

Discussion Group Problems for Week 8

For: Oct. 17, 2019

Problem 1. Urban Planning Nightmares with Mr. Dnivog

You have just landed an internship at a top urban planning company. Your boss, Mr.Dnivog, has asked you to look into certain road planning proposals to help improve connectivity within the city, specifically between the transport hubs in the city. Since you are new (and probably naïve), you agree to help him without hesitation, only to realise that you have been hoodwinked as millions of proposals are given to you to look through. You decide to call on your trusty knowledge of algorithms to help you bulldoze through this mountain of work. You know you cannot fail in this endeavour (especially since Mr Dnivog's Russian aura frightens you even when he is in a good mood).

Problem 1.a. There are multiple sets of road proposals. Within each set, you are given a road network, full of bidirectional roads. The road network consists of two transport hubs, marked S and T . Each bidirectional road has a non-negative weight attached to it, representing its length. Following this, you are given multiple queries of the form X, Y where you need to figure out whether adding a road between X and Y will shorten the distance between S and T . You should consider each query independently (i.e. No two roads from the set of queries will be added at the same time). For each query, you need to indicate whether the given road proposal will shorten the distance between S and T . How would you do so given that there are many many queries for each set?

Problem 1.b. Mr. Dnivog was very impressed with how fast you managed to look through all the road proposals in one night. He decides to continue to the torturous workload. This time round, he wants you to optimise his coffee routine. One problem he faces is that after finishing a meeting somewhere within the city, Mr.Dnivog wants to get coffee. However, he also does not want to walk too long in the heat so he would prefer to know which of the two biggest coffee chains, Barstucks or Lejew has an outlet nearest to him.

You are given a network that shows all the locations of coffee stores of which Mr Dnivog likes (there may be many of them). Mr Dnivog has marked out certain nodes as points of interest. At these nodes, Mr Dnivog wants to know whether he is nearest to a Lewej or a Barstucks outlet. How would you do this quickly?

Problem 1.c. Mr. Dnivog was extremely impressed at your ability to complete the previous two tasks. He decides to take things a step further, by going back to a classic problem, finding the shortest path between two points. However, there is a twist: there may be multiple shortest paths! Hence given a network Mr.Dnivog wants to find the number of distinct shortest paths from point A to point B . How can you do so?

Extension: In relation to this, how would you find out the k shortest paths within a graph, assuming that they are not of equal length?

Problem 2. Shortest Paths 1

Next, we will attempt the **Shortest Paths** problem on Kattis

<https://nus.kattis.com/problems/shortestpaths1>

Problem 3. Island Escape

After many days of exploration, you have decided to leave treasure island, with the plenty of treasures to sell back in your hometown. However, when you have reached the path to exit the island, you realise that the island has changed! Now, in order to leave the island you need to pass through a series of obstacles.

Problem 3.a. First, there is a treacherous maze of bridges to cross. Each bridge is guarded by goblins, who are under strict orders to not let anyone pass. However, you know that goblins can be bribed and you decide to make use of some of the looted treasure coins in order to convince them for you to pass through a particular bridge. However, you being your thrifty self, you want to ensure that you do not pay much more than is required in order to cross the bridge. You also are very careful to not deceive the goblins in anyway, for the moment they see you on the bridge and they have not received a sufficient bribe, they will not hesitate to destroy the bridge and send you to your fiery depths.

In addition to this, one of idols gathered from the your plunders allows you to teleport across a bridge without bribing the goblin. However, this idol can only be used **once**. By your supreme powers of estimation, you know roughly how many coins you need to bribe each goblin. What is the cheapest way across the bridge maze given there are n islands connected by m bridges, where each Goblin charges w_i for $i \in [1 \dots m]$?

Hint: *We can use the idol at any point in time, but which edge do we use it on? Is it always the most expensive edge in the maze?*

Problem 3.b. In the previous problem, can we find a better algorithm if the amount charged by each Goblin was bounded by some small constant k (e.g. $k = 5$) What if each Goblin can be bribed using only 0 or 1 coins?