NATIONAL UNIVERSITY OF SINGAPORE

CS1231 - DISCRETE STRUCTURES

(SEMESTER 2 AY 2015/2016)

Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This assessment paper contains **FIVE** questions and comprises **EIGHT** printed pages, including this page.
2. Answer **ALL** questions within the space in this booklet.
3. This is a Closed Book assessment.
4. Candidates are allowed to bring in an A4-sized help sheet, written on both sides.
5. Calculators are allowed.
6. Please write your Student Number below. Do not write your name.

Student NO: _____

Question	Marks	Remarks
A(Pg 2)		
A(Pg 3)		
B		
C		
D		
E		
Total		

Question A [40 marks]. *For each of the following, just write down the answers in the spaces provided. Detailed workings are not required. Also numerical answers are to be written as integers or powers of a single integer. For example, you can write 2300 or* 3^{27} *but not* $\binom{5}{1}\binom{3}{1}$.

(1) Find -5633 **Mod** 13.

(2) Is 1693 a prime number? Yes/ No

(3) Find the coefficient of x^{17} in the expansion of $(3x^7 + 2x^5 - 1)^{20}$.

(4) Find the number of integers in $\{1, 2, \ldots, 2016\}$ which are

(i) multiples of at least two of the integers 3, 5 or 7.

(ii) multiples of 7 and are also multiples exactly one of 3 and 5.

(5) How many ways are there to choose 3 integers a, b, c from $1, 2, \ldots, 36$ so that $a < b < c$ and $b - a \geq 3$, $c - b \geq 5$.

(6) How many strings of length 8 formed using letters from $\{a, b, c, d\}$ contain exactly one pair of adjacent letters that are the same?

(7) Balloons are to be distributed to 5 boys sitting in a circle so that each boy is to receive balloons of one colour and adjacent boys are to receive balloons of different colours. If there are balloons of 10 different colours, how many ways can this be done?

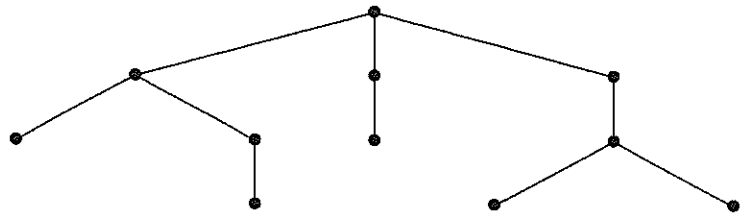
(8) Let G be a connected simple graph with 7 vertices, 12 edges and the degree of each vertex is 2, 3 or 4, all inclusive.

(i) How many vertices of degree 2 are there?

(ii) Is it true that such a G must have an Euler path? Yes / No

(9) In the hypercube Q_4, find a simple path from 0101 to 1100. (You only need to name the vertices in the path.)

(10) Label the following tree using the Universal Address system. (Write the labels beside the vertices.)



Order the vertices using the lexicographic ordering.

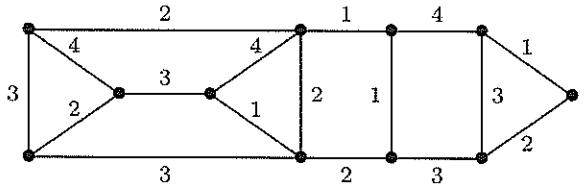
(11) Let T be a full 3-ary tree.

(i) How many vertices does T have if it has 100 internal vertices?

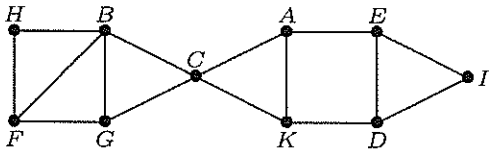
(ii) How many among the numbers 101, 222, 333, 666, can be the number of leaves of T? (Your answer ranges from 0 to 4.)

(12) A full m-ary tree has 81 leaves and height 4. Find the minimum and maximum values of m. min: max:

(13) Find the weight of a minimum spanning tree in the following graph.



(14)



Let G be the graph above. Using the alphabetical ordering, find a spanning tree by depth first search. Draw the tree below.

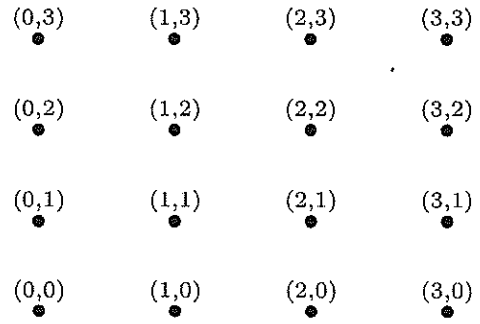
Question B [5 marks]. Prove by using mathematical induction that for any integer $n \geq 2$

$$\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}}.$$

Question C [5 marks]. The integers from 1 to 1000 are written in order around a circle. Starting at 1, every 15^{th} is marked (that is 1, 16, 31, etc). This process is continued until a number is reached which has already been marked. How many unmarked numbers remain? Justify your answer.

Question D [5 marks]. Consider the simple graph with m^2 vertices where each vertex is an ordered pair (i, j) with $0 \leq i, j \leq m-1$ and the vertex (i, j) is connected to 4 vertices (x, y) where either $i = x$ and $j \equiv y \pm 1 \pmod{m}$ or $j = y$ and $i \equiv x \pm 1 \pmod{m}$.

(i) Draw the graph for $m = 4$. (You only need to draw the edges.)



(ii) Find a shortest path from $(1, 1)$ to $(m-1, m-1)$.

(iii) Show that the length of a shortest path between any two vertices is at most m.

Question E [5 marks]. Let the public key of a RSA cryptosystem be $(n, e) = (4819, 37)$.

(i) Find the decryption key.

(ii) Decrypt the message 3763 using $A = 01, B = 02, \ldots, Z = 26$.

—END OF PAPER—