Query Result Size Estimation

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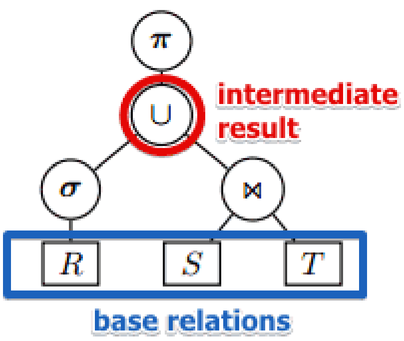
Choosing a physical operator for a Relational Algebra operator depends on

- a particular case and statistics kept in Database System Catalog
- note that this data is kept only for base relations, not for sub-results
  - but we need to be able to estimate them for sub-results as well!



- note that these measures depend only on
  - statistics
  - and Logical Query Plan and not on Physical Query Plan (no matter what physical algorithm we apply we will end with exactly same result)

So the goal:

- for every internal node n estimate parameters
  - $B(n)$ - the number of blocks,
  - $T(n)$ - the number of tuples,
  - $V(n, A_1, \ldots, A_k)$ - the number of distinct values
- note that we can compute $B(n)$ given (1) $T(n)$ (2) size of each tuple in n and (3) size of a block

Projection

for *bag-based* projection $\pi_L(R)$ the general formula is

- $T(\pi_L(R)) = T(R)$: tuples are not eliminated
- but $B(\pi_L(R))$ can change since the size of each tuple changes

Example

Relation $R(A, B, C)$

- A, B - 4 bytes int, C - 100 bytes string
- each tuple has header 12 bytes
- block size: 1024 bytes, and block header is 24 bytes
- $T(R) = 10000, B(R) = 1250$
- how many blocks needed to store $\pi_{A,B}(R)$

Solution

- 1024 - 24 = 1000 bytes per block
- 12 + 4 + 4 = 20 bytes per projected record
- 1000 / 20 = 50 tuples per block
- $B(\pi_{A,B}(R)) = T(\pi_{A,B}(R))/50 = 10000/50 = 200$

If size of records is variable, it's harder.

- In this case usually keep some statistics to estimate the avg size of a projected record

Selection

$\sigma_p(R)$ for some filtering predicate p

- estimation is $T(\sigma_p(R)) = T(R) \times \text{sel}_p(R)$
- where $\text{sel}_p(R)$ is selectivity of predicate p on relation R
  - or the probability that a tuple $t \in R$ will satisfy p
- calculating sel_p depends on the type of predicate p

Equality

Selection $\sigma_{A=c}(R)$ where c is a constant

- $\text{sel}_{A=c}(R) = \dfrac{1}{V(R, A)}$
- where $V(R, A)$ is the number of distinct values in R
- in this case for simplicity we assume the uniform distribution of values in R

Example

- Given: $R(A, B, C), T(R) = 10000, V(R, A) = 50$
- $T(\sigma_{A=10}(R)) = \dfrac{T(R)}{V(R, A)} = \dfrac{10000}{50} = 200$

But typically Databases collect some statistics in the Database System Catalog

range	$[1, 10)$	$[11, 20)$	$[21, 30)$	$[31, 40)$	$[41, 50)$
# of tuples	50	2000	2000	3000	2950

- suppose we have equal-width histogram on A:
- then we can estimate $\text{sel}_{A=10} = \underbrace{\dfrac{50}{10000}}_{50 values} \times \underbrace{\dfrac{1}{10}}_{10 possible values}$

Inequality

Selection $\sigma_{A<c}(R)$ where c is constant

Suppose we don't have any statistics

- in this case we apply a simple following heuristic
  - $\text{sel}_{A<c} = \dfrac{1}{2} or \text{sel}_{A<c}(R) = \dfrac{1}{3}$
- rationale: queries with inequalities usually retrieve a small fraction of the possible tuples, not all of them

Example

- Given: $R(A, B, C), T(R) = 10000$
- estimation: $T(\sigma_{B<100}(R)) = T(R)/3 = 3334$

Better estimates are possible if we have some statistics

- Given: $R(A, B, C), T(R) = 10000$, values of B lay in range $[8, 57]$ distributed uniformly
- Therefore $V(R, B) \leqslant 57 - 8 + 1$ - that many values of B are possible
- Estimate $\text{sel}_{B<10}(R)$
- only $B = 8$ and $B = 9$ satisfy $B < 10$
- therefore $\text{sel}_{B<10}(R) = \dfrac{2}{50} = 0.04$

- and $T(\sigma_{B<10}(R)) = T(R) \times 0.04 = 400$

Inequality

Selection $\sigma_{A \neq c}(R)$ where c is constant

- this is the inverse of $\sigma_{A=c}(R)$
- $\text{sel}_{A \neq c}(R) = \dfrac{V(R, A) - 1}{V(R, A)}$
- this is estimated probability that a tuple doesn't satisfy the predicate $A = c$

Inversion (Not)

Selection $\sigma_{\text{not}(p)}(R)$

- same as for Inequality
- $\text{sel}_{\text{not}(p)}(R) = 1 - \text{sel}_p(R)$

And

Selection $\sigma_{p_1 \wedge p_2}(R)$

- $\sigma_{p_1 \wedge p_2}(R) = \sigma_{p_1} \sigma_{p_2}(R) = \sigma_{p_2} \sigma_{p_1}(R)$ (order doesn't matter)
- in this case, $\text{sel}_{p_1 \wedge p_2}(R) = \text{sel}_{p_1}(R) \times \text{sel}_{p_2}(R)$
- important assumption: p_1 and p_2 are independent
  - for example, doesn't hold for $A > 100 \wedge A < 200$ - because the conditions are correlated in this case

Example

- $T(R) = 10000$, $V(R, A) = 50$
- estimate $T(\sigma_{A=10 \wedge B<10(R)})$:
- $T(R) \times \text{sel}_{A=10}(R) \times \text{sel}_{B<10}(R) = \dfrac{T(R)}{V(R, A) \times 3} = 67$

Or

Selection $\sigma_{p_1 \vee p_2}(R)$

- $\text{sel}_{p_1 \vee p_2}(R) = \min(\text{sel}_{p_1}(R) + \text{sel}_{p_2}(R), 1)$
  - it cannot be greater than 1
- assumptions
  - p_1 and p_2 are independent
  - also they select disjoint sets of tuples (otherwise we would count some tuples twice)

Another way: to use De-Morgan Rule

- $p_1 \vee p_2 \equiv \overline{\overline{p_1} \wedge \overline{p_2}}$ (the line over means **not**)
- $\text{sel}_{p_1 \vee p_2}(R) = 1 - (1 - \text{sel}_{p_1}(R)) \times (1 - \text{sel}_{p_2}(R))$
- in this case we also have the same assumptions

Cartesian Product

$R \times S$

The general formula is:

- $T(R \times S) = T(R) \times T(S)$

Joins

Simple Cases

$R \bowtie S, R(X, Y), S(Y, Z)$ (i.e. we join on Y)

1. R and S have no tuples in common
   - $T(R \bowtie S) = 0$
2. Y is a key in S and a foreign key of R
   - each tuple of R joins exactly with one tuple in S
   - $T(R \bowtie S) = T(R)$
3. almost all tuples of R and S have the same Y value
   - then $T(R \bowtie S) \approx T(R) \times T(S)$ (degenerates to a Cartesian product)

One Join Attribute

$R \bowtie S, R(X, Y), S(Y, Z)$ (i.e. we join on Y)

- it's same as selection with predicate $R.Y = S.Y$

Simplifications

For other harder cases we need the following simplifications:

Containment of Value Sets

- if $R(V, Y) \leqslant V(S, Y)$
- then every value of $Y \in R$ will have a joining tuple with $Y \in S$
- that means: all matched values in X will have a corresponding value in Y - or vice-versa

Preservation of Value Sets

- when joining two relations, all non-matching attributes are not lost
- i.e. they get transfered to the results

- (if we join two relations on Y, R has X and S has Z, then all possible values are going to occur in the output)
- i.e. $V(R \bowtie S, X) = V(R, X)$ and $V(R \bowtie S, Z) = V(S, Z)$

Under there simplification we will consider two cases

Case 1

$V(R, Y) \leqslant V(S, Y)$ (say one-to-many relationship)

- every tuple if R has a match is S by the containment assumption
- or each tuple in R has $\approx \dfrac{T(S)}{V(S, Y)}$ tuples in S (assuming uniform distribution)
- therefore $T(R \bowtie S) = T(R) \times \dfrac{T(S)}{V(S, Y)}$

Case 2

$V(R, Y) \geqslant V(S, Y)$ (say many-to-one relationship)

- each tuple in S has $\approx \dfrac{T(R)}{V(R, Y)}$ tuples in R
- therefore $T(R \bowtie S) = T(S) \times \dfrac{T(R)}{V(R, Y)}$

General Case

$$T(R \bowtie S) = \frac{T(S) \times T(R)}{\min(V(R, Y), V(S, Y))}$$

More Join Attributes

Now assume that we join on two attributes Y_1, Y_2:

- $R(X, Y_1, Y_2) \bowtie S(Y_1, Y_2, Z)$
- (under the same assumptions)
- same as selection with predicate with AND: $R.Y_1 = S.Y_1 \land R.Y_2 = S.Y_2$

Case 1

$V(R, Y_1) \leqslant V(S, Y_1)$ and $V(R, Y_2) \leqslant V(S, Y_2)$

- a tuple in R has $\dfrac{1}{V(S, Y_1)} \times \dfrac{1}{V(S, Y_2)}$ chance of joining with a tuple in S
  - (again assuming uniform distribution)

- therefore $T(R \bowtie S) = T(R) \times \dfrac{T(S)}{V(S, Y_1) \times V(S, Y_2)}$

Case 2

$V(S, Y_1) \leqslant V(R, Y_1)$ and $V(S, Y_2) \leqslant V(R, Y_2)$

- symmetric to Case 1
- chance of tuple from S joining with R is $\dfrac{1}{V(R, Y_1)} \times \dfrac{1}{V(R, Y_2)}$
- therefore $T(R \bowtie S) = T(S) \times \dfrac{T(R)}{V(R, Y_1) \times V(R, Y_2)}$

Case 3

$V(R, Y_1) \leqslant V(S, Y_1)$ but $V(S, Y_2) \leqslant V(R, Y_2)$

- chance of tuple from R joining with S is $\dfrac{1}{V(R, Y_1)} \times \dfrac{1}{V(S, Y_2)}$
- therefore $T(R \bowtie S) = T(R) \times \dfrac{T(S)}{V(R, Y_1) \times V(S, Y_2)}$

Case 4

$V(S, Y_1) \leqslant V(R, Y_1)$ but $V(R, Y_2) \leqslant V(S, Y_2)$

- chance of tuple from R joining with S is $\dfrac{1}{V(S, Y_1)} \times \dfrac{1}{V(R, Y_2)}$
- therefore $T(R \bowtie S) = T(R) \times \dfrac{T(S)}{V(S, Y_1) \times V(R, Y_2)}$

General Formula

$$T(R \bowtie S) = \frac{T(R) \times T(S)}{\max(V(R, Y_1), V(S, Y_1) \times \max(V(R, Y_2), V(S, Y_2)}$$

This formula generalizes to more than 2 joining attributes

See also

- Database System Catalog
- Relational Algebra
- Physical Operators (databases)

Sources

- Database Systems Architecture (ULB)