CS1231–Midterm 2, 2019

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1. $|S| = 3$, $P(\{a\}) = \{\emptyset, \{a\}\}$. So $|P(S)| = 2^3 = 8$.

2. (i) 0100001111, (ii) $\{2, 4, 5, 6, 7\}$

3. Method 1. $(A \cup B) - (A \cap B) = (A \cup B) \cap \overline{(A \cap B)} = (A \cup B) \cap (\overline{A} \cup \overline{B}) = (A \cap (\overline{A} \cup \overline{B})) \cup (B \cap (\overline{A} \cup \overline{B}))$.

Note $A \cap (\overline{A} \cup \overline{B}) = (A \cap \overline{A}) \cup (A \cap \overline{B}) = \emptyset \cup (A \cap \overline{B}) = A \cap \overline{B} = A - B$ and $B \cap (\overline{A} \cup \overline{B}) = (B \cap \overline{A}) \cup (B \cap \overline{B}) = (B \cap \overline{A}) \cup \emptyset = B \cap \overline{A} = B - A$.

Therefore, $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$.

Method 2. Suppose $x \in$ LHS. Then $x \in A \cup B$ and $x \notin A \cap B$. Since $x \in A \cup B$, we consider 2 cases: $x \in A$, or $x \in B$

Case 1. $x \in A$.

Because $x \notin A \cap B$, we get $x \notin B$. So $x \in A - B$. Then $x \in$ RHS

Case 2. $x \in B$.

Because $x \notin A \cap B$, we get $x \notin A$. So $x \in B - A$. Then $x \in$ RHS.

Now suppose $x \in RHS$. Then $x \in (A - B) \cup (B - A)$. We also consider 2 cases: $x \in A - B$ or $x \in B - A$.

Case 1. $x \in A - B$.

Then $x \in A$ and $x \notin B$. Because $x \in A$, $x \in A \cup B$. Because $x \notin B$, $x \notin A \cap B$. Then $x \in$ LHS.

Case 1. $x \in B - A$.

Then $x \in B$ and $x \notin A$. Because $x \in B$, $x \in A \cup B$. Because $x \notin A$, $x \notin A \cap B$. Then $x \in$ LHS.

4. Y, N, Y. (i) is absorption law. (ii) If $A = B = \emptyset$ and $C = \mathbb{R}$, then LHS=$\mathbb{R}$ and RHS=$\emptyset$. (iii) $(x, y) \in$LHS iff $x \in A \cap B$ and $y \in C \cap D$ iff $x \in A$ and $x \in B$ and $y \in C$ and $y \in D$ iff ($x \in A$ and $y \in C$) and ($x \in B$ and $y \in D$) iff $(x, y) \in A \times C$ and $(x, y) \in B \times D$ iff $(x, y) \in$RHS.

5. (a) Y (b) N. $f(1)$ is not defined.

6. (a) Domain: S, Range: $\mathbb{Z}$. (b) Domain: $\mathbb{Z}^+$. Range $\mathbb{Z}^+$.

7. f is 1-1: $f(a) = f(b) \Rightarrow 3a - 2 = 3b - 2 \Rightarrow a = b$.

f is onto: $f(x) = y \Rightarrow 3x - 2 = y \Rightarrow x = (2 + y)/3$.

The inverse function $f^{-1}(y) = (2 + y)/3$

8. Ans: No.

Justification: For example: $A = \{1, 2, 3\}$, $B = \{1, 2\}$ and $C = \{1\}$. Let g be from A to B so that for any $x \in A$, $g(x) = 1$. Let f be from B to C so that $f(x) = 1$ for any $x \in B$. Then $f \circ g(x) = 1$ for any $x \in A$. $f \circ g$ is onto and f is onto, but g is not onto.

9. Let $\lfloor \sqrt{x} \rfloor = n$. Then $n \le \sqrt{x} < n + 1 \Rightarrow n^2 \le x < (n+1)^2 \Rightarrow n^2 \le \lfloor x \rfloor \le x < (n+1)^2 \Rightarrow n \le \sqrt{\lfloor x \rfloor} < n + 1 \Rightarrow \lfloor \sqrt{\lfloor x \rfloor} \rfloor = n$.