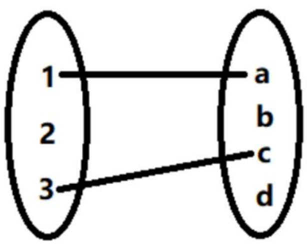
CS1231 Review 10

1. A **function** f from X to Y, $f : X \to Y$, is an assignment of exactly one element of Y to each element of X.

   Are the followings functions from X to Y?

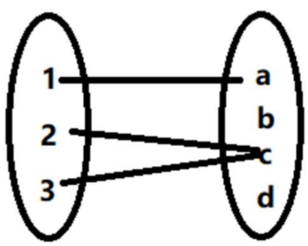
   - $X = \{1, 2, 3\}$. $Y = \{a, b, c, d\}$. f is defined in the arrow diagram below.

     Not function

     $2 \in X$ has no image

   - $X = \{1, 2, 3\}$. $Y = \{a, b, c, d\}$. f is defined in the arrow diagram below.

     Yes, it is a function.

   - $X = \mathbb{R}$. $Y = \mathbb{R}$. $f(x) = x^2$. Yes.
   - $X = \mathbb{R}$. $Y = \mathbb{R}^*$. $f(x) = x^2$. Yes.
   - $X \overset{+ve}{=} \mathbb{R}$. $Y = \mathbb{R}$. $f(x) = \sqrt{x}$. No. $f(-1)$ has no value
   - $X = \mathbb{R}^*$. $Y = \mathbb{R}$. $f(x) = \sqrt{x}$. Yes

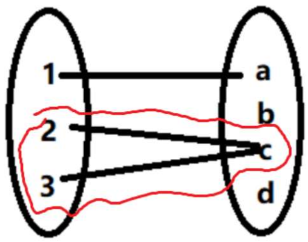
     $f(9) = \sqrt{9} = 3$

2. Identity function $i_A : A \to A$. $i_A(x) =$ __x__.

3. A function $f : X \to Y$ is **one-to-one** or **injective** if $\forall a, b \ (f(a) = f(b) \to a = b)$

   Are the followings one-to-one functions from X to Y?

- $X = \{1, 2, 3\}$. $Y = \{a, b, c, d\}$. f is defined in the arrow diagram below.

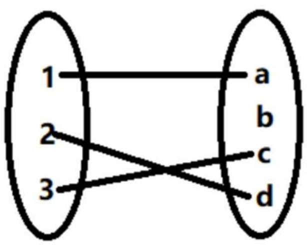


f is not 1-1

$f(2) = f(3) \rightarrow 2 = 3$

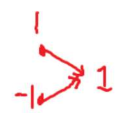
T F

- $X = \{1, 2, 3\}$. $Y = \{a, b, c, d\}$. f is defined in the arrow diagram below.

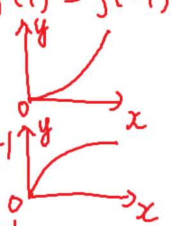


Yes. f is 1-1

- $X = \mathbb{R}$. $Y = \mathbb{R}^*$. $f(x) = x^2$. No. $f(1) = f(-1)$ but $1 \neq -1$



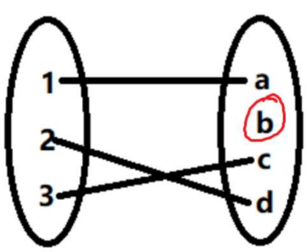
- $X = \mathbb{R}^*$. $Y = \mathbb{R}$. $f(x) = x^2$. Yes.
- $X = \mathbb{R}^*$. $Y = \mathbb{R}$. $f(x) = \sqrt{x}$. Yes 1-1
- $X = \mathbb{R}^*$. $Y = \mathbb{R}^*$. $f(x) = \sqrt{x}$. Yes 1-1



4. A function $f : X \to Y$ is **onto** or **surjective** if $\forall y \in Y\ \exists x \in X\ f(x) = y$.

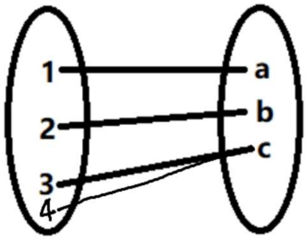
Are the followings onto functions from X to Y?

- $X = \{1, 2, 3\}$. $Y = \{a, b, c, d\}$. f is defined in the arrow diagram below.

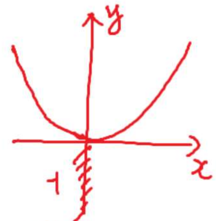


Not onto

- $X = \{1, 2, 3, 4\}$. $Y = \{a, b, c\}$. f is defined in the arrow diagram below.

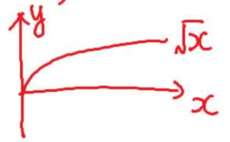


Yes, f is onto.



onto.

- No • $X = \mathbb{R}$. $Y = \mathbb{R}$. $f(x) = x^2$. (-1 is not an image of any x)
- Yes • $X = \mathbb{R}$. $Y = \mathbb{R}^*$. $f(x) = x^2$.



- No • $X = \mathbb{R}^*$. $Y = \mathbb{R}$. $f(x) = \sqrt{x}$. (-1 is not an image of any x)
  $-\sqrt{1} = -1$ $\pm\sqrt{1} = \pm 1$ $\sqrt{1} = 1$
- Yes • $X = \mathbb{R}^*$. $Y = \mathbb{R}^*$. $f(x) = \sqrt{x}$.

5. A function f is a **bijection** if it is both <u>1-1</u> and <u>onto.</u>.

   Are the followings bijections from X to Y?

   bijection

		1-1	onto	
No.	$X = \mathbb{R}$. $Y = \mathbb{R}$. $f(x) = x^2$.	$f(1)=f(-1)$ ✗	✗	-1 is not an image of any x
No	$X = \mathbb{R}$. $Y = \mathbb{R}^*$. $f(x) = x^2$.	$f(1)=f(-1)$ ✗	✓	
No	$X = \mathbb{R}^*$. $Y = \mathbb{R}$. $f(x) = \sqrt{x}$.	✓	✗	-1 is not an image of any x
Yes.	$X = \mathbb{R}^*$. $Y = \mathbb{R}^*$. $f(x) = \sqrt{x}$.	✓	✓	
Yes	$X = Y = A$. Identity function i_A.	✓	✓	

6. Let $f : X \to Y$ be bijection. Its **inverse function** f^{-1} is defined by

$$f^{-1}(y) = x \Leftrightarrow \underline{f(x) = y}.$$

7. **Composition Function.**

   $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \to \mathbb{R}$ are defined by $f(x) = x - 1$, $g(x) = 2x$. Then

   $(f \circ g)(x) =$ <u>$f(g(x)) = f(2x) = 2x-1$</u>

   $\neq$

   $(g \circ f)(x) =$ <u>$g(f(x)) = g(x-1) = 2(x-1) = 2x-2$</u>

8. Find the **inverse** of the function $f : \mathbb{R} \to \mathbb{R}$, where $f(x) = 2x - 1$ for all $x \in \mathbb{R}$, if it **exists**.

bijection ① 1–1 $f(a) = f(b)$
$\Rightarrow 2a-1 = 2b-1$
$\Rightarrow 2a = 2b$
$\Rightarrow a = b$
$\therefore f$ is 1–1

$f^{-1}(y) = \frac{y+1}{2}$

$f^{-1}(y) = x$

② onto For every $y \in \mathbb{R}$, find x such that $f(x) = y$
(solve out)
$\Rightarrow 2x - 1 = y$
$\Rightarrow \boxed{x = \frac{y+1}{2}}$

$\therefore f$ is onto

9. **Floor and Ceiling Functions.**

- $\lfloor -2 \rfloor =$ -2 $\lceil -2 \rceil =$ -2
- $\lfloor 0.5 \rfloor =$ 0 $\lceil 0.5 \rceil =$ 1
- $\lfloor -0.5 \rfloor =$ -1 $\lceil -0.5 \rceil =$ 0
- Given $n \in \mathbb{Z}$. $\lfloor x \rfloor = n \Leftrightarrow$ $n \leq x < n+1$
- Given $n \in \mathbb{Z}$. $\lceil x \rceil = n \Leftrightarrow$ $n-1 < x \leq n$