NUS National University of Singapore
School of Computing

Graph Colouring
(Other Cool Applications)
Video 5.8

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Experience the fun of problem solving

Modelling the Tourist Problem

Recap: TP with Graph Colouring

Graph Model	Tourist Problem			
Nodes	*places*			
Edges	*conflicts (between pairs of places)*			
Colors	*days*			
Others	*tourists*			

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A natural question to ask…

Q: Where else can we use Graph Colouring?

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Many Application of GC

Where is Graph Colouring used?

- The Tourist Problem – [DONE]
- Map Colouring,
- Fish in a Tank,
- Frequency assignment for radio stations,
- Time Table Scheduling,
- And a whole lot more…

Graph Colouring has many uses…

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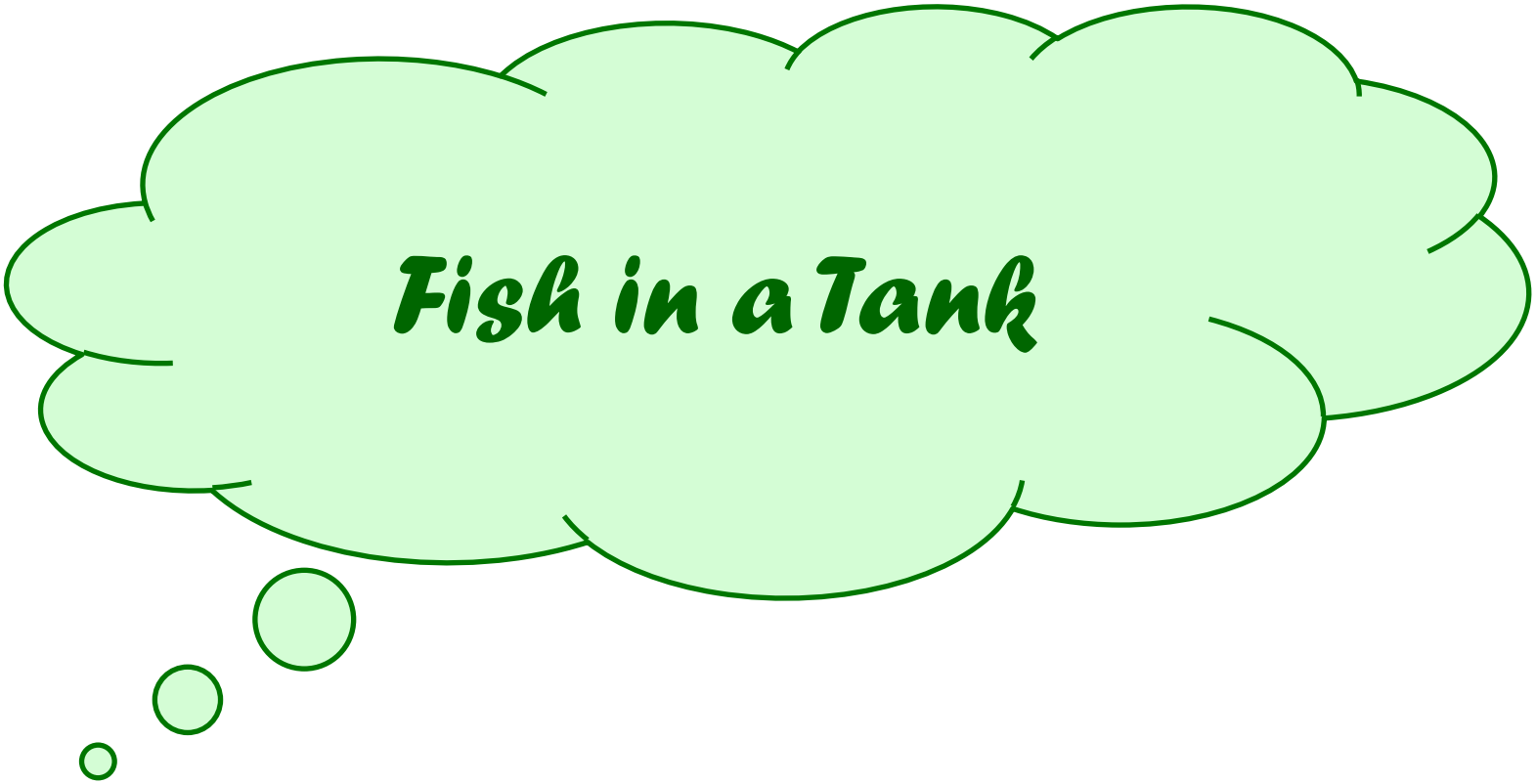

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Fish in a Tank (1)

A tropical fish hobbyist has six different species of fishes: *Angelfish, Betta, Catfish, Danio, Eel and Fingerfish* (denoted here by A, B, C, D, E, and F respectively).

Some fish can be kept in the same tank, while others *cannot* be together.

(many factors, including predator-prey relationship, water conditions, and size, etc)

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Fish in a Tank (2a)

He quickly did some research and produced the following table:

Fish	Cannot be together with
A	*B, C*
B	*A, C, E*
C	*A, B, D, E*
D	*C, F*
E	*B, C, F*
F	*D, E*

"Conflict" here is NOT exactly the same as for TP.

Here, C conflicts with B,
C conflicts with D, but,
B does not conflict with D.

PQ: Have you seen the problem before? …similar problem? …similar unknown?

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Fish in a Tank (2b)

You quickly did some research and produced the following table:

Fish	Cannot be together with
A	*B, C*
B	*A, C, E*
C	*A, B, D, E*
D	*C, F*
E	*B, C, F*
F	*D, E*

Q: Can we use graph colouring?

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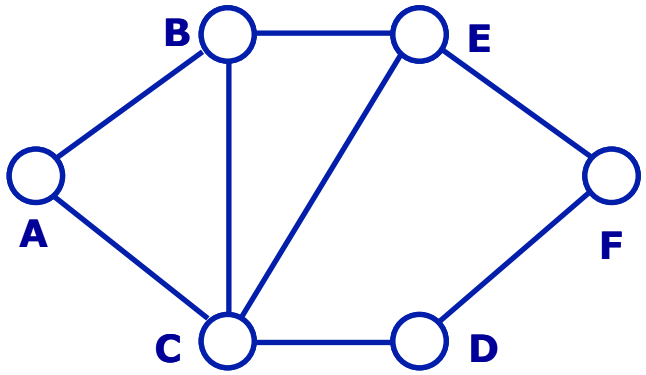
Fish in a Tank (3)

Fish	Cannot be together with
A	*B, C*
B	*A, C, E*
C	*A, B, D, E*
D	*C, F*
E	*B, C, F*
F	*D, E*



$G = (V, E)$

Q: If we use graph colouring, what are vertices, edges?

Vertices = Fishes

Edges are conflicts between pairs of fishes.
("Cannot be together with")

The edges: (A,B), (A,C),
(B,A), (B,C), (B,E),
(C,A), (C,B), (C,D), (C,E),…

And you can finish the rest…

Some Observations…

Asking Question via
Polya PS Process works!

For "Fish in a Tank"…

Obvious what are vertices (the fish)

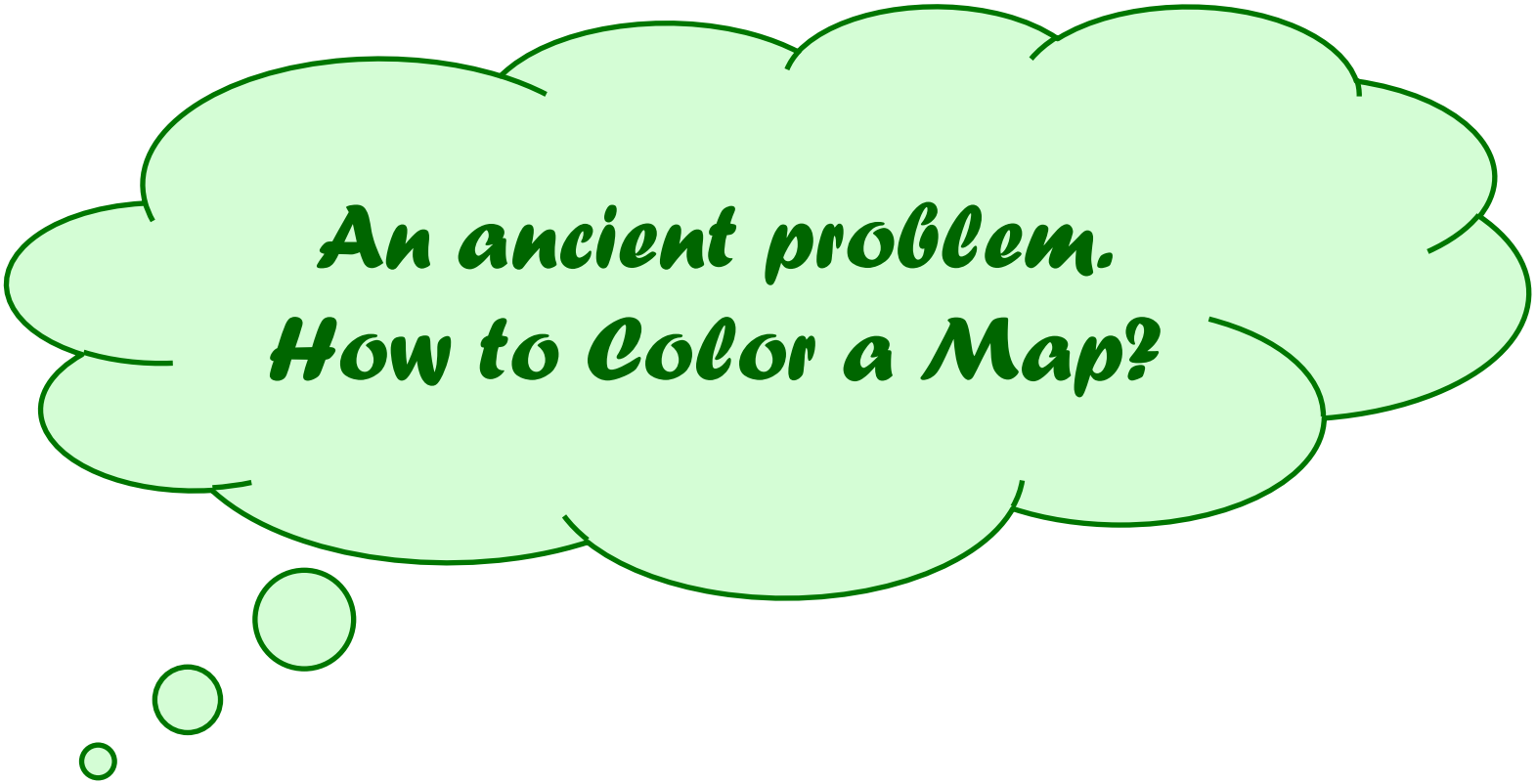
What about the edges?

Edges connect pairs of conflict vertices
(Here, "conflict" table "looks" similar,
but is not the same as table for TP.)

To model problems with GC,

must carefully decide what are the vertices, &
what are the edges (*this part may be tricky*)

One application done, NEXT…



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The Map Coloring Problem (1)

We want to color countries (oceans, lakes, and islands) on a map so that no two adjacent countries have the same colour.

Q1: How many colours do we need?

Problem first posed in 1852
(by Guthrie → deMorgan)

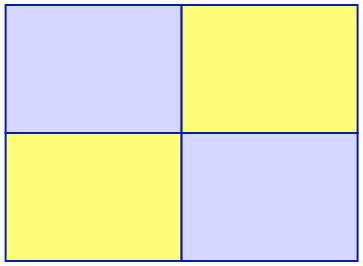
The Map Coloring Problem (2a)

We want to color countries (oceans, lakes, and islands) on a map so that no two adjacent countries have the same colour.

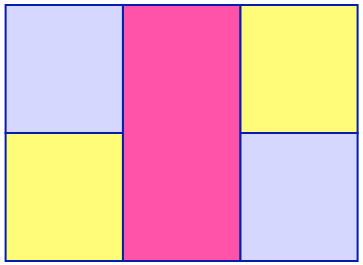
PQ: Let's first try some small examples.

The Map Coloring Problem (2b)

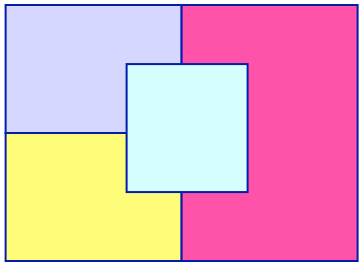
We want to color countries (oceans, lakes, and islands) on a map so that no two adjacent countries have the same colour.



Two colors



Three colors



Four colors

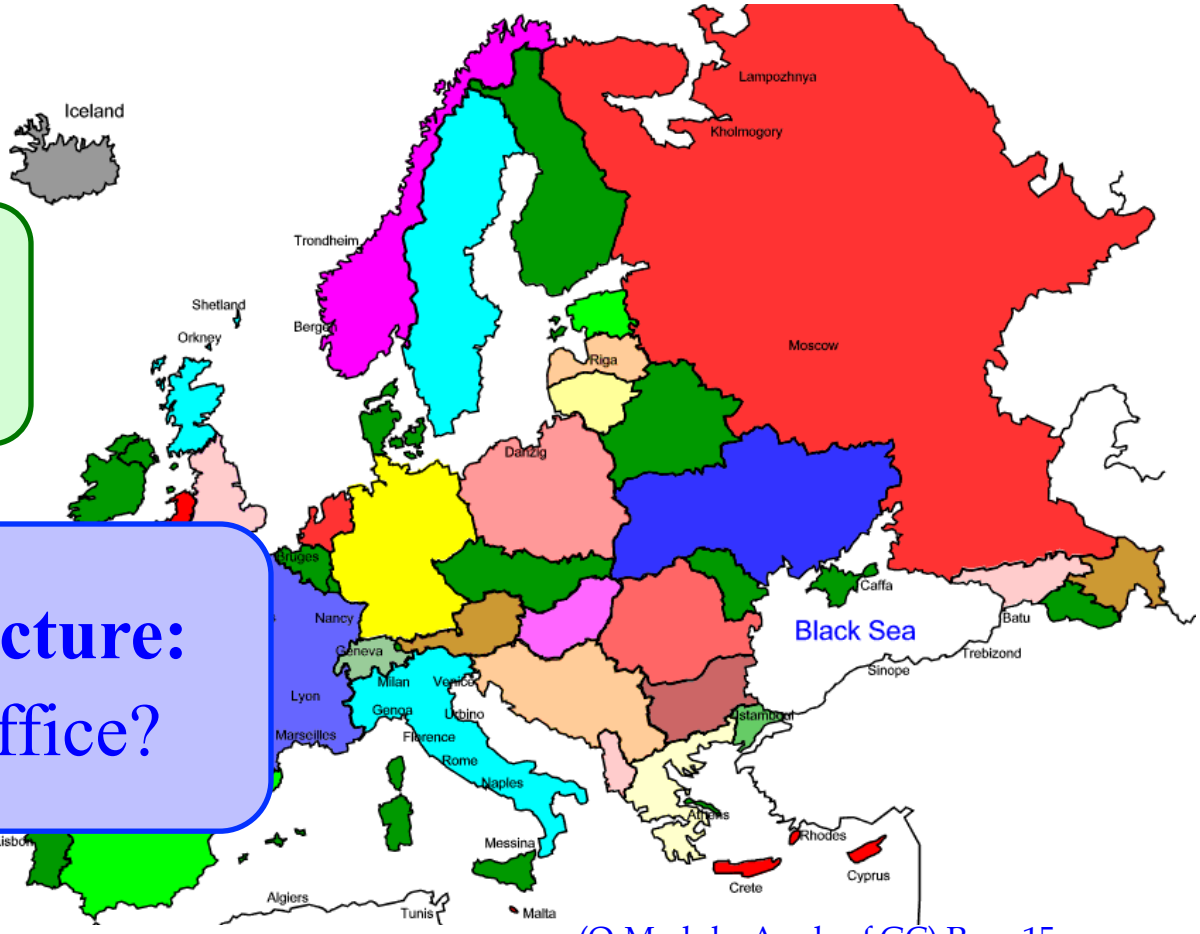
What about 5 colours?
(This proved to be difficult!)

The Map Coloring Problem (3)

We want to color countries (oceans, lakes, and islands) on a map so that no two adjacent countries have the same colour.

A legal colouring.
But uses > 10 colours!

Four Color Conjecture:
Does 4 colours suffice?



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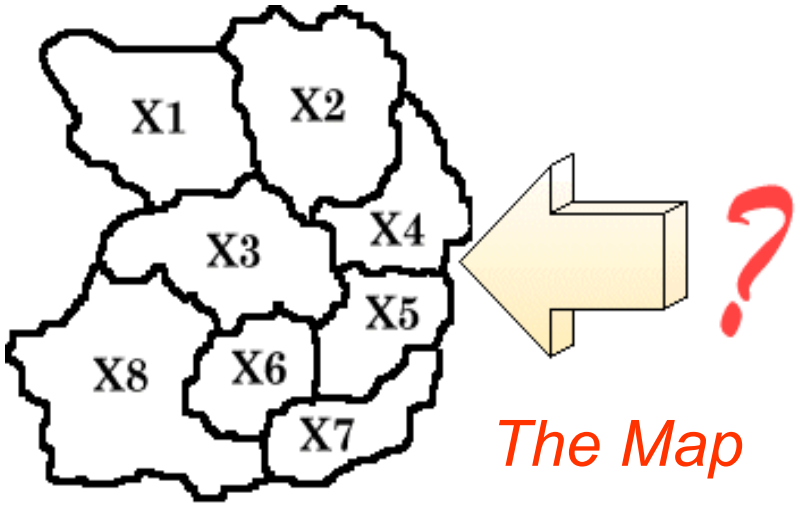
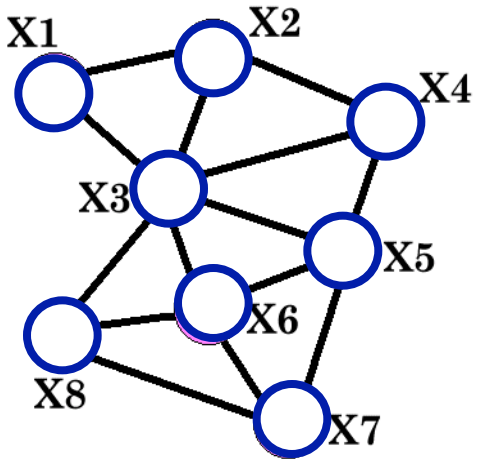
Map and Graph Coloring (4)



The Map



The Graph model

Use Graph Model:
What are vertices, edges?

vertices are *countries*
(or lakes, islands, etc)

edge (x,y) *iff* x and y share a ***common border***

[a common point is *not counted*]

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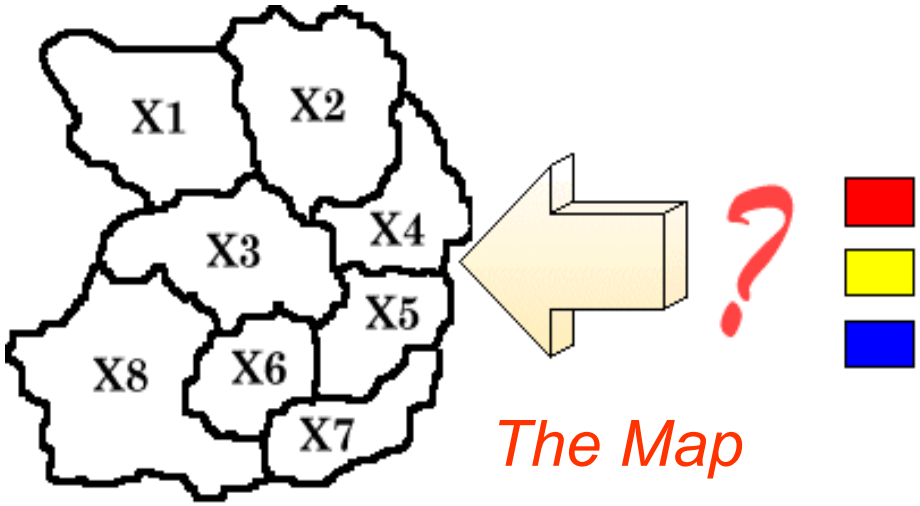
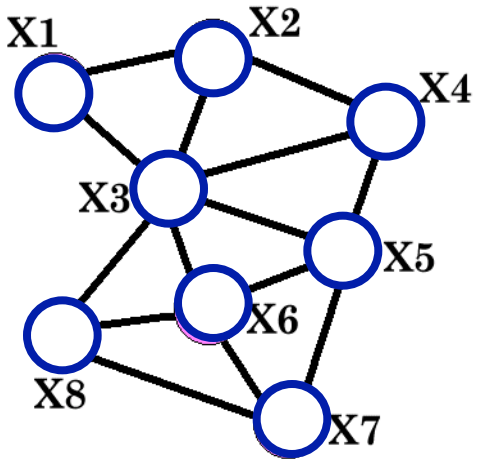
Map and Graph Coloring (5)



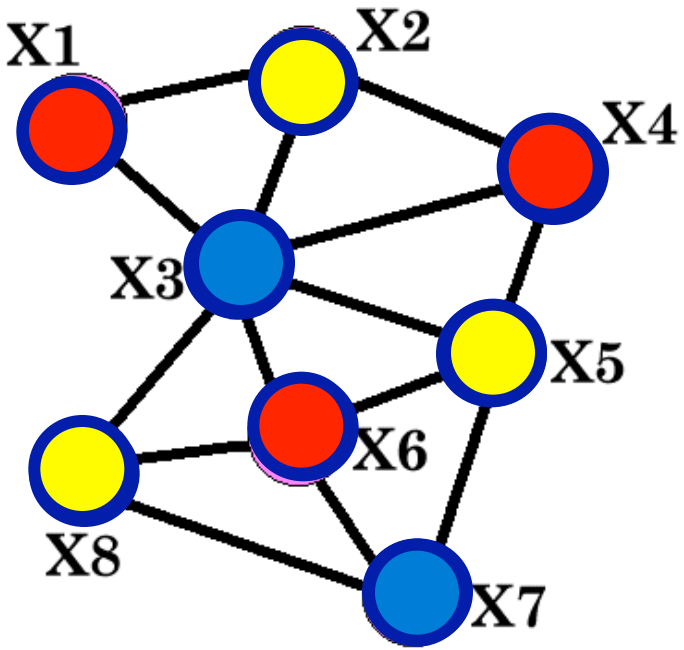
The Map

?

Now, to colour the graph.
Use an informal algorithm…



Graph model



Coloring the graph

The Four Color Conjecture (6)

Question: (from 1852…)

Does 4 colours suffice to colour *all* maps?

HISTORY :

1852 Conjecture (*Guthrie* → *DeMorgan*)

1878 Publication (*Cayley*)

1879-1891 …Many incorrect "proof", but "5 suffices"

1913 …Reducibility, connexity (*Birkhoff*)

1969 Discharging (*Heesch*)

1976 Four Color Theorem (*Appel & Haken*) @UIUC

1995 Streamlining (*Robertson & al.*)

2005 COQ proof (*Gonthier*)

The 1976 proof (6b)

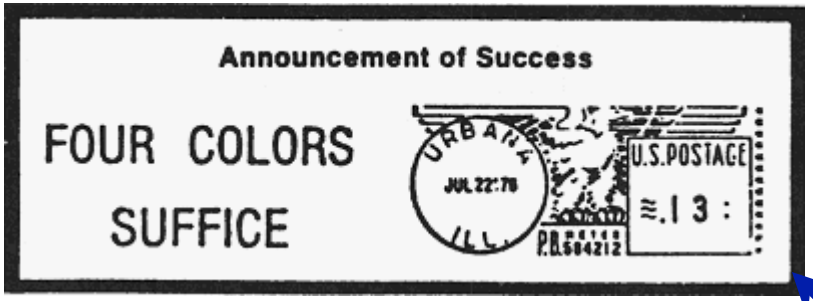
Many mathematician did not accept the result

It required computer assistance (with some parts of the proof – taking several thousand hours of computer time).

Detractors say these cannot be mathematically validated…

To know more, see my references on FCT.

Postage Stamp @ UIUC



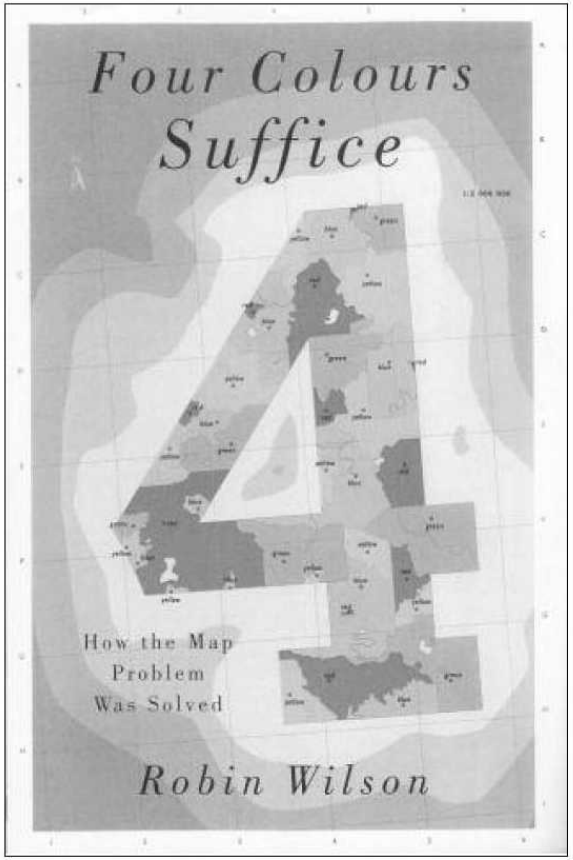
University of Illinois
at Urbana-Champaign

DEPARTMENT OF MATHEMATICS
273 Altgeld Hall
1409 West Green Street
Urbana, Illinois 61801

FOUR COLORS
SUFFICE

Mr. Donald White
120 E. 13th Ave. N.
Columbus, OH 43201

My first semester, grad school @ UIUC




Kenneth Appel and Wofgang Haken in the 1970s

In Fall 1979,
I took a course
MA313 *Combinatorics*
taught by Ken Appel

Ken Appel & Wolfgang Haken @UIUC
(University of Illinois at Urbana-Champaign

The other applications?

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Activity #3: Optional

Fun with Other Applications of Graph Colouring.

Instruction:

Download and print a copy of GC-Activity-3.pdf.

Try out these other applications of Graph Colouring for FUN!

Can you find more applications of Graph Colouring?

Post them on the IVLE Forum!

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Modeling with Graph Colouring

	Tourist Problem	Fish in a tank	Frequency Assignment	Map Coloring
Nodes	*places*	*fishes*	*radio stations*	*Countries*
Edges / Conflicts	*tourist want to visit both places*	*cannot be placed in same tank*	*interference if placed too near*	*share a common border*
Colors	*bus trips to places*	*fish tanks*	*signal frequencies*	*color*
Others	*The tourists*	--		

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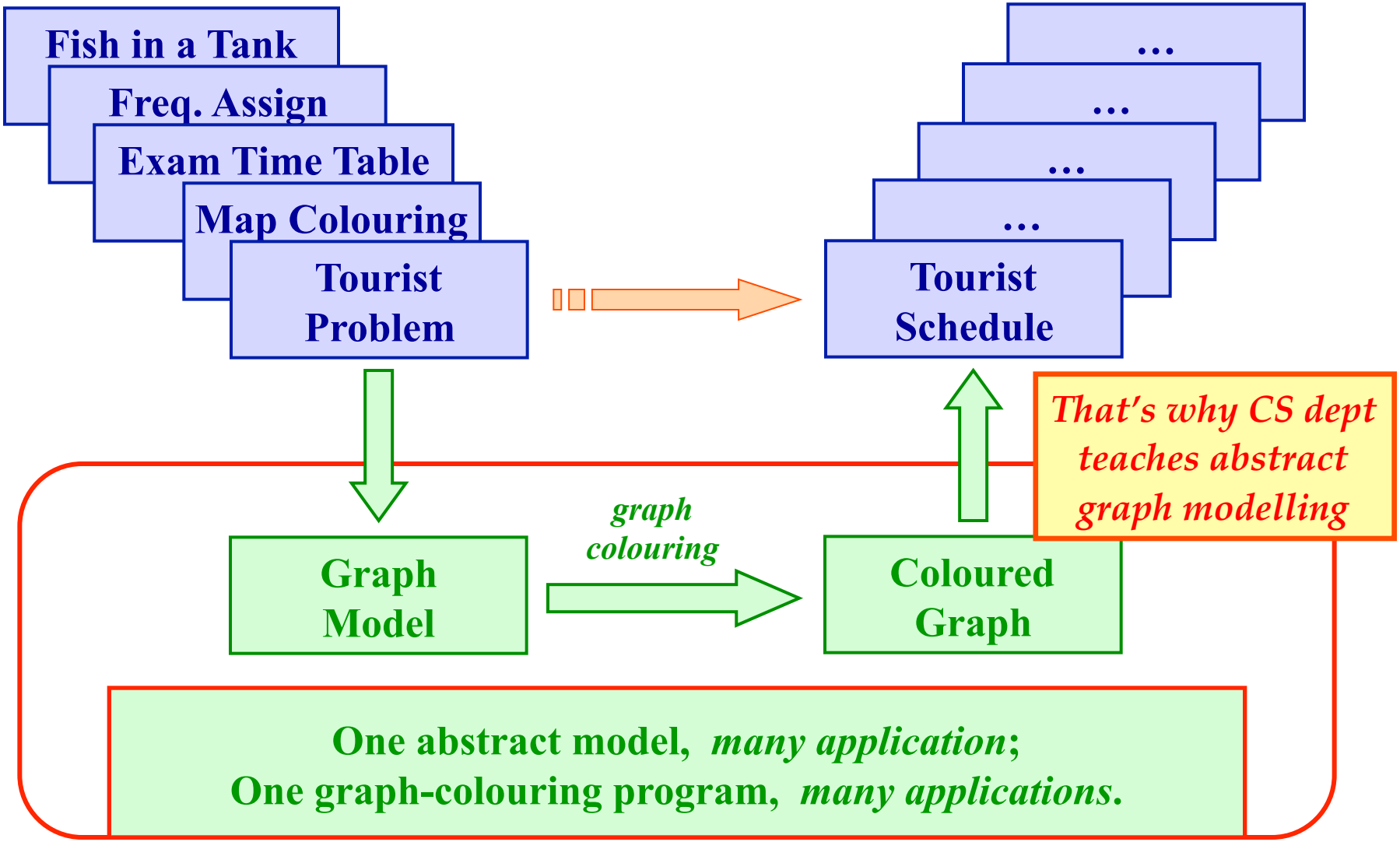
Why CS dept teach abstract problems?



References…

On Graph Coloring and Applications:

1. https://en.wikipedia.org/wiki/Graph_coloring
2. http://www.geom.uiuc.edu/~zarembe/graph3.html
3. http://www.colorado.edu/education/DMP/activities/graph/ddghnd03.html
4. https://www.youtube.com/watch?v=y4RAYQjKb5Y
5. Lots of other links available

On the Four Color Theorem:

1. http://en.wikipedia.org/wiki/Four_color_theorem
2. http://www.maa.org/reviews/fourcolors.html
3. http://www.math.gatech.edu/~thomas/FC/fourcolor.html
4. http://www.mathpages.com/home/kmath266/kmath266.htm

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Computational Thinking involves

Problem Formulation

Abstraction

Thinking Abstractly, Algorithmically, Recursively…

Algorithm Design

Decomposition Composition

Finding a Pattern

…

Developing ITeMS

(End of video 5.8)

If you want to contact me,

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School of Computing

Oct 2015, at UIUC Quad