CS1231: Discrete Structures

Tutorial 6

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11 March, 2019

Quick Review

- **Division Algorithm.** The Remainder is Never Negative.
- If $a, b \in \mathbb{Z}$ and $m \in \mathbb{Z}^+$, then a is **congruent** to b modulo m if $m|(a-b)$. We write $a \equiv b \pmod{m}$.

Menu

Question 1

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Question 7

1. Prove that for any odd integer n

$$\left\lfloor \frac{n^2}{4} \right\rfloor = \frac{n^2-1}{4} \qquad \left\lceil \frac{n^2}{4} \right\rceil = \frac{n^2+3}{4}$$

Answer. Since n is odd, $n =$ Thus

$$\frac{n^2}{4} = \qquad , \quad \frac{n^2-1}{4} = \qquad , \quad \frac{n^2+3}{4} = \qquad .$$

Hence

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Answer. Since n is odd, $n = 2k+1$ for some $k \in \mathbb{Z}$. Thus

$$\frac{n^2}{4} = \qquad\qquad, \quad \frac{n^2-1}{4} = \qquad\qquad, \quad \frac{n^2+3}{4} = \qquad\qquad.$$

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Answer. Since n is odd, $n = 2k+1$ for some $k \in \mathbb{Z}$. Thus

$$\frac{n^2}{4} = k^2 + k + \frac{1}{4}, \quad \frac{n^2-1}{4} = \qquad\qquad, \quad \frac{n^2+3}{4} = \qquad\qquad.$$

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2. Show that if n and k are positive integers, then

$$\left\lceil \frac{n}{k} \right\rceil = \left\lfloor \frac{n-1}{k} \right\rfloor + 1$$

Idea. Suppose $(n-1) \div k = q \ldots\ldots r$, $0 \leqslant r \leqslant k-1$.

$\Rightarrow n - 1 =$ $\qquad\qquad$ $\Rightarrow n =$

$\Rightarrow \dfrac{n}{k} =$ $\qquad\qquad\qquad$,

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(a) $(44, 8)$. Idea. Quotient: $\lfloor 44/8 \rfloor = \quad$.
$44 = (\quad) \times 8 + \quad$. Remainder:

(b) $(777, 21)$. Idea. Quotient: $\lfloor 777/21 \rfloor = \quad$.
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(d) $(0, 17)$. Idea. Quotient: $\lfloor 0/17 \rfloor = \quad$.
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Recall

Let $n \in \mathbb{Z}$ and $d \in \mathbb{Z}^+$. $n = dq + r \quad 0 \leqslant r < d$

✍ q: **quotient**. Notation: $q = n$ Div $d = \lfloor n/d \rfloor$;

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(b) $(777, 21)$. Idea. Quotient: $\lfloor 777/21 \rfloor = 37$.
$777 = (37) \times 21 + 0$. Remainder: 0

(c) $(-123, 19)$. Idea. Quotient: $\lfloor -123/19 \rfloor = -7$.
$-123 = (-7) \times 19 + 10$. Remainder: 10

(d) $(0, 17)$. Idea. Quotient: $\lfloor 0/17 \rfloor = 0$.
$0 = (0) \times 17 + 0$. Remainder: 0

(e) $(-100, 101)$. Idea. Quotient: $\lfloor -100/101 \rfloor = -1$.
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Recall

Let $n \in \mathbb{Z}$ and $d \in \mathbb{Z}^+$. $n = dq + r \quad 0 \leqslant r < d$

✍ q: **quotient**. Notation: $q = n$ Div $d = \lfloor n/d \rfloor$;

✍ r: **remainder**. Notation: $r = n$ Mod d

3. What are the quotient and remainder when x is divided by y where $(x, y) =$

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Answer. $(5, 4)$, $(37, 0)$, $(-7, 10)$, $(0, 0)$, $(-1, 1)$.

4. Show that if $a|b$ and $b|a$, then $a = b$ or $a = -b$.

Recall

- ✍ Divisor is never 0.
- ✍ $x|y$ means: $\exists k(y = kx)$;
- ✍ if $x|y$ and $y \neq 0$, then $|x| \leqslant |y|$.

$a|b$

$\Rightarrow$

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Answer. $a|b$ implies $|a|||b|$ implies $|a| \leqslant |b|$. Similarly, $b|a$ implies $|b|||a|$. Thus $|a| = |b|$. This means $a = b$ or $a = -b$.

5. Prove or disprove that if $a \mid bc$, where a, b, and c are positive integers and $a \neq 0$, then $a \mid b$ or $a \mid c$.

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Ans.

5. Prove or disprove that if $a \mid bc$, where a, b, and c are positive integers and $a \neq 0$, then $a \mid b$ or $a \mid c$.
Ans.Disprove: $a = 4$, $b = 6$, $c = 10$.

6. Prove that if d is a positive divisor of a, b and m and $a \equiv b \pmod{m}$, then

$$a/d \equiv b/d \pmod{m/d}$$

Recall

✍ Let $a, b \in \mathbb{Z}$ and $m \in \mathbb{Z}^+$. Then $a \equiv b \pmod{m}$ iff $\exists k \in \mathbb{Z}$ such that $a = b + km$.

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9. Then
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11. $\therefore$
12. $\therefore$
    (Substitute 2,5,8 into 11)
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7. Find counter examples to each of these statements.

(a) If $ac \equiv bc \mod m$, then $a \equiv b \mod m$.

(b) If $a \equiv b \mod m$ and $c \equiv d \mod m$, then $a^c \equiv b^d \mod m$.

Answer.

Recall

Compare with the following

✍ If $a \equiv b \mod m$ and $c \equiv d \mod m$, then $a + b \equiv b + d \mod m$ and $ab \equiv bd \mod m$.

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Answer.

(a) $a = 1$, $b = 2$, $c = 3$, $m = 3$.

Recall

Compare with the following

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Answer.

(a) $a = 1$, $b = 2$, $c = 3$, $m = 3$.

(b) $a = b = 2$, $c = 1$, $d = 4$, $m = 3$.

Recall

Compare with the following

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