CS1231: Discrete Structures

Tutorial 7

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Quick Review

- prime number, composite number
- If n is composite, then it has a divisor d with $1 < d \leqslant \sqrt{n}$.
- Euclidean Algorithm.
- Base b Expansion.
- Modular Exponentiation. b^n **Mod** m

Menu

Question 1

Question 2

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Question 6

Question 7

1. Determine whether 107 and 113 are primes.

Recall

- ✍ If n is composite, then it has a prime divisor $\leqslant \sqrt{n}$.
- ✍ n is prime if and only if it has no prime divisor $\leqslant \sqrt{n}$

Idea.

$\sqrt{107} \approx 10.344080432788600469738599442627$;

$\sqrt{113} \approx 10.630145812734649407999121914929$.

$\Rightarrow \lfloor\sqrt{107}\rfloor = \quad , \lfloor\sqrt{113}\rfloor = \quad .$

the primes no more than are , , , .

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the primes no more than 10 are 2, 3, 5, 7.

None of these is a divisor of 107 or 113. Thus they are both primes.

2. An integer n is a perfect square if $n = k^2$ for some $k \in \mathbb{Z}$. Prove that a positive integer is a perfect square if and only if it has an odd number of positive divisors.

Idea.

$$\begin{aligned} & \{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n\} \\ = \; & \{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d < \sqrt{n}\} \cup \\ & \{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d > \sqrt{n}\} \cup \\ & \{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d = \sqrt{n}\} \end{aligned}$$

Therefore,

$$\begin{aligned} & |\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n\}| \\ = \; & |\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d < \sqrt{n}\}| + \\ & |\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d > \sqrt{n}\}| + \\ & |\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d = \sqrt{n}\}| \end{aligned}$$

$$\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d = \sqrt{n}\}$$

$$= \begin{cases} \{\sqrt{n}\} & \text{if } n \text{ is a perfect square} \\ \varnothing & \text{if } n \text{ is not a perfect square} \end{cases}$$

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$$= \begin{cases} 1 & \text{if } n \text{ is a perfect square} \\ 0 & \text{if } n \text{ is not a perfect square} \end{cases}$$

$|\{d \in \mathbb{Z}^+ \mid d$ is a divisor of $n \wedge d < \sqrt{n}\}|$
$= |\{d \in \mathbb{Z}^+ \mid d$ is a divisor of $n \wedge d > \sqrt{n}\}|$

Recall

✍ $|A| = |B|$ if and only if there is a bijection between A and B.

Let $f : \{d \in \mathbb{Z}^+ \mid d|n \wedge d < \sqrt{n}\} \to \{d \in \mathbb{Z}^+ \mid d|n \wedge d > \sqrt{n}\}$,
be $f(d) = n/d$. This f is a bijection.

$1-1$:
$f(a) = f(b)$
$\Rightarrow$
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Onto:
For any
$y \in$,
we will show there is a x such that $f(x) = y$, i.e. $n/x = y$
Let $x =$
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$\Rightarrow x|n$ and $x < \sqrt{n}$ (as $y|n$ and $y > \sqrt{n}$)

$$
\begin{aligned}
& |\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n\}| \\
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$$= \qquad +$$

$$|\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d = \sqrt{n}\}|$$

$$= \qquad +$$

if n is a perfect square;

or $= \qquad +$

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& |\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d = \sqrt{n}\}| \\
= \;& 2|\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d < \sqrt{n}\}| + \\
& |\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d = \sqrt{n}\}| \\
& = 2|\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d < \sqrt{n}\}| + \\
& \text{ if } n \text{ is a perfect square;} \\
\text{or} \quad & = 2|\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d < \sqrt{n}\}| + \\
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& = 2|\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d < \sqrt{n}\}| + 1 \\
& \text{ if } n \text{ is a perfect square;} \\
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& = 2|\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d < \sqrt{n}\}| + 1 \\
& \text{ if } n \text{ is a perfect square;} \\
\text{or} \quad & = 2|\{d \in \mathbb{Z}^+ \mid d \text{ is a divisor of } n \wedge d < \sqrt{n}\}| + 0 \\
& \text{ if } n \text{ is not a perfect square}
\end{aligned}
$$

Answer.

If $d \mid n$ and $\sqrt{n} < d \leqslant n$, then $d' \mid n$ where $d' = n/d$ and $1 \leqslant d' < \sqrt{n}$. Thus each positive divisor of n which is less than $\sqrt{n}$ can be paired up with a positive divisor $> \sqrt{n}$. Hence the number of divisors of n that are different from $\sqrt{n}$ is even. $\sqrt{n}$ itself is a divisor iff n is a perfect square. Thus n is a perfect square iff it has an odd number of positive divisors.

3. Find the binary expansion of 321.

$$
\begin{aligned}
321 \div 2 &= \cdots\cdots \quad \text{(the} \qquad\qquad\qquad \text{digit)}\\
\div 2 &= \cdots\cdots\\
\div 2 &= \cdots\cdots\\
\div 2 &= \cdots\cdots\\
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\end{aligned}
$$

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$$
\begin{array}{rcll}
321 \div 2 & = & 160 \cdots\cdots & \text{(the} \qquad\qquad\qquad \text{digit)} \\
160 \div 2 & = & \quad \cdots\cdots & \\
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321 \div 2 &= 160 \cdots\cdots 1 \text{ (the} \qquad\qquad\qquad \text{digit)} \\
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3. Find the binary expansion of 321.

$$
\begin{array}{rcl}
321 \div 2 & = & 160 \cdots\cdots 1 \text{ (the \qquad\qquad digit)} \\
160 \div 2 & = & 80 \cdots\cdots \\
80 \div 2 & = & \quad \cdots\cdots \\
\div 2 & = & \quad \cdots\cdots \\
\div 2 & = & \quad \cdots\cdots \\
\div 2 & = & \quad \cdots\cdots \\
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$$\begin{array}{rcl}
321 \div 2 & = & 160 \cdots\cdots 1 \text{ (the } \qquad\qquad \text{ digit)} \\
160 \div 2 & = & 80 \cdots\cdots 0 \\
80 \div 2 & = & \quad \cdots\cdots \\
\div 2 & = & \quad \cdots\cdots \\
\div 2 & = & \quad \cdots\cdots \\
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10 \div 2 & = & \quad \cdots\cdots \\
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\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)}
\end{aligned}
$$

3. Find the binary expansion of 321.

$$\begin{aligned}
321 \div 2 &= 160 \cdots\cdots 1 \text{ (the } \qquad\qquad\qquad \text{digit)} \\
160 \div 2 &= 80 \cdots\cdots 0 \\
80 \div 2 &= 40 \cdots\cdots 0 \\
40 \div 2 &= 20 \cdots\cdots 0 \\
20 \div 2 &= 10 \cdots\cdots 0 \\
10 \div 2 &= 5 \cdots\cdots 0 \\
5 \div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \quad \text{(the } \qquad\qquad\qquad \text{digit)}
\end{aligned}$$

3. Find the binary expansion of 321.

$$\begin{array}{rcl}
321 \div 2 & = & 160 \cdots\cdots 1 \text{ (the} \qquad\qquad\qquad\quad \text{digit)} \\
160 \div 2 & = & 80 \cdots\cdots 0 \\
80 \div 2 & = & 40 \cdots\cdots 0 \\
40 \div 2 & = & 20 \cdots\cdots 0 \\
20 \div 2 & = & 10 \cdots\cdots 0 \\
10 \div 2 & = & 5 \cdots\cdots 0 \\
5 \div 2 & = & 2 \cdots\cdots \\
2 \div 2 & = & \quad \cdots\cdots \\
\div 2 & = & \quad \cdots\cdots \quad \text{(the} \qquad\qquad\qquad\quad \text{digit)}
\end{array}$$

3. Find the binary expansion of 321.

$$
\begin{aligned}
321 \div 2 &= 160 \cdots\cdots 1 \quad \text{(the} \qquad\qquad\qquad \text{digit)} \\
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40 \div 2 &= 20 \cdots\cdots 0 \\
20 \div 2 &= 10 \cdots\cdots 0 \\
10 \div 2 &= 5 \cdots\cdots 0 \\
5 \div 2 &= 2 \cdots\cdots 1 \\
2 \div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad\qquad \text{digit)}
\end{aligned}
$$

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$$
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40 \div 2 &= 20 \cdots\cdots 0 \\
20 \div 2 &= 10 \cdots\cdots 0 \\
10 \div 2 &= 5 \cdots\cdots 0 \\
5 \div 2 &= 2 \cdots\cdots 1 \\
2 \div 2 &= 1 \cdots\cdots \\
1 \div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad\qquad\qquad \text{digit)}
\end{aligned}
$$

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10 \div 2 &= 5 \cdots\cdots 0 \\
5 \div 2 &= 2 \cdots\cdots 1 \\
2 \div 2 &= 1 \cdots\cdots 0 \\
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5 \div 2 &= 2 \cdots\cdots 1 \\
2 \div 2 &= 1 \cdots\cdots 0 \\
1 \div 2 &= 0 \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)}
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10 \div 2 &= 5 \cdots\cdots 0 \\
5 \div 2 &= 2 \cdots\cdots 1 \\
2 \div 2 &= 1 \cdots\cdots 0 \\
1 \div 2 &= 0 \cdots\cdots 1 \text{ (the } \qquad\qquad \text{digit)}
\end{aligned}
$$

3. Find the binary expansion of 321.

$$\begin{array}{rcl}
321 \div 2 & = & 160 \cdots\cdots 1 \text{ (the right most digit)} \\
160 \div 2 & = & 80 \cdots\cdots 0 \\
80 \div 2 & = & 40 \cdots\cdots 0 \\
40 \div 2 & = & 20 \cdots\cdots 0 \\
20 \div 2 & = & 10 \cdots\cdots 0 \\
10 \div 2 & = & 5 \cdots\cdots 0 \\
5 \div 2 & = & 2 \cdots\cdots 1 \\
2 \div 2 & = & 1 \cdots\cdots 0 \\
1 \div 2 & = & 0 \cdots\cdots 1 \text{ (the} \qquad\qquad\qquad \text{digit)}
\end{array}$$

3. Find the binary expansion of 321.

$$\begin{aligned}
321 \div 2 &= 160 \cdots\cdots 1 \text{ (the right most digit)} \\
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80 \div 2 &= 40 \cdots\cdots 0 \\
40 \div 2 &= 20 \cdots\cdots 0 \\
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10 \div 2 &= 5 \cdots\cdots 0 \\
5 \div 2 &= 2 \cdots\cdots 1 \\
2 \div 2 &= 1 \cdots\cdots 0 \\
1 \div 2 &= 0 \cdots\cdots 1 \text{ (the left most digit)}
\end{aligned}$$

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20 \div 2 &= 10 \cdots\cdots 0 \\
10 \div 2 &= 5 \cdots\cdots 0 \\
5 \div 2 &= 2 \cdots\cdots 1 \\
2 \div 2 &= 1 \cdots\cdots 0 \\
1 \div 2 &= 0 \cdots\cdots 1 \text{ (the left most digit)}
\end{aligned}$$

Answer. $(101000001)_2$.

4. What is the decimal representation of $(101011111)_2$?

$$
\begin{aligned}
&= 2^0 \times \underline{1} \text{ (the \underline{right most} digit)} \\
&= 2^1 \times 1 \\
&= 2^2 \times 1 \\
&= 2^3 \times 1 \\
&= 2^4 \times 1 \\
&= 2^5 \times 0 \\
&= 2^6 \times 1 \\
&= 2^7 \times 0 \\
&= 2^8 \times \underline{1} \text{ (the \underline{left most} digit)} \\
\text{Sum} &=
\end{aligned}
$$

4. What is the decimal representation of $(101011111)_2$?

$$\begin{aligned}
1 &= 2^0 \times \underline{1} \text{ (the \underline{right most} digit)} \\
&= 2^1 \times 1 \\
&= 2^2 \times 1 \\
&= 2^3 \times 1 \\
&= 2^4 \times 1 \\
&= 2^5 \times 0 \\
&= 2^6 \times 1 \\
&= 2^7 \times 0 \\
&= 2^8 \times \underline{1} \text{ (the \underline{left most} digit)} \\
\text{Sum} &=
\end{aligned}$$

4. What is the decimal representation of $(101011111)_2$?

$$\begin{aligned}
1 &= 2^0 \times \underline{1} \text{ (the \underline{right most} digit)} \\
2 &= 2^1 \times 1 \\
&= 2^2 \times 1 \\
&= 2^3 \times 1 \\
&= 2^4 \times 1 \\
&= 2^5 \times 0 \\
&= 2^6 \times 1 \\
&= 2^7 \times 0 \\
&= 2^8 \times \underline{1} \text{ (the \underline{left most} digit)} \\
\text{Sum} &=
\end{aligned}$$

4. What is the decimal representation of $(101011111)_2$?

$$
\begin{array}{rcl}
1 & = & 2^0 \times \underline{1} \text{ (the \underline{right most} digit)} \\
2 & = & 2^1 \times 1 \\
4 & = & 2^2 \times 1 \\
 & = & 2^3 \times 1 \\
 & = & 2^4 \times 1 \\
 & = & 2^5 \times 0 \\
 & = & 2^6 \times 1 \\
 & = & 2^7 \times 0 \\
 & = & 2^8 \times \underline{1} \text{ (the \underline{left most} digit)} \\
\text{Sum} & = &
\end{array}
$$

4. What is the decimal representation of $(101011111)_2$?

$$
\begin{aligned}
1 &= 2^0 \times \underline{1} \text{ (the \underline{right most} digit)} \\
2 &= 2^1 \times 1 \\
4 &= 2^2 \times 1 \\
8 &= 2^3 \times 1 \\
&= 2^4 \times 1 \\
&= 2^5 \times 0 \\
&= 2^6 \times 1 \\
&= 2^7 \times 0 \\
&= 2^8 \times \underline{1} \text{ (the \underline{left most} digit)} \\
\text{Sum} &=
\end{aligned}
$$

4. What is the decimal representation of $(101011111)_2$?

$$
\begin{aligned}
1 &= 2^0 \times \underline{1} \text{ (the \underline{right most} digit)} \\
2 &= 2^1 \times 1 \\
4 &= 2^2 \times 1 \\
8 &= 2^3 \times 1 \\
16 &= 2^4 \times 1 \\
&= 2^5 \times 0 \\
&= 2^6 \times 1 \\
&= 2^7 \times 0 \\
&= 2^8 \times \underline{1} \text{ (the \underline{left most} digit)} \\
\text{Sum} &=
\end{aligned}
$$

4. What is the decimal representation of $(101011111)_2$?

$$
\begin{aligned}
1 &= 2^0 \times \underline{1} \text{ (the \underline{right most} digit)} \\
2 &= 2^1 \times 1 \\
4 &= 2^2 \times 1 \\
8 &= 2^3 \times 1 \\
16 &= 2^4 \times 1 \\
0 &= 2^5 \times 0 \\
&= 2^6 \times 1 \\
&= 2^7 \times 0 \\
&= 2^8 \times \underline{1} \text{ (the \underline{left most} digit)} \\
\text{Sum} &=
\end{aligned}
$$

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$$
\begin{aligned}
1 &= 2^0 \times \underline{1} \text{ (the \underline{right most} digit)} \\
2 &= 2^1 \times 1 \\
4 &= 2^2 \times 1 \\
8 &= 2^3 \times 1 \\
16 &= 2^4 \times 1 \\
0 &= 2^5 \times 0 \\
64 &= 2^6 \times 1 \\
&= 2^7 \times 0 \\
&= 2^8 \times \underline{1} \text{ (the \underline{left most} digit)} \\
\text{Sum} &=
\end{aligned}
$$

4. What is the decimal representation of $(101011111)_2$?

$$
\begin{aligned}
1 &= 2^0 \times \underline{1} \text{ (the } \underline{\text{right most}} \text{ digit)} \\
2 &= 2^1 \times 1 \\
4 &= 2^2 \times 1 \\
8 &= 2^3 \times 1 \\
16 &= 2^4 \times 1 \\
0 &= 2^5 \times 0 \\
64 &= 2^6 \times 1 \\
0 &= 2^7 \times 0 \\
&= 2^8 \times \underline{1} \text{ (the } \underline{\text{left most}} \text{ digit)} \\
\text{Sum} &=
\end{aligned}
$$

4. What is the decimal representation of $(101011111)_2$?

$$\begin{aligned}
1 &= 2^0 \times \underline{1} \text{ (the \underline{right most} digit)} \\
2 &= 2^1 \times 1 \\
4 &= 2^2 \times 1 \\
8 &= 2^3 \times 1 \\
16 &= 2^4 \times 1 \\
0 &= 2^5 \times 0 \\
64 &= 2^6 \times 1 \\
0 &= 2^7 \times 0 \\
256 &= 2^8 \times \underline{1} \text{ (the \underline{left most} digit)} \\
\text{Sum} &=
\end{aligned}$$

4. What is the decimal representation of $(101011111)_2$?

$$\begin{aligned}
1 &= 2^0 \times \underline{1} \text{ (the \underline{right most} digit)} \\
2 &= 2^1 \times 1 \\
4 &= 2^2 \times 1 \\
8 &= 2^3 \times 1 \\
16 &= 2^4 \times 1 \\
0 &= 2^5 \times 0 \\
64 &= 2^6 \times 1 \\
0 &= 2^7 \times 0 \\
256 &= 2^8 \times \underline{1} \text{ (the \underline{left most} digit)} \\
\text{Sum} &= 1 + 2 + 4 + 8 + 16 + 0 + 64 + 0 + 256 =
\end{aligned}$$

4. What is the decimal representation of $(101011111)_2$?

$$
\begin{aligned}
1 &= 2^0 \times \underline{1} \text{ (the \underline{right most} digit)} \\
2 &= 2^1 \times 1 \\
4 &= 2^2 \times 1 \\
8 &= 2^3 \times 1 \\
16 &= 2^4 \times 1 \\
0 &= 2^5 \times 0 \\
64 &= 2^6 \times 1 \\
0 &= 2^7 \times 0 \\
256 &= 2^8 \times \underline{1} \text{ (the \underline{left most} digit)} \\
\text{Sum} &= 1 + 2 + 4 + 8 + 16 + 0 + 64 + 0 + 256 = 351
\end{aligned}
$$

4. What is the decimal representation of $(101011111)_2$?

$$\begin{aligned}
1 &= 2^0 \times \underline{1} \text{ (the \underline{right most} digit)} \\
2 &= 2^1 \times 1 \\
4 &= 2^2 \times 1 \\
8 &= 2^3 \times 1 \\
16 &= 2^4 \times 1 \\
0 &= 2^5 \times 0 \\
64 &= 2^6 \times 1 \\
0 &= 2^7 \times 0 \\
256 &= 2^8 \times \underline{1} \text{ (the \underline{left most} digit)} \\
\text{Sum} &= 1 + 2 + 4 + 8 + 16 + 0 + 64 + 0 + 256 = 351
\end{aligned}$$

Answer. 351.

5. Find the hexadecimal representation of 175627.

Recall

- ✍ Hexadecimal expansion: Base 16.
- ✍ Use A for 10. Use B for 11. Use C for 12. Use D for 13. Use E for 14. Use F for 15.

$$
\begin{aligned}
175627 \div 16 &= \qquad\qquad \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)} \\
\div 16 &= \qquad \cdots\cdots \\
\div 16 &= \quad \cdots\cdots \\
\div 16 &= \cdots\cdots \\
\div 16 &= \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)}
\end{aligned}
$$

5. Find the hexadecimal representation of 175627.

Recall

- ✍ Hexadecimal expansion: Base 16.
- ✍ Use A for 10. Use B for 11. Use C for 12. Use D for 13. Use E for 14. Use F for 15.

$$
\begin{aligned}
175627 \div 16 &= 10976 \cdots\cdots & &\text{(the} \qquad\qquad \text{digit)} \\
10976 \div 16 &= \qquad \cdots\cdots \\
\div 16 &= \qquad \cdots\cdots \\
\div 16 &= \qquad \cdots\cdots \\
\div 16 &= \qquad \cdots\cdots & &\text{(the} \qquad\qquad \text{digit)}
\end{aligned}
$$

5. Find the hexadecimal representation of 175627.

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- ✍ Hexadecimal expansion: Base 16.
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$$
\begin{aligned}
175627 \div 16 &= 10976 \cdots\cdots 11 \text{ (the} \qquad\qquad\qquad \text{digit)} \\
10976 \div 16 &= \qquad \cdots\cdots \\
\div 16 &= \qquad \cdots\cdots \\
\div 16 &= \qquad \cdots\cdots \\
\div 16 &= \qquad \cdots\cdots \quad \text{(the} \qquad\qquad\qquad \text{digit)}
\end{aligned}
$$

5. Find the hexadecimal representation of 175627.

Recall

- ✍ Hexadecimal expansion: Base 16.
- ✍ Use A for 10. Use B for 11. Use C for 12. Use D for 13. Use E for 14. Use F for 15.

$$\begin{aligned}
175627 \div 16 &= 10976 \cdots\cdots 11 \text{ (the} \qquad\qquad\qquad \text{digit)} \\
10976 \div 16 &= 686 \cdots\cdots \\
686 \div 16 &= \quad \cdots\cdots \\
\div 16 &= \quad \cdots\cdots \\
\div 16 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad\qquad \text{digit)}
\end{aligned}$$

5. Find the hexadecimal representation of 175627.

Recall

- ✍ Hexadecimal expansion: Base 16.
- ✍ Use A for 10. Use B for 11. Use C for 12. Use D for 13. Use E for 14. Use F for 15.

$$\begin{aligned}
175627 \div 16 &= 10976 \cdots\cdots 11 \text{ (the} \qquad\qquad \text{digit)} \\
10976 \div 16 &= 686 \cdots\cdots 0 \\
686 \div 16 &= \quad \cdots\cdots \\
\div 16 &= \quad \cdots\cdots \\
\div 16 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)}
\end{aligned}$$

5. Find the hexadecimal representation of 175627.

Recall

- ✍ Hexadecimal expansion: Base 16.
- ✍ Use A for 10. Use B for 11. Use C for 12. Use D for 13. Use E for 14. Use F for 15.

$$
\begin{aligned}
175627 \div 16 &= 10976 \cdots\cdots 11 \text{ (the \qquad\qquad digit)} \\
10976 \div 16 &= 686 \cdots\cdots 0 \\
686 \div 16 &= 42 \cdots\cdots \\
42 \div 16 &= \quad \cdots\cdots \\
\div 16 &= \quad \cdots\cdots \quad \text{ (the \qquad\qquad digit)}
\end{aligned}
$$

5. Find the hexadecimal representation of 175627.

Recall

- ✍ Hexadecimal expansion: Base 16.
- ✍ Use A for 10. Use B for 11. Use C for 12. Use D for 13. Use E for 14. Use F for 15.

$$\begin{aligned}
175627 \div 16 &= 10976 \cdots\cdots 11 \text{ (the \qquad\qquad digit)} \\
10976 \div 16 &= 686 \cdots\cdots 0 \\
686 \div 16 &= 42 \cdots\cdots 14 \\
42 \div 16 &= \quad \cdots\cdots \\
\div 16 &= \quad \cdots\cdots \quad \text{(the \qquad\qquad digit)}
\end{aligned}$$

5. Find the hexadecimal representation of 175627.

Recall

- ✍ Hexadecimal expansion: Base 16.
- ✍ Use A for 10. Use B for 11. Use C for 12. Use D for 13. Use E for 14. Use F for 15.

$$
\begin{aligned}
175627 \div 16 &= 10976 \cdots\cdots 11 \text{ (the digit)} \\
10976 \div 16 &= 686 \cdots\cdots 0 \\
686 \div 16 &= 42 \cdots\cdots 14 \\
42 \div 16 &= 2 \cdots\cdots \\
2 \div 16 &= \quad \cdots\cdots \quad \text{ (the digit)}
\end{aligned}
$$

5. Find the hexadecimal representation of 175627.

Recall

- Hexadecimal expansion: Base 16.
- Use A for 10. Use B for 11. Use C for 12. Use D for 13. Use E for 14. Use F for 15.

$$
\begin{aligned}
175627 \div 16 &= 10976 \cdots\cdots 11 \text{ (the} \qquad\qquad \text{digit)}\\
10976 \div 16 &= 686 \cdots\cdots 0\\
686 \div 16 &= 42 \cdots\cdots 14\\
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\end{aligned}$$

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10976 \div 16 &= 686 \cdots\cdots 0 \\
686 \div 16 &= 42 \cdots\cdots 14 \\
42 \div 16 &= 2 \cdots\cdots 10 \\
2 \div 16 &= 0 \cdots\cdots 2 \text{ (the \qquad\qquad digit)}
\end{aligned}
$$

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Recall

- ✍ Hexadecimal expansion: Base 16.
- ✍ Use A for 10. Use B for 11. Use C for 12. Use D for 13. Use E for 14. Use F for 15.

$$
\begin{aligned}
175627 \div 16 &= 10976 \cdots\cdots 11 \text{ (the right most digit)}\\
10976 \div 16 &= 686 \cdots\cdots 0\\
686 \div 16 &= 42 \cdots\cdots 14\\
42 \div 16 &= 2 \cdots\cdots 10\\
2 \div 16 &= 0 \cdots\cdots 2 \text{ (the} \qquad\qquad \text{digit)}
\end{aligned}
$$

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Recall

- ✍ Hexadecimal expansion: Base 16.
- ✍ Use A for 10. Use B for 11. Use C for 12. Use D for 13. Use E for 14. Use F for 15.

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42 \div 16 &= 2 \cdots\cdots 10 \\
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\begin{aligned}
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42 \div 16 &= 2 \cdots\cdots 10 \\
2 \div 16 &= 0 \cdots\cdots 2 \text{ (the left most digit)}
\end{aligned}
$$

Base 16 representation. $(2(10)(14)0(11))_{16}$.

5. Find the hexadecimal representation of 175627.

Recall

- ✍ Hexadecimal expansion: Base 16.
- ✍ Use A for 10. Use B for 11. Use C for 12. Use D for 13. Use E for 14. Use F for 15.

$$
\begin{aligned}
175627 \div 16 &= 10976 \cdots\cdots 11 \text{ (the right most digit)} \\
10976 \div 16 &= 686 \cdots\cdots 0 \\
686 \div 16 &= 42 \cdots\cdots 14 \\
42 \div 16 &= 2 \cdots\cdots 10 \\
2 \div 16 &= 0 \cdots\cdots 2 \text{ (the left most digit)}
\end{aligned}
$$

Base 16 representation. $(2(10)(14)0(11))_{16}$.

Answer. Hexadecimal representation $(2AE0B)_{16}$.

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b =$, $n =$, $m =$.

(1) Compute the binary expansion of $n =$.

$$
\begin{aligned}
644 \div 2 &= \qquad \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)} \\
\div 2 &= \qquad \cdots\cdots \\
\div 2 &= \qquad \cdots\cdots \\
\div 2 &= \qquad \cdots\cdots \\
\div 2 &= \qquad \cdots\cdots \\
\div 2 &= \qquad \cdots\cdots \\
\div 2 &= \qquad \cdots\cdots \\
\div 2 &= \qquad \cdots\cdots \\
\div 2 &= \qquad \cdots\cdots \\
\div 2 &= \qquad \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)}
\end{aligned}
$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = \quad$, $m = \quad$.

(1) Compute the binary expansion of $n = \quad$.

$$
\begin{aligned}
644 \div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)} \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)}
\end{aligned}
$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m =$.

(1) Compute the binary expansion of $n = 644$.

$$
\begin{aligned}
644 \div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad \text{digit)} \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad \text{digit)}
\end{aligned}
$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$
\begin{aligned}
644 \div 2 &= \quad \cdots\cdots \quad \text{(the \qquad\qquad digit)} \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \quad \text{(the \qquad\qquad digit)}
\end{aligned}
$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$\begin{array}{rcll}
644 \div 2 & = & 322 \cdots\cdots & \text{(the} \qquad\qquad \text{digit)} \\
322 \div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \text{(the} \qquad\qquad \text{digit)}
\end{array}$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$\begin{aligned}
644 \div 2 &= 322 \cdots\cdots 0 \text{ (the} \qquad\qquad \text{digit)} \\
322 \div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)}
\end{aligned}$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$
\begin{aligned}
644 \div 2 &= 322 \cdots\cdots 0 \text{ (the} \qquad\qquad \text{digit)} \\
322 \div 2 &= 161 \cdots\cdots \\
161 \div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)}
\end{aligned}
$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$\begin{array}{rcll}
644 \div 2 & = & 322 \cdots\cdots 0 & \text{(the} \qquad\qquad\qquad \text{digit)} \\
322 \div 2 & = & 161 \cdots\cdots 0 & \\
161 \div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \text{(the} \qquad\qquad\qquad \text{digit)}
\end{array}$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$\begin{array}{rcll}
644 \div 2 & = & 322 \cdots\cdots 0 & \text{(the} \qquad\qquad\qquad \text{digit)} \\
322 \div 2 & = & 161 \cdots\cdots 0 & \\
161 \div 2 & = & 80 \cdots\cdots & \\
80 \div 2 & = & \quad \cdots\cdots & \\
\div 2 & = & \quad \cdots\cdots & \\
\div 2 & = & \quad \cdots\cdots & \\
\div 2 & = & \quad \cdots\cdots & \\
\div 2 & = & \quad \cdots\cdots & \\
\div 2 & = & \quad \cdots\cdots & \\
\div 2 & = & \quad \cdots\cdots & \text{(the} \qquad\qquad\qquad \text{digit)}
\end{array}$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$\begin{array}{rcll}
644 \div 2 & = & 322 \cdots\cdots 0 & \text{(the} \qquad\qquad\qquad \text{digit)} \\
322 \div 2 & = & 161 \cdots\cdots 0 & \\
161 \div 2 & = & 80 \cdots\cdots 1 & \\
80 \div 2 & = & \quad \cdots\cdots & \\
\div 2 & = & \quad \cdots\cdots & \\
\div 2 & = & \quad \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \\
\div 2 & = & \cdots\cdots & \text{(the} \qquad\qquad\qquad \text{digit)}
\end{array}$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$
\begin{array}{rcl}
644 \div 2 & = & 322 \cdots\cdots 0 \text{ (the \qquad\qquad digit)} \\
322 \div 2 & = & 161 \cdots\cdots 0 \\
161 \div 2 & = & 80 \cdots\cdots 1 \\
80 \div 2 & = & 40 \cdots\cdots \\
40 \div 2 & = & \quad \cdots\cdots \\
\div 2 & = & \quad \cdots\cdots \\
\div 2 & = & \cdots\cdots \\
\div 2 & = & \cdots\cdots \\
\div 2 & = & \cdots\cdots \\
\div 2 & = & \cdots\cdots \quad \text{(the \qquad\qquad digit)}
\end{array}
$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$
\begin{aligned}
644 \div 2 &= 322 \cdots\cdots 0 \quad \text{(the} \qquad\qquad\qquad \text{digit)} \\
322 \div 2 &= 161 \cdots\cdots 0 \\
161 \div 2 &= 80 \cdots\cdots 1 \\
80 \div 2 &= 40 \cdots\cdots 0 \\
40 \div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad\qquad \text{digit)}
\end{aligned}
$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$
\begin{aligned}
644 \div 2 &= 322 \cdots\cdots 0 \quad \text{(the} \qquad\qquad \text{digit)} \\
322 \div 2 &= 161 \cdots\cdots 0 \\
161 \div 2 &= 80 \cdots\cdots 1 \\
80 \div 2 &= 40 \cdots\cdots 0 \\
40 \div 2 &= 20 \cdots\cdots \\
20 \div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)}
\end{aligned}
$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$
\begin{aligned}
644 \div 2 &= 322 \cdots\cdots 0 \text{ (the} \qquad\qquad\qquad \text{digit)} \\
322 \div 2 &= 161 \cdots\cdots 0 \\
161 \div 2 &= 80 \cdots\cdots 1 \\
80 \div 2 &= 40 \cdots\cdots 0 \\
40 \div 2 &= 20 \cdots\cdots 0 \\
20 \div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad\qquad \text{digit)}
\end{aligned}
$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$
\begin{array}{rcl}
644 \div 2 & = & 322 \cdots\cdots 0 \text{ (the} \qquad\qquad \text{digit)} \\
322 \div 2 & = & 161 \cdots\cdots 0 \\
161 \div 2 & = & 80 \cdots\cdots 1 \\
80 \div 2 & = & 40 \cdots\cdots 0 \\
40 \div 2 & = & 20 \cdots\cdots 0 \\
20 \div 2 & = & 10 \cdots\cdots \\
10 \div 2 & = & \quad \cdots\cdots \\
\div 2 & = & \quad \cdots\cdots \\
\div 2 & = & \quad \cdots\cdots \\
\div 2 & = & \quad \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)}
\end{array}
$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$\begin{array}{rcl}
644 \div 2 & = & 322 \cdots\cdots 0 \text{ (the} \qquad\qquad \text{digit)} \\
322 \div 2 & = & 161 \cdots\cdots 0 \\
161 \div 2 & = & 80 \cdots\cdots 1 \\
80 \div 2 & = & 40 \cdots\cdots 0 \\
40 \div 2 & = & 20 \cdots\cdots 0 \\
20 \div 2 & = & 10 \cdots\cdots 0 \\
10 \div 2 & = & \cdots\cdots \\
\div 2 & = & \cdots\cdots \\
\div 2 & = & \cdots\cdots \\
\div 2 & = & \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)}
\end{array}$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$
\begin{array}{rcl}
644 \div 2 & = & 322 \cdots\cdots 0 \text{ (the} \qquad\qquad \text{digit)} \\
322 \div 2 & = & 161 \cdots\cdots 0 \\
161 \div 2 & = & 80 \cdots\cdots 1 \\
80 \div 2 & = & 40 \cdots\cdots 0 \\
40 \div 2 & = & 20 \cdots\cdots 0 \\
20 \div 2 & = & 10 \cdots\cdots 0 \\
10 \div 2 & = & 5 \cdots\cdots \\
5 \div 2 & = & \cdots\cdots \\
\div 2 & = & \cdots\cdots \\
\div 2 & = & \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)}
\end{array}
$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$
\begin{aligned}
644 \div 2 &= 322 \cdots\cdots 0 \quad \text{(the} \qquad\qquad \text{digit)} \\
322 \div 2 &= 161 \cdots\cdots 0 \\
161 \div 2 &= 80 \cdots\cdots 1 \\
80 \div 2 &= 40 \cdots\cdots 0 \\
40 \div 2 &= 20 \cdots\cdots 0 \\
20 \div 2 &= 10 \cdots\cdots 0 \\
10 \div 2 &= 5 \cdots\cdots 0 \\
5 \div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)}
\end{aligned}
$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$
\begin{aligned}
644 \div 2 &= 322 \cdots\cdots 0 \quad \text{(the} \qquad\qquad\qquad \text{digit)} \\
322 \div 2 &= 161 \cdots\cdots 0 \\
161 \div 2 &= 80 \cdots\cdots 1 \\
80 \div 2 &= 40 \cdots\cdots 0 \\
40 \div 2 &= 20 \cdots\cdots 0 \\
20 \div 2 &= 10 \cdots\cdots 0 \\
10 \div 2 &= 5 \cdots\cdots 0 \\
5 \div 2 &= 2 \cdots\cdots \\
2 \div 2 &= \quad \cdots\cdots \\
\div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad\qquad \text{digit)}
\end{aligned}
$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$
\begin{array}{rcl}
644 \div 2 & = & 322 \cdots\cdots 0 \text{ (the} \qquad\qquad\qquad \text{digit)} \\
322 \div 2 & = & 161 \cdots\cdots 0 \\
161 \div 2 & = & 80 \cdots\cdots 1 \\
80 \div 2 & = & 40 \cdots\cdots 0 \\
40 \div 2 & = & 20 \cdots\cdots 0 \\
20 \div 2 & = & 10 \cdots\cdots 0 \\
10 \div 2 & = & 5 \cdots\cdots 0 \\
5 \div 2 & = & 2 \cdots\cdots 1 \\
2 \div 2 & = & \quad \cdots\cdots \\
\div 2 & = & \quad \cdots\cdots \quad \text{(the} \qquad\qquad\qquad \text{digit)}
\end{array}
$$

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(1) Compute the binary expansion of $n = 644$.

$$
\begin{array}{rcl}
644 \div 2 & = & 322 \cdots\cdots 0 \text{ (the} \qquad\qquad\qquad \text{digit)} \\
322 \div 2 & = & 161 \cdots\cdots 0 \\
161 \div 2 & = & 80 \cdots\cdots 1 \\
80 \div 2 & = & 40 \cdots\cdots 0 \\
40 \div 2 & = & 20 \cdots\cdots 0 \\
20 \div 2 & = & 10 \cdots\cdots 0 \\
10 \div 2 & = & 5 \cdots\cdots 0 \\
5 \div 2 & = & 2 \cdots\cdots 1 \\
2 \div 2 & = & 1 \cdots\cdots \\
1 \div 2 & = & \quad \cdots\cdots \quad \text{(the} \qquad\qquad\qquad \text{digit)}
\end{array}
$$

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(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$\begin{aligned}
644 \div 2 &= 322 \cdots\cdots 0 \text{ (the} \qquad\qquad \text{digit)} \\
322 \div 2 &= 161 \cdots\cdots 0 \\
161 \div 2 &= 80 \cdots\cdots 1 \\
80 \div 2 &= 40 \cdots\cdots 0 \\
40 \div 2 &= 20 \cdots\cdots 0 \\
20 \div 2 &= 10 \cdots\cdots 0 \\
10 \div 2 &= 5 \cdots\cdots 0 \\
5 \div 2 &= 2 \cdots\cdots 1 \\
2 \div 2 &= 1 \cdots\cdots 0 \\
1 \div 2 &= \quad \cdots\cdots \quad \text{(the} \qquad\qquad \text{digit)}
\end{aligned}$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$\begin{array}{rcll}
644 \div 2 & = & 322 \cdots\cdots 0 & \text{(the} \qquad\qquad\qquad \text{digit)} \\
322 \div 2 & = & 161 \cdots\cdots 0 & \\
161 \div 2 & = & 80 \cdots\cdots 1 & \\
80 \div 2 & = & 40 \cdots\cdots 0 & \\
40 \div 2 & = & 20 \cdots\cdots 0 & \\
20 \div 2 & = & 10 \cdots\cdots 0 & \\
10 \div 2 & = & 5 \cdots\cdots 0 & \\
5 \div 2 & = & 2 \cdots\cdots 1 & \\
2 \div 2 & = & 1 \cdots\cdots 0 & \\
1 \div 2 & = & 0 \cdots\cdots & \text{(the} \qquad\qquad\qquad \text{digit)}
\end{array}$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$\begin{aligned}
644 \div 2 &= 322 \cdots\cdots 0 \text{ (the} \qquad\qquad\qquad \text{digit)} \\
322 \div 2 &= 161 \cdots\cdots 0 \\
161 \div 2 &= 80 \cdots\cdots 1 \\
80 \div 2 &= 40 \cdots\cdots 0 \\
40 \div 2 &= 20 \cdots\cdots 0 \\
20 \div 2 &= 10 \cdots\cdots 0 \\
10 \div 2 &= 5 \cdots\cdots 0 \\
5 \div 2 &= 2 \cdots\cdots 1 \\
2 \div 2 &= 1 \cdots\cdots 0 \\
1 \div 2 &= 0 \cdots\cdots 1 \text{ (the} \qquad\qquad\qquad \text{digit)}
\end{aligned}$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$
\begin{array}{rcl}
644 \div 2 & = & 322 \cdots\cdots 0 \text{ (the right most digit)} \\
322 \div 2 & = & 161 \cdots\cdots 0 \\
161 \div 2 & = & 80 \cdots\cdots 1 \\
80 \div 2 & = & 40 \cdots\cdots 0 \\
40 \div 2 & = & 20 \cdots\cdots 0 \\
20 \div 2 & = & 10 \cdots\cdots 0 \\
10 \div 2 & = & 5 \cdots\cdots 0 \\
5 \div 2 & = & 2 \cdots\cdots 1 \\
2 \div 2 & = & 1 \cdots\cdots 0 \\
1 \div 2 & = & 0 \cdots\cdots 1 \text{ (the} \qquad\qquad \text{digit)}
\end{array}
$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$\begin{aligned}
644 \div 2 &= 322 \cdots\cdots 0 \text{ (the right most digit)} \\
322 \div 2 &= 161 \cdots\cdots 0 \\
161 \div 2 &= 80 \cdots\cdots 1 \\
80 \div 2 &= 40 \cdots\cdots 0 \\
40 \div 2 &= 20 \cdots\cdots 0 \\
20 \div 2 &= 10 \cdots\cdots 0 \\
10 \div 2 &= 5 \cdots\cdots 0 \\
5 \div 2 &= 2 \cdots\cdots 1 \\
2 \div 2 &= 1 \cdots\cdots 0 \\
1 \div 2 &= 0 \cdots\cdots 1 \text{ (the left most digit)}
\end{aligned}$$

6. Find 11^{644} **Mod** 645.

(0) Find b^n **Mod** m

$b = 11$, $n = 644$, $m = 645$.

(1) Compute the binary expansion of $n = 644$.

$$\begin{array}{rcl} 644 \div 2 & = & 322 \cdots\cdots 0 \text{ (the right most digit)} \\ 322 \div 2 & = & 161 \cdots\cdots 0 \\ 161 \div 2 & = & 80 \cdots\cdots 1 \\ 80 \div 2 & = & 40 \cdots\cdots 0 \\ 40 \div 2 & = & 20 \cdots\cdots 0 \\ 20 \div 2 & = & 10 \cdots\cdots 0 \\ 10 \div 2 & = & 5 \cdots\cdots 0 \\ 5 \div 2 & = & 2 \cdots\cdots 1 \\ 2 \div 2 & = & 1 \cdots\cdots 0 \\ 1 \div 2 & = & 0 \cdots\cdots 1 \text{ (the left most digit)} \end{array}$$

Binary Expansion $(1010000100)_2$.

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

(2)

$$
\begin{aligned}
r_0 &= b \textbf{ Mod } m = 11 \textbf{ Mod } 645 = \\
r_1 &= r_0^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 = \\
r_2 &= r_1^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 = \\
r_3 &= r_2^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 = \\
r_4 &= r_3^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 = \\
r_5 &= r_4^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 = \\
r_6 &= r_5^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 = \\
r_7 &= r_6^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 = \\
r_8 &= r_7^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 = \\
r_9 &= r_8^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 =
\end{aligned}
$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

(2)

$$
\begin{aligned}
r_0 &= b \textbf{ Mod } m = 11 \textbf{ Mod } 645 = 11 \\
r_1 &= r_0^2 \textbf{ Mod } m = 11^2 \textbf{ Mod } 645 = \\
r_2 &= r_1^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_3 &= r_2^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_4 &= r_3^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_5 &= r_4^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_6 &= r_5^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_7 &= r_6^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_8 &= r_7^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_9 &= r_8^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 =
\end{aligned}
$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

(2)

$$
\begin{aligned}
r_0 &= b \textbf{ Mod } m = 11 \textbf{ Mod } 645 = 11 \\
r_1 &= r_0^2 \textbf{ Mod } m = 11^2 \textbf{ Mod } 645 = 121 \\
r_2 &= r_1^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = \\
r_3 &= r_2^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_4 &= r_3^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_5 &= r_4^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_6 &= r_5^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_7 &= r_6^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_8 &= r_7^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_9 &= r_8^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 =
\end{aligned}
$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

(2)

$$
\begin{aligned}
r_0 &= b \textbf{ Mod } m = 11 \textbf{ Mod } 645 = 11 \\
r_1 &= r_0^2 \textbf{ Mod } m = 11^2 \textbf{ Mod } 645 = 121 \\
r_2 &= r_1^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451 \\
r_3 &= r_2^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 = \\
r_4 &= r_3^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 = \\
r_5 &= r_4^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 = \\
r_6 &= r_5^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 = \\
r_7 &= r_6^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 = \\
r_8 &= r_7^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 = \\
r_9 &= r_8^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 =
\end{aligned}
$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

(2)

$$\begin{aligned}
r_0 &= b \textbf{ Mod } m = 11 \textbf{ Mod } 645 = 11 \\
r_1 &= r_0^2 \textbf{ Mod } m = 11^2 \textbf{ Mod } 645 = 121 \\
r_2 &= r_1^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451 \\
r_3 &= r_2^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 = 226 \\
r_4 &= r_3^2 \textbf{ Mod } m = 226^2 \textbf{ Mod } 645 = \\
r_5 &= r_4^2 \textbf{ Mod } m = \quad{}^2 \textbf{ Mod } 645 = \\
r_6 &= r_5^2 \textbf{ Mod } m = \quad{}^2 \textbf{ Mod } 645 = \\
r_7 &= r_6^2 \textbf{ Mod } m = \quad{}^2 \textbf{ Mod } 645 = \\
r_8 &= r_7^2 \textbf{ Mod } m = \quad{}^2 \textbf{ Mod } 645 = \\
r_9 &= r_8^2 \textbf{ Mod } m = \quad{}^2 \textbf{ Mod } 645 =
\end{aligned}$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

(2)

$$
\begin{aligned}
r_0 &= b \textbf{ Mod } m = 11 \textbf{ Mod } 645 = 11 \\
r_1 &= r_0^2 \textbf{ Mod } m = 11^2 \textbf{ Mod } 645 = 121 \\
r_2 &= r_1^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451 \\
r_3 &= r_2^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 = 226 \\
r_4 &= r_3^2 \textbf{ Mod } m = 226^2 \textbf{ Mod } 645 = 121 \\
r_5 &= r_4^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = \\
r_6 &= r_5^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_7 &= r_6^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_8 &= r_7^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_9 &= r_8^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 =
\end{aligned}
$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

(2)

$$\begin{aligned}
r_0 &= b \textbf{ Mod } m = 11 \textbf{ Mod } 645 = 11\\
r_1 &= r_0^2 \textbf{ Mod } m = 11^2 \textbf{ Mod } 645 = 121\\
r_2 &= r_1^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451\\
r_3 &= r_2^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 = 226\\
r_4 &= r_3^2 \textbf{ Mod } m = 226^2 \textbf{ Mod } 645 = 121\\
r_5 &= r_4^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451\\
r_6 &= r_5^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 =\\
r_7 &= r_6^2 \textbf{ Mod } m = \quad{}^2 \textbf{ Mod } 645 =\\
r_8 &= r_7^2 \textbf{ Mod } m = \quad{}^2 \textbf{ Mod } 645 =\\
r_9 &= r_8^2 \textbf{ Mod } m = \quad{}^2 \textbf{ Mod } 645 =
\end{aligned}$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

(2)

$$
\begin{aligned}
r_0 &= b \textbf{ Mod } m = 11 \textbf{ Mod } 645 = 11 \\
r_1 &= r_0^2 \textbf{ Mod } m = 11^2 \textbf{ Mod } 645 = 121 \\
r_2 &= r_1^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451 \\
r_3 &= r_2^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 = 226 \\
r_4 &= r_3^2 \textbf{ Mod } m = 226^2 \textbf{ Mod } 645 = 121 \\
r_5 &= r_4^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451 \\
r_6 &= r_5^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 = 226 \\
r_7 &= r_6^2 \textbf{ Mod } m = 226^2 \textbf{ Mod } 645 = \\
r_8 &= r_7^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 = \\
r_9 &= r_8^2 \textbf{ Mod } m = \quad ^2 \textbf{ Mod } 645 =
\end{aligned}
$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

(2)

$$\begin{aligned}
r_0 &= b \textbf{ Mod } m = 11 \textbf{ Mod } 645 = 11 \\
r_1 &= r_0^2 \textbf{ Mod } m = 11^2 \textbf{ Mod } 645 = 121 \\
r_2 &= r_1^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451 \\
r_3 &= r_2^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 = 226 \\
r_4 &= r_3^2 \textbf{ Mod } m = 226^2 \textbf{ Mod } 645 = 121 \\
r_5 &= r_4^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451 \\
r_6 &= r_5^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 = 226 \\
r_7 &= r_6^2 \textbf{ Mod } m = 226^2 \textbf{ Mod } 645 = 121 \\
r_8 &= r_7^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = \\
r_9 &= r_8^2 \textbf{ Mod } m = \quad {}^2 \textbf{ Mod } 645 =
\end{aligned}$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

(2)

$$
\begin{aligned}
r_0 &= b \textbf{ Mod } m = 11 \textbf{ Mod } 645 = 11 \\
r_1 &= r_0^2 \textbf{ Mod } m = 11^2 \textbf{ Mod } 645 = 121 \\
r_2 &= r_1^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451 \\
r_3 &= r_2^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 = 226 \\
r_4 &= r_3^2 \textbf{ Mod } m = 226^2 \textbf{ Mod } 645 = 121 \\
r_5 &= r_4^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451 \\
r_6 &= r_5^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 = 226 \\
r_7 &= r_6^2 \textbf{ Mod } m = 226^2 \textbf{ Mod } 645 = 121 \\
r_8 &= r_7^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451 \\
r_9 &= r_8^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 =
\end{aligned}
$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

(2)

$$
\begin{aligned}
r_0 &= b \textbf{ Mod } m = 11 \textbf{ Mod } 645 = 11 \\
r_1 &= r_0^2 \textbf{ Mod } m = 11^2 \textbf{ Mod } 645 = 121 \\
r_2 &= r_1^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451 \\
r_3 &= r_2^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 = 226 \\
r_4 &= r_3^2 \textbf{ Mod } m = 226^2 \textbf{ Mod } 645 = 121 \\
r_5 &= r_4^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451 \\
r_6 &= r_5^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 = 226 \\
r_7 &= r_6^2 \textbf{ Mod } m = 226^2 \textbf{ Mod } 645 = 121 \\
r_8 &= r_7^2 \textbf{ Mod } m = 121^2 \textbf{ Mod } 645 = 451 \\
r_9 &= r_8^2 \textbf{ Mod } m = 451^2 \textbf{ Mod } 645 = 226
\end{aligned}
$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

r_9	r_8	r_7	r_6	r_5	r_4	r_3	r_2	r_1	r_0
226	451	121	226	451	121	226	451	121	11

(3)

$$11^{644} \textbf{ Mod } 645 \quad = \quad r_0^{a_0} r_1^{a_1} r_2^{a_2} r_3^{a_3} r_4^{a_4} r_5^{a_5} r_6^{a_6} r_7^{a_7} r_8^{a_8} r_9^{a_9} \textbf{ Mod } 645$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

r_9	r_8	r_7	r_6	r_5	r_4	r_3	r_2	r_1	r_0
226	451	121	226	451	121	226	451	121	11

(3)

$$\begin{aligned} 11^{644} \textbf{ Mod } 645 &= r_0^{a_0} r_1^{a_1} r_2^{a_2} r_3^{a_3} r_4^{a_4} r_5^{a_5} r_6^{a_6} r_7^{a_7} r_8^{a_8} r_9^{a_9} \textbf{ Mod } 645 \\ &= r_0^0 r_1^0 r_2^1 r_3^0 r_4^0 r_5^0 r_6^0 r_7^1 r_8^0 r_9^1 \textbf{ Mod } 645 \end{aligned}$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

r_9	r_8	r_7	r_6	r_5	r_4	r_3	r_2	r_1	r_0
226	451	121	226	451	121	226	451	121	11

(3)

$$\begin{aligned} 11^{644} \textbf{ Mod } 645 &= r_0^{a_0} r_1^{a_1} r_2^{a_2} r_3^{a_3} r_4^{a_4} r_5^{a_5} r_6^{a_6} r_7^{a_7} r_8^{a_8} r_9^{a_9} \textbf{ Mod } 645 \\ &= r_0^0 r_1^0 r_2^1 r_3^0 r_4^0 r_5^0 r_6^0 r_7^1 r_8^0 r_9^1 \textbf{ Mod } 645 \\ &= r_2^1 r_7^1 r_9^1 \textbf{ Mod } 645 = r_2 r_7 r_9 \textbf{ Mod } 645 \end{aligned}$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

r_9	r_8	r_7	r_6	r_5	r_4	r_3	r_2	r_1	r_0
226	451	121	226	451	121	226	451	121	11

(3)

$$\begin{aligned}
11^{644} \textbf{ Mod } 645 &= r_0^{a_0} r_1^{a_1} r_2^{a_2} r_3^{a_3} r_4^{a_4} r_5^{a_5} r_6^{a_6} r_7^{a_7} r_8^{a_8} r_9^{a_9} \textbf{ Mod } 645 \\
&= r_0^0 r_1^0 r_2^1 r_3^0 r_4^0 r_5^0 r_6^0 r_7^1 r_8^0 r_9^1 \textbf{ Mod } 645 \\
&= r_2^1 r_7^1 r_9^1 \textbf{ Mod } 645 = r_2 r_7 r_9 \textbf{ Mod } 645 \\
&= 451 \times 121 \times 226 \textbf{ Mod } 645
\end{aligned}$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
r_9	r_8	r_7	r_6	r_5	r_4	r_3	r_2	r_1	r_0
226	451	121	226	451	121	226	451	121	11

(3)

$$\begin{aligned}
11^{644} \textbf{ Mod } 645 &= r_0^{a_0} r_1^{a_1} r_2^{a_2} r_3^{a_3} r_4^{a_4} r_5^{a_5} r_6^{a_6} r_7^{a_7} r_8^{a_8} r_9^{a_9} \textbf{ Mod } 645 \\
&= r_0^0 r_1^0 r_2^1 r_3^0 r_4^0 r_5^0 r_6^0 r_7^1 r_8^0 r_9^1 \textbf{ Mod } 645 \\
&= r_2^1 r_7^1 r_9^1 \textbf{ Mod } 645 = r_2 r_7 r_9 \textbf{ Mod } 645 \\
&= 451 \times 121 \times 226 \textbf{ Mod } 645 \\
&= 12333046 \textbf{ Mod } 645
\end{aligned}$$

Find b^n **Mod** m: $b = 11$, $n = 644$, $m = 645$.

$644 = (1010000100)_2$.

1	0	1	0	0	0	0	1	0	0
a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

r_9	r_8	r_7	r_6	r_5	r_4	r_3	r_2	r_1	r_0
226	451	121	226	451	121	226	451	121	11

(3)

$$\begin{aligned}
11^{644} \textbf{ Mod } 645 &= r_0^{a_0} r_1^{a_1} r_2^{a_2} r_3^{a_3} r_4^{a_4} r_5^{a_5} r_6^{a_6} r_7^{a_7} r_8^{a_8} r_9^{a_9} \textbf{ Mod } 645 \\
&= r_0^0 r_1^0 r_2^1 r_3^0 r_4^0 r_5^0 r_6^0 r_7^1 r_8^0 r_9^1 \textbf{ Mod } 645 \\
&= r_2^1 r_7^1 r_9^1 \textbf{ Mod } 645 = r_2 r_7 r_9 \textbf{ Mod } 645 \\
&= 451 \times 121 \times 226 \textbf{ Mod } 645 \\
&= 12333046 \textbf{ Mod } 645 \\
&= 1
\end{aligned}$$

7. Find $\gcd(14039, 1529) = d$. Find s, t so that $d = 14039s + 1529t$.

Idea. Euclidean Algorithm.

$$\begin{aligned} 14039 &= \underline{1529} \times \quad + \\ \underline{1529} &= \qquad \times \quad + \\ &= \qquad \times \quad + \end{aligned}$$

Remark: the procedure stops when the remainder becomes 0; the <u>last remainder above 0</u> is the greatest common divisor.

$$\begin{aligned} &= \\ &= \\ &= 14039 \times \qquad + 1529 \times \end{aligned}$$

7. Find $\gcd(14039, 1529) = d$. Find s, t so that $d = 14039s + 1529t$.

Idea. Euclidean Algorithm.

$$\begin{aligned} 14039 &= \underline{1529} \times 9 + \\ \underline{1529} &= \qquad \times \quad + \\ &= \qquad \times \quad + \end{aligned}$$

Remark: the procedure stops when the remainder becomes 0; the <u>last remainder above 0</u> is the greatest common divisor.

$$\begin{aligned} &= \\ &= \\ &= 14039 \times \qquad + 1529 \times \end{aligned}$$

7. Find $\gcd(14039, 1529) = d$. Find s, t so that $d = 14039s + 1529t$.

Idea. Euclidean Algorithm.

$$
\begin{aligned}
14039 &= \underline{1529} \times 9 + \underline{278} \\
\underline{1529} &= \underline{278} \times \quad + \\
278 &= \qquad \times \quad +
\end{aligned}
$$

Remark: the procedure stops when the remainder becomes 0; the <u>last remainder above 0</u> is the greatest common divisor.

$$
\begin{aligned}
&= \\
&= \\
&= 14039 \times \qquad + 1529 \times
\end{aligned}
$$

7. Find $\gcd(14039, 1529) = d$. Find s, t so that $d = 14039s + 1529t$.

Idea. Euclidean Algorithm.

$$\begin{array}{rcl} 14039 & = & \underline{1529} \times 9 + \underline{278} \\ \underline{1529} & = & \underline{278} \times 5 + \\ 278 & = & \quad\;\; \times \quad + \end{array}$$

Remark: the procedure stops when the remainder becomes 0; the <u>last remainder above 0</u> is the greatest common divisor.

$$\begin{array}{rl} = & \\ = & \\ = & 14039 \times \qquad\quad + 1529 \times \end{array}$$

7. Find $\gcd(14039, 1529) = d$. Find s, t so that $d = 14039s + 1529t$.

Idea. Euclidean Algorithm.

$$\begin{aligned}
14039 &= \underline{1529} \times 9 + \underline{278} \\
\underline{1529} &= \underline{278} \times 5 + 139 \\
278 &= 139 \times \quad +
\end{aligned}$$

Remark: the procedure stops when the remainder becomes 0; the <u>last remainder above 0</u> is the greatest common divisor.

$$\begin{aligned}
139 &= \\
&= \\
&= 14039 \times \qquad\quad + 1529 \times
\end{aligned}$$

7. Find $\gcd(14039, 1529) = d$. Find s, t so that $d = 14039s + 1529t$.
Idea. Euclidean Algorithm.

$$\begin{aligned} 14039 &= \underline{1529} \times 9 + \underline{278} \\ \underline{1529} &= \underline{278} \times 5 + 139 \\ 278 &= 139 \times 2 + \end{aligned}$$

Remark: the procedure stops when the remainder becomes 0; the <u>last remainder above 0</u> is the greatest common divisor.

$$\begin{aligned} 139 &= \\ &= \\ &= 14039 \times \qquad + 1529 \times \end{aligned}$$

7. Find $\gcd(14039, 1529) = d$. Find s, t so that $d = 14039s + 1529t$.

Idea. Euclidean Algorithm.

$$\begin{aligned} 14039 &= \underline{1529} \times 9 + \underline{278} \\ \underline{1529} &= \underline{278} \times 5 + 139 \\ 278 &= 139 \times 2 + 0 \end{aligned}$$

Remark: the procedure stops when the remainder becomes 0; the <u>last remainder above 0</u> is the greatest common divisor.

$$\begin{aligned} 139 &= \\ &= \\ &= 14039 \times \qquad + 1529 \times \end{aligned}$$

7. Find $\gcd(14039, 1529) = d$. Find s, t so that $d = 14039s + 1529t$.

Idea. Euclidean Algorithm.

$$\begin{aligned}
14039 &= \underline{1529} \times 9 + \underline{278} \\
\underline{1529} &= \underline{278} \times 5 + 139 \\
278 &= 139 \times 2 + 0
\end{aligned}$$

Remark: the procedure stops when the remainder becomes 0; the <u>last remainder above 0</u> is the greatest common divisor.

$$\begin{aligned}
139 &= 1529 - 278 \times 5 \\
&= \\
&= 14039 \times \qquad + 1529 \times
\end{aligned}$$

7. Find $\gcd(14039, 1529) = d$. Find s, t so that $d = 14039s + 1529t$.

Idea. Euclidean Algorithm.

$$\begin{aligned} 14039 &= \underline{1529} \times 9 + \underline{278} \\ \underline{1529} &= \underline{278} \times 5 + 139 \\ 278 &= 139 \times 2 + 0 \end{aligned}$$

Remark: the procedure stops when the remainder becomes 0; the <u>last remainder above 0</u> is the greatest common divisor.

$$\begin{aligned} 139 &= 1529 - 278 \times 5 \\ &= 1529 - \qquad\qquad\qquad\qquad \times 5 \\ &= 14039 \times \qquad + 1529 \times \end{aligned}$$

7. Find $\gcd(14039, 1529) = d$. Find s, t so that $d = 14039s + 1529t$.
Idea. Euclidean Algorithm.

$$\begin{aligned}
14039 &= \underline{1529} \times 9 + \underline{278} \\
\underline{1529} &= \underline{278} \times 5 + 139 \\
278 &= 139 \times 2 + 0
\end{aligned}$$

Remark: the procedure stops when the remainder becomes 0; the <u>last remainder above 0</u> is the greatest common divisor.

$$\begin{aligned}
139 &= 1529 - 278 \times 5 \\
&= 1529 - (14039 - 1529 \times 9) \times 5 \\
&= 14039 \times \qquad + 1529 \times
\end{aligned}$$

7. Find $\gcd(14039, 1529) = d$. Find s, t so that $d = 14039s + 1529t$.

Idea. Euclidean Algorithm.

$$\begin{aligned} 14039 &= \underline{1529} \times 9 + \underline{278} \\ \underline{1529} &= \underline{278} \times 5 + 139 \\ 278 &= 139 \times 2 + 0 \end{aligned}$$

Remark: the procedure stops when the remainder becomes 0; the <u>last remainder above 0</u> is the greatest common divisor.

$$\begin{aligned} 139 &= 1529 - 278 \times 5 \\ &= 1529 - (14039 - 1529 \times 9) \times 5 \\ &= 14039 \times (-5) + 1529 \times \end{aligned}$$

7. Find $\gcd(14039, 1529) = d$. Find s, t so that $d = 14039s + 1529t$.

Idea. Euclidean Algorithm.

$$
\begin{aligned}
14039 &= \underline{1529} \times 9 + \underline{278} \\
\underline{1529} &= \underline{278} \times 5 + 139 \\
278 &= 139 \times 2 + 0
\end{aligned}
$$

Remark: the procedure stops when the remainder becomes 0; the <u>last remainder above 0</u> is the greatest common divisor.

$$
\begin{aligned}
139 &= 1529 - 278 \times 5 \\
&= 1529 - (14039 - 1529 \times 9) \times 5 \\
&= 14039 \times (-5) + 1529 \times 46
\end{aligned}
$$

Answer.

$$
\begin{aligned}
14039 \textbf{ Mod } 1529 &= 278\\
1529 \textbf{ Mod } 278 &= 139\\
278 \textbf{ Mod } 139 &= 0
\end{aligned}
$$

Thus $\gcd(14039, 1529) = 139$.
1529 **Div** $278 = 5$ and 14039 **Div** $1529 = 9$. Thus
$1529 = 5 \cdot 278 + 139$ and $14039 = 9 \cdot 1529 + 278$. Hence
$139 = 1529 - 5 \cdot 278 = 1529 - 5(14039 - 9 \cdot 1529) = 46 \cdot 1529 - 5 \cdot 14039$