CS1231: Discrete Structures

Tutorial 1

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Quick Review

- Proposition V.S. Non-propositions.
- Translation of $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$. Their order precedence.
- Only if. Sufficient. Necessary.
- Truth Table. $(p \rightarrow q)$.
- Tautology and Contradiction.
- Logically Equivalent.
- Logical Equivalence Laws.

Menu

1. Which of these are propositions?

(a) Do not go.

(b) Can you answer the question?

(c) There are no black flies in Maine.

(d) $4 + x = 5$.

(e) $2^n \geqslant 100$.

(f) The moon is made of green cheese.

Recall

- ✍ A **proposition** is a sentence that declares a fact, that is true or false.
- ✍ **Non-propositions** include questions, commands, sentences with undefined words, etc.

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Answer. (c) and (f).

2. Let p and q be the propositions "The election is decided" and "The votes have been counted", respectively. Express each of the following as an English sentence.

(a) $\neg p \wedge q$. (b) $q \to p$. (c) $p \leftrightarrow q$. (d) $\neg q \vee (\neg p \wedge q)$.

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The order of precedence, listed in decreasing order:

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Answer.

(a) The election is not decided and the votes have been counted.

(b) If the votes have been counted, then the election is decided.

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(a) The election is not decided and the votes have been counted.

(b) If the votes have been counted, then the election is decided.

(c) The election is decided iff the votes have been counted.

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The order of precedence, listed in decreasing order:

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(a) The election is not decided and the votes have been counted.
(b) If the votes have been counted, then the election is decided.
(c) The election is decided iff the votes have been counted.
(d) Either the votes have not been counted, or else the election is not decided and the votes have been counted.

3. Determine whether these conditional propositions are true or false.

(a) If $1 + 1 = 3$, then unicorns exist.

(b) If $1 + 1 = 3$, then dogs can fly.

(c) If $1 + 1 = 2$, then dogs can fly.

(d) If $2 + 2 = 4$, then $1 + 2 = 3$.

Recall

The truth table for $p \to q$ is

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
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Answer. T, T, F, T.

4. Consider the proposition "You will get an A in this module only if either you do every exercise in the text book or you score at least 80 marks in the final."

(a) Determine all the situations in which the proposition is true.

(b) Given that the proposition is true, would you get an A if you did not do exercise 5 and scored 79 marks for the final?

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p only if q means: $p \to q$.

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p only if q means: $p \to q$.

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Recall

p only if q means: $p \to q$.

Answer.

(a) The proposition is true when you

  (i) do not get A or

  (ii) get A and either do every exercise in the text book or you score at least 80 marks in the final.

(b) No.

5. Are the following pairs logically equivalent?

(a) $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$

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p	q	r	$p \vee q$	$p \wedge r$	$(p \vee q) \vee (p \wedge r)$	$(p \vee q) \wedge r$
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

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T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F
F	F	T	F	F	F	F
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$$
\begin{aligned}
(p \vee q) \vee (p \wedge r) &\equiv ((p \vee q) \vee p) \wedge ((p \vee q) \vee r) \text{ (Distributive Law)} \\
&\equiv (p \vee q) \wedge ((p \vee q) \vee r) \text{ (Idempotent Law)} \\
&\equiv p \vee q \text{ (Absorption Law)} \\
&\not\equiv (p \vee q) \wedge r
\end{aligned}
$$

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p	q	r	$\neg r$	$r \vee p$	$p \wedge q$	$r \vee q$	$\neg r \vee (p \wedge q)$	
					RHS			LHS
T	T	T	F	T	T	T	T	T
T	T	F	T	T	T	T	T	T
T	F	T	F	T	F	T	F	F
T	F	F	T	T	F	F	T	F
F	T	T	F	T	F	T	F	F
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F	F	T	F	T	F	T	F	F
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5. Are the following pairs logically equivalent?

(b) $(r \vee p) \wedge (\neg r \vee (p \wedge q)) \wedge (r \vee q)$ and $(p \wedge q)$

p	q	r	$\neg r$	$r \vee p$	$p \wedge q$ RHS	$r \vee q$	$\neg r \vee (p \wedge q)$	LHS
T	T	T	F	T	T	T	T	T
T	T	F	T	T	T	T	T	T
T	F	T	F	T	F	T	F	F
T	F	F	T	T	F	F	T	F
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$$(r \vee p) \wedge (\neg r \vee (p \wedge q)) \wedge (r \vee q)$$

$$\equiv (r \vee p) \wedge (\neg r \vee p) \wedge (\neg r \vee q) \wedge (r \vee q) \text{ (Distributive Law)}$$

$$\equiv [(r \vee p) \wedge (\neg r \vee p)] \wedge [(\neg r \vee q) \wedge (r \vee q)] \text{ (Associative Law)}$$

$$\equiv [(r \wedge \neg r) \vee p] \wedge [(r \wedge \neg r) \vee q] \text{ (Distributive Law)}$$

$$\equiv [\mathbf{C} \vee p] \wedge [\mathbf{C} \vee q] \text{ (Negation Law)}$$

$$\equiv p \wedge q \text{ (Identity Law)}$$

6. Complete the following truth table.

p	q	r	$p \rightarrow q$	$\neg p \rightarrow r$	$t = (p \rightarrow q) \wedge (\neg p \rightarrow r)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	F
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Let $t = (p \to q) \wedge (\neg p \to r)$. Are the following true or false?

(a) t is false if p and q are both true.

(b) If t is false then $p \wedge q$ is false.

(c) A sufficient condition for t to be true is that p is true and r is false.

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(e) $(p \wedge q) \to t$ is a tautology.

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Answers. F,T,F,T,T

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- The 99th proposition says: "Exactly 99 of the propositions in this list are false"

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- The 3rd proposition says: "Exactly 3 of the propositions in this list are false"
- ......
- The 98th proposition says: "Exactly 98 of the propositions in this list are false"
- The 99th proposition says: "Exactly 99 of the propositions in this list are false"
- The 100th proposition says: "Exactly 100 of the propositions in this list are false"

Q(2). Exactly how many of the proposition in this list are false?

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- Is it true or false?

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- What should we do?
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  See which ones are false and which ones are true

  Count how many are false.
- But the 1st proposition says: "Exactly 1 of the propositions in this list is false"
- Is it true or false?
- We need to know the truth values of all propositions! It's a loop!

Imagine: We know the truth values of all propositions and exactly x propositions are false

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The xth proposition is true. Other propositions are false, because they give a different number of false propositions.

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Q(5). Count: How many are false?

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The xth proposition is true. Other propositions are false, because they give a different number of false propositions.

Q(5). Count: How many are false? Ans: 99.

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Q(6). What is x?

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Q(3). How many propositions are true?Ans: $100 - x$

Q(4). Which propositions are true and which are false?

- The 1st proposition says: "Exactly 1 of the propositions in this list is false"
- ......
- The xth proposition says: "Exactly x of the propositions in this list are false"
- ......
- The 100th proposition says: "Exactly 100 of the propositions in this list are false"

The xth proposition is true. Other propositions are false, because they give a different number of false propositions.

Q(5). Count: How many are false? Ans: 99.

Q(6). What is x?Ans: $x = 99$. Only the 99th proposition is true, others all false.

(b) Answer part (a) if the n^{th} proposition is "At least n of the propositions in this list are false".

Q(1). What are the propositions?

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Q(1). What are the propositions?

- The 1st proposition says: "At least 1 of the propositions in this list are false"
- The 2nd proposition says: "At least 2 of the propositions in this list are false"

(b) Answer part (a) if the n^{th} proposition is "At least n of the propositions in this list are false".

Q(1). What are the propositions?

- The 1st proposition says: "At least 1 of the propositions in this list are false"
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- The 3rd proposition says: "At least 3 of the propositions in this list are false"

(b) Answer part (a) if the n^{th} proposition is "At least n of the propositions in this list are false".

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- ......
- The 98th proposition says: "At least 98 of the propositions in this list are false"

(b) Answer part (a) if the n^{th} proposition is "At least n of the propositions in this list are false".

Q(1). What are the propositions?

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- The 2nd proposition says: "At least 2 of the propositions in this list are false"
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- The 98th proposition says: "At least 98 of the propositions in this list are false"
- The 99th proposition says: "At least 99 of the propositions in this list are false"

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- ......
- The 98th proposition says: "At least 98 of the propositions in this list are false"
- The 99th proposition says: "At least 99 of the propositions in this list are false"
- The 100th proposition says: "At least 100 of the propositions in this list are false"

Imagine: We know the truth values of all propositions and exactly x propositions are false

Q(2). Which propositions are true and which are false?

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The 1st up to the xth propositions are true. Other propositions are false, because they give a bigger number of false propositions.

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Q(3). Count: How many are false?

Imagine: We know the truth values of all propositions and exactly x propositions are false

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The 1st up to the xth propositions are true. Other propositions are false, because they give a bigger number of false propositions.

Q(3). Count: How many are false? Ans: x proportions true and $100 - x$ false.

Imagine: We know the truth values of all propositions and exactly x propositions are false

Q(2). Which propositions are true and which are false?

- The 1st proposition says: "At least 1 of the propositions in this list is false"
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The 1st up to the xth propositions are true. Other propositions are false, because they give a bigger number of false propositions.

Q(3). Count: How many are false? Ans: x proportions true and $100 - x$ false.

Q(4). What is x?

Imagine: We know the truth values of all propositions and exactly x propositions are false

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- The 1st proposition says: "At least 1 of the propositions in this list is false"
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- ......
- The 100th proposition says: "At least 100 of the propositions in this list are false"

The 1st up to the xth propositions are true. Other propositions are false, because they give a bigger number of false propositions.

Q(3). Count: How many are false? Ans: x proportions true and $100 - x$ false.

Q(4). What is x?Ans: $x = 100 - x$, so $x = 50$. The 1st up to the 50th propositions are true. Other propositions are false.

(c) Answer part (b) assuming that there are 101 propositions in the list.

- We follow the same procedure as (b) and get

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- We follow the same procedure as (b) and get $x = 101 - x$. No integer solution.

Answer.

(a) The 99^{th} is true and the rest are false.

(b) $1 - 50$ are true and $51 - 100$ are false.

(c) This is a paradox.

8. Show that $[(p \to q) \wedge q] \to p$ is not a tautology.

Recall

A **tautology** is a compound proposition that is always true.

Idea.

p	q	$p \to q$	$(p \to q) \wedge q$	$[(p \to q) \wedge q] \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

9. Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known?

(i) At least one of Kevin and Heather is chatting;

(ii) Exactly one of Randy and Vijay is chatting;

(iii) If Abby is chatting, then so it Randy;

(iv) Vijay and Kevin are either both chatting or both not chatting;

(v) If Heather is chatting, then so are Abby and Kevin.

Idea. Let A: Abby is chatting; H: Heather is chatting; K: Kevin is chatting; R: Randy is chatting; V: Vijay is chatting. so the information is

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Idea. Let A: Abby is chatting; H: Heather is chatting; K: Kevin is chatting; R: Randy is chatting; V: Vijay is chatting. so the information is

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Now we make an assumption that H is true

Then is true, by (v).

That is, ___ are true.

Then R is , by (iii).

So V is , by (ii).

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So it is to make H be true.

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