

# CS1010S Programming Methodology

## Lecture 6

# Working with Sequences

19 Sep 2018

# Midterm Exam

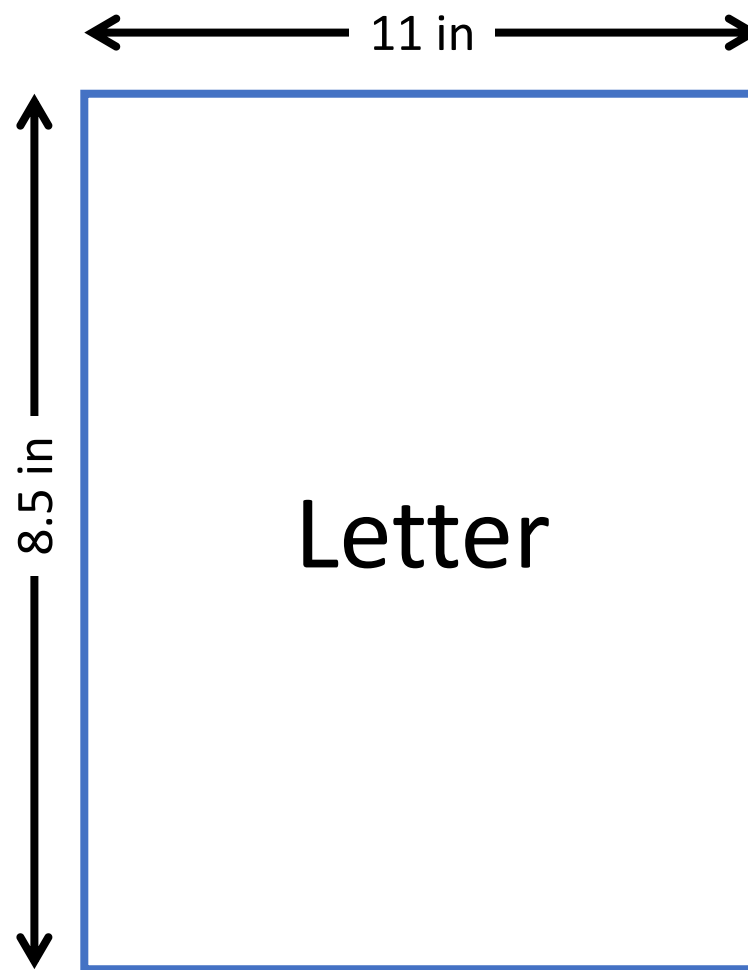
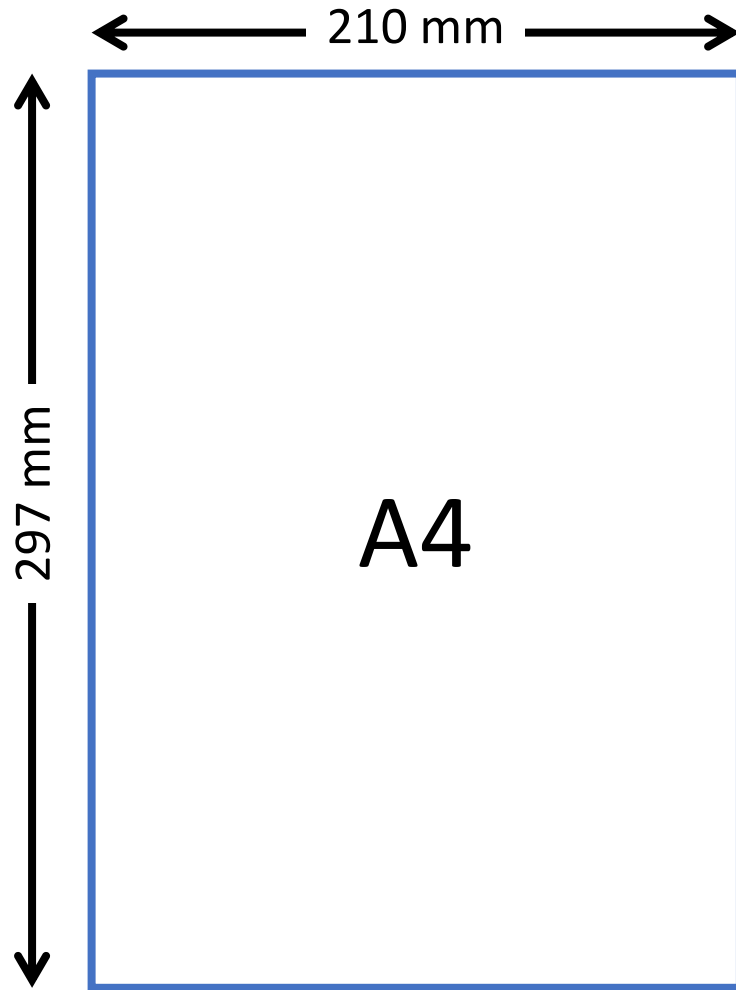
- Date: 3 October
- Time: 6:45pm – 8:15pm
- Venue: MPSH 2



# Midterm Exam

- Date: 3 October
- Time: 6:45pm – 8:15pm
- Venue: MPSH 2
- Open-sheet exam (no laptops!)
  - 1 x A4 sheet (both sides)

# Cheat Sheet



# Midterm Exam

- Date: 3 October
- Time: 6:45pm – 8:15pm
- Venue: MPSH 2
- Open-sheet exam (no laptops!)
  - 1 x A4 sheet (both sides)
  - Printed or Handwritten
  - Monochrome

- ordered sequence
  - list
  - tuple
    - Non modifiable values
    - str by
- key containers, n
  - dictionary dict
    - (key/value associations)
  - collection set
    - keys → hashable value

```

("15") → 15
("3f", 16) → 63
(15.56) → 15
at("-11.24e8")
nd(15.56, 1)
l(x)
(x) →

```

```
(64) →
r(x) →
es{[72
t("abc
t([3,
(["one
rator st
:'.join
splitted o
words with
splitted on separate
1, 4, 8, 2".split
ence of one type → 1
print(x) for x in
```

```
len(lst) # index from 0 (here from 0)
```

```

→ [50, 40, 30, 20, 10]
→ [50, 30, 10]
0, 20, 30, 40, 50]
[3: 5] and modify with

```

#### Statements Block

```

statement :
element block 1...
:
statement :
element block2
:
element after block2

editor to insert & space
double-click on each

```

```

=====
medians
with import sin,
(4)→0.707...
pi/3)→-0.4999.
)→9.0      ✓
(2)→-2.0
(2.5)→-13
(2.5)→-12
h, statistics, random,
tions, numpy, etc. (cf.

```

- ordered sequences, last index access
- break
- continue
- Non mutable values (immutable)
- str bytes (ordered)
- key containers, no a priori order
- dictionary dict {"key": "value"}
- (key/value associations) {1: "one", 3: "three"}
- collection set {"key1": "value1", "key2": "value2"}
- 4 keys - hashable values (non bytes, non dict)

```

{"15"} → 15
{"3F", 16} → 63
(15.56) → 15
{"-11.24e8"} → -1124000000
(15.56, 1) → 15.6
False for null x, empty string
"=" representation string

```



```
len(lst) → 5
```

~~Operations on Sets~~

~~$\{50, 40, 30, 20, 10\}$~~

~~$\{50, 30, 10\}$~~

```

(1) *copy()
State: die Röhre ist leer

```

**L**

[illegible]

**Loop Control**  
*Immediate exit*  
*next iteration*  
*for normal loop exit*

$\sum_{i=1}^{100} i^2$   
 =====  
**Display**

**for var**  
 ———→ **statement**

Go over sequence's  
 s = "Some text"  
 cnt = 0  
     loop variable, assign  
 for 'c' in s:  
     if c == 'e':  
         cnt = 0  
     print("found",  
         loop on dict/set c loop on keys

```

# use slices to loop on a subset of
# Go over sequence's Index
# = modify item at index
# = access items around index
lst = [11, 10, 9, 12, 23,
lost = []
for idx in range(len(
    val = lst[idx]

```

```

range(10)
range(1, 10)
range(1, 10, 2)
range(5) → 0 1 2 3 4
range(3, 8) → 3 4 5 6 7
range(1, len(seq))
# range provides an iterator

function name (identifier)
named parameters
def fct(x, y, z):
    """documenta

```

```

def f(x, y, z):
    # statements block;
    return res-

# parameters and all
variables of this block exist on
call (think of a "black box")
Advanced def f(x, y, z, *args)
#args variable positional args

```

```

10 r = fct(3, 1+2, 2*)
11 # storage of returned value use a param
12 # this is the use of function name with parentheses which does the call

```

```

    e.startsWith(prefix, start)

```

```

s.endswith(suffix_surfl, en
s.count(subj_surfl, end]]
s.index(subj_surfl, end]]
s.isalpha() tests on chars cango
s.upper() s.lower()
s.capitalize() s.capitalize()
s.ljust([width, fill]) s.rjust([width, fill])
s.encode(encoding) s.decode(encoding)

```

formatting directives

```

"modele{} {} {}".format(
    "selection : formatting 1 con
    Selection :

```

```

2      nom
3      0, nom
4      4[key]
5      0[2]
6
7  = Formating :
8  fill char alignment sign mini
9
10 <> ^ = + = space 0 a
11 integer: b binary, c char, d de
12 float: a or E exponential, f or
13 string: a ...
14
15 = Conversion : a (readable us

```

Page 10 of 10

[illegible][illegible]

110

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	-----

[illegible]

188

101

# Midterm Exam

- Scope: everything up to and including Lecture 6 (Today!)
- Past Year Exams have been uploaded to Coursemology



# Midterm Exam

1. Python Expressions
2. Solving Problems with Recursion/Iteration
  - Order of Growth
3. Higher Order Functions
4. Data Abstraction
  - Define new Abstract Data Type + Operations

# Makeup Midterm Exam

- If you miss the midterm with valid reason
  - MC, Leave of Absence, etc.
- Makeup Midterm
  - Date: Friday 19 October
  - Time: 6:45pm – 8:15pm

Only 15%

Don't Stress

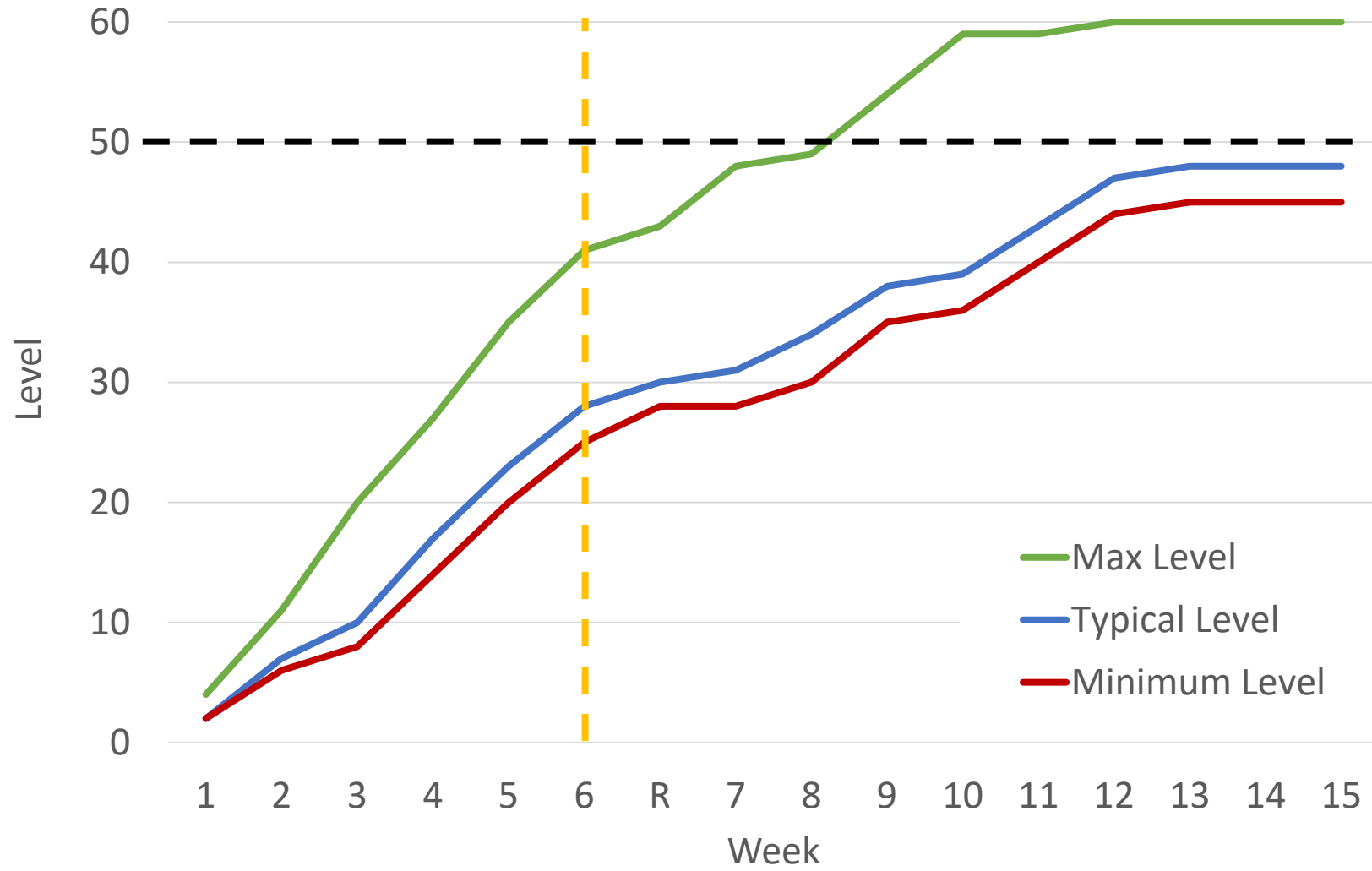
# Help is Coming

- Remedial Sessions
  - 19 Sep (Wed), SR8, 6:30 – 8:30 pm
- Past-Exam Review
  - TBA during recess week
- “Desperado” Session
  - 1 Oct (Mon), 6:30 – 8:30 pm
  - 2 Oct (Tues), 6:30 – 8:30 pm

# No Tutorials & Recitations

- No Recitations on midterm week
- Tutors will still be in class on Mon/Tues during tutorial times for consultation

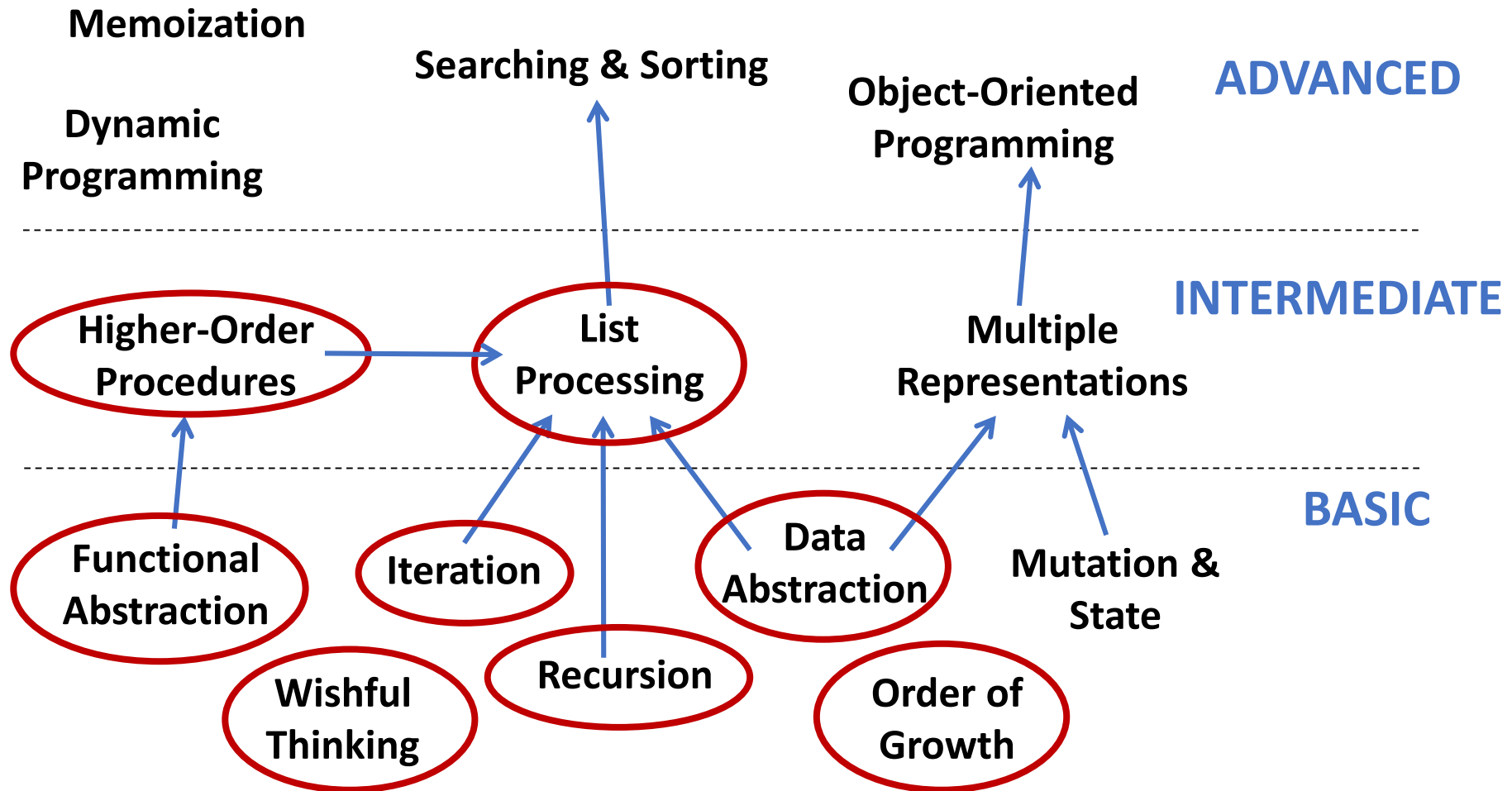
# Expected Level Progression



# Today's Agenda

- Processing Sequences
  - Recursion & Iteration
- Tree as nested sequences
  - Hierarchical structures
- Signal-processing view of Computations
- Working with Files

# CS1010S Road Map



**Fundamental concepts of computer programming**



# Recap: Data Abstraction

- Abstracts away irrelevant details, exposes what is necessary
- Separates usage from implementation.
- Captures common programming patterns
- Serves as a building block for other **compound data**.

# Key idea

- Decide on an internal representation of the Abstract Data Type (ADT) **Tuple!**
- Write functions that operate on that new ADT

Key insight: nobody needs to know your internal representation to use your ADT

# Guidelines for Creating Compound Data

- Constructors
  - To create compound data from primitive data
- Selector (Accessors)
  - To access individual components of compound data
- Predicates
  - To ask (true/false) questions about compound data
- Printers
  - To display compound data in human-readable form

# Sequences

- Sequential data, represented by tuples

- Get the first element of the list:

`seq[0]`

- Get the rest of the elements:

`seq[1:]`

- If a seq. is a tuple containing a single integer 4:

`seq = (4, )`

`seq[0] → 4`

`seq[1:] → ()`

# Reversing a Sequence

```
def reverse(seq):  
    if seq == ():  
        return ()  
    else:  
        return reverse(seq[1:]) + (seq[0],)
```

- Notice that `(seq[0],)` is a tuple and not an integer
- Can only concatenate tuples with tuples

# Reverse Example

reverse(1, 2, 3, 4)  
reverse(2, 3, 4) + (1,)  
reverse(3, 4) + (2,) + (1,)  
reverse(4) + (3,) + (2,) + (1,)  
( ) + (4,) + (3,) + (2,) + (1,)  
(4,) + (3,) + (2,) + (1,)  
(4, 3) + (2,) + (1,)  
(4, 3, 2) + (1,)  
(4, 3, ,2 ,1)

```
def reverse(seq):  
    if seq == ():  
        return ()  
    else:  
        return reverse(seq[1:]) + \  
            (seq[0],)
```

Recursive

Order of Growth?

# Visualizing Space

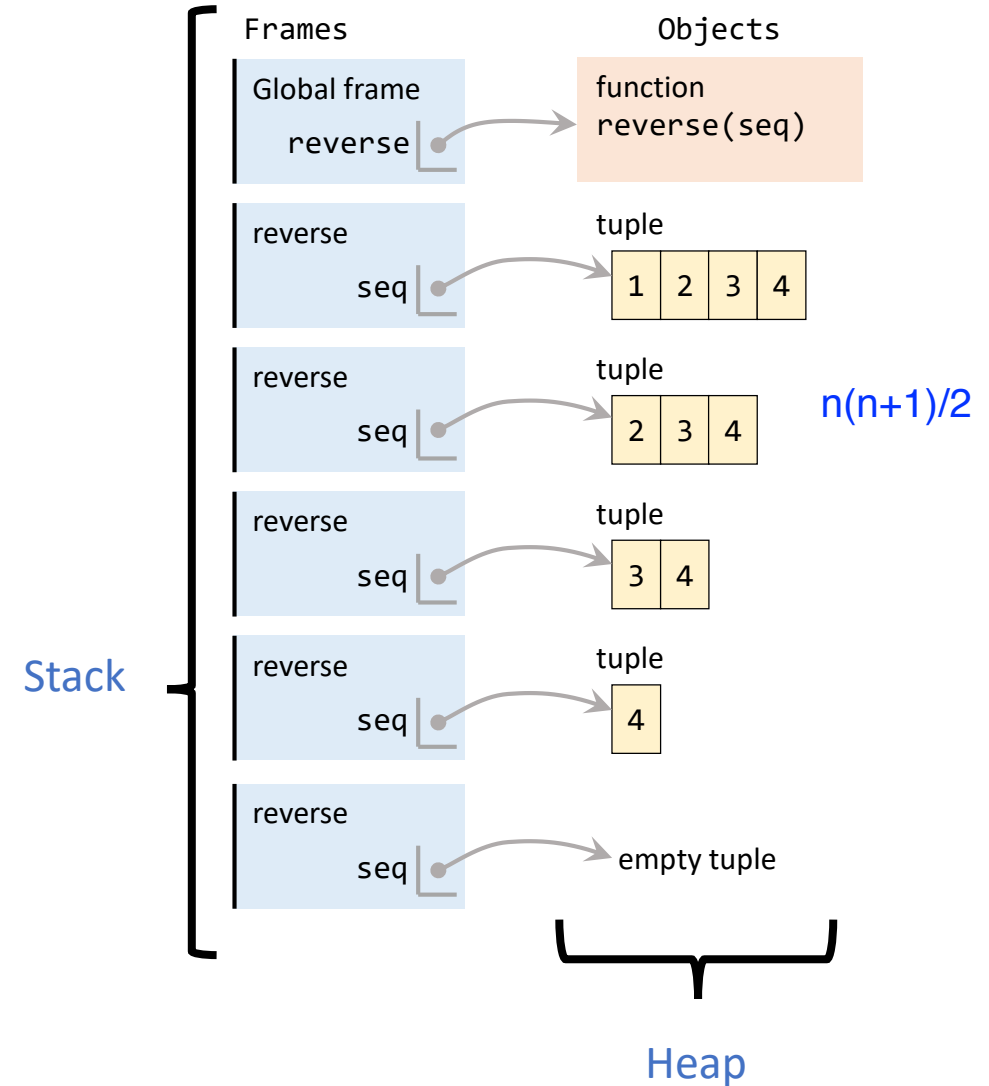
```
def reverse(seq):  
    if seq == ():  
        return ()  
    else:  
        return reverse(seq[1:]) +  
            (seq[0],)
```

reverse((1, 2, 3, 4))

## Order of Growth

Time:  $O(n + n^2) = O(n^2)$

Space:  $O(n + n^2) = O(n^2)$



# Orders of Growth

```
def reverse(seq):  
    result = ()  
    for item in seq:  
        result = (item,) + result  
    return result
```

Iterative

$\text{tuple1} + \text{tuple2}$  takes  $\text{len}(\text{tuple1}) + \text{len}(\text{tuple2})$  steps!

• Orders of growth:	Time	Space
- Recursive version:	$O(n^2)$	$O(n^2)$
- Iterative version:	$O(n^2)$	$O(n)$



Key Idea:

Handle the First Element  
and then the Rest

Iterate/recurse down the sequence!

# Scaling a sequence

Suppose we want to scale all the elements of a sequence by some factor

`scale_seq((1, 2, 3, 4), 3) → (3, 6, 9, 12)`

```
def scale_seq(seq, factor):
```

```
    if seq == ():
```

```
        return ()
```

```
    else:
```

```
        return (seq[0] * factor,) +
```

```
                scale_seq(seq[1:], factor)
```

Time?  $O(n^2)$

Space?  $O(n^2)$

# Scaling a sequence (iterative)

Suppose we want to scale all the elements of a sequence by some factor

`scale_seq((1, 2, 3, 4), 3) → (3, 6, 9, 12)`

```
def scale_seq(seq, factor):  
    result = ()  
    for element in seq:  
        result = result + (element * factor,)   
    return result
```

Time?  $O(n^2)$

Space?  $O(n)$

# Squaring a sequence

Given a sequence, we want to return a sequence of the squares of all elements.

`square_seq((1, 2, 3, 4)) → (1, 4, 9, 16)`

```
def square_seq(seq):
```

```
    if seq == ():
```

```
        return ()
```

```
    else:
```

```
        return (seq[0] ** 2, ) +  
                square_seq(seq[1:])
```

Time?  $O(n^2)$

Space?  $O(n^2)$

Homework: Do this iteratively

# Looking for patterns ....

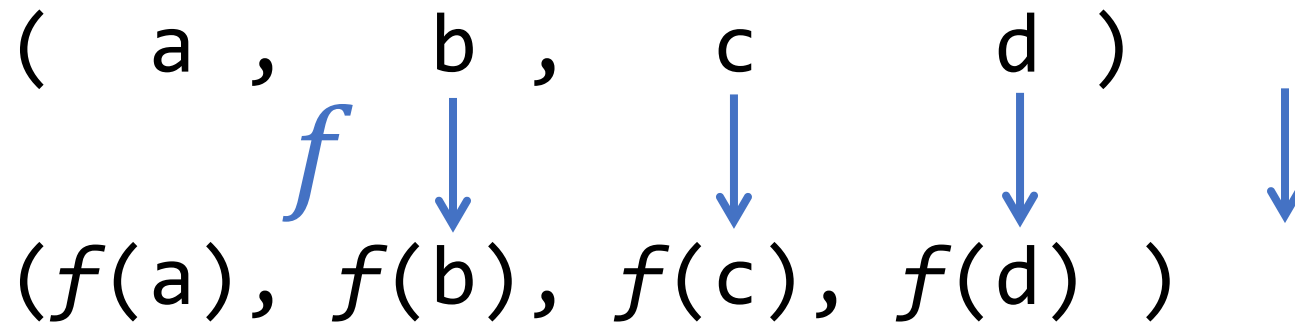
```
def scale_seq(seq, factor):  
    if seq == ():  
        return ()  
    else:  
        return (seq[0] * factor,) +  
                scale_seq(seq[1:], factor)
```

```
def square_seq(seq):  
    if seq == ():  
        return ()  
    else:  
        return (seq[0] ** 2,) +  
                square_seq(seq[1:])
```

Higher-order  
function!!

# Mapping

Often, we want to perform the same operation on every element of a list.



This is called *mapping*.

# Mapping

```
def map(fn, seq):  
    if seq == ():  
        return ()  
    else:  
        return (fn(seq[0]), ) + map(fn, seq[1:])
```

Note: this will overwrite  
the default Python map function!

## Scaling a list by a factor

```
def scale_seq(seq, factor):  
    return map(lambda x: x * factor, seq)
```

# Examples

`map(abs, (-10, 2.5, -11.6, 17))`

`→ (10, 2.5, 11.6, 17)`

`map(square, (1, 2, 3, 4))`

`→ (1, 4, 9, 16)`

`map(cube, (1, 2, 3, 4))`

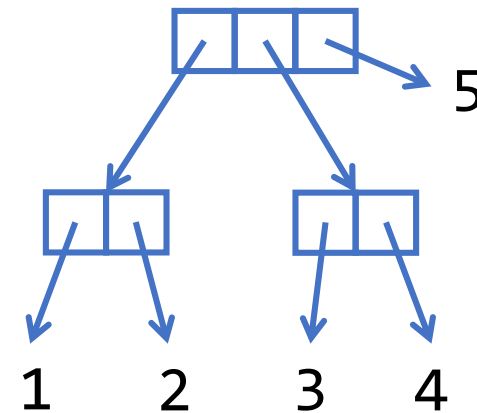
`→ (1, 8, 27, 64)`



# Trees

Trees are sequences of sequences and single elements

- This is possible because of the closure property: we can include a sequence as an element of another sequence
- This allows us to build hierarchical structures, e.g. trees.  
((1, 2), (3, 4), 5)



# Examples

```
>>> x = ((1, 2), 3, 4)
```

```
>>> len(x)
```

```
3
```

```
>>> count_leaves(x)
```

```
4
```

```
>>> (x, x)
```

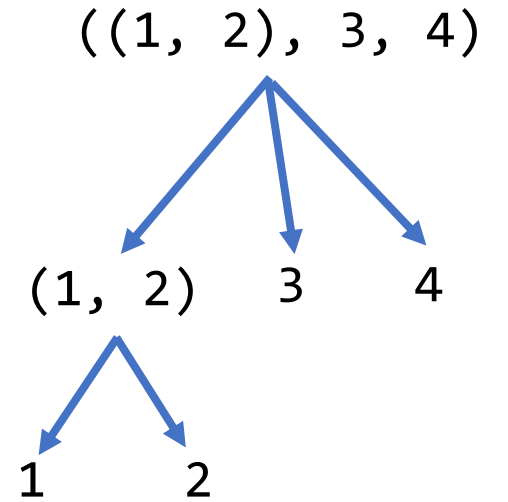
```
((1, 2), 3, 4), ((1, 2), 3, 4))
```

```
>>> len((x, x))
```

```
2
```

```
>>> count_leaves((x, x))
```

```
8
```

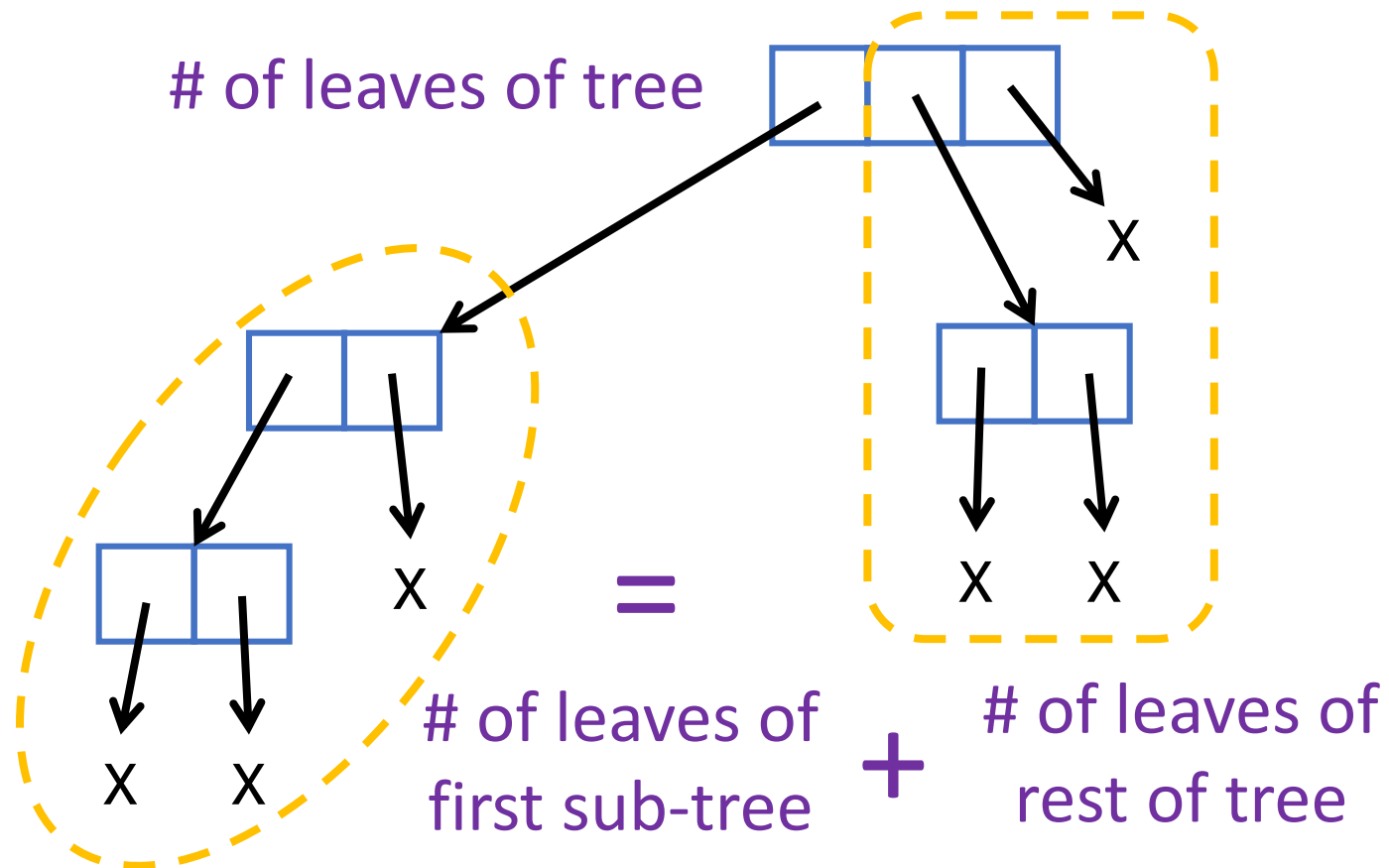


How would we count  
the leaves?

RECURSION!

# Recurrence Relation

Observation:



# Recursion

In other words,

```
count_leaves(tree) =  
    count_leaves(tree[0]) +  
    count_leaves(tree[1:])
```

Base Case:

If tree is empty

Zero!

# Another Base Case

Observe:

Possible for the head or tail to be a leaf!

Leaf  $\Rightarrow +1$

# Summary

## Strategy:

- If tree is empty, then 0
- Another base case:
  - tree is a leaf, then count as 1
- Count this, and add to:
  - `tail` also a tree, so recursively count this



# Count Leaves

```
def count_leaves(tree):  
    if tree == ():  
        return 0  
    elif is_leaf(tree):  
        return 1  
    else:  
        return count_leaves(tree[0])  
        + count_leaves(tree[1:])
```

# What are leaves

Remember `type()` in Lecture 1:

```
>>> t = (1, 2, 3)
```

```
>>> type(t)
```

```
<class 'tuple'>
```

```
>>> type(t) == tuple
```

```
True
```

```
def is_leaf(item):
```

```
    return type(item) != tuple
```

# Mapping over trees

Suppose we want to scale each leaf by a factor, i.e.

`mytree`  $\rightarrow$  `(1, (2, (3, 4), 5), (6, 7))`

`scale_tree(mytree, 10)`

$\rightarrow$  `(10, (20, (30, 40), 50), (60, 70))`

# Strategy

- Since tree is a **sequence of sequences**, we can map over each element in a tree.
- Each element is a subtree, which we recursively scale, and return sequence of results.
- **Base case:** if tree is a leaf, multiply by factor

# Mapping over trees

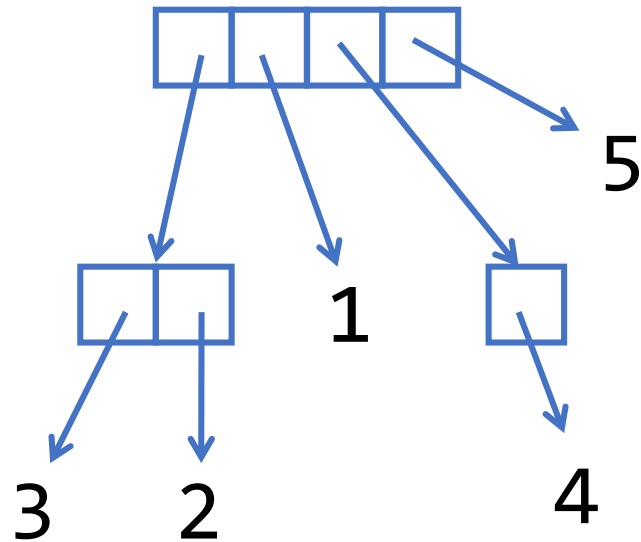
```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor * subtree  
        else:  
            return scale_tree(subtree, factor)  
    return map(scale_func, tree)
```

## Compare with:

```
def count_leaves(tree):  
    if tree == ():  
        return 0  
    elif is_leaf(tree):  
        return 1  
    else:  
        return count_leaves(tree[0]) + count_leaves(tree[1:])
```

# Let's see what `scale_tree` does

```
tree = ((3, 2), 1, (4,), 5)
```



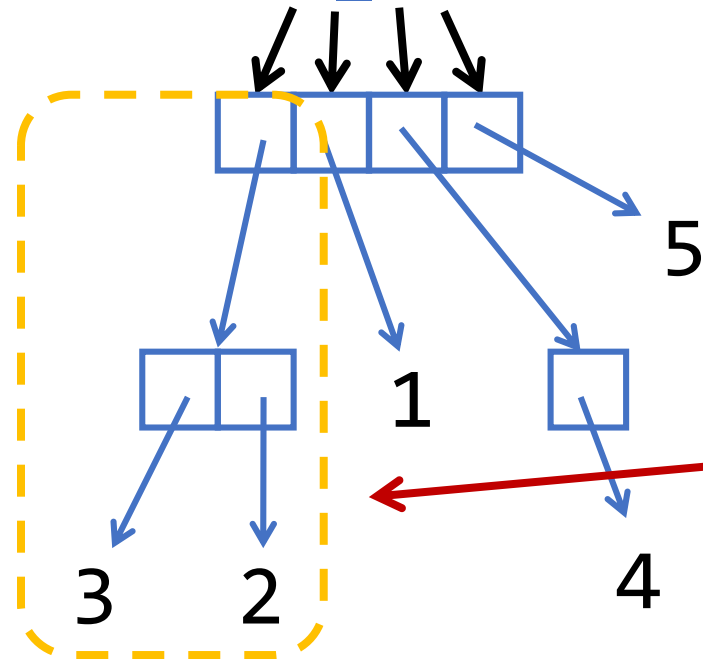
```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor*subtree  
        else:  
            return scale_tree(subtree, factor)  
    return map(scale_func, tree)
```

Suppose we do `scale_tree(tree, 2)`

# Let's see what `scale_tree` does

```
tree = ((3, 2), 1, (4,), 5)
```

Apply `scale_func` to each element



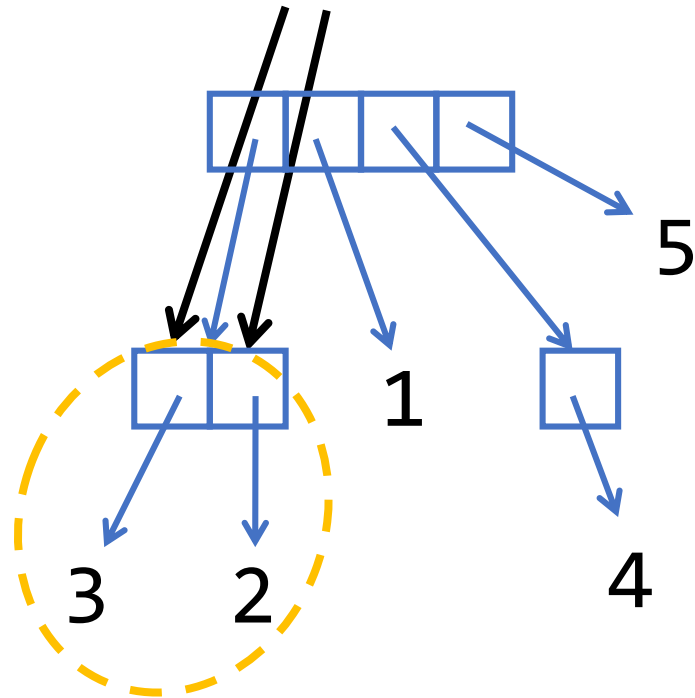
Not a leaf

```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor*subtree  
        else:  
            return scale_tree(subtree,  
                               factor)  
    return map(scale_func, tree)
```

# Let's see what `scale_tree` does

```
tree = ((3, 2), 1, (4,), 5)
```

Apply `scale_func` to each element

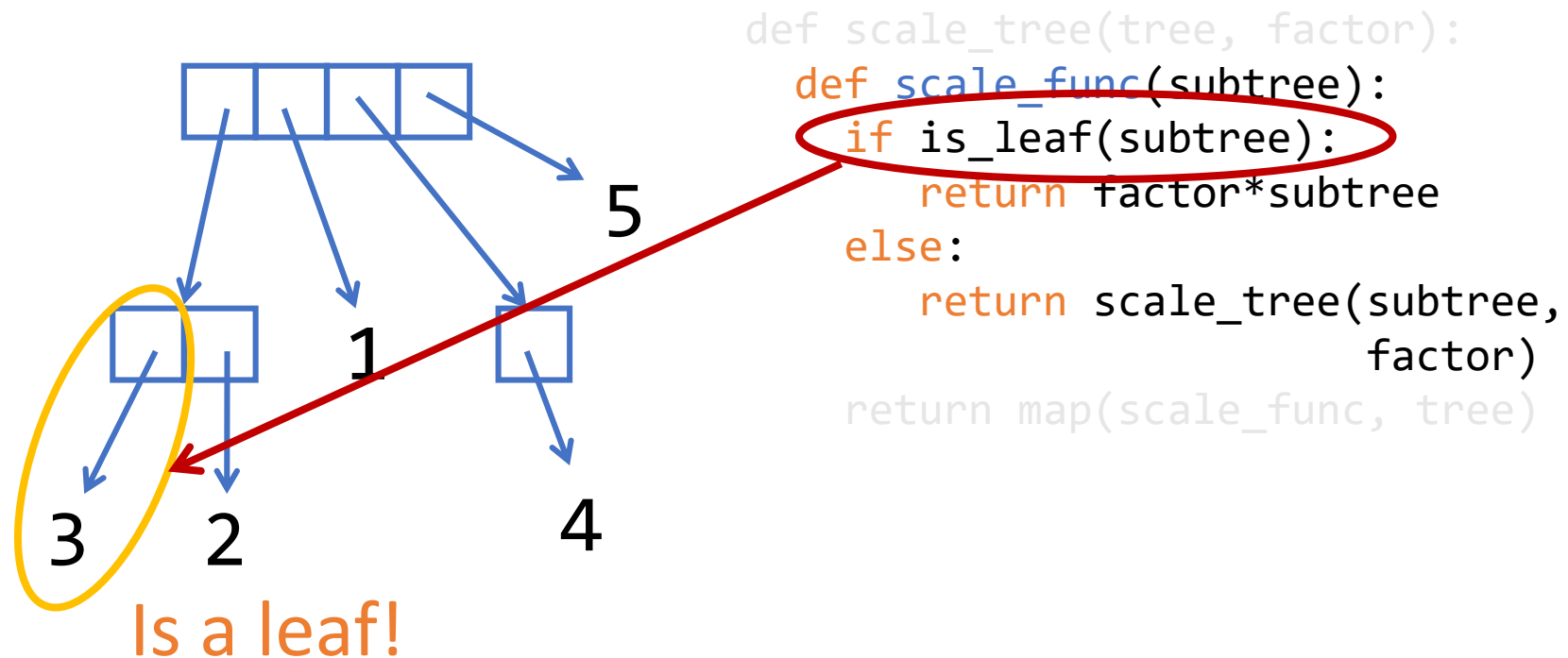


```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor*subtree  
        else:  
            return scale_tree(subtree,  
                               factor)  
    return map(scale_func, tree)
```



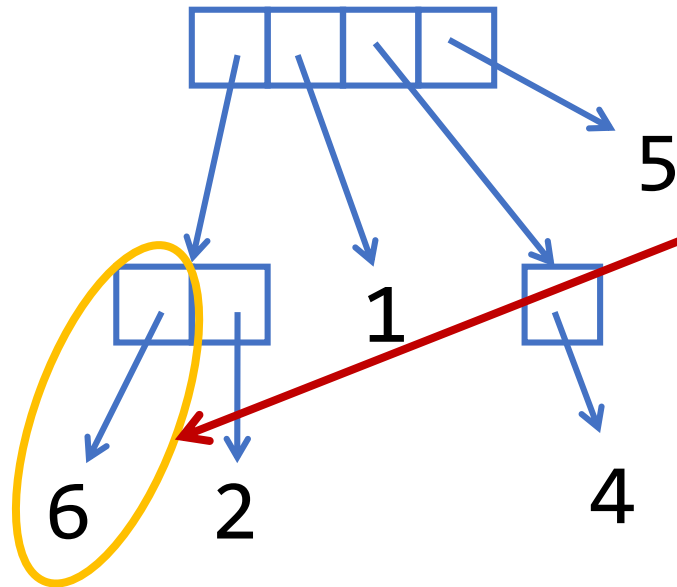
# Let's see what `scale_tree` does

```
tree = ((3, 2), 1, (4,), 5)
```



# Let's see what `scale_tree` does

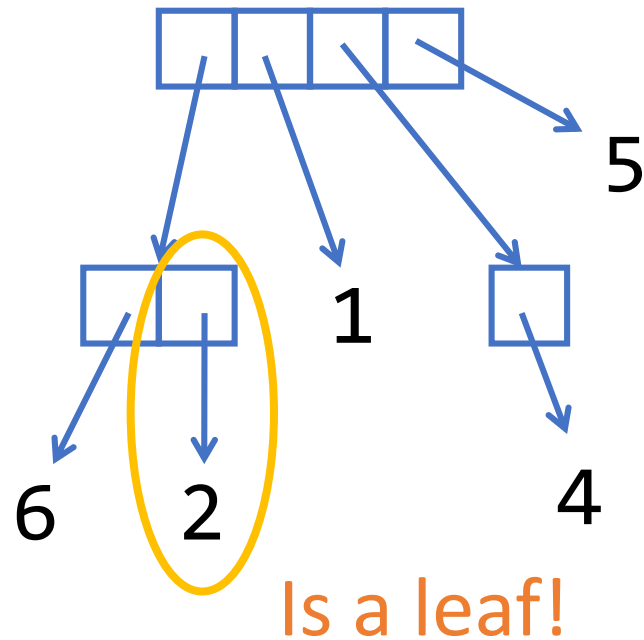
```
tree = ((3, 2), 1, (4,), 5)
```



```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor*subtree  
        else:  
            return scale_tree(subtree,  
                               factor)  
    return map(scale_func, tree)
```

# Let's see what `scale_tree` does

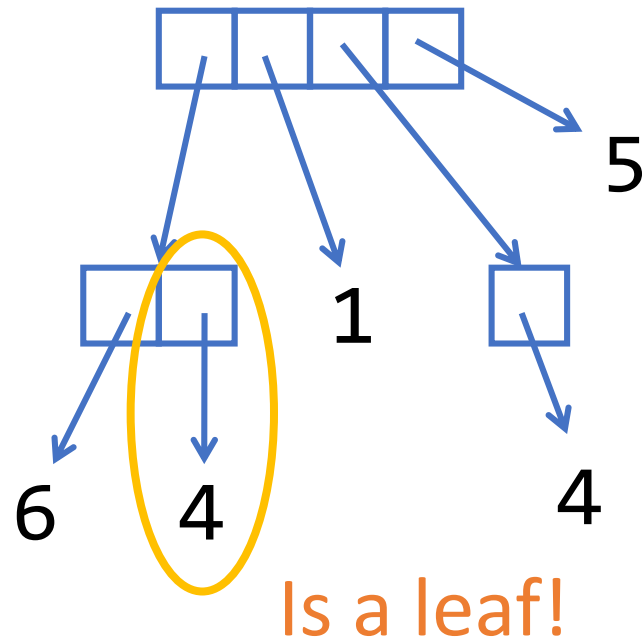
```
tree = ((3, 2), 1, (4,), 5)
```



```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor*subtree  
        else:  
            return scale_tree(subtree,  
                               factor)  
    return map(scale_func, tree)
```

# Let's see what `scale_tree` does

```
tree = ((3, 2), 1, (4,), 5)
```

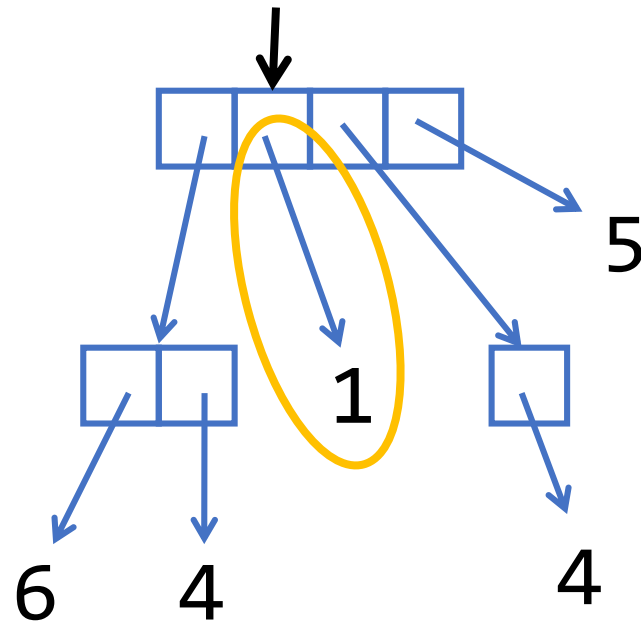


```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor*subtree  
        else:  
            return scale_tree(subtree,  
                               factor)  
    return map(scale_func, tree)
```

# Let's see what `scale_tree` does

```
tree = ((3, 2), 1, (4,), 5)
```

Apply `scale_func` to each element



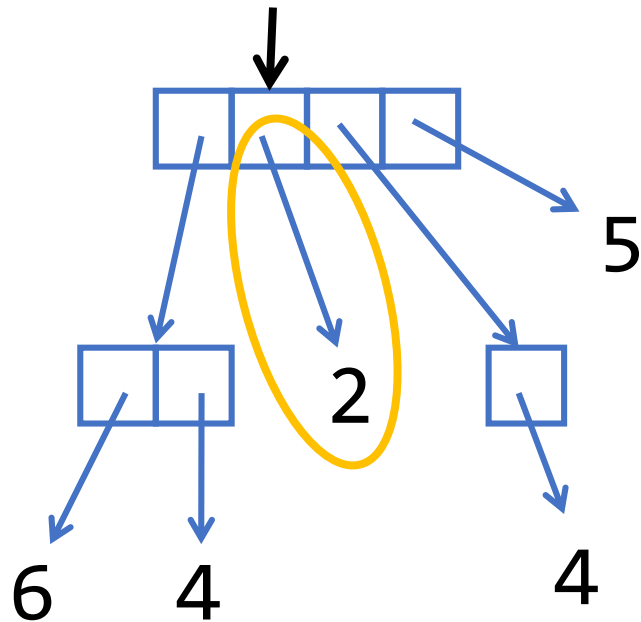
Is a leaf!

```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor*subtree  
        else:  
            return scale_tree(subtree,  
                               factor)  
    return map(scale_func, tree)
```

# Let's see what `scale_tree` does

```
tree = ((3, 2), 1, (4,), 5)
```

Apply `scale_func` to each element

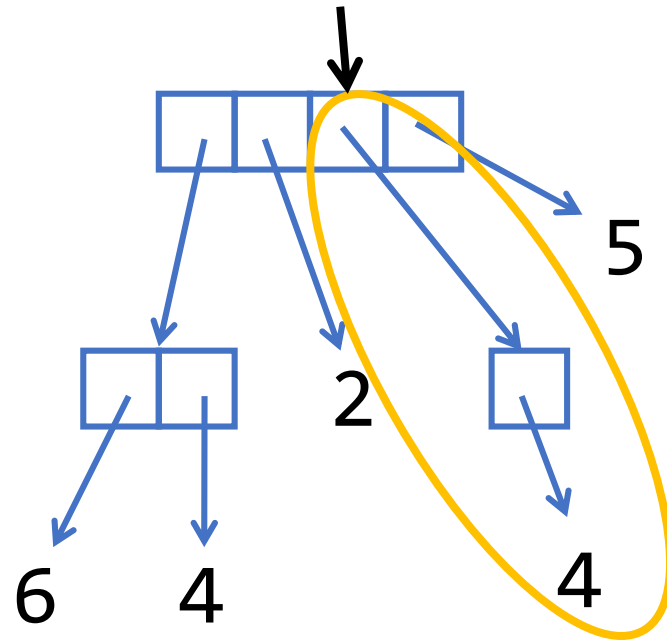


```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor*subtree  
        else:  
            return scale_tree(subtree,  
                               factor)  
    return map(scale_func, tree)
```

# Let's see what `scale_tree` does

```
tree = ((3, 2), 1, (4,), 5)
```

Apply `scale_func` to each element



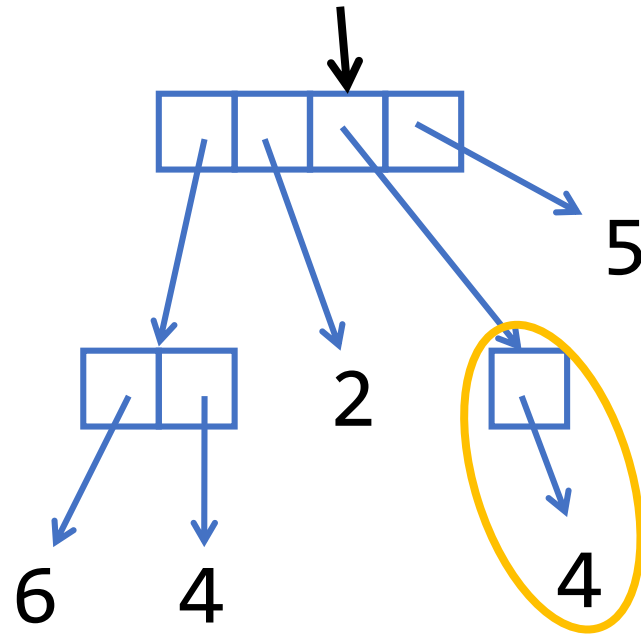
Not a leaf!

```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor*subtree  
        else:  
            return scale_tree(subtree,  
                               factor)  
    return map(scale_func, tree)
```

# Let's see what `scale_tree` does

```
tree = ((3, 2), 1, (4,), 5)
```

Apply `scale_func` to each element



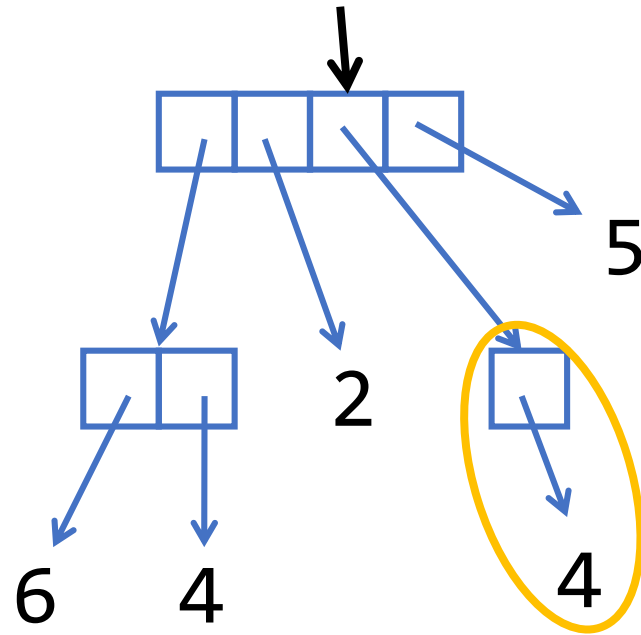
```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor*subtree  
        else:  
            return scale_tree(subtree, factor)  
    return map(scale_func, tree)
```



# Let's see what `scale_tree` does

```
tree = ((3, 2), 1, (4,), 5)
```

Apply `scale_func` to each element



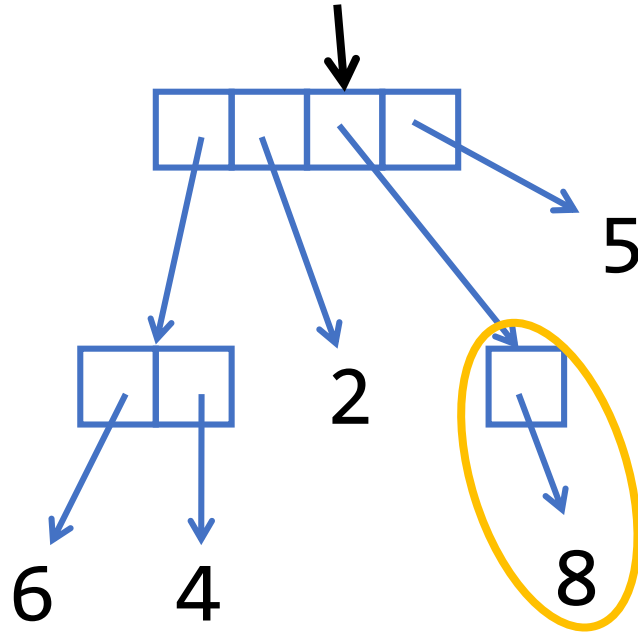
Is a leaf!

```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor*subtree  
        else:  
            return scale_tree(subtree, factor)  
    return map(scale_func, tree)
```

# Let's see what `scale_tree` does

```
tree = ((3, 2), 1, (4,), 5)
```

Apply `scale_func` to each element

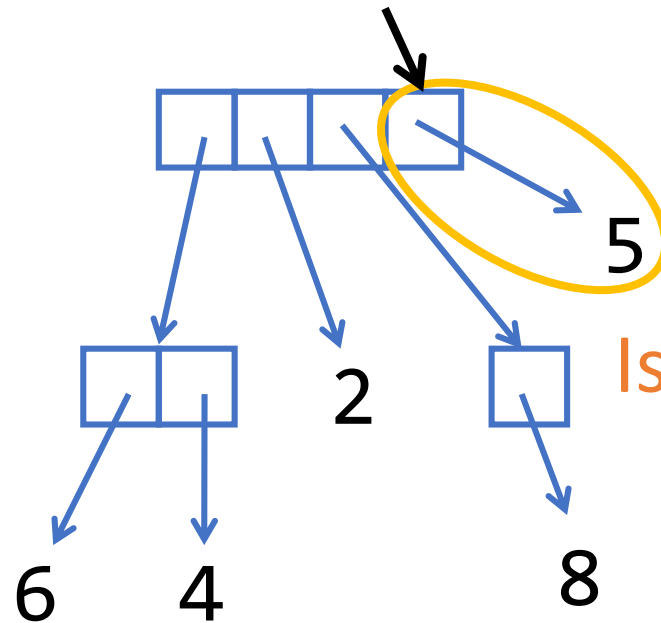


```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor*subtree  
        else:  
            return scale_tree(subtree,  
                               factor)  
    return map(scale_func, tree)
```

# Let's see what `scale_tree` does

```
tree = ((3, 2), 1, (4,), 5)
```

Apply `scale_func` to each element



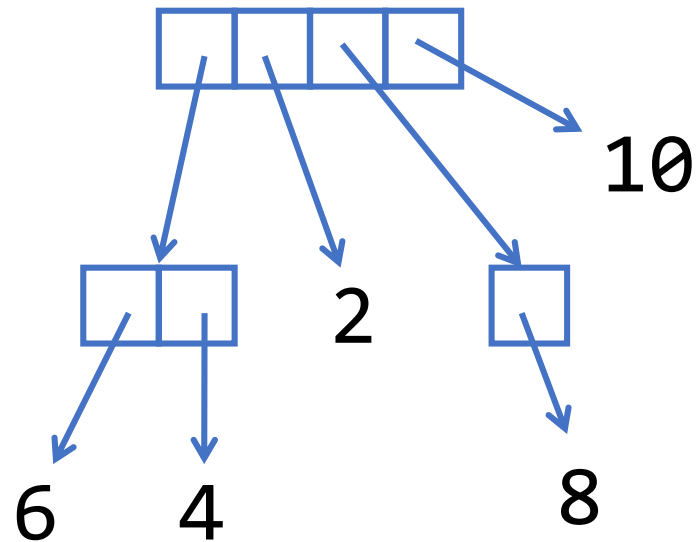
```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor*subtree  
        else:  
            return scale_tree(subtree, factor)  
    return map(scale_func, tree)
```

Is a leaf!

# Let's see what `scale_tree` does

```
tree = ((3, 2), 1, (4,), 5)
```

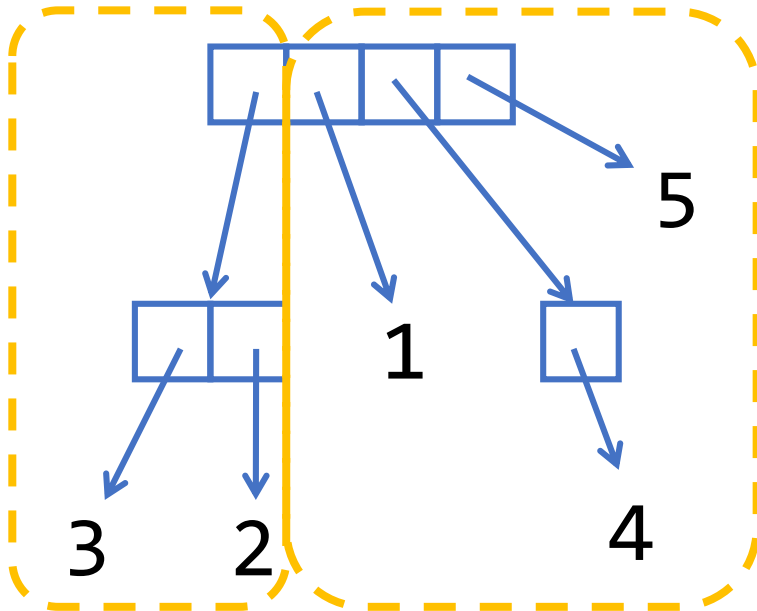
Done applying `scale_func` to each element



```
def scale_tree(tree, factor):  
    def scale_func(subtree):  
        if is_leaf(subtree):  
            return factor*subtree  
        else:  
            return scale_tree(subtree,  
                               factor)  
    return map(scale_func, tree)
```

# Let's compare with `count_leaves`

`tree = ((3, 2), 1, (4,), 5)`

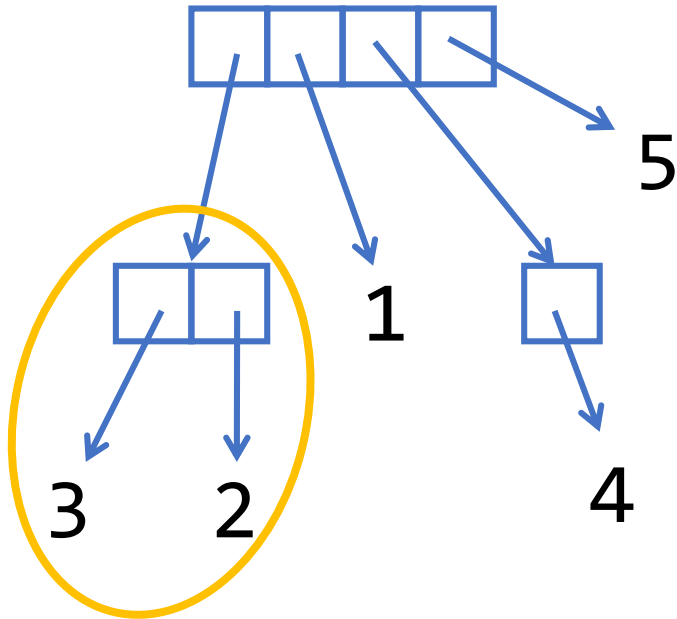


`count_leaves` + `count_leaves`

```
def count_leaves(tree):  
    if tree == ():  
        return 0  
    elif is_leaf(tree):  
        return 1  
    else:  
        return count_leaves(tree[0])  
               + count_leaves(tree[1:])
```

# Let's compare with `count_leaves`

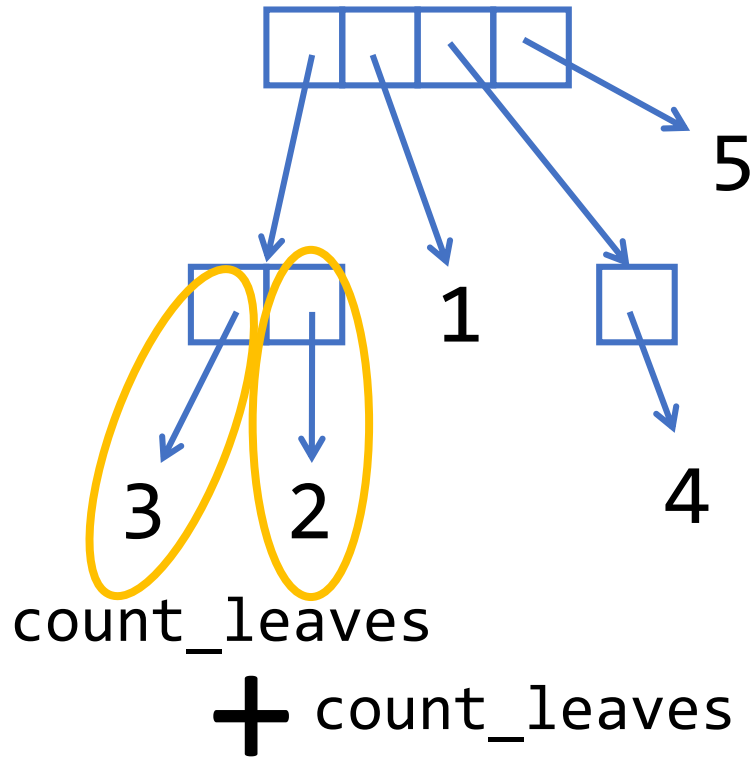
```
tree = ((3, 2), 1, (4,), 5)
```



```
def count_leaves(tree):  
    if tree == ():  
        return 0  
    elif is_leaf(tree):  
        return 1  
    else:  
        return count_leaves(tree[0])  
               + count_leaves(tree[1:])
```

# Let's compare with `count_leaves`

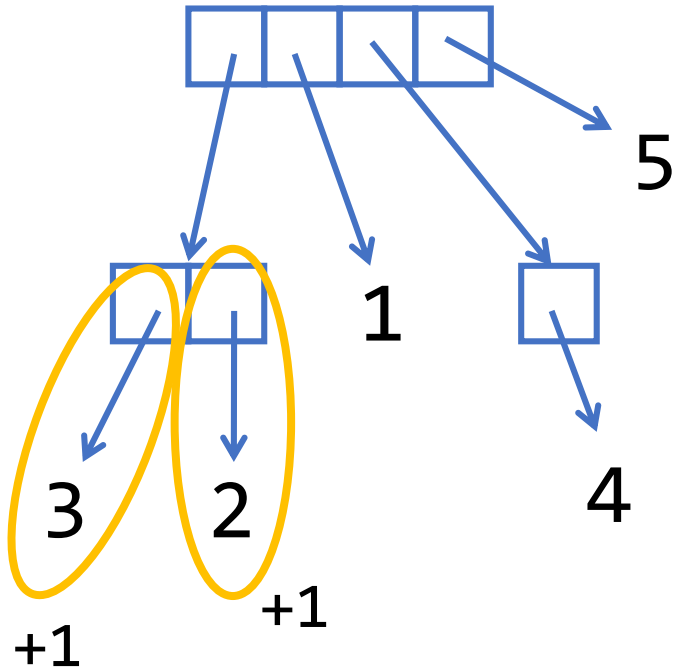
```
tree = ((3, 2), 1, (4,), 5)
```



```
def count_leaves(tree):  
    if tree == ():  
        return 0  
    elif is_leaf(tree):  
        return 1  
    else:  
        return count_leaves(tree[0])  
               + count_leaves(tree[1:])
```

# Let's compare with `count_leaves`

```
tree = ((3, 2), 1, (4,), 5)
```

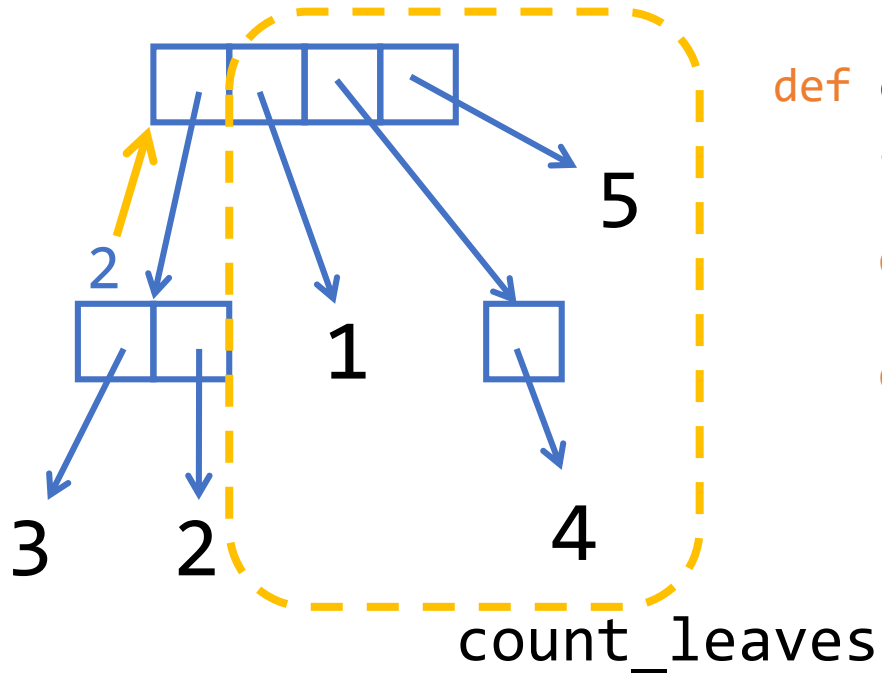


```
def count_leaves(tree):  
    if tree == ():  
        return 0  
    elif is_leaf(tree):  
        return 1  
    else:  
        return count_leaves(tree[0])  
               + count_leaves(tree[1:])
```



# Let's compare with `count_leaves`

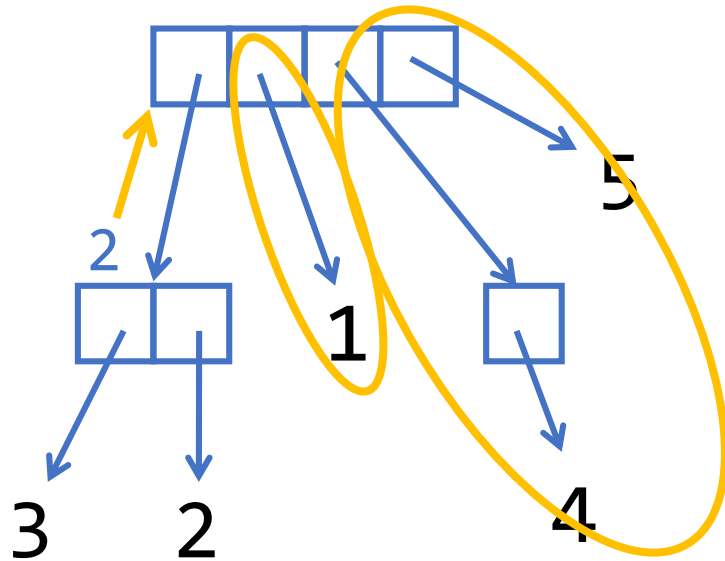
```
tree = ((3, 2), 1, (4,), 5)
```



```
def count_leaves(tree):  
    if tree == ():  
        return 0  
    elif is_leaf(tree):  
        return 1  
    else:  
        return count_leaves(tree[0])  
               + count_leaves(tree[1:])
```

# Let's compare with `count_leaves`

```
tree = ((3, 2), 1, (4,), 5)
```

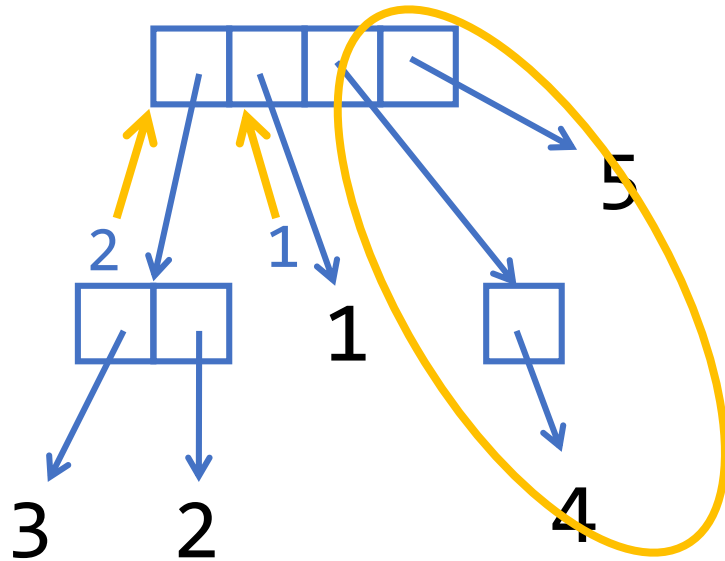


```
def count_leaves(tree):  
    if tree == ():  
        return 0  
    elif is_leaf(tree):  
        return 1  
    else:  
        return count_leaves(tree[0])  
               + count_leaves(tree[1:])
```

`count_leaves` + `count_leaves`

# Let's compare with `count_leaves`

```
tree = ((3, 2), 1, (4,), 5)
```

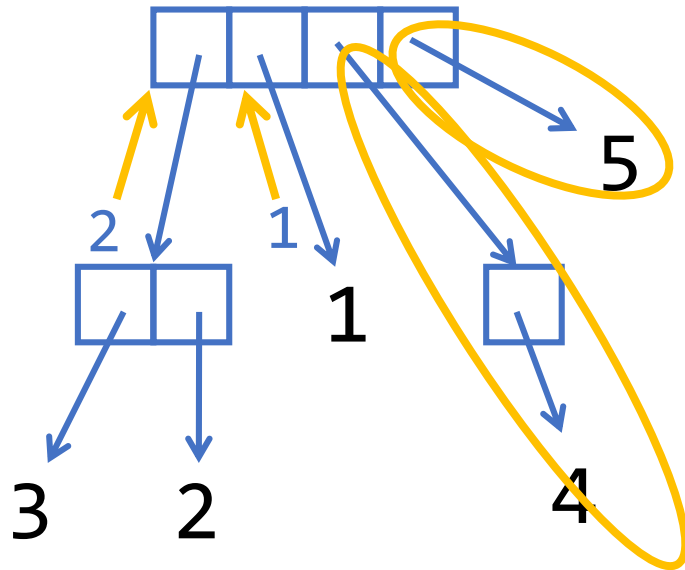


```
def count_leaves(tree):
    if tree == ():
        return 0
    elif is_leaf(tree):
        return 1
    else:
        return count_leaves(tree[0])
            + count_leaves(tree[1:])
```

`count_leaves`

# Let's compare with `count_leaves`

```
tree = ((3, 2), 1, (4,), 5)
```

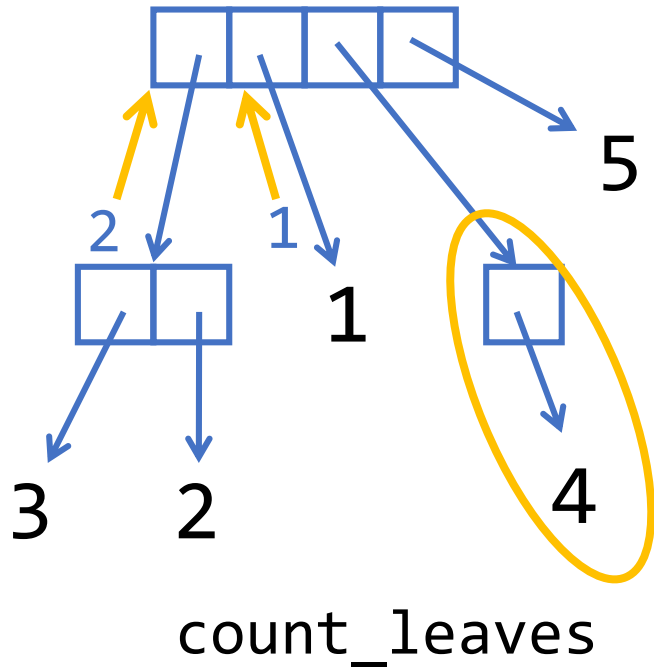


```
def count_leaves(tree):  
    if tree == ():  
        return 0  
    elif is_leaf(tree):  
        return 1  
    else:  
        return count_leaves(tree[0])  
               + count_leaves(tree[1:])
```

`count_leaves` + `count_leaves`

# Let's compare with `count_leaves`

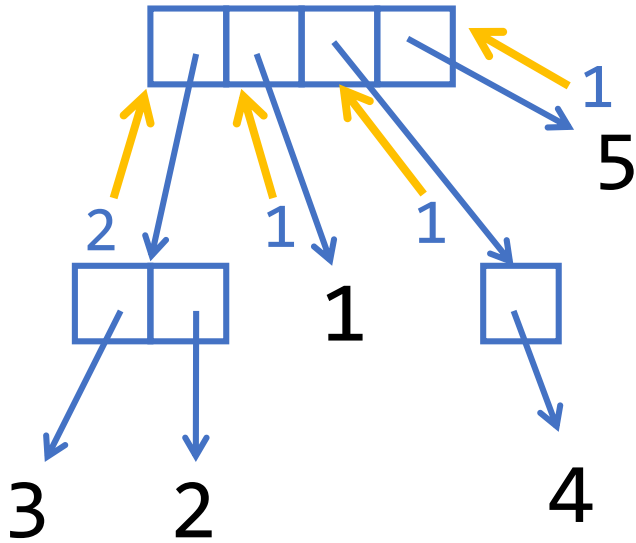
```
tree = ((3, 2), 1, (4,), 5)
```



```
def count_leaves(tree):  
    if tree == ():  
        return 0  
    elif is_leaf(tree):  
        return 1  
    else:  
        return count_leaves(tree[0])  
               + count_leaves(tree[1:])
```

# Let's compare with `count_leaves`

```
tree = ((3, 2), 1, (4,), 5)
```



```
def count_leaves(tree):  
    if tree == ():  
        return 0  
    elif is_leaf(tree):  
        return 1  
    else:  
        return count_leaves(tree[0])  
               + count_leaves(tree[1:])
```

# Key Idea:

# Traverse tree with recursion

Check for leaf!

# Sanity Check (QOTD)

How do you write a function `copy_tree` that takes a tree and returns a copy of that tree?



# Sanity Check (QOTD)

```
def copy_tree(tree):  
    return tree # is NOT acceptable!
```

```
>>> t = (1, 2, 3)
```

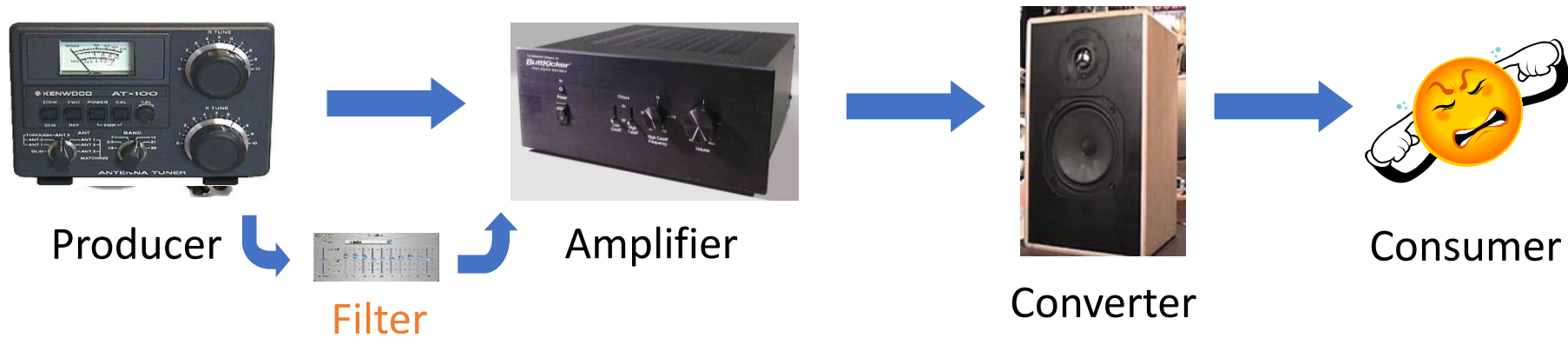
```
>>> t_copy = copy_tree(t)
```

```
t == t_copy → True
```

```
t is t_copy → False
```

# Listening to Music

- Signal goes through various stages of “processing” .
- Additional component can be inserted.
- Easy to change component.
- Components interface via **signals**.



# Modeling Computation as Signal Processing

- Producer (enumerator) creates signal.
- Filter removes some elements.
- Mapper modifies signal.
- Consumer (accumulator) consumes signal.

# Benefits

1. Modularity: each component independent of others; components may be re-used.
2. Clarity: separates data from processes
3. Flexibility: new component can be added

## Example:

### Sum of squares of odd leaves

Given a tree, want to add the squares of  
(only) leaves of odd numbers:

`sum_odd_squares(((1, 2), (3, 4)))`  $\rightarrow$  10

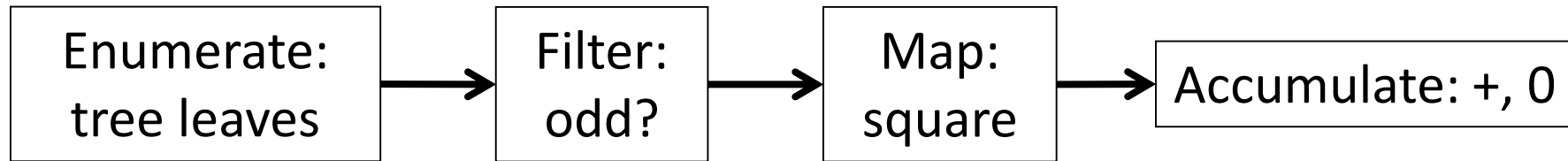
# Example:

## Sum of squares of odd leaves

```
def sum_odd_squares(tree):  
    if tree == ():  
        return 0  
    elif is_leaf(tree):  
        if tree % 2 == 0:  
            return 0  
        else:  
            return tree ** 2  
    else:  
        return sum_odd_squares(tree[0]) +  
               sum_odd_squares(tree[1:])
```

# Alternative Approach

View it as signal processing computation!



How to represent “signals”?

- Sequences

# Enumerating leaves

What does the following function do?

```
def enumerate_tree(tree):  
    if tree == ():  
        return ()  
    elif is_leaf(tree):  
        return (tree,)  
    else:  
        return enumerate_tree(tree[0]) +  
               enumerate_tree(tree[1:])
```

`enumerate_tree((1, (2, (3, 4)), 5))`

→ `(1, 2, 3, 4, 5)`

Also known as flattening the tree.



# Filtering a sequence

```
def filter(pred, seq):
```

```
    if seq == ():
```

```
        return ()
```

Note: we are overwriting  
the default Python filter function!

```
    elif pred(seq[0]):
```

```
        return (seq[0],)
```

```
            + filter(pred, seq[1:])
```

```
    else:
```

```
        return filter(pred, seq[1:])
```

```
is_odd = lambda x: x%2 != 0
```

```
filter(is_odd, (1, 2, 3, 4, 5)) → (1, 3, 5)
```

# Accumulating a sequence

```
def accumulate(fn, initial, seq):  
    if seq == ():  
        return initial  
    else:  
        return fn(seq[0],  
                    accumulate(fn, initial,  
                               seq[1:]))
```

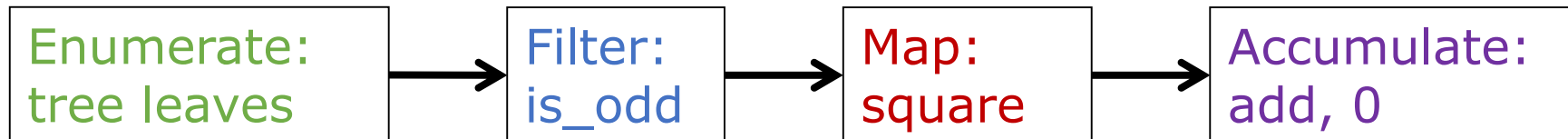
```
add = lambda x, y: x+y  
accumulate(add, 0, (1, 2, 3, 4, 5))  
accumulate(lambda x, y:(x, y), (),  
            (1, 2, 3, 4, 5))
```

```
→ (1, (2, (3, (4, (5, ())))))
```

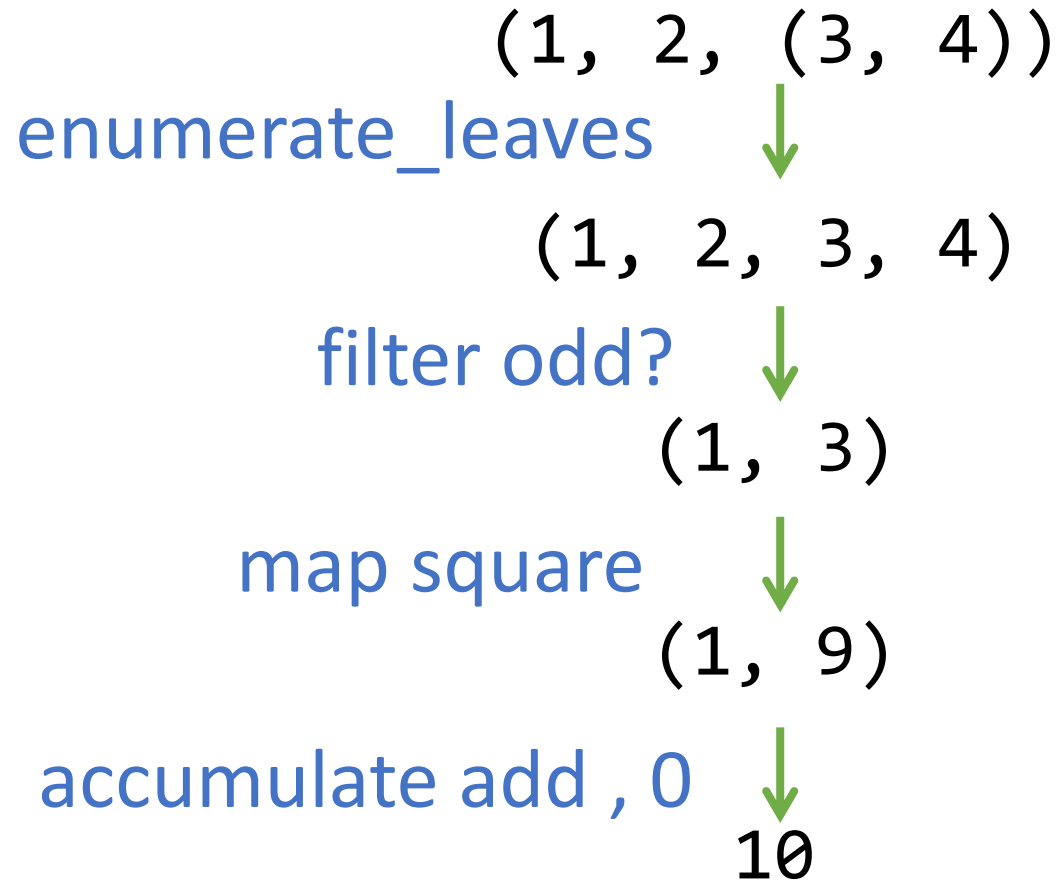
→ 15

# Putting it together

```
def sum_odd_squares(tree):  
    return \  
    accumulate(add, 0,  
               map(square,  
                   filter(is_odd,  
                           enumerate_tree(tree))))
```



# Putting it together



# Another Example: Tuple of even Fib

Want a list of even  $\text{fib}(k)$  for  
all  $k$  up to given integer  $n$ .

# “Usual” Way

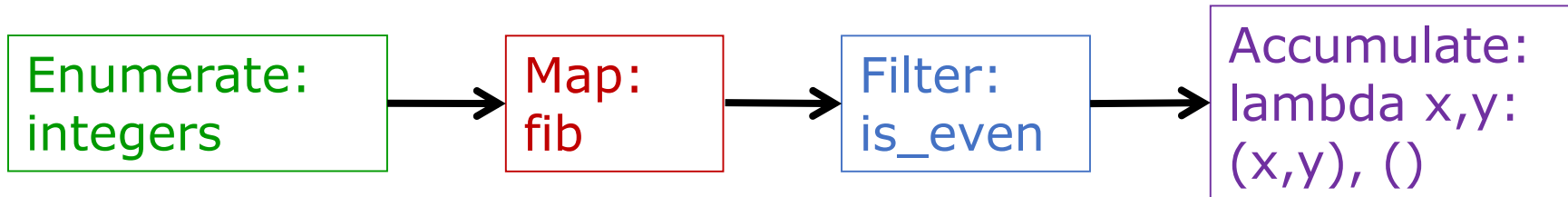
```
def even_fibs(n):  
    result = ()  
    for k in range(0, n + 1):  
        f = fib(k)  
        if is_even(f):  
            result = result + (f, )  
    return result
```

```
is_even = lambda x: x % 2 == 0
```

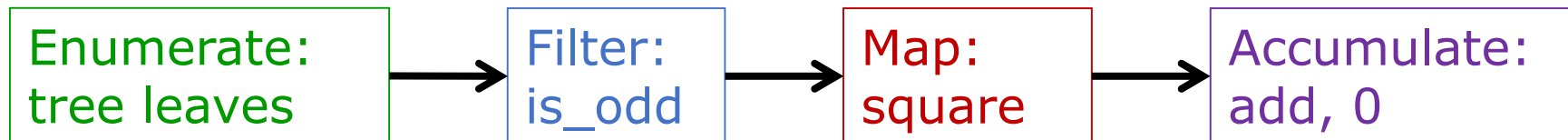
```
>>> even_fibs(30)  
(0, 2, 8, 34, 144, 610, 2584, 10946, 46368, 196418, 832040)
```

# Signal processing view

- Even fib



- Compare: sum square odd leaves



# Enumerate integers

```
def enumerate_interval(low, high):  
    return tuple(range(low, high+1))
```

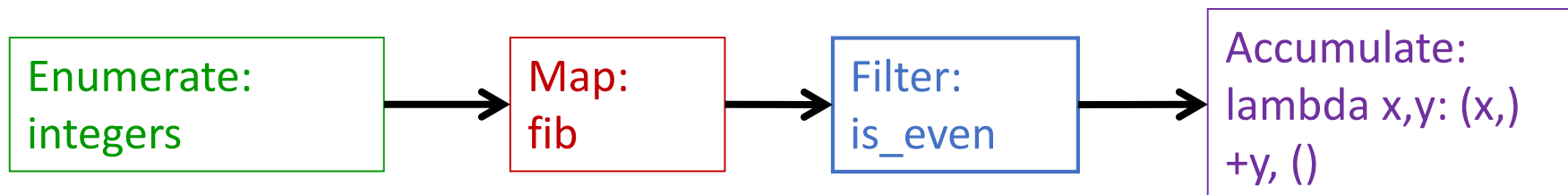
```
enumerate_interval(2, 7)
```

```
→ (2, 3, 4, 5, 6, 7)
```



# Even Fibs

```
def even_fibs(n):  
    return  
        accumulate(lambda x, y: (x,) + y, (),  
                    filter(is_even,  
                            map(fib,  
                                enumerate_interval(0, n))))
```



# Signal Processing View

- Modular components:
  - Enumerate, Filter, Map, Accumulate
  - Each is independent of others.
  - Modularity is a powerful strategy for controlling complexity.

# Signal Processing View

- Build a library of components.
- Sequences used to interface between components.



# Other Uses

- Suppose we have a sequence of personnel records.
- Want to find salary of highest-paid programmer.

```
def salary_of_highest_paid_programmer(records):  
    return accumulate(max, 0,  
                       map(salary,  
                           filter(is_programmer,  
                                records)))
```

## Default Python `map` and `filter` functions

Returns an “iterable” instead of tuple, but you can force it into a tuple.

```
>>> a = (1, 2, 3, 4, 5)
>>> b = filter(lambda x: x%2 == 0, a)
>>> b
<filter object at 0x02EC4710>
```

Think of iterable as a “one-time” use  
sequence

```
>>> b  
<filter object at 0x02EC4710>
```

```
>>> for i in b:  
    print(i)
```

```
2
```

```
4
```

```
>>> tuple(b)  
()
```



```
>>> b = filter(lambda x: x%2 == 0, a)
```

```
>>> b
```

```
<filter object at 0x02E42C10>
```

```
>>> c = tuple(b)
```

```
>>> c
```

```
(2, 4)
```

```
>>> tuple(b)
```

```
()
```

# Comprehension

If `map` and `filter` is too complicated...

`(<expr> for <var> in <seq> if <cond>)`

is equivalent to

```
map(lambda <var>: <expr>,  
    filter(lambda <var>:<cond>, <seq>))
```

```
a = (1,2,3,4,5)
```

```
>>> b = tuple((x*2 for x in a))
```

```
>>> b
```

```
(2, 4, 6, 8, 10)
```

$2x, \forall x \in a$

```
>>> b = tuple((x*2 for x in a if x%2 == 0))
```

```
>>> b
```

```
(4, 8)
```

$2x, \forall x \in a, x \bmod 2 = 0$

# Working with Files

## Reading a File:

```
input = open('inputfilename.txt', 'r')  
some_line = input.readline()
```

We can check for end of file by checking whether

```
some_line == '' #empty string
```

## Writing to a File:

```
output = open('outputfilename.txt', 'w')  
output.write('HELLO WORLD')
```

# Example

```
def metrics(dictfile):  
    dict = open(dictfile, 'r')  
  
    currword = dict.readline()  
    longest_word = currword  
    shortest_word = currword  
  
    while currword != '':  
        if(len(currword) < len(shortest_word)):  
            shortest_word = currword  
        if(len(currword) > len(longest_word)):  
            longest_word = currword  
        currword = dict.readline()  
  
    output = open("output.txt", "w")  
    output.write("longest word: " + longest_word)  
    output.write("shortest word: " + shortest_word)
```



Find longest and  
shortest word



write to file

# Example

```
dictionary.txt >>
```

```
CS1010S
```

```
BEST
```

```
MODULE
```

```
WORLD
```

```
metrics("dictionary.txt")
```

```
output.txt >>
```

```
longest word: CS1010S
```

```
shortest word: BEST
```

# Summary

- Data often comes in the form of sequences
  - Easy to manipulate using recursion/iteration
  - Can be nested
- Closure property allows us to build hierarchical structures, e.g. trees, with tuples
  - Can use recursion to traverse such structures

# Summary

- “Signal-processing” view of computation.
  - Powerful way to organize computation.
  - Sequences as interfaces
  - Components: (i) Enumeration, (ii) Map, (iii) Filter, (iv) Accumulate