CS1231 TUTORIAL 11(Last Tutorial)

1. Prove that Q_n is bipartite.

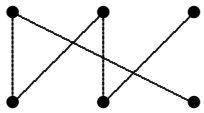
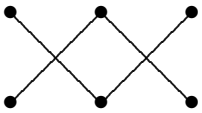
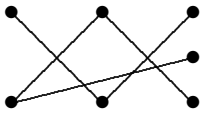
2. Suppose that each vertex of Q_n represents a processor and two processors are connected if they are connected by an edge in the graph. Show that any two processors are connected by a simple of length at most n.

3. Given that A is the adjacency matrix of a graph G and B is the incidence matrix of a graph H. Draw G and H

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

4. The graph H does not have an Euler circuit. What is the minimum number of new edges that can be added so that it has an Euler circuit?

5. Which of the following are trees?

(a)  (b)  (c) 

6. Draw all trees with up to 5 vertices.

7. Is there a tree with 10 vertices with the degrees of its vertices sum to 24?

8. Consider the following rooted tree. Answer the following.

(a) What is the root?

(b) What are the internal vertices?

(c) What are the leaves?

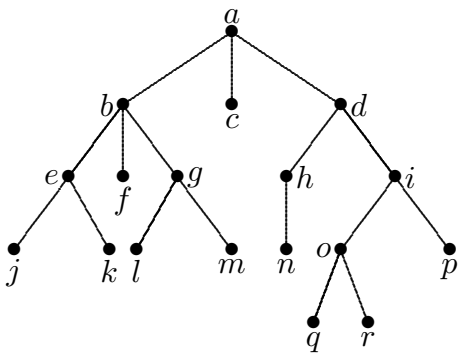
(d) What are the children of b?

(e) What is the parent of k?

(f) What are the ancestors of o?

(g) What are the descendants of d?

(h) What is the height?



9. What can you deduce about the height of a binary tree if you know that it has the following properties?

(a) Twenty-five leaves (b) forty leaves (c) Sixty leaves

10. Prove by mathematical induction that a full binary tree with i internal vertices has $i + 1$ leaves.