Decision Analysis

CS4246/CS5446

AI Planning and Decision Making

Sem 1, AY2021-22

Topics

- The decision analysis framework
  - Formulating decision models
    - Decision networks: Influence diagrams (16.5)
    - Decision trees
  - Analyzing decision networks
    - Sensitivity analysis and robust decision models (16.6.6)
- Information value theory (16.6)
  - Expected value of perfect information (16.6.1-16.6.3)
  - Implementing an information gathering agent (16.6.4)

Types of Decision Theory

- Normative decision theory
  - Describes how ideal, rational agents should behave
- Descriptive decision theory
  - Describes how actual agents (humans) really behave
- Prescriptive decision theory
  - Prescribes guidelines for agents to behave rationally

Recall: Solving Decision Problems

- Decision Problem or Model
  - Appropriate abstraction of states, actions, uncertain effects, and goals (wrt costs and values or preferences)
- Decision Algorithm
  - Input: a problem
  - Output: a solution in the form of an action sequence
    - Optimal action at each decision or choice point
- Decision Solution
  - An action sequence or solution from an initial state to the goal state(s)
    - An optional solution or action sequence; OR
    - An optimal policy that specifies "best" action in each state wrt to costs or values or preferences
  - (Optional) A goal state that satisfies certain properties

Recall: Decision Making under Uncertainty

- Decision Model:
  - Actions: $a \in A$
  - Uncertain current state: $s \in S$ with probability of reaching: $P(s)$
  - Transition model of uncertain action outcome or effects:
    $P(s'|s, a)$ – probability that action a in state s reaches state s'
  - Outcome of applying action a:
    $\text{Result}(a)$ – random variable whose values are outcome states
  - Probability of outcome state s', conditioning on that action a is executed:
    $P(\text{Result}(a) = s') = \sum_s P(s)P(s'|s, a)$
  - Preferences captured by a utility function:
    $U(s)$ – assigns a single number to express the desirability of a state s

Recall: Fundamentals of Decision Theory

- Decision theory
  - Choosing among actions based on desirability of outcomes
- In non-deterministic, partially observable, episodic environments:
  - An agent can act rationally – consistently with its preferences – only by choosing an action that maximizes expected utility according to MEU principle:
  - A rational agent should choose the action that maximizes its expected utility

$$action = \underset{a}{\operatorname{argmax}}\, EU(a)$$

  - Expected utility of an action a is the average utility value of the outcomes, weighted by the probability that the outcome occurs

$$EU(a) = \sum_{s'} P(\text{Result}(a) = s')U(s') = \sum_{s'} \sum_{s} P(s)P(s'|s,a)U(s')$$

Decision Analysis

A prescriptive framework for decision making

What is Decision Analysis?

- Emerged in the 1960s from operations research and game theory
  - (Howard, 1966)
  - (Raiffa and Abbas, 2016)
- A prescriptive framework
  - Vs. Normative decision theory - Assumes decision makers as ideally rational agents
  - Vs. Descriptive decision theory - Describes how people actually make decisions
- Provides structure and guidance for thinking systematically and recommends alternatives about hard decisions
  - Can help make better decisions, but not improve luck nor guarantee good outcomes!
- Why does it matter for AI Planning and Decision Making?

How to Improve Decision Making?

- Complexity Management
  - Effective methods for problem organization
  - Vocabulary on elements of decision structure
  - Graphical structuring tools
  - Solution and analysis procedures
- Uncertainty Reduction
  - Identification of sources of uncertainty
  - Quantitative representation of uncertainty
- Multiple Objectives Specification
  - A framework and specific tools for dealing with multiple objectives
- Multiple Perspectives Resolution
  - A framework and specific tools to sort through and resolve differences

Decision Model: Decision Basis Formulation

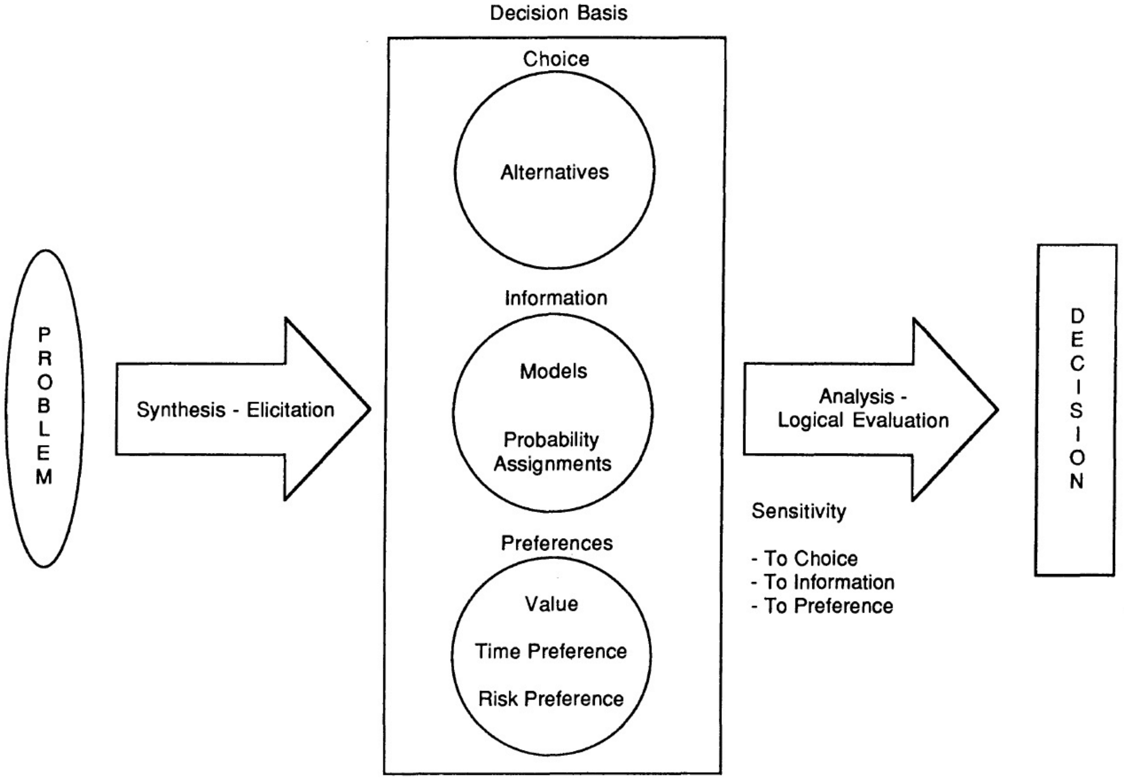
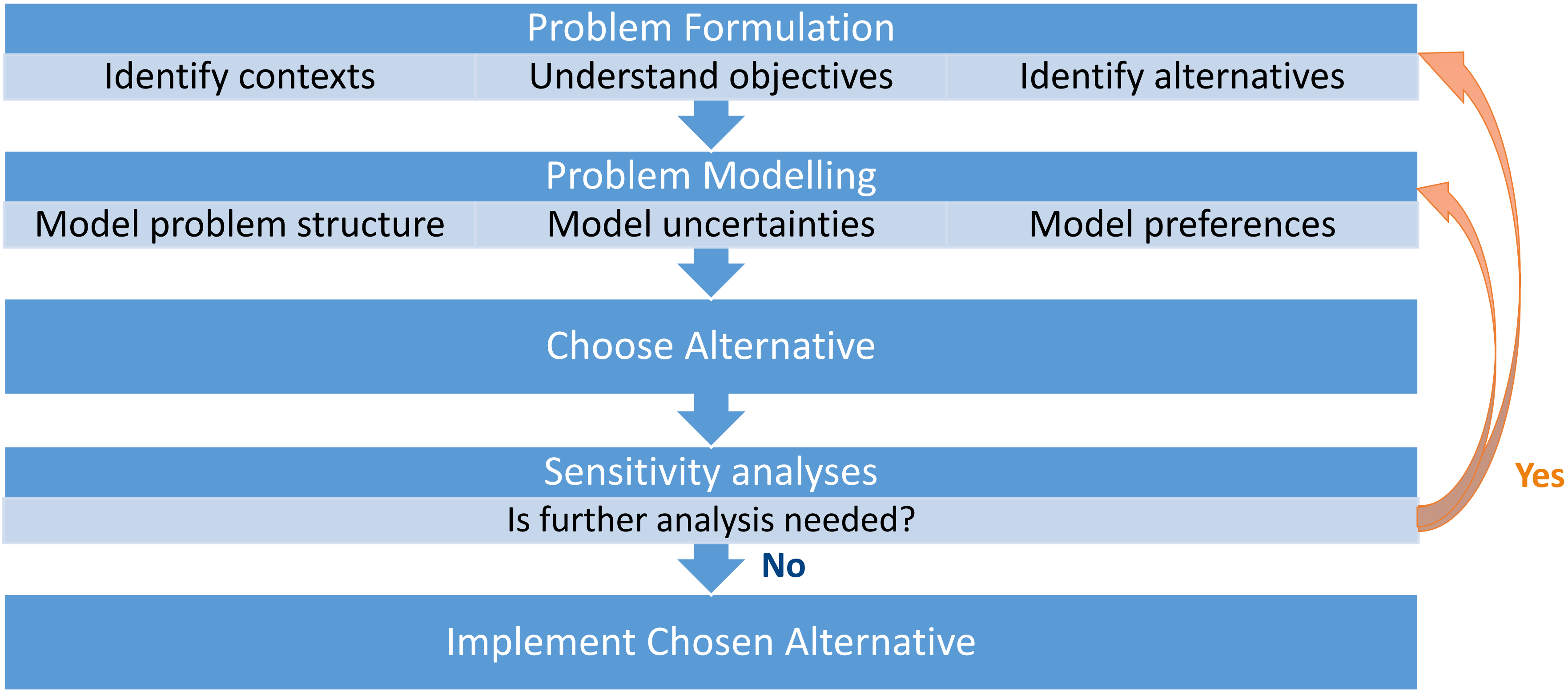


FIGURE 2. Elicitation and Evaluation of the Decision Basis.

Source: Howard, R. A. 1988

Howard, R. A., Decision Analysis: Practice and Promise, *Management Science*, Vol. 34, No. 6 (Jun., 1988) , pp. 679-695

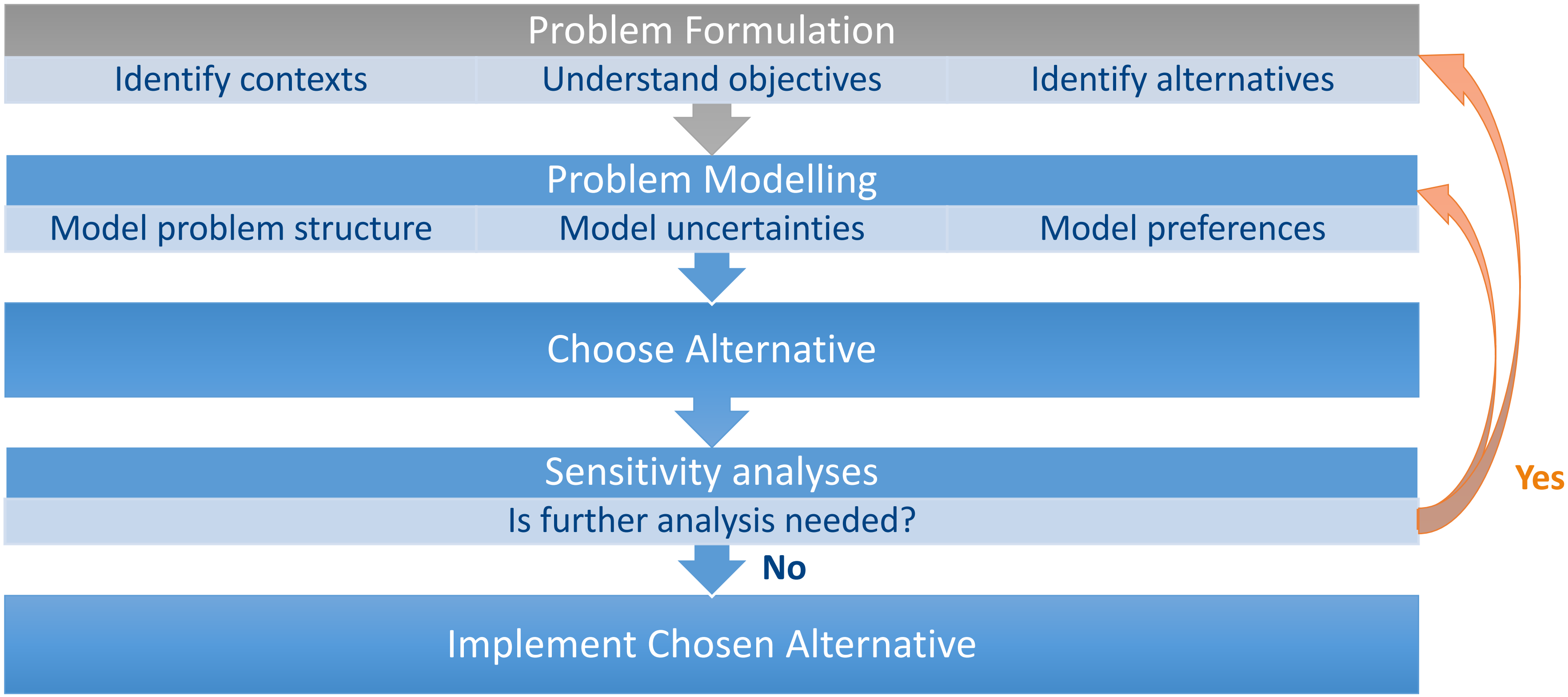
The Decision Analysis Process



Example: Cryptocurrency Investment Problem

- Decision:
  - Whether or not to invest in a new cryptocurrency – KittyCoin
- Objective:
  - To maximize profit
- Considerations:
  - Technology and exchange platforms have great credentials
  - Proposed project is more risky than most other cryptocurrencies
- Possible consequences:
  - If invest and value of KittyCoin rises, there will be high monetary returns
  - If not, capital may be put in stock market or other investment options

Problem Formulation



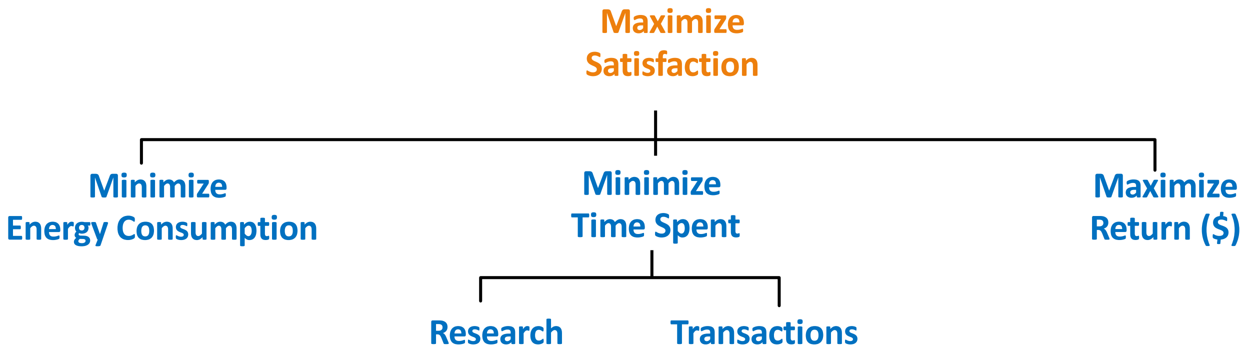
Problem Formulation

- Identify the decision context
  - Define precise problem to solve
  - Identify the decision maker and perspective
- Identify objectives
  - What are the goals of the decision?
- Identify alternatives or actions
  - Careful inspection of all aspects of a problem can lead to discovery of new alternatives
- Using:
  - Fundamental objectives
  - Other objectives (e.g., means objectives)

Organizing Objectives

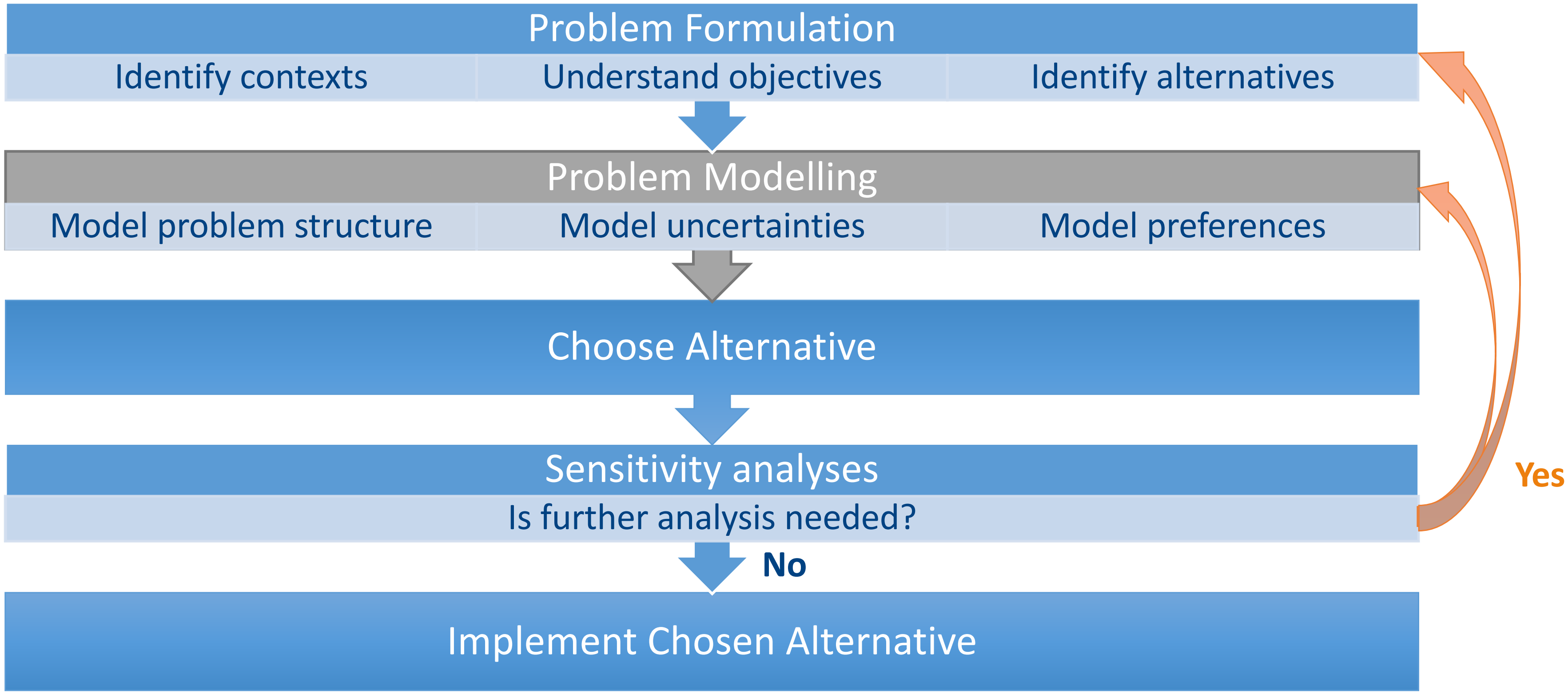
- Structure or organize objectives to
  - reveal all relevant details to be achieved
  - allow direct incorporation into problem definition
- Fundamental Objectives
  - Reflect things that the problem solver needs to achieve
  - Organized into hierarchies

Example: Cryptocurrency Investment Problem



- How to construct the fundamental objectives hierarchy?
  - Ultimately, for any objective: Why is it important?
  - To move downward: What do you mean by that?
  - To move upward: Of what more general objective is this an aspect?
- The lowest level form the basis to measure consequences
- May use network to identify alternatives to achieve objectives

Problem Formulation

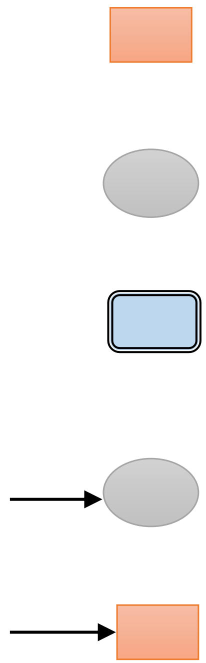


Decision Model Formulation

- Use "divide and conquer" method
  - To understand its structure
  - To measure the uncertainty, and
  - To specify the preferences
- Modeling Structure
  - Construct graphical, mathematical model to denote structure of decision problem
  - Influence Diagram
  - Decision Tree

Basic Elements of a Decision Model

- Decision nodes
  - Decision points with several choices or alternatives
- Chance nodes
  - Uncertain or chance events with several outcomes
- Value or utility nodes
  - Deterministic monetary value or utility functions
  - Measure desirability of final objectives
- Probabilistic dependencies
  - Represent conditional dependence of chance events
- Informational dependencies
  - Represent information available at decision points



Relevance and Sequence

- Relevance Arcs
  - Predecessor is relevant for assessing the chances associated with the uncertain event or the values of the value (consequence) node

Outcomes of event B are probabilistically dependent on outcomes of event A

Outcomes of event B are probabilistically dependent on choices of decision A

- Sequence Arcs
  - Decision is made knowing the outcome of the predecessor node

The decision maker knows the outcome of event A when making decision B

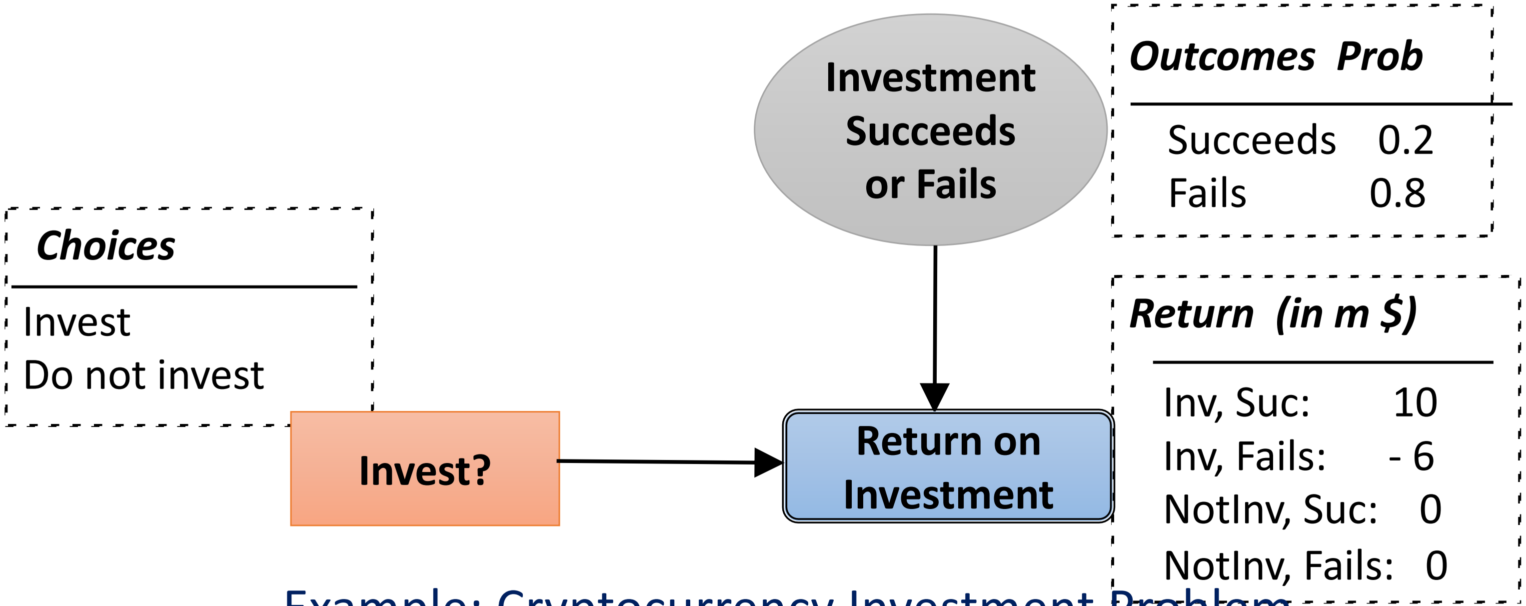
Decision A is made before decision B

Decision Networks

Influence Diagrams

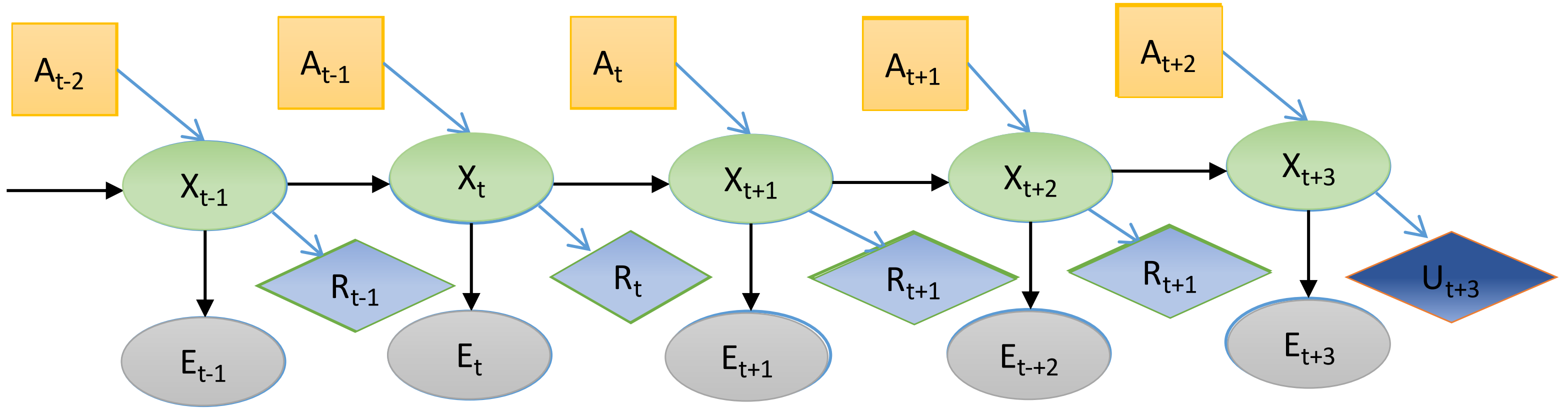
The Basic Risky Decision

- No information is available when decision is made



Example: Cryptocurrency Investment Problem

Sequential Decisions



Sequential decision problem as dynamic decision network

- Sequential structure explicitly shown
- No cycles allowed
- “No-forgetting” arcs implied but not shown
- Final value is a function of individual values over all stages

Influence Diagram

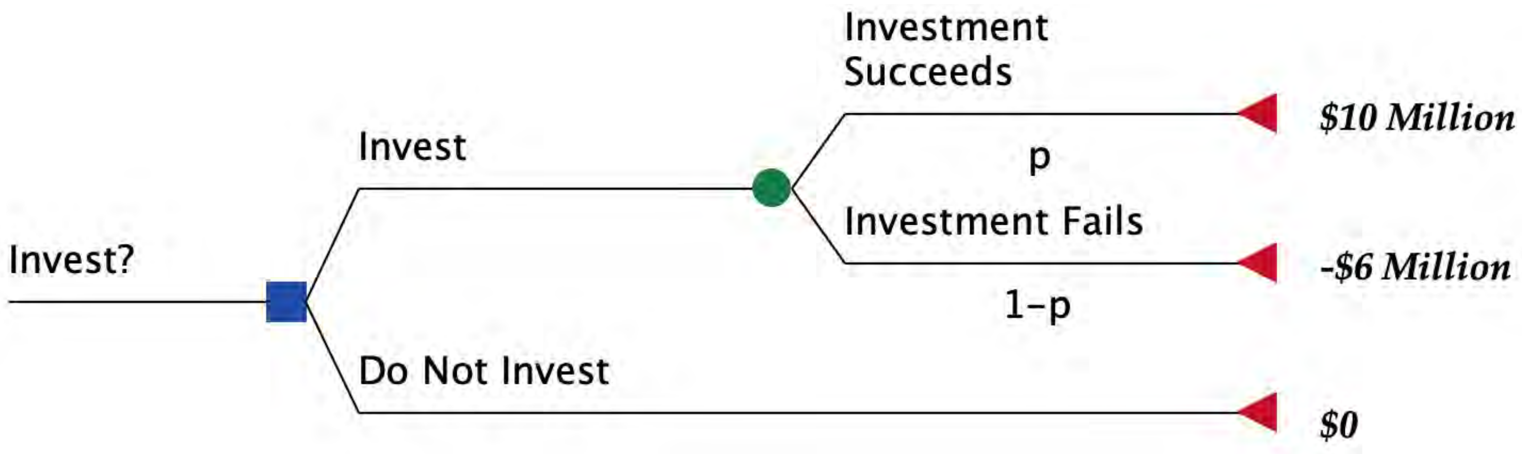
- Characteristics:
  - Hierarchical representation captures current state of knowledge of decision maker
  - Arcs indicate probabilistic dependence or relevance if leading into chance nodes or value nodes
  - Arcs indicate informational dependence if leading into decision nodes
  - A node at the beginning of an arc is a predecessor
  - A node at the end of an arc is a successor
  - A proper influence diagram has no cycles
  - Factorized utility functions represented by chance or deterministic nodes leading into final unity node

Building Influence Diagrams

- Hierarchical representation to facilitate communication
  - Captures current state of knowledge of decision maker
  - Details hidden beneath the structure to favor simplicity
- No best strategy for building influence diagrams; best approach is:
  - Put together a simple version of diagram
  - Add details as necessary until all relevant aspects are included
- No fixed order of “reasoning”
- Some Common Mistakes:
  - Influence diagrams (like BNs) are not flowcharts
    - An influence diagram is a snapshot of the decision situation
    - An arrow from a chance node to a decision node means that the chance event outcome is known at the time of decision
  - No cycles are allowed in an influence diagram

Decision Trees

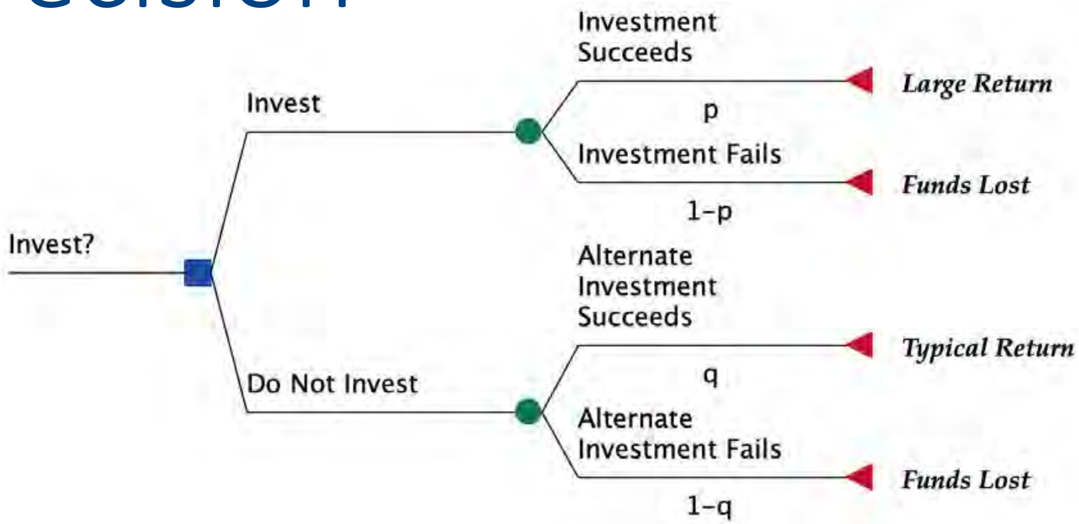
The Basic Risky Decision



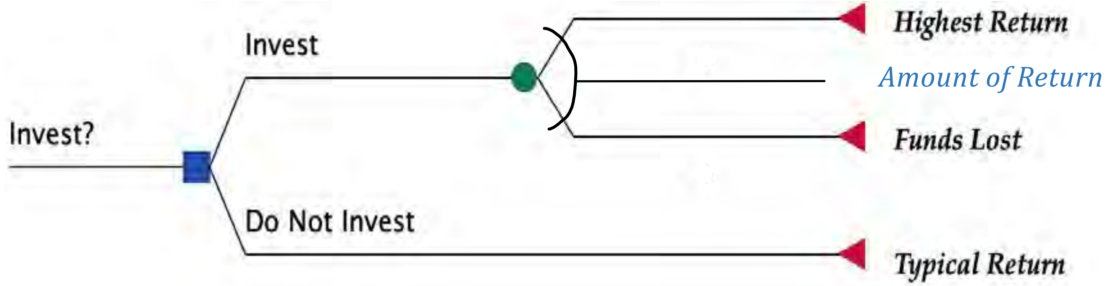
Example: Cryptocurrency Investment Problem

The Basic Risky Decision

- Double Risky Decision
  - e.g., if the investor decides not to invest, she may lose the money in the stock market or the less risky investment

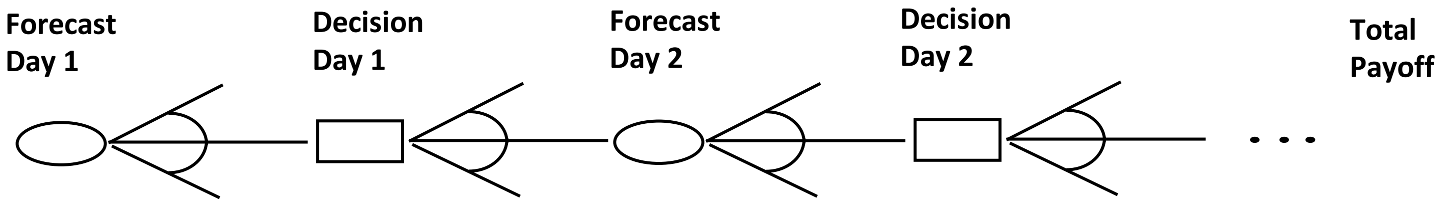


- Range of Risk Decision
  - e.g., Return of the original risky decision may take a range of values



Sequential Decisions

- Number of branches increase exponentially as number of decisions and chance events increase
- May use "skeleton" version of decision tree to represent sequential decisions if:
  - 1) the sequential problem repeats itself; and
  - 2) the decision tree is symmetric at each stage



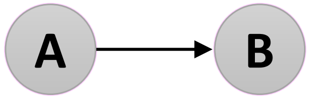
Decision Tree

- Characteristics:
  - Arcs denote decision alternatives or chance outcomes
  - Only one option can be chosen at each decision node
  - Each chance node has a set of mutually exclusive and collectively exhaustive outcomes
  - Represents all possible paths through time
  - Decisions and chance events are most naturally placed in a time order from left to right
  - Implicit probabilistic and information dependencies
  - Utility (terminal) node represents conditional utility associated with path of action-alternative-chance-outcome combinations
  - Collapse multidimensional objective description into a single score for final consequence

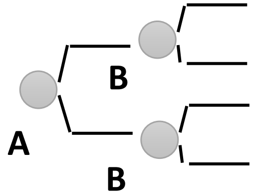
Building Decision Trees

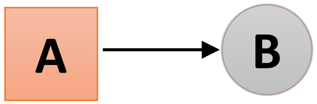
- Start with the first (leftmost) node:
  - For decision node, draw a branch for each alternative
  - For chance node, draw a branch for each possible outcome
- Label each branch emanating from a chance node with:
  - a unique outcome
  - the corresponding path dependent probability, and
  - the path dependent value for that branch
- Label each branch emanating from a decision node with:
  - a unique alternative, and
  - the value for that alternative
- Place final values on terminal nodes at the end of each path

Correspondence between ID and DT

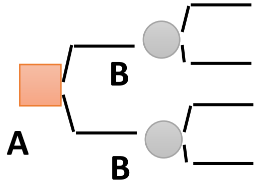


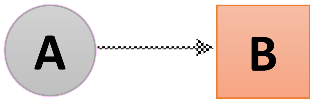
Outcomes of event B are probabilistically dependent on outcomes of event A



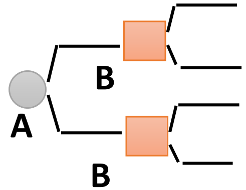


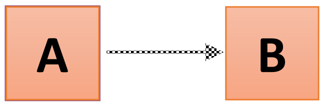
Outcomes of event B are probabilistically dependent on choices of decision A



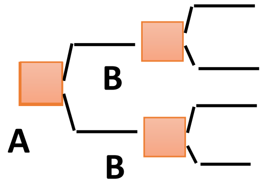


The decision maker knows the outcome of event A when making decision B





Decision A is made before decision B



Modeling Uncertainty

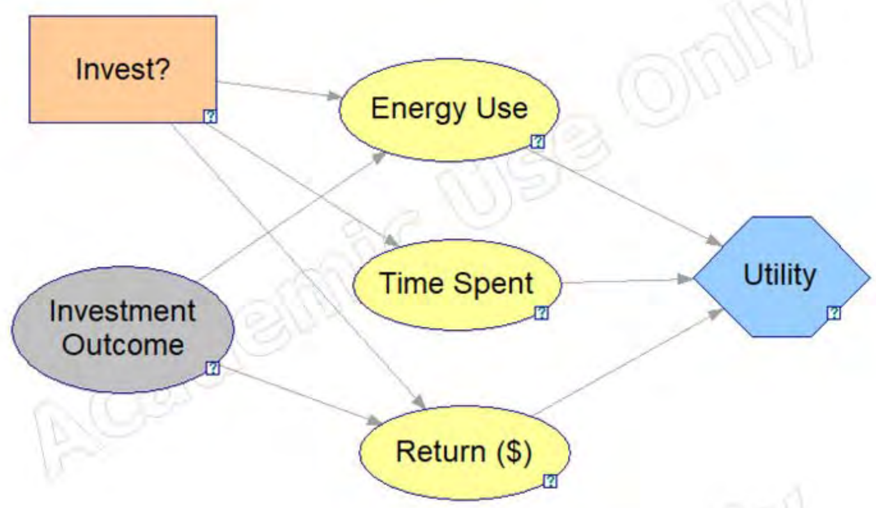
- Apply probability theory to represent uncertainty
- Approaches:
  - Using subjective judgment in assessing probabilities – difficult!
  - Using theoretical probability models
  - Using data

Modeling Preferences

- Assess utility or value functions to measure “desirability” of different outcomes and trade-off situations
- Possible scales:
  - Monetary cost
  - Revenue, profit
  - Life expectancy
  - Jobs saved
  - etc.

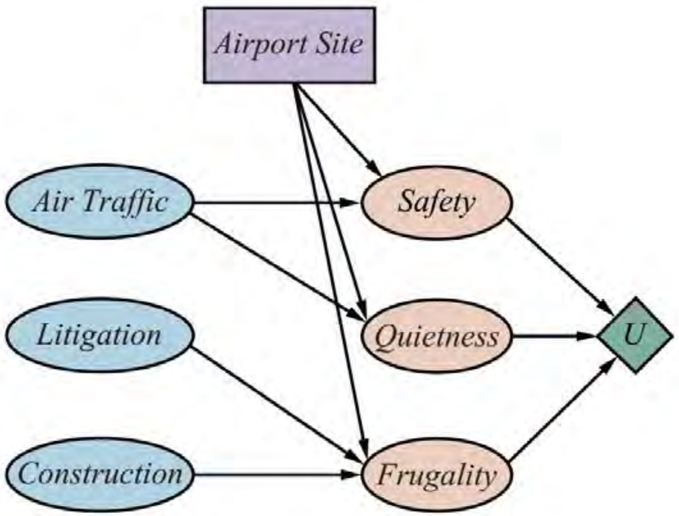
Representing Fundamental Objectives

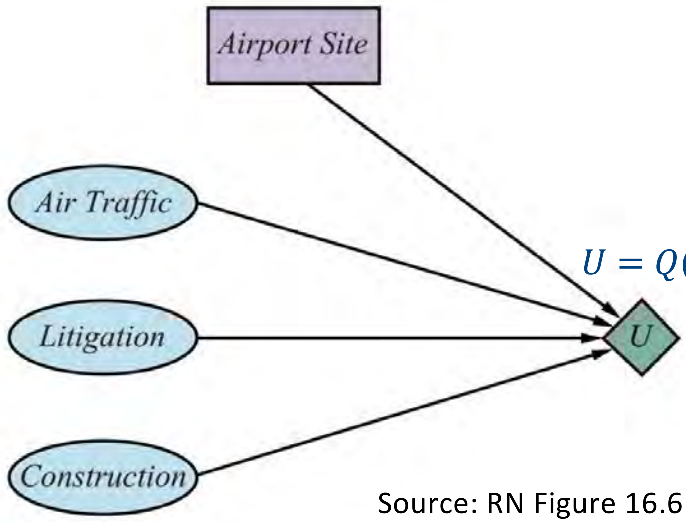
- In decision networks:
- To represent multiple attributes:
  - Use fundamental objectives hierarchy or utility node structure to represent multiple attributes
- To capture trade-offs:
  - Aggregate function for individual attribute utilities in “overall” utility node
  - Incorporate appropriate trade-offs among the different objectives



Example: Multi-attribute Utility Function

- Utility node:
  - Represents the utility function; parents are all the variables (attributes) that directly affect utility
  - A special kind of deterministic nodes - outcome is certain given values of the parents
  - Simplified form: Utility node represents expected utility associated with each action: the action-utility function or Q-function





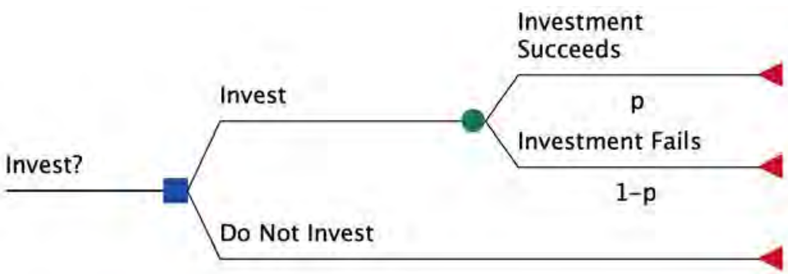
$U = Q(AT, L, C, AS)$

Source: RN Figure 16.6 and 17.7

Representing Fundamental Objectives

In decision trees:

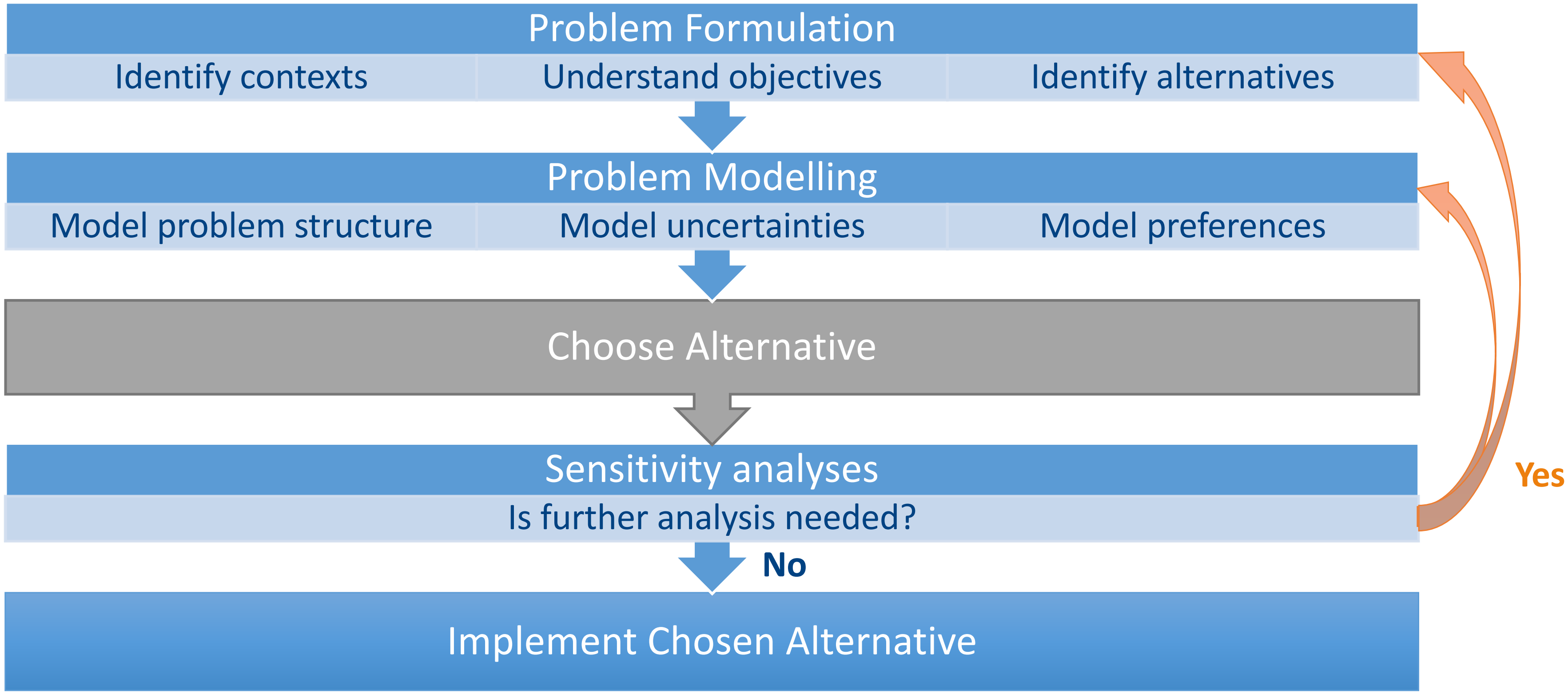
- Normally assign only one final value to each terminal node outcome or branch in consequence matrix
- Collapse multidimensional objective description into single score for final consequence
- Require assessment and application of appropriate trade-off weight
  - e.g., High return from KIttyCoin takes a long time; typical return from stock market takes a shorter time



Overall Utility		
Energy	Time	Return($)

Consequence Matrix

Model Solution

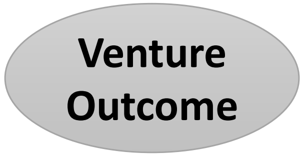


Solving a Decision Model

- Solution algorithms often find optimal decisions by:
  - Chance values expectation
  - Decision value maximization
- Common solution methods for decision trees:
  - Rollback algorithm
  - Monte Carlo simulation
- Common solution methods for influence diagrams:
  - Reduction algorithm
  - Sampling
  - Inference in Bayesian networks

Chance Values Expectation

- Determine expected value for a chance node
  - e.g. Consider a chance event “Venture Outcome” with possible outcomes “Gain,” “Unchanged,” “Loss”



Outcome	Probability	Value (utility)
Gain	0.2	1.0
Unchanged	0.3	0.5
Loss	0.5	0

  - EU [Investment outcome] =

Decision Values Maximization

- Determine maximum value for a decision node
  - e.g. Consider a decision point “Invest?” with possible alternatives “Invest” and “Not Invest”

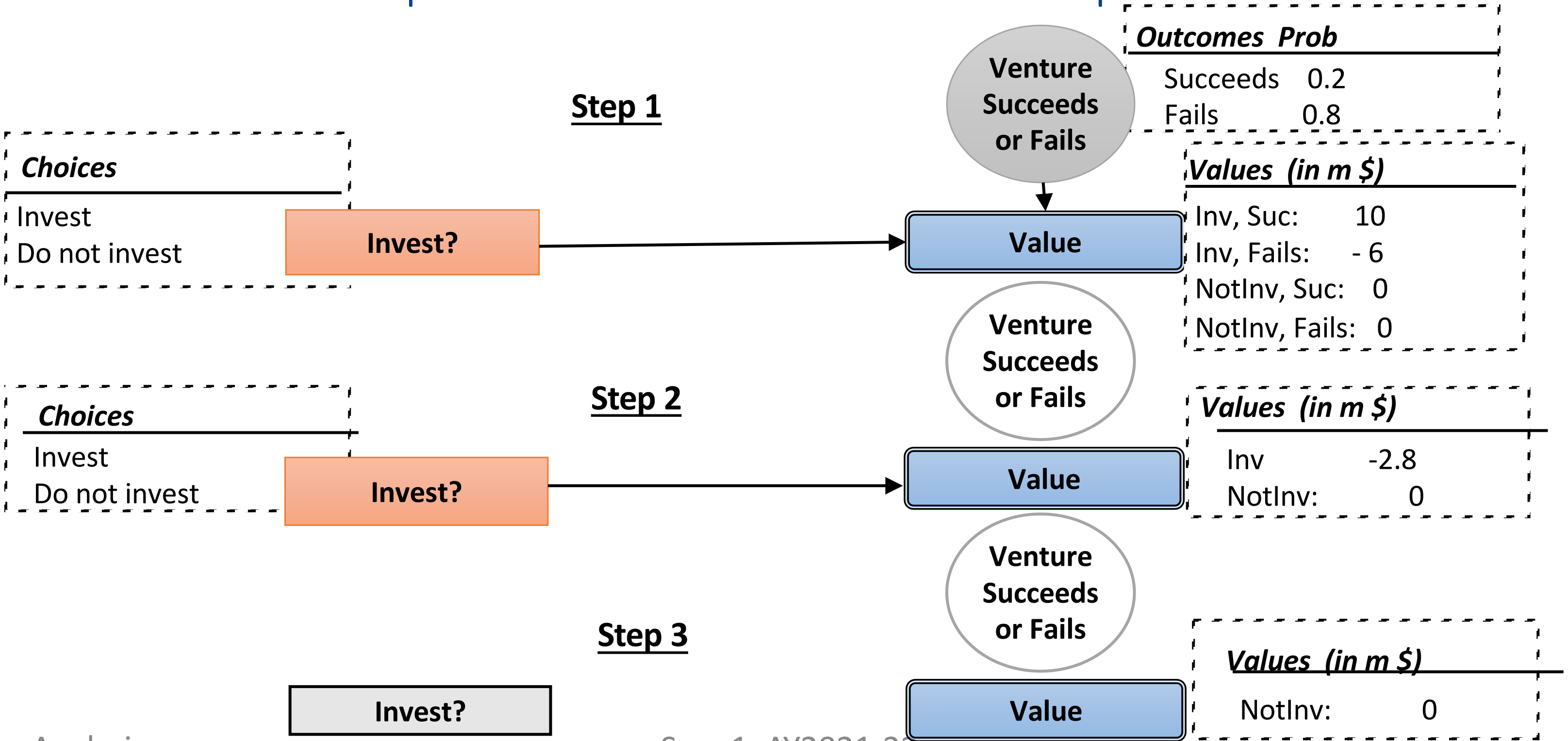
Invest?

Alternative	Value ($ in M)
Invest	6
Not Invest	1

Max[Invest?] =

Solving an Influence Diagram

- Solution obtained by reducing diagram, while recording optimal choices, into single value node
- Final value node contains expected maximum value achievable with optimal decisions

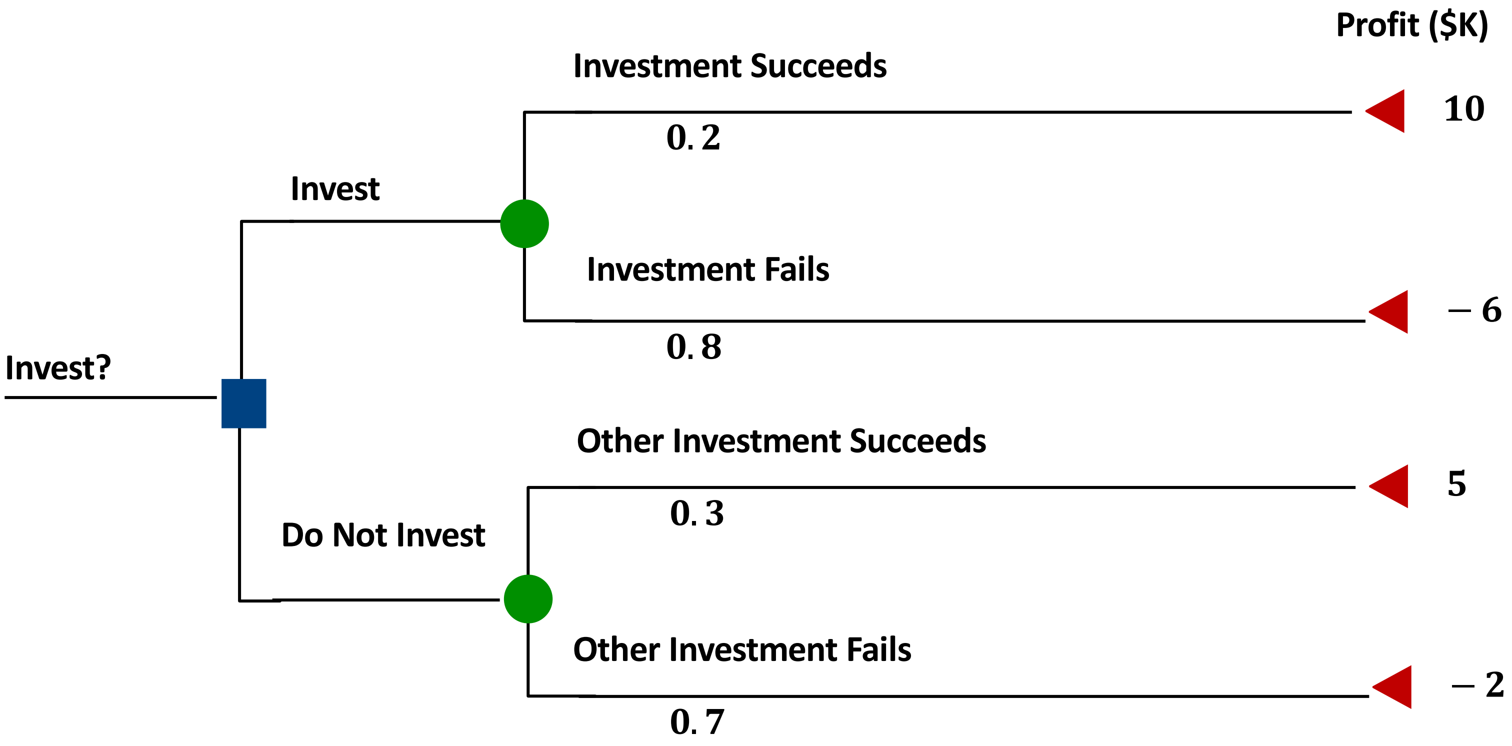


Solving an Influence Diagram: Common Approach

- Set evidence variable to the observed state
- For each possible value of the decision node:
  - Set the decision node to that value
  - Calculate the posterior probability of the parents of the utility using probabilistic inference algorithm
  - Calculate the resulting utility for the action
- Return the action with the highest utility

Use efficient Bayesian Network inference algorithms

Solving a Decision Tree (Rollback)



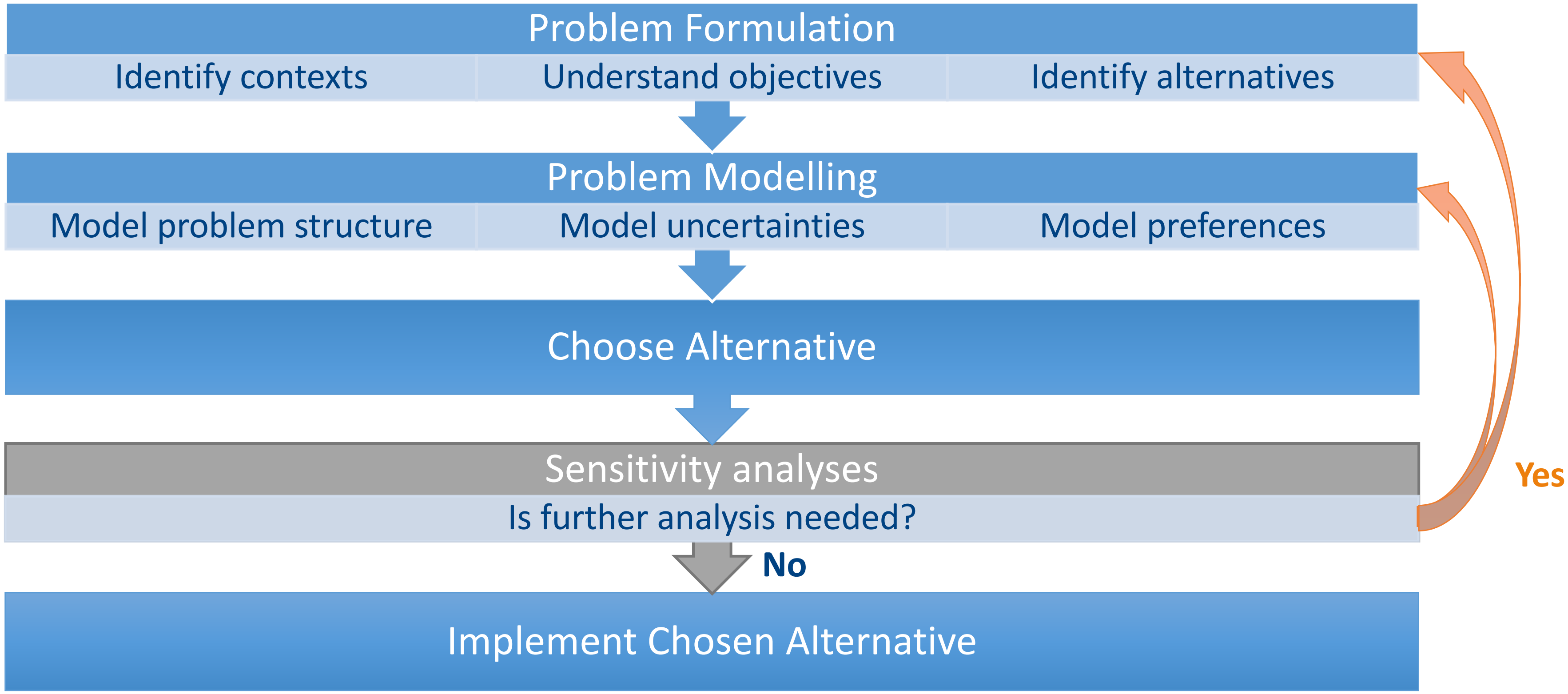
Solving a Decision Tree

- Choose among risky alternatives
  - Pick alternative with the highest EMV or utility
- Folding or rolling back the tree:
  - Begin at end points of branches on far right-hand side and move to left
  - Chance value expectation
    - Calculate expected values when encountering a chance node; OR
  - Decision value maximization
    - Choose the branch with the highest value or expected value when encountering a decision node
  - Record the value accumulated and alternative selected for each decision node along the optimal path

Influence Diagrams vs. Decision Trees

Feature \ Model	Influence Diagram	Decision Tree
Compact Representation	Yes	No
Explicit informational and probabilistic dependencies	Yes	No
Explicit value function structure	Yes	No
Explicit labels and bindings	No	Yes
Explicit representation of asymmetric situations	No	Yes
Straightforward solution algorithm(s)	No	Yes
Good for:	Communication; sequential decisions	Sensitivity analysis

Model Analysis

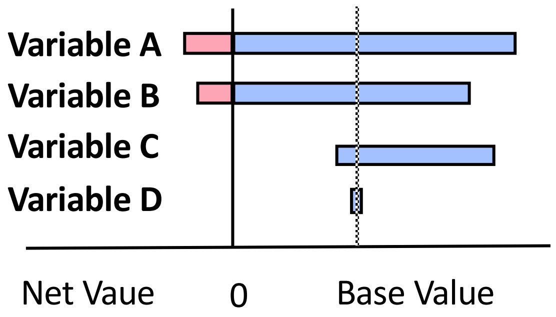


Sensitivity Analysis

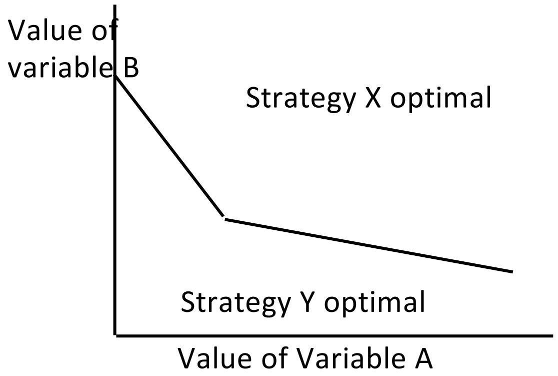
- Answers the question:
  - “What if … is changed in this decision”?
- Determines what matters and what does not throughout the modeling process
- Provides guidance to develop requisite decision model
- Integral part of decision analysis, may lead to:
  - Problem re-definition
  - New objectives, alternatives, structure, uncertainty, preference
- No optimal procedures, useful techniques facilitate :
  - identifying and structuring problems
  - detecting dominance among alternatives
  - assessing probabilities and utilities

Sensitivity Analysis Methods

- Common Methods:
  - Structural analysis methods
  - The clarity test
  - Tornado diagrams
  - One, two, three-way analysis



A Tornado Diagram



Two-Way Sensitivity Analysis

- The Clarity Test:
  - Resolve ambiguities
- Definition:
  - Imagine a clairvoyant who has access to all future information
  - Would the clairvoyant be able to determine unequivocally what the outcome would be for every event in the decision model?
  - No interpretation or judgment should be required of the clairvoyant
- Example:
  - Does “Temperature” of “high, medium, or low” pass the clarity test?

Robust Decisions

- Robust or minimax decision – Extreme case
  - One that gives the best result in the worst case.
  - "Worst case" means worst with respect to all plausible variations in the parameter values of the model
  - Letting θ stand for all the parameters in the model, the robust decision is defined by:

$$a^* = \max_a \min_\theta EU(a;\theta)$$

- For parametric uncertainty
  - Bayesian decision theory: model with hyperparameters
  - Analyze performance of hyperparameters or ranges of parameters
- For structural uncertainty
  - Examine ensemble of models
  - Use domain knowledge to propose alternatives

Requisite Decision Model

- When no new intuition emerges about the problem
- When it contains everything essential for solving the problem
- The decision maker’s
  - thoughts about the problem
  - beliefs regarding uncertainty and preferences

  are fully developed

Value of Information

Information and Decision Making

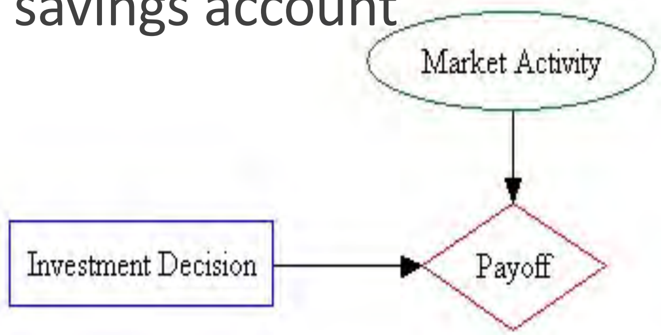
- Information is gathered to reduce uncertainty in decision making
  - Consult experts
  - Conduct surveys
  - Perform mathematical or statistical analyses
  - Do research
  - Read books, journals, newspapers
  - Learn from past data
- Relevant questions:
  - How to evaluate or measure the “value” or usefulness of the information?
  - What does it mean for a knowledge source to provide perfect information?
  - Shall we invest effort or pay $X for additional information to help problem solving?

Value of Information

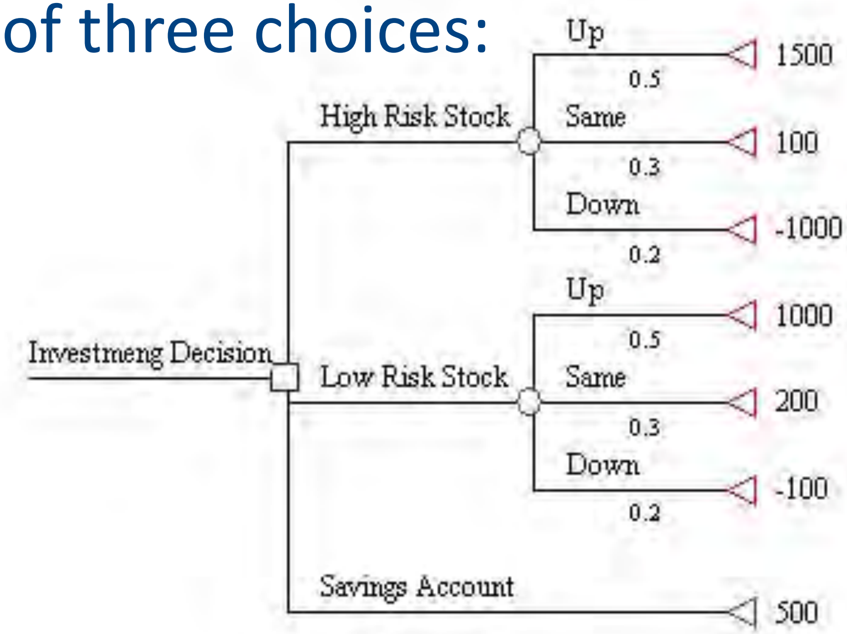
- Costly information gathered to reduce uncertainty
  - How to place value on information in a problem?
  - How to decide whether or not to gather more information?
- Main ideas:
  - Information has value to the extent that it is likely to cause a change of plan and to the extent that the new plan is significantly better than the old one
  - Use conditional probabilities and Bayes' Theorem to model the expected value of information

Example: Stock Investment

- An investor may invest in one of three choices:
  - A high-risk stock
  - A low-risk stock
  - A savings account



Influence diagram



Decision tree

Stock Investment Example (cont.)

- If the expert always provides perfect information:
  - P(Exp says "Up" | Market Up) = 1
  - P(Exp says "Down" | Market Up) = 0
  - P(Exp says "Up" | Market Down) = 0
- To show there is no uncertainty after hearing the expert, apply Bayes' theorem:

P(Market Up | Exp says "Up")

$$= \frac{P(\text{Exp says "Up"} \mid \text{Market Up})\, P(\text{Market Up})}{[P(\text{Exp says "Up"} \mid \text{Market Up}) P(\text{Market Up}) + P(\text{Exp says "Up"} \mid \text{Market Down}) P(\text{Market Down})]}$$

$= 1$

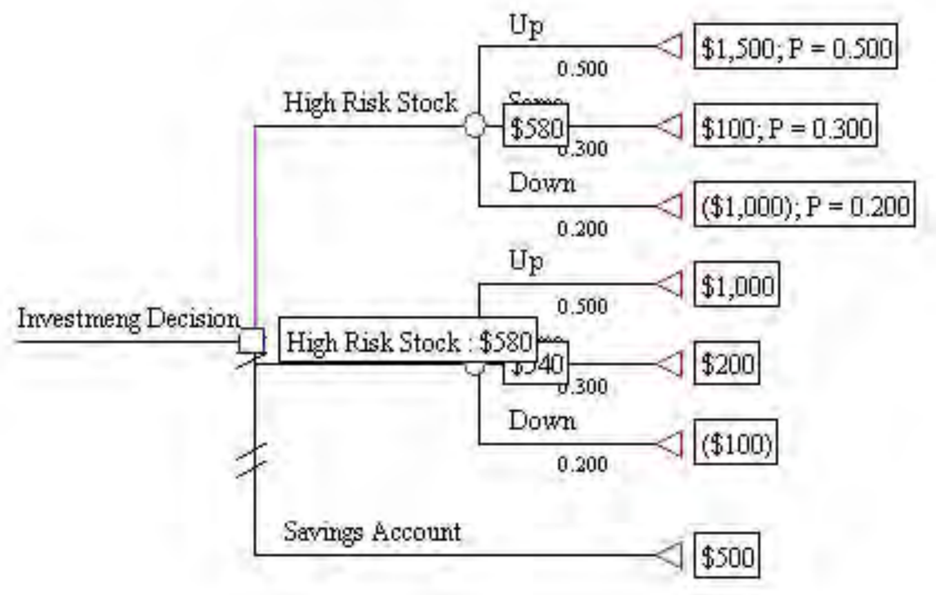
- Note:
  - The posterior probability is equal to 1 regardless of the prior probability
  - How do real problems differ from the above situation?

Expected Value of Information

- Expected value of information (EVI)
  - Indicates if information is worth gathering
  - Lower bound: zero expected value
  - Upper bound: expected value of perfect information
- Information has:
  - no value or zero expected value if the same choice will be made before and after obtaining information
  - positive expected value if it leads to a different choice
  - maximum expected value if information is perfect
- EVI is defined in terms of the decision context
  - Different people in different situations may place different values on the same information

Stock Investment Example (cont.)

- The optimal choice is the high-risk stock with EMV $580
- Assumption: optimistic about market (Up, 0.5 prob)
- How much would the investor be willing to pay for information about the market activity?

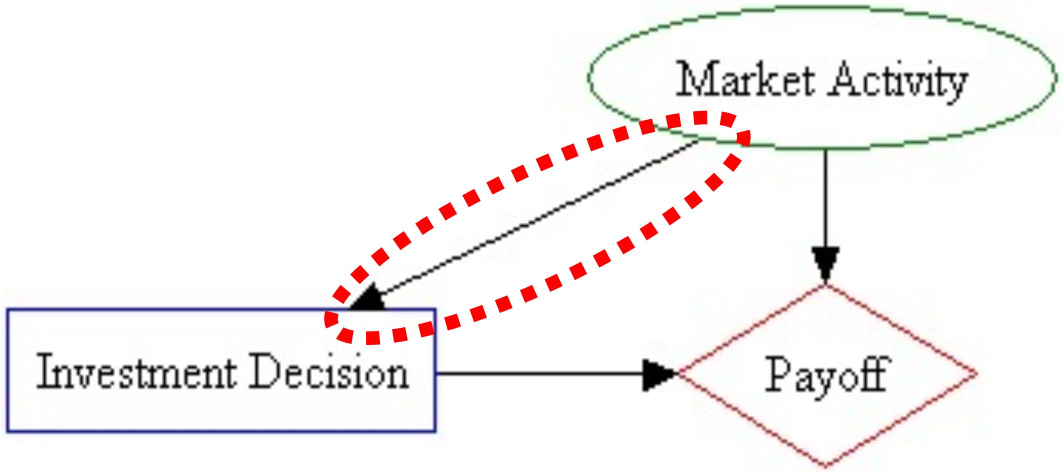


Expected Value of Perfect Information

- Max amount that the decision maker is willing to pay for perfect information
- To find expected value of perfect information (EVPI):
  - Modify the decision model to indicate perfect information
  - Solve the model and find the EMV ($1000)
  - EVPI = EMV (with perfect information) - EMV(original)
    =

Modifying Influence Diagram for EVPI

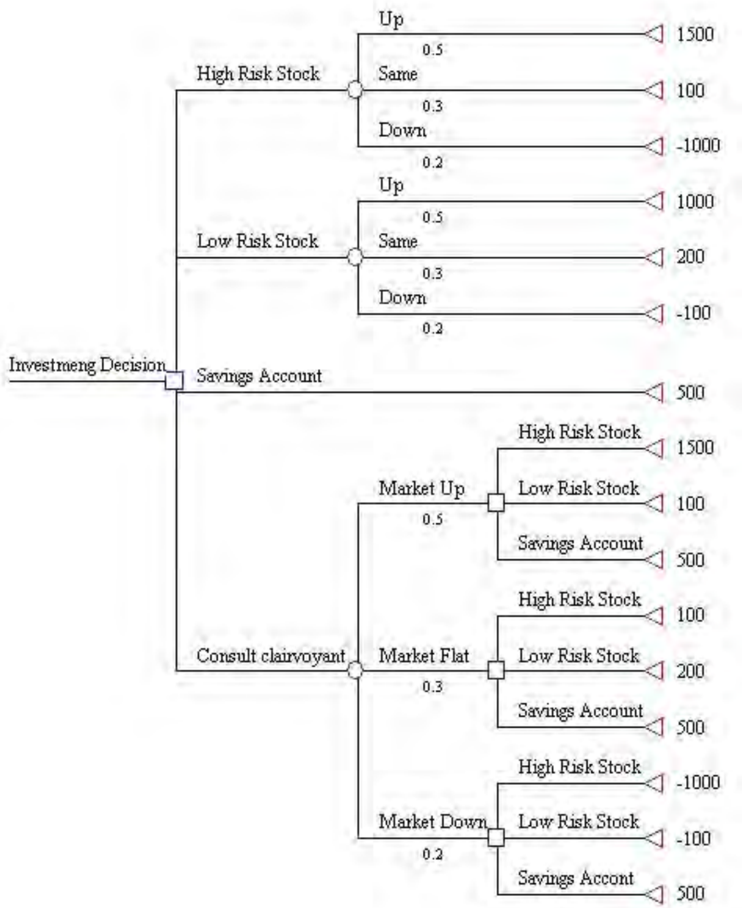
- Impose order on the decision and uncertain event nodes
- The uncertainty nodes for which perfect information is available comes before the decision node



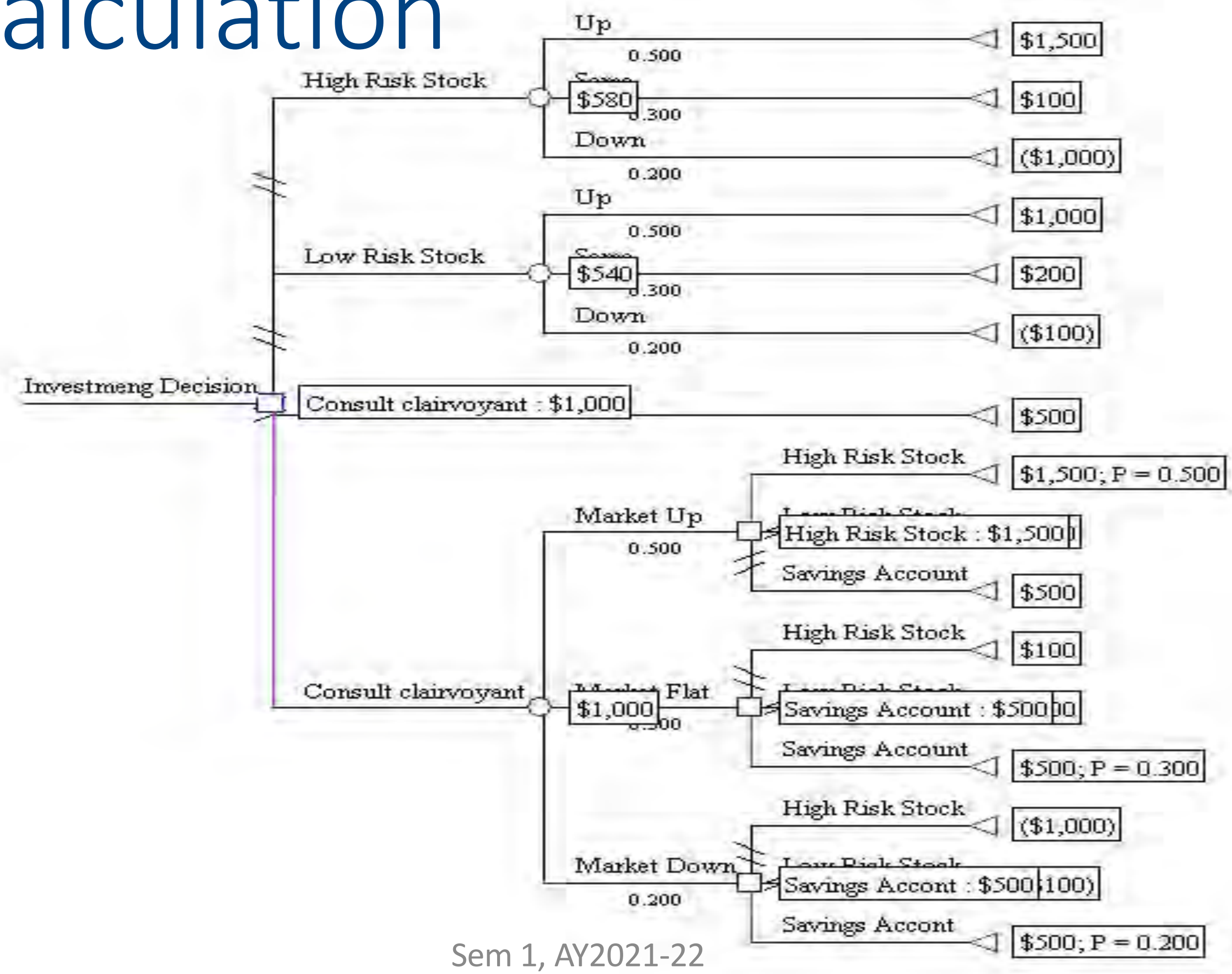
Influence diagram with perfect information

Modifying Decision Tree for EVPI

- Reorder the decision and the uncertain event nodes
- The uncertainty nodes for which perfect information is available comes before the decision node



EVPI Calculation



(Expected) Value of Perfect Information

Assume exact evidence about variable E_j; compute value of perfect information (VPI)

- Given expected utility with current best action α:

$$EU(\alpha) = \max_{\mathrm{a}} \sum_{s'} P(Result(a) = s')U(s')$$

- Value of the best new action after $E_j = e_j$ is obtained

$$EU\left(\alpha_{e_j} \middle| e_j\right) = \max_{a} \sum_{s'} P\left(Result(a) = s' \middle| a, e_j\right)U(s')$$

- Variable E_j can take multiple values e_{jk}, so on averaging:

$$VPI\left(E_j\right) = \sum_{e_j} P(E_j = e_j)EU\left(\alpha_{e_j} \middle| e_j\right) - EU(\alpha)$$

Properties of Value of information

- Expected value of information is always non-negative

$$\forall \mathbf{e}, E_j \quad VPI(E_j) \geq 0$$

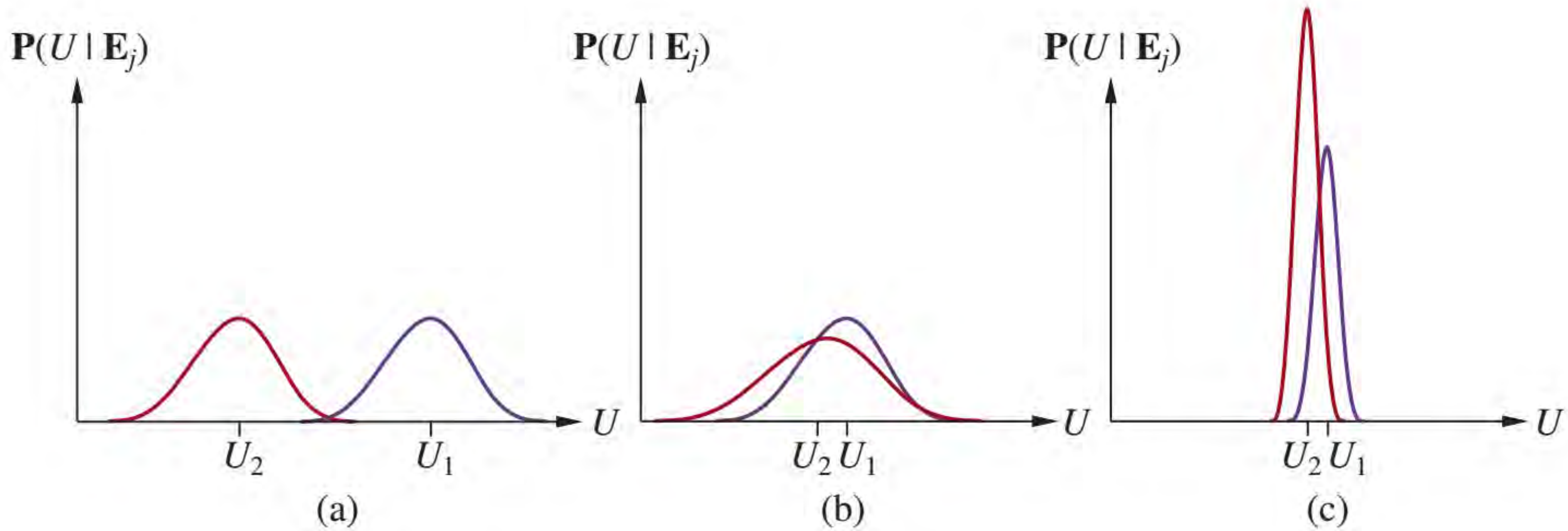
- VPI is not additive

$$VPI(E_j, E_k) \neq VPI(E_j) + VPI(E_k)$$

- VPI is order independent

$$VPI(E_j, E_k) = VPI(E_j) + VPI(E_k|E_j) = VPI(E_k) + VPI(E_j|E_k) \quad = VPI(E_k, E_j)$$

When to Gather More Information?



Source: RN Figure 16.8

Information Gathering, Decision-Theoretic Agent

- Agent should gather information before taking actions, if possible
  - Cost associated with getting the information, so how to choose which variable to get more information about?

function INFORMATION-GATHERING-AGENT(*percept*) **returns** an *action*
  persistent: D, a decision network

  integrate *percept* into D
  $j \leftarrow$ the value that maximizes $VPI(E_j) \,/\, C(E_j)$
  if $VPI(E_j) > C(E_j)$
    then return $Request(E_j)$
  else return the best action from D

Source: RN Figure 16.9

Summary

- Decision analysis
  - A prescriptive framework for guiding systematic, rational decision making
  - Involve formulation of explainable decision models and solutions
  - Extensive applications in practice
  - Theoretical foundations and methodological bases for AI decision systems – decision-theoretic AI
- Challenges and opportunities for AI
  - Human-machine collaboration in decision making
  - Uncertainty modeling with expert judgment and observational data
  - Preference modeling in complex, changing and uncertain conditions
  - Responsible AI – ethical, governance, and regulatory conditions

Homework

- Readings:
  - RN: 16.5 (Decision networks)
  - RN: 16.6.1-16.6.4 (Value of information)
  - RN: 16.6.6 (Sensitivity analysis)
  - *Optional:*
    - Howard, R.A., Decision Analysis: Practice and Promise. Management Science, 1988. 34(6): p. 679-695. [Accessible through NUS Library e-Resources]
- Reviews:
  - RN: 13.2-13.5; 14.2-14.4 (Conditional probability and Bayesian networks)
  - Charniak, E., Bayesian networks without tears: making Bayesian networks more accessible to the probabilistically unsophisticated. AI Mag., 1991. 12(4): p. 50–63.

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  - Howard, Ronald, A. (1966). "Decision Analysis: Applied Decision Theory" (PDF). Proceedings of the Fourth International Conference on Operational Research, Wiley-Interscience.
  - Howard, R.A., Decision Analysis: Practice and Promise. Management Science, 1988. 34(6): p. 679-695.
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  - Clemen, R.T. and T. Reilly, *Making Hard Decisions with DecisionTools*. 2013: Cengage Learning.
    - “Theoretical probability models,” Chapter 9
    - “Using data,” Chapter 10
    - “Value of information,” Chapter 12
  - Howard, R.A. and A.E. Abbas, Foundations of Decision Analysis. 2016: Pearson.
  - Abbas, A.E., Foundations of Multiattribute Utility. 2018, Cambridge: Cambridge University Press.
  - Fenton, N. and M. Neil, Risk Assessment and Decision Analysis with Bayesian Networks. 2nd ed. 2019: CRC Press, Inc.
- Website:
  - Decision Analysis Section of the Institute For Operations Research and Management Science (INFORMS):
    - http://decision-analysis.society.informs.org/index.html

Computing Tools

- Commercial Products – mostly derived from academic research work
  - DPL – Decision Programming Language
  - Palisade PrecisionTree
  - Treeage Pro
  - Netica
  - Hugin Expert
  - Supertree
  - …
- Commercial Products With FREE academic version
  - Bayesfusion: GeNIe and SMILE
  - …
- Also check out survey at:
  - INFORMS Decision Analysis Software Survey (2020)
  - https://pubsonline.informs.org/do/10.1287/orms.2020.06.04/full/
  - https://pubsonline.informs.org/magazine/orms-today/2020-decision-analysis-software-survey