

1. Which of these are propositions?

- (a) Do not go. (b) Can you answer the question? (c) There are no black flies in Maine.
 (d) $4 + x = 5$. (e) $2^n \geq 100$. (f) The moon is made of green cheese.

2. Let p and q be the propositions “The election is decided” and “The votes have been counted”, respectively. Express each of the following as an English sentence.

- (a) $\neg p \wedge q$. (b) $q \rightarrow p$. (c) $p \leftrightarrow q$. (d) $\neg q \vee (\neg p \wedge q)$.

3. Determine whether these conditional propositions are true or false.

- (a) If $1 + 1 = 3$, then unicorns exist.
 (b) If $1 + 1 = 3$, then dogs can fly.
 (c) If $1 + 1 = 2$, then dogs can fly.
 (d) If $2 + 2 = 4$, then $1 + 2 = 3$.

4. Consider the proposition “You will get an A in this module only if either you do every exercise in the text book or you score at least 80 marks in the final.”

- (a) Determine all the situations in which the proposition is true.
 (b) Given that the proposition is true, would you get an A if you did not do exercise 5 and scored 79 marks for the final?

5. Are the following pairs logically equivalent?

- (a) $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$.
 (b) $(r \vee p) \wedge (\neg r \vee (p \wedge q)) \wedge (r \vee q)$ and $p \wedge q$.

6. Complete the following truth table.

p	q	r	$p \rightarrow q$	$\neg p \rightarrow r$	$(p \rightarrow q) \wedge (\neg p \rightarrow r)$
T	T	T			
T	T	F			
T	F	T			
F	T	T			
T	F	F			
F	T	F			
F	F	T			
F	F	F			

Let $t = (p \rightarrow q) \wedge (\neg p \rightarrow r)$.

Are the following true or false?

- (a) t is false if p and q are both true.
- (b) If t is false then $p \wedge q$ is false.
- (c) A sufficient condition for t to be true is that p is true and r is false.
- (d) t is true only if $p \vee r$ is true.
- (e) $(p \wedge q) \rightarrow t$ is a tautology.

7. The n^{th} proposition in a list of 100 propositions is “Exactly n of the propositions in this list are false”.

- (a) What conclusion can you draw?
- (b) Answer part (a) if the n^{th} proposition is “At least n of the propositions in this list are false”.
- (c) What can you say about part (b) if the list has 101 items?

8. Show that $[(p \rightarrow q) \wedge q] \rightarrow p$ is **not** a tautology.

9. Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known? (i) At least one of Kevin and Heather is chatting; (ii) Exactly one of Randy and Vijay is chatting; (iii) If Abby is chatting, then so is Randy; (iv) Vijay and Kevin are either both chatting or both not chatting; (v) If Heather is chatting, then so are Abby and Kevin.