GEQ1000 Asking Questions
Engineering Segment

Lecture 1.3
The Activity of Modelling
How do Models Help Engineers to Understand and to Solve Problems?

The activity of modelling is a significant aspect of the engineer's practice. Engineers use models to approximate a real engineering system. Models can function as a safe and less expensive proxy, or stand-in, for the true system. They can be used to find solutions to the problem at hand, examine the effects that uncertainties in some variables have on other variables (say, a key performance indicator).

This is important as engineers are often required to solve problems under severe time and budget constraints. This means that we must to be efficient at acquiring and organizing relevant information.

Models come in all shapes and sizes. Sometimes, engineers use physical models to represent the real-world. Take, for example, a physical model representing NUS or UTown. We create this to help people understand the local geography.

For us, engineers, building mathematical models is a very effective way of capturing our current technical knowledge, field information, and objectives. The modeling effort will help us discover what additional information is needed so that the problem can be satisfactorily solved. These mathematical models codify our appreciation of basic and inviolable laws of nature.

Let me take you through some of the modelling principles engineers typically employ. We begin first with the **Principles of Conservation**, namely the conservation of mass, energy, momentum, electric charge etc.

This is a way for engineers to keep an account of (1) what goes in, (2) what comes out, (3) what stays inside (i.e. accumulates within), and (4) what gets transformed to some other form.

For example, we can model the accumulation of energy within our bodies to understand weight gain or loss. We can account for the overall weight gain/loss based on the energy intake (through food) and energy expenditure (for basic metabolism and external activities).

When the energy intake exceeds the expenditure, there is an accumulation of energy which the body converts into fat and/or muscle. This ultimately results in weight gain over time.

In addition to ***conservation principles***, models also incorporate mathematical equations deduced via experiments. These equations are termed ***constitutive relationships***. They do not emerge from conservation principles but they do the critical job of connecting variables that one sees in the conservation principles.

The famous German physicist Georg Ohm experimentally observed that the two conserved quantities (voltage, V and current, I) follow a linear relationship in most cases and proposed the well-known Ohm's law, $V = I R$, with R representing the resistance of the conductor.

Here's an example of how this model is useful for engineers. Let's say we have a circuit where the voltage drop is 220 volts, and we need a current of 5 amperes to flow through it. Based on this model (i.e. ohms law), we know that a resistance of 44 ohms (R = V/I = 220/5) is needed for the circuit to function well.

Now, the resistance value of the conductor depends on several factors. From this model, knowing that we need 44 ohms, we can now ask: What should be the material of the resistor element? How long should it be? What must be the cross-sectional area? These choices should be made such that the resistance R of the chosen material will be 44 ohms.

This is how a simple model represented by Ohm's law can help in the design of electrical circuits.

To summarize, one route engineers use when creating models is to employ conservation equations, and their related constitutive relationships. This is often referred to as ***first principles modelling***.

There is another route engineers use. We do this when we want to create a model quickly, or if we do not understand the scientific principles behind the systems in question.

Imagine that you have never driven a car before. You don't need to learn automobile engineering, or the mechanics of how the internal combustion engine works in order to drive a car. Instead, you can develop a working knowledge of how to drive it by doing careful experiments with the steering wheel, brakes, and accelerator of their car, and seeing the effect it has on the speed and direction of the car.

Likewise, we can still develop a good representation of a system's behaviour without a proper knowledge of the mechanisms involved. We do this by collecting data from carefully designed experiments that will help us gain specific insights on the system's behaviour. We can create a mathematical model using regression techniques and machine learning tools.

Models created using this experimental route are known as ***empirical models***.

Consider this example: an engineer wants to improve the yield (Y) of a chemical product and knows that the yield is affected by the processing temperature (T), Catalyst Type (C) and Purity of an additive (P).

Let's imagine that the engineer established the following empirical model, through an understanding of the basic process and by constructing experiments:

$$Y = 38.1 - 3.6\,T + 8.1\,P - 7.4\,(C \times P)$$

The above model will be incredibly useful for the engineer in answering questions like, "Given that T, C, and P may vary over a certain range of values, what would be the best values of T, C, and P that will maximize the yield Y of the product?"

As I have shown, with empirical models, we don't have to possess proper knowledge of the mechanisms involved. A good representation of the system's behaviour will suffice!

Yet, despite the utility and popularity of empirical models, there is one problem engineers typically face. Let me illustrate this with a simple example.

Let's say we are modelling the relationships between variables relevant to driving a car. We shall use U to include the various inputs, e.g. u_1 = steering wheel position, u_2 = accelerator pedal position and u_3 = brake pedal position, etc. And we shall use Y to include the various outputs, e.g. y_1 = speed of car and y_2 = direction of the car, etc.

Let us use F to denote the correct relationship. The correct relationship between the inputs and outputs of the car may be written as $Y = F(U)$.

However, one common problem engineers face is that nature often does not allow us to discover the true relationship between a system's inputs and its outputs. Hence, we might have found a different modelled relationship, e.g. $Y = G(U)$.

Why is there such a ***mismatch***?

For one, we may not know all the mechanisms that are at work in the real system. To some extent, ignorance about phenomena underlying our system is not uncommon.

Second, whenever we model, we need to make ***defendable assumptions*** to simplify the complexity of the real world. Thus, there is a deliberate move to capture only the "essence" of the real world in the model, and not the whole "truth." This ***model reduction*** keeps the model solvable, even with powerful computers, for real-time applications.

Third, the mismatch between the truth and our model can arise because of ***experimental flaws*** as well. Inputs, unknown and/or unrecorded may be affecting the output variables significantly during the time we conduct our experiments. Mismatch can stem from the ***stochastic noise*** that contaminates all the measurements we make.

As we do not have access to the true values of measurements because of noise, errors will find its way into the model. It suffices to say that engineers are challenged both by errors in models (arising out of the need for simplification, ignorance etc.) and in measurements. Yet, good decisions must be made in all aspects of engineering design and operations. Certainly, the need to make ***decisions under uncertainty*** makes the engineering profession very interesting.

We expect errors to be present in our models, and for things like environmental factors, to change over time. Thus, the actual system performance may not match the performance that our models promised! This compels us to ask questions like: By how much should I over-design my solution to override uncertainties?

Typically, engineers over-design their solutions by 10 or 20%, depending on the magnitude of the estimated uncertainty. The larger the uncertainty, the larger the overdesign factor.

There is so much more that I wish to share with you about the process of modelling. But what is important is this: regardless of the type of model we employ, we must continuously validate the model's performance against the data we collect from a real system. When there is a significant gap between the actual system output and predictions from the current model, the engineer needs to return to the drawing board again to adjust the model and make it reflect the current reality.

We engineers must keep an ***open mind*** to question the assumptions and update the model as we understand more about the real system and/or gain access to more data. Model building is therefore an ***iterative*** *process.* For some systems, this is a never-ending process.