ST2334 (2018/2019 Semester 2) Solutions to Questions in Tutorial 1

Question 1

(a) $S = \{123, 124, 125, 13, 14, 15, 213, 214, 215, 23, 24, 25, 3, 4, 5\}$
(b) $A = \{3, 4, 5\}$
(c) $B = \{5, 15, 25, 125, 215\}$
(d) $C = \{23, 24, 25, 3, 4, 5\}$
(e) $A \cap B = \{5\} \neq \varnothing$. Hence A and B are not mutually exclusive events.

Question 2

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. $A = \{2, 4, 6, 8, 10\}$. $B = \{1, 3, 5, 7, 9\}$. $C = \{2, 3, 4, 5\}$ and $D = \{1, 6, 7\}$.

(a) $A \cup C = \{2, 3, 4, 5, 6, 8, 10\}$.
(b) $A \cap B = \varnothing$.
(c) $C' = \{1, 6, 7, 8, 9, 10\}$.
(d) Since $A \cap C = \{2, 4\}$ and $D' = \{2, 3, 4, 5, 8, 9, 10\}$. Hence $A \cap C \cap D' = \{2, 4\}$.

Question 3

(a) Number of choices for the hundreds, tens and units positions are 5, 5 and 4 respectively. Hence the number of 3-digit numbers formed = $5\times5\times4 = 100$.
(b) Number of choices for the units, hundreds and tens positions are 1, 4 and 4 respectively. Hence the number of odd 3-digit numbers formed = $4\times4\times1 = 16$.
(c) Number of odd 3-digit numbers > 620 with hundreds position > 6 = $1\times4\times1 = 4$.
Number of odd 3-digit numbers > 620 with hundreds position being 6 = $1\times3\times1 = 3$.
Hence the number of 3-digit numbers > 620 = 4 + 3 = 7.

Question 4

(a) ${}_8P_8 = 8! = 40320$.
(b) Let A, B, C, D represent the four couples. Number of ways to permute these four couples = ${}_4P_4 = 4! = 24$.
For each of these permutations, we can permute the husband and wife in each couple, hence the number of ways to permute = $2!2!2!2! = 16$.
Therefore, the number of ways that they can be seated if each couple is to sit together = $4!\times(2!2!2!2!) = 384$.
(c) Number of ways to permute husbands = 4! and number of ways to permute wives = 4!. Hence the number of ways that they can be seated together if all the men sit together to the right of all the women = $4!\times4! = 576$.

Question 5

(a) $n = 7$ and $r = 5$. Number of choices is given by ${}_7C_5 = 7!/(5!2!) = 21$.
(b) Number of ways to choose three questions from the remaining 5 questions = ${}_5C_3 = 5!/(3!2!) = 10$.
(c) Number of choices for selecting 1 question from the first 2 questions and 4 from the remaining 5 questions = ${}_2C_1 \times {}_5C_4 = (2)(5) = 10$.
Number of choices for selecting 2 question from the first 2 questions and 3 from the remaining 5 questions = ${}_2C_2 \times {}_5C_3 = (1)(10) = 10$.
Therefore, the number of choices if at least one of the first two questions must be answered = 10 + 10 = 20.
(d) Number of choices for selecting exactly 2 questions from the first 3 questions and 3 from the remaining 4 questions = ${}_3C_2 \times {}_4C_3 = (3)(4) = 12$.

Question 6

(a) Each path from A to B can be represented by a permutation of 8 U's and 13 R's (or choose 8 steps (or numbers) out of 21 steps (or numbers) for the U's).
For example,
8 numbers (2, 4, 5, 6, 9, 13, 18, 19) represent the path
RURUUURRURRRURRRRUURR
Number of ways from A to B = ${}_{21}C_8$ = 21!/(13!8!) = 203490.

(b) Number of ways from A to X = ${}_4C_2$ = 4!/(2!2!) = 6. (Choose 2 U's out of 4 steps.)
Number of ways from X to B = ${}_{17}C_6$ = 17!/(6!11!) = 12376. (Choose 6 U's out of 17 steps.)
Hence the number of ways from A to B stopping at X = 6(12376) = 74256.
Therefore, the number of ways from A to B without stopping at X = 203490 – 74256 = 129234.

(c) Number of ways from A to B stopping at X and Y = ${}_4C_2 \times {}_{12}C_4 \times {}_5C_2$ = [4!/(2!2!)]×[12!/(4!8!)]×[5!/(2!3!)] = 29700.

Question 7

(a) ${}_9C_1 \times {}_{27}C_1$ = 9(27) = 243.

(b) ${}_9C_1 \times {}_{27}C_1 \times {}_{15}C_1$ = 9(27)(15) = 3645 ≈ 10 (years).

Question 8

(a) The number of permutations begin with a consonant = ${}_3P_1 \times {}_4P_4$ = 3(4!) = 72.

(b) The number of permutations end with a vowel = ${}_2P_1 \times {}_4P_4$ = 2(4!) = 48.

(c) The number of permutations have the consonants and vowels alternating = 3(2)(2)(1)(1) = 3!2! = 12.
Alternatively, as there is only one pattern CVCVC to meet the specification, we may consider to permute the 3 consonants and to permute the 2 vowels. Hence the number of permutations = ${}_3P_3 \times {}_2P_2$ = (3!)(2!) =12.

Question 9

Number of ways to select 6 houses to be on 1 side of the street = ${}_9C_6$ = 9!/(6!3!) = 84.
For each of these selection, the number of ways to arrange the houses = ${}_6P_6 \times {}_3P_3$ = 6!3! = 4320.
Therefore the number of ways to place these houses = ${}_9C_6 \times {}_6P_6 \times {}_3P_3$ = 362880.

Question 10

Number of ways to arrange 3 oaks, 4 pines and 2 maples = 9!/(3!4!2!) = 1260.