School of Computing

The Tourist Problem:
Graph Model and Graph Coloring
Video 5.4

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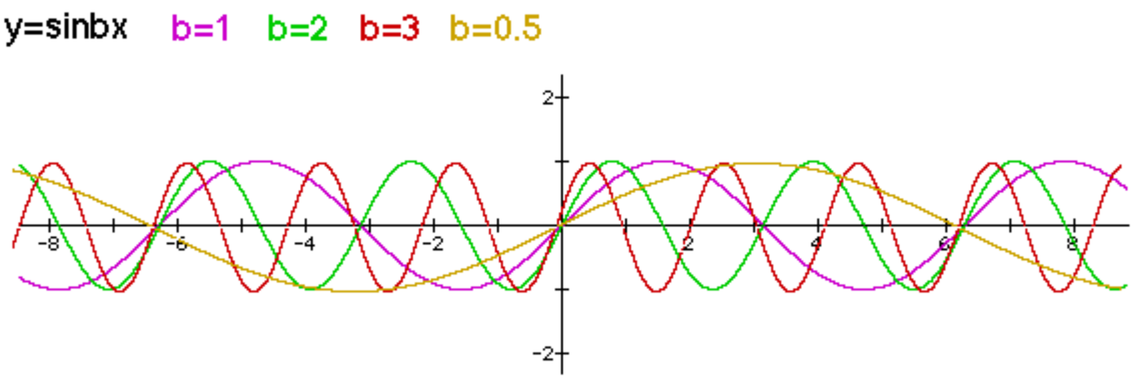
Experience the fun of problem solving

Re-Cap…

- **Solved TP v1.0**
  - **Used notion of non-conflict**
  - **Done Activity 1 to solve TP instance**
- **The method works, but**
  - **tedious, error-prone, *does not scale***
- **We seek a better method**
  - **Answer: Use a GRAPH**

The Graph Model

- What is a graph?
  - eg: $y = sin(bx)$



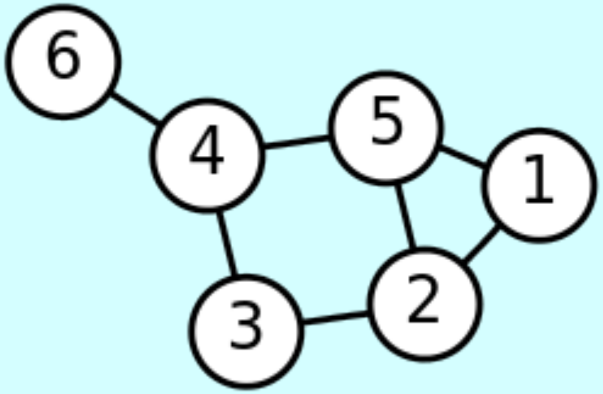
- No. Not this type of graph.

A Graph Model

- **Graph $G = (V, E)$**
  - *V* is a set of vertices, (aka nodes, circles)
  - *E* is a set of edges (aka arcs, links)

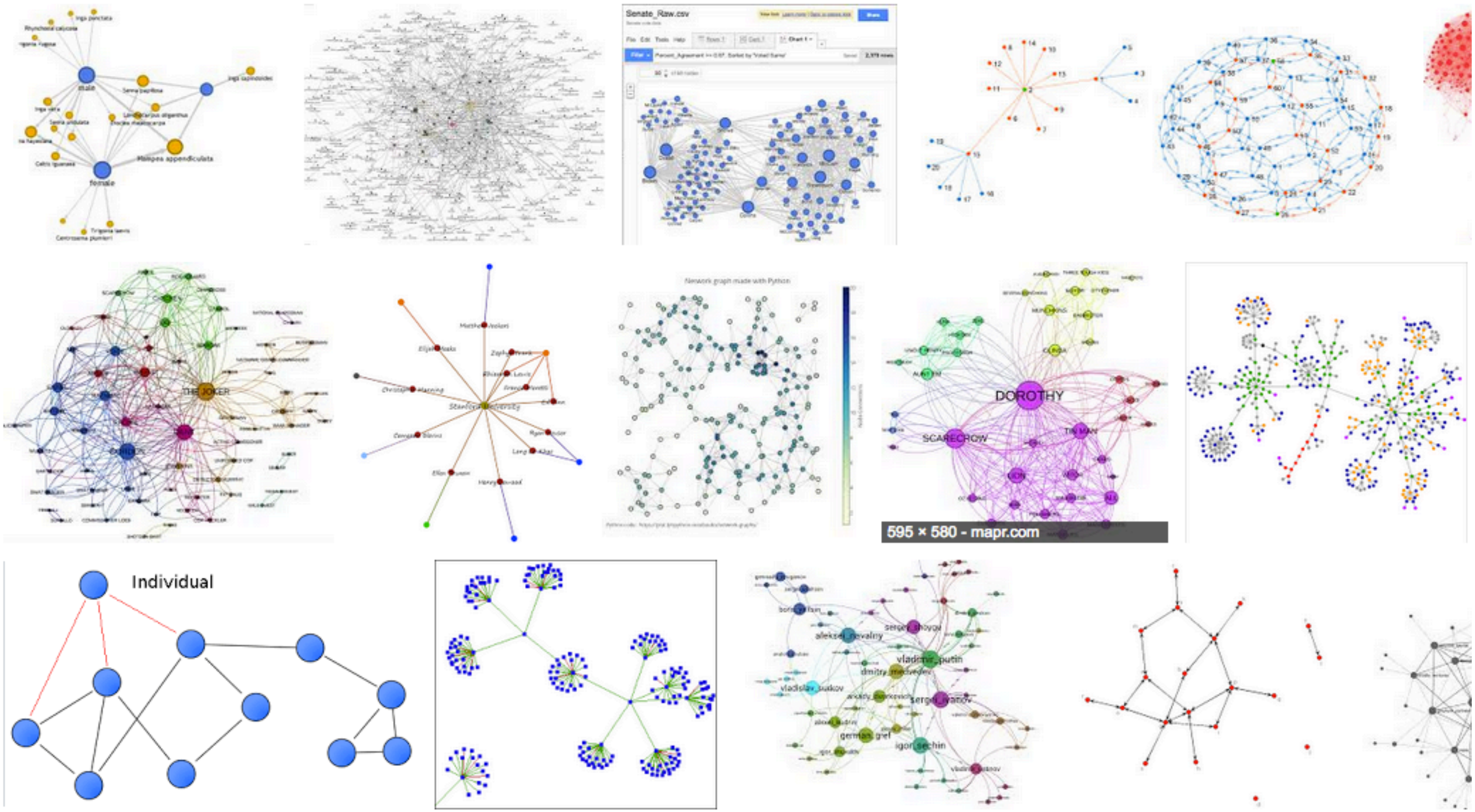
V = {1, 2, 3, 4, 5, 6}

E = { (1,2), (1,5), (2,3),
(2,5), (3,4), (4,5),
(4,6) }



A graph is a model with vertices and edges (arc) that connect pairs of vertices.

Graphs (or networks) are A&E

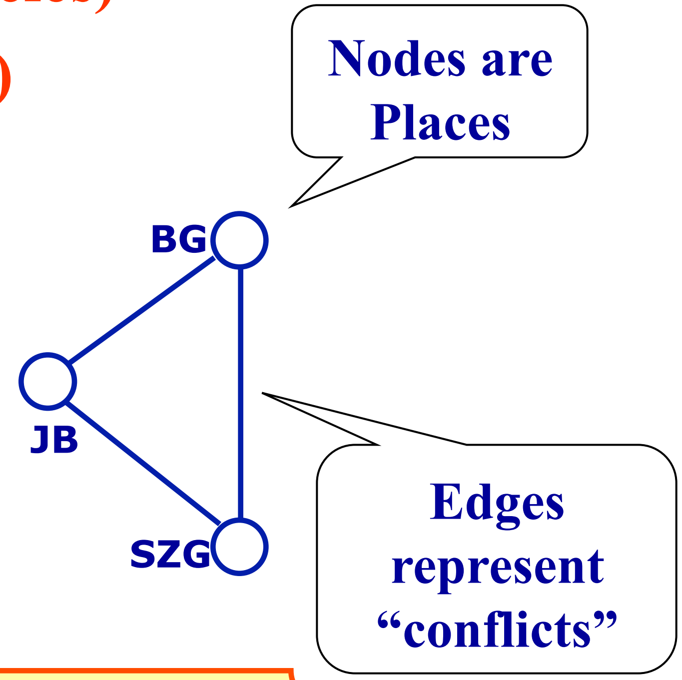


The Graph Model

- Graph $G = (V, E)$
  - V is a set of vertices, nodes (circles)
  - E is a set of edges (connections)

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG



In our graph, nodes are places, and edges in the graph means conflicts.

Graph Model for the Tourist Problem

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
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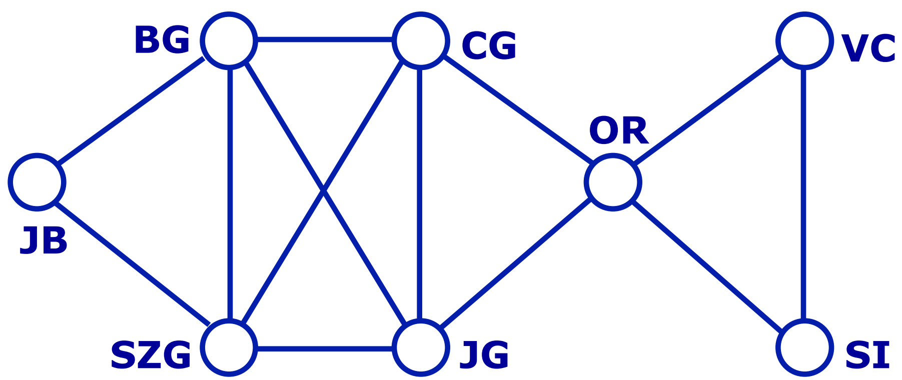
$G = (V, E)$



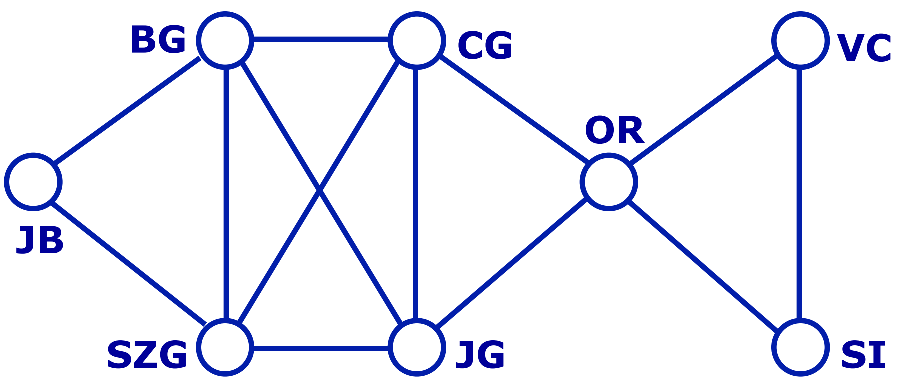
The graph $G = (V, E)$ captures all the conflicts for our tourist problem instance.

Graph Model for the Tourist Problem

- **What's good about the graph model?**
  - *very simple !*
  - **easy to spot the conflicts**

    and the *non-conflicts*

$G = (V, E)$



Two places that are *adjacent* (connected by an edge) conflicts with each other;

Two places that are not adjacent have no conflict;

Can verify with
JB and BG (conflict)
OR and SZG. (no conflict)

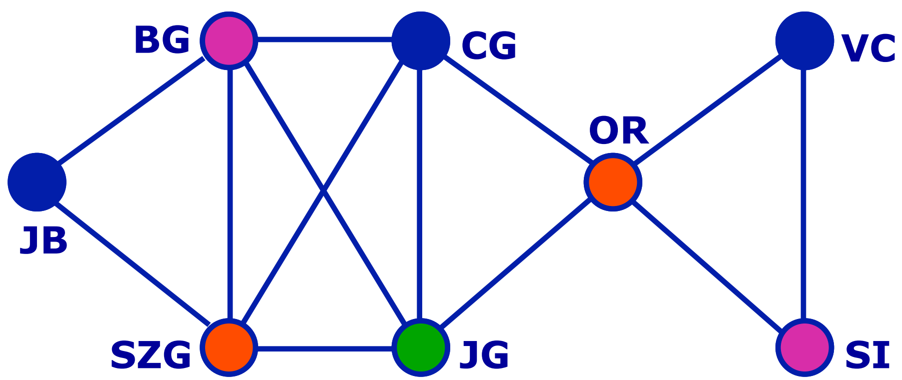
Graph Model for the Tourist Problem

- **What's good about the graph model?**
  - *very simple* **!**
  - **easy to spot conflicts**

    and the *non-conflicts*

Now we use conflict graph
To do bus scheduling

$G = (V, E)$



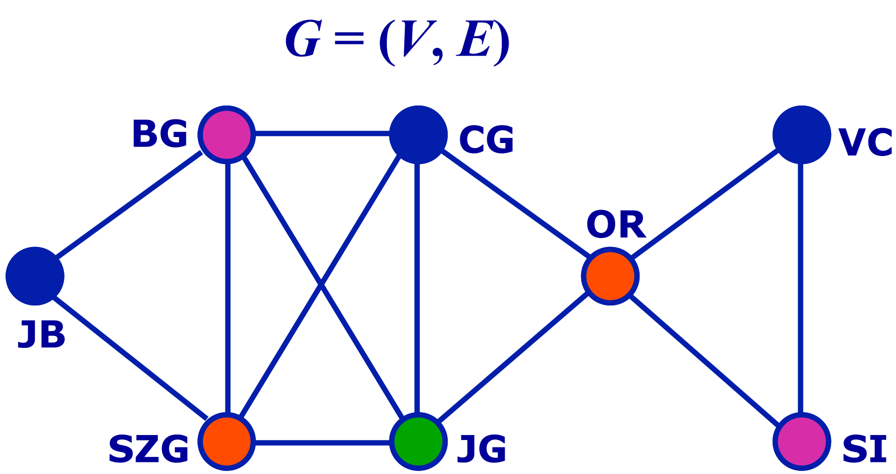
On Day 1, **can schedule SZG, OR** **[Any more? Why?]**
On Day 2, **can schedule JB, CG, VC**
On Day 3, **can schedule BG, SI**
On Day 4, **can schedule JG**

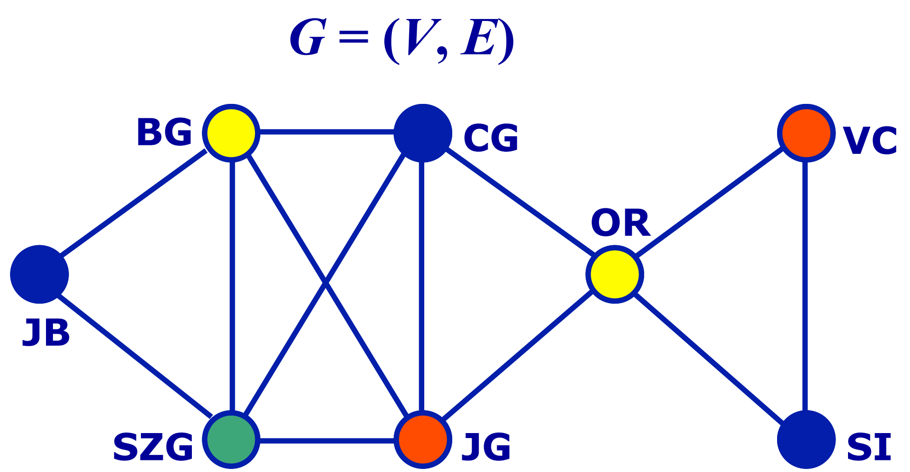
WOW!
"Bus scheduling" become
"colouring of vertices in G".

Graph Coloring Problem

- Given a graph $G = (V, E)$, colour the vertices in V so that any two vertices that are connected by an edge in E will have *different* colors.

We want to *minimize* the number of colors.





Number of colours used to colour the graph G = *Number of days needed to complete the schedule*

TP Activity #2:

Bus Scheduling via Graph Colouring

(8 minutes)

Instruction:

Download and print a copy of TP-Activity-2.pdf.

Then complete the activity – colour the graphs with the minimum number of colours.

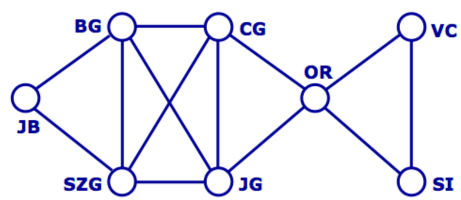
Have your answers ready at hand before continuing to Video **5.5**

TP Activity 2: (10 minutes) [Graph Colouring]

The Tourist Problem **Your Name:** ______________________________

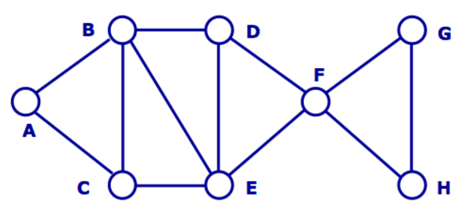
The tourist problem instance in the lecture can be modeled with the following conflict graph. Two possible colorings of the graph are given in the lecture.

Q1: Give a *different* way to colour the vertices of the graph on the left. How many colours?



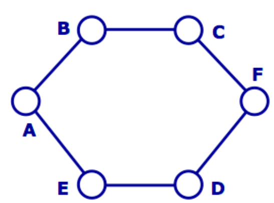
Q1: # colors: ______

Q2: What about this one?

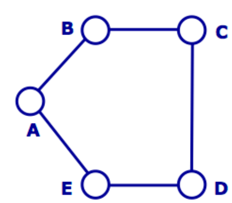


Q2: # colors: ______

Q3: Try coloring the following graphs with the minimum number of colors.



Q3(a): # colors: ______



Q3(b): # colors: ______

(End of video 5.4)

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