CS1231: Discrete Structures

Tutorial 9

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Quick Review

- Product Rule; Sum Rule.
- $|A \cup B| = |A| + |B| - |A \cap B|$;
  $|A \cup B \cup C| =$
  $|A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$.
- A **permutation** of a set of distinct objects is an ordering of the objects.
  1. The number of permutations of n distinct objects is $n!$.
  2. The number of r-permutations of a set of n elements is denoted $P(n, r) = n!/(n-r)!$.
     The number of r-permutation $\underline{\text{(repetition allowed)}}$ of a set of n distinct objects is n^r.
- Let n, r be integers with $0 \leqslant r \leqslant n$. An r-**combination** of a set of n (distinct) objects is a subset of r objects.
  1. The number of r-combinations of a set of n elements is $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$.
  2. The number of r-combinations $\underline{\text{(repetition allowed)}}$ of a set of n elements is $\binom{n+r-1}{r} = \dfrac{(n+r-1)!}{r!(n-1)!}$.

Menu

Question 1

Question 2

Question 3

Question 4

Question 5

Question 6

Question 7

Question 8

Question 9

1.

(a) How many integers from 1000 through 9999 have distinct digits?

(b) How many odd integers from 1000 through 9999 have distinct digits?

(c) How many odd integers from 5000 through 9999 have distinct digits?

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Idea. We always start from the one with most restriction (the least choices).

Answer.

(a) Pick

(b) Pick

(c) Pick the

1st digit is and 1st digit is

Ans $= \times \times \times + \times \times \times$.

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Answer.

(a) Pick Thousands,

(b) Pick

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Ans $= \times \quad \times \quad \times \quad + \quad \times \quad \times \quad \times \quad .$

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Idea. We always start from the one with most restriction (the least choices).

Answer.

(a) Pick Thousands, Hundreds, Tens, Unit digits in that order:

(b) Pick

(c) Pick the

1st digit is and 1st digit is

Ans $= \times \times \times + \times \times \times$.

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Idea. We always start from the one with most restriction (the least choices).

Answer.

(a) Pick Thousands, Hundreds, Tens, Unit digits in that order:
$9 \times$

(b) Pick

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Answer.

(a) Pick Thousands, Hundreds, Tens, Unit digits in that order: $9 \times 9 \times 8 \times$

(b) Pick

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Answer.

(a) Pick Thousands, Hundreds, Tens, Unit digits in that order: $9 \times 9 \times 8 \times 7 = 4536.$

(b) Pick

(c) Pick the

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Ans $=$ $\times$ $\times$ $\times$ $+$ $\times$ $\times$ $\times$.

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Answer.

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(b) Pick unit,

(c) Pick the

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(b) Pick unit, thousands, hundred, tens digits in that order: $5 \times$

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(b) Pick unit, thousands, hundred, tens digits in that order: $5 \times 8 \times 8 \times 7 = 2240.$

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(c) Pick the thousands, unit, hundreds and tens digit in that order. There are two cases: 1st digit is and 1st digit is

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(c) Pick the thousands, unit, hundreds and tens digit in that order. There are two cases: 1st digit is odd: $5,$ and 1st digit is even:

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(c) Pick the thousands, unit, hundreds and tens digit in that order. There are two cases: 1st digit is odd: 5, 7 or and 1st digit is even:

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(c) Pick the thousands, unit, hundreds and tens digit in that order. There are two cases: 1st digit is odd: $5, 7$ or 9 and 1st digit is even:

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(c) Pick the thousands, unit, hundreds and tens digit in that order. There are two cases: 1st digit is odd: 5, 7 or 9 and 1st digit is even: 6 or

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Ans $= \quad \times \quad \times \quad \times \quad + \quad \times \quad \times \quad \times \qquad .$

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$\text{Ans} = 3 \times \quad \times \quad \times \quad + \quad \times \quad \times \quad \times \qquad .$

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Ans $= 3 \times 4 \times \quad \times \quad + \quad \times \quad \times \quad \times \qquad .$

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Ans $= 3 \times 4 \times 8 \times \quad + \quad \times \quad \times \quad \times \qquad .$

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(c) Pick the thousands, unit, hundreds and tens digit in that order. There are two cases: 1st digit is odd: 5, 7 or 9 and 1st digit is even: 6 or 8.
Ans $= 3 \times 4 \times 8 \times 7 + \quad \times \quad \times \quad \times \qquad .$

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(c) Pick the thousands, unit, hundreds and tens digit in that order. There are two cases: 1st digit is odd: 5, 7 or 9 and 1st digit is even: 6 or 8.
Ans $= 3 \times 4 \times 8 \times 7 + 2 \times \quad \times \quad \times \qquad .$

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(b) Pick unit, thousands, hundred, tens digits in that order: $5 \times 8 \times 8 \times 7 = 2240.$

(c) Pick the thousands, unit, hundreds and tens digit in that order. There are two cases: 1st digit is odd: 5, 7 or 9 and 1st digit is even: 6 or 8.
Ans $= 3 \times 4 \times 8 \times 7 + 2 \times 5 \times \quad \times \qquad .$

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(c) Pick the thousands, unit, hundreds and tens digit in that order. There are two cases: 1st digit is odd: 5, 7 or 9 and 1st digit is even: 6 or 8.

Ans $= 3 \times 4 \times 8 \times 7 + 2 \times 5 \times 8 \times$.

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(b) Pick unit, thousands, hundred, tens digits in that order: $5 \times 8 \times 8 \times 7 = 2240.$

(c) Pick the thousands, unit, hundreds and tens digit in that order. There are two cases: 1st digit is odd: 5, 7 or 9 and 1st digit is even: 6 or 8.
Ans $= 3 \times 4 \times 8 \times 7 + 2 \times 5 \times 8 \times 7 = 1232.$

2. How many ways are there for 10 women and 6 men to sit in a row so that no two men are next to each other?

Idea.

(1) How many ways women to form a row?

? How many positions for men? (Insert them in the row of women.)

(2) How many ways to choose the positions for men?

(3) How many ways to seat men in these positions?

ans = .

2. How many ways are there for 10 women and 6 men to sit in a row so that no two men are next to each other?

Idea.

(1) How many ways women to form a row?
$10!$

? How many positions for men? (Insert them in the row of women.)

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ans = .

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11

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ans = .

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(2) How many ways to choose the positions for men?
$\binom{11}{6}$

(3) How many ways to seat men in these positions?

ans = .

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ans = .

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(2) How many ways to choose the positions for men?

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(3) How many ways to seat men in these positions?

$6!$

Answer. First arrange the woman ($10!$ ways) and then insert the men ($\binom{11}{6}6!$ ways). ans = .

2. How many ways are there for 10 women and 6 men to sit in a row so that no two men are next to each other?

Idea.

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? How many positions for men? (Insert them in the row of women.)
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(2) How many ways to choose the positions for men?
$\binom{11}{6}$

(3) How many ways to seat men in these positions?
$6!$

Answer. First arrange the woman ($10!$ ways) and then insert the men ($\binom{11}{6}6!$ ways). ans $= 10! \times 6! \times \binom{11}{6}$.

3. Consider strings of length n over the set $\{a, b, c, d\}$. How many such strings contain at least one pair of adjacent characters that are the same?

Idea.

(1) How many strings of length n over the set $\{a, b, c, d\}$?

(2) How many strings contain no pair of adjacent characters that are the same?

sub(1) How many choices for the first bit?

sub(2) How many choices for the second bit?

sub(3) How many choices for the third bit?

sub(4) ...

sub (n-1) How many choices for the $n-1$ th bit?

sub (n) How many choices for the n th bit?

ans for (2)=

Ans =

3. Consider strings of length n over the set $\{a, b, c, d\}$. How many such strings contain at least one pair of adjacent characters that are the same?

Idea.

(1) How many strings of length n over the set $\{a, b, c, d\}$?
4^n

(2) How many strings contain no pair of adjacent characters that are the same?

sub(1) How many choices for the first bit?

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4

sub(2) How many choices for the second bit?

sub(3) How many choices for the third bit?

sub(4) ...

sub (n-1) How many choices for the $n-1$ th bit?

sub (n) How many choices for the n th bit?

ans for (2)=

Ans =

3. Consider strings of length n over the set $\{a, b, c, d\}$. How many such strings contain at least one pair of adjacent characters that are the same?

Idea.

(1) How many strings of length n over the set $\{a, b, c, d\}$?
4^n

(2) How many strings contain no pair of adjacent characters that are the same?

sub(1) How many choices for the first bit?
4

sub(2) How many choices for the second bit?
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sub (n) How many choices for the n th bit?
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ans for (2)= $4 \times 3^{n-1}$

Ans =

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- sub(1) How many choices for the first bit?
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ans for (2)= $4 \times 3^{n-1}$

Ans $= 4^n - 4 \times 3^{n-1}$

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(2) Let U be the set of integers from 0 through 999999.

(3) Let A_1 be the set of integers in U that **do not contain the digit** 1. $|A_1| =$

(4) Let A_2 be the set of integers in U that **do not contain the digit** 2. $|A_2| =$

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(9) $A_1 \cap A_2 \cap A_3 =$ the set of integers in U that

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(10) $A_1 \cup A_2 \cup A_3 =$ the set of integers in U that

$|A_1 \cup A_2 \cup A_3| =$

(11) What is the set of integers from 0 through 999999 contain each of the digits 1, 2, 3 at least once?

Its cardinality is?

Ans.

4. How many integers from 1 through 999999 contain each of the digits 1, 2, 3 at least once?

(9) $A_1 \cap A_2 \cap A_3 =$ the set of integers in U that **do not contain the digit 1 and do not contain 2 and do not contain 3.**
$|A_1 \cap A_2 \cap A_3| =$

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(9) $A_1 \cap A_2 \cap A_3 =$ the set of integers in U that **do not contain the digit 1 and do not contain 2 and do not contain 3.**
$|A_1 \cap A_2 \cap A_3| = 7^6$

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(10) $A_1 \cup A_2 \cup A_3 =$ the set of integers in U that **do not contain 1 or do not contain 2 or do not contain 3.**
$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_2 \cap A_3| - |A_1 \cap A_3| + |A_1 \cap A_2 \cap A_3| =$

(11) What is the set of integers from 0 through 999999 contain each of the digits 1, 2, 3 at least once?

Its cardinality is?

Ans.

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(10) $A_1 \cup A_2 \cup A_3 =$ the set of integers in U that **do not contain 1 or do not contain 2 or do not contain 3.**
$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_2 \cap A_3| - |A_1 \cap A_3| + |A_1 \cap A_2 \cap A_3| = 3 \times 9^6 - 3 \times 8^6 + 7^6$

(11) What is the set of integers from 0 through 999999 contain each of the digits 1, 2, 3 at least once?

Its cardinality is?

Ans.

4. How many integers from 1 through 999999 contain each of the digits 1, 2, 3 at least once?

(9) $\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}$ = the set of integers in U that **do not contain the digit 1 and do not contain 2 and do not contain 3.**
$|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}| = 7^6$

(10) $\overline{A_1} \cup \overline{A_2} \cup \overline{A_3}$ = the set of integers in U that **do not contain 1 or do not contain 2 or do not contain 3.**
$|\overline{A_1} \cup \overline{A_2} \cup \overline{A_3}| = |\overline{A_1}| + |\overline{A_2}| + |\overline{A_3}| - |\overline{A_1} \cap \overline{A_2}| - |\overline{A_2} \cap \overline{A_3}| - |\overline{A_1} \cap \overline{A_3}| + |\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}| = 3 \times 9^6 - 3 \times 8^6 + 7^6$

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$\overline{\overline{A_1} \cup \overline{A_2} \cup \overline{A_3}}$. Its cardinality is?
Ans.

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(11) What is the set of integers from 0 through 999999 contain each of the digits 1, 2, 3 at least once?
$\overline{A_1 \cup A_2 \cup A_3}$. Its cardinality is?
Ans. $10^6 - (3 \times 9^6 - 3 \times 8^6 + 7^6) = 74460$.

5. In how many ways can two distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is (a) even? (b) odd?

Idea. What is the sum of the following two integers, if

(1) both of them are odd;

(2) both of them are even;

(3) one is even, and one is odd

How many ways to find two distinct integers in $\{1, \ldots, 100\}$ such that

(1) both of them are odd;

(2) both of them are even;

(3) one is even, and one is odd

So the answer is

5. In how many ways can two distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is (a) even? (b) odd?

Idea. What is the sum of the following two integers, if

(1) both of them are odd; even

(2) both of them are even;

(3) one is even, and one is odd

How many ways to find two distinct integers in $\{1, \ldots, 100\}$ such that

(1) both of them are odd;

(2) both of them are even;

(3) one is even, and one is odd

So the answer is

5. In how many ways can two distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is (a) even? (b) odd?

Idea. What is the sum of the following two integers, if

(1) both of them are odd; even

(2) both of them are even; even

(3) one is even, and one is odd

How many ways to find two distinct integers in $\{1, \ldots, 100\}$ such that

(1) both of them are odd;

(2) both of them are even;

(3) one is even, and one is odd

So the answer is

5. In how many ways can two distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is (a) even? (b) odd?

Idea. What is the sum of the following two integers, if

(1) both of them are odd; even

(2) both of them are even; even

(3) one is even, and one is odd odd

How many ways to find two distinct integers in $\{1, \ldots, 100\}$ such that

(1) both of them are odd;

(2) both of them are even;

(3) one is even, and one is odd

So the answer is

5. In how many ways can two distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is (a) even? (b) odd?

Idea. What is the sum of the following two integers, if

(1) both of them are odd; even

(2) both of them are even; even

(3) one is even, and one is odd odd

How many ways to find two distinct integers in $\{1, \ldots, 100\}$ such that

(1) both of them are odd; $\binom{50}{2}$

(2) both of them are even;

(3) one is even, and one is odd

So the answer is

5. In how many ways can two distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is (a) even? (b) odd?

Idea. What is the sum of the following two integers, if

(1) both of them are odd; even

(2) both of them are even; even

(3) one is even, and one is odd odd

How many ways to find two distinct integers in $\{1, \ldots, 100\}$ such that

(1) both of them are odd; $\binom{50}{2}$

(2) both of them are even; $\binom{50}{2}$

(3) one is even, and one is odd

So the answer is

5. In how many ways can two distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is (a) even? (b) odd?

Idea. What is the sum of the following two integers, if

(1) both of them are odd; even

(2) both of them are even; even

(3) one is even, and one is odd odd

How many ways to find two distinct integers in $\{1, \ldots, 100\}$ such that

(1) both of them are odd; $\binom{50}{2}$

(2) both of them are even; $\binom{50}{2}$

(3) one is even, and one is odd 50×50

So the answer is

5. In how many ways can two distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is (a) even? (b) odd?

Idea. What is the sum of the following two integers, if

(1) both of them are odd; even

(2) both of them are even; even

(3) one is even, and one is odd odd

How many ways to find two distinct integers in $\{1, \ldots, 100\}$ such that

(1) both of them are odd; $\binom{50}{2}$

(2) both of them are even; $\binom{50}{2}$

(3) one is even, and one is odd 50×50

So the answer is

(a) $\binom{50}{2} + \binom{50}{2} = 2450$

5. In how many ways can two distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is (a) even? (b) odd?

Idea. What is the sum of the following two integers, if

(1) both of them are odd; even

(2) both of them are even; even

(3) one is even, and one is odd odd

How many ways to find two distinct integers in $\{1, \ldots, 100\}$ such that

(1) both of them are odd; $\binom{50}{2}$

(2) both of them are even; $\binom{50}{2}$

(3) one is even, and one is odd 50×50

So the answer is

(a) $\binom{50}{2} + \binom{50}{2} = 2450$

(b) $50 \times 50 = 2500$

Answer.

(a) Either both are odd or both are even: $\binom{50}{2} + \binom{50}{2} = 2450$

(b) one odd and one even: $\binom{50}{1} \times \binom{50}{1} = 2500.$

6. In how many ways can three, not necessarily distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is (a) even? (b) odd?

Idea. What is the sum of the following three integers, if

(1) all of them are odd;

(2) all of them are even;

(3) one is even, and the other two are odd;

(4) one is odd, and the other two are even

6. In how many ways can three, not necessarily distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is (a) even? (b) odd?
Idea. What is the sum of the following three integers, if

(1) all of them are odd; odd

(2) all of them are even;

(3) one is even, and the other two are odd;

(4) one is odd, and the other two are even

6. In how many ways can three, not necessarily distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is (a) even? (b) odd?
Idea. What is the sum of the following three integers, if

(1) all of them are odd; odd

(2) all of them are even; even

(3) one is even, and the other two are odd;

(4) one is odd, and the other two are even

6. In how many ways can three, not necessarily distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is (a) even? (b) odd?

Idea. What is the sum of the following three integers, if

(1) all of them are odd; odd

(2) all of them are even; even

(3) one is even, and the other two are odd; even

(4) one is odd, and the other two are even

6. In how many ways can three, not necessarily distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is (a) even? (b) odd?
Idea. What is the sum of the following three integers, if

(1) all of them are odd; odd

(2) all of them are even; even

(3) one is even, and the other two are odd; even

(4) one is odd, and the other two are even odd

How many ways to find three integers (repetition allowed) in $\{1, \ldots, 100\}$ such that

(1) all of them are odd;

(2) all of them are even;

(3) one is even, and the other two are odd;

(4) one is odd, and the other two are even

Summary. In how many ways can three, not necessarily distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is

(a) even?

(b) odd?

How many ways to find three integers (repetition allowed) in $\{1, \ldots, 100\}$ such that

(1) all of them are odd; $\binom{50+3-1}{3} = 22100$

(2) all of them are even;

(3) one is even, and the other two are odd;

(4) one is odd, and the other two are even

Summary. In how many ways can three, not necessarily distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is

(a) even?

(b) odd?

How many ways to find three integers (repetition allowed) in $\{1, \dots, 100\}$ such that

(1) all of them are odd; $\binom{50+3-1}{3} = 22100$

(2) all of them are even; $\binom{50+3-1}{3} = 22100$

(3) one is even, and the other two are odd;

(4) one is odd, and the other two are even

Summary. In how many ways can three, not necessarily distinct integers be chosen from $\{1, \dots, 100\}$ so that their sum is

(a) even?

(b) odd?

How many ways to find three integers (repetition allowed) in $\{1, \ldots, 100\}$ such that

(1) all of them are odd; $\binom{50+3-1}{3} = 22100$

(2) all of them are even; $\binom{50+3-1}{3} = 22100$

(3) one is even, and the other two are odd; $50 \times \binom{50+2-1}{2} = 63750$

(4) one is odd, and the other two are even

Summary. In how many ways can three, not necessarily distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is

(a) even?

(b) odd?

How many ways to find three integers (repetition allowed) in $\{1, \ldots, 100\}$ such that

(1) all of them are odd; $\binom{50+3-1}{3} = 22100$

(2) all of them are even; $\binom{50+3-1}{3} = 22100$

(3) one is even, and the other two are odd; $50 \times \binom{50+2-1}{2} = 63750$

(4) one is odd, and the other two are even $50 \times \binom{50+2-1}{2} = 63750$

Summary. In how many ways can three, not necessarily distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is

(a) even?

(b) odd?

How many ways to find three integers (repetition allowed) in $\{1, \ldots, 100\}$ such that

(1) all of them are odd; $\binom{50+3-1}{3} = 22100$

(2) all of them are even; $\binom{50+3-1}{3} = 22100$

(3) one is even, and the other two are odd; $50 \times \binom{50+2-1}{2} = 63750$

(4) one is odd, and the other two are even $50 \times \binom{50+2-1}{2} = 63750$

Summary. In how many ways can three, not necessarily distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is

(a) even? $22100 + 63750 = 85850$

(b) odd?

How many ways to find three integers (repetition allowed) in $\{1, \ldots, 100\}$ such that

(1) all of them are odd; $\binom{50+3-1}{3} = 22100$

(2) all of them are even; $\binom{50+3-1}{3} = 22100$

(3) one is even, and the other two are odd; $50 \times \binom{50+2-1}{2} = 63750$

(4) one is odd, and the other two are even $50 \times \binom{50+2-1}{2} = 63750$

Summary. In how many ways can three, not necessarily distinct integers be chosen from $\{1, \ldots, 100\}$ so that their sum is

(a) even? $22100 + 63750 = 85850$

(b) odd? $22100 + 63750 = 85850$

7. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 2, 3, 4\}$. How many onto functions $f : X \to Y$ are there?

Idea.

(1) What is an onto function?

(6) Ans.

7. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 2, 3, 4\}$. How many onto functions $f : X \to Y$ are there?

Idea.

(1) What is an onto function?
Every element in Y has at least one preimage.

(2) How many Preimages does one element of Y have?

(6) Ans.

7. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 2, 3, 4\}$. How many onto functions $f : X \to Y$ are there?

Idea.

(1) What is an onto function?
Every element in Y has at least one preimage.

(2) How many Preimages does one element of Y have?
One element in Y has exactly 2 preimages; other elements in Y has exactly 1 preimages.

(3) Step 1. How many ways are there to choose the element in Y with 2 preimages?

(6) Ans.

7. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 2, 3, 4\}$. How many onto functions $f : X \to Y$ are there?

Idea.

(1) What is an onto function?
Every element in Y has at least one preimage.

(2) How many Preimages does one element of Y have?
One element in Y has exactly 2 preimages; other elements in Y has exactly 1 preimages.

(3) Step 1. How many ways are there to choose the element in Y with 2 preimages? 4.

(4) Step 2. How many ways are there to choose preimages for the element in Y determined in Step 1?

(6) Ans.

7. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 2, 3, 4\}$. How many onto functions $f : X \to Y$ are there?

Idea.

(1) What is an onto function?
Every element in Y has at least one preimage.

(2) How many Preimages does one element of Y have?
One element in Y has exactly 2 preimages; other elements in Y has exactly 1 preimages.

(3) Step 1. How many ways are there to choose the element in Y with 2 preimages? 4.

(4) Step 2. How many ways are there to choose preimages for the element in Y determined in Step 1? $\binom{5}{2}$.

(6) Ans.

7. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 2, 3, 4\}$. How many onto functions $f : X \to Y$ are there?

Idea.

(1) What is an onto function?
Every element in Y has at least one preimage.

(2) How many Preimages does one element of Y have?
One element in Y has exactly 2 preimages; other elements in Y has exactly 1 preimages.

(3) Step 1. How many ways are there to choose the element in Y with 2 preimages? 4.

(4) Step 2. How many ways are there to choose preimages for the element in Y determined in Step 1? $\binom{5}{2}$.

(5) Step 3. How many ways are there to choose preimages for other elements in Y?

(6) Ans.

7. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 2, 3, 4\}$. How many onto functions $f : X \to Y$ are there?

Idea.

(1) What is an onto function?
Every element in Y has at least one preimage.

(2) How many Preimages does one element of Y have?
One element in Y has exactly 2 preimages; other elements in Y has exactly 1 preimages.

(3) Step 1. How many ways are there to choose the element in Y with 2 preimages? 4.

(4) Step 2. How many ways are there to choose preimages for the element in Y determined in Step 1? $\binom{5}{2}$.

(5) Step 3. How many ways are there to choose preimages for other elements in Y? $3 \times 2 \times 1 = 3!$.

(6) Ans.

7. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 2, 3, 4\}$. How many onto functions $f : X \to Y$ are there?

Idea.

(1) What is an onto function?
Every element in Y has at least one preimage.

(2) How many Preimages does one element of Y have?
One element in Y has exactly 2 preimages; other elements in Y has exactly 1 preimages.

(3) Step 1. How many ways are there to choose the element in Y with 2 preimages? 4.

(4) Step 2. How many ways are there to choose preimages for the element in Y determined in Step 1? $\binom{5}{2}$.

(5) Step 3. How many ways are there to choose preimages for other elements in Y? $3 \times 2 \times 1 = 3!$.

(6) Ans. $4 \times \binom{5}{2} \times 3! = 240.$

8. In how many ways can 5 integers be chosen from $1, 2 \ldots, 100$ so that no two are consecutive?

(1) Let A be a subset of $\{1, 2 \ldots, 100\}$ such that A has 5 integers and no two in A are consecutive.

(2) Represent A by a bit string.

(3) How many such bit strings? I.e. how many without consecutive 1 bits?

(4) How many such A's?
Ans.

8. In how many ways can 5 integers be chosen from $1, 2 \ldots, 100$ so that no two are consecutive?

(1) Let A be a subset of $\{1, 2 \ldots, 100\}$ such that A has 5 integers and no two in A are consecutive.

(2) Represent A by a bit string.

(3) How many such bit strings? I.e. how many without consecutive 1 bits? $\binom{95+1}{5}$

(4) How many such A's?
Ans.

8. In how many ways can 5 integers be chosen from $1, 2 \ldots, 100$ so that no two are consecutive?

(1) Let A be a subset of $\{1, 2 \ldots, 100\}$ such that A has 5 integers and no two in A are consecutive.

(2) Represent A by a bit string.

(3) How many such bit strings? I.e. how many without consecutive 1 bits? $\binom{95+1}{5}$

(4) How many such A's?
Ans. $\binom{96}{5} = 61124064$

9. How many integers from 1 through 1000 are:

(a) multiples 2 or multiples of 9?

(b) neither multiples 2 nor multiples of 9?

Recall

✍ There are $\lfloor n/k \rfloor$ many integers in $[1, n]$ are multiple of k.

Idea. Let U: integers from 1 through 1000, A: multiples of 2, B: multiples of 9.

Then $A \cap B$: , i.e.

$A \cup B$:

$\overline{A \cup B} = U - A \cup B$:

$|A| =$

$|B| =$

$|A \cap B| =$

$|A \cup B| =$

$|U| =$

$|\overline{A \cup B}| =$.

9. How many integers from 1 through 1000 are:

(a) multiples 2 or multiples of 9?

(b) neither multiples 2 nor multiples of 9?

Recall

✍ There are $\lfloor n/k \rfloor$ many integers in $[1, n]$ are multiple of k.

Idea. Let U: integers from 1 through 1000, A: multiples of 2, B: multiples of 9.

Then $A \cap B$: multiples of both 2 and 9, i.e.

$A \cup B$:

$\overline{A \cup B} = U - A \cup B$:

$|A| =$

$|B| =$

$|A \cap B| =$

$|A \cup B| =$

$|U| =$

$|\overline{A \cup B}| =$.

9. How many integers from 1 through 1000 are:

(a) multiples 2 or multiples of 9?

(b) neither multiples 2 nor multiples of 9?

Recall

✍ There are $\lfloor n/k \rfloor$ many integers in $[1, n]$ are multiple of k.

Idea. Let U: integers from 1 through 1000, A: multiples of 2, B: multiples of 9.
Then $A \cap B$: multiples of both 2 and 9, i.e. multiples of 18
$A \cup B$:
$\overline{A \cup B} = U - A \cup B$:
$|A| =$
$|B| =$
$|A \cap B| =$
$|A \cup B| =$
$|U| =$
$|\overline{A \cup B}| =$.

9. How many integers from 1 through 1000 are:

(a) multiples 2 or multiples of 9?

(b) neither multiples 2 nor multiples of 9?

Recall

✍ There are $\lfloor n/k \rfloor$ many integers in $[1, n]$ are multiple of k.

Idea. Let U: integers from 1 through 1000, A: multiples of 2, B: multiples of 9.
Then $A \cap B$: multiples of both 2 and 9, i.e. multiples of 18
$A \cup B$: multiples of 2 or 9
$\overline{A \cup B} = U - A \cup B$:
$|A| =$
$|B| =$
$|A \cap B| =$
$|A \cup B| =$
$|U| =$
$|\overline{A \cup B}| =$.

9. How many integers from 1 through 1000 are:

(a) multiples 2 or multiples of 9?

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✍ There are $\lfloor n/k \rfloor$ many integers in $[1, n]$ are multiple of k.

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$|A| = \lfloor 1000/2 \rfloor =$
$|B| =$
$|A \cap B| =$
$|A \cup B| =$
$|U| =$
$|\overline{A \cup B}| =$.

9. How many integers from 1 through 1000 are:

(a) multiples 2 or multiples of 9?

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Recall

✍ There are $\lfloor n/k \rfloor$ many integers in $[1, n]$ are multiple of k.

Idea. Let U: integers from 1 through 1000, A: multiples of 2, B: multiples of 9.

Then $A \cap B$: multiples of both 2 and 9, i.e. multiples of 18

$A \cup B$: multiples of 2 or 9

$\overline{A \cup B} = U - A \cup B$: neither multiples 2 nor multiples of 9

$|A| = \lfloor 1000/2 \rfloor = 500$

$|B| =$

$|A \cap B| =$

$|A \cup B| =$

$|U| =$

$|\overline{A \cup B}| =$.

9. How many integers from 1 through 1000 are:

(a) multiples 2 or multiples of 9?

(b) neither multiples 2 nor multiples of 9?

Recall

✍ There are $\lfloor n/k \rfloor$ many integers in $[1, n]$ are multiple of k.

Idea. Let U: integers from 1 through 1000, A: multiples of 2, B: multiples of 9.

Then $A \cap B$: multiples of both 2 and 9, i.e. multiples of 18

$A \cup B$: multiples of 2 or 9

$\overline{A \cup B} = U - A \cup B$: neither multiples 2 nor multiples of 9

$|A| = \lfloor 1000/2 \rfloor = 500$

$|B| = \lfloor 1000/9 \rfloor =$

$|A \cap B| =$

$|A \cup B| =$

$|U| =$

$|\overline{A \cup B}| =$.

9. How many integers from 1 through 1000 are:

(a) multiples 2 or multiples of 9?

(b) neither multiples 2 nor multiples of 9?

Recall

✍ There are $\lfloor n/k \rfloor$ many integers in $[1, n]$ are multiple of k.

Idea. Let U: integers from 1 through 1000, A: multiples of 2, B: multiples of 9.
Then $A \cap B$: multiples of both 2 and 9, i.e. multiples of 18
$A \cup B$: multiples of 2 or 9
$\overline{A \cup B} = U - A \cup B$: neither multiples 2 nor multiples of 9
$|A| = \lfloor 1000/2 \rfloor = 500$
$|B| = \lfloor 1000/9 \rfloor = 111$
$|A \cap B| =$
$|A \cup B| =$
$|U| =$
$|\overline{A \cup B}| =$.

9. How many integers from 1 through 1000 are:

(a) multiples 2 or multiples of 9?

(b) neither multiples 2 nor multiples of 9?

Recall

✍ There are $\lfloor n/k \rfloor$ many integers in $[1, n]$ are multiple of k.

Idea. Let U: integers from 1 through 1000, A: multiples of 2, B: multiples of 9.
Then $A \cap B$: multiples of both 2 and 9, i.e. multiples of 18
$A \cup B$: multiples of 2 or 9
$\overline{A \cup B} = U - A \cup B$: neither multiples 2 nor multiples of 9
$|A| = \lfloor 1000/2 \rfloor = 500$
$|B| = \lfloor 1000/9 \rfloor = 111$
$|A \cap B| = \lfloor 1000/18 \rfloor =$
$|A \cup B| =$
$|U| =$
$|\overline{A \cup B}| =$.

9. How many integers from 1 through 1000 are:

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Idea. Let U: integers from 1 through 1000, A: multiples of 2, B: multiples of 9.
Then $A \cap B$: multiples of both 2 and 9, i.e. multiples of 18
$A \cup B$: multiples of 2 or 9
$\overline{A \cup B} = U - A \cup B$: neither multiples 2 nor multiples of 9
$|A| = \lfloor 1000/2 \rfloor = 500$
$|B| = \lfloor 1000/9 \rfloor = 111$
$|A \cap B| = \lfloor 1000/18 \rfloor = 55$
$|A \cup B| =$
$|U| =$
$|\overline{A \cup B}| =$.

9. How many integers from 1 through 1000 are:

(a) multiples 2 or multiples of 9?

(b) neither multiples 2 nor multiples of 9?

Recall

✍ There are $\lfloor n/k \rfloor$ many integers in $[1, n]$ are multiple of k.

Idea. Let U: integers from 1 through 1000, A: multiples of 2, B: multiples of 9.
Then $A \cap B$: multiples of both 2 and 9, i.e. multiples of 18
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$|A| = \lfloor 1000/2 \rfloor = 500$
$|B| = \lfloor 1000/9 \rfloor = 111$
$|A \cap B| = \lfloor 1000/18 \rfloor = 55$
$|A \cup B| = |A| + |B| - |A \cap B| =$
$|U| =$
$|\overline{A \cup B}| =$.

9. How many integers from 1 through 1000 are:

(a) multiples 2 or multiples of 9?

(b) neither multiples 2 nor multiples of 9?

Recall

✍ There are $\lfloor n/k \rfloor$ many integers in $[1, n]$ are multiple of k.

Idea. Let U: integers from 1 through 1000, A: multiples of 2, B: multiples of 9.
Then $A \cap B$: multiples of both 2 and 9, i.e. multiples of 18
$A \cup B$: multiples of 2 or 9
$\overline{A \cup B} = U - A \cup B$: neither multiples 2 nor multiples of 9
$|A| = \lfloor 1000/2 \rfloor = 500$
$|B| = \lfloor 1000/9 \rfloor = 111$
$|A \cap B| = \lfloor 1000/18 \rfloor = 55$
$|A \cup B| = |A| + |B| - |A \cap B| = 556$
$|U| =$
$|\overline{A \cup B}| =$.

9. How many integers from 1 through 1000 are:

(a) multiples 2 or multiples of 9?

(b) neither multiples 2 nor multiples of 9?

Recall

✍ There are $\lfloor n/k \rfloor$ many integers in $[1, n]$ are multiple of k.

Idea. Let U: integers from 1 through 1000, A: multiples of 2, B: multiples of 9.
Then $A \cap B$: multiples of both 2 and 9, i.e. multiples of 18
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$|A| = \lfloor 1000/2 \rfloor = 500$
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$|A \cap B| = \lfloor 1000/18 \rfloor = 55$
$|A \cup B| = |A| + |B| - |A \cap B| = 556$
$|U| = 1000$
$|\overline{A \cup B}| =$.

9. How many integers from 1 through 1000 are:

(a) multiples 2 or multiples of 9?

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✍ There are $\lfloor n/k \rfloor$ many integers in $[1, n]$ are multiple of k.

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Then $A \cap B$: multiples of both 2 and 9, i.e. multiples of 18
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$|A| = \lfloor 1000/2 \rfloor = 500$
$|B| = \lfloor 1000/9 \rfloor = 111$
$|A \cap B| = \lfloor 1000/18 \rfloor = 55$
$|A \cup B| = |A| + |B| - |A \cap B| = 556$
$|U| = 1000$
$|\overline{A \cup B}| = |U| - |A \cup B| =$.

9. How many integers from 1 through 1000 are:

(a) multiples 2 or multiples of 9?

(b) neither multiples 2 nor multiples of 9?

Recall

✍ There are $\lfloor n/k \rfloor$ many integers in $[1, n]$ are multiple of k.

Idea. Let U: integers from 1 through 1000, A: multiples of 2, B: multiples of 9.

Then $A \cap B$: multiples of both 2 and 9, i.e. multiples of 18

$A \cup B$: multiples of 2 or 9

$\overline{A \cup B} = U - A \cup B$: neither multiples 2 nor multiples of 9

$|A| = \lfloor 1000/2 \rfloor = 500$

$|B| = \lfloor 1000/9 \rfloor = 111$

$|A \cap B| = \lfloor 1000/18 \rfloor = 55$

$|A \cup B| = |A| + |B| - |A \cap B| = 556$

$|U| = 1000$

$|\overline{A \cup B}| = |U| - |A \cup B| = 444.$