NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
MA1101R Linear Algebra I

2018-2019 (Semester 1) **Tutorial 6**

1. Let $S = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_r\} \subset \mathbb{R}^n$.

   (a) Show that if S is linearly independent then any non-empty subset T of S is linearly independent.

   (b) If any non-empty proper subset of S is linearly independent, is S linearly independent? Justify your answer.

2. Let $S = \{\boldsymbol{u}_1 = (2, -1, 1), \boldsymbol{u}_2 = (-1, 2, 3), \boldsymbol{u}_3 = (2, 1, -2), \boldsymbol{u}_4 = (1, 2, -9)\}$ and $V = \text{span}(S)$.

   (a) Is S a basis of V? Justify.

   (b) Find a basis of V.

   (c) Determine the dimension of V.

3. (a) Let U be a subspace of V. Show that if $\dim(U) = \dim(V)$ then $U = V$.

   (b) For the vector space V in Question 2, show that $V = \mathbb{R}^3$. (This question provides us an alternative way to determined whether $V = \mathbb{R}^n$ or not.)

4. Let V be the solution space of the following homogeneous linear system:

$$\begin{cases} x_1 + x_2 + 3x_3 - 2x_4 + 4x_5 = 0 \\ x_1 + x_2 + 2x_3 - x_4 + 2x_5 = 0 \end{cases}$$

   (a) Find a basis S of V;

   (b) Determine the dimension of V;

   (c) Find the coordinate vector of $\boldsymbol{u} = (-1, 1, 0, 2, 1)$ relative to the basis S found in part (a).

   (d) Find a vector $\boldsymbol{v}$ such that $(\boldsymbol{v})_S = (3, 2, 1)$ (relative to the basis S obtained in part (a).)

5. Find a subspace W of $\mathbb{R}^5$ such that W contains the solution space V in Question 4 and $\dim(W) = 4$.