CS1231: Discrete Structures

Tutorial 3

Li Wei

Department of Mathematics
National University of Singapore

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Quick Review

- Testing for a valid argument form
- Valid argument forms

Menu

Question 1(a)

Question 1(b)

Question 1(c)

Question 2

Question 3

Question 4

1. Use a truth table to determine whether the following argument forms are valid.

Recall

✍ **Testing for a valid argument form**

1. Identify the premises and conclusion.
2. Construct a truth table.
3. If the truth table contains a row in which all the premises are true and the conclusion is false, the argument form is invalid. Otherwise the form is valid.

1. Use a truth table to determine whether the following argument forms are valid.

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(a) $p \to r,\ q \to r\ \therefore\ p \vee q \to r$.

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(a) $p \to r,\ q \to r\ \therefore\ p \vee q \to r$.

p	q	r	$p \to r$	$q \to r$	$p \vee q$	$p \vee q \to r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	T	T
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	T	F	T	F	T	F
F	F	T	T	T	F	T
F	F	F	T	T	F	T

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(a) $p \to r,\ q \to r\ \therefore\ p \vee q \to r$. Answer. Valid

p	q	r	$p \to r$	$q \to r$	$p \vee q$	$p \vee q \to r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	T	T
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	T	F	T	F	T	F
F	F	T	T	T	F	T
F	F	F	T	T	F	T

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(b) $p \to q \vee r,\ \neg q \vee \neg r\ \therefore \neg p \vee \neg r.$

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(b) $p \to q \vee r,\ \neg q \vee \neg r\ \therefore\ \neg p \vee \neg r.$

p	q	r	$p \to q \vee r$	$\neg q \vee \neg r$	$\neg p \vee \neg r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	T	F
T	F	F	F	T	T
F	T	T	T	F	T
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(b) $p \to q \vee r,\ \neg q \vee \neg r\ \therefore\ \neg p \vee \neg r$. Answer. Invalid

p	q	r	$p \to q \vee r$	$\neg q \vee \neg r$	$\neg p \vee \neg r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	T	F
T	F	F	F	T	T
F	T	T	T	F	T
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(c) $p \vee q$

$p \to r$

$q \to s$

$\therefore r \vee s$

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(c) $p \vee q$

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$\therefore r \vee s$

Answer. Valid.

p	q	r	s	$p \vee q$	$p \rightarrow r$	$q \rightarrow s$	$r \vee s$
T	T	T	T	T	T	T	T
T	T	T	F	T	T	T	T
T	T	F	T	T	F	T	T
T	T	F	F	T	F	F	F
T	F	T	T	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	T	T	F	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	T	F	T	T	F	T
F	T	F	T	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	T	F	T	T	T
F	F	T	F	F	T	T	T
F	F	F	T	F	T	T	T
F	F	F	F	F	T	T	F

2. Use valid argument forms to derive the conclusion $\neg q$ from the given premises:

$\neg p \vee q \rightarrow r, \qquad s \vee \neg q, \qquad \neg t, \qquad p \rightarrow t, \qquad \neg p \wedge r \rightarrow \neg s$

Recall

- **Modus Tollen (Method of Denying)** $p \rightarrow q, \neg q \quad \therefore \neg p$
- **Generalization** $p \quad \therefore p \vee q$
- **Modus Pollen (Method of Affirming)** $p \rightarrow q, p \quad \therefore q$
- **Elimination** $p \vee q, \neg p \quad \therefore q$

1. $p \rightarrow t$
2. $\neg t$
3. $\therefore$ From 1, 2 (modus tollen)
4. $\therefore$ From 3(generalization)
5. $\neg p \vee q \rightarrow r$
6. $\therefore$ From 4,5 (modus ponens)
7. $\therefore$ From 3, 6(conjunction)
8. $\neg p \wedge r \rightarrow \neg s$
9. $\therefore$ From 7, 8 (modus ponens)
10. $s \vee \neg q$
11. $\therefore$ From 9, 10 (elimination)

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4. $\therefore \neg p \vee q$ From 3(generalization)
5. $\neg p \vee q \rightarrow r$
6. $\therefore r$ From 4,5 (modus ponens)
7. $\therefore$ From 3, 6(conjunction)
8. $\neg p \wedge r \rightarrow \neg s$
9. $\therefore$ From 7, 8 (modus ponens)
10. $s \vee \neg q$
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6. $\therefore r$ From 4,5 (modus ponens)
7. $\therefore \neg p \wedge r$ From 3, 6(conjunction)
8. $\neg p \wedge r \rightarrow \neg s$
9. $\therefore$ From 7, 8 (modus ponens)
10. $s \vee \neg q$
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5. $\neg p \vee q \rightarrow r$
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9. $\therefore \neg s$ From 7, 8 (modus ponens)
10. $s \vee \neg q$
11. $\therefore \neg q$ From 9, 10 (elimination)

3. Solve the following using argument forms.
Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known?

(i) At least one of Kevin and Heather is chatting;
(ii) Exactly one of Randy and Vijay is chatting;
(iii) If Abby is chatting, then so is Randy;
(iv) Vijay and Kevin are either both chatting or both not chatting;
(v) If Heather is chatting, then so are Abby and Kevin.

Idea. Let A: Abby is chatting; H: Heather is chatting; K: Kevin is chatting; R: Randy is chatting; V: Vijay is chatting. so the information is

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(i) $K \vee H$;

(ii) $R \to \neg V$, $\neg R \to V$;

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(iii) $A \to R$;

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Idea. Let A: Abby is chatting; H: Heather is chatting; K: Kevin is chatting; R: Randy is chatting; V: Vijay is chatting. so the information is

(i) $K \vee H$;
(ii) $R \to \neg V$, $\neg R \to V$;
(iii) $A \to R$;
(iv) $V \leftrightarrow K$;
(v) $H \to A \wedge K$.

$K \vee H$; $R \to \neg V$, $\neg R \to V$; $A \to R$; $V \leftrightarrow K$; $H \to A \wedge K$.

Recall

✍ **Transitivity** $p \to q, q \to r \quad \therefore p \to r$

1. $K \vee H \equiv$;
2. $H \to A \wedge K$
3. $\therefore$ From 1, 2 (transitivity)
4. $\therefore$ From 3
5. $K \to V$;
6. $\therefore$ From 4,5 (modus ponens)
7. $R \to \neg V$
8. $\therefore$ From 6, 7(modus tollens)
9. $A \to R$
10. $\therefore$ From 8, 9(modus tollens)
11. $\therefore$ From 10 (generalization)
12. $\therefore$ From 2, 11 (modus tollen)

$K \vee H$; $R \to \neg V$, $\neg R \to V$; $A \to R$; $V \leftrightarrow K$; $H \to A \wedge K$.

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8. $\therefore \neg R$ From 6, 7(modus tollens)
9. $A \rightarrow R$
10. $\therefore \neg A$ From 8, 9(modus tollens)
11. $\therefore \neg(A \wedge K)$ From 10 (generalization)
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Recall

✍ **Transitivity** $p \to q, q \to r \quad \therefore p \to r$

1. $K \vee H \equiv \neg K \to H$;
2. $H \to A \wedge K$
3. $\therefore \neg K \to A \wedge K$ From 1, 2 (transitivity)
4. $\therefore \neg K \to A \wedge K \equiv K \vee (A \wedge K) \equiv K$ From 3
5. $K \to V$;
6. $\therefore V$ From 4,5 (modus ponens)
7. $R \to \neg V$
8. $\therefore \neg R$ From 6, 7(modus tollens)
9. $A \to R$
10. $\therefore \neg A$ From 8, 9(modus tollens)
11. $\therefore \neg(A \wedge K)$ From 10 (generalization)
12. $\therefore \neg H$ From 2, 11 (modus tollen)

4. You are given the following. Use it to prove that superman does not exist. (You need to use argument forms.)

(i) If Superman were able and willing to prevent evil, he would do so.

(ii) If Superman were unable to prevent evil, he would be impotent.

(iii) If he were unwilling to prevent evil, he would be malevolent.

(iv) Superman does not prevent evil.

(v) If Superman exists, he is neither impotent nor malevolent.

- 'a' is Superman is able to prevent evil.
- 'w' is Superman is willing to prevent evil.
- 'p' is Superman prevents evil.
- 'i' is Superman is impotent.
- 'm' is Superman is malevolent.
- 'e' is Superman exists.

4. You are given the following. Use it to prove that superman does not exist. (You need to use argument forms.)

(i) If Superman were able and willing to prevent evil, he would do so. $a \wedge w \rightarrow p$

(ii) If Superman were unable to prevent evil, he would be impotent.

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(i) If Superman were able and willing to prevent evil, he would do so. $a \wedge w \rightarrow p$

(ii) If Superman were unable to prevent evil, he would be impotent. $\neg a \rightarrow i$

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(v) If Superman exists, he is neither impotent nor malevolent. $e \rightarrow \neg(i \vee m)$

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$a \wedge w \rightarrow p,\ \neg a \rightarrow i,\ \neg w \rightarrow m,\ \neg p,\ e \rightarrow \neg(i \vee m)$

Recall

✍ Question 1(c) $p \vee q,\ p \rightarrow r,\ q \rightarrow s \quad \therefore r \vee s$

1. $\neg p$
2. $a \wedge w \rightarrow p$
3. $\therefore$ From 1,2 (modus tollen)
4. $\neg a \rightarrow i$
5. $\neg w \rightarrow m$
6. $\therefore$ from 3, 4, 5. (Question 1(c))
7. $e \rightarrow \neg(i \vee m)$
8. $\therefore$ from 6,7 (modus tollen)

$a \wedge w \to p$, $\neg a \to i$, $\neg w \to m$, $\neg p$, $e \to \neg(i \vee m)$

Recall

✍ Question 1(c) $p \vee q$, $p \to r$, $q \to s \quad \therefore r \vee s$

1. $\neg p$
2. $a \wedge w \to p$
3. $\therefore \neg(a \wedge w) \equiv$ From 1,2 (modus tollen)
4. $\neg a \to i$
5. $\neg w \to m$
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7. $e \to \neg(i \vee m)$
8. $\therefore \neg e$ from 6,7 (modus tollen)