 &= \max\{0.28 \times 0.1, 0.02 \times 0.2, 0.0\} \times 0.0 \\
 &= \max\{0.028, 0.004, 0.0\} \times 0.0 = 0.0
 \end{aligned}$$

Example

Fruit flies like bananas

Tags: NN, VBZ, IN

	¹ Fruit	² Flies	³ Like	⁴ Bananas	</s>
NN	0.28	0.0224	0.00168		
VBZ	0.02	0.0336			
IN	0.0	0.0			

$$\begin{aligned}
 v_3(NN) &= \max_{i=1 \dots N} v_2(i) \times p(NN|q_i) \times p(Like|NN) \\
 &= \max\{0.0224 \times p(NN|NN), 0.0336 \times p(NN|VBZ), 0.0\} \times p(Like|NN) \\
 &= \max\{0.0224 \times 0.4, 0.0336 \times 0.5, 0.0\} \times 0.1 \\
 &= \max\{0.00896, 0.0168, 0.0\} \times 0.1 = 0.0168
 \end{aligned}$$

Example

Fruit flies like bananas

Tags: NN, VBZ, IN

	¹ Fruit	² Flies	³ Like	⁴ Bananas	</s>
NN	0.28	0.0224	0.00168	0.00014112	?
VBZ	0.02	0.0336	0.002688	0.0000504	
IN	0.0	0.0	0.002016	0.00037632	

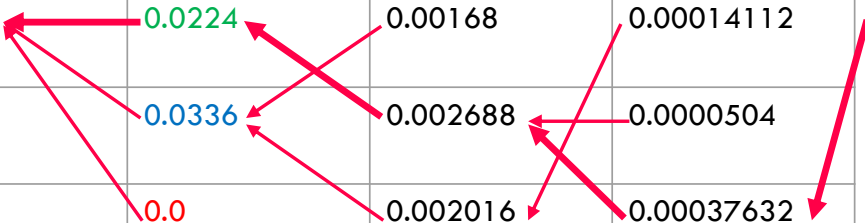
$$\begin{aligned}
 v_5(\langle/s\rangle) &= \max_{i=1\dots N} v_4(i) \times p(\langle/s\rangle|q_i) \\
 &= \max\{0.00014112 \times p(\langle/s\rangle|NN), 0.0000504 \times p(\langle/s\rangle|VBZ), 0.00037632 \times p(\langle/s\rangle|IN)\} \\
 &= \max\{0.00014112 \times 0.2, 0.0000504 \times 0.2, 0.00037632 \times 0.1\} \\
 &= \max\{0.000028224, 0.00001008, 0.000037632\} = 0.000037632
 \end{aligned}$$

Example

Fruit flies like bananas

Tags: NN, VBZ, IN

	¹ Fruit	² Flies	³ Like	⁴ Bananas	</s>
NN	0.28	0.0224	0.00168	0.00014112	0.000037632
VBZ	0.02	0.0336	0.002688	0.0000504	
IN	0.0	0.0	0.002016	0.00037632	



The optimal path: Fruit flies like bananas
 NN NN VBZ IN

Sequences

A primary form of natural language data with many applications.

The classic model of the **Hidden Markov Model**, where a latent (unobserved) variable is key aspect of the inference.

- What's the likelihood? Solved by Forward
- What's the best path? Solved by Viterbi Decoding