(2018/19) Semester 2   ST2334 Probability and Statistics   Tutorial 1

1. The NUS library has five copies of a certain text on reserve. Two copies (1 and 2) are first edition, and the other three (3, 4 and 5) are second edition. A student examines these books in random order, stopping only when a second edition has been selected. One possible outcome is 5, and another is 213.
    (a) List the outcomes in the sample space S.
    (b) Let A denote the event that exactly one book must be examined. What outcomes are in A?
    (c) Let B be the event that book 5 is the one selected. What outcomes are in B?
    (d) Let C be the event that book 1 is not examined. What outcomes are in C?
    (e) Are events A and B mutually exclusive events? Explain.

2. Suppose a number is chosen randomly from the set of ten numbers ranging from one to ten. Let A be the event that an even number is drawn, B be the event that an odd number is drawn, C be the event that the number drawn is greater than one but less than six and D be the event that the number drawn is either 1, 6 or 7. List the sample points corresponding to the following events:
    (a) A or C;
    (b) A and B;
    (c) the complement of C;
    (d) A, C and the complement of D.

3. Consider the digits 0, 2, 4, 6, 8 and 9. If each digit can be used only once,
    (a) how many three-digit numbers can be formed from the digits 0, 2, 4, 6, 8 and 9?
    (b) how many of these are odd numbers?
    (c) how many of these odd numbers are greater than or equal to 620?

4. Four married couples have bought 8 seats in a row for a concert. In how many different ways can they be seated
    (a) with no restrictions?
    (b) if each couple is to sit together?
    (c) if all the men sit together to the right of all the women?

5. An exam paper consists of seven questions. Candidates are asked to answer five questions. Find the number of choices of the five questions if
    (a) no restriction on the choices;
    (b) the first two questions must be answered;
    (c) at least one of the first two questions must be answered and
    (d) exactly two from the first three questions must be answered.

6.

Y(10, 6)   B (13, 8)

X (2,2)   A (0, 0)
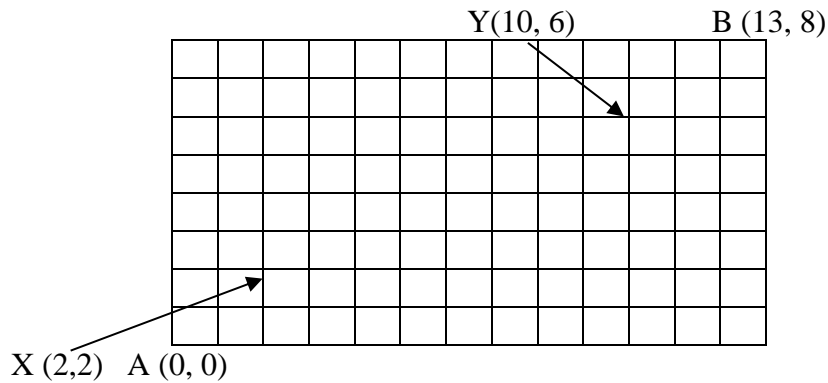
Suppose that we want to move from the bottom left hand corner (point A: (0, 0)) of the grid to the top right hand corner (point B: (13, 8)). At each step, we can either move up or move to the right.
   (a) How many ways are there to move from the point A to the point B?
   (b) How many ways are there to move from the point A to the point B if one must not stop at the point X (2, 2)?
   (c) How many ways are there to move from the point A to the point B if one must stop at points X and Y (10, 6)?
   [*Hint*: A path is represented by an arrangement of 8 Up's and 13 Right's]

7. Beethoven wrote 9 symphonies, Mozart wrote 27 piano concertos and Schubert wrote 15 string quartets.
   (a) If a deejay of a radio station wishes to play a Beethoven symphony and then a Mozart concerto, in how many ways can this be done?
   (b) The station manager decides that on each successive night (7 nights per week), a Beethoven symphony will be played, followed by a Mozart piano concerto, then followed by a Schubert string quartet. For roughly how many years could this policy be continued before exactly the same program would have to be repeated?

8. How many permutations can be made from the word "white" if
   (a) it begins with a consonant;
   (b) it ends with a vowel;
   (c) it has the consonants and vowels alternating.

9. A contractor wishes to build 9 houses, each of different in design. In how many ways can be place these houses on a street if 6 lots are on one side of the street and 3 lots are on the opposite side?

10. In how many ways can 3 oaks, 4 pines, and 2 maples be arranged along a property line if one does not distinguish among trees of the same kind?

Answers to selected problems:
1. (a) $S = \{123, 124, 125, 13, 14, 15, 213, 214, 215, 23, 24, 25, 3, 4, 5\}$.
    (b) $A = \{3, 4, 5\}$. (c) $B = \{5, 15, 25, 125, 215\}$. (d) $C = \{23, 24, 25, 3, 4, 5\}$.
    (e) $A \cap B = \{5\}$.
2. (a) $A \cup C = \{2, 3, 4, 5, 6, 8, 10\}$. (b) $A \cap B = \emptyset$. (c) $C' = \{1, 6, 7, 8, 9, 10\}$.
    (d) $A \cap C \cap D' = \{2, 4\}$.
3. (a) $5 \times 5 \times 4 = 100$. (b) $4 \times 4 \times 1 = 16$. (c) $1 \times 4 \times 1 + 1 \times 3 \times 1 = 7$.
4. (a) $_8P_8 = 40320$. (b) $_4P_4 \times (_2P_1 \times _2P_1 \times _2P_1 \times _2P_1) = 384$. (c) $_4P_4 \times _4P_4 = 576$.
5. (a) $_7C_5 = 21$. (b) $_2C_2 \times _5C_3 = 10$. (c) $_2C_1 \times _5C_4 + _2C_2 \times _5C_3 = 20$. (d) $_3C_2 \times _4C_3 = 12$.
6. (a) $_{21}C_8 = 203490$. (b) $_{21}C_8 - _4C_2 \times _{17}C_6 = 129234$. (c) $_4C_2 \times _{12}C_4 \times _5C_2 = 29700$.
7. (a) $_9C_1 \times _{27}C_1 = 243$. (b) $_9C_1 \times _{27}C_1 \times _{15}C_1 = 3654 \approx 10$ years.
8. (a) $_3P_1 \times _4P_4 = 72$. (b) $_2P_1 \times _4P_4 = 48$. (c) $_3P_3 \times _2P_2 = 12$.
9. $_9C_6 \times _6P_6 \times _3P_3 = 362880$.
10. $9! / (3!\, 4!\, 2!) = 1260$.