CS4246 / CS5446

Tutorial Week 10

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First

Review

- Linear Regression
- Neural Networks
- Gradient Descent
- Common Issues

Linear Regression

Single variable

f(x) = wx+b

(linear) function approximator

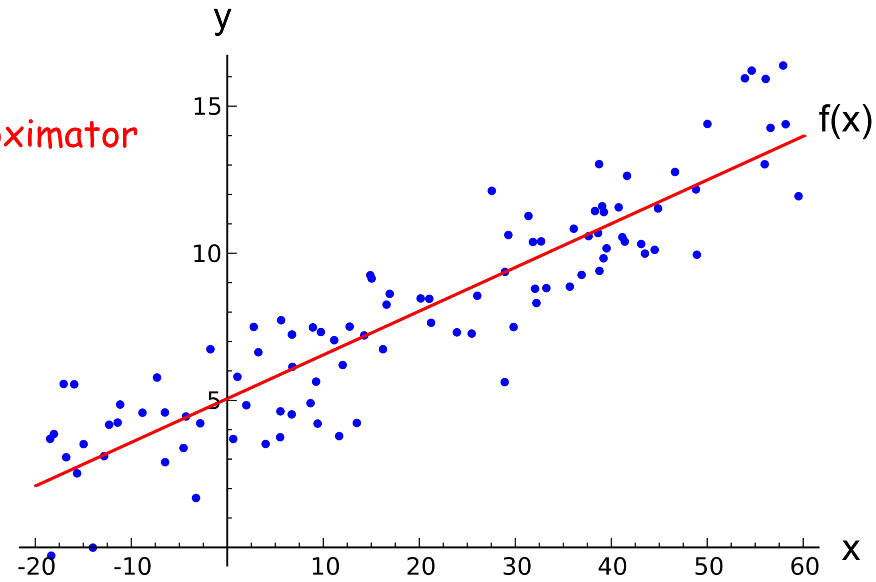


Image source: https://en.wikipedia.org/wiki/Linear_regression#/media/File:Linear_regression.svg

Linear Regression

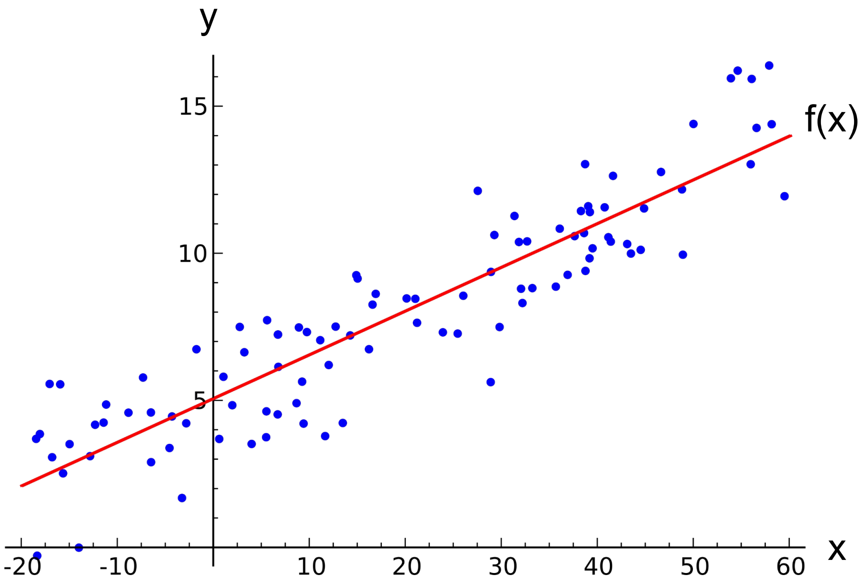
Single variable

$f(x) = wx+b$

Multiple variables:

$f(\mathbf{x}) = \mathbf{w}^T\mathbf{x}+b$

where $\mathbf{w} = [w_1,...,w_F]^T$, $\mathbf{x} = [x_1,...,x_F]^T$



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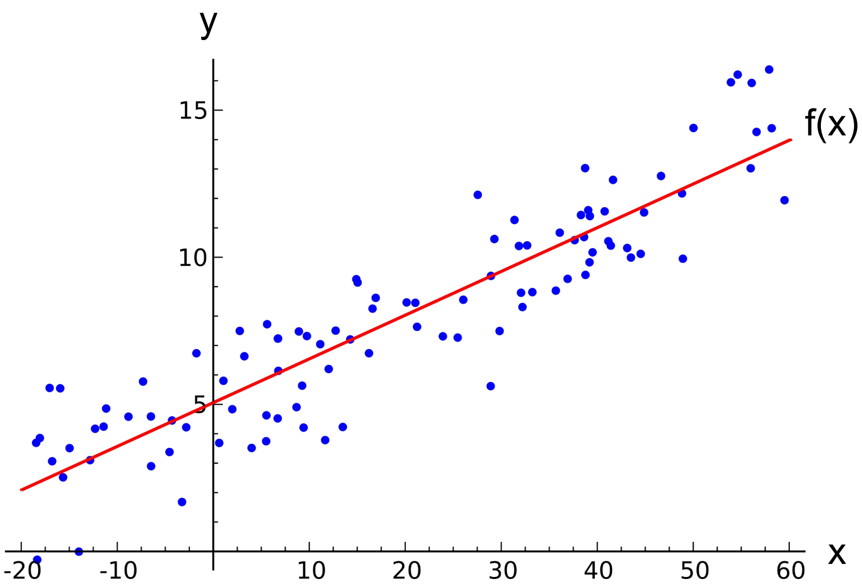

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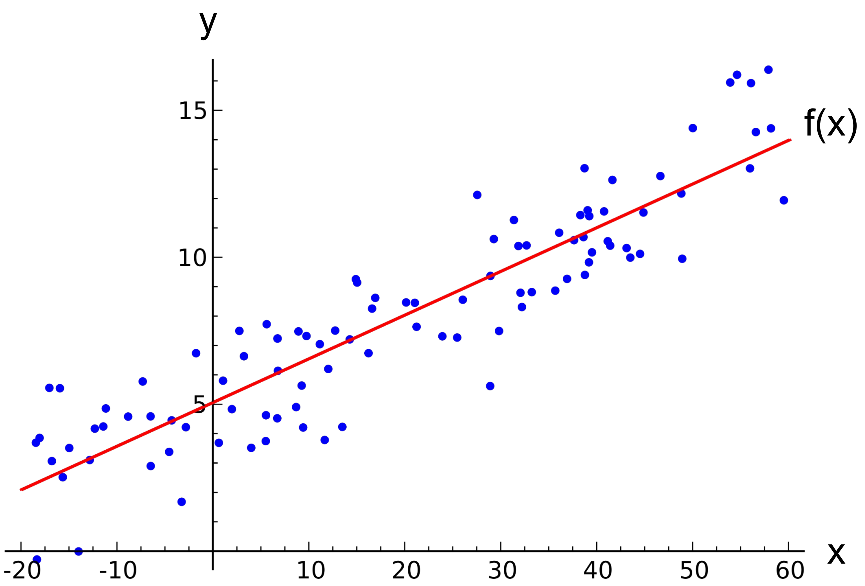

Goal: minimize $\sum_{(x,y)} [\ f(x) - y\ ]^2$

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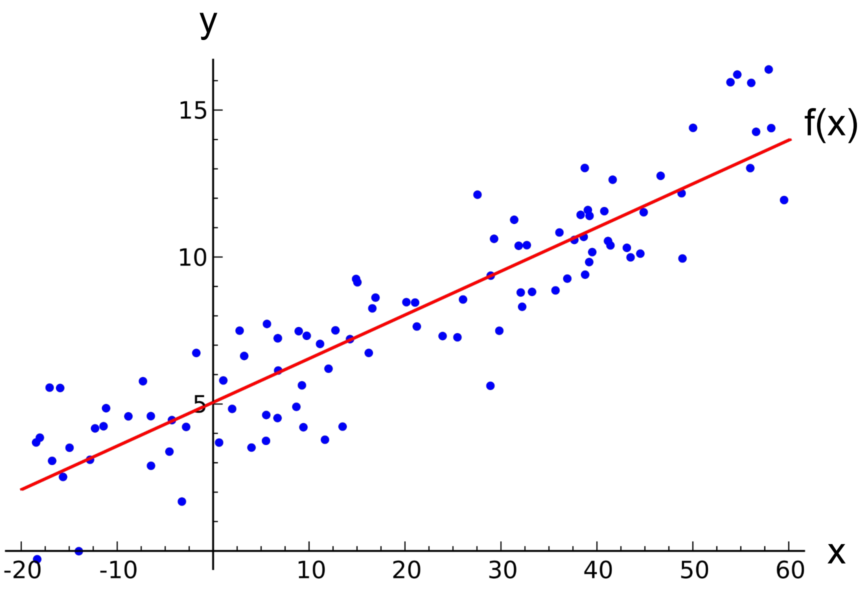

Goal: minimize $\underbrace{\sum_{(x,y)} [\, f(x) - y \,]^2}_{\text{Squared distance}}$

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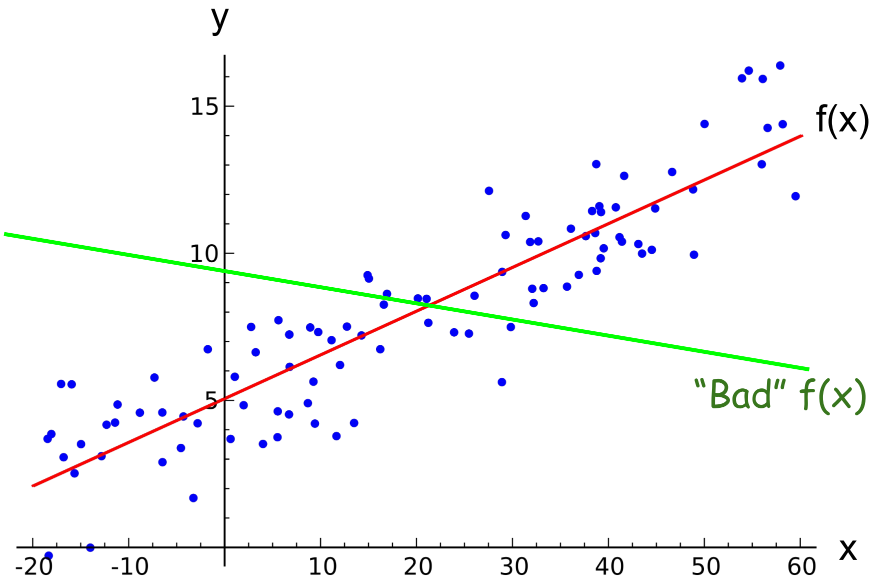

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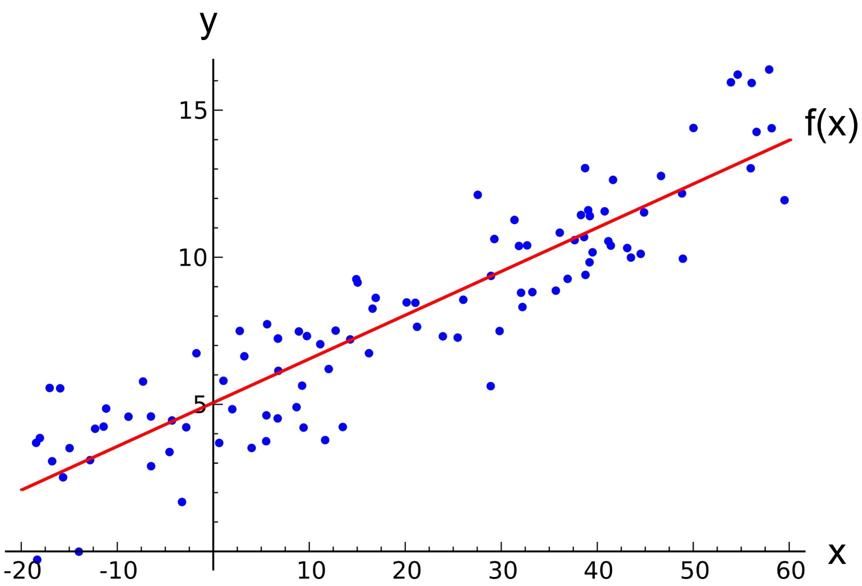

Goal: minimize $\sum_{(x,y)} [\, f(x) - y \,]^2 = L_{(x,y)}(\, f(x)\, ,\, y\,)$

Generally: loss function

Image source: https://en.wikipedia.org/wiki/Linear_regression#/media/File:Linear_regression.svg

Neural Networks (NN)

(another) function approximator

Neural Networks (NN)

Recall our linear function

$f(\mathbf{x}) = \quad \mathbf{w}^T\mathbf{x}+b$

where $\mathbf{w} = [w_1,\ldots,w_F]^T$, $\mathbf{x} = [x_1,\ldots,x_F]^T$

Neural Networks (NN)

Simple NN (Neuron)

Now it goes **neural**!

$$f(\mathbf{x}) = \varphi(\mathbf{w}^T\mathbf{x}+b)$$

where $\mathbf{w} = [w_1,\ldots,w_F]^T$, $\mathbf{x} = [x_1,\ldots,x_F]^T$

Neural Networks (NN)

Simple NN

$f(\mathbf{x}) = \varphi(\mathbf{w}^T\mathbf{x}+b)$

where $\mathbf{w} = [w_1,...,w_F]^T$, $\mathbf{x} = [x_1,...,x_F]^T$,

φ non-linear activation function

Neural Networks (NN)

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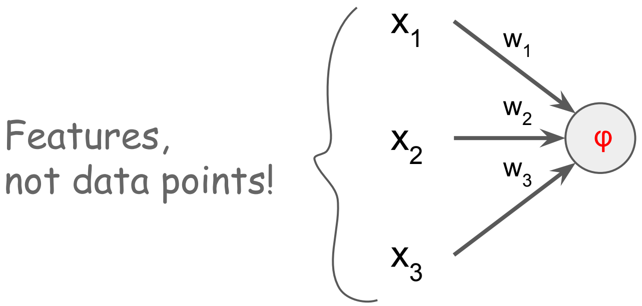

Neural Networks (NN)

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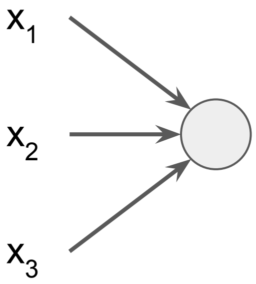

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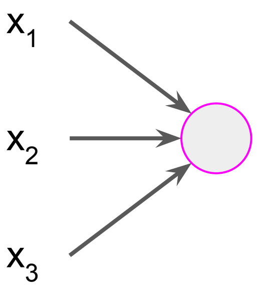
"Deep" NN

$f(\mathbf{x}) =$ $f_{i1}(\mathbf{x})$



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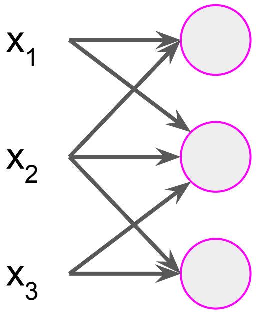
"Deep" NN

$f(\mathbf{x}) =$ $f_{i1}(\mathbf{x})\ldots$ $\ldots$



Neural Networks (NN)

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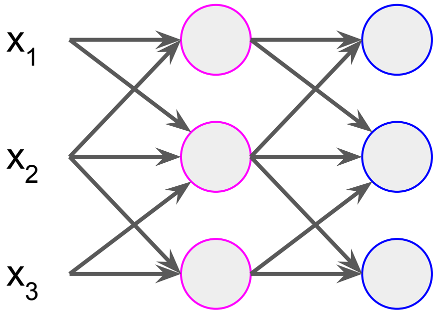
"Deep" NN

$f(\mathbf{x}) = f_{H1}([f_{i1}(\mathbf{x})\ldots]^T) , \ldots, f_{HM}([\ldots]^T)$



Neural Networks (NN)

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$f(\mathbf{x}) = \varphi(\mathbf{w}^T\mathbf{x}+b)$

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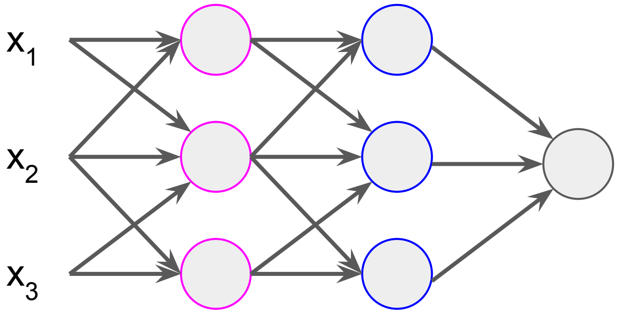
"Deep" NN

$f(\mathbf{x}) = f_O([\, f_{H1}([f_{i1}(\mathbf{x})\ldots]^T)\, ,\, \ldots,\, f_{HM}([\ldots]^T)\,]^T)$



Neural Networks (NN)

“building blocks”

Simple NN

$f(\mathbf{x}) = \varphi(\mathbf{w}^T\mathbf{x}+b)$

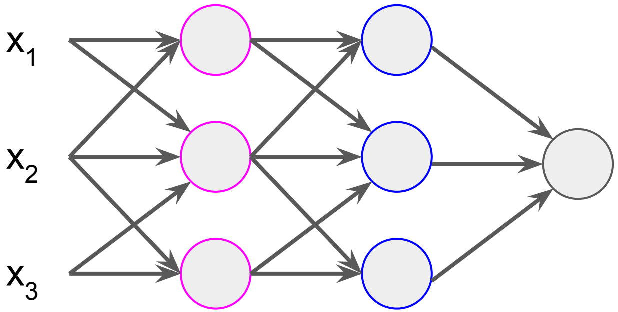
where $\mathbf{w} = [w_1,\ldots,w_F]^T$, $\mathbf{x} = [x_1,\ldots,x_F]^T$,

φ non-linear activation function

Convolutional NN

“Deep” NN

$f(\mathbf{x}) = f_O([\ f_{H1}([f_{i1}(\mathbf{x})\ldots]^T)\ ,\ \ldots,\ f_{HM}([\ldots]^T)\ ]^T)$



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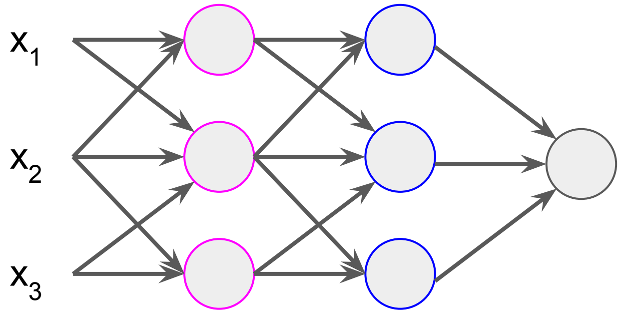
Convolutional NN

$f(\mathbf{x}) = \varphi(\mathbf{w}^T \ast \mathbf{x})$

where $\mathbf{w} = [w_1,...,w_K]^T$, $K \leq F$

“Deep” NN

$f(\mathbf{x}) = f_O([\ f_{H1}([f_{i1}(\mathbf{x})...]^T)\ , \ldots, f_{HM}([...]^T)\ ]^T)$



Neural Networks (NN)

Simple NN

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where $\mathbf{w} = [w_1,\ldots,w_F]^T$, $\mathbf{x} = [x_1,\ldots,x_F]^T$,

φ non-linear activation function

Convolutional NN

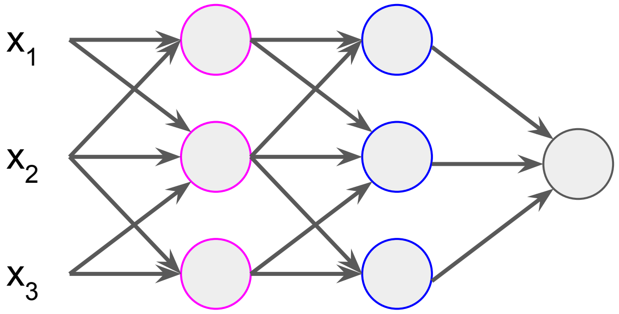
$$f(\mathbf{x}) = \varphi(\mathbf{w}^T * \mathbf{x})$$

where $\mathbf{w} = [w_1,\ldots,w_K]^T$, $K \leq F$

x f(**x**)

"Deep" NN

$$f(\mathbf{x}) = f_O([\ f_{H1}([f_{i1}(\mathbf{x})\ldots]^T)\ ,\ \ldots,\ f_{HM}([\ldots]^T)\ ]^T)$$



Source: https://towardsdatascience.com/intuitively-understanding-convolutions-for-deep-learning-1f6f42faee1

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Convolutional NN

$f(\mathbf{x}) = \varphi(\mathbf{w}^T * \mathbf{x})$

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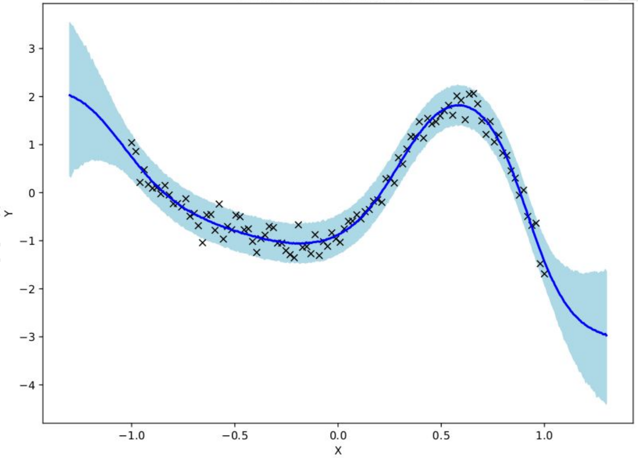
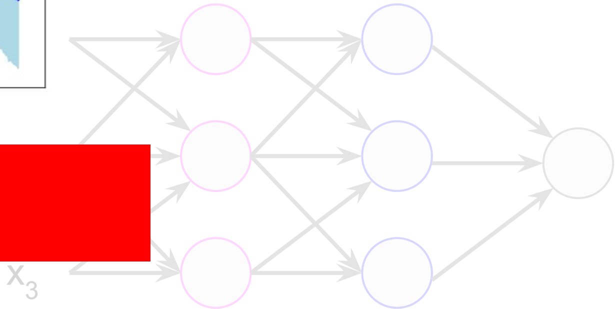
"Deep" NN

$= f_O([\, f_{H1}([f_{i1}(\mathbf{x})\ldots]^T) , \ldots , f_{HM}([\ldots]^T)$



Goal: Minimize Loss



$\mathbf{x}$ $f(\mathbf{x})$

http://num.pyro.ai/en/0.5.0/examples/bnn.html

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Minimizing Loss

Consider:

- Loss = $\sum_{(x,y)} [\ f(x) - y\ ]^2$
- $f(x) = wx$

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The only independent variable!

Minimizing Loss

Consider:

- Loss = $\sum_{(x,y)} [\ f(x) - y\ ]^2$
- $f(x) = wx$

How to find w such that we get minimum loss?

$L_{(x,y)} = [\ f(x) - y\ ]^2 = [\ wx - y\ ]^2$

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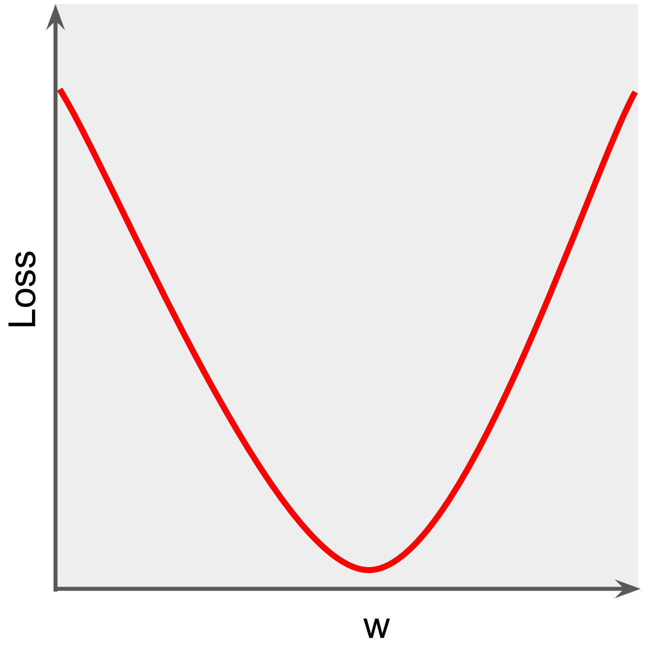

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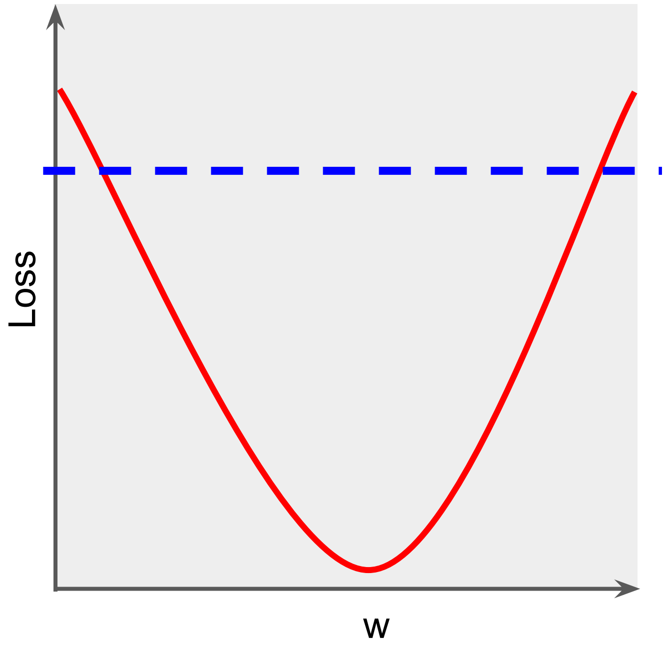

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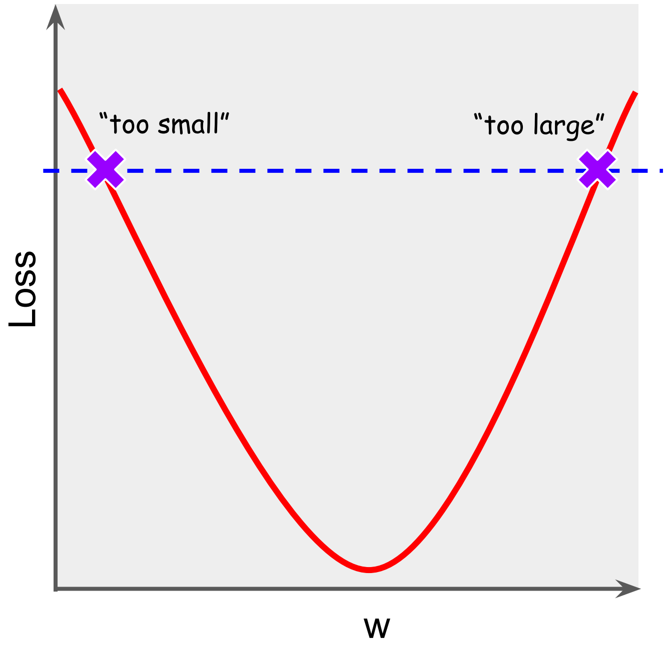
$L_{(x,y)} = [\, f(x) - y \,]^2 = [\, wx - y \,]^2$

wx < y → "too small"
wx > y → "too large"



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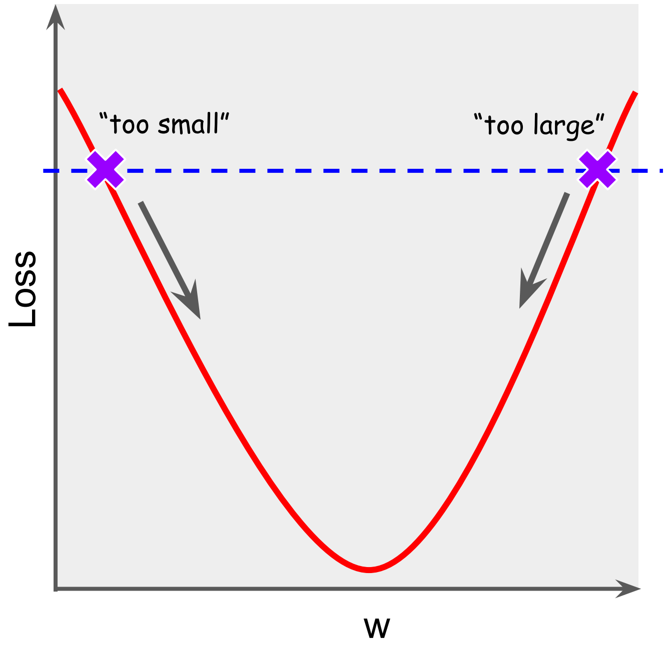

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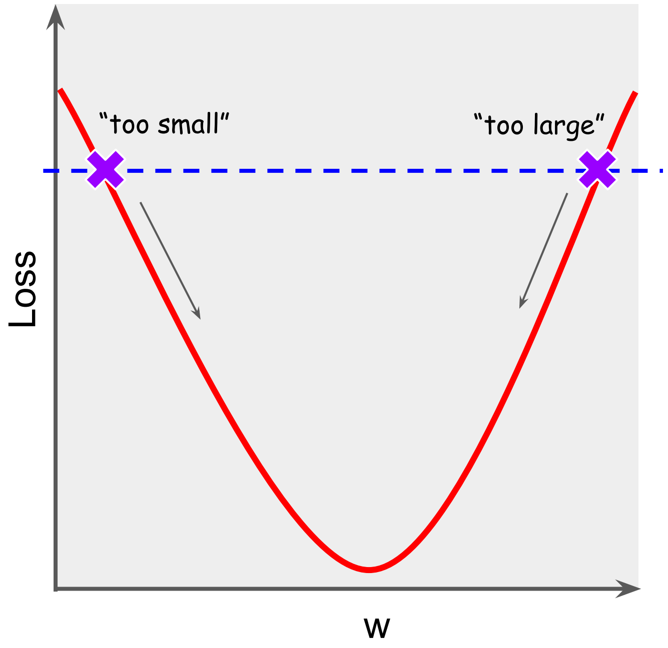

How do we know which one?

(should be general enough for any function)

Minimizing Loss

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How to find w such that we get minimum loss?

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wx < y → "too small"
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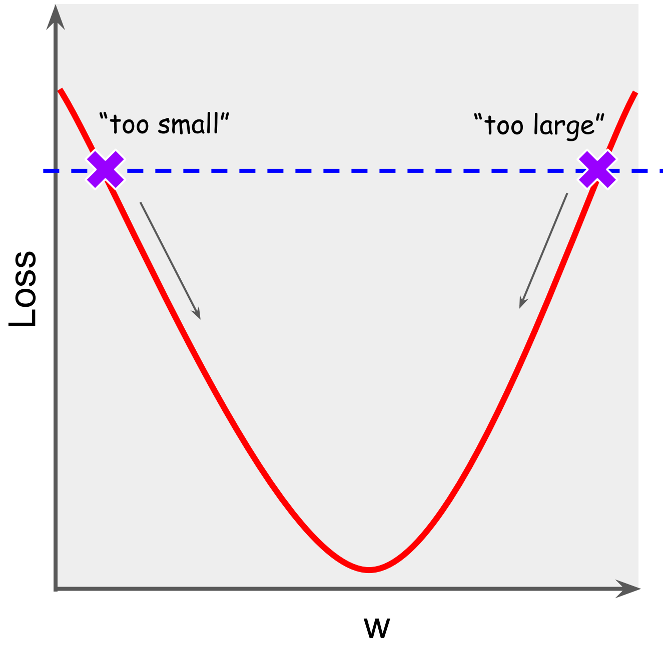
How do we know which one?

$dL_{(x,y)}/dw = 2[\ wx - y\ ]x$

"too small" → gradient -ve
"too large" → gradient +ve



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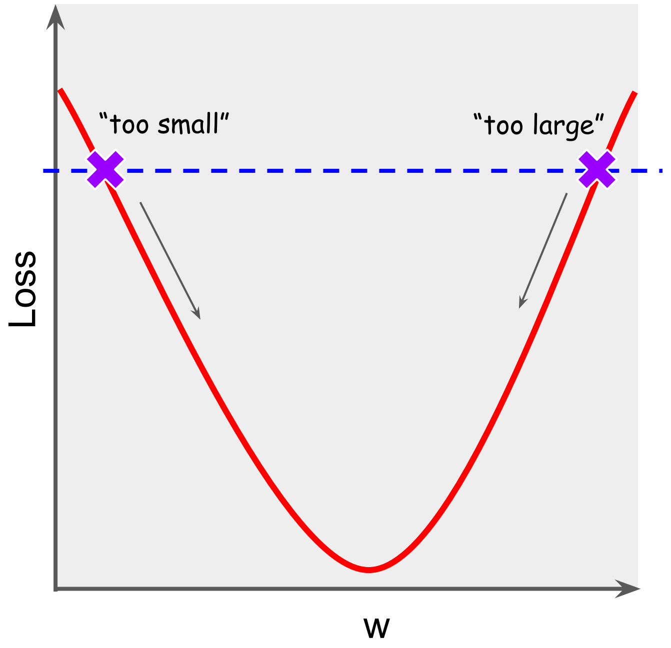
Update:

$w_t = w_{t-1}$ - gradient



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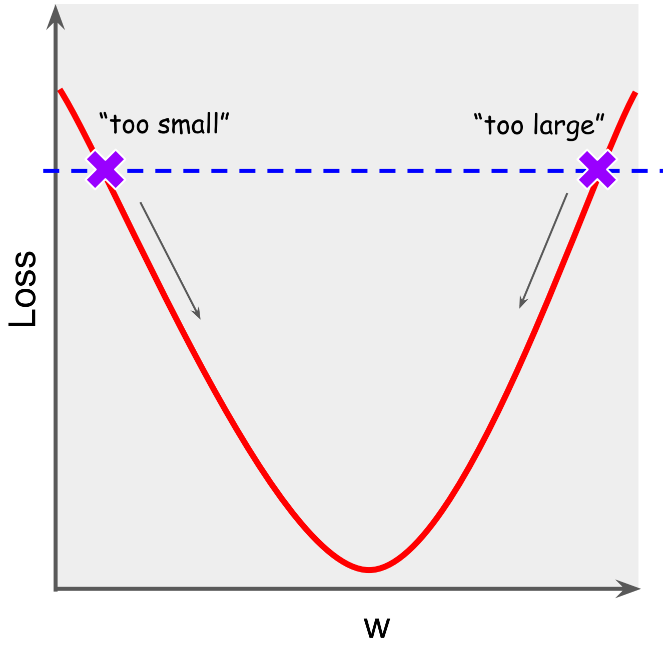
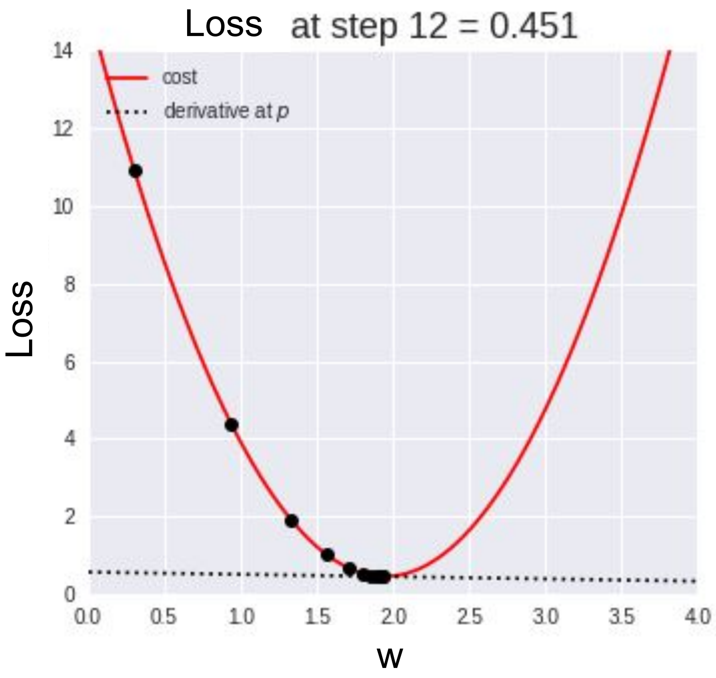
"too small" $\rightarrow$ gradient -ve
"too large" $\rightarrow$ gradient +ve

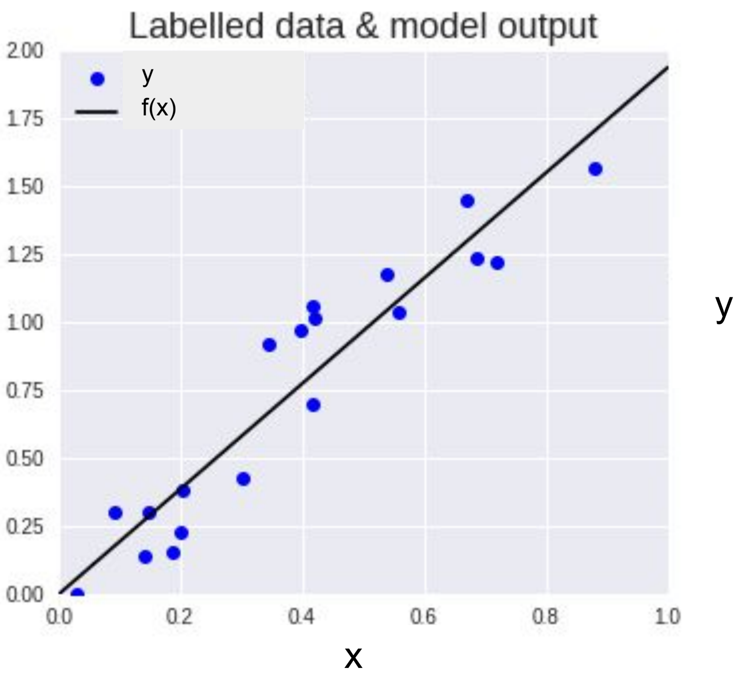
Update:

$w_t = w_{t-1}$ - gradient (do in a loop) ← Gradient Descent



Gradient Descent: Illustration





Source: https://camo.githubusercontent.com/a401a48f5503c52004369148a784e779aa7e3411/68747470733a2f2f63646e2d696d616765732d312e6d656469756d2e636f6d2f6d61782f313630302f312a70775049472d475748796150564d564747354f6841512e676966

Gradient Descent: Backpropagation

What about complicated functions (e.g., neural networks)?

Gradient Descent: Backpropagation

What about complicated functions (e.g., neural networks)?

Backpropagation = Chain Rule of Calculus

$$\frac{da}{dx}a(b(c(d(e(f(g(x)))))))$$

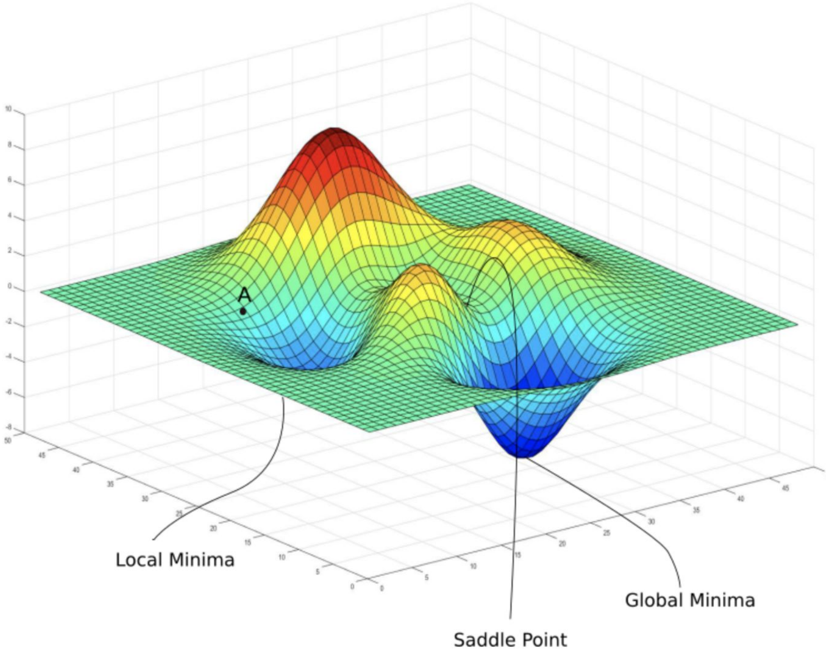
$$\frac{da}{\partial x} = \frac{da}{db} \times \frac{db}{dc} \times \frac{dc}{dd} \times \frac{dd}{de} \times \frac{de}{df} \times \frac{df}{\mathrm{dg}} \times \frac{\mathrm{dg}}{dx}$$

Source: https://ai.plainenglish.io/calculus-for-backpropagation-doesnt-have-to-be-scary-16595d76e744

Gradient Descent: Issues with Deep Neural Networks

Gradient Descent: Issues with Deep Neural Networks

Local Minima



Sources:
https://medium.com/analytics-vidhya/journey-of-gradient-descent-from-local-to-global-c851eba3d367

Gradient Descent: Issues with Deep Neural Networks

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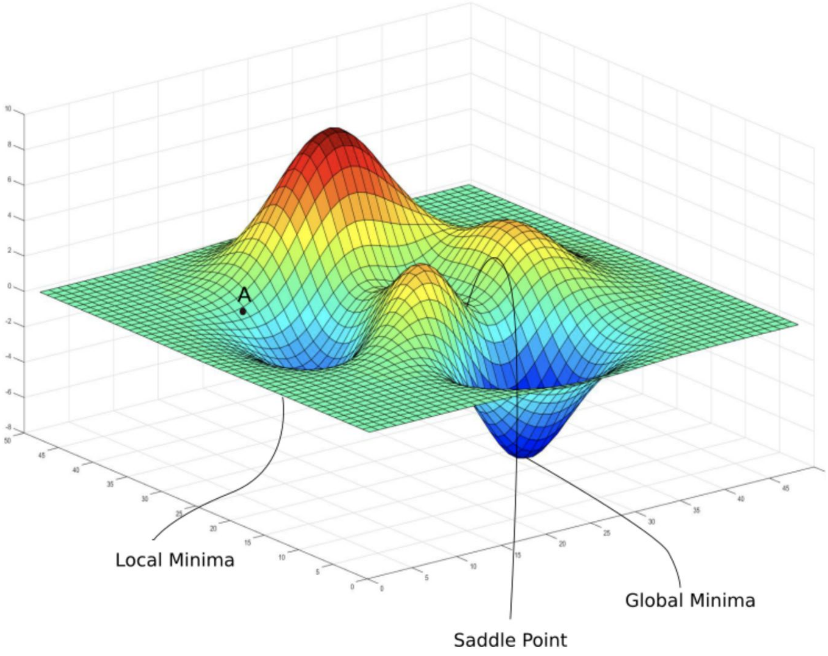
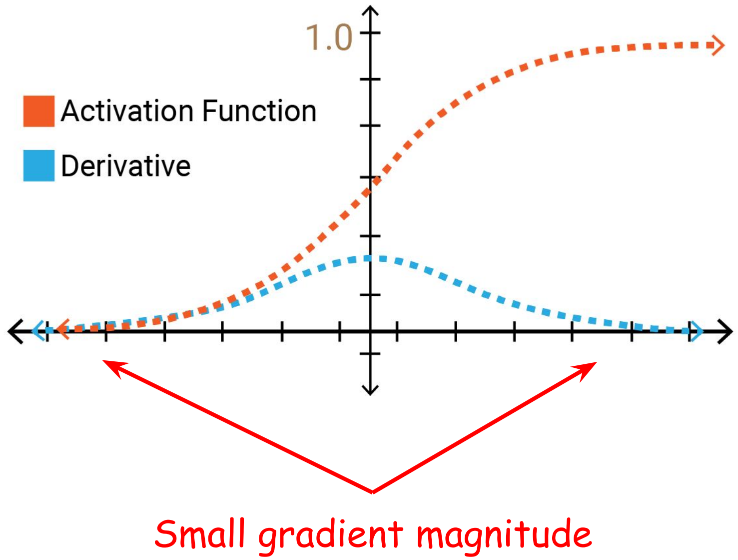


Vanishing Gradient



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https://towardsdatascience.com/the-problem-of-vanishing-gradients-68cea05e2625

Issue with Function Approximation in RL

The Deadly Triad in RL:

- **Function Approximation**
- Bootstrapping
- Off-policy Learning

Second

[RN 21.4] Write out the parameter update equations for TD learning with

$$\hat{U}_\theta(x,y) = \theta_0 + \theta_1 x + \theta_2 y + \theta_3 \sqrt{(x - x_g) + (y - y_g)}$$

[RN 21.4] Write out the parameter update equations for TD learning with

$$\hat{U}_\theta(x, y) = \theta_0 + \theta_1 x + \theta_2 y + \theta_3 \sqrt{(x - x_g) + (y - y_g)} \qquad s = (x, y)$$

$$L = \frac{1}{2}\left(R(s \quad) + \gamma \hat{U}_\theta(s') - \hat{U}_\theta(s)\right)^2$$

[RN 21.4] Write out the parameter update equations for TD learning with

$$\hat{U}_\theta(x,y) = \theta_0 + \theta_1 x + \theta_2 y + \theta_3 \sqrt{(x - x_g) + (y - y_g)} \qquad s = (x,y)$$

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Third

Consider a system with a single state variable x that can take value 0 or 1 and actions a_1 and a_2. An agent can observe the value of the state variable as well as the reward in the observed state. Assume a discount factor $\gamma = 0.9$.

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a_1	0	0
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Question

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Q	$x=0$	$x=1$
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Q	$x=0$	$x=1$
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ii. Second observed transition: from $x = 1$, observed reward $r = -5$, action a_2, next state $x = 0$.

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$$Q(1,a_2) \leftarrow 0 + 0.5(-5 + 0.9 \max(5,0) - 0) = -0.25$$

Q	$x = 0$	$x = 1$
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(b) Now perform Q-learning with function approximation using

- $Q(x, a_1) = \beta_1 x$
- $Q(x, a_2) = \beta_2 x$
- $\alpha = 0.5$
- $\beta_1 = 0$
- $\beta_2 = 0$

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$$\beta_1 \leftarrow 0 + 0.5(10 + 0.9 \max(0, 0) - 0)1 = 5$$

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$Q(x, a_2) = \beta_2 x$

$$\beta_1 \leftarrow 5 + 0.5(-5 + 0.9 \max(0, 0) - 0)0 = 5$$

$$\beta_2 \leftarrow 0 + 0.5(-5 + 0.9 \max(0, 0) - 0)1 = -2.5$$

Consider a system with a single state variable x that can take value 0 or 1 and actions a_1 and a_2. An agent can observe the value of the state variable as well as the reward in the observed state. Assume a discount factor $\gamma = 0.9$.

(c) After enough data is observed, which method would give better performance, the tabular method in (a) or the function approximation method in (b)? Why? Suggest how the poorer performing method can be improved.

Question

Consider a system with a single state variable x that can take value 0 or 1 and actions a_1 and a_2. An agent can observe the value of the state variable as well as the reward in the observed state. Assume a discount factor $\gamma = 0.9$.

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Tabular is better

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Failure case of the function approximation

$$Q(x, a1) = \beta_1 x \text{ and } Q(x, a2) = \beta_2 x$$

Can't have non-zero value for x=0

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Improvements

$$Q(x, a_1) = \beta_1 x + \delta_1 \text{ and } Q(x, a2) = \beta_2 x + \delta_2$$

Question?

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