CS1231 TUTORIAL 4

1. T,T,F,T,T,T,F

2. Yes. If $n \in B$, then $n = 3j+2$, for some $j \in \mathbb{Z}$. But then $n = 3(j+1)-1$, thus $n \in D$. Conversely, if $n \in D$, then $n = 3j-1$ for some $j \in \mathbb{Z}$. But then $n = 3(j-1)+2$. Thus $n \in B$. So $B = D$.

3. $|A| = 5$

4. $\{a\}$ and $\{a, b, c\}$. Not equal.

5. (a) Let $x \in T_{P \vee Q}$. Then $P(x) \vee Q(x)$ is true. Thus either $P(x)$ is true or $Q(x)$ is true. Hence $x \in T_P$ or $x \in T_Q$, i.e., $x \in T_P \cup T_Q$. The converse is similar and write it up yourself. The second part is also similar.

(b) We only need to note that $P \to Q \equiv \neg P \vee Q$ and $T_{\neg P} = \overline{T_P}$.

6.

$$
\begin{aligned}
(A \times B) \times C = \{&((1,u),m), ((1,u),n), ((1,v),m), ((1,v),n), ((2,u),m), ((2,u),n), \\
&((2,v),m), ((2,v),n), ((3,u),m), ((3,u),n), ((3,v),m), ((3,v),n)\}. \\
A \times B \times C = \{&(1,u,m), (1,u,n), (1,v,m), (1,v,n), (2,u,m), (2,u,n), \\
&(2,v,m), (2,v,n), (3,u,m), (3,u,n), (3,v,m), (3,v,n)\}.
\end{aligned}
$$

Not Equal.

7. $x \notin A$ or $x \notin B$ does not imply $x \notin A \cup B$. Counter example: $A = \{1\}$, $B = \{2\}$, $x = 1$.

8. Suppose $\exists x \in (A - C) \cap (B - C) \cap (A - B)$. Then $x \in A - C$ and $x \in B - C$ and $x \in A - B$. $x \in B - C$ implies $x \in B$ and $x \in A - B$ implies $(x \notin B)$. That's a contradiction. Thus no such x exists, i.e., $(A - C) \cap (B - C) \cap (A - B) = \emptyset$.

9. Suppose $\exists (x, y) \in (A \times B) \cap (C \times D)$. Then $(x, y) \in A \times B$ and $(x, y) \in C \times D$. Thus $x \in A$ and $x \in C$, i.e., $x \in A \cap C$, a contradiction. Thus no such x exists, i.e., $(A \times B) \cap (C \times D) = \emptyset$.

10. (a) False. $A = \{1, 2, 3\}$, $B = \{3\}$, $C = \{2\}$.

(b) False. Same counter example.

(c) True. By definition $A \cup B \subseteq U$. Thus we need to prove $U \subseteq A \cup B$.

Let $x \in U = A \cup \overline{A}$. If $x \in A$, then $x \in A \cup B$. If $x \in \overline{A}$, then $x \in B$. Thus $x \in A \cup B$. In both cases, we have $x \in A \cup B$. Thus $U \subseteq A \cup B$. Thus $A \cup B = U$.

(d) True. Let $X \in P(A \cap B)$. Then $X \subseteq A \cap B$. Therefore $X \subseteq A$ and $X \subseteq B$, i.e., $X \in P(A) \cap P(B)$. Thus we have proved that $P(A \cap B) \subseteq P(A) \cap P(B)$.

Now let $X \in P(A) \cap P(B)$. Then $X \subseteq A$ and $X \subseteq B$, i.e., $X \subseteq A \cap B$. Thus $X \in P(A \cap B)$. This proves $P(A \cap B) \subseteq P(A) \cap P(B)$ and the proof is complete.

11. (a) $\{1, 2, 7, 8\}$. (b) Let $x \in A$. We have 2 cases: (i) $x \in C$. Then $x \notin A \oplus C$. If $x \notin B$, Then $x \in C - B$. Thus $x \in B \oplus C$, a contradiction. Thus $x \in B$. (ii) Direct proof:$x \notin C$. Then $x \in A - C$, and thus $x \in A \oplus C = B \oplus C = (B - C) \cup (C - B)$. $x \notin C$ implies $x \notin C - B$. Thus $x \in B - C$ and hence $x \in B$. In both cases, we have $x \in B$. Thus $A \subseteq B$.

Reversing the role of A and B, we have $B \subseteq A$. Thus $A = B$.

Contrapositive proof for (b): Suppose $A \neq B$. Then either (i) $\exists x \in A$ but $x \notin B$ or (ii) $\exists x \in B$ but $x \notin A$.

(i) If $x \in C$, then $x \notin A \oplus C$ and $x \in B \oplus C$. Thus $A \oplus C \neq B \oplus C$. If $x \notin C$, then $x \in A \oplus C$ and $x \notin B \oplus C$. Thus $A \oplus C \neq B \oplus C$.

Case (ii) is similar. Thus $A \oplus C \neq B \oplus C$ in both cases.