Review of Chapter 2

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Definition of Matrix

1. An $m \times n$ matrix can be written as

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}. \tag{1}$$

2. Simply denoted by $A = (a_{ij})_{m \times n}$, where a_{ij} is the $(i, j)-$entry of $\mathbf{A}$.
3. A **column matrix (vector)** is a matrix with only one column. A **row matrix (vector)** is a matrix with only one row.
4. **Square matrix**, $m = n$.
5. $\mathbf{A}$ is **diagonal** matrix $\Leftrightarrow$ $a_{ij} = 0$ whenever $i \neq j$.
6. $\mathbf{A}$ is **scalar** matrix $\Leftrightarrow$ $\mathbf{A}$ is diagonal matrix and $a_{ii} = c, \forall i, c$ constant.

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- Two matrices are equal if they have the same size and their corresponding entries are equal.
- $\mathbf{A} = (a_{ij})_{m\times n}, \mathbf{B} = (b_{ij})_{m\times n}$.
  1. (Matrix Addition) $\mathbf{A} + \mathbf{B} = (a_{ij} + b_{ij})_{m\times n}$;
  2. (Matrix Subtraction) $\mathbf{A} - \mathbf{B} = (a_{ij} - b_{ij})_{m\times n}$;
  3. (Scalar Multiplication) $c\mathbf{A} = (ca_{ij})_{m\times n}$.
- $-\mathbf{A} = (-1)\mathbf{A}$, $\mathbf{A} - \mathbf{B} = \mathbf{A} + (-1)\mathbf{B}$.
- Note that all these operations required the sizes of matrices to be equal.

Theorem 2.2.6

Theorem

Let $\mathbf{A}, \mathbf{B}, \mathbf{C}$ *be matrices of the same size and* c, d *scalars. Then*

1. *(Commutative Law for Matrix Addition)* $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$,
2. *(Associative Law for Matrix Addition)* $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$,
3. $c(\mathbf{A} + \mathbf{B}) = c\mathbf{A} + c\mathbf{B}$,
4. $(c + d)\mathbf{A} = c\mathbf{A} + d\mathbf{A}$,
5. $c(d\mathbf{A}) = (cd)\mathbf{A} = d(c\mathbf{A})$,
6. $A + \mathbf{0} = \mathbf{0} + \mathbf{A} = \mathbf{A}$,
7. $\mathbf{A} - \mathbf{A} = \mathbf{0}$, *and*
8. $0\mathbf{A} = \mathbf{0}$.

Matrix Multiplication

Let $\mathbf{A} = (a_{ij})_{m \times p}$ and $\mathbf{B} = (b_{ij})_{p \times n}$ be two matrices. The product $\mathbf{AB}$ is defined to be an $m \times n$ matrix whose $(i,j)-$entry is

$$a_{i1}b1j + a_{i2}b_{2j} + \cdots + a_{ip}b_{pj} = \sum_{k=1}^{p} a_{ik}b_{kj},$$

$$i = 1, 2, \cdots, m, \quad j = 1, 2, \cdots, n.$$

Note that

1. Be careful of the sizes of matrices.
2. **AB** is not equal to **BA** (if the multiplications are defined) in general.
3. **Pre-multiplication** of **A** to **B**: **AB**.
4. **Post-multiplication** of **A** to **B**: **BA**.

Theorem 2.2.11

If the following all multiplications are defined.

Theorem

1. *(Associative Law for Matrix Multiplication)* $\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$.
2. *(Distributive Law for Matrix Addition and Matrix Multiplication)* $\mathbf{A}(\mathbf{B}_1 + \mathbf{B}_2) = \mathbf{AB}_1 + \mathbf{AB}_2$ *and* $(\mathbf{C}_1 + \mathbf{C}_2)\mathbf{A} = \mathbf{C}_1\mathbf{A} + \mathbf{C}_2\mathbf{A}$.
3. $c(\mathbf{AB}) = (c\mathbf{A})\mathbf{B} = \mathbf{A}(c\mathbf{B})$.
4. $\mathbf{A0} = \mathbf{0}, \mathbf{0A} = \mathbf{0}, \mathbf{IA} = \mathbf{AI} = \mathbf{A}$.