LECTURE 7: BINARY SEARCH TREES

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ADMINISTRATIVE ISSUES: PROBLEM SET 2

Start 2019-09-08 15:59 UTC **Problem Set 2** **End** 2019-09-23 16:00 UTC

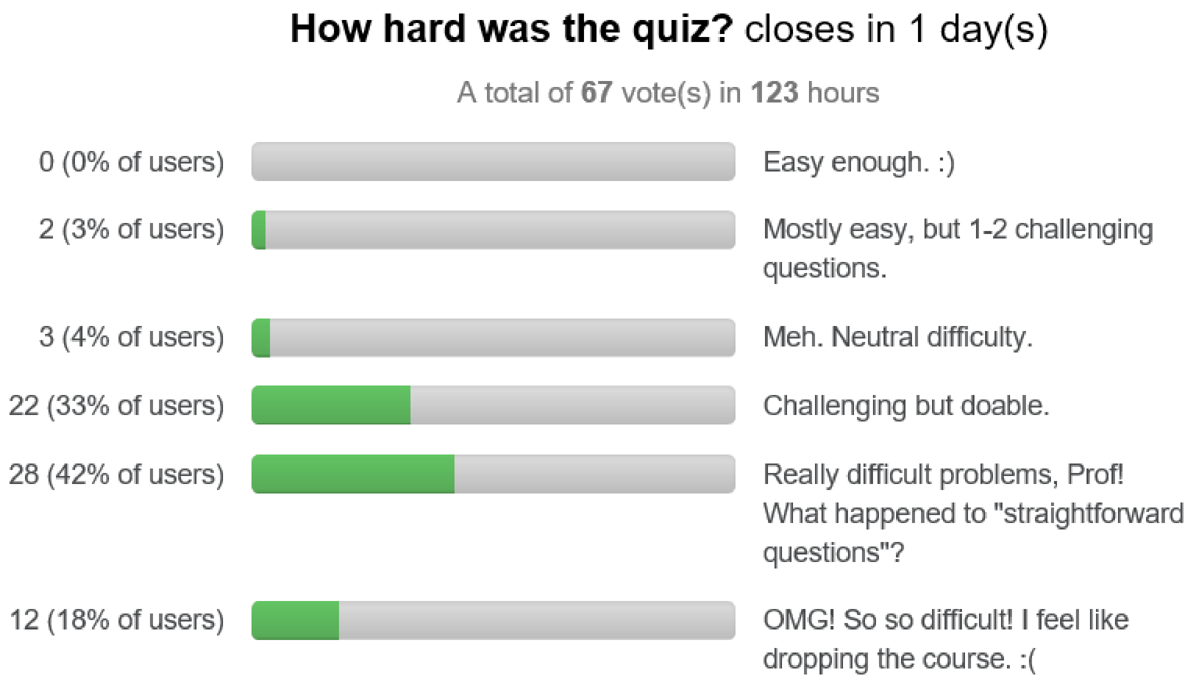
Time elapsed 14:19:35 **Time remaining** 345:40:26

- Deadline is Monday 23rd 23:59:19
- 3 graded problems
  - Kattis Quest
  - Cookie Selection
  - GCPC
- 2 challenge (ungraded) problems
  - BST
  - Shortsell

ADMINISTRATIVE ISSUES

Quiz 1

- Still grading
- "Absorbed material" means knowing how to apply and change alg/data structure to match problem.



QUIZ 1: SOME COMMON ISSUES

Problem 1 (Asymptotic Analysis):

- Problem 1c is a bonus. (requires induction to prove)
- Practice needed with Big-O
- We will organize a special class

Problem 2 (RPN) and **Problem 3** (Quicksort):

- Overly complex algorithms
- We will focus on associations and problem-solving techniques

General:

- Do not need to write too much.
  - Trying to smoke through will probably fail. :P
- Don’t expect us to “extrapolate” your answer.

LEARNING OUTCOMES

By the end of this session, students should be able to:

- Describe the **Binary Search Tree (BST)** and its operations
- Analyze **performance of BST operations**
- Relate the **importance of balance in a BST** to enable efficient operations.
- Explain the **pre-order, post-order, and in-order tree traversal** algorithms.

PROBLEM: POVERTY IDENTIFICATION

The Stop-Poverty charity calls:

To provide financial aid, Help identify families:

- earning exactly $a amount
- earning less than $a amount
- earning more than $a amount

What operations would we need to perform?

THE ORDERED DICTIONARY ADT

What Data Structures can we use to implement the Dictionary ADT?

Operations with k = key, v = value:

`insert(k, v)`: inserts an element with value v and key k

`search(k)`: returns the value with key k

`delete(k)`: deletes the element with key k

`contains(k)`: true if the dictionary contains an element with key k

`floor(k)`: returns next key $\leq$ k

`ceiling(k)`: returns next key $\geq$ k

`size()`: returns the size of the dictionary

*also called "Ordered Symbol Tables"

EXPLOITING STRUCTURE

The more "ordered" your structure:

- the more prior information you can exploit to speed up certain operations.

But: you will likely have to pay to maintain this order.

- some operations (e.g., that change the data) will become more expensive

EXPLOITING STRUCTURE

Unsorted array

30	70	12	50	6

Insert: O(1)
Max: O(n)
Search: O(n)

Sorted array

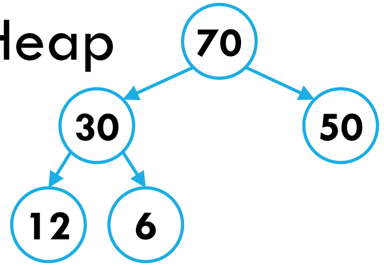
6	12	30	50	70

Insert: O(n)
Max: O(1)
Search: O(logn)

Unordered **Ordered**

EXPLOITING STRUCTURE

Binary Heap



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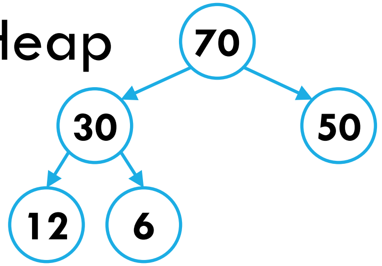

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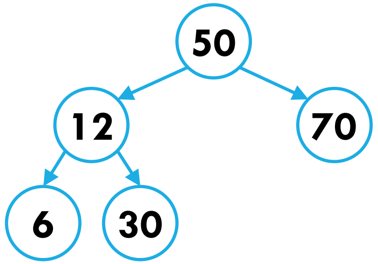
Balanced Binary **Search** Tree



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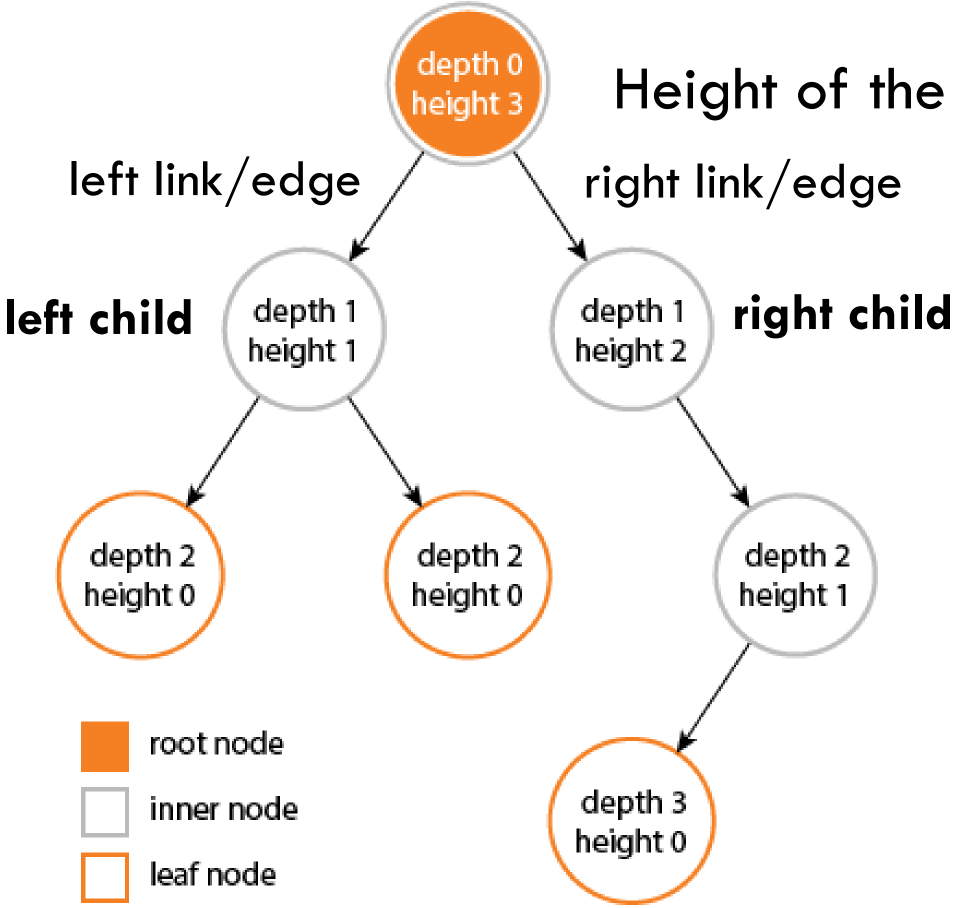
A BINARY TREE



Height of the tree = depth of its deepest node

Height of a node: number of edges on the longest path from node to a leaf.

Depth of a node: number of edges to the root

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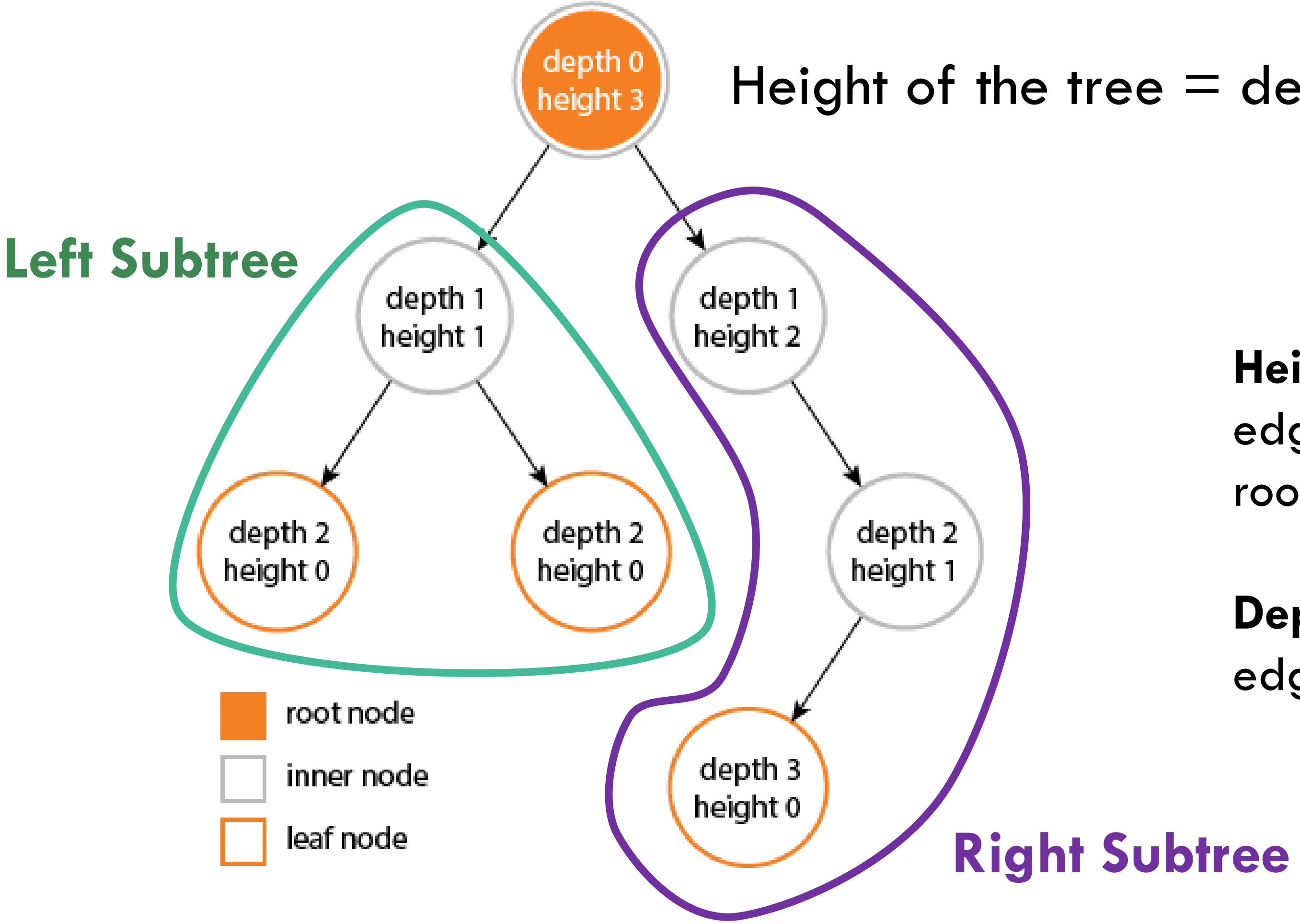

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Height of a tree: number of edges on the longest path from root to a leaf.

Depth of a node: number of edges to the root

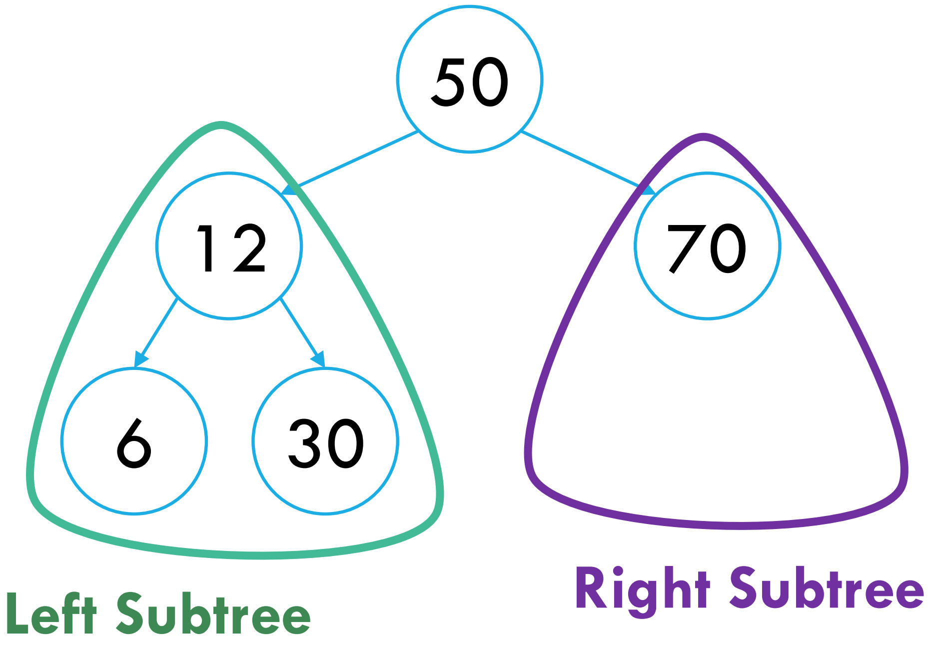
THE BINARY TREE JAVA CLASS

```
public class BinaryTree<Key extends Comparable<Key>, Value> {

      private BinaryTree<Key, Value> m_leftTree;
      private BinaryTree<Key, Value> m_rightTree;
      private BinaryTree<Key, Value> m_parent;


      private Key m_key;
      private Value m_value;

      // Remainder of binary tree implementation
}
```

BINARY SEARCH TREES

Binary Search Tree Properties:

- A node's left subtree only contains nodes with keys strictly less than the node's key
- A node's right subtree only contains nodes with keys strictly larger than the node's key
- The left and right subtrees are binary trees.
- All keys belong to a total order*.



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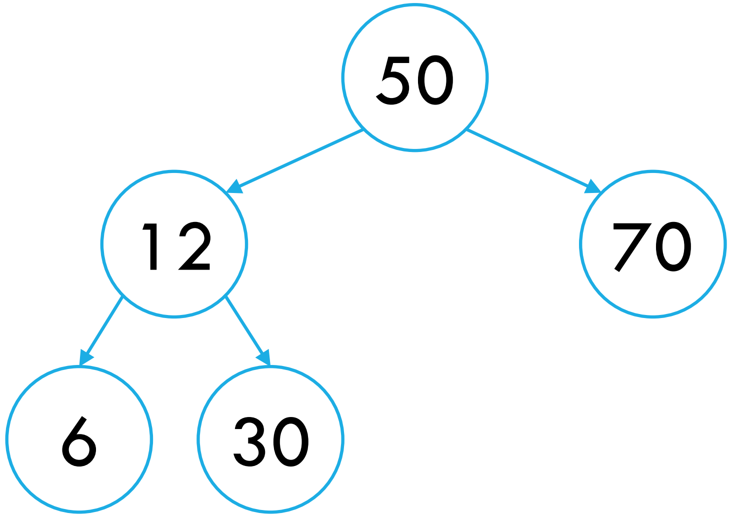

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C. Maybe…

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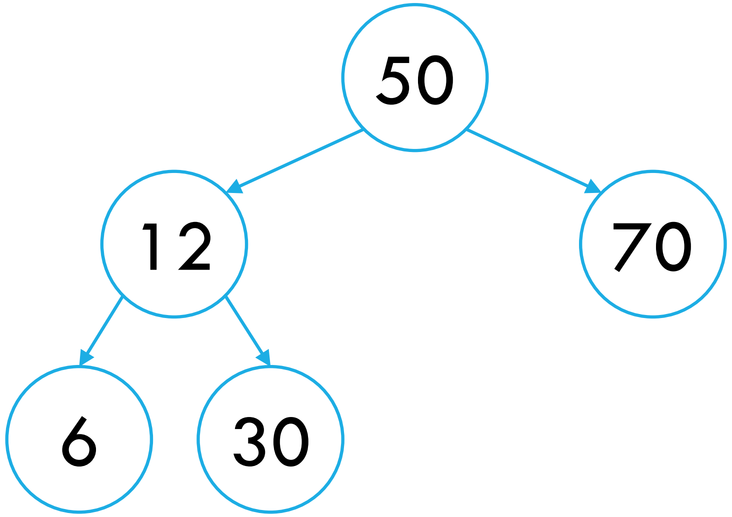

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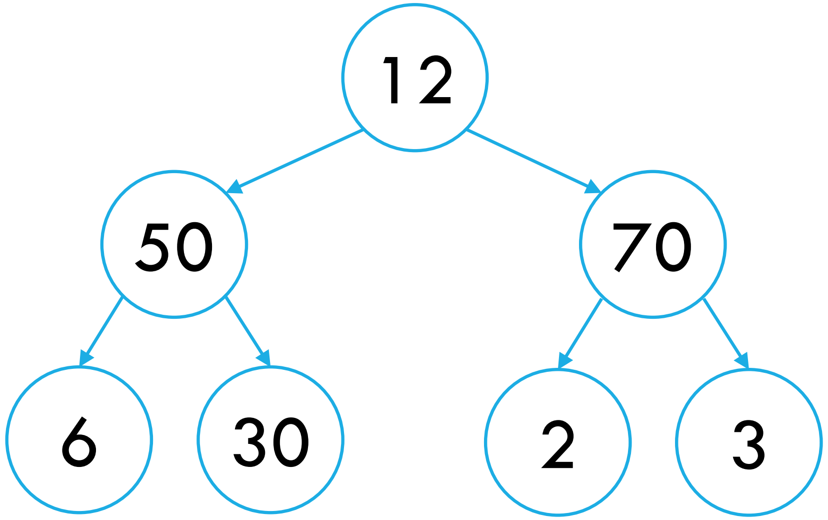

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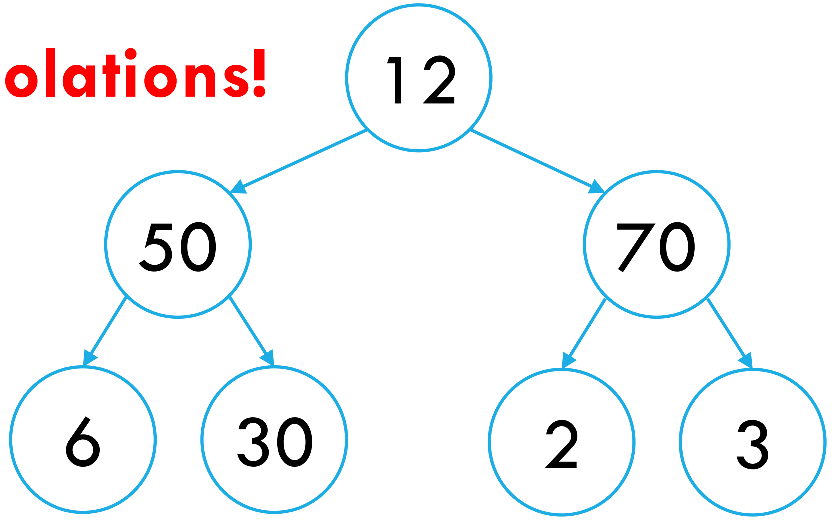
Lots of violations!



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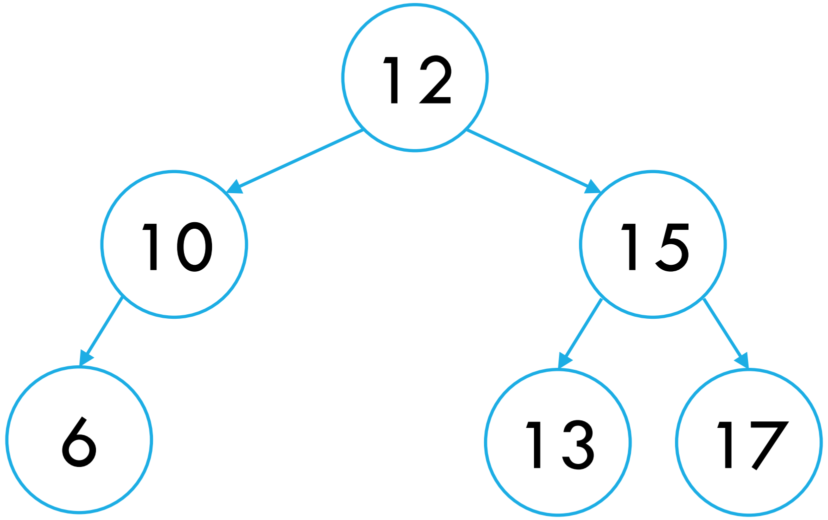

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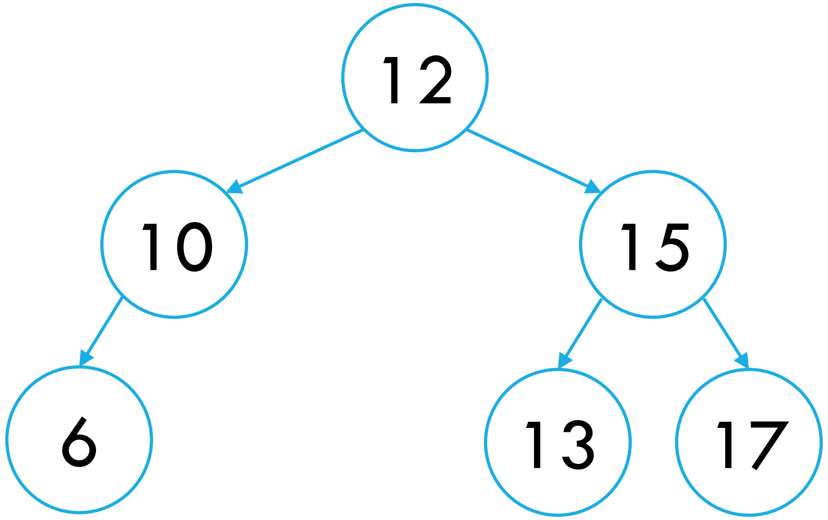

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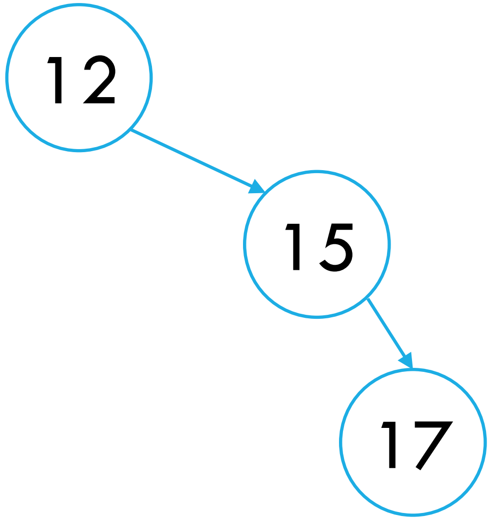

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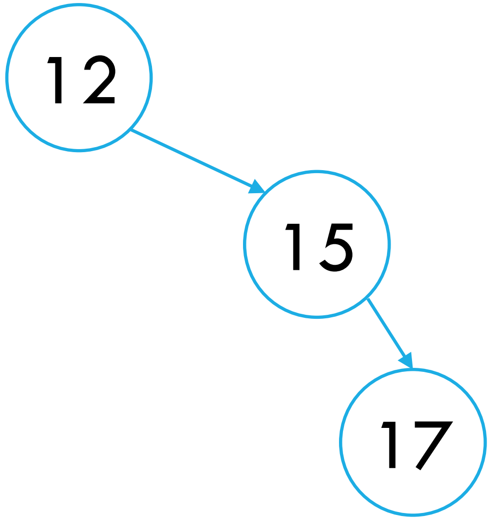

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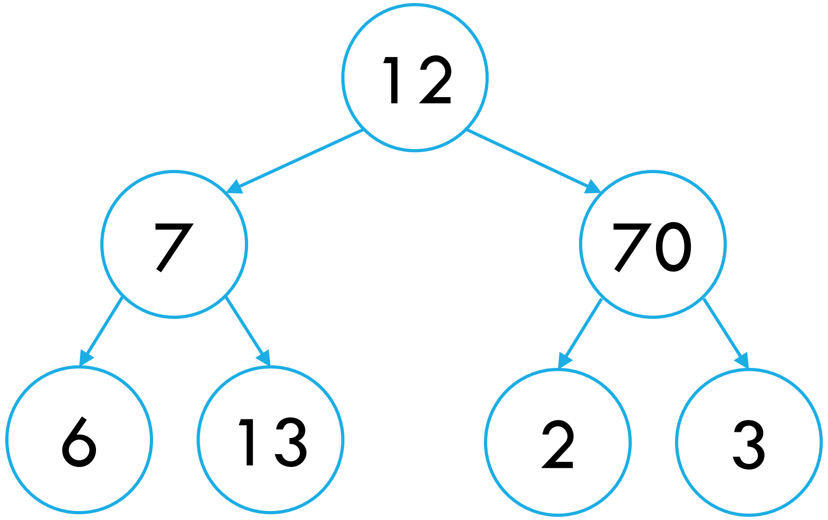

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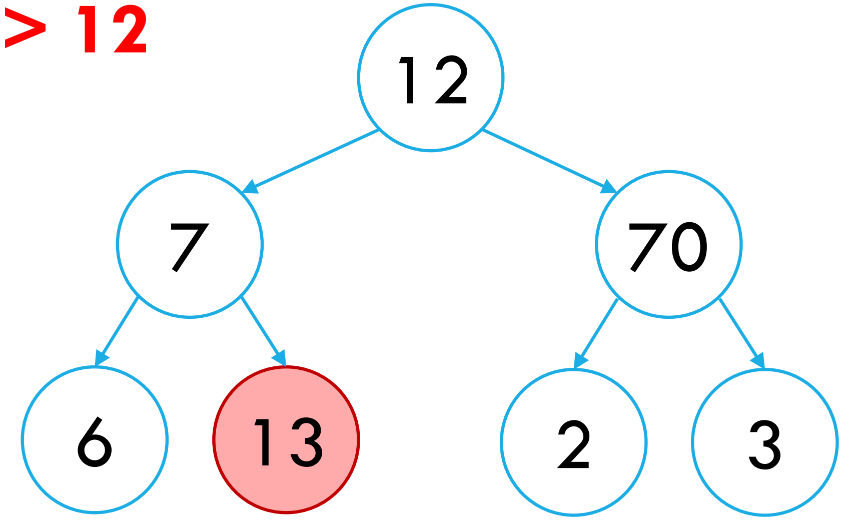
13 > 12



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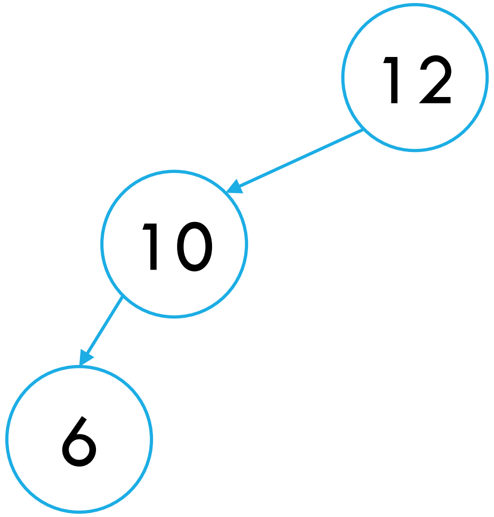

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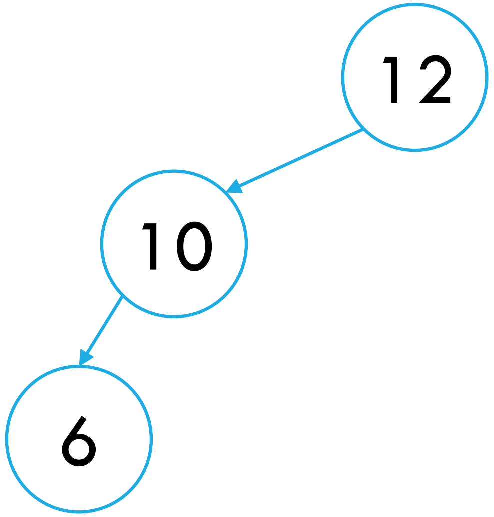

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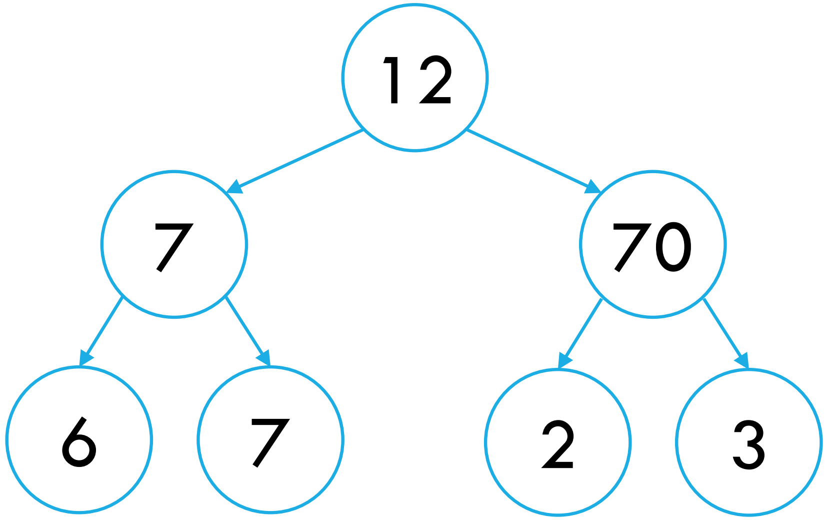

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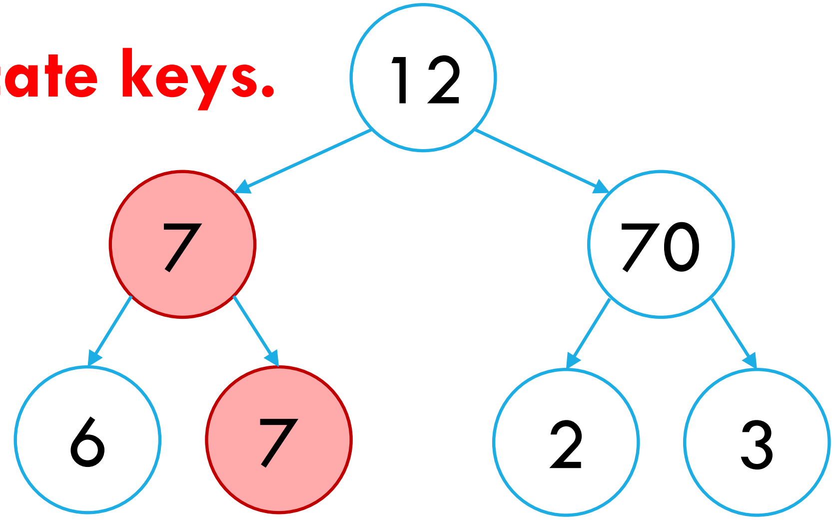
BINARY SEARCH TREES

no duplicate keys.



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BINARY SEARCH TREES (BST): BASIC OPERATIONS

`height()`: returns the height of the node

`search(k)` : returns an object v with key k

`searchMin()` : finds the minimum key k

`searchMax()` : finds the maximum key k

`successor()`: returns next key > k

`predecessor()`: returns next key < k

`insert(k,v)`: inserts an object v with key k

`delete(k)` : deletes the node with key k

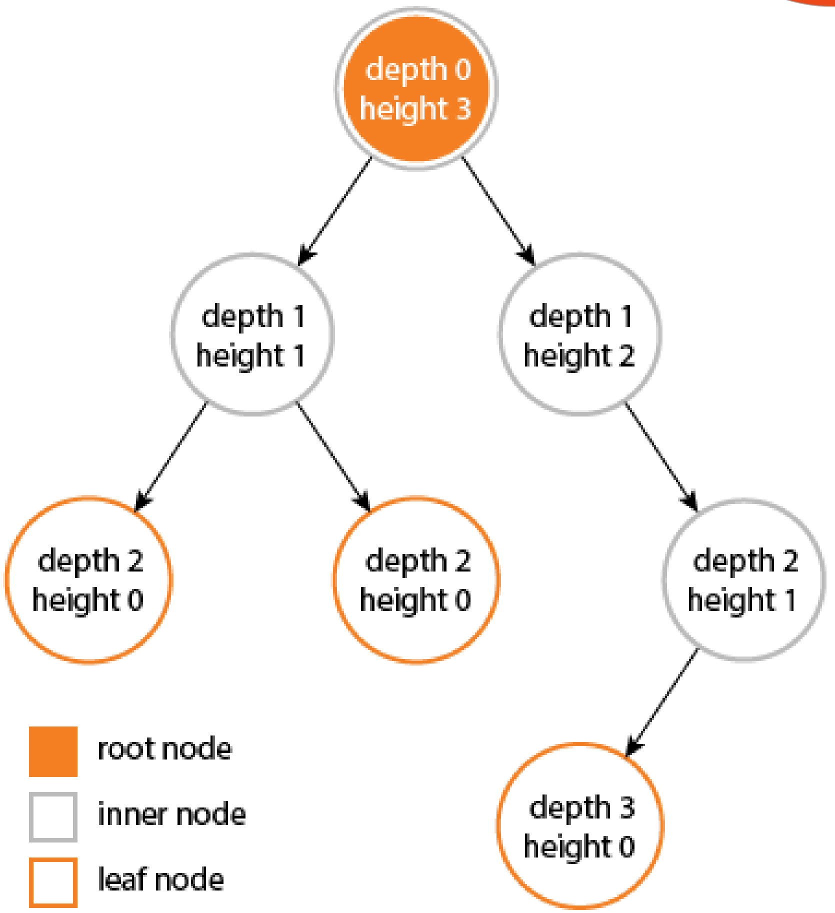
HEIGHT OF A BST

Number of edges on longest path from root to leaf

or Depth of the tree's deepest node

How do we compute it?

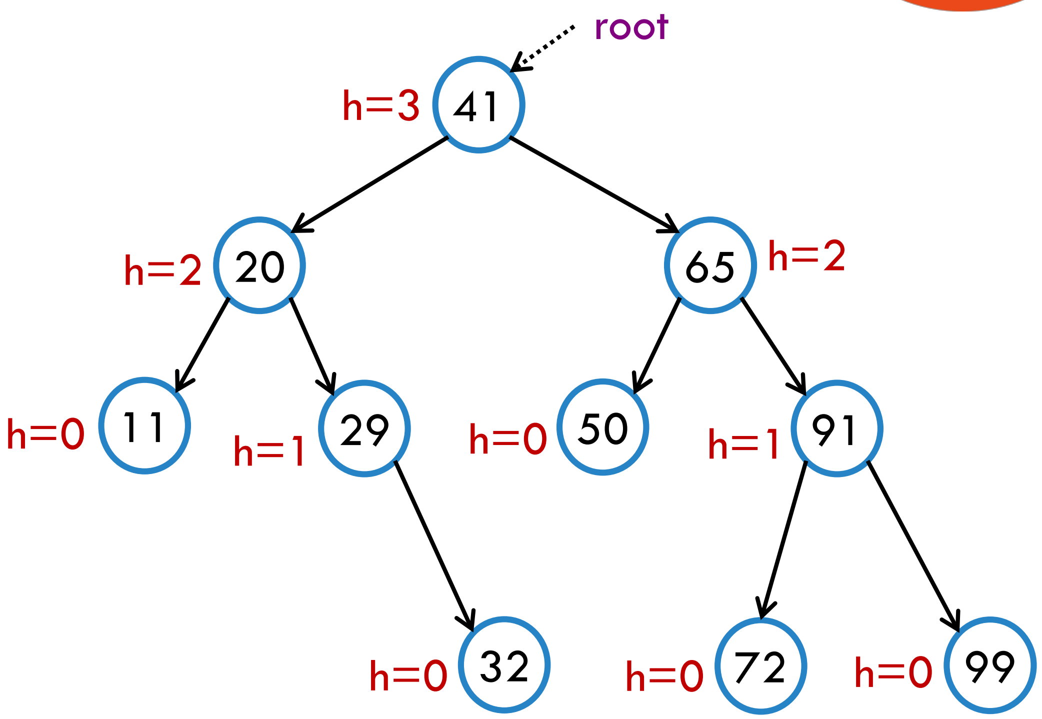
$$h(o) = 0 \text{ if } o \text{ is a leaf}$$
$$h(o) = \max\big(h(o.\text{left}), h(o.\text{right})\big) + 1$$



HEIGHT OF A BST: PSEUDOCODE

Call root.height()

```
function height()
    leftHeight = -1
    rightHeight = -1
    if m_leftTree is not null
        leftHeight = m_leftTree.height()
    if m_rightTree is not null
        rightHeight = m_rightTree.height()
    return max(leftHeight, rightHeight) + 1
```



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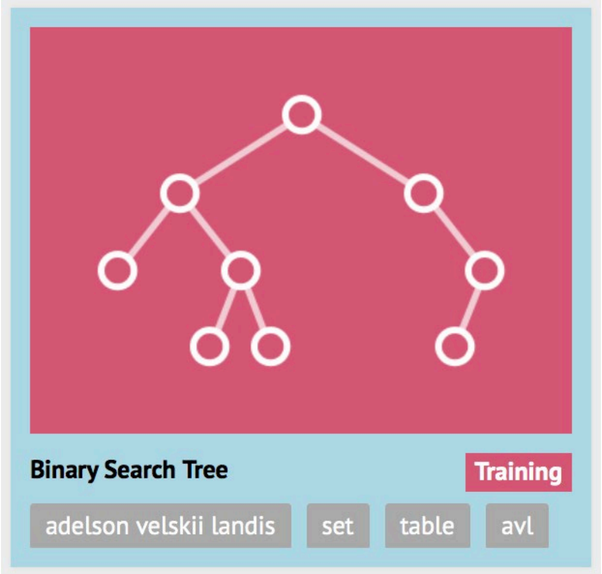
covered in Visualgo!



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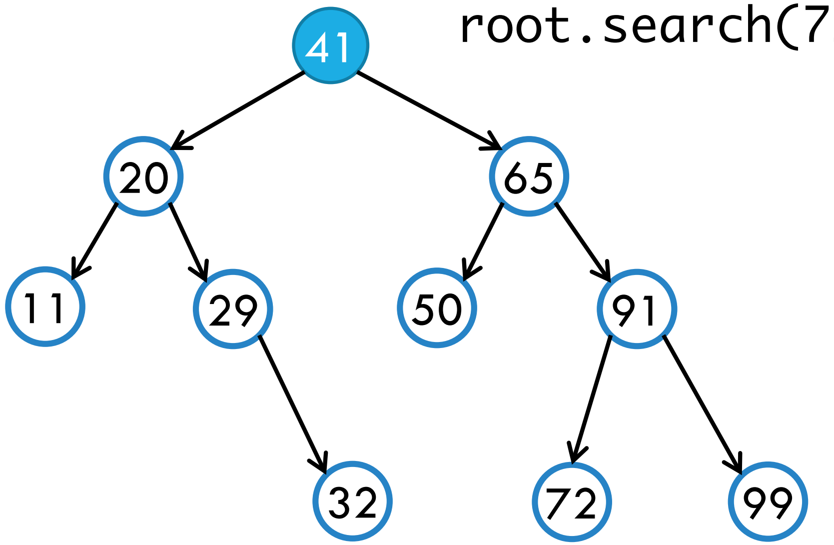
BST SEARCHING

Idea: We **binary search** inside a BST.

Given key k that we want to find and current node curr

Call `curr.search(k)`




```
function search(Key k)
  if k < m_key


  else if k > m_key


  return this
```

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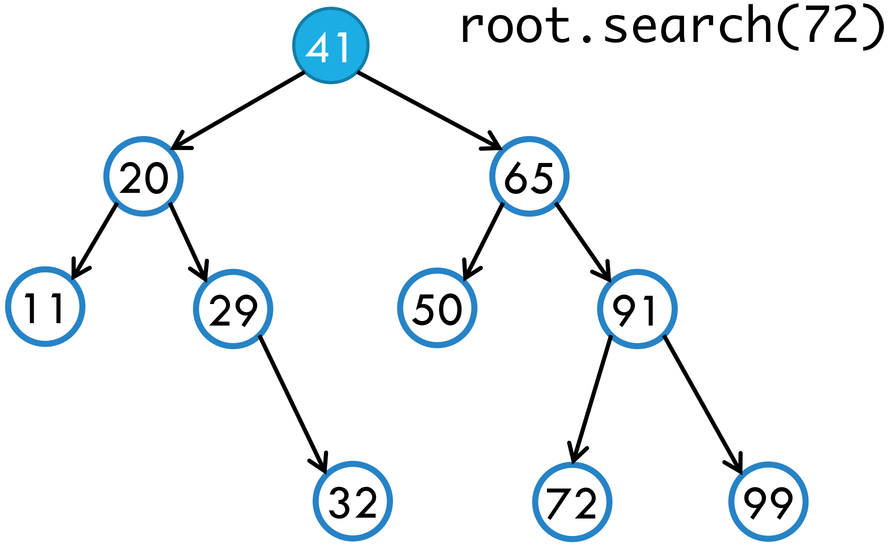


```
function search(Key k)
  if k < m_key


  else if k > m_key

      return m_rightTree.search(k)


  return this
```

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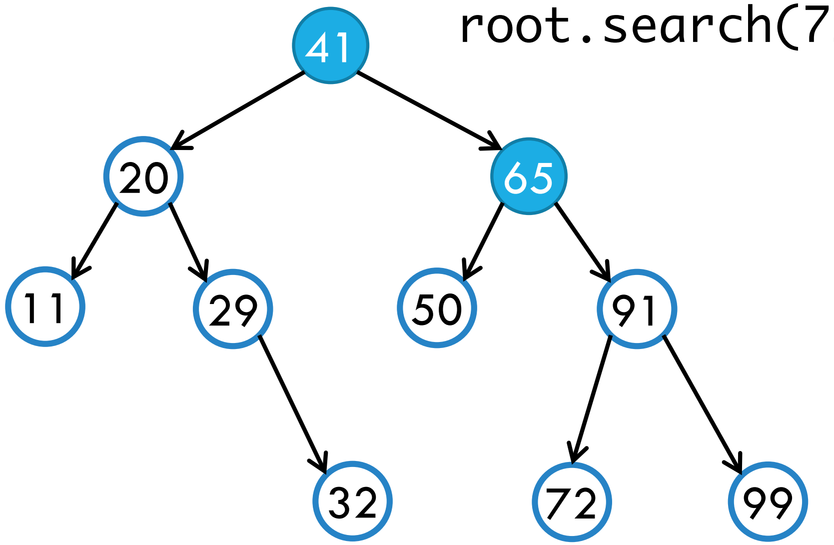


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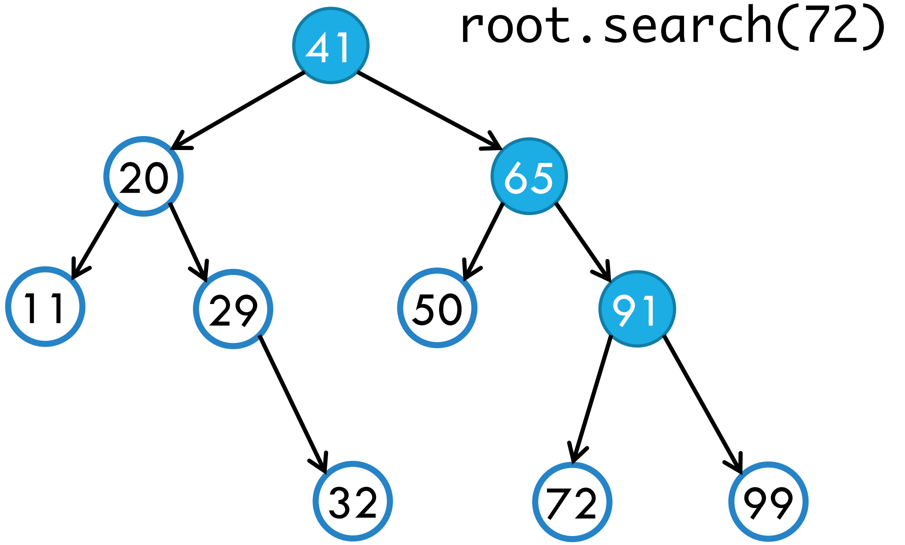


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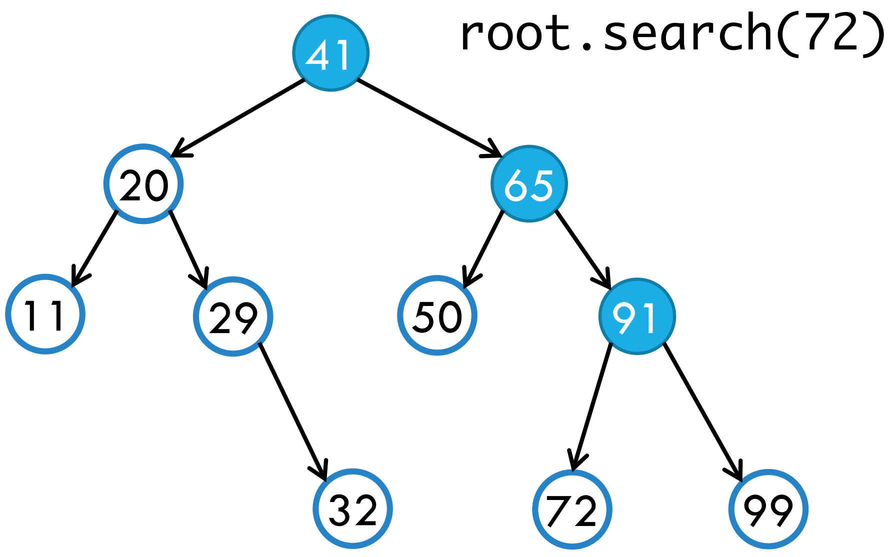


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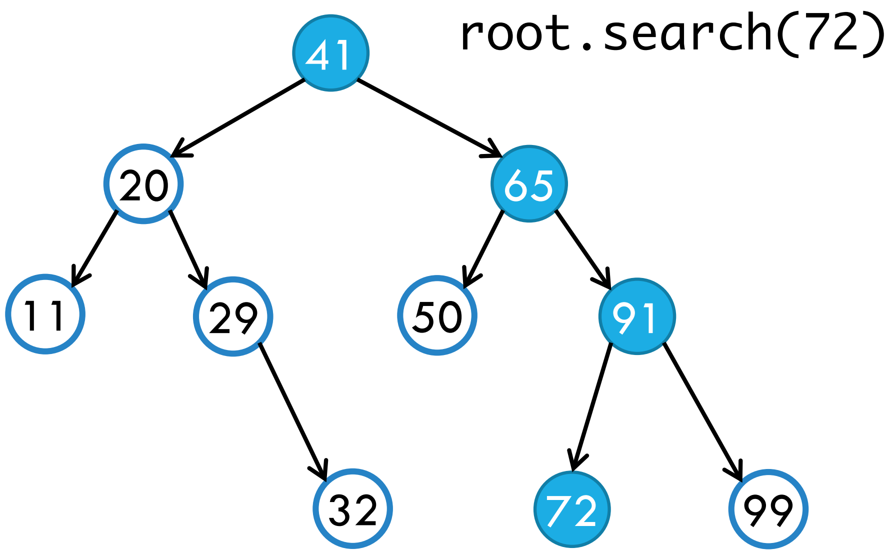
Is the code correct?

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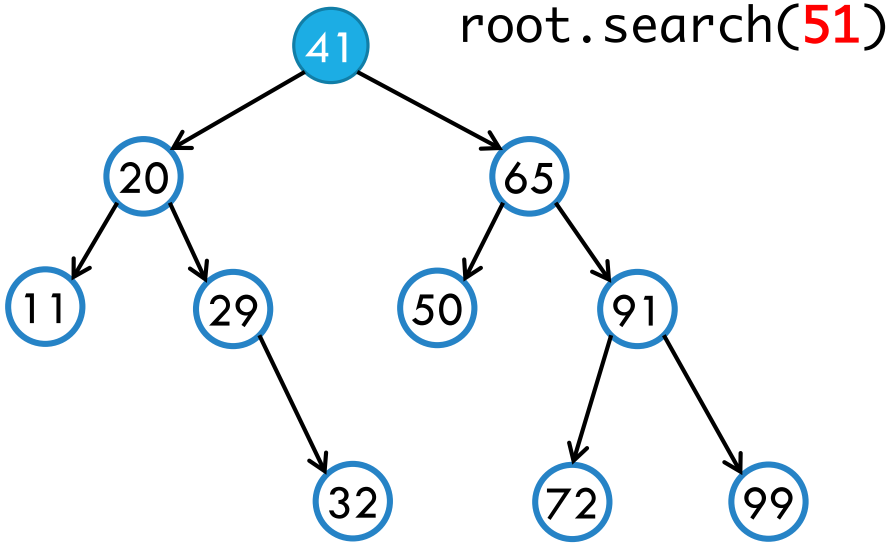


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      return m_rightTree.search(k)

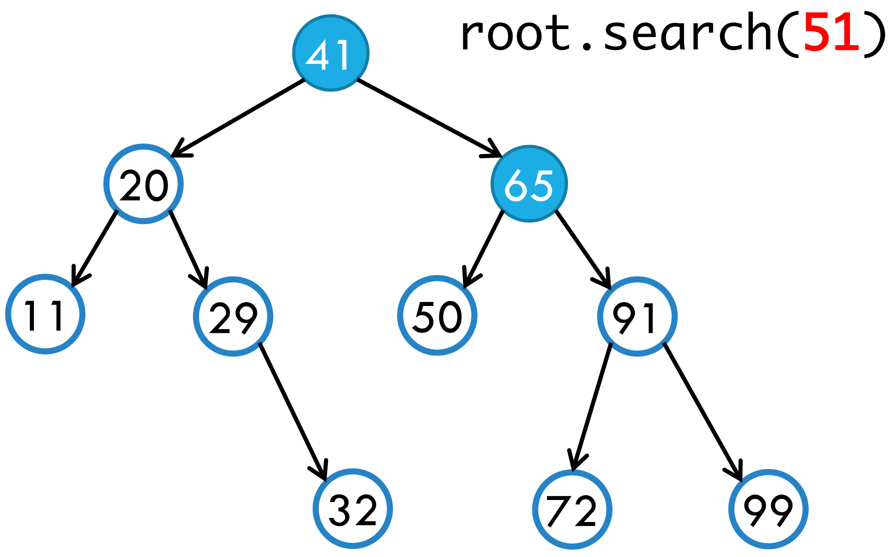

  return this
```

BST SEARCHING

Idea: We **binary search** inside a BST.

Given key k that we want to find and current node curr

Call `curr.search(k)`

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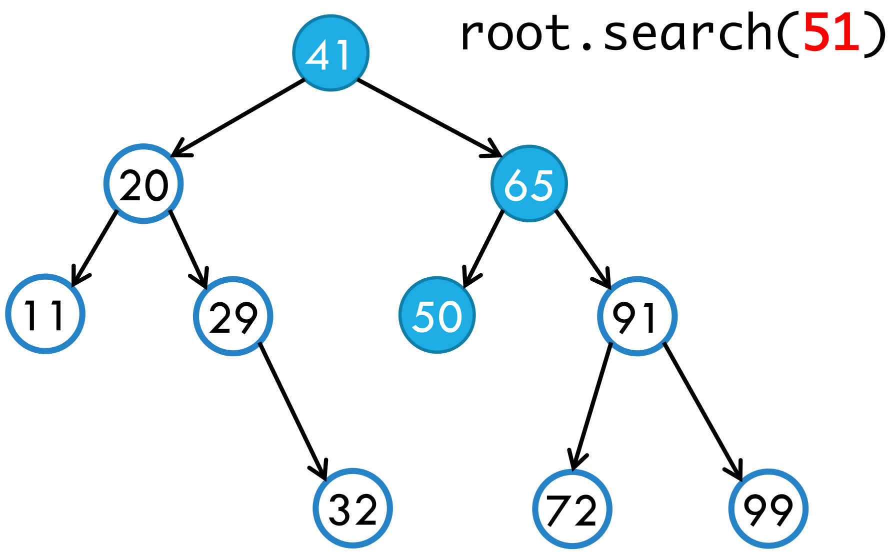


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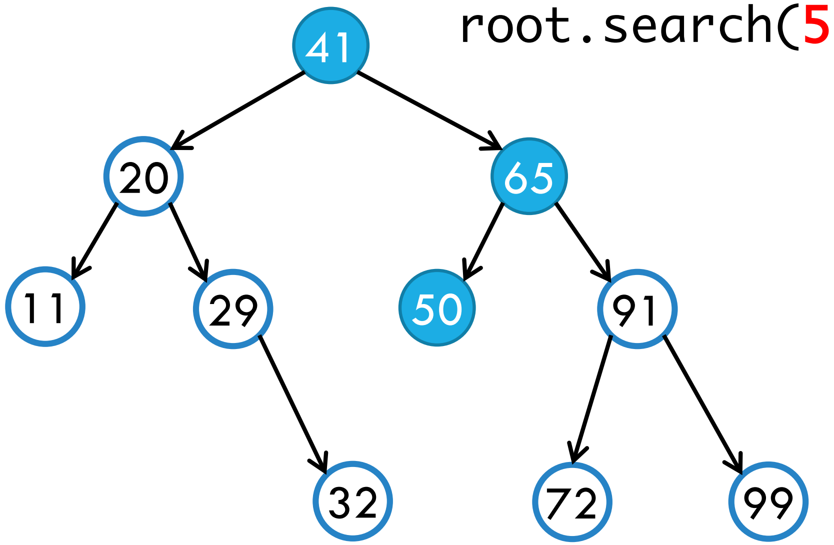
root.search(51)

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    else return null
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  return this
```

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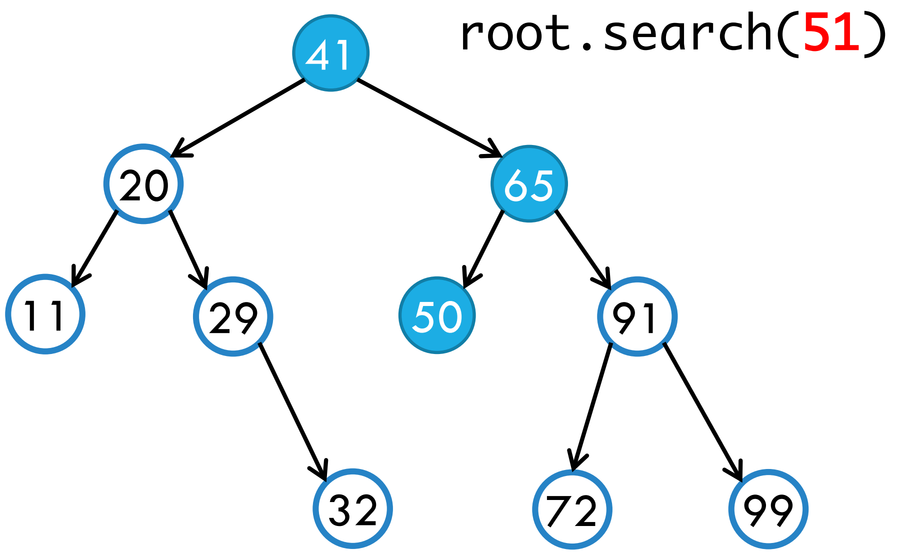


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  return this
```

BINARY SEARCH TREES (BST): BASIC OPERATIONS

`height()`: returns the height of the node

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`searchMax()` : finds the maximum key k

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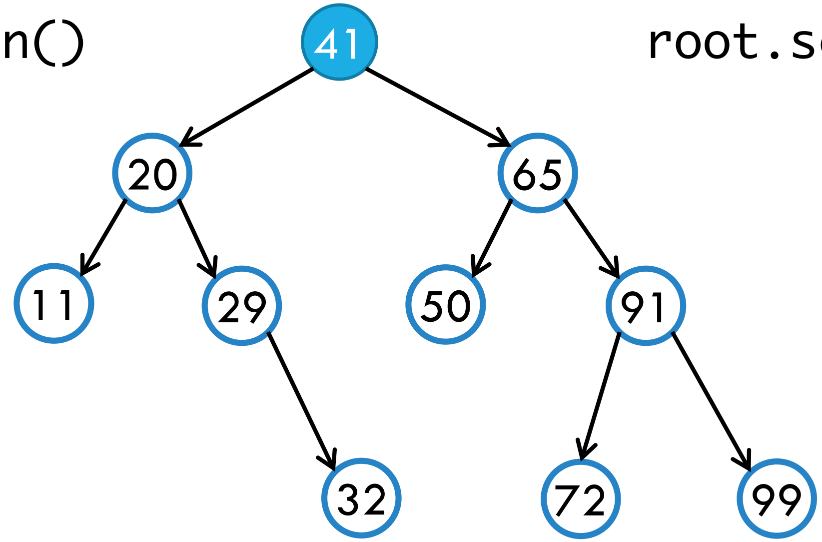
SEARCHING FOR THE MINIMUM & MAXIMUM KEYS

Question: Where is the minimum key located?

Question: Where is the maximum key located?

`root.searchMin()`



`root.searchMax()`

SEARCHING FOR THE MINIMUM & MAXIMUM KEYS

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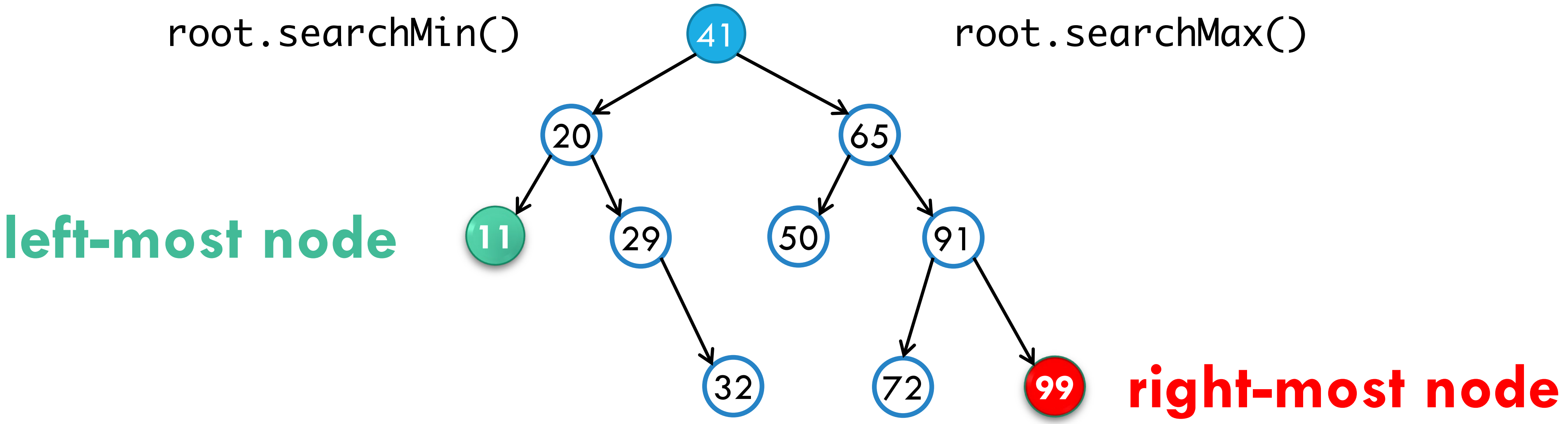
Question: Where is the maximum key located?



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`root.searchMin()`

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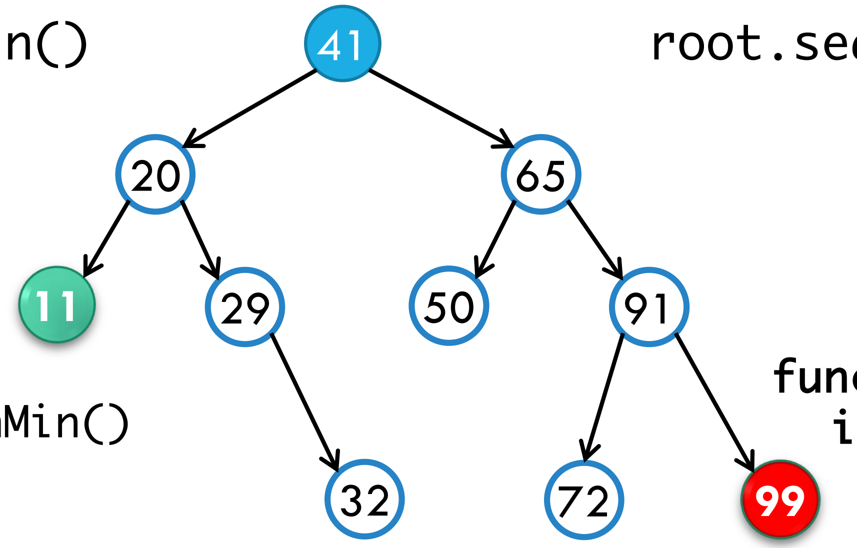


```
function searchMin()
  if m_leftTree is not null
    return m_leftTree.searchMin()

  return this
```

```
function searchMax()
  if m_rightTree is not null
    return m_rightTree.searchMax()

  return this
```

Poll Everywhere

https://bit.ly/2LvG9bq

SEARCHING: WHICH COSTS THE MOST?

In a standard BST, which of the following has the highest worst-case time complexity?

A. searchMin()
B. searchMax()
C. search(k)
D. Both A & B
E. They are all the same.

Poll Everywhere
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A. searchMin()
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$O(h)$ where h is the height of the BST

Poll Everywhere

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SEARCHING: WHICH COSTS THE MOST?

In a standard BST with *n* nodes, what is the worst case height?

A. $O(n)$

B. $O(\log n)$

C. $O(\lfloor \log n \rfloor)$

D. $O(n^2)$

E. 

Poll Everywhere

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SEARCHING: WHICH COSTS THE MOST?

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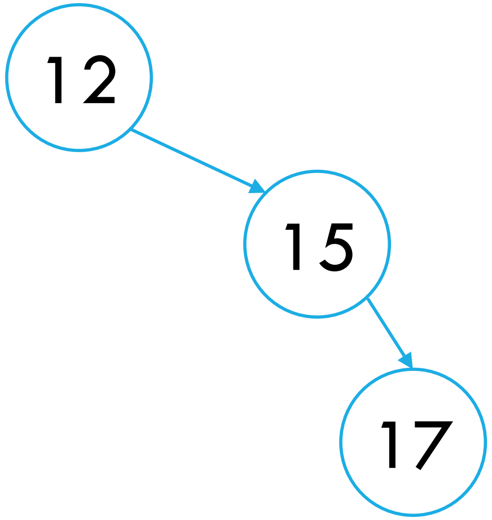
BSTs can be lop-sided and form a chain.



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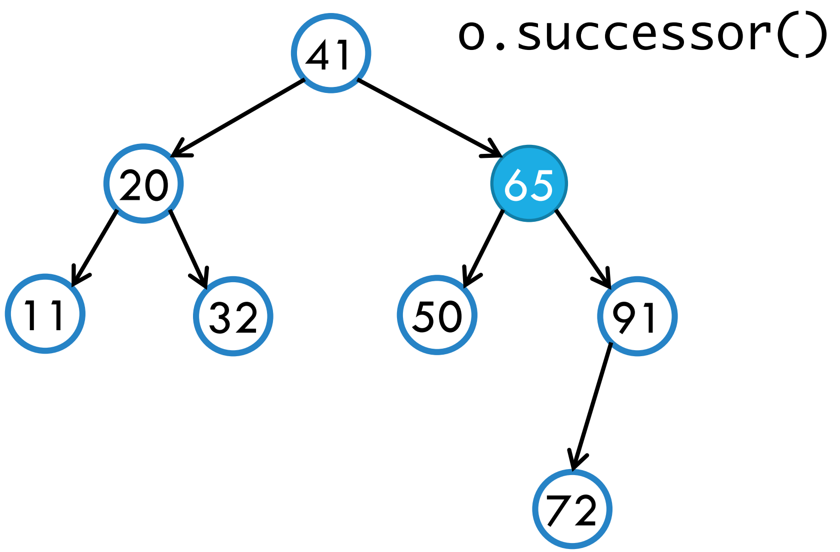
FINDING THE SUCCESSOR NODE

Given a node o with key k:

o's successor = the node with the smallest key that is larger than k



o is the node with key 65. Which of the following is the key of o's successor node?

A. 41
B. 91
C. 72
D. 50
E. Impossible to determine

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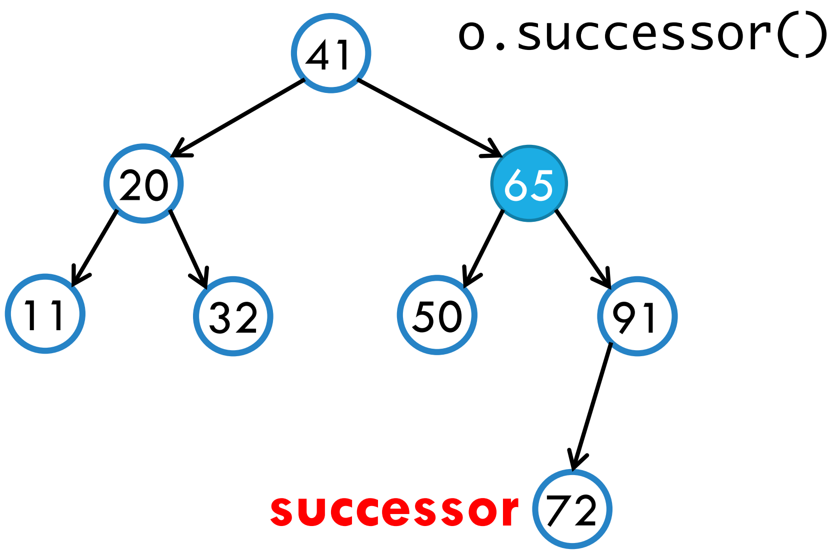
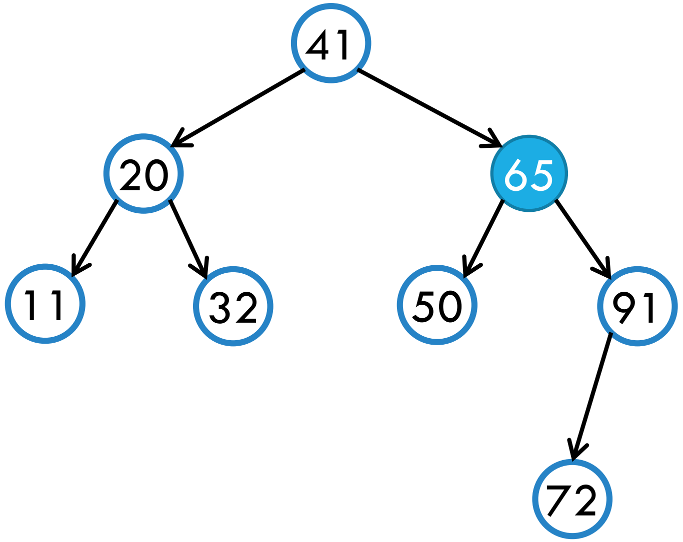


FINDING THE SUCCESSOR NODE

Given a node o, there are 2 cases:

- it has a right subtree
    -
- it doesn't have a right subtree



FINDING THE SUCCESSOR NODE

Given a node o, there are 2 cases:

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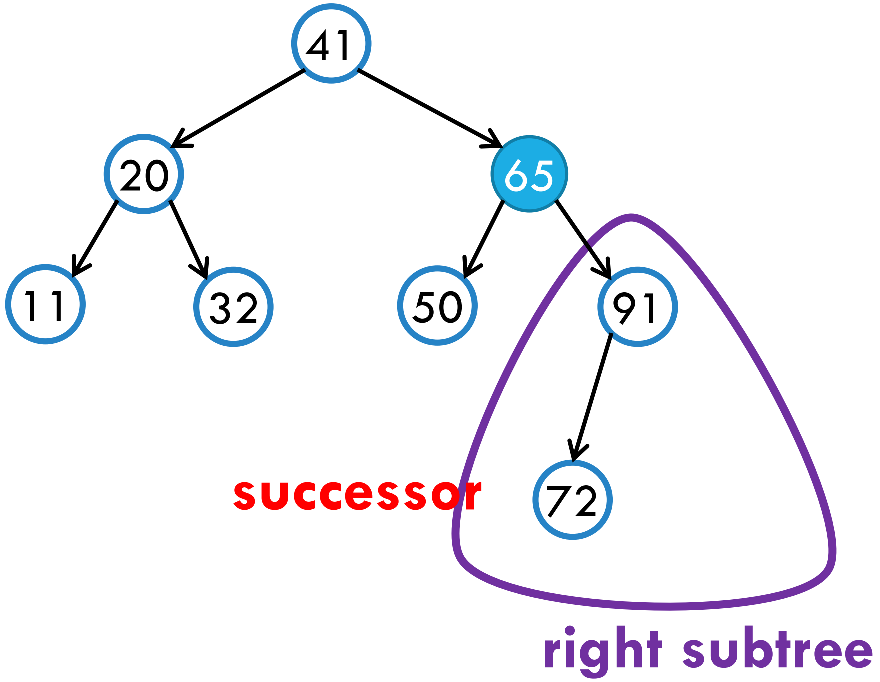
  - **return searchMin(o.m_rightTree)**
- it doesn't have a right subtree



FINDING THE SUCCESSOR NODE

Given a node o, there are 2 cases:

- it has a right subtree
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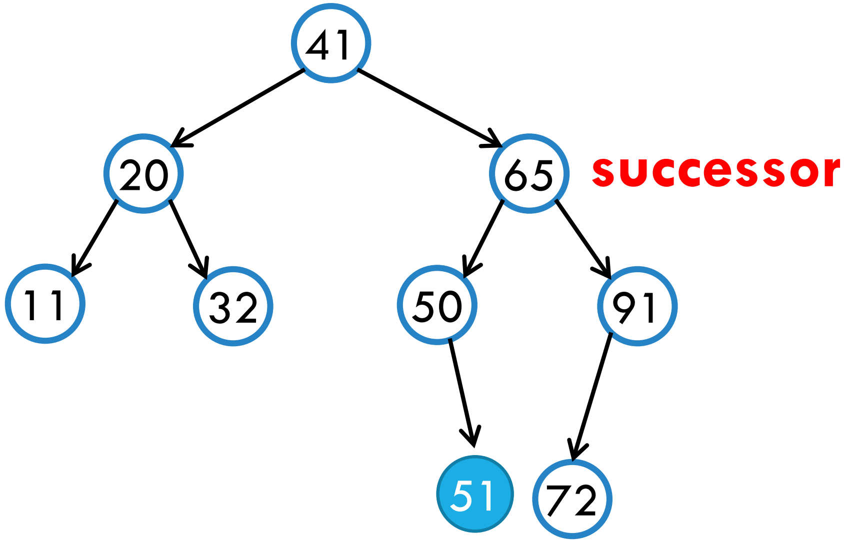
- it doesn't have a right subtree
  - **Where is the successor?**



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Given a node o, there are 2 cases:

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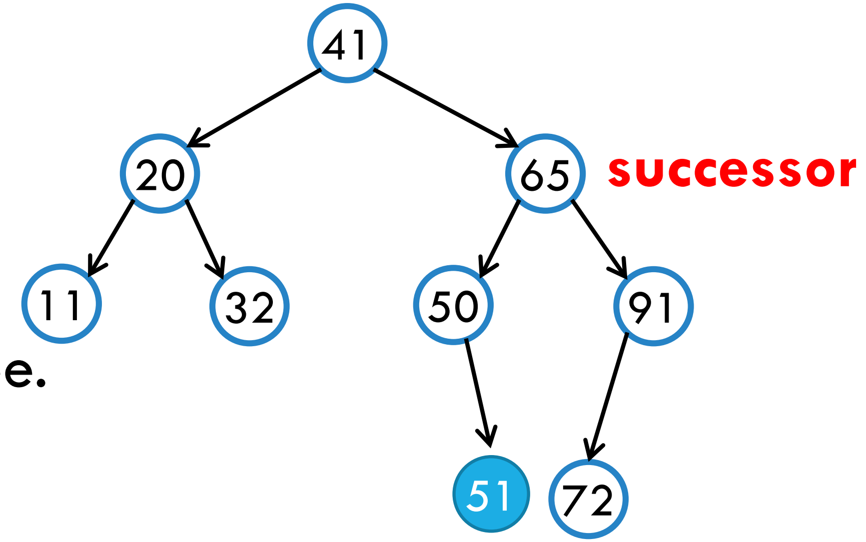
  - 2 cases:
    - o is the left child: ______________
    - o is the right child: ______________



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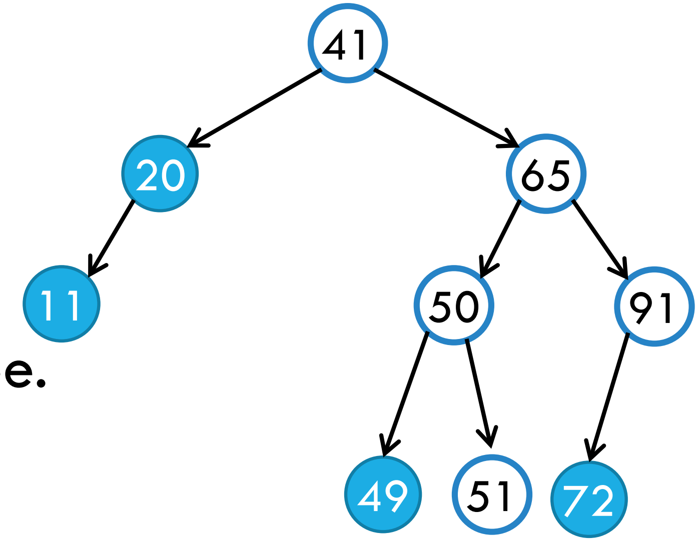

do you see a pattern?

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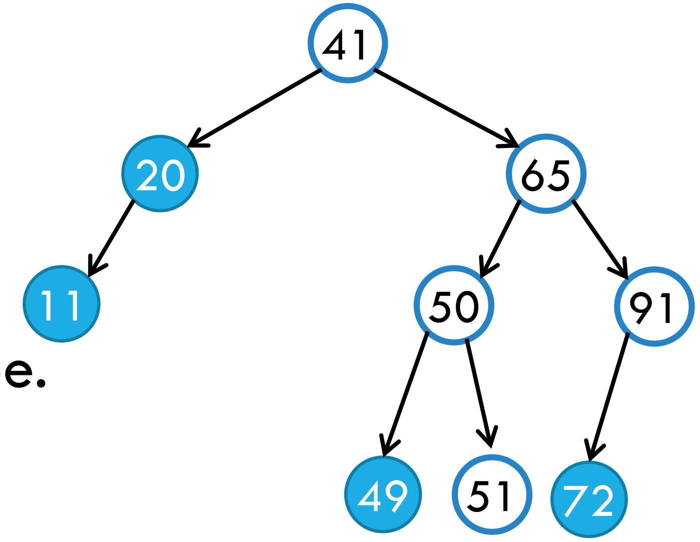

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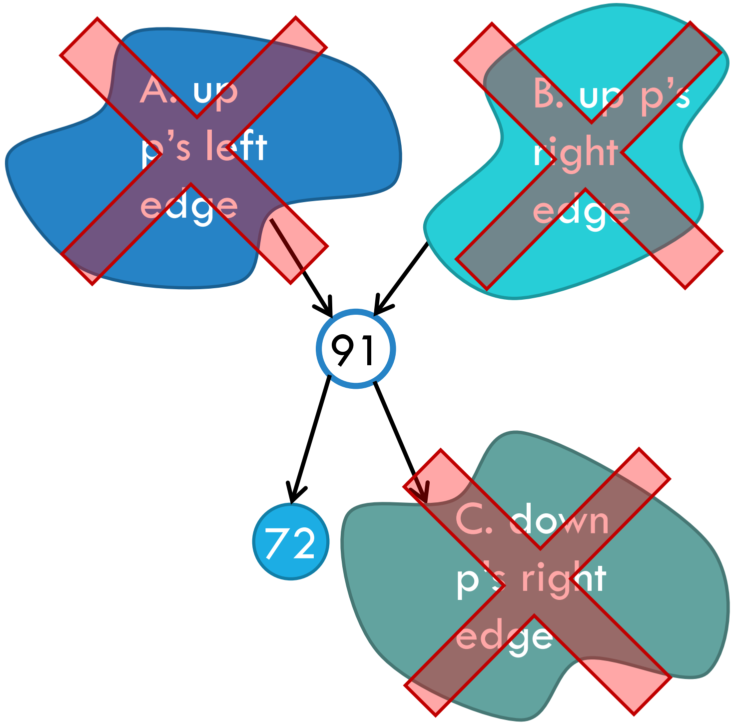

Examine each in turn

FINDING THE SUCCESSOR NODE

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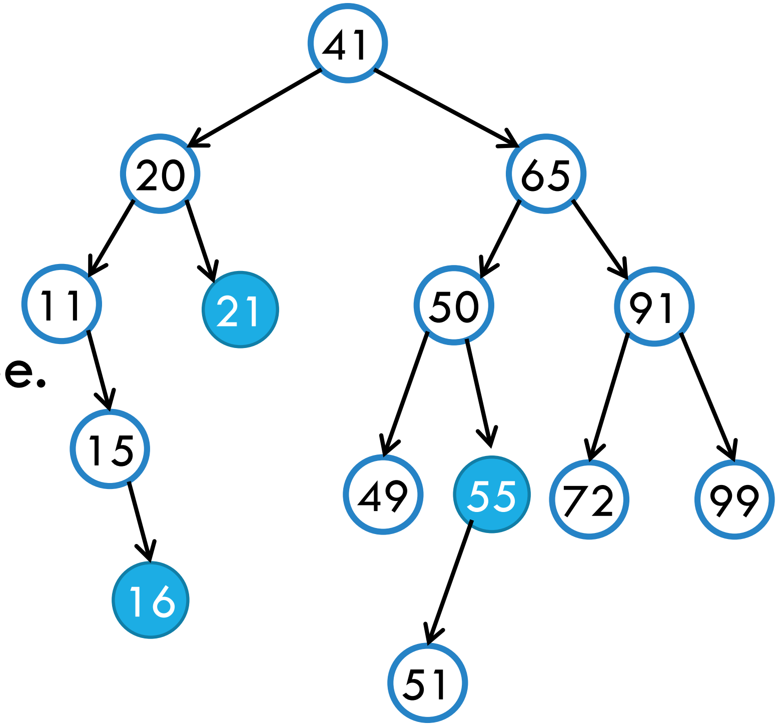

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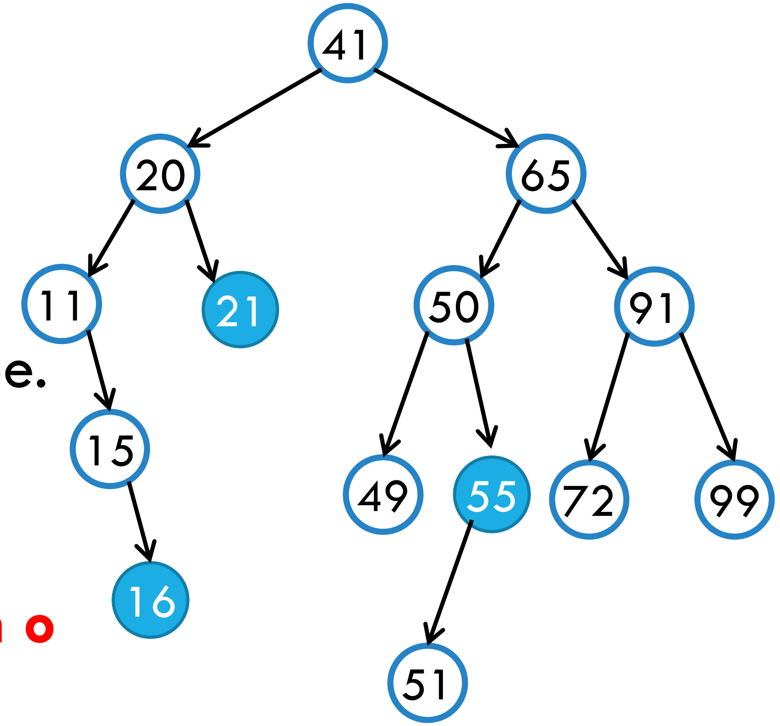

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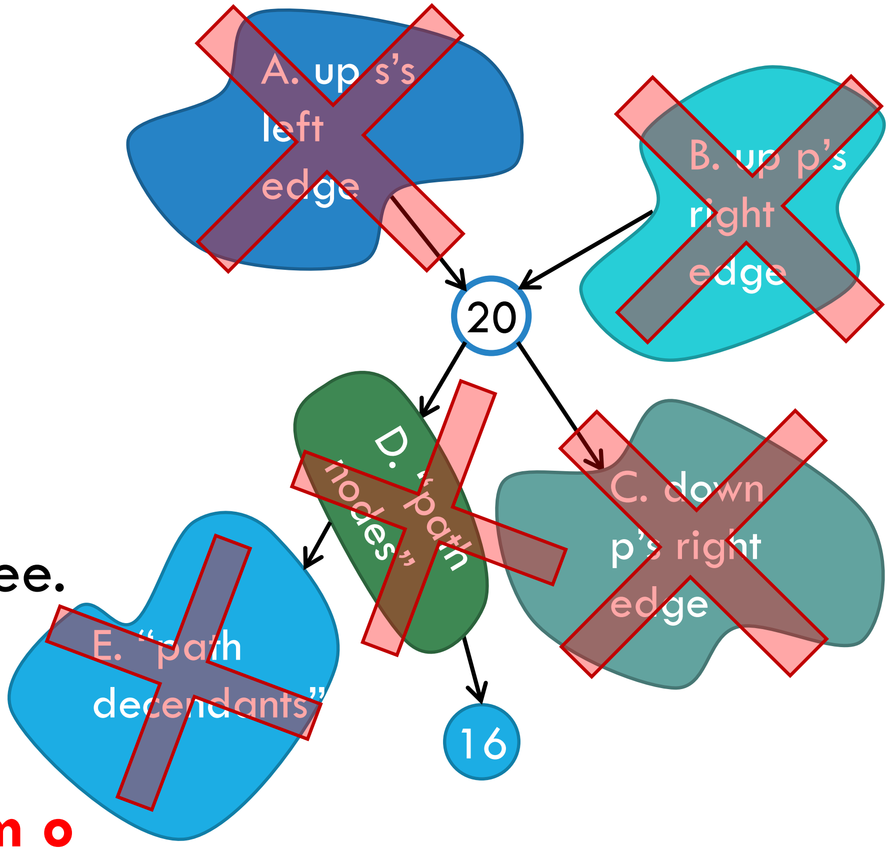
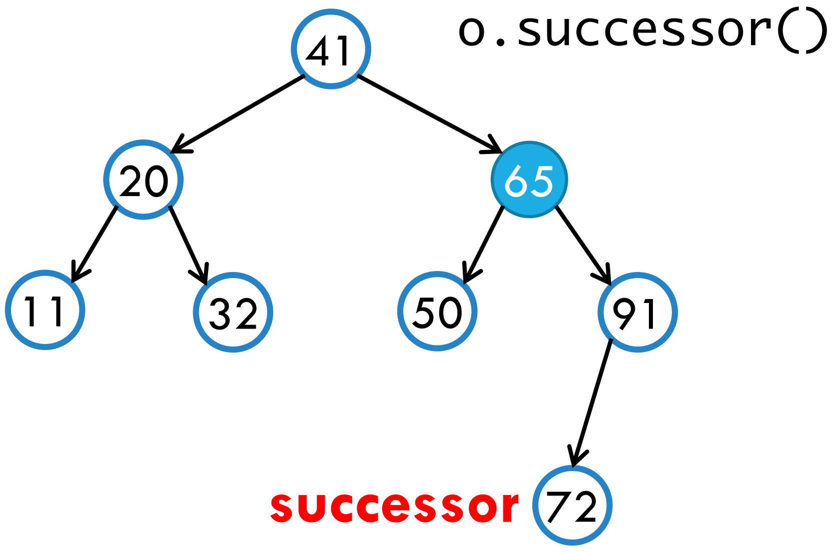


SUCCESSOR PSEUDOCODE




```
function successor()
    if m_rightTree is not null then
      return m_rightTree.searchMin()
    else
      p = m_parent, temp = this
      while (p is not null) and (temp == p.rightTree)
        temp = p, p = temp.m_parent
      if p is null then return -1
      else return p
```

if m has no rightsubtree and m is right child

if m has no right subtree and m is left child

very similar for predecessor(); just reason about it slowly.

BINARY SEARCH TREES (BST): BASIC OPERATIONS

`height()`: returns the height of the node

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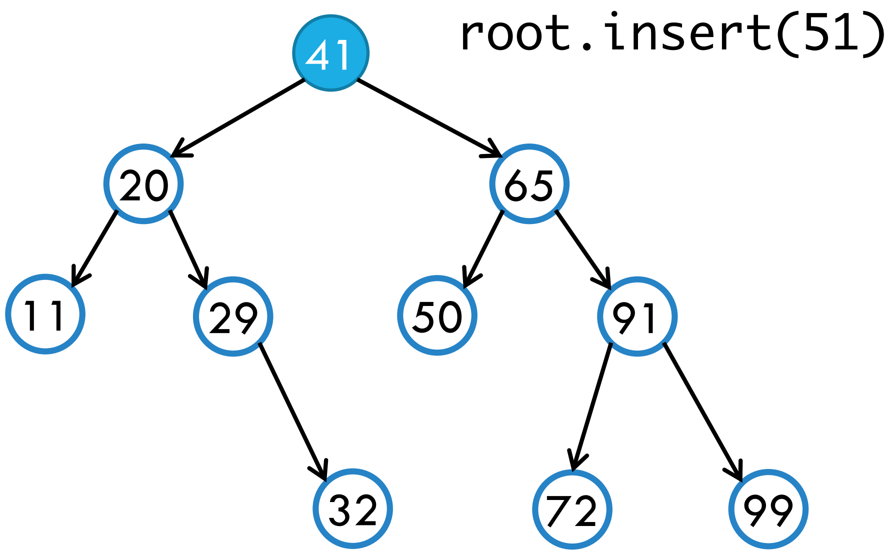
BST INSERTION

Need to insert the key in the “right” place to preserve the BST properties.

Idea: We search until we can’t go any further and add the node there.

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function insert(Key k)
  if k < m_key



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  else return
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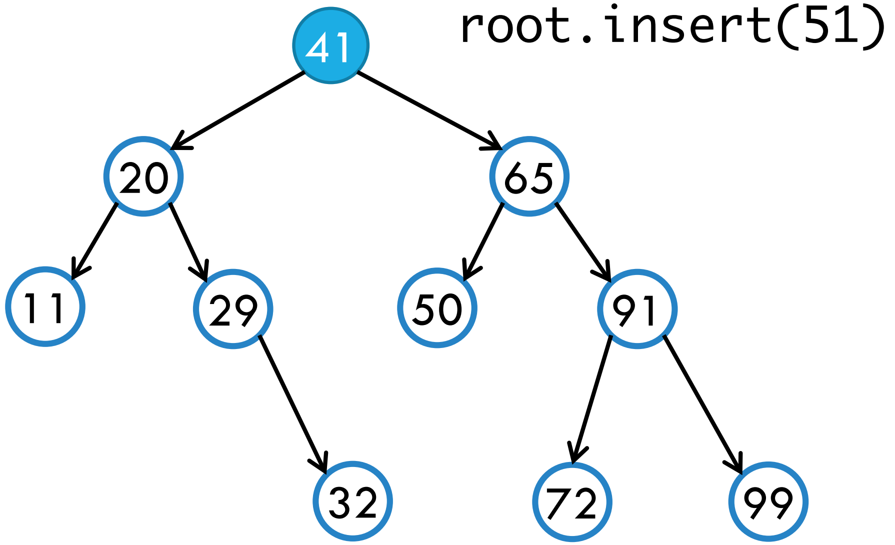


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function insert(Key k)
  if k < m_key
    if m_leftTree is not null
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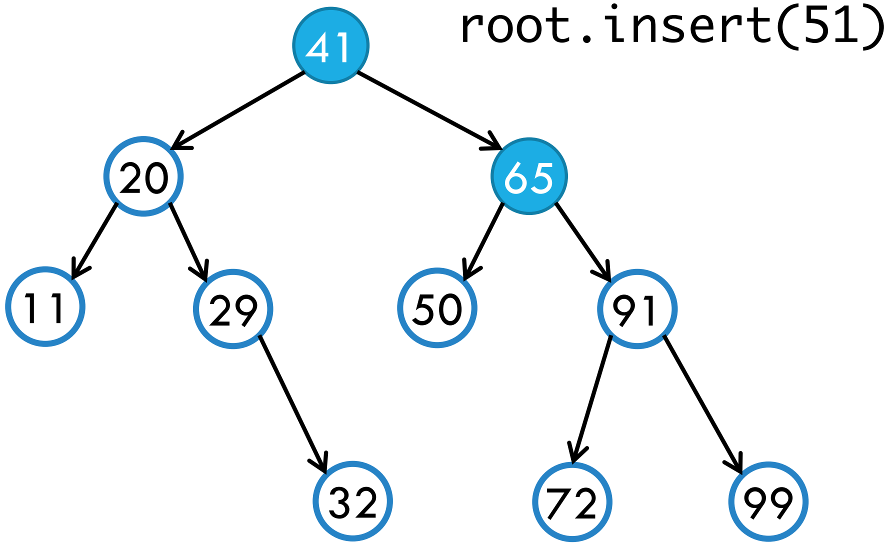


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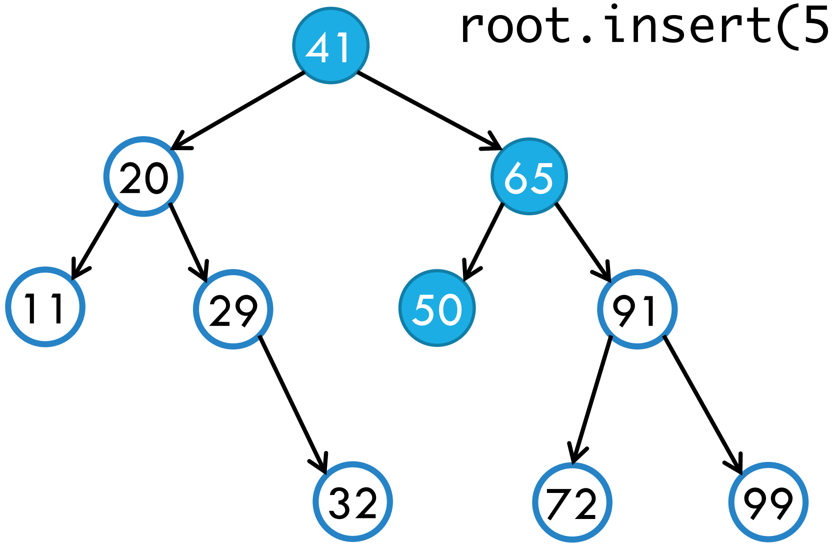


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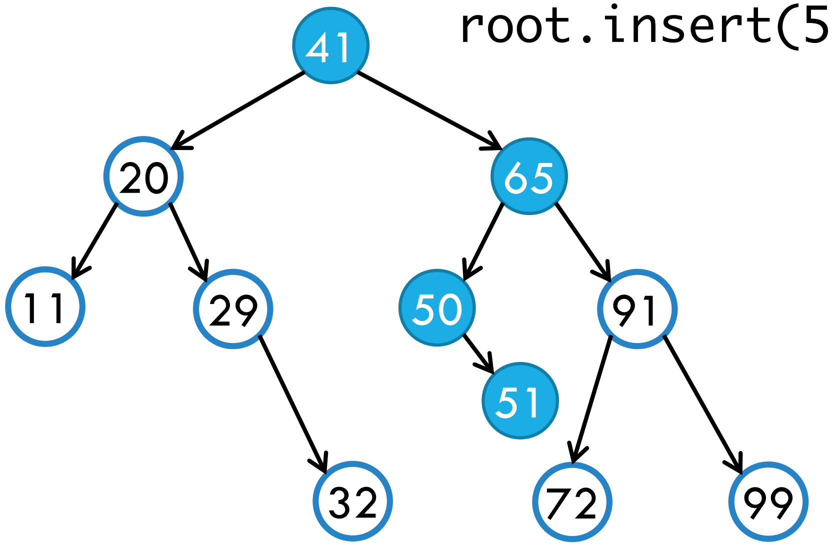


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```
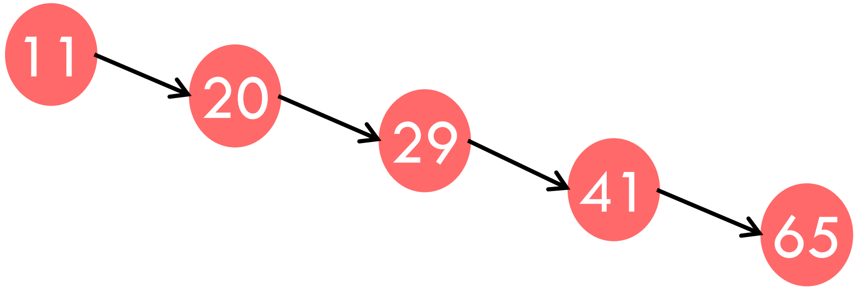
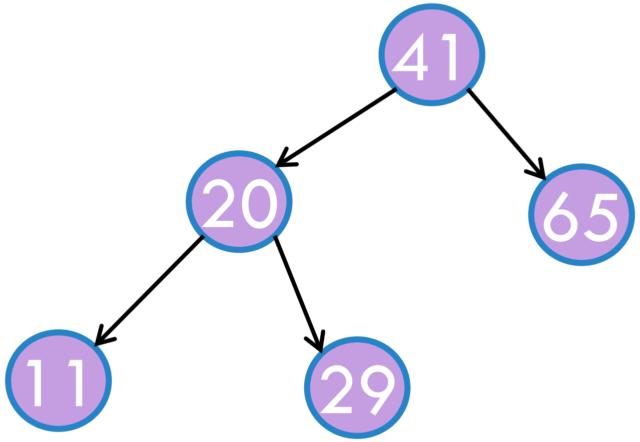

SHAPE OF THE TREE & ORDER OF INSERTION

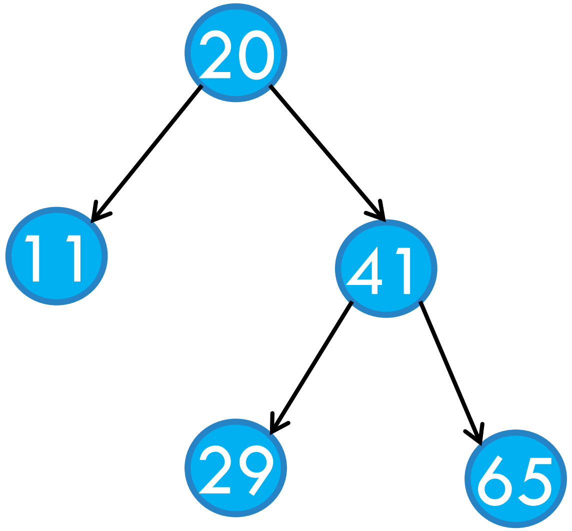
Performance of searching and inserts depends on height.

Height depends on shape

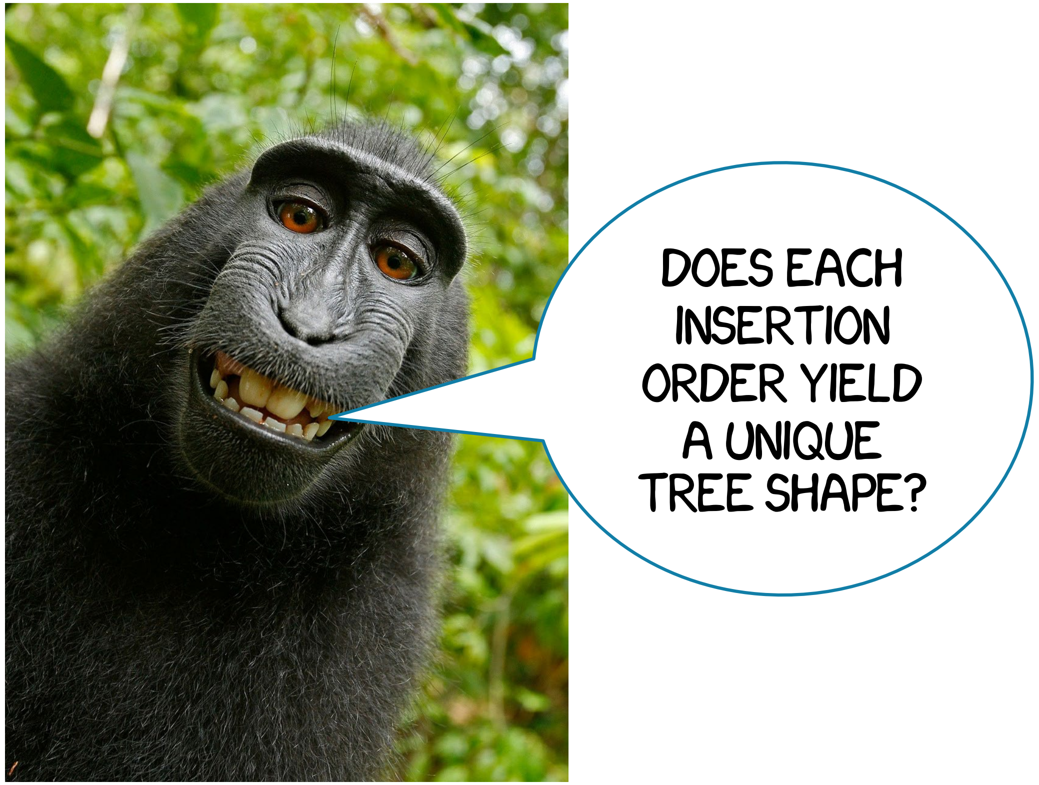
Shape depends on order of insertion (and deletions)



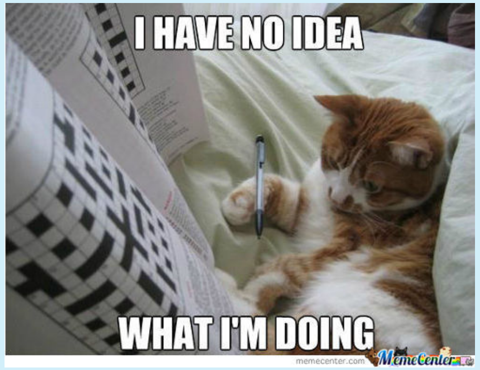




NARUTO PUZZLE: SHAPES AND INSERTIONS



Does each insertion order yield a *unique* shape?

A. Yes!
B. No!
C. Umm.. maybe.
D. 

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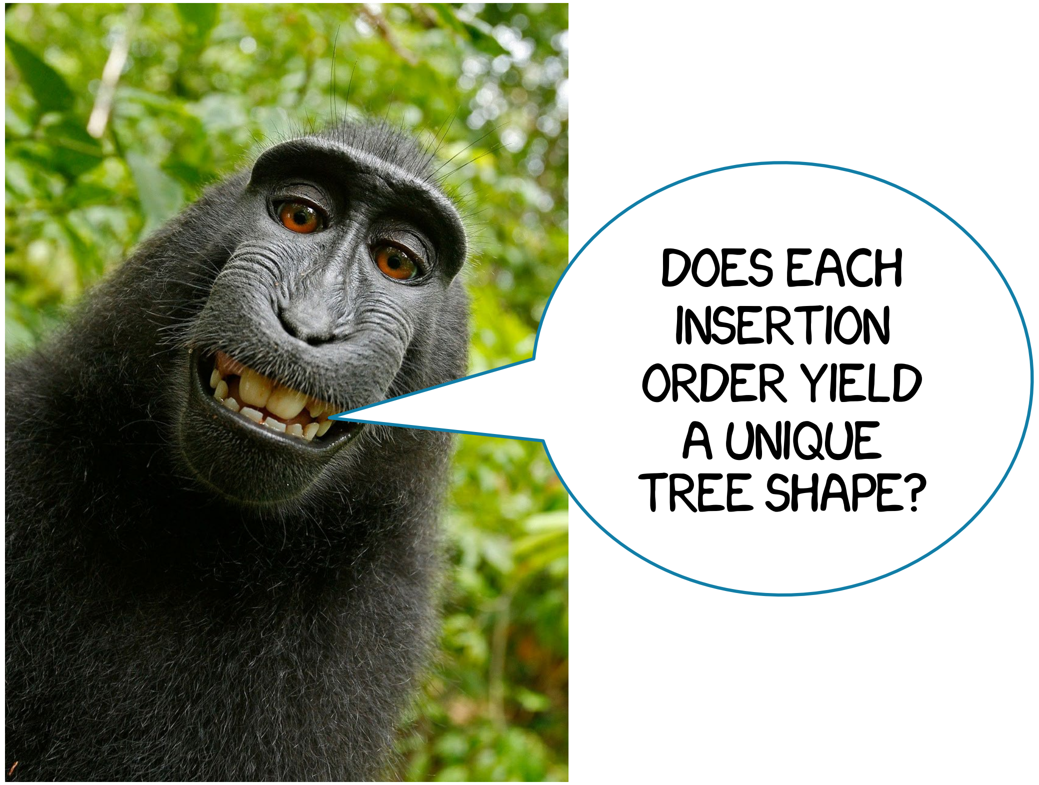

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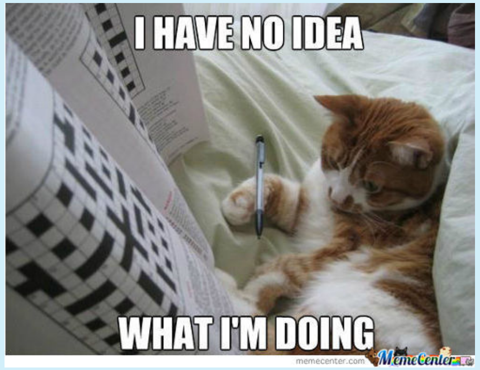
D. 

NARUTO PUZZLE: SHAPES AND INSERTIONS

How many ways to order insertions? $n!$

How many shapes of a binary tree?

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

More at :
http://cs.lmu.edu/~ray/notes/binarytrees/

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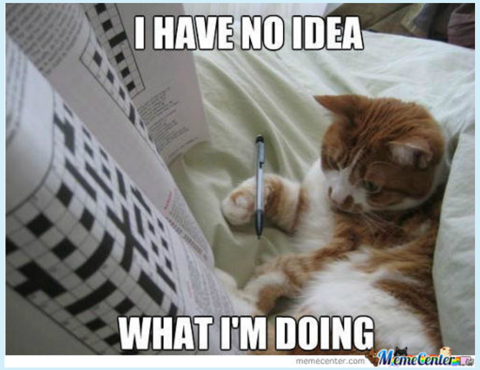
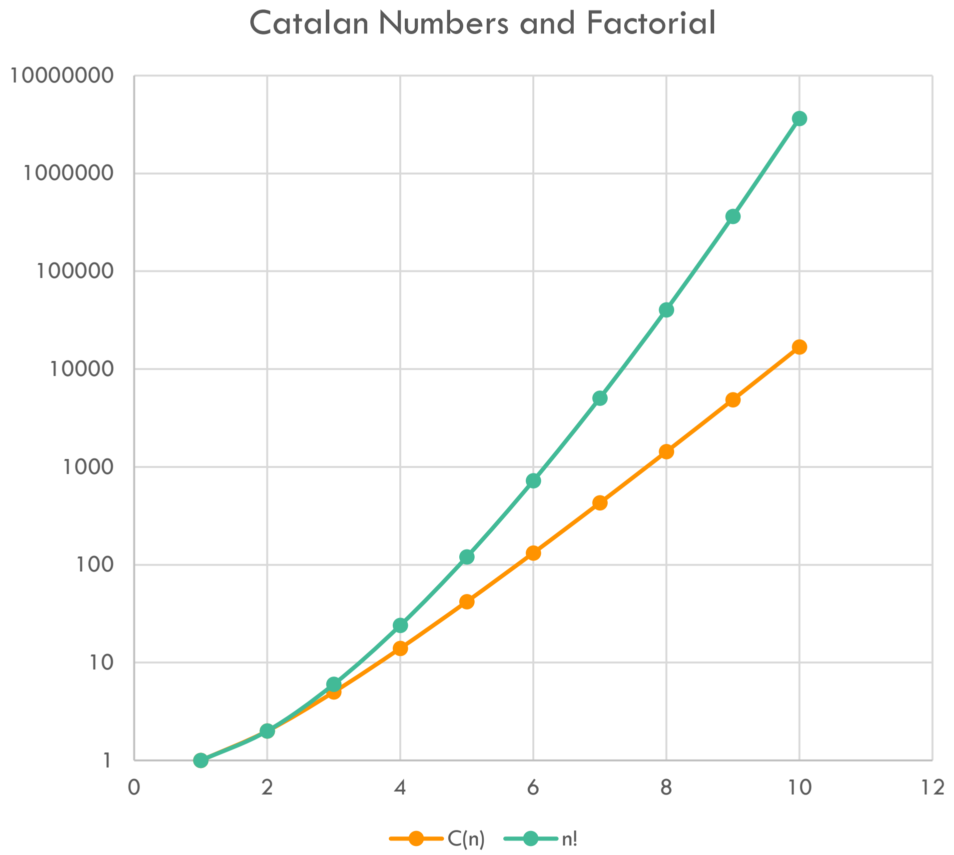


NARUTO PUZZLE: SHAPES AND INSERTIONS



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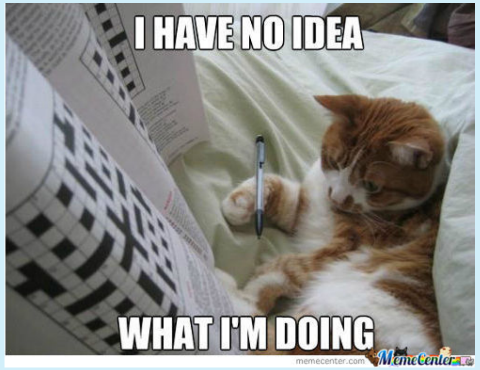
D. 

WHAT IF THE KEYS ARE INSERTED IN RANDOM ORDER?

What is the average-case time complexity of n distinct keys inserted into a BST in *random* order?

A. $O(1)$
B. $O(n)$
C. $O(\log n)$
D. 

WHAT IF THE KEYS ARE INSERTED IN **RANDOM** ORDER?

The proof for this one is long.

For interested readers: look at [CLRS Section 12.4] or [Sedgewick & Wayne, Section 3.2]

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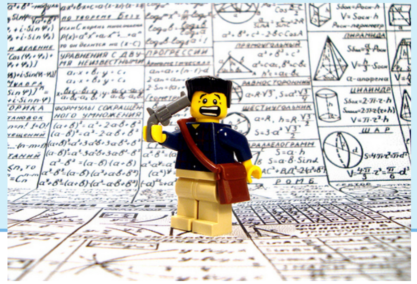
D. 

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`predecessor()`: returns next key $< k$

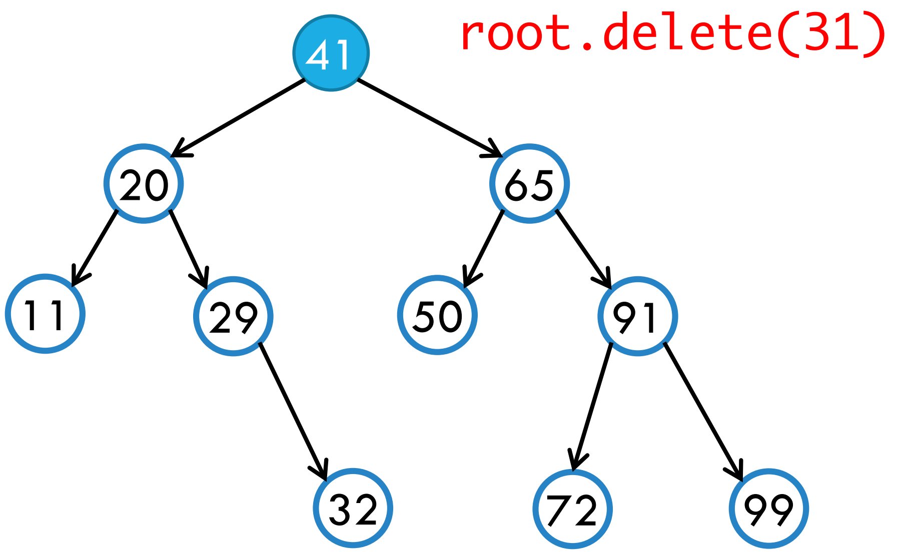
`insert(k,v)`: inserts an object v with key k

`delete(k)` : deletes the node with key k

DELETING NODES

Idea: Search for the node, then handle one of 4 cases:

- The node is not found: __________
- Node is at a leaf: _____________
- Node is an internal node with 1 child: ______________
- Node is an internal node with 2 children: _______________



DELETING NODES

Idea: Search for the node, then handle one of 4 cases:

- The node is not found: **do nothing!**
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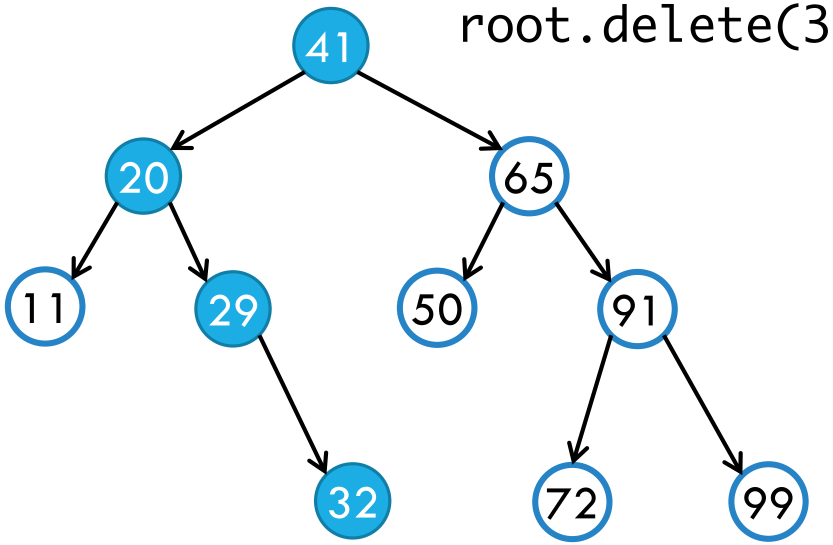
- Node is an internal node with 1 child: ______________
- Node is an internal node with 2 children: ______________

`root.delete(31)`



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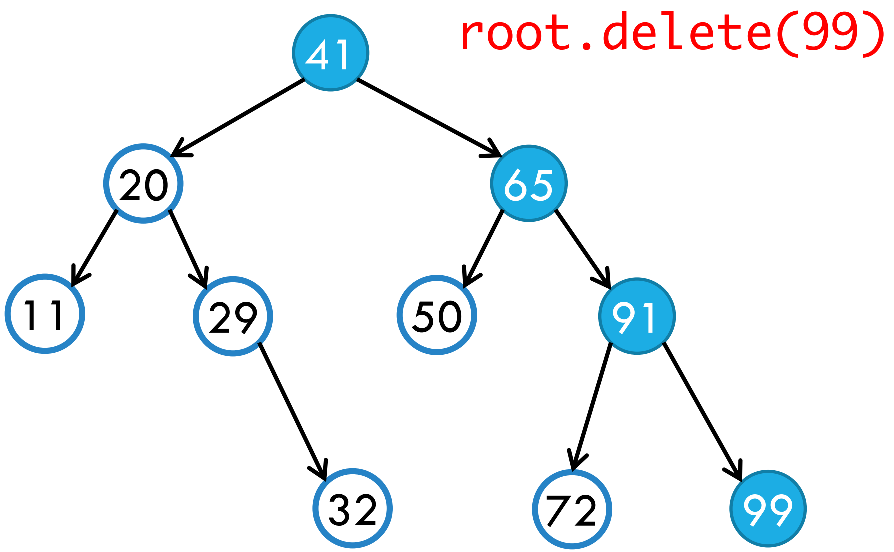
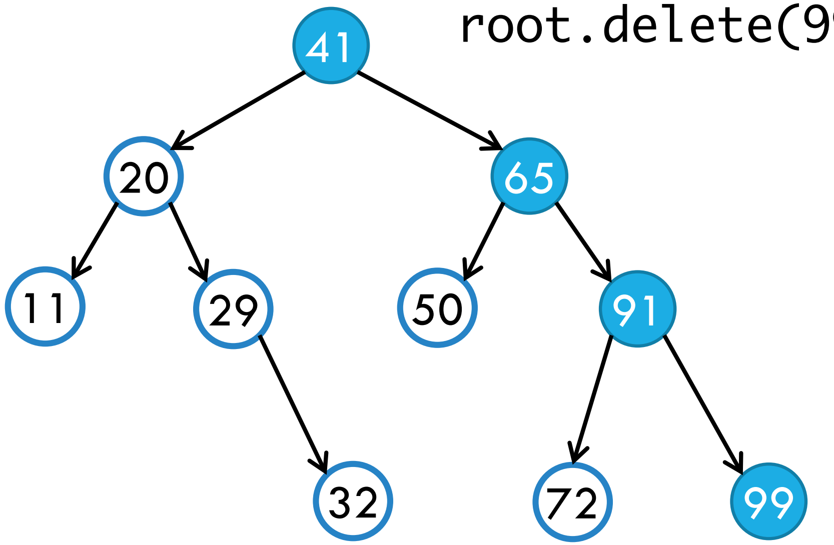


DELETING NODES

Idea: Search for the node, then handle one of 4 cases:

- The node is not found: **do nothing!**
- Node is at a leaf: **just remove!**
- Node is an internal node with 1 child: ________________
- Node is an internal node with 2 children: ________________

`root.delete(99)`



DELETING NODES

Idea: Search for the node, then handle one of 4 cases:

- The node is not found: **do nothing!**
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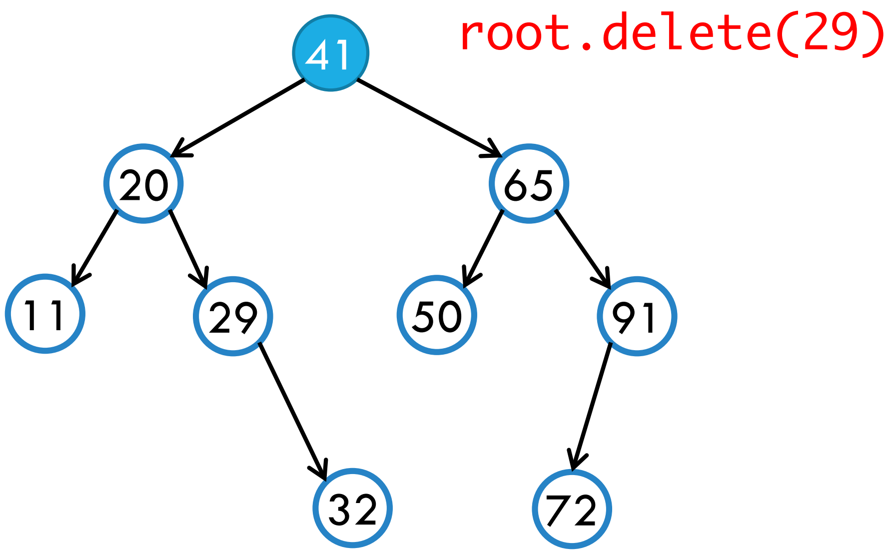
- Node is an internal node with 2 children: _________________



DELETING NODES

Idea: Search for the node, then handle one of 4 cases:

- The node is not found: **do nothing!**
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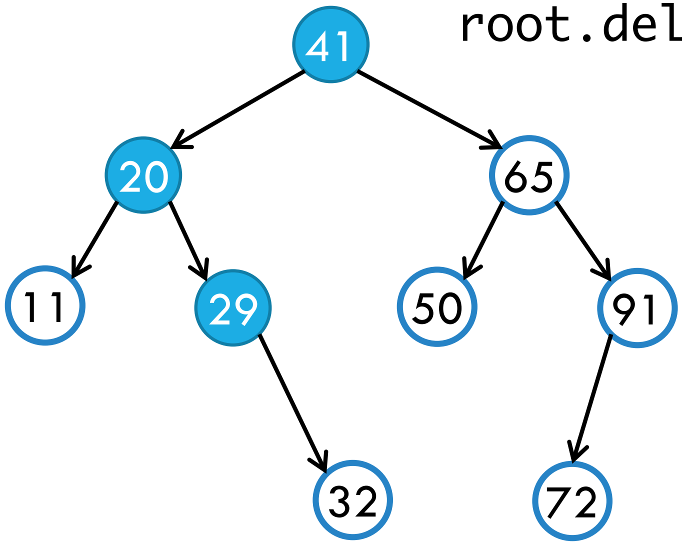
- Node is an internal node with 2 children: ________________

`root.delete(29)`

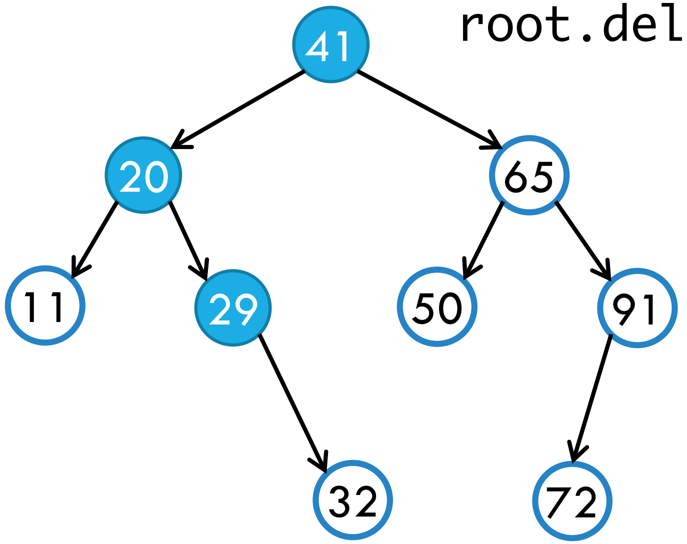


DELETING NODES

Idea: Search for the node, then handle one of 4 cases:

- The node is not found: **do nothing!**
- Node is at a leaf: **just remove!**
- Node is an internal node with 1 child: **remove node and connect child to parent**
- Node is an internal node with 2 children: ________________

root.delete(29)



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- Node is an internal node with 2 children:

____

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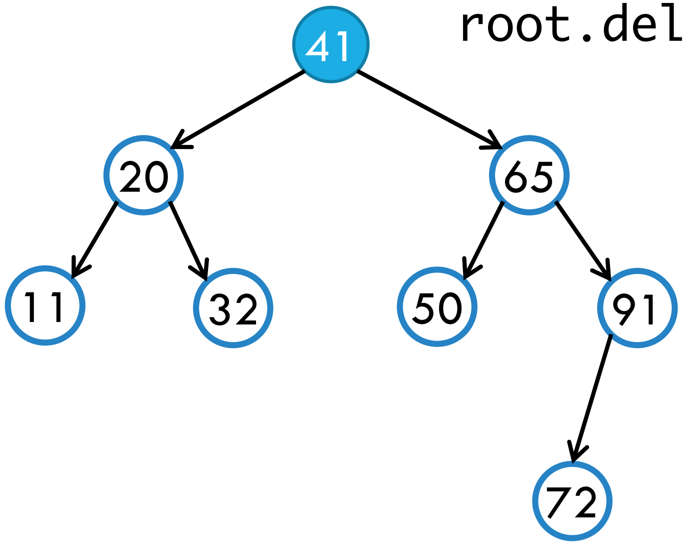

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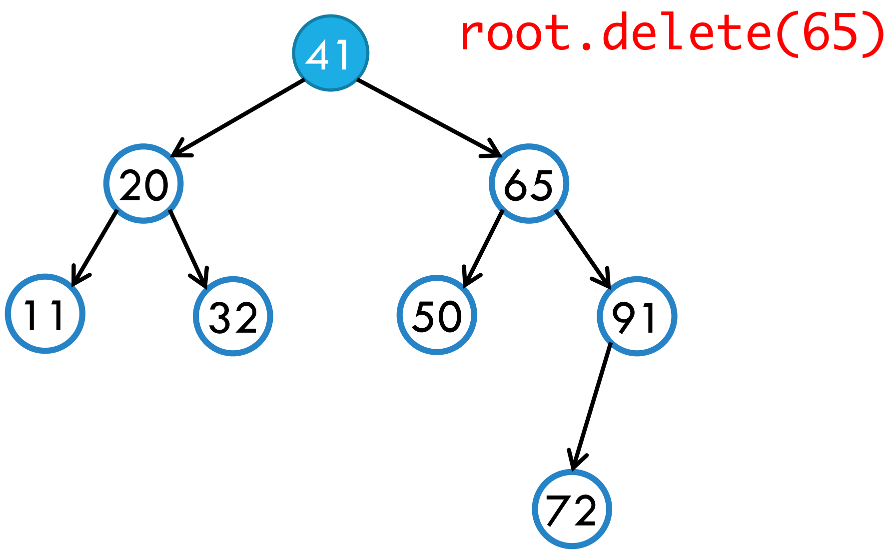

DELETING NODES

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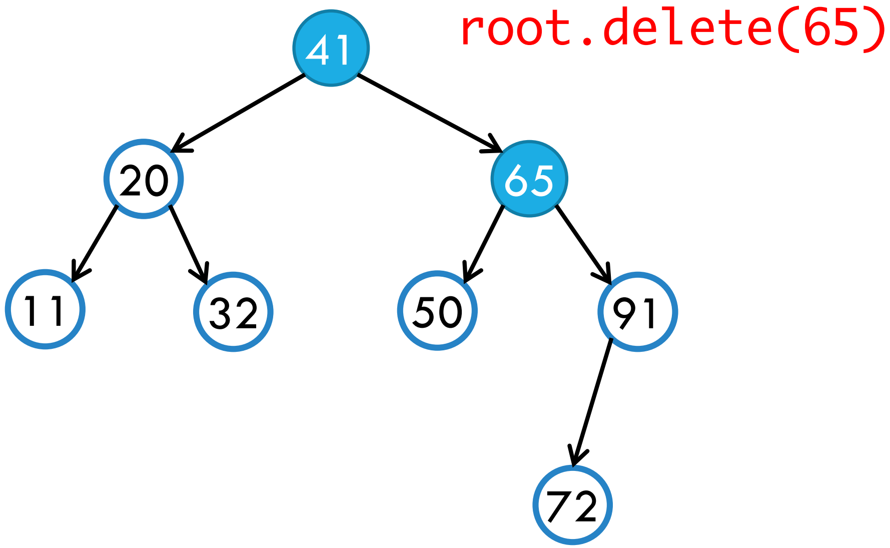
- Node is an internal node with 2 children:

____

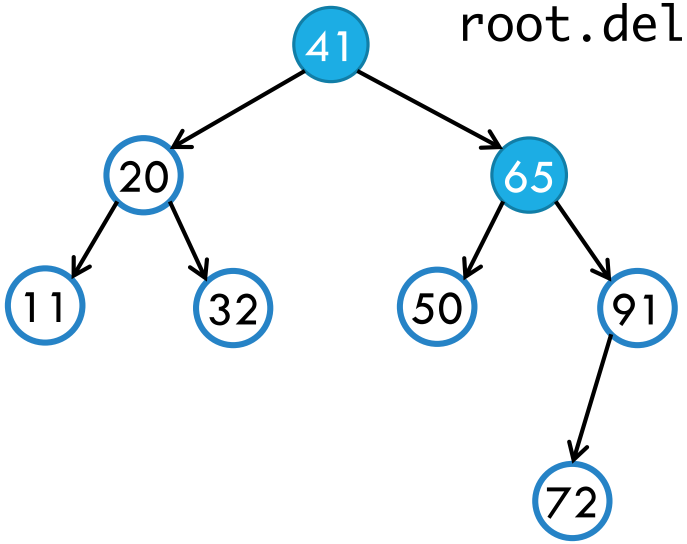


DELETING NODES

Idea: Search for the node, then handle one of 4 cases:

- The node is not found: **do nothing!**
- Node is at a leaf: **just remove!**
- Node is an internal node with 1 child: **remove node and connect child to parent**
- Node is an internal node with 2 children: **replace node with its successor**

`root.delete(65)`



Successor = node with the smallest key larger than k

DELETING NODES

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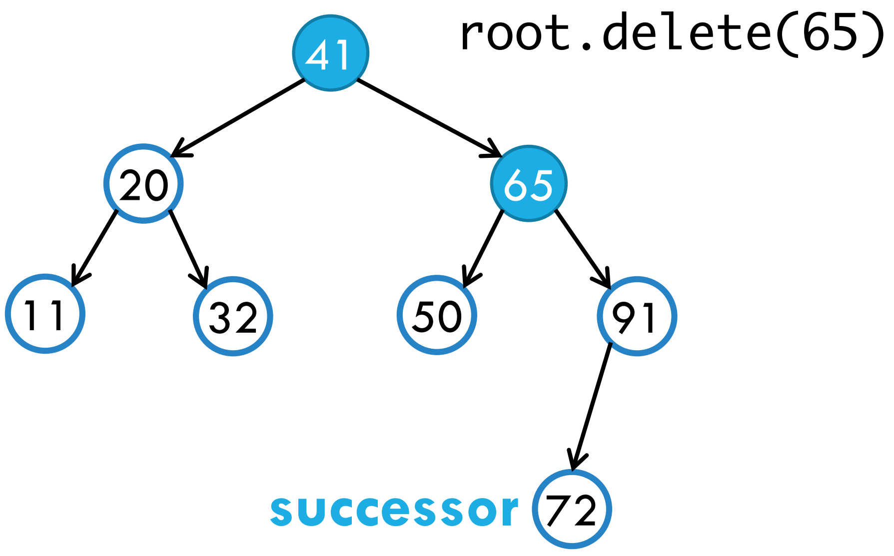

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DELETING NODES

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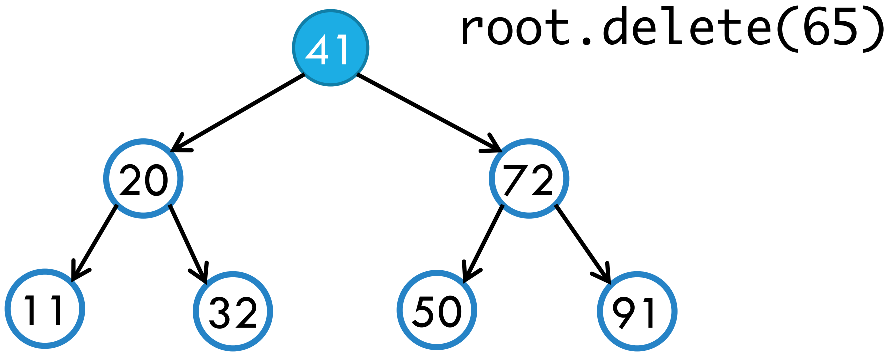


Does replacing the node with the successor always work?

Successor = node with the smallest key larger than k

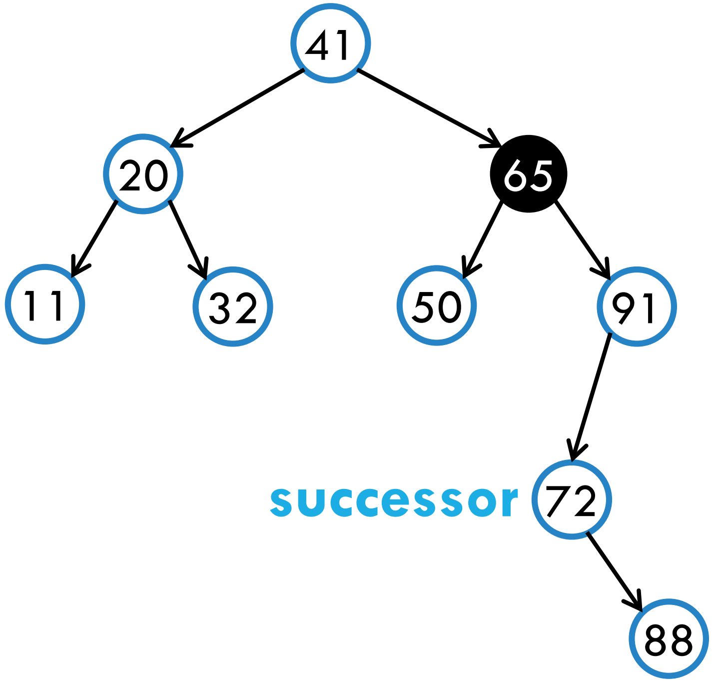
YES, IT DOES.

For this to work:

1. The successor *s* maintains the BST property when inserted to where k used to be.
2. The successor *s* has at most 1 child

Why do we want condition 2?

Because we can then just re-attach the successor's child to s's parent



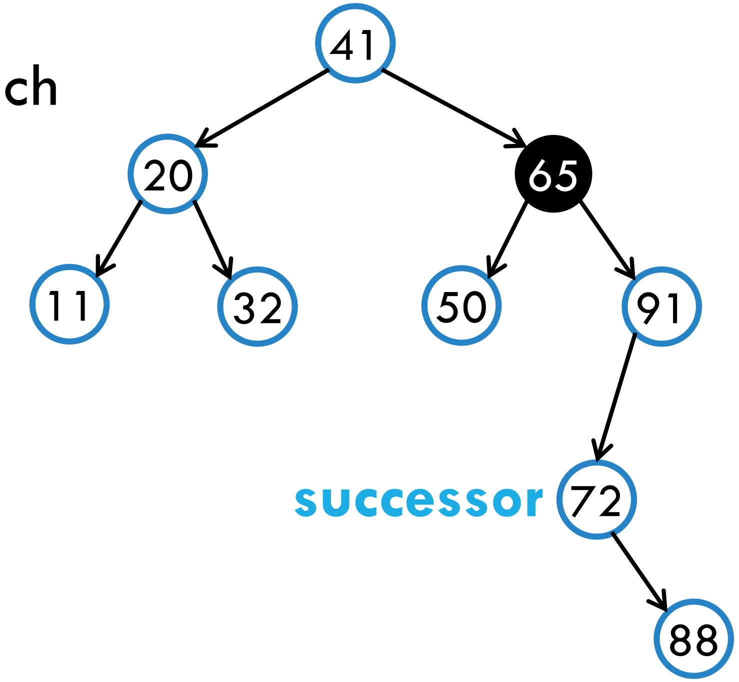
YES, IT DOES.

For this to work:

1. The successor *s* maintains the BST property when inserted to where k used to be.
2. The successor *s* has at most 1 child, so we can re-attach

Why does 1 hold?

- By definition $s > k$ so, s must be in k's right subtree.
- nodes in k's left subtree must be $< s$
- nodes k's right subtree (excluding s) must be $> s$
- Hence, inserting s to k's position is valid.



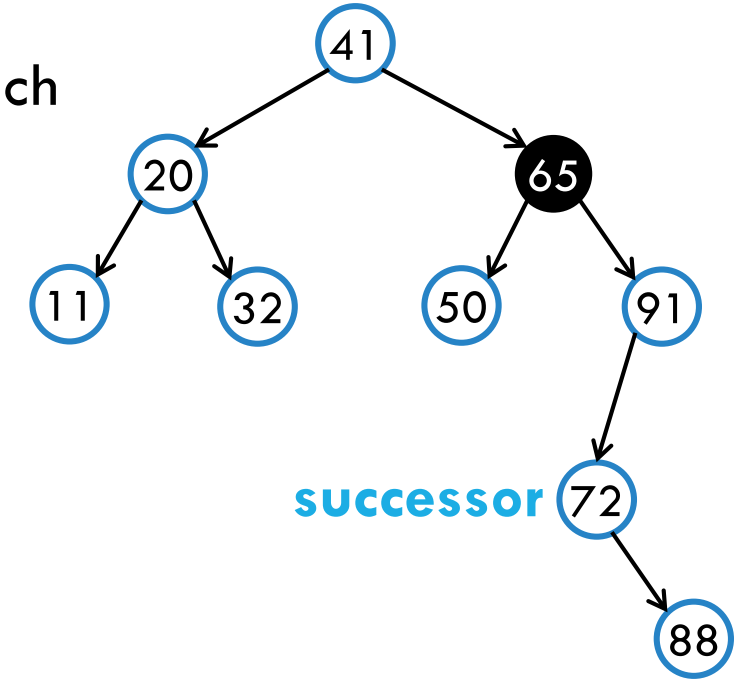
YES, IT DOES.

For this to work:

1. The successor *s* maintains the BST property when inserted to where k used to be.
2. The successor *s* has at most 1 child, so we can re-attach

Why does 2 hold?

- *s* is the minimum element of *k*'s right subtree.
- Can the minimum element of a BST have a left child? ______



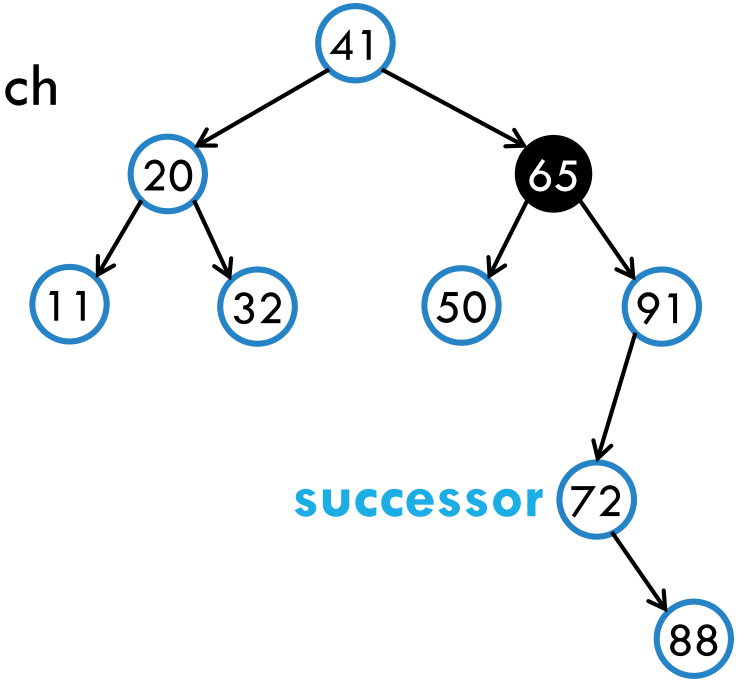
YES, IT DOES.

For this to work:

1. The successor *s* maintains the BST property when inserted to where k used to be.
2. The successor *s* has at most 1 child, so we can re-attach

Why does 2 hold?

- *s* is the minimum element of *k*'s right subtree.
- Can the minimum element of a BST have a left child?
  No
- So, *s* has no children or a right child.



YES, IT DOES.

For this to work:

1. The successor *s* maintains the BST property when inserted to where k used to be.
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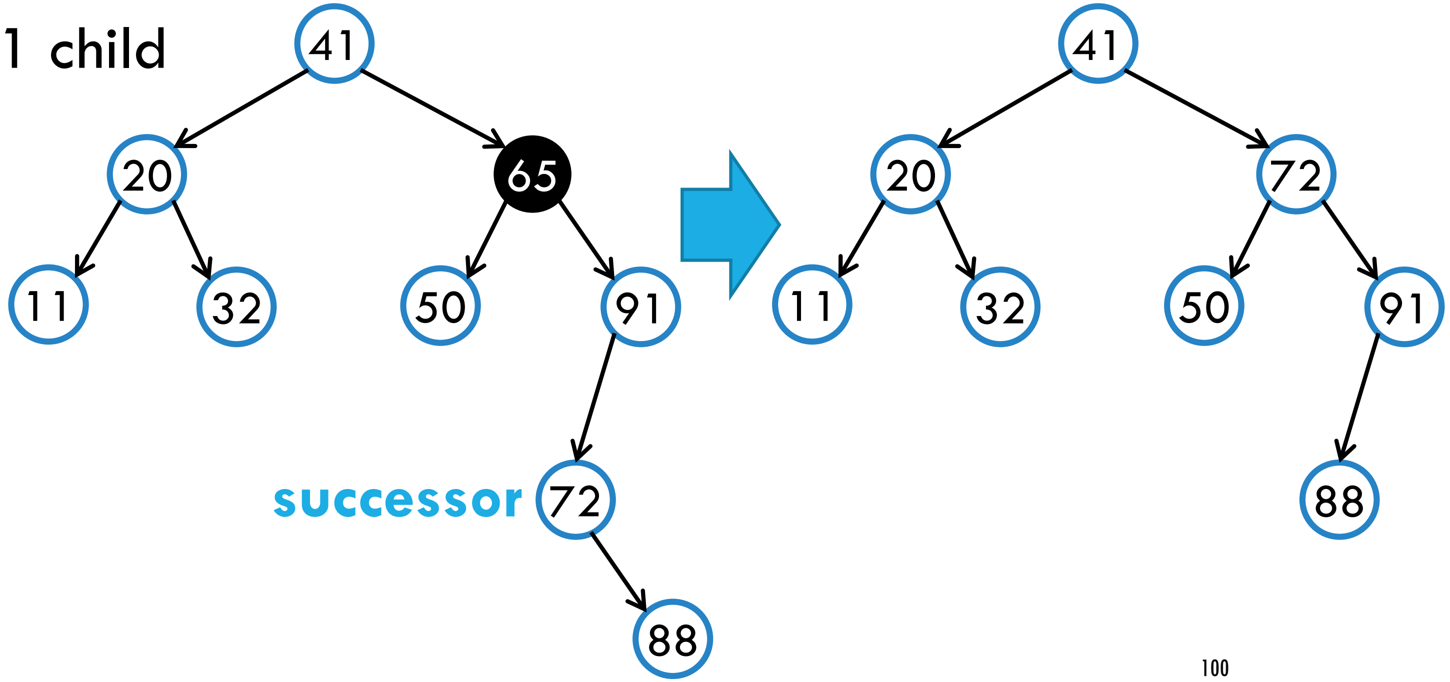


BINARY SEARCH TREES (BST): BASIC OPERATIONS

`height()`: returns the height of the node

`search(k)` : returns an object v with key k

`searchMin()` : finds the minimum key k

`searchMax()` : finds the maximum key k

`successor()`: returns next key > k

`predecessor()`: returns next key < k

`insert(k,v)`: inserts an object v with key k

`delete(k)` : deletes the node with key k

OPERATIONS & COSTS

height(): $O(n)$

search(k) : $O(h)$

searchMin() : $O(h)$

searchMax() : $O(h)$

successor(): $O(h)$

predecessor(): $O(h)$

insert(k,v): $O(h)$

delete(k) : $O(h)$

if the tree is imbalanced, $O(n)$

THE ORDERED DICTIONARY ADT

Operations with k = key, v = value:

✓ `insert(k, v)`: inserts an element with value v and key k

✓ `search(k)`: returns the value with key k

✓ `delete(k)`: deletes the element with key k

✓ `contains(k)`: true if the dictionary contains an element with key k

`floor(k)`: returns next key $\leq$ k

`ceiling(k)`: returns next key $\geq$ k

✓ `size()`: returns the size of the dictionary

*also called "Ordered Symbol Tables"

PROBLEM: POVERTY IDENTIFICATION

The Stop-Poverty charity calls:

To provide financial aid, Help identify families:

- earning exactly $a amount
- earning less than or equal to $a
- earning more than or equal to $a

NOT SO BASIC BST OPERATIONS

`floor(k)`: returns next key $\leq$ k

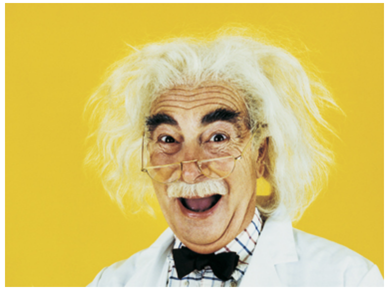
`ceiling(k)`: returns next key $\geq$ k

`inorder(Node o)`: returns nodes of the tree rooted at o in order

`copyTree(Node o)`: returns a copy of the tree rooted at o

`deleteTree(Node o)`: deletes the tree rooted at o

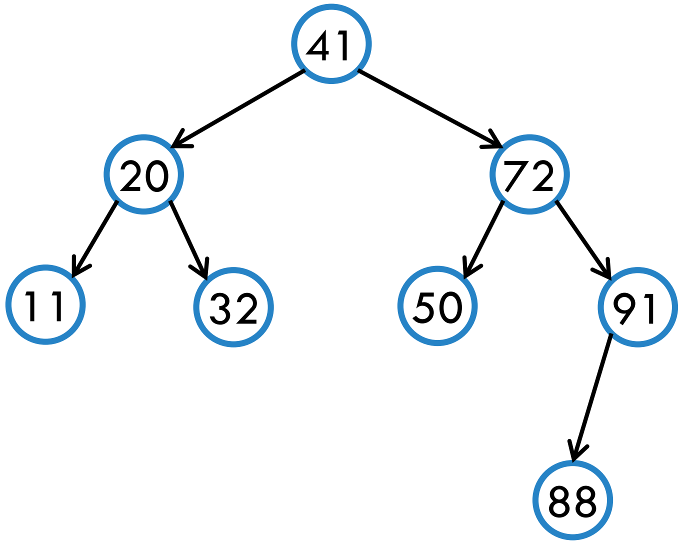
Just for fun!



CEILING FOR KEY K

Question: for a given key k, how do we find the ceiling (the smallest key in the BST larger or equal to k)?

```
ceiling(root, k)
```

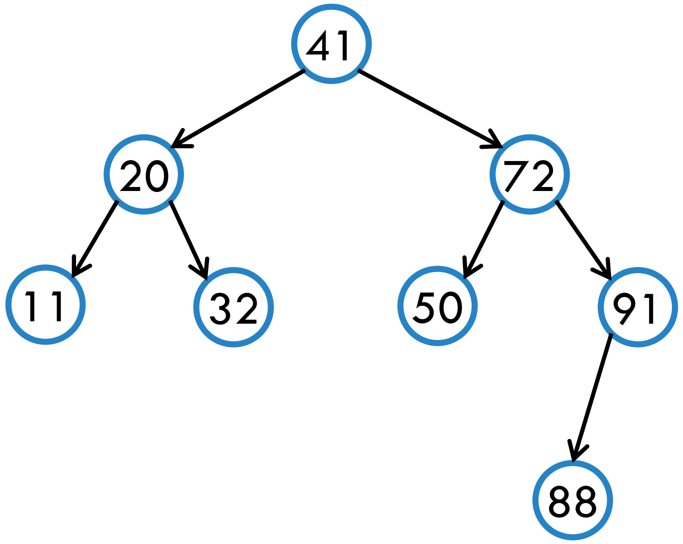


CEILING FOR KEY K

Question: for a given key k, how do we find the ceiling (the smallest key in the BST larger or equal to k)?

```
ceiling(root, k)

function ceiling(Node x, k)
    if x is null then return null
    if x.key == k then ?
    if k > x.key then
        ?
    if k < x.key then
        ?
```



CEILING FOR KEY K

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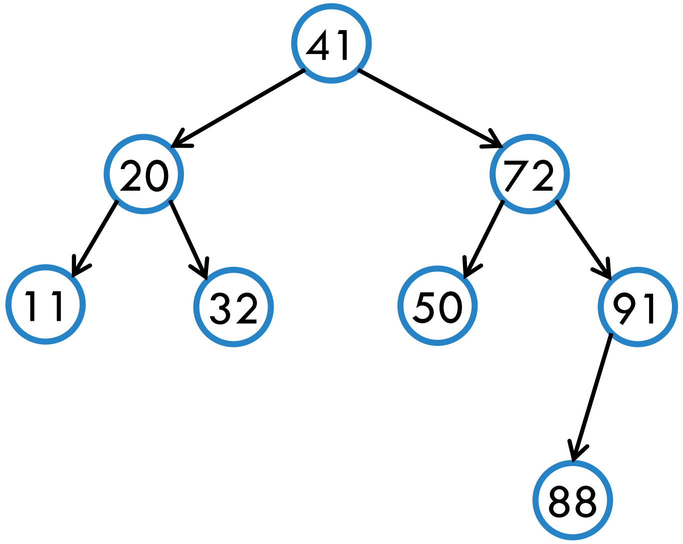
```
ceiling(root, k)

function ceiling(Node x, k)
    if x is null then return null
    if x.key == k then return x
    if k > x.key then
        ?
    if k < x.key then
        ?
```

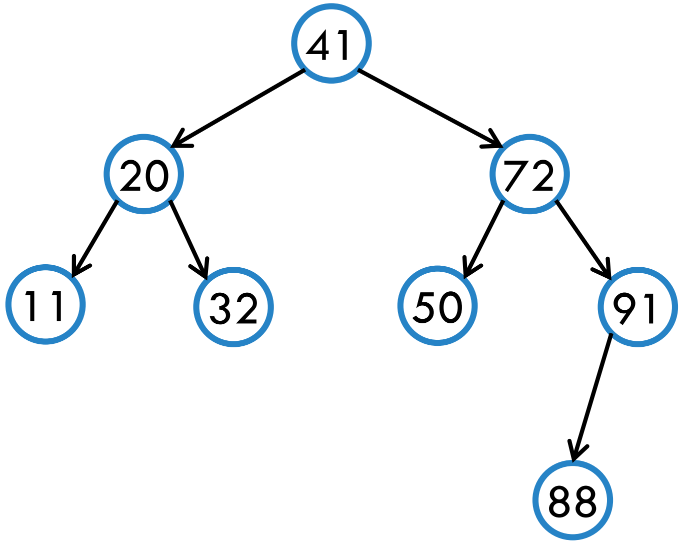


CEILING FOR KEY K

Question: for a given key k, how do we find the ceiling (the smallest key in the BST larger or equal to k)?

```
ceiling(root, k)

function ceiling(Node x, k)
    if x is null then return null
    if x.key == k then return x
    if k > x.key then
        return ceiling(x.rightTree, k)
    if k < x.key then
        ?
```



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Question: for a given key k, how do we find the ceiling (the smallest key in the BST larger or equal to k)?

```
ceiling(root, k)

function ceiling(Node x, k)
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    if x.key == k then return x
    if k > x.key then
        return ceiling(x.rightTree, k)
    if k < x.key then
        t = ceiling(x.leftTree, k)
        if t is not null then return t
        else return x
```
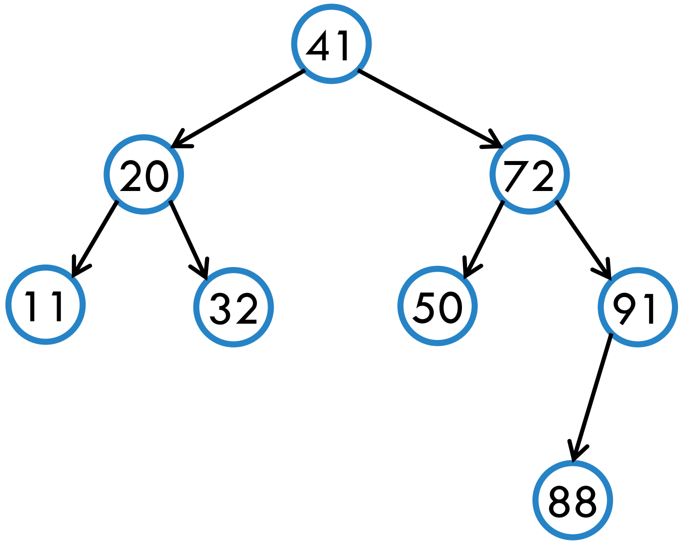



Floor is similar. Try it out!

NOT SO BASIC BST OPERATIONS

`floor(k)`: returns next key $\leq$ k

`ceiling(k)`: returns next key $\geq$ k

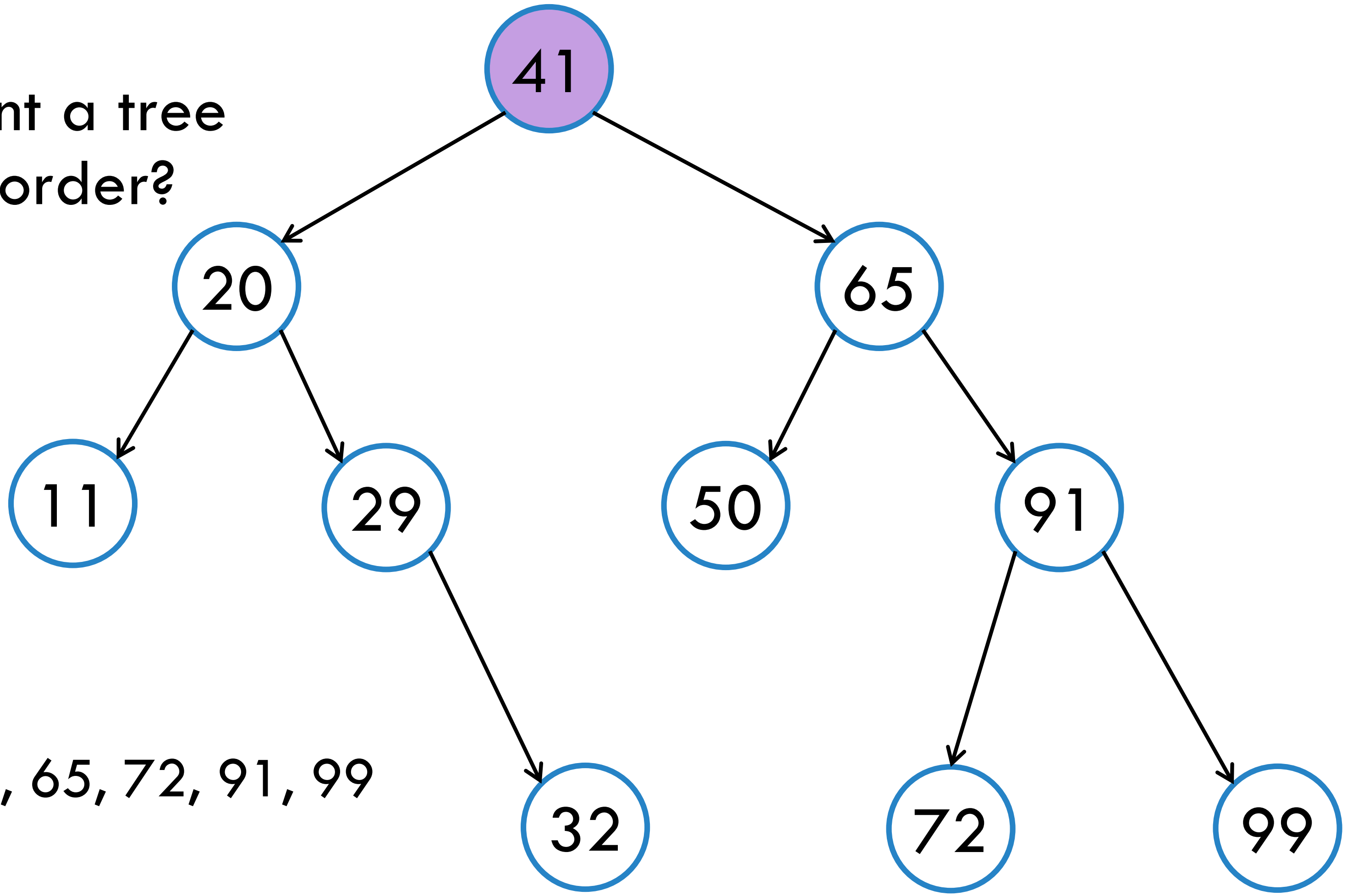
`inorder(Node o)`: returns nodes of the tree rooted at o in order

`copyTree(Node o)`: returns a copy of the tree rooted at o

`deleteTree(Node o)`: deletes the tree rooted at o

PRINTING THE NODES IN ORDER

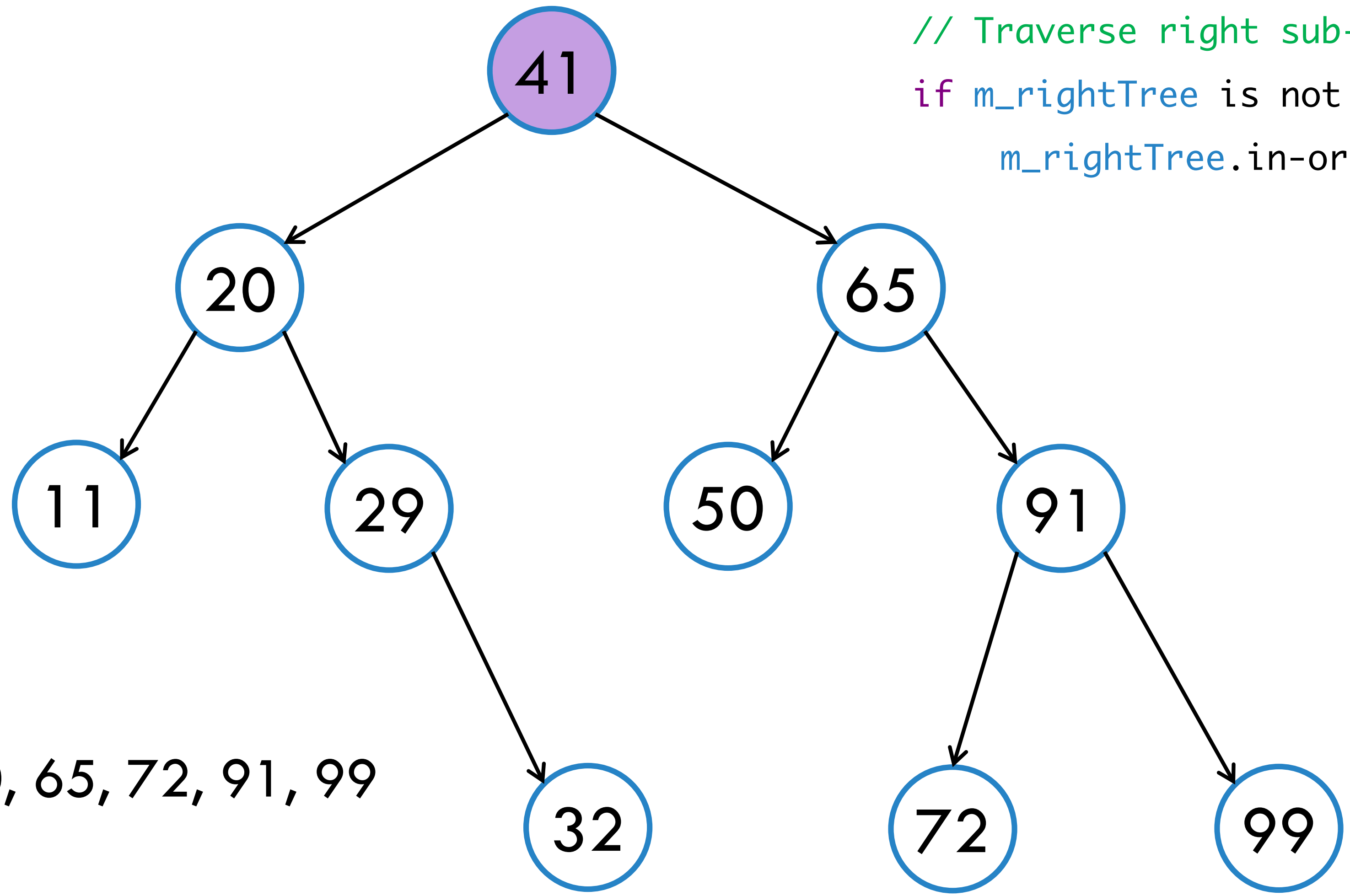
How can we print a tree rooted at 41 in order?



11, 20, 29, 32, 41, 50, 65, 72, 91, 99

PRINTING THE NODES IN ORDER

```
function in-order-traversal()
    // Traverse left sub-tree
    if m_leftTree is not null
        m_leftTree.in-order-traversal()
    visit(this)
    // Traverse right sub-tree
    if m_rightTree is not null then
        m_rightTree.in-order-traversal()
```



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INORDER EXAMPLE:

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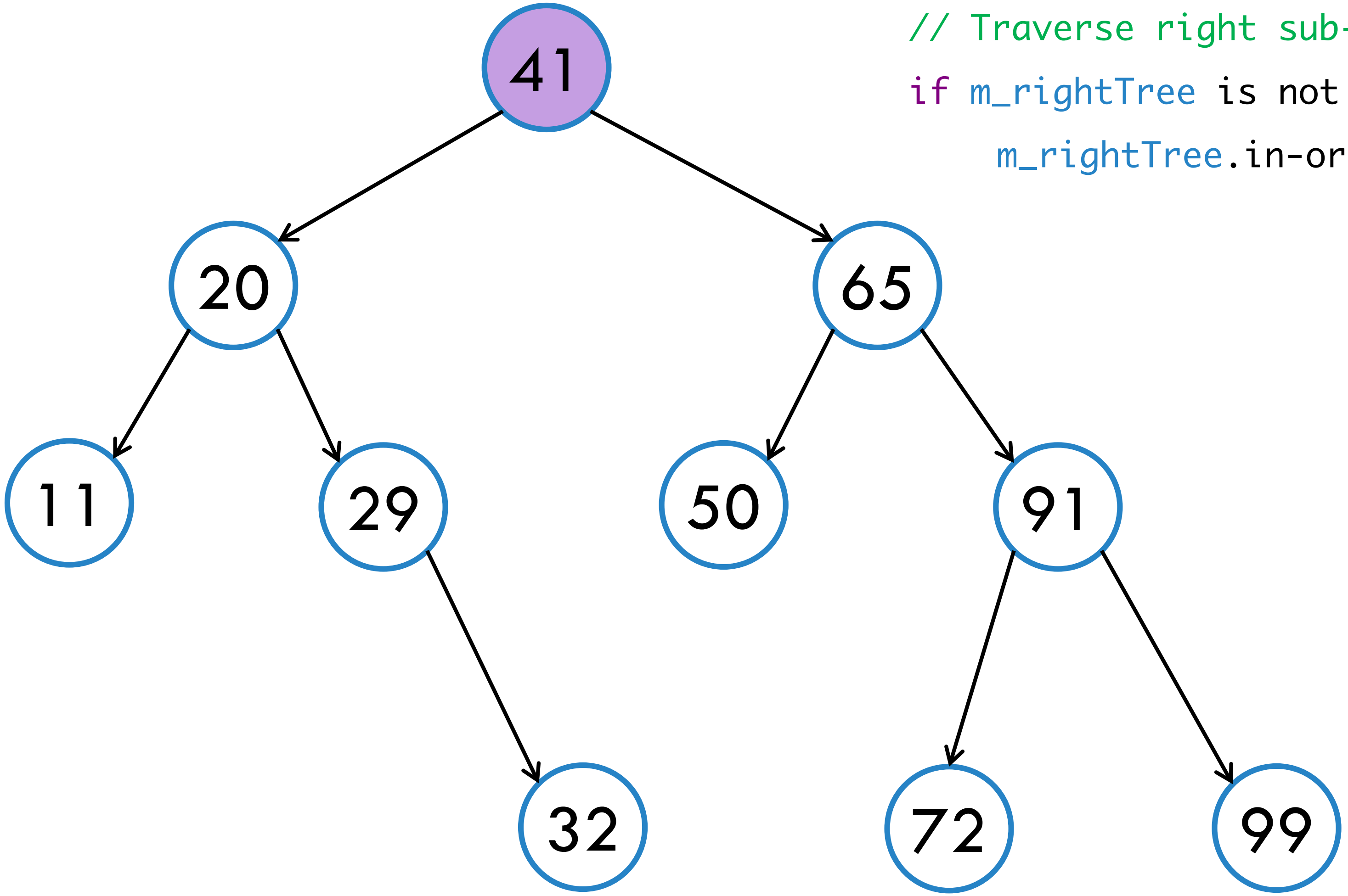

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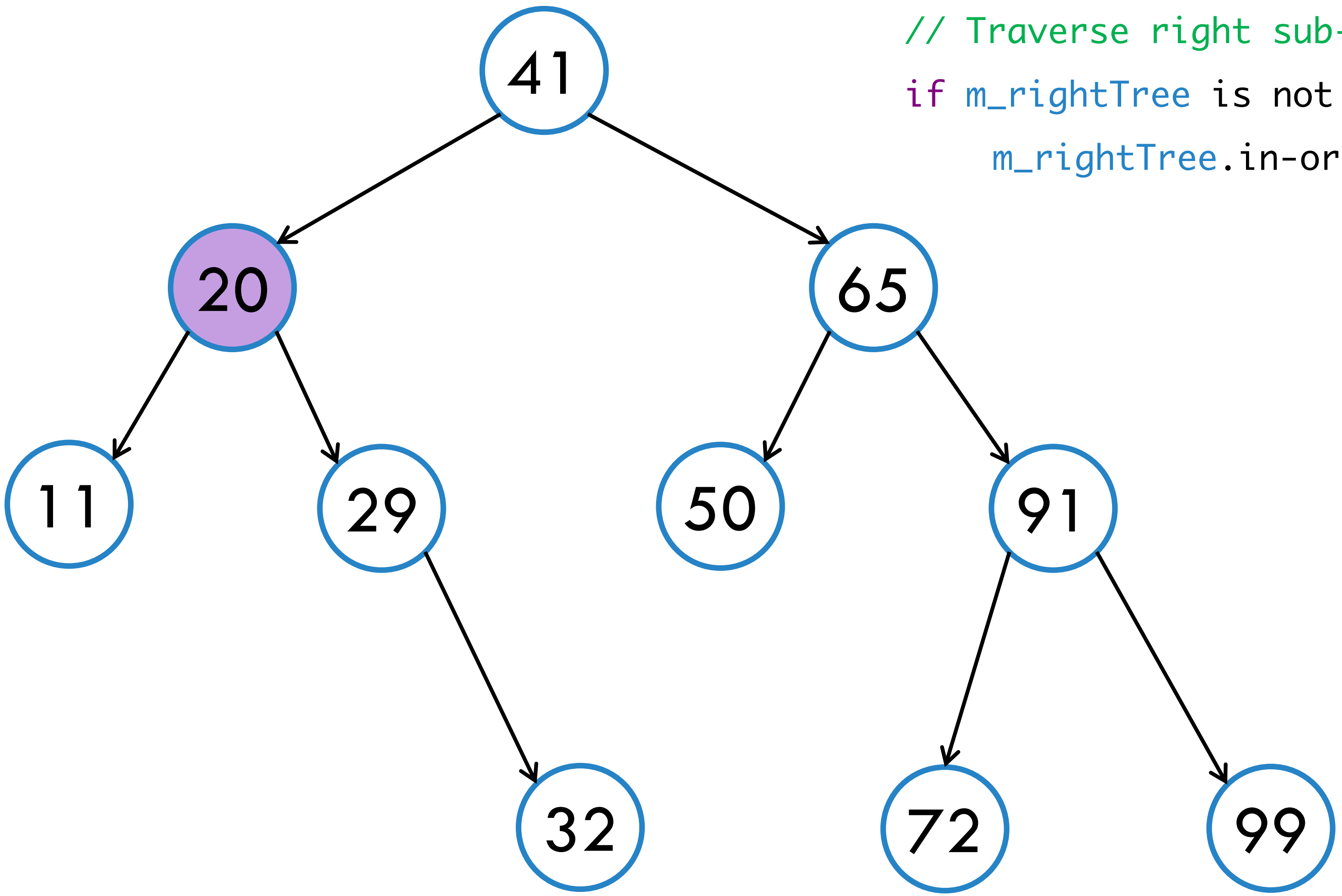

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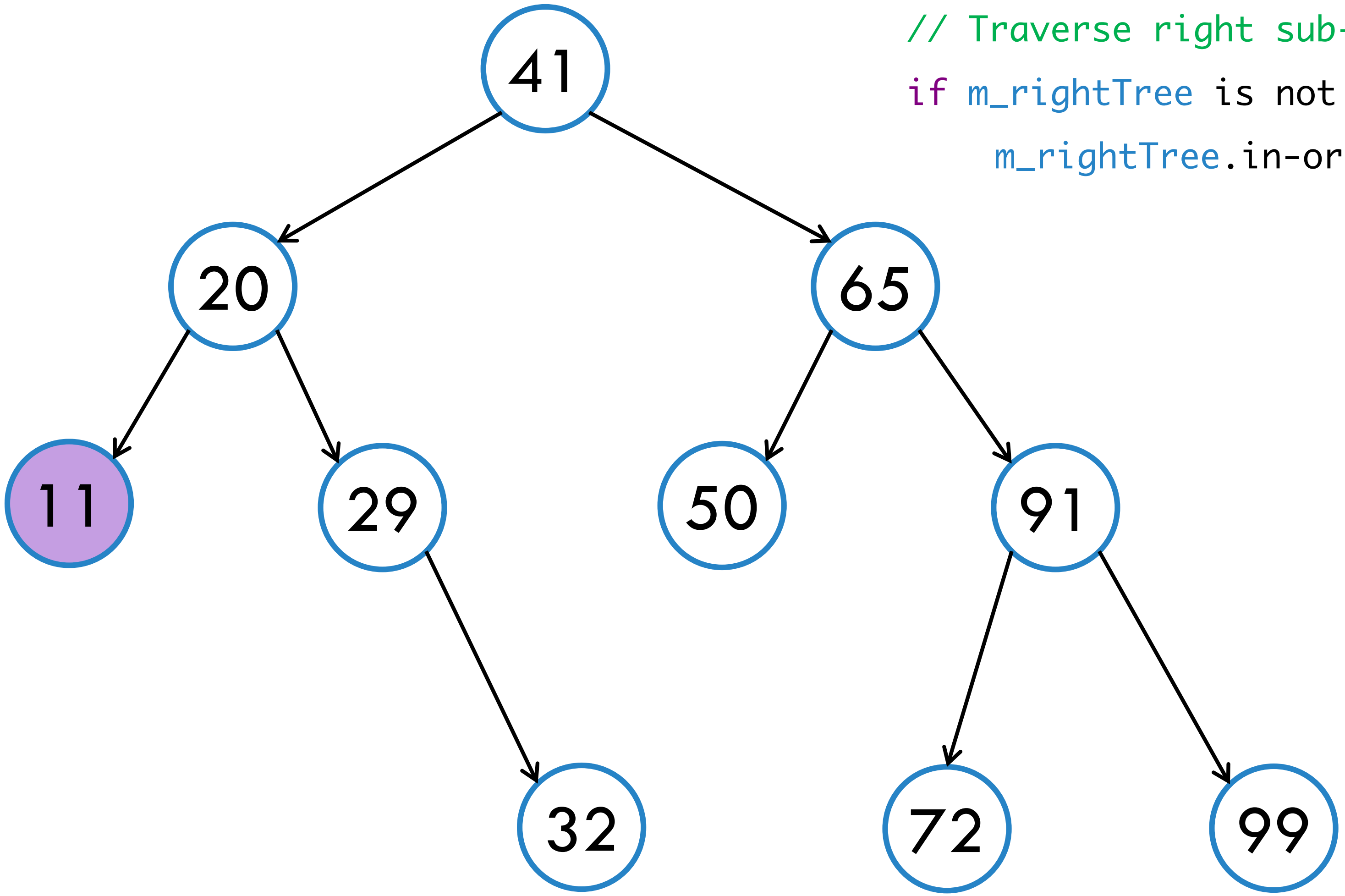

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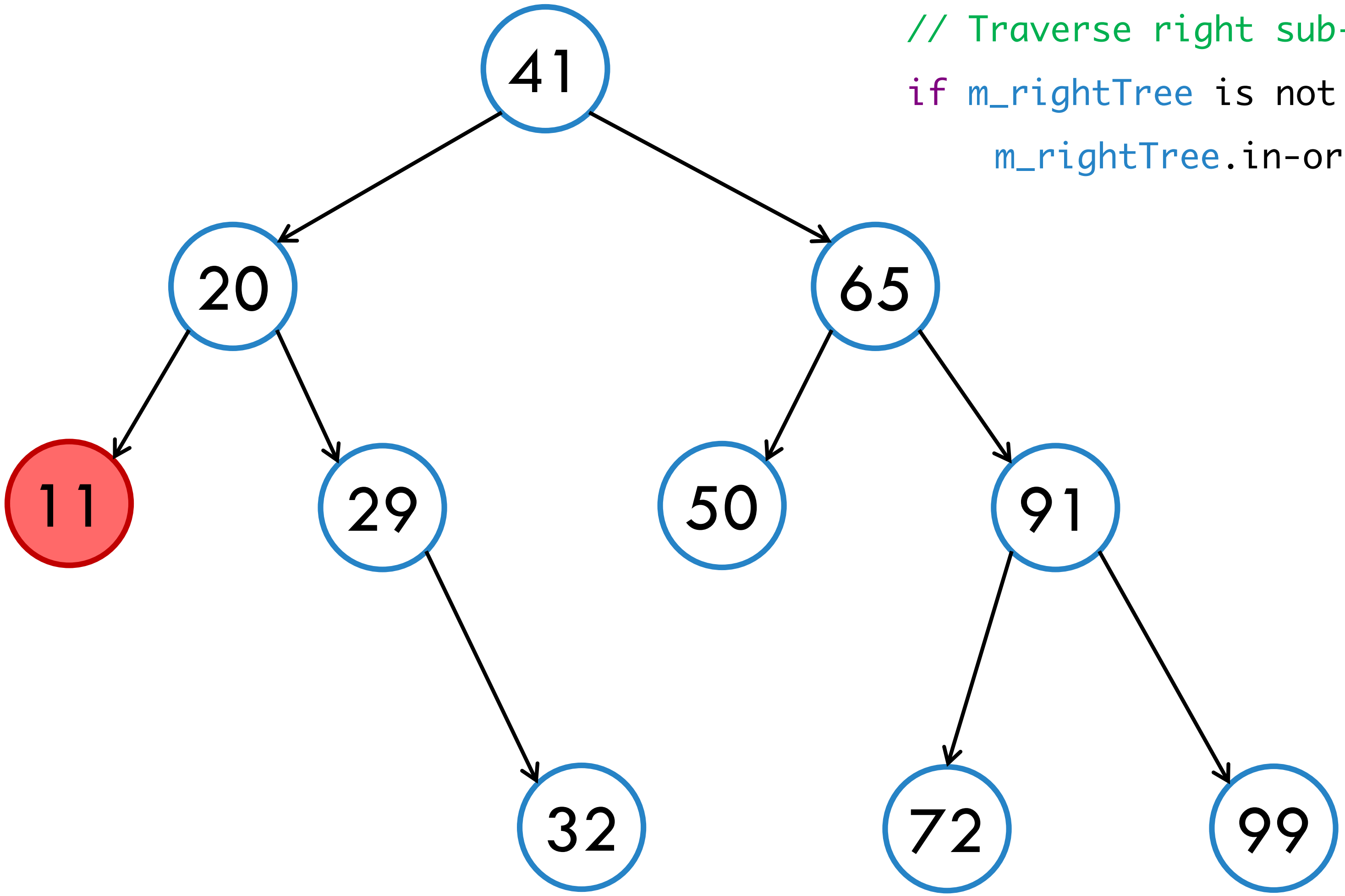

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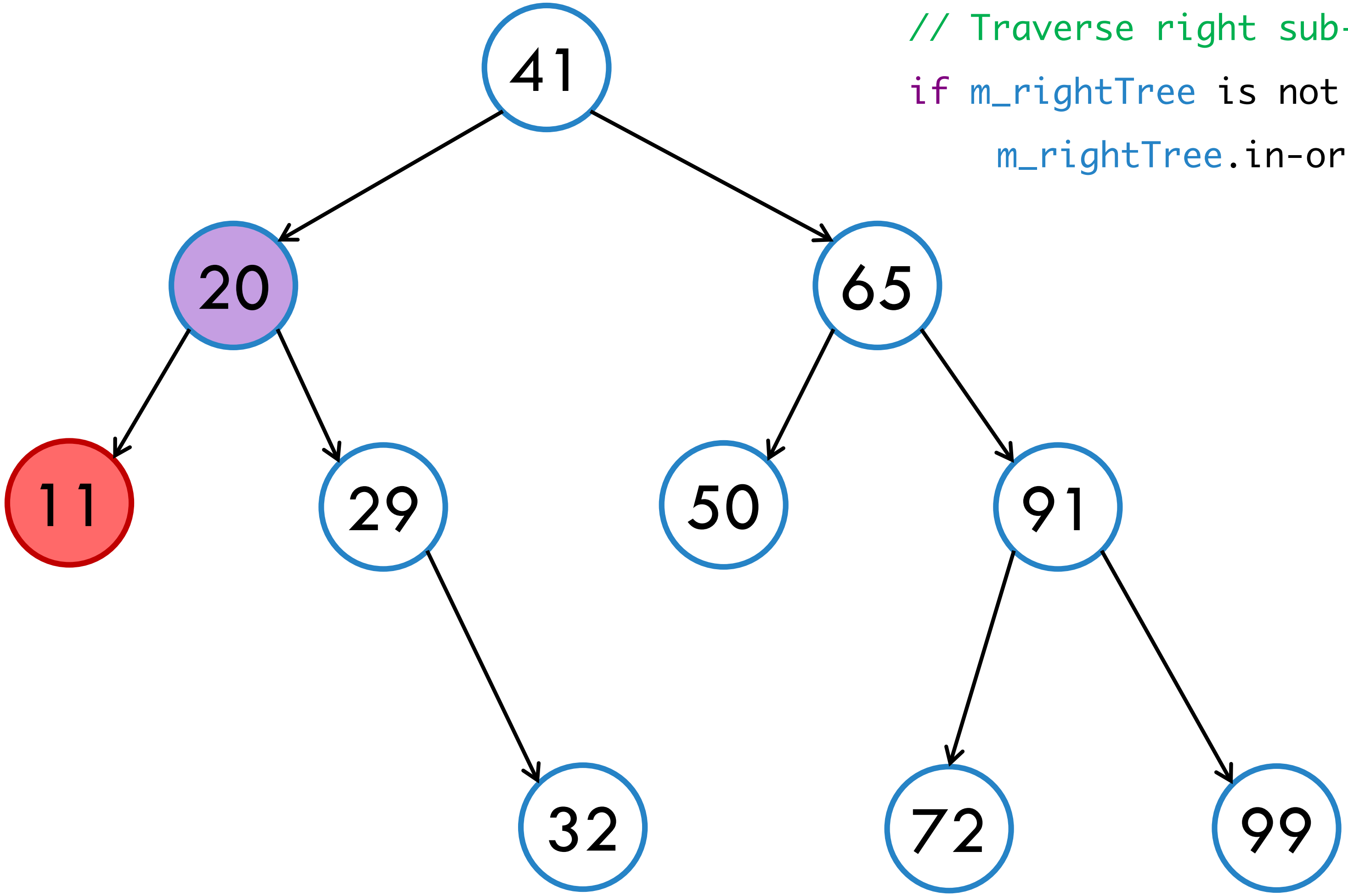

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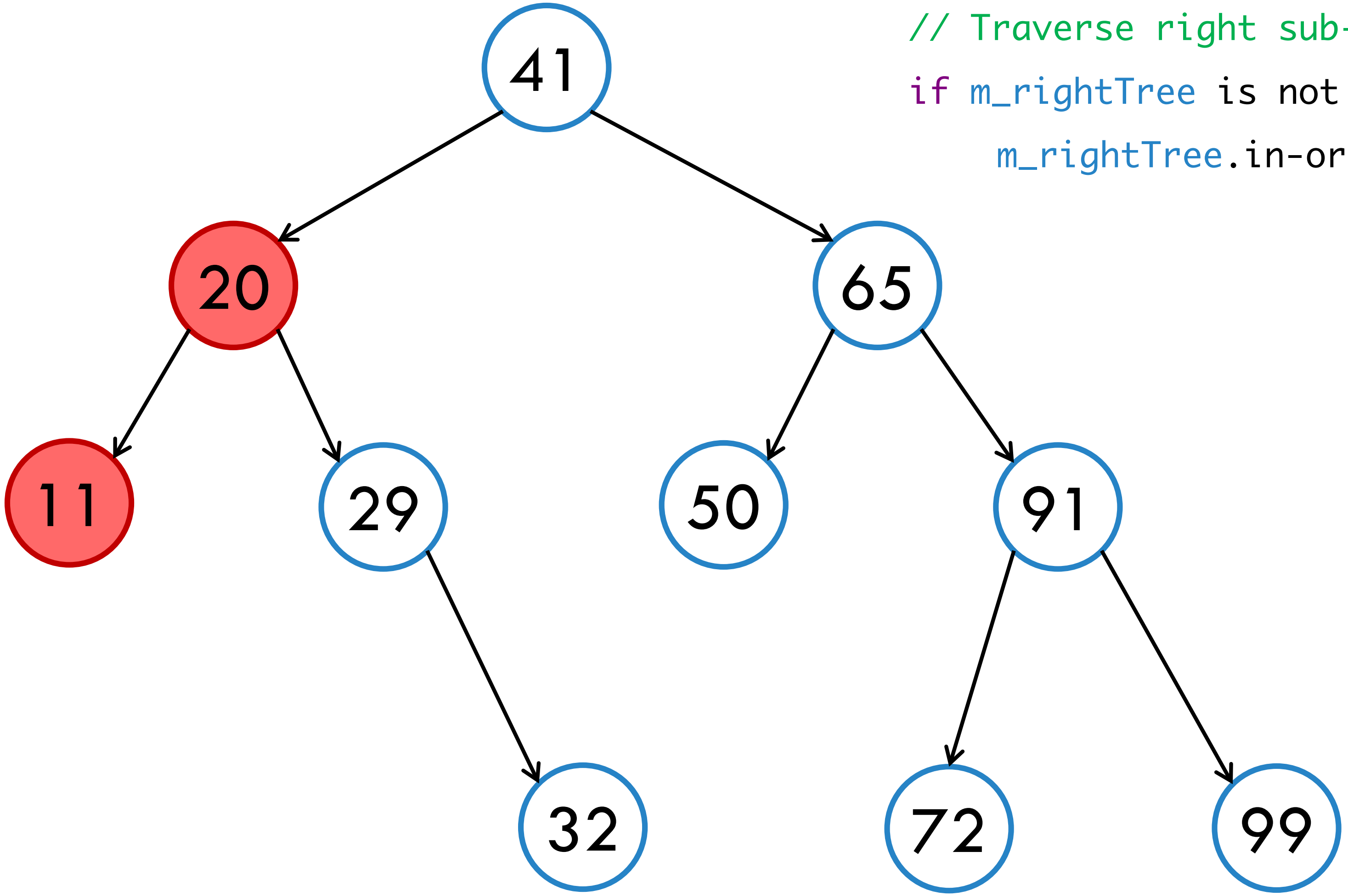

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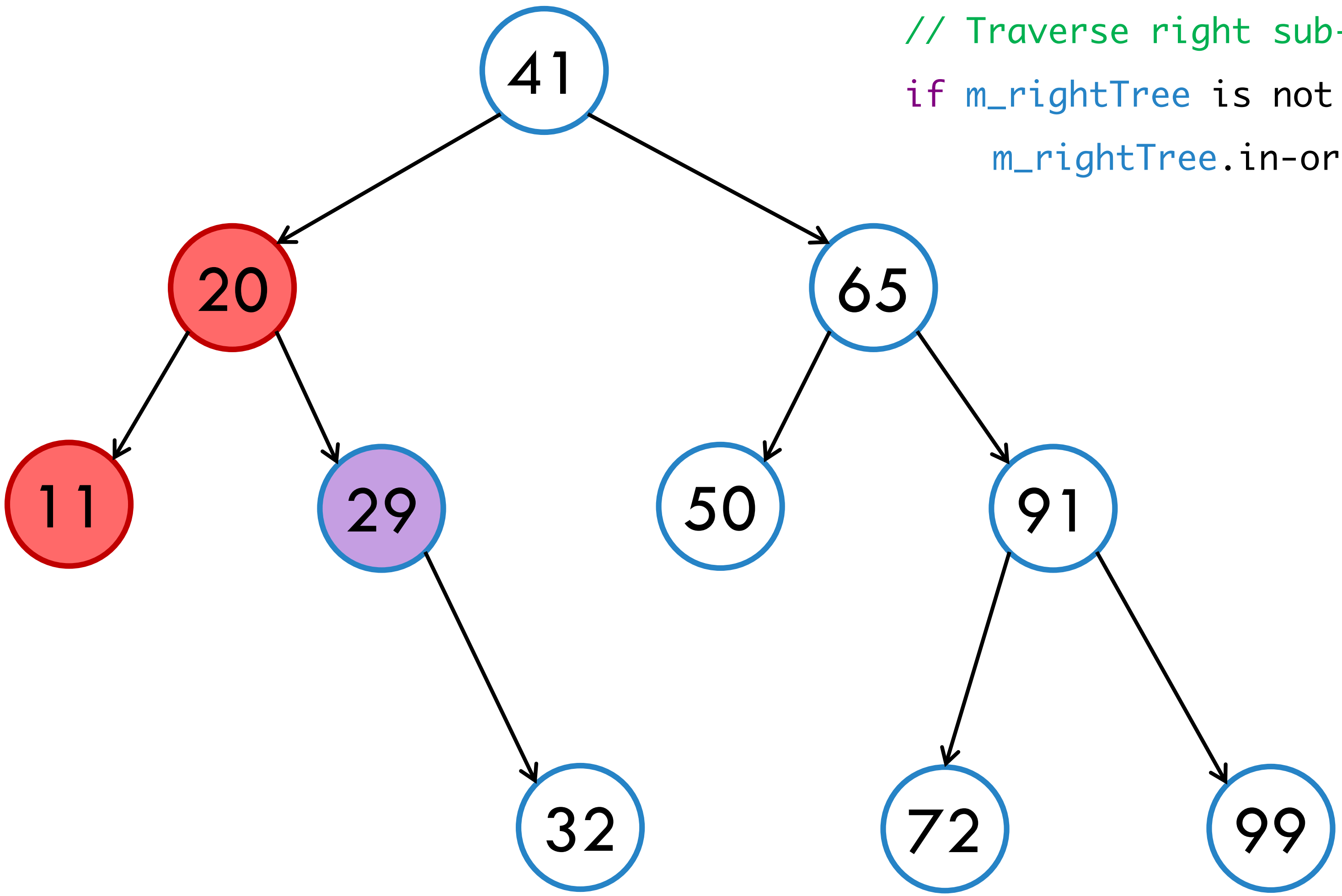

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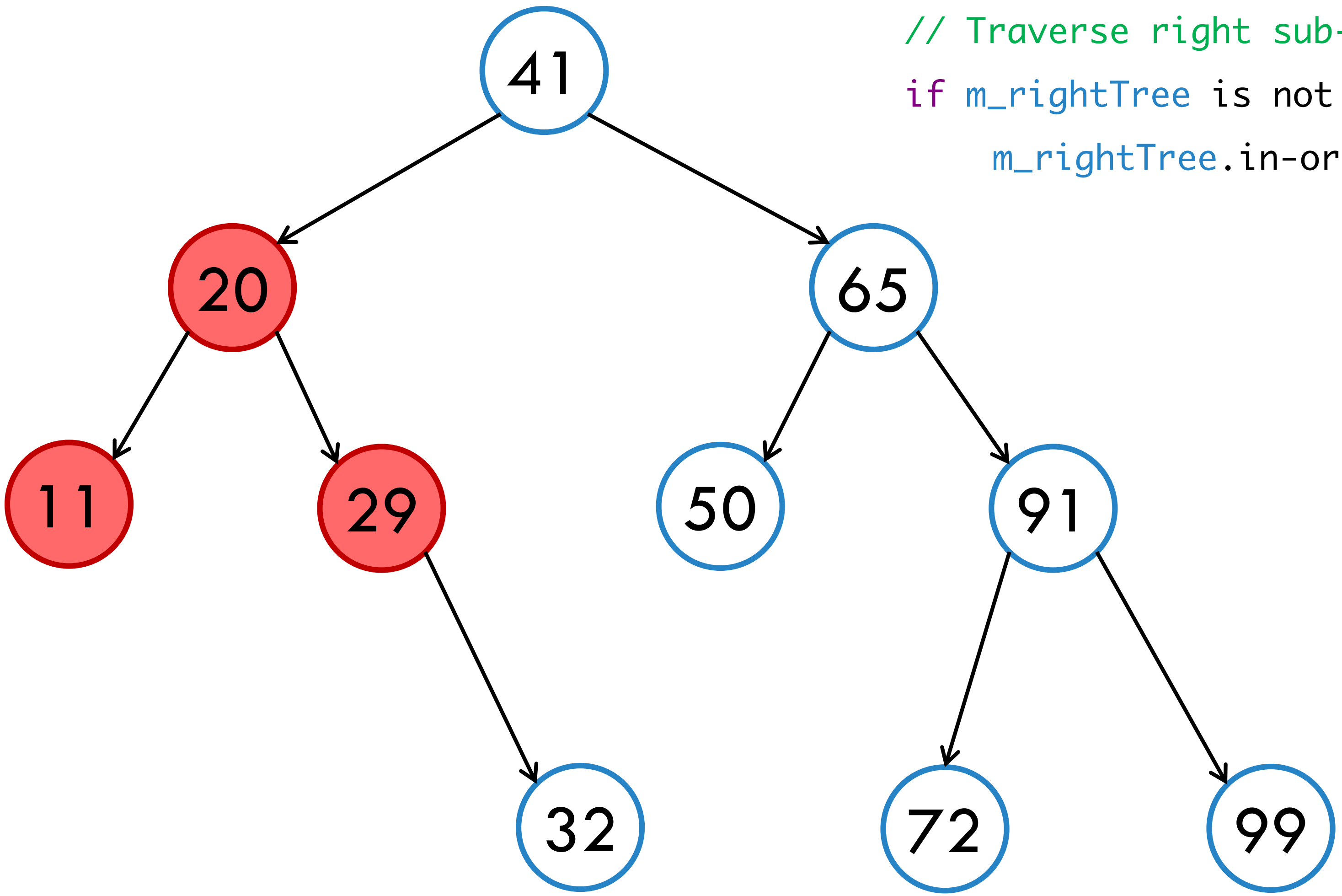

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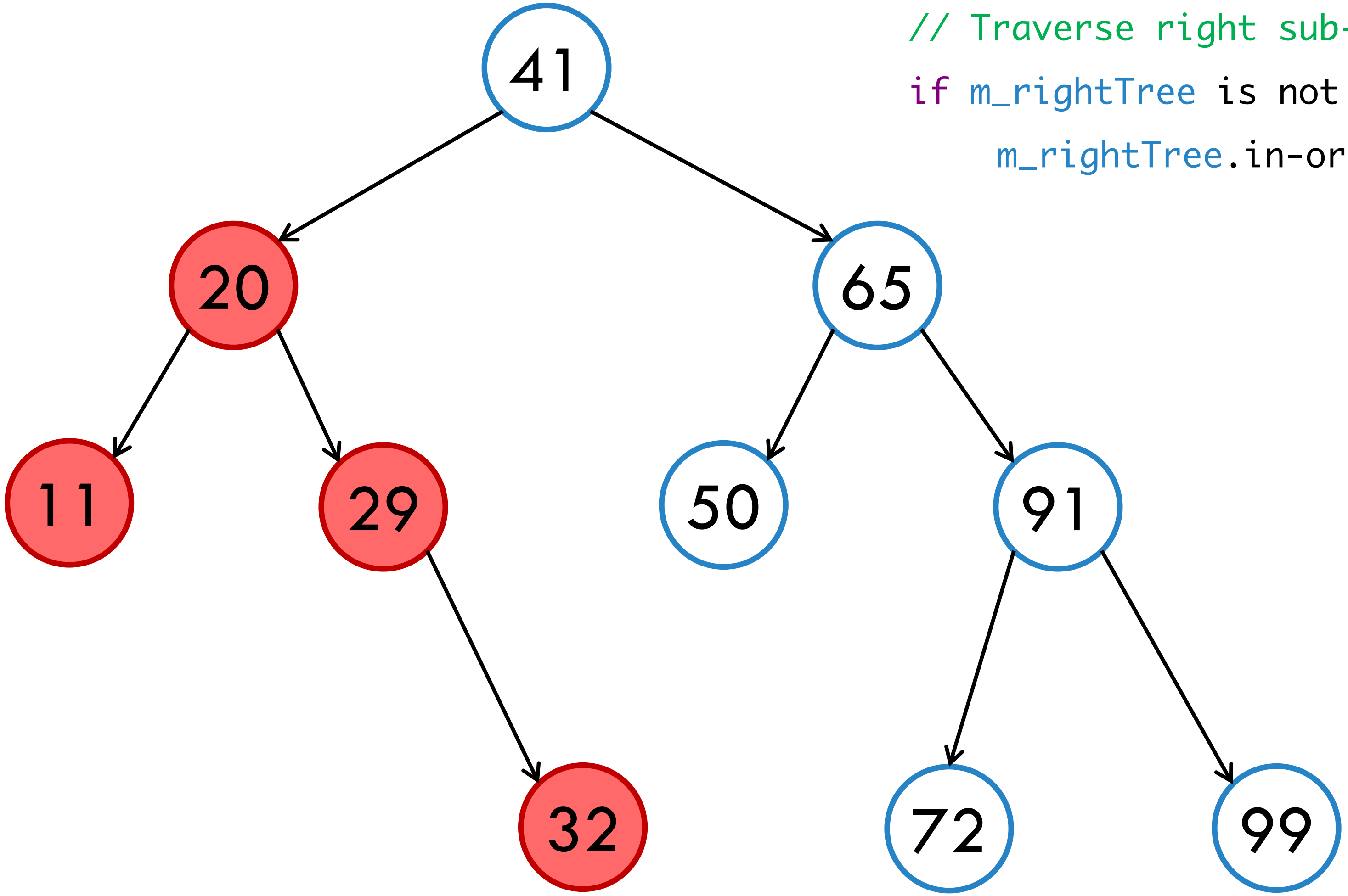

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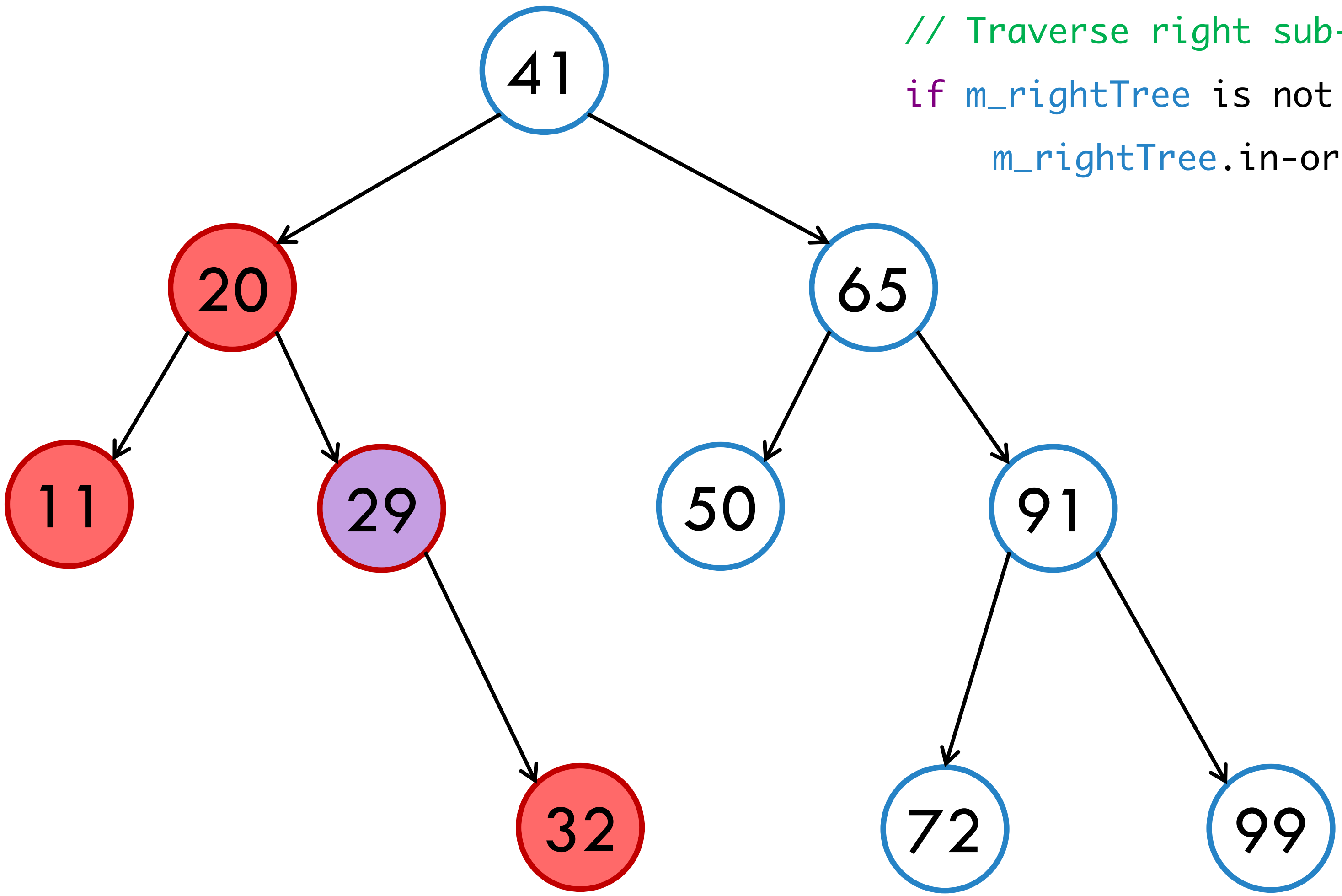

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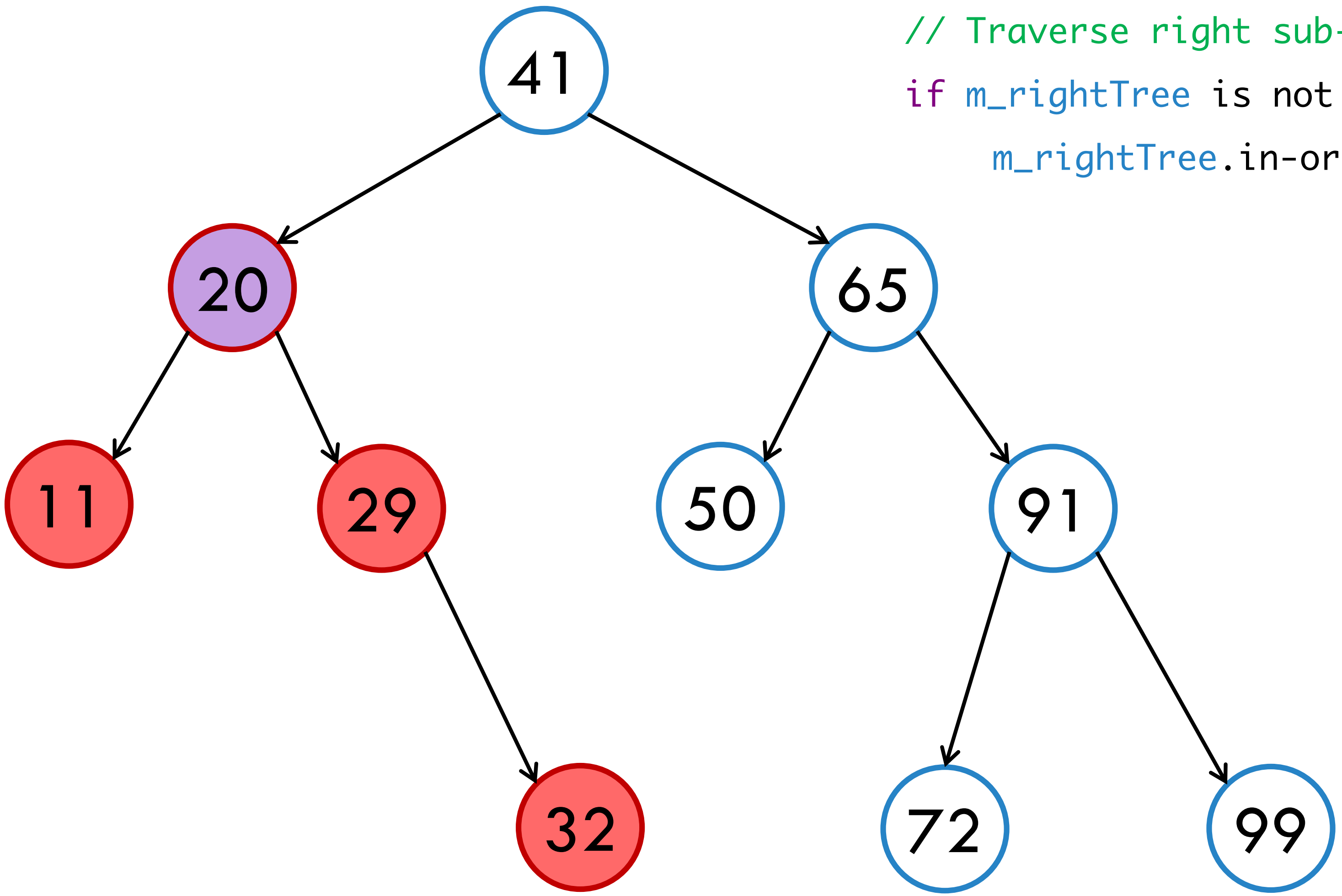

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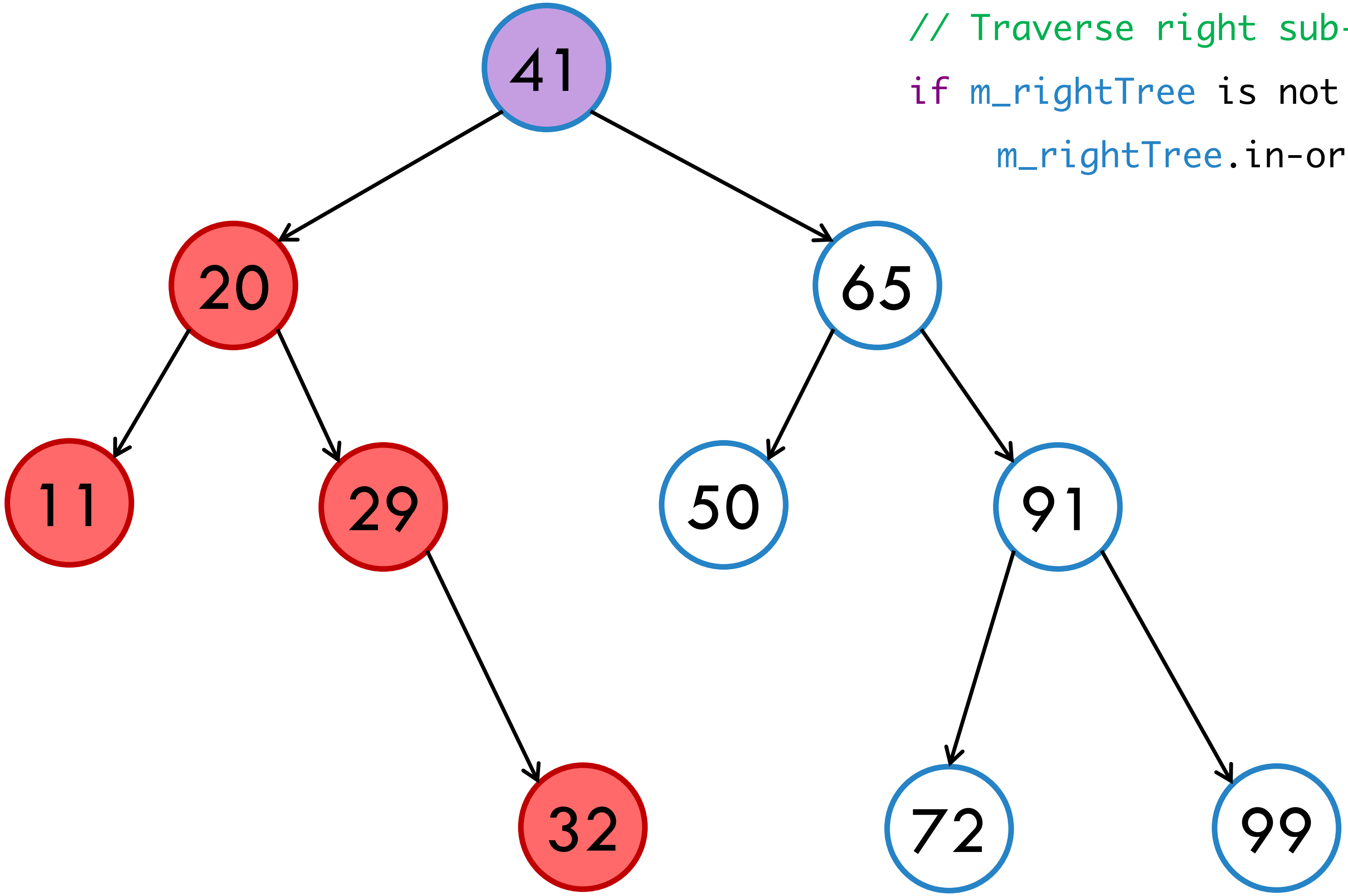
    // Traverse right sub-tree
    if m_rightTree is not null then
        m_rightTree.in-order-traversal()
```



INORDER EXAMPLE:

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function in-order-traversal()
    // Traverse left sub-tree
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    // Traverse right sub-tree
    if m_rightTree is not null then
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```

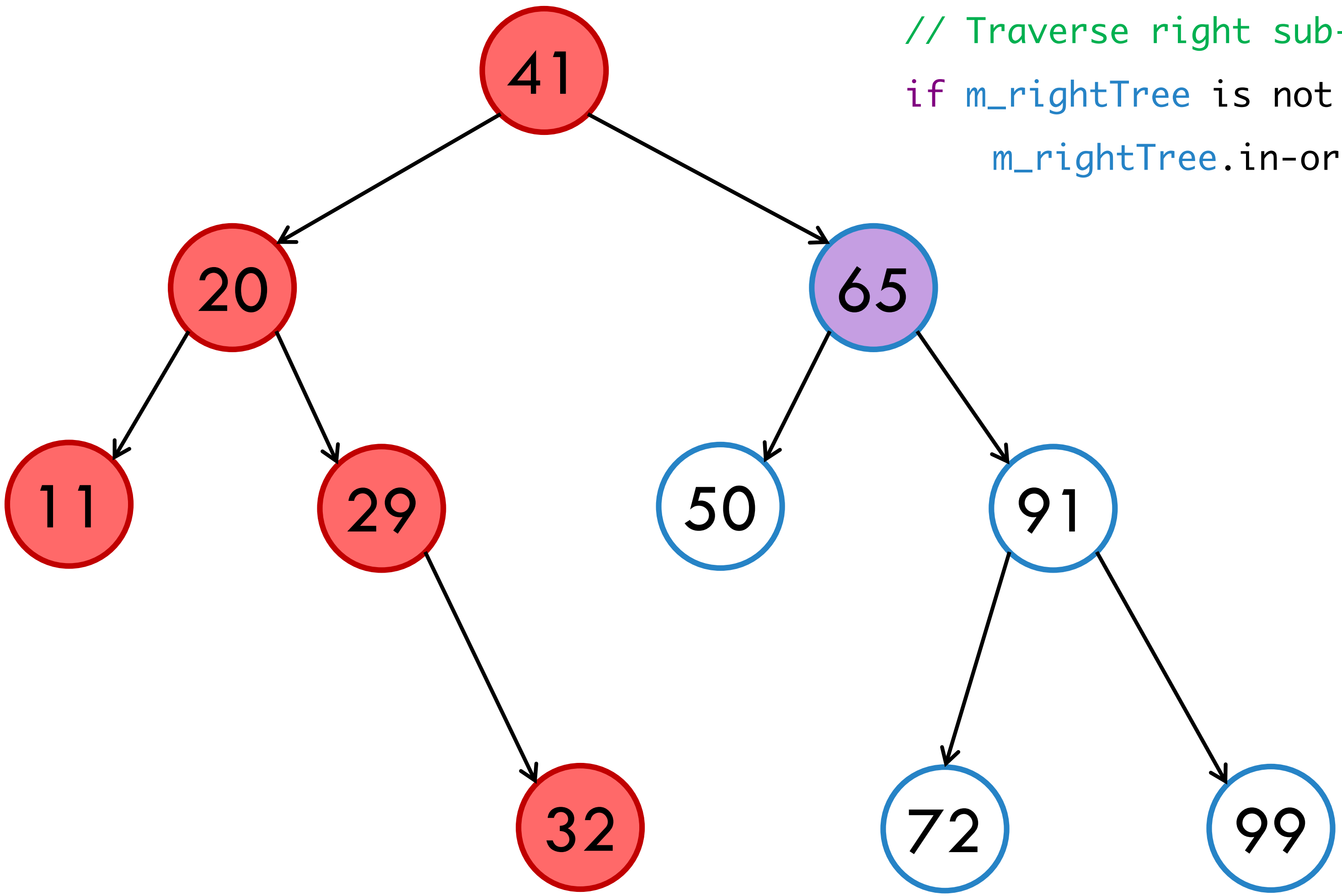

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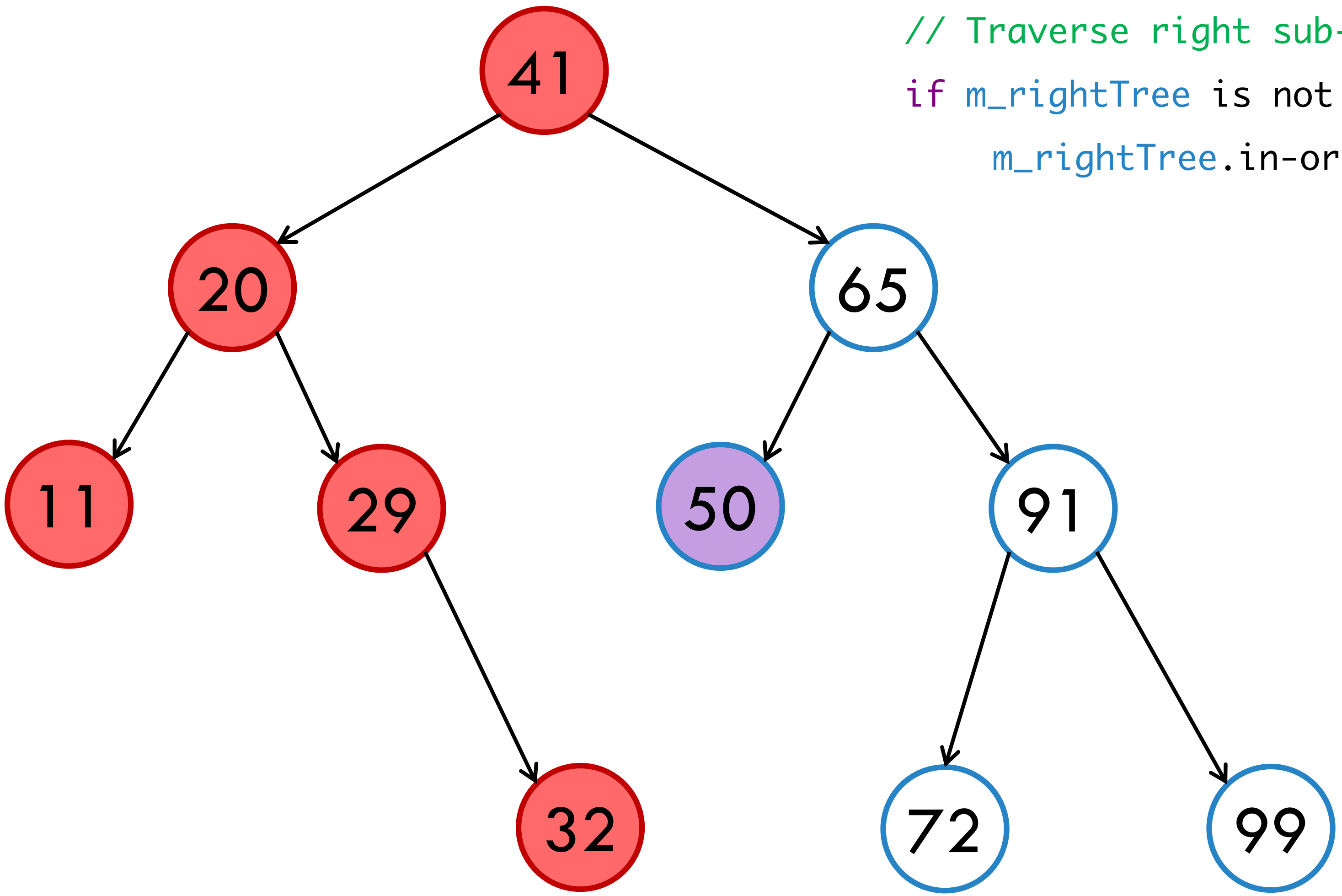

INORDER EXAMPLE:

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    if m_leftTree is not null
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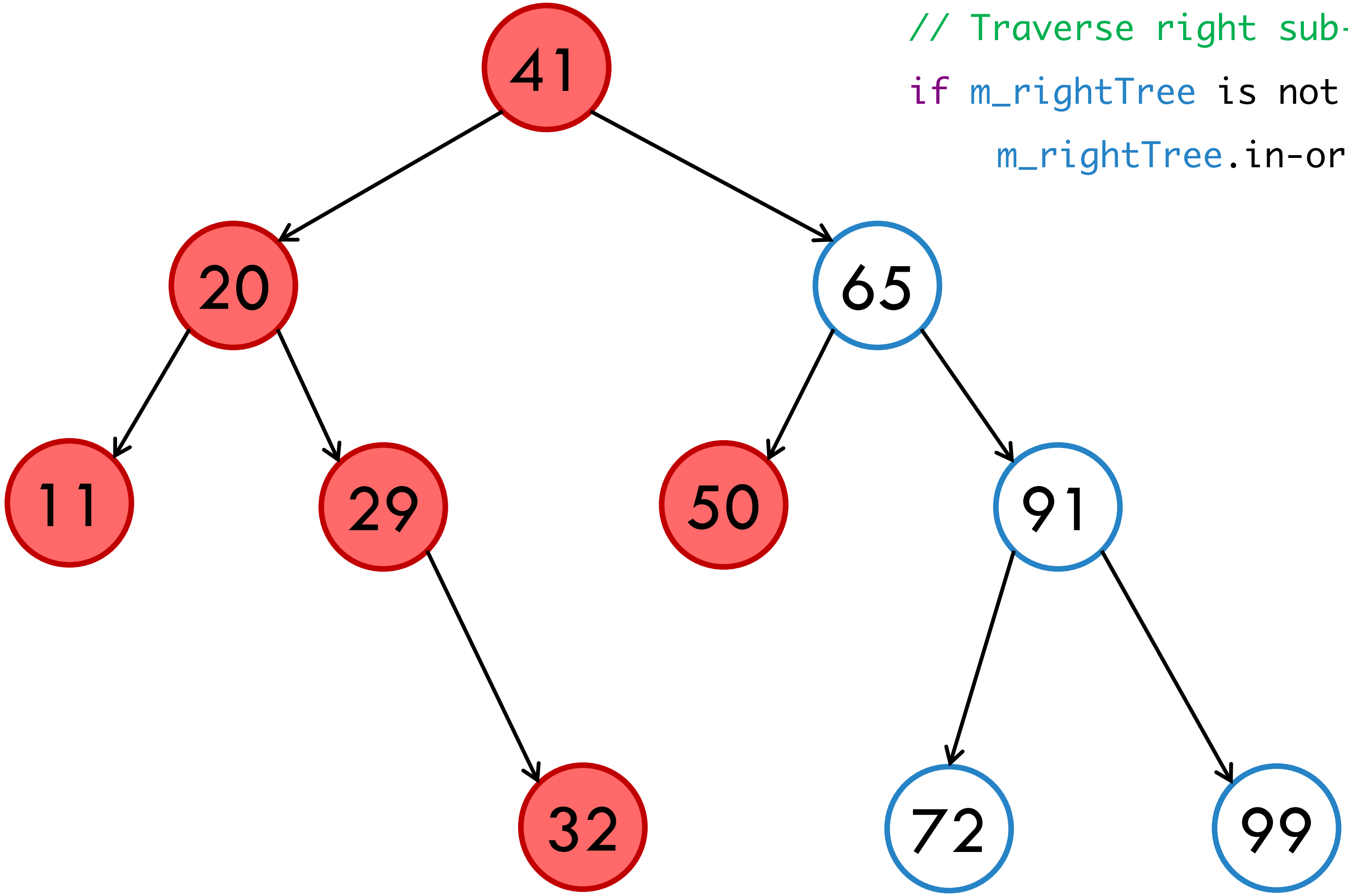

INORDER EXAMPLE:

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    // Traverse left sub-tree
    if m_leftTree is not null
        m_leftTree.in-order-traversal()
    visit(this)
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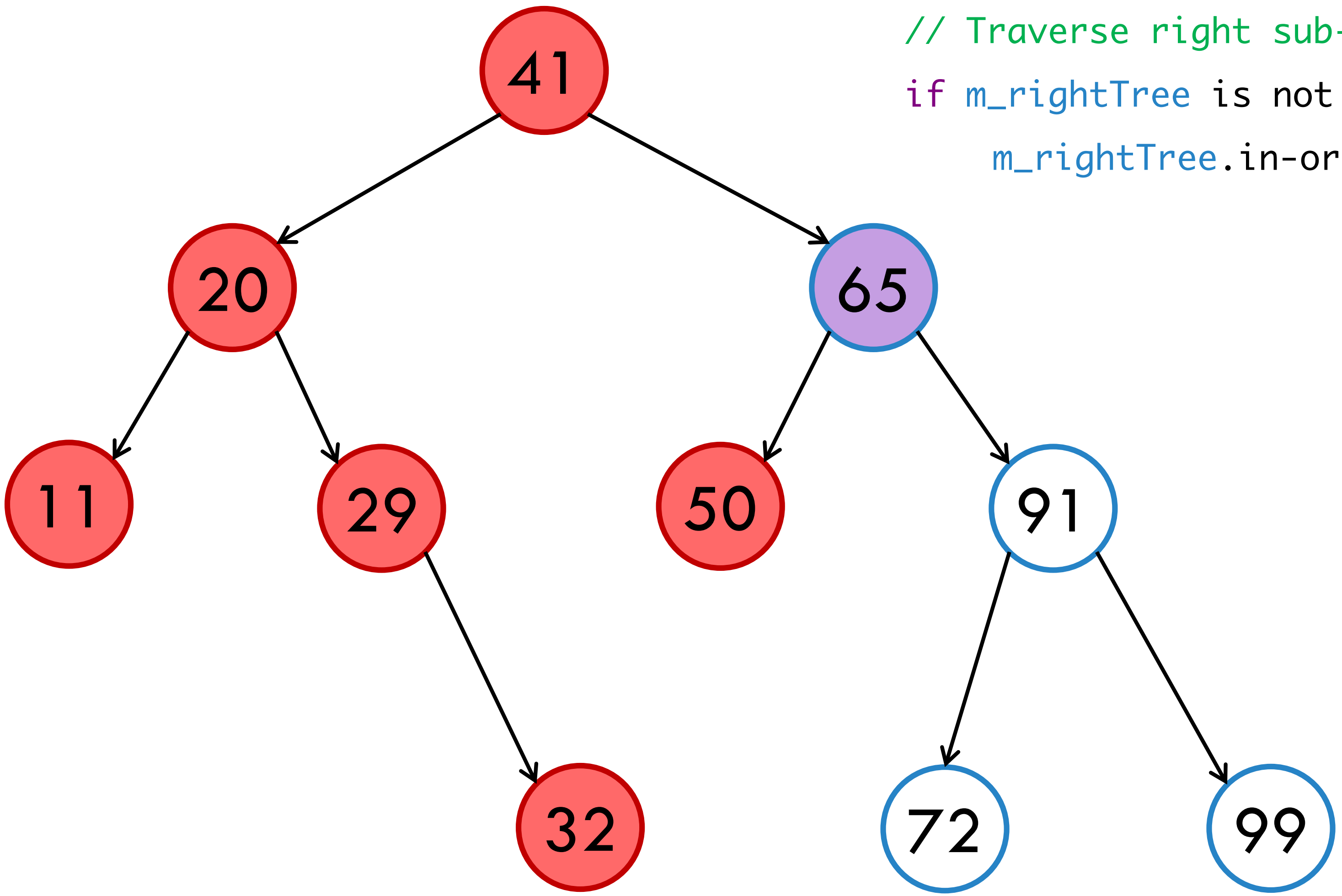

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    if m_leftTree is not null
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    if m_rightTree is not null then
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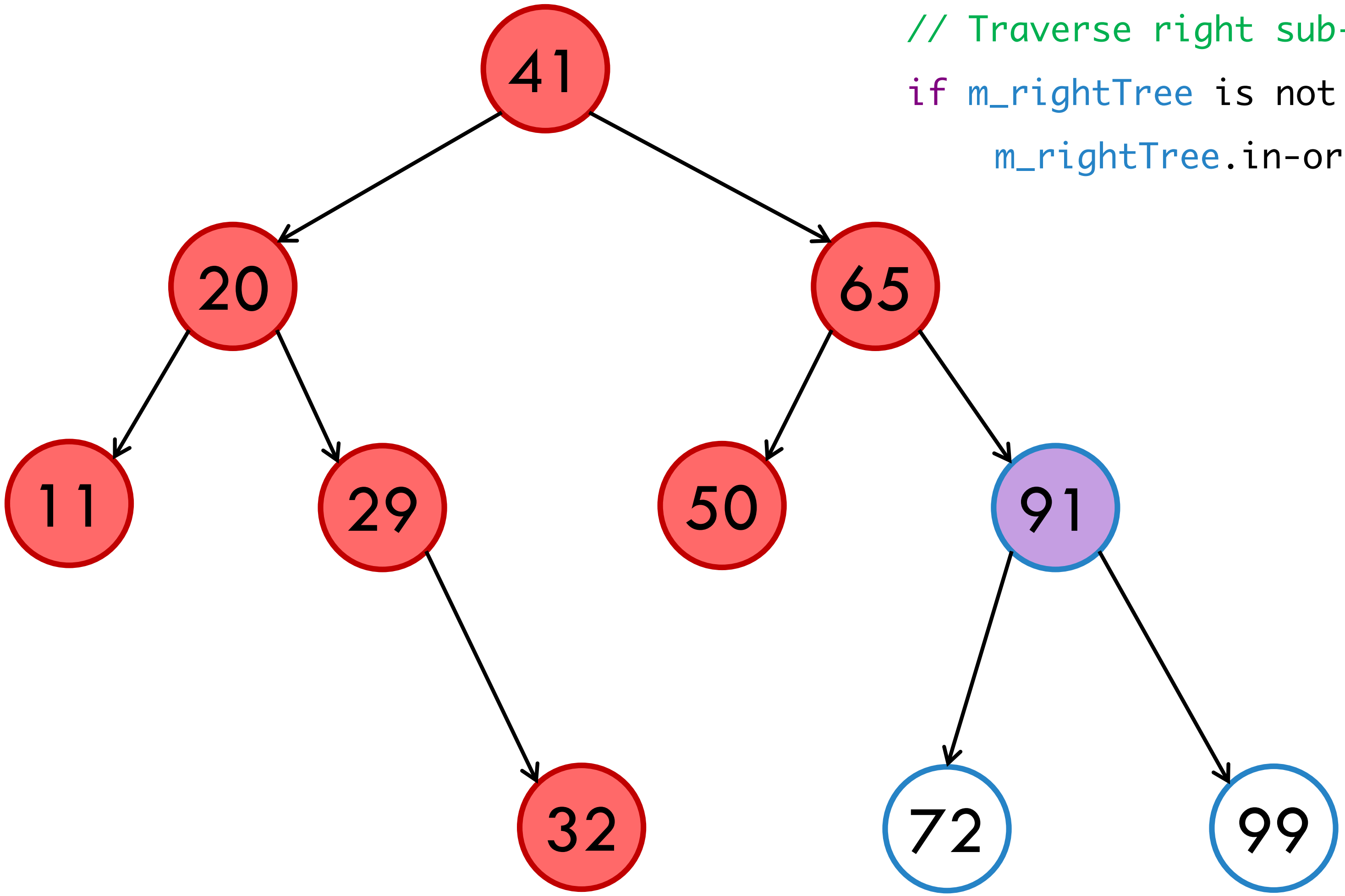

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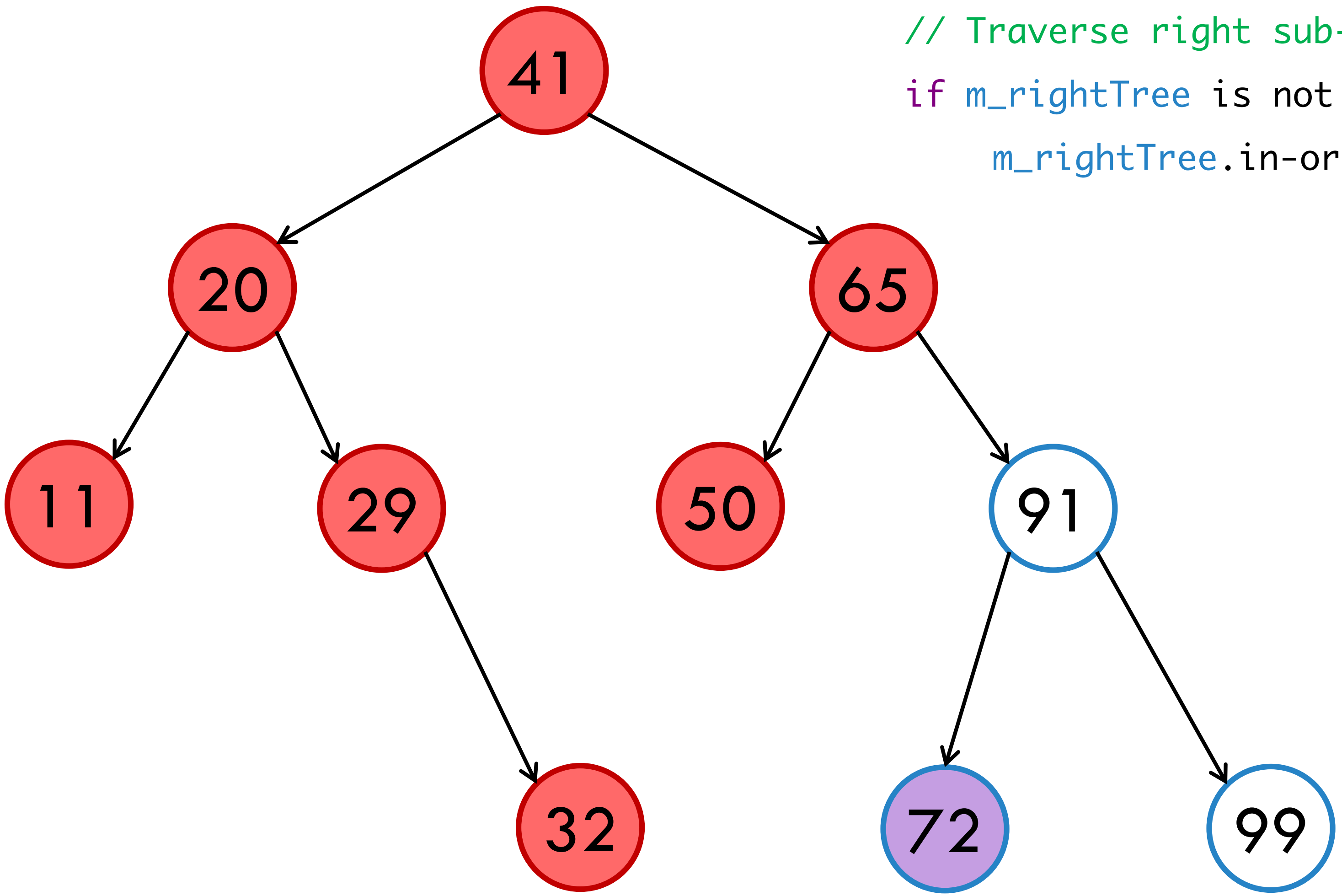

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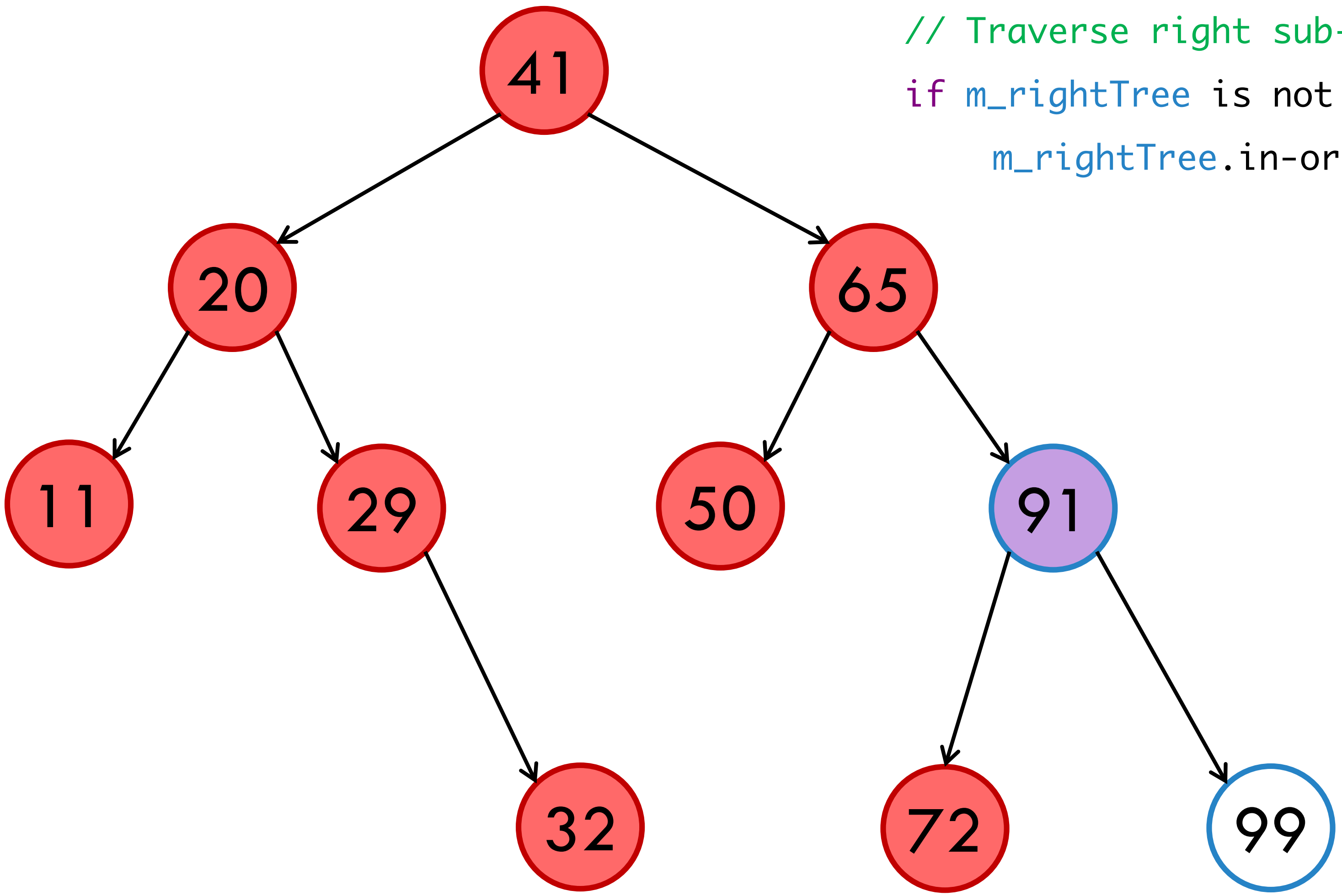
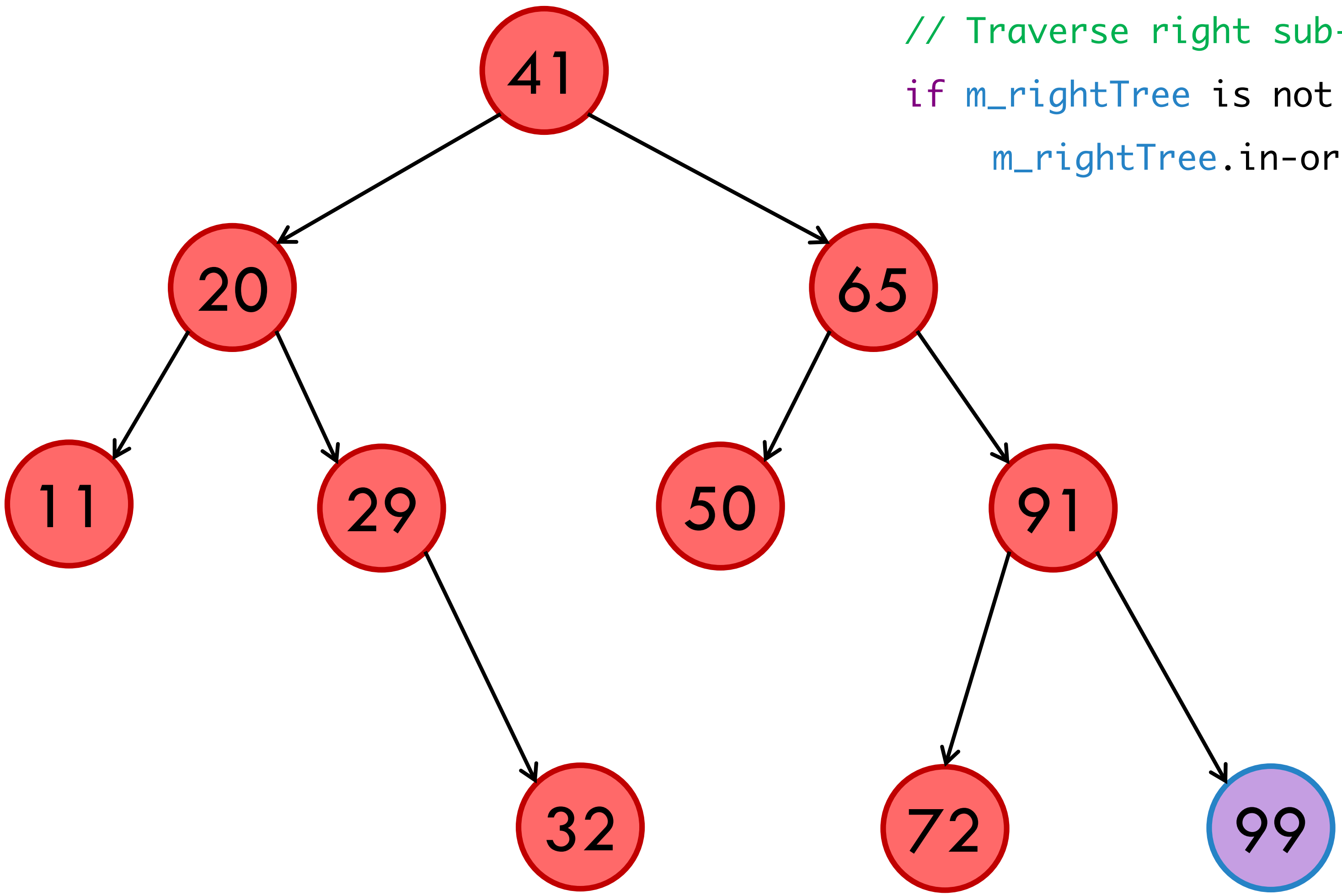


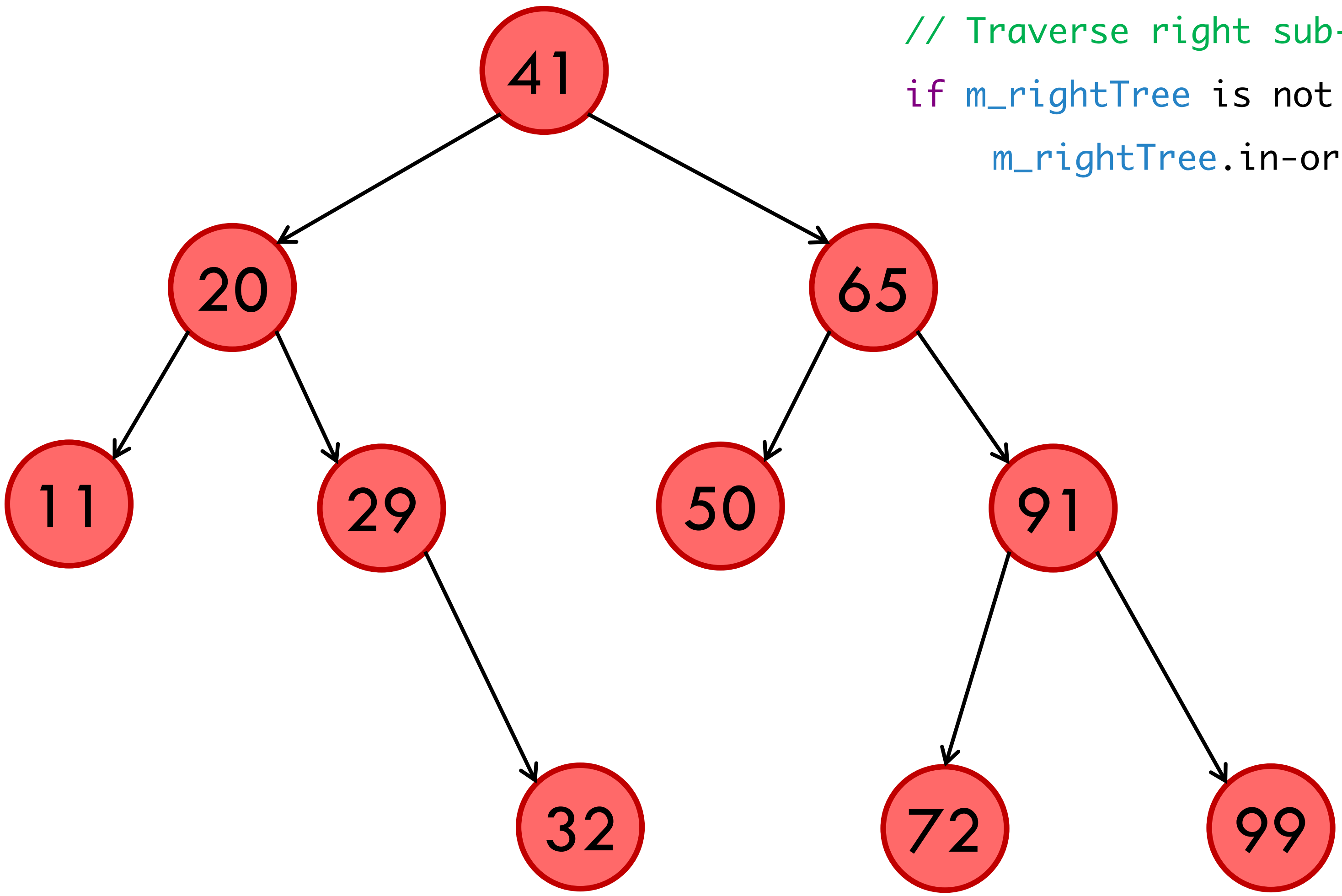
INORDER EXAMPLE:

```
function in-order-traversal()
    // Traverse left sub-tree
    if m_leftTree is not null
        m_leftTree.in-order-traversal()
    visit(this)
    // Traverse right sub-tree
    if m_rightTree is not null then
        m_rightTree.in-order-traversal()
```



INORDER EXAMPLE:

```
function in-order-traversal()
    // Traverse left sub-tree
    if m_leftTree is not null
        m_leftTree.in-order-traversal()
    visit(this)
    // Traverse right sub-tree
    if m_rightTree is not null then
        m_rightTree.in-order-traversal()
```



INORDER: JAVA CODE

```
public void in-order-traversal(){
    // Traverse left sub-tree
    if (m_leftTree != null)
         m_leftTree.in-order-traversal();

    visit(this);

    // Traverse right sub-tree
    if (m_rightTree != null)
        m_rightTree.in-order-traversal();
}
```

TREE TRAVERSALS

Pre-Order	In-Order	Post-order
SELF LEFT-SUBTREE RIGHT-SUBTREE	LEFT-SUBTREE SELF RIGHT-SUBTREE	LEFT-SUBTREE RIGHT-SUBTREE SELF

NOT SO BASIC BST OPERATIONS

`floor(k)`: returns next key $\leq$ k

`ceiling(k)`: returns next key $\geq$ k

`inorder(Node o)`: returns nodes of the tree rooted at o in order

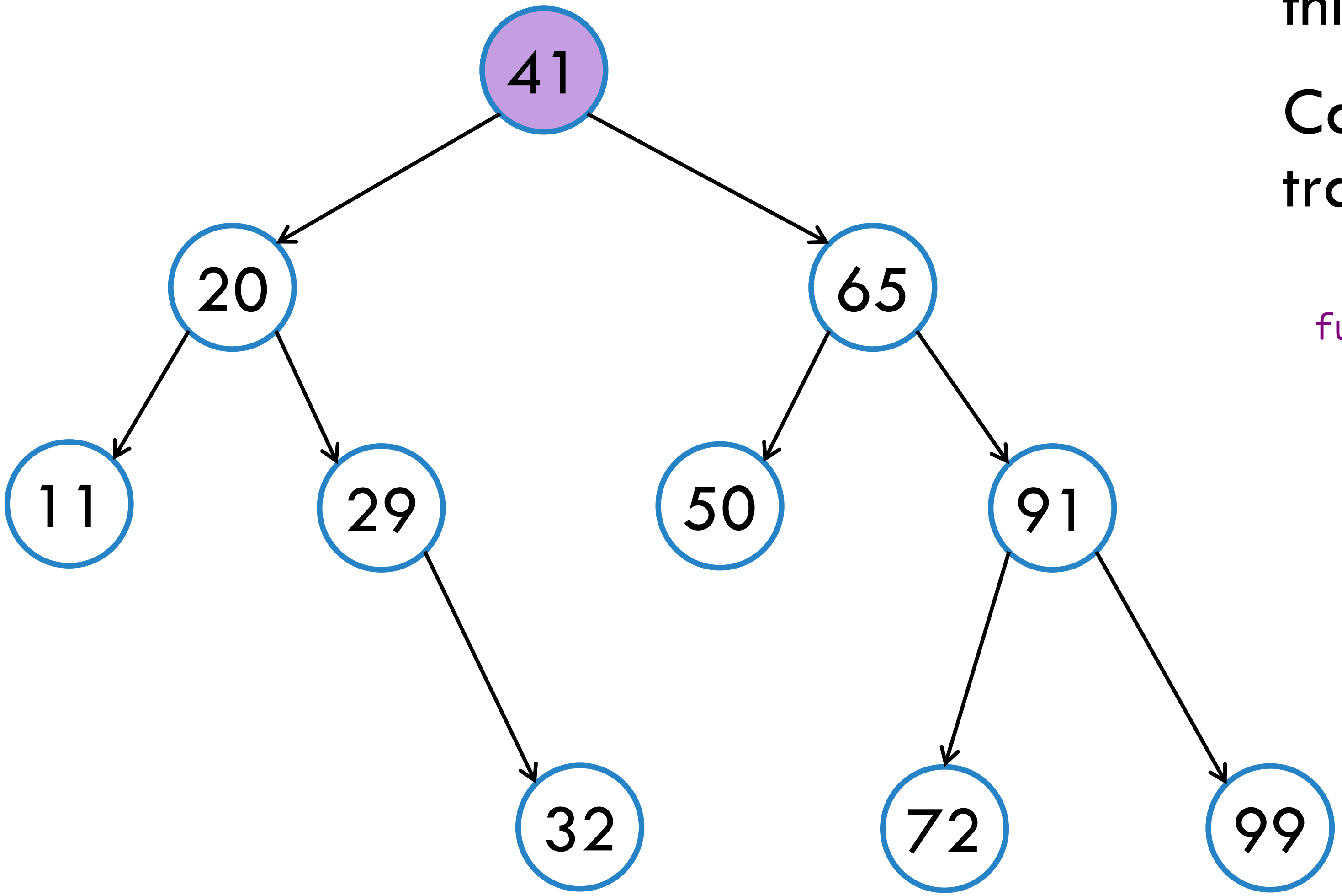
`copyTree(Node o)`: returns a copy of the tree rooted at o

`deleteTree(Node o)`: deletes the tree rooted at o

TREE TRAVERSALS

Pre-Order	In-Order	Post-order
SELF LEFT-SUBTREE RIGHT-SUBTREE	LEFT-SUBTREE SELF RIGHT-SUBTREE	LEFT-SUBTREE RIGHT-SUBTREE SELF

DEEP-COPYING A TREE

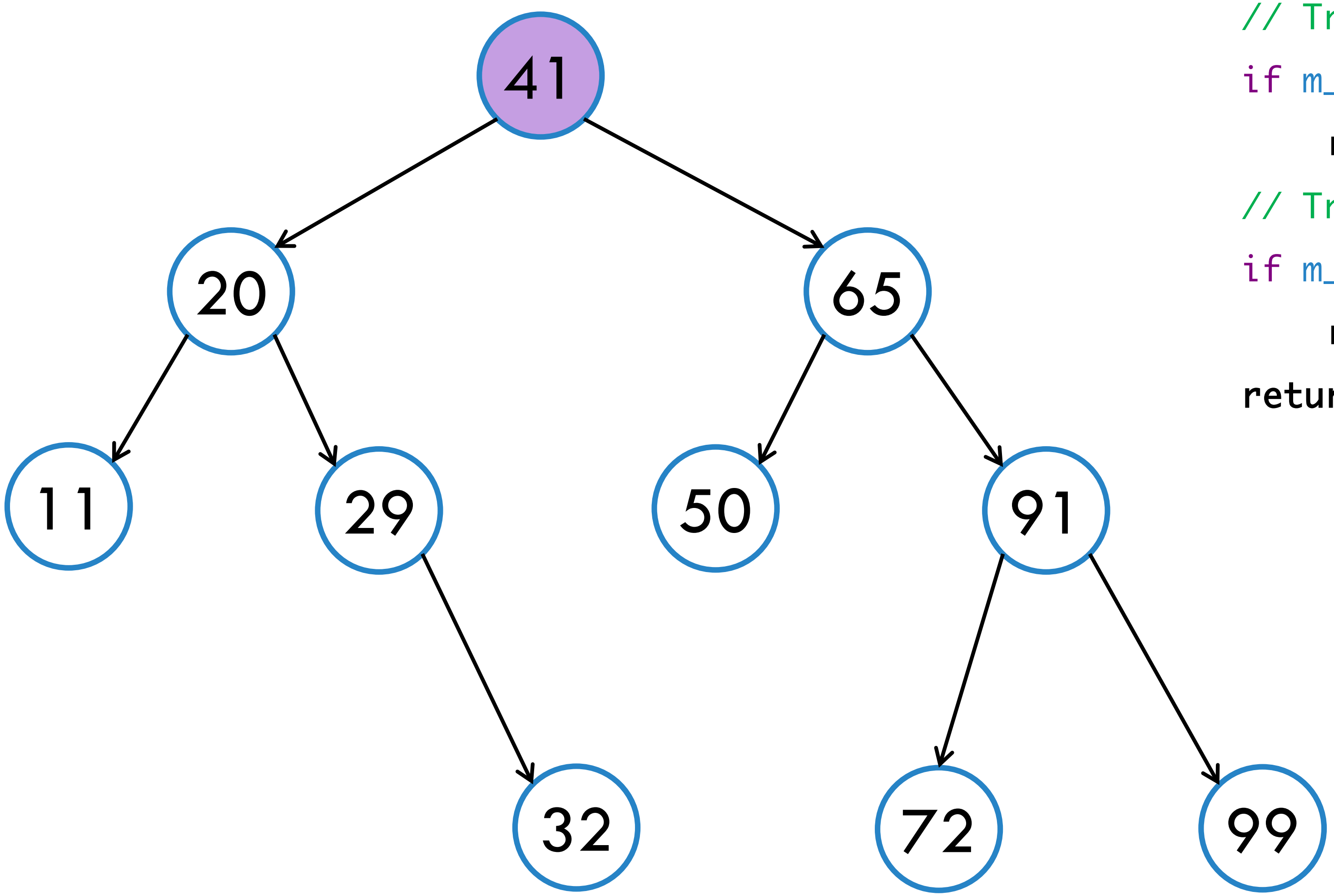


Want to make a copy of this tree.

Can do it via pre-order traversal.

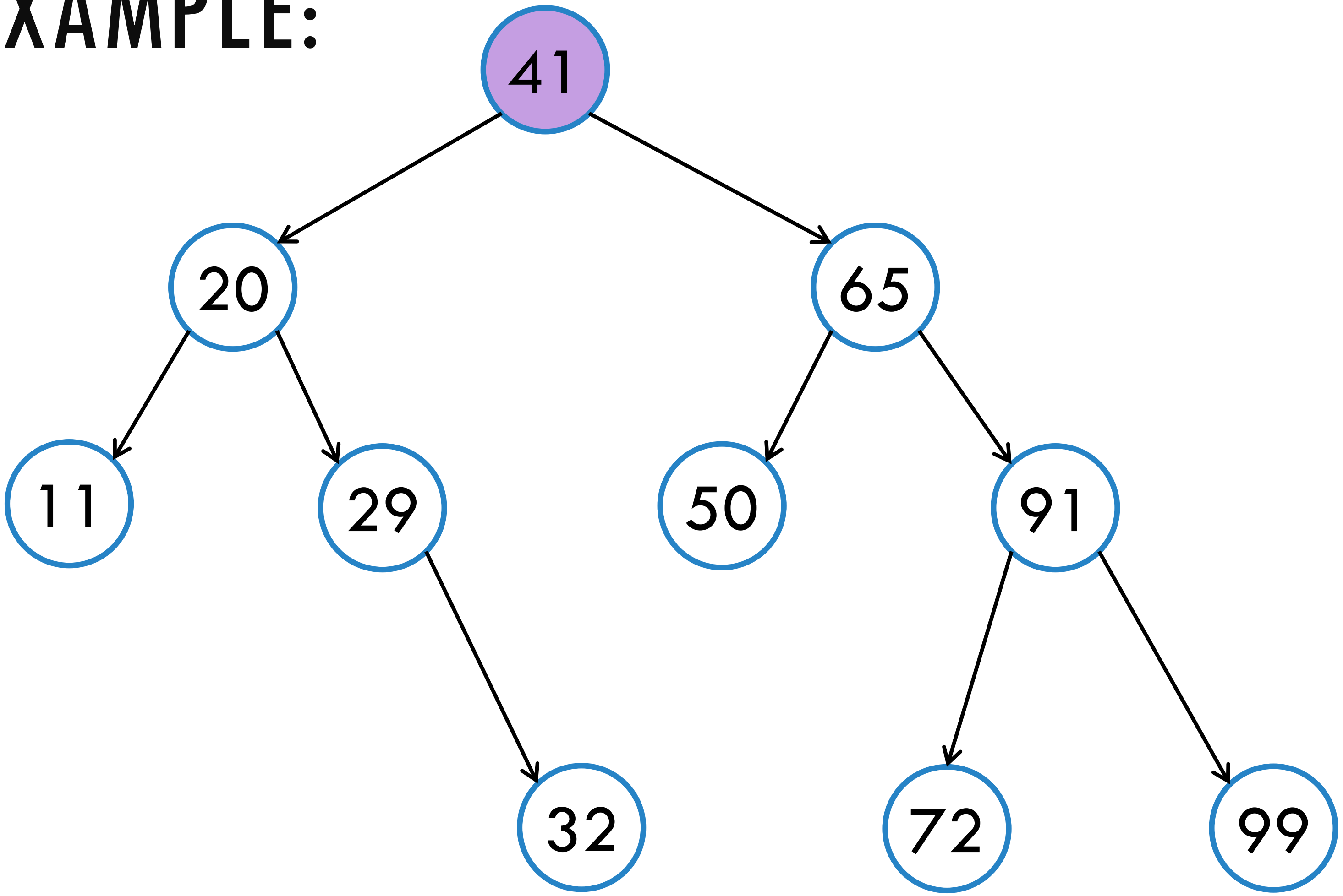
```
function pre-order-traversal()
    visit(this)
    // Traverse left sub-tree
    if m_leftTree is not null
        m_leftTree.pre-order-traversal()
    // Traverse right sub-tree
    if m_rightTree is not null then
        m_rightTree.pre-order-traversal()
```

DEEP-COPYING A TREE



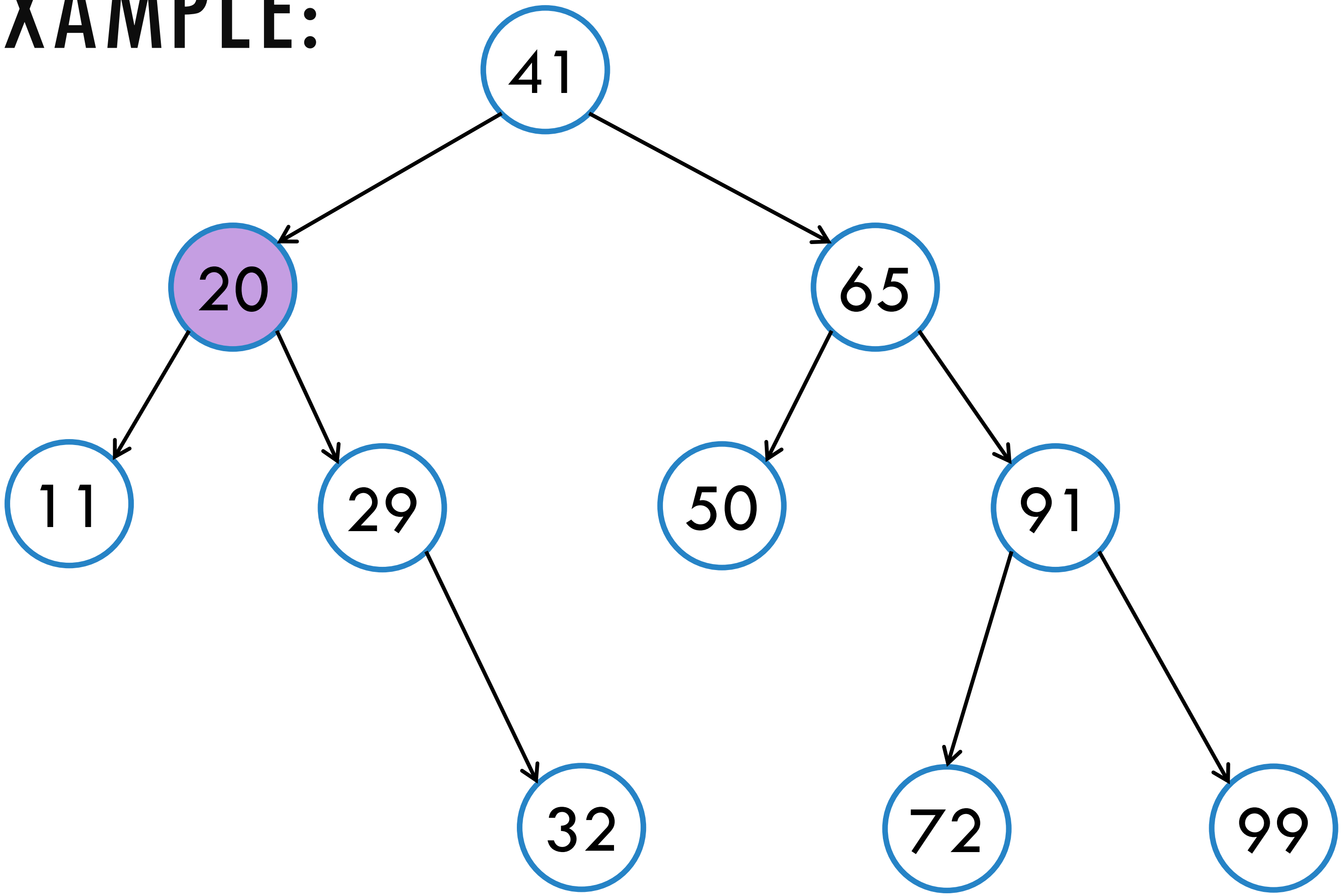

```
function deepCopy()
    n = new Node(key, value)
    // Traverse left sub-tree
    if m_leftTree is not null
        n.m_leftTree = m_leftTree.deepCopy()
    // Traverse right sub-tree
    if m_rightTree is not null
        n.m_rightTree = m_rightTree.deepCopy()
    return n
```

PREORDER EXAMPLE:



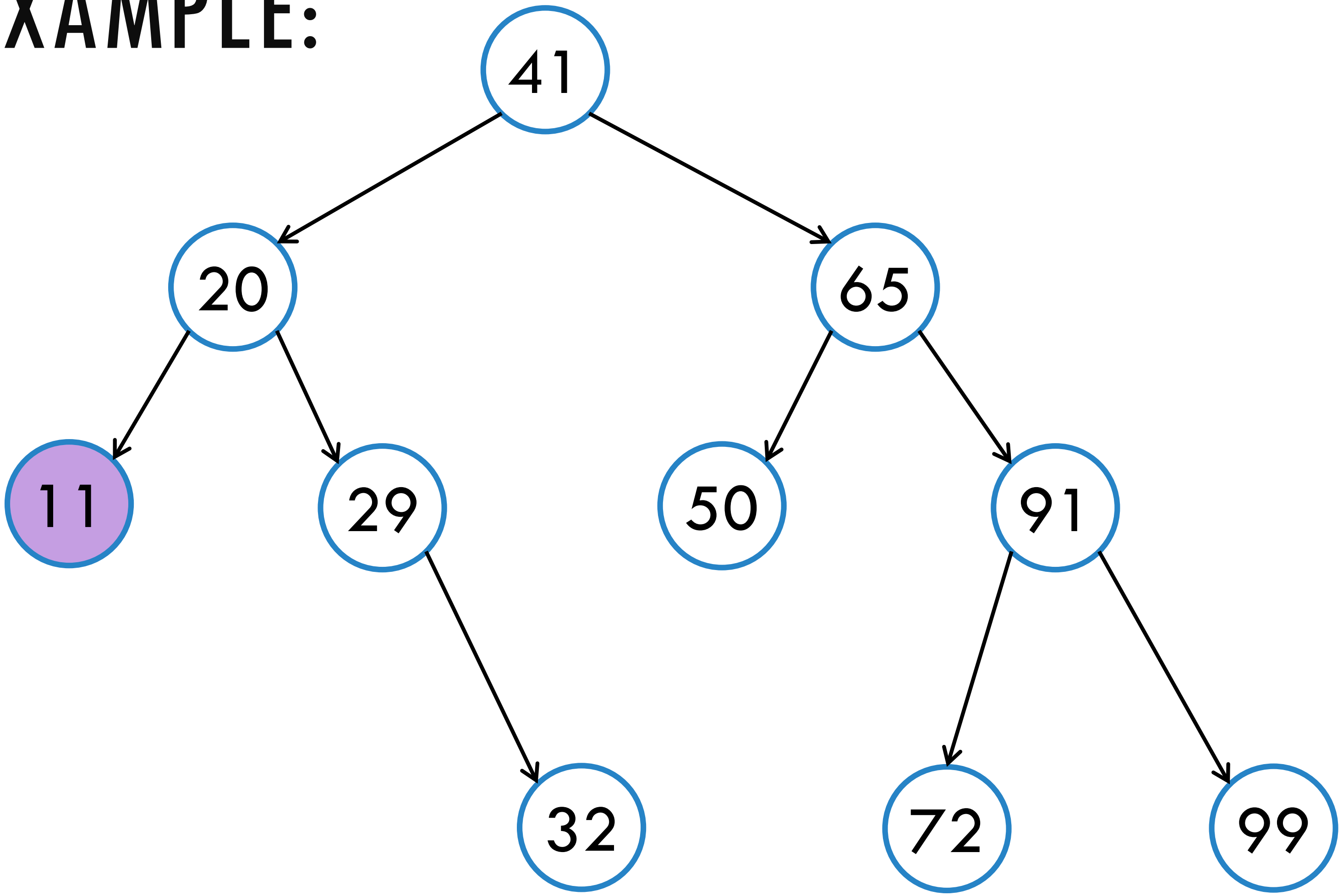
41

PREORDER EXAMPLE:



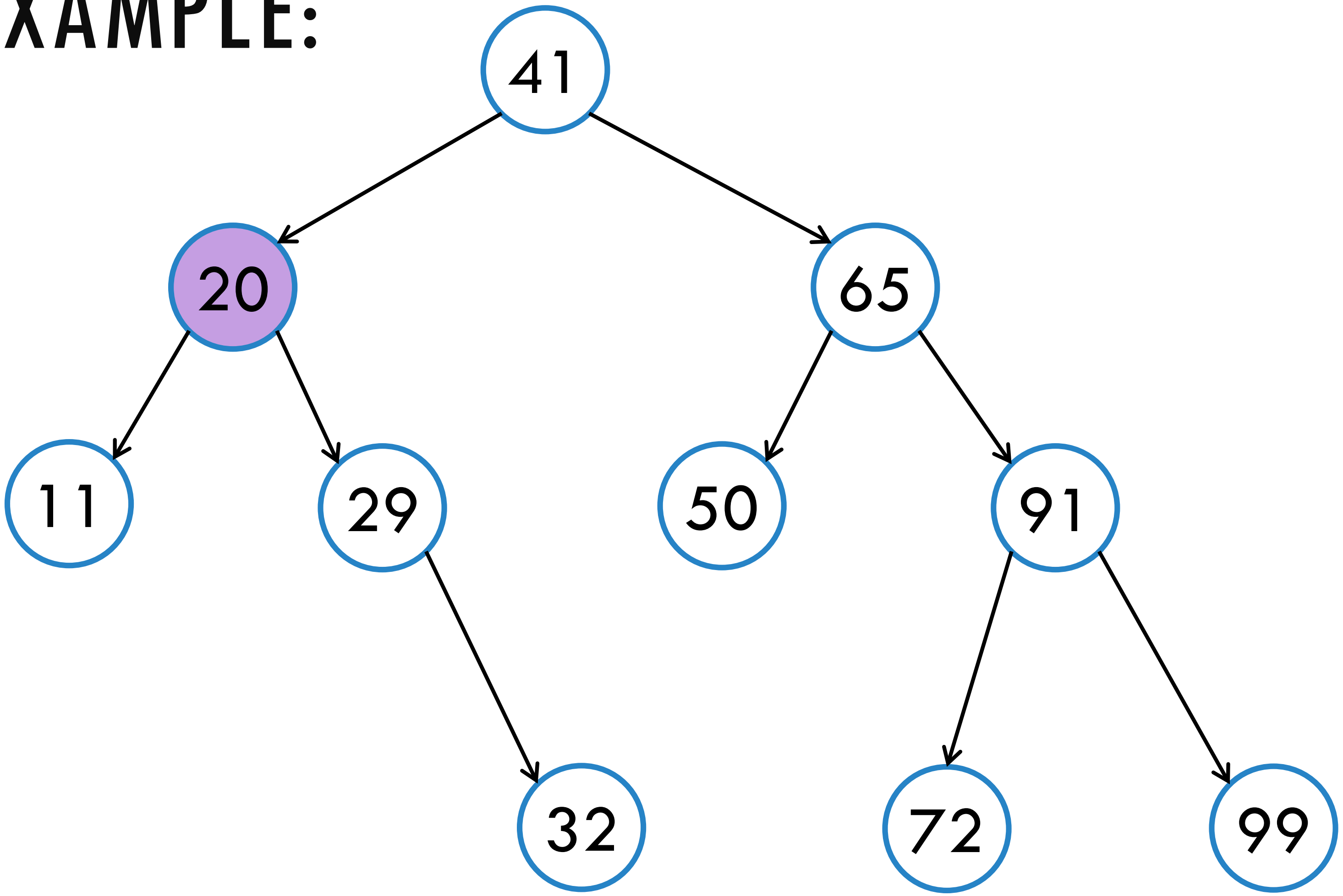
41 20

PREORDER EXAMPLE:



41 20 11

PREORDER EXAMPLE:



41 20 11

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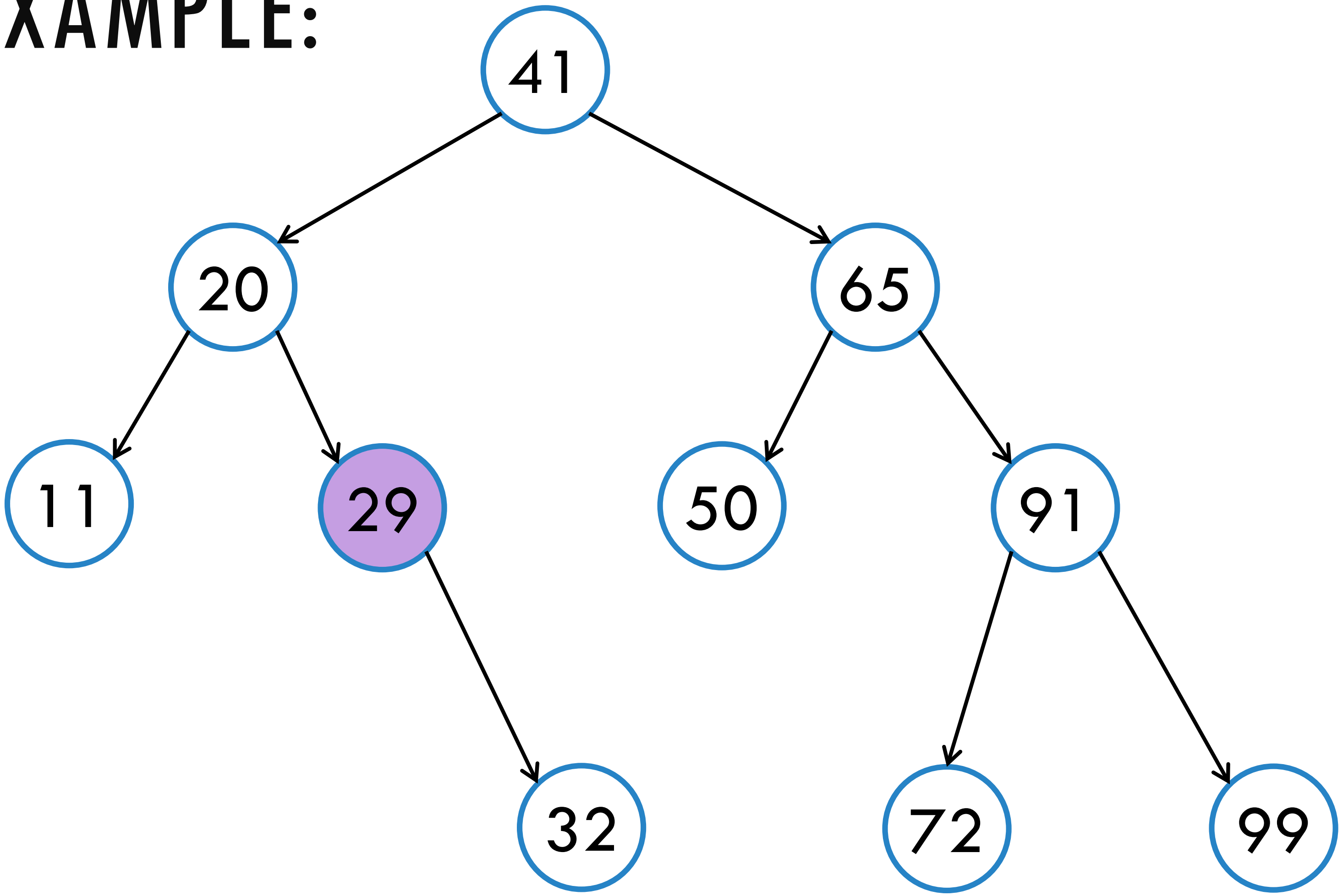

41 20 11 29

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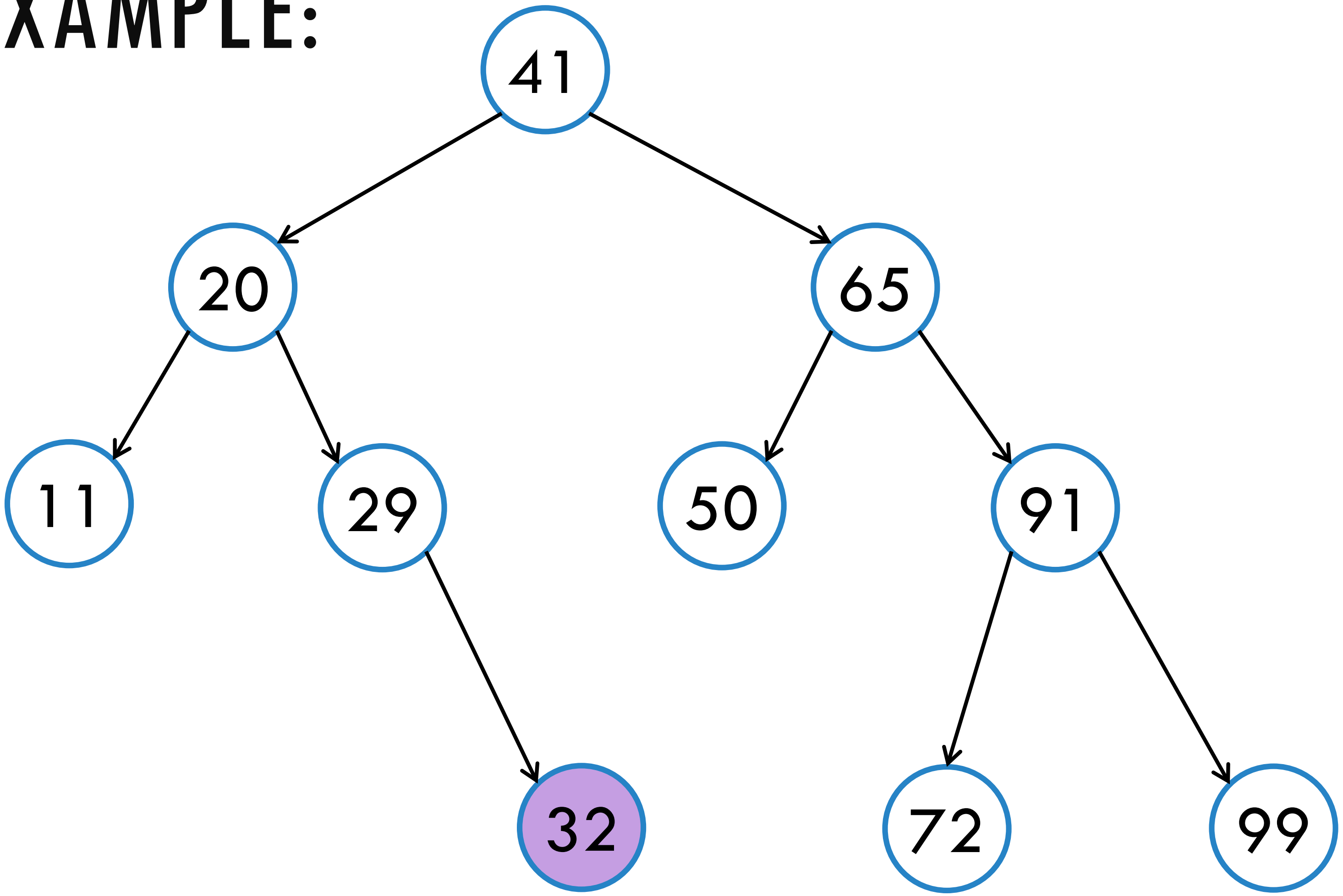
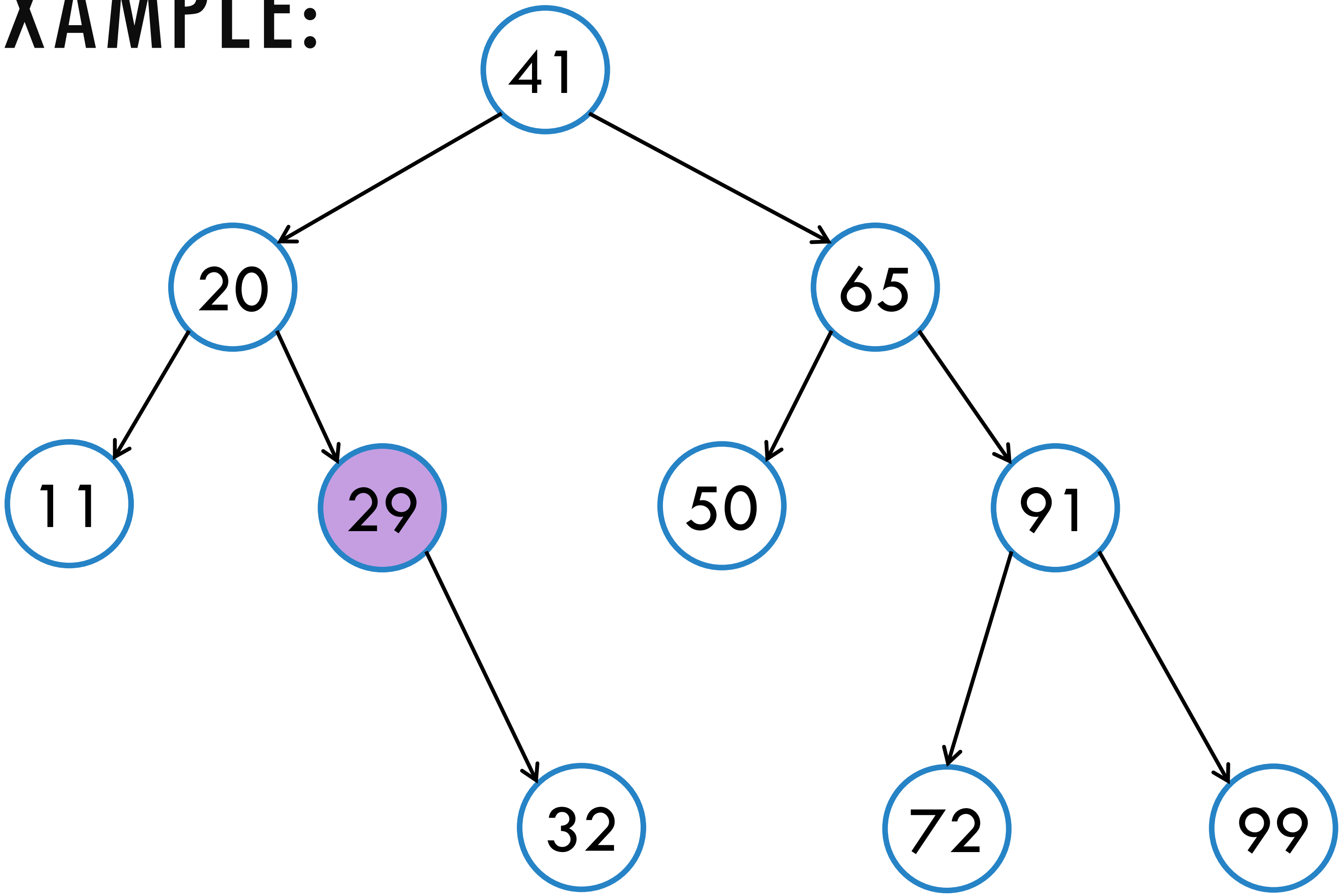


41 20 11 29 32

PREORDER EXAMPLE:



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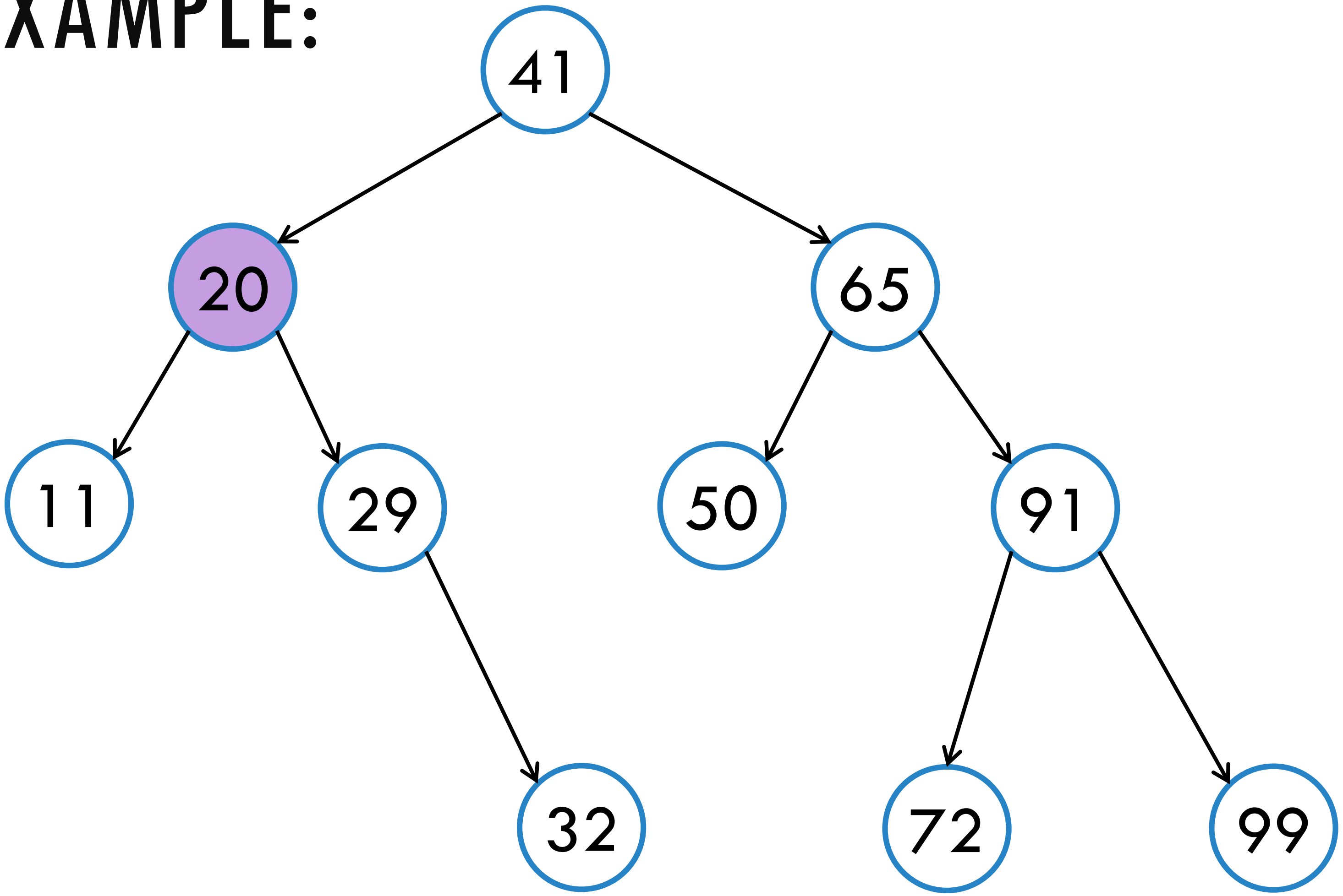

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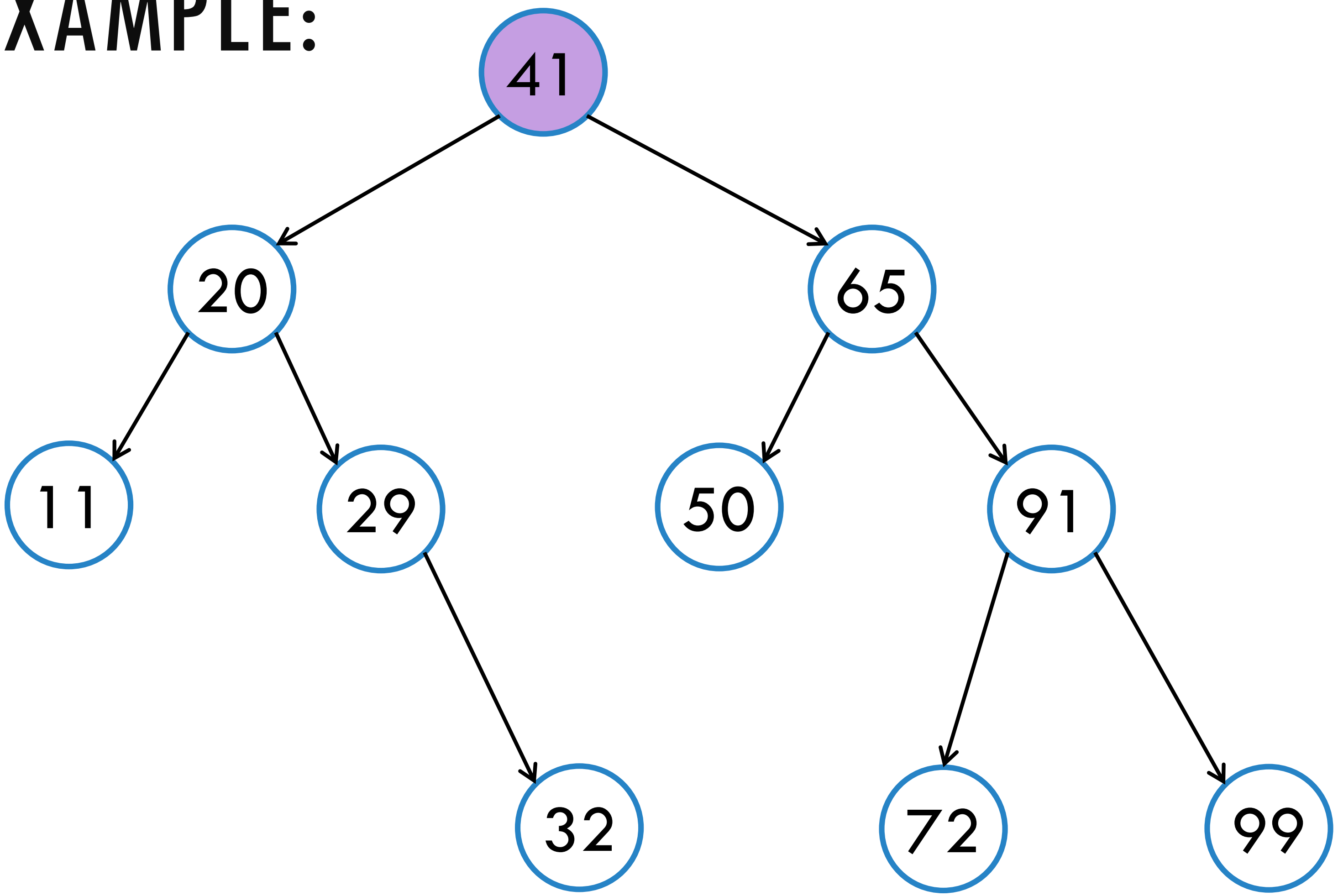

41 20 11 29 32

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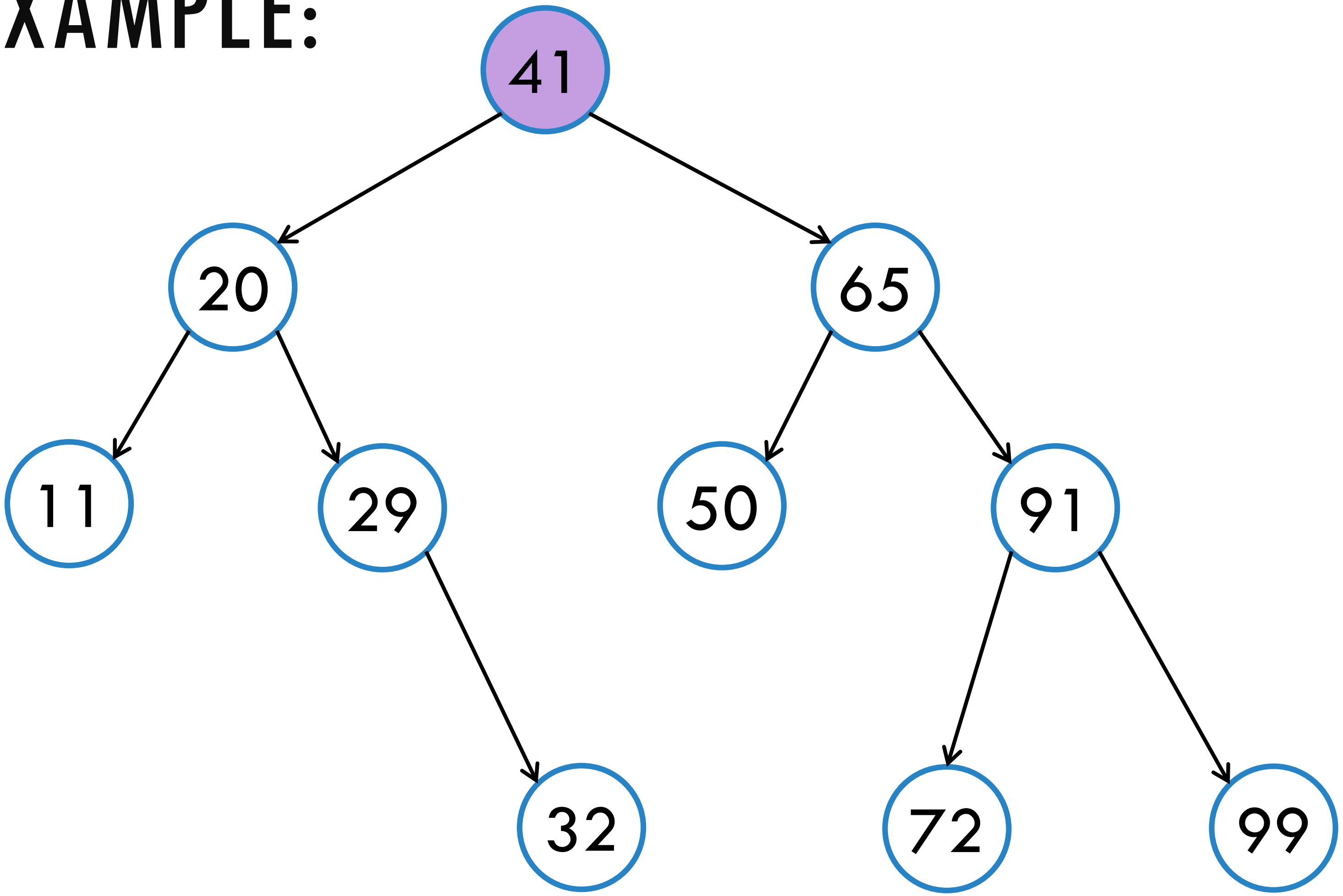

41 20 11 29 32 65 50 91 72 99

NOT SO BASIC BST OPERATIONS

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`inorder(Node o):` returns nodes of the tree rooted at o in order

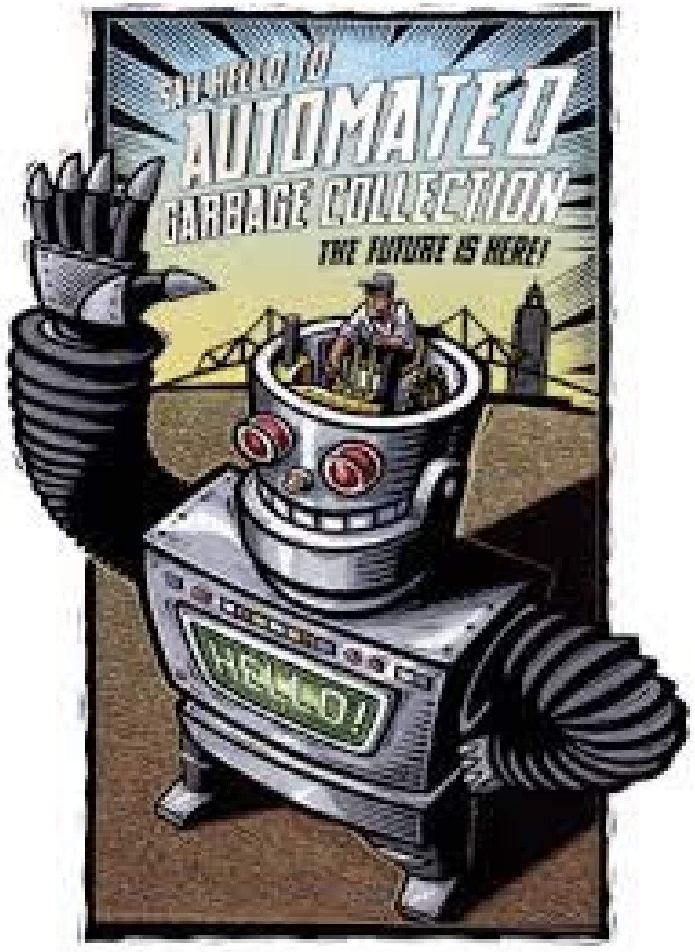
`copyTree(Node o):` returns a copy of the tree rooted at o

`deleteTree(Node o):` deletes the tree rooted at o

TODAY WITH JAVA

To Delete a Tree, you can just do this:

```java
root = null;
```



IN THE OLD DAYS... OR WITH C/C++

with no automatic garbage collection.

We have to delete node by node via some traversal

TREE TRAVERSALS

Pre-Order	In-Order	Post-order
SELF LEFT-SUBTREE RIGHT-SUBTREE	LEFT-SUBTREE SELF RIGHT-SUBTREE	LEFT-SUBTREE RIGHT-SUBTREE SELF

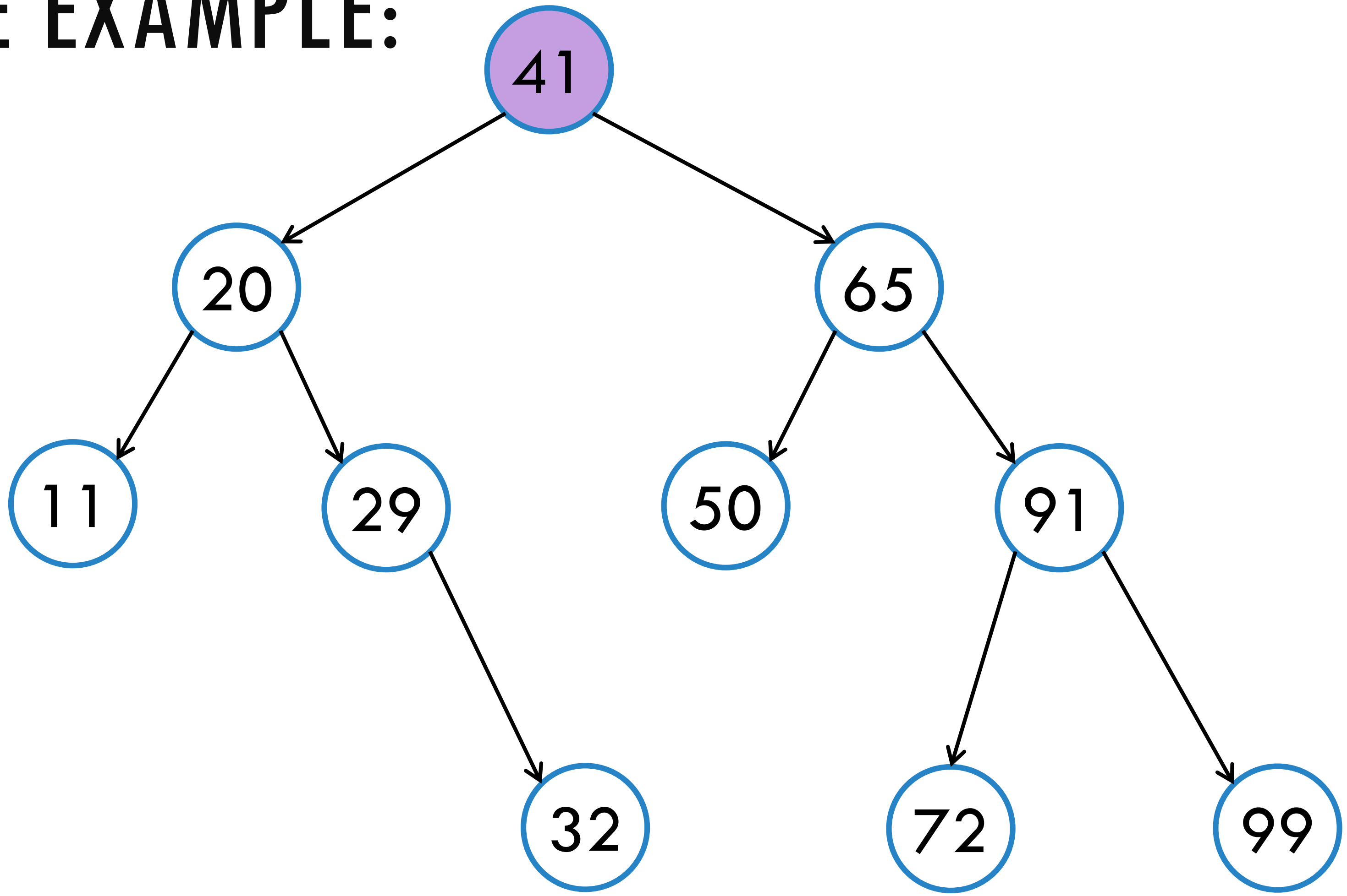
IN THE OLD DAYS… OR WITH C/C++

with no automatic garbage collection.

We have to delete node by node via **post-order** traversal

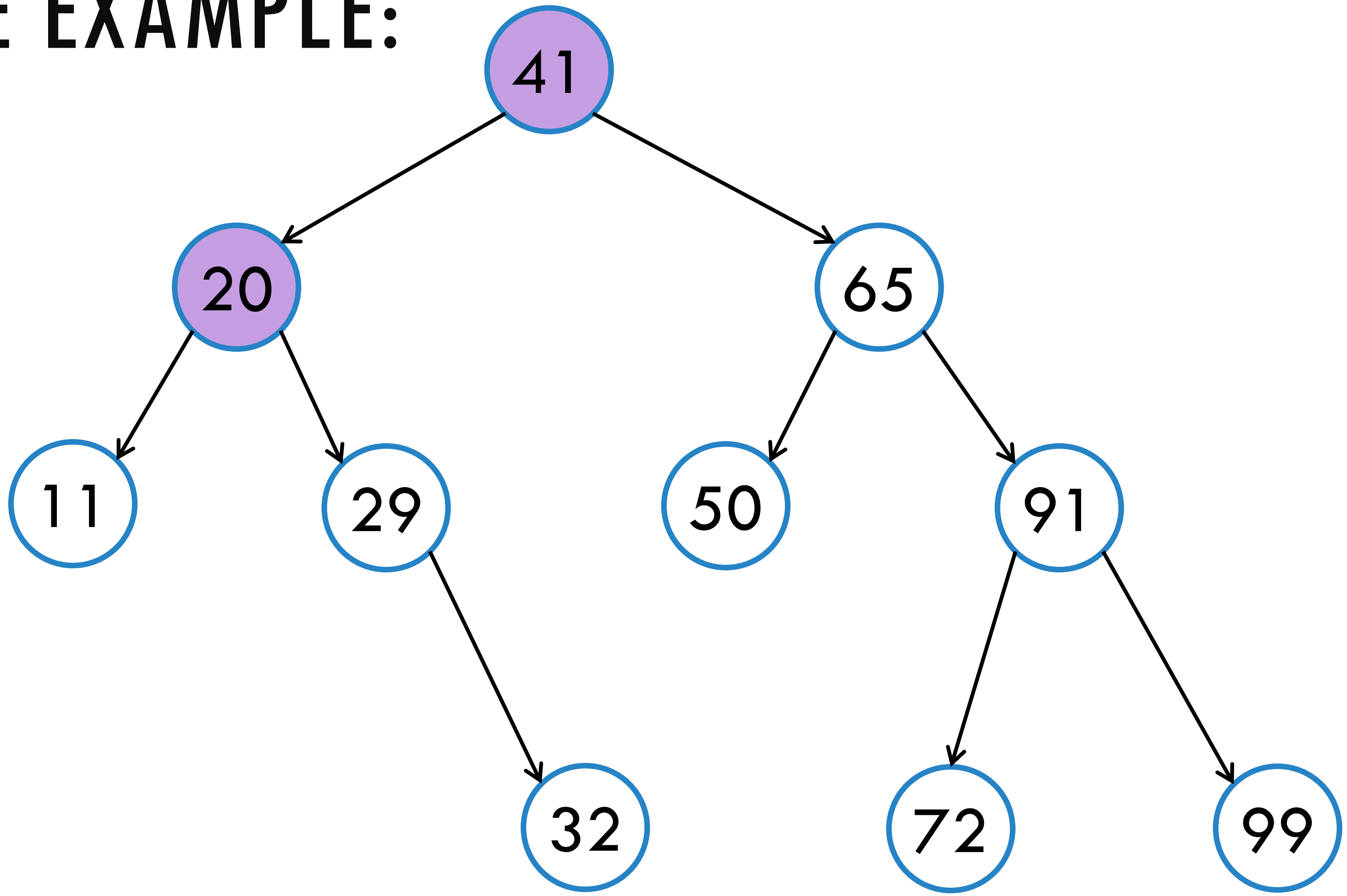
```
function deleteTree(Node n)
    // Traverse left sub-tree
    if n.m_leftTree is not null
        deleteTree(n.m_leftTree)
    // Traverse right sub-tree
    if n.m_rightTree is not null then
        deleteTree(n.m_rightTree)
    free(n)
```

DELETE TREE EXAMPLE:



11 32 29 20 50 72 99 91 65 41

DELETE TREE EXAMPLE:



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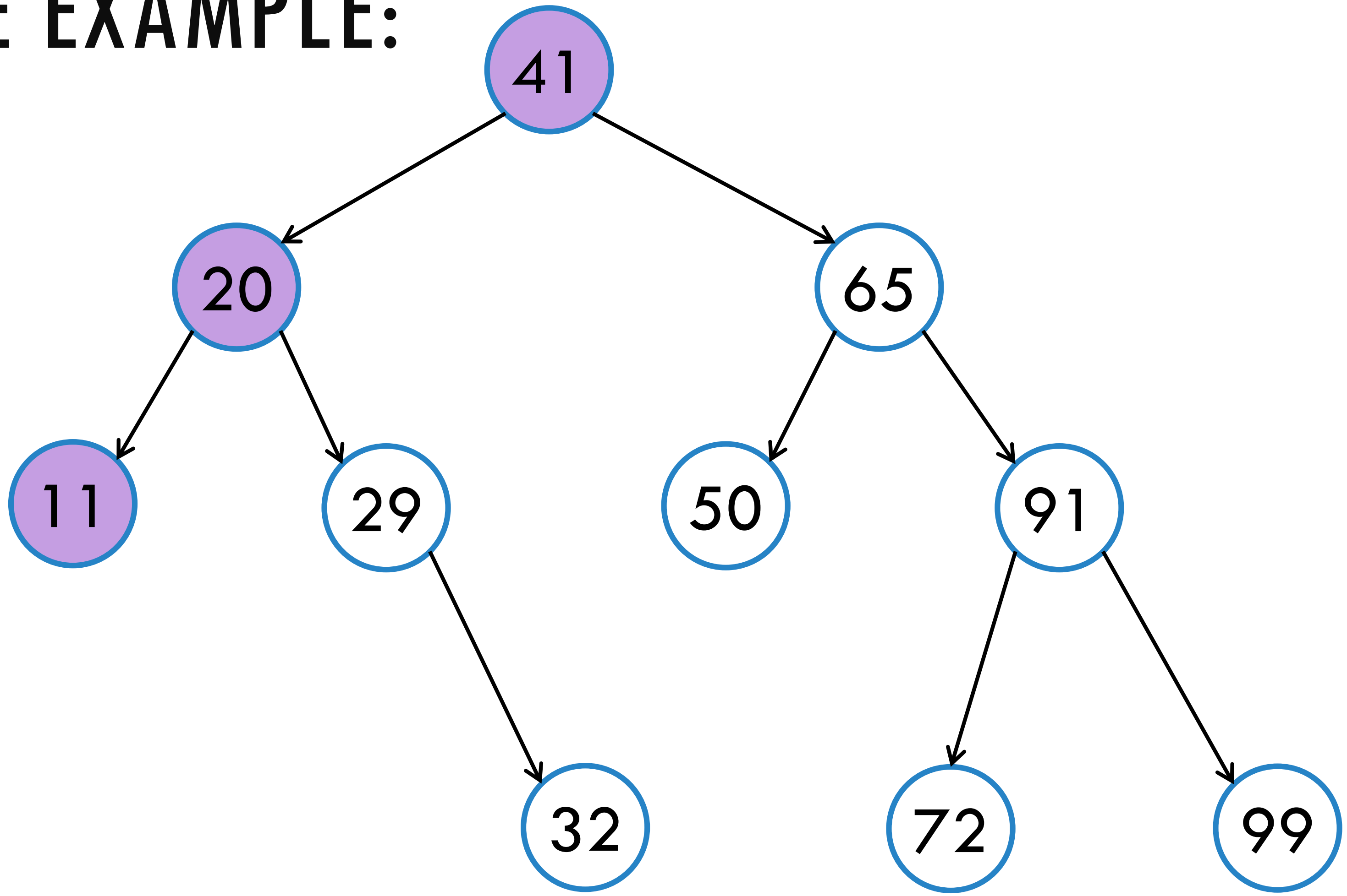

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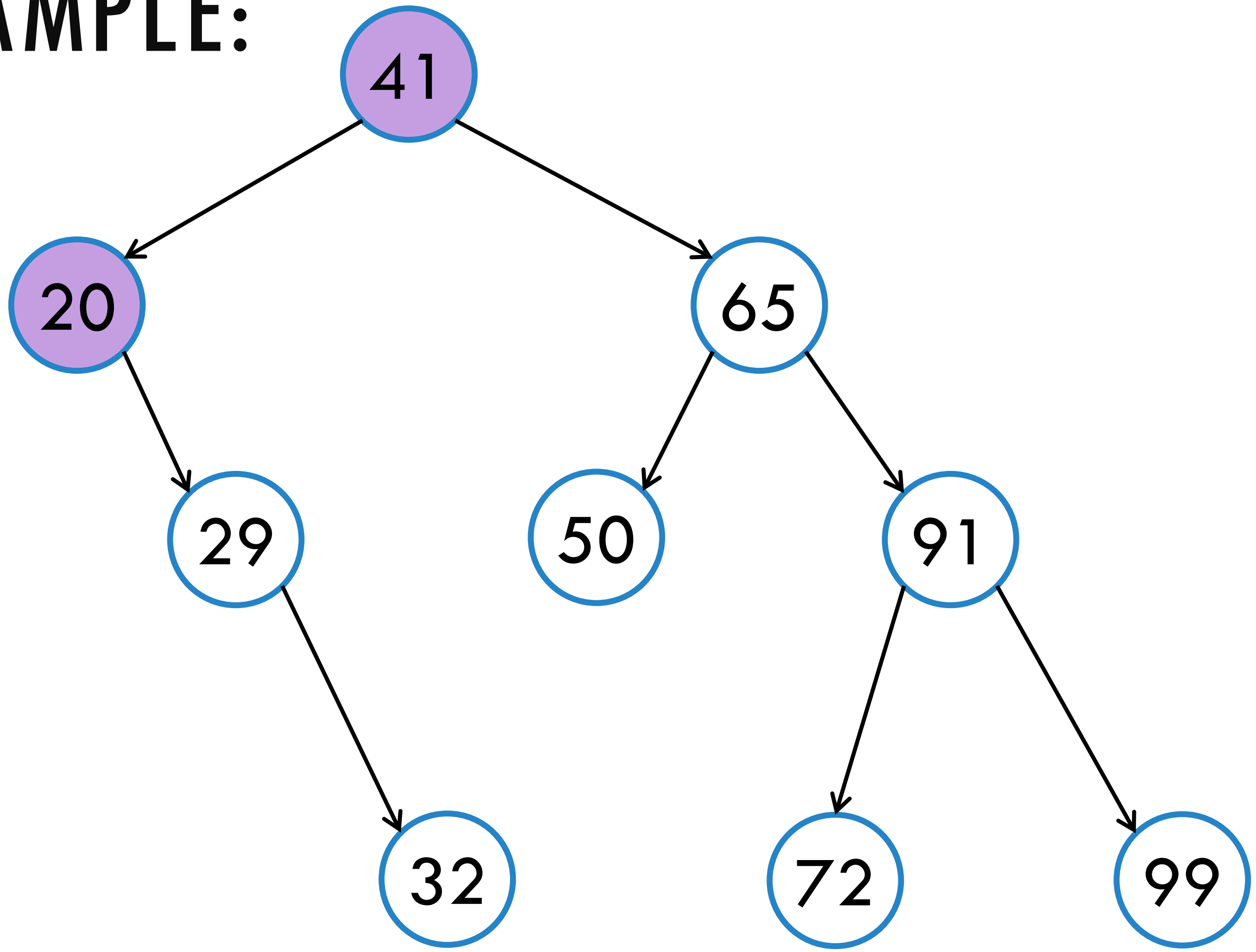

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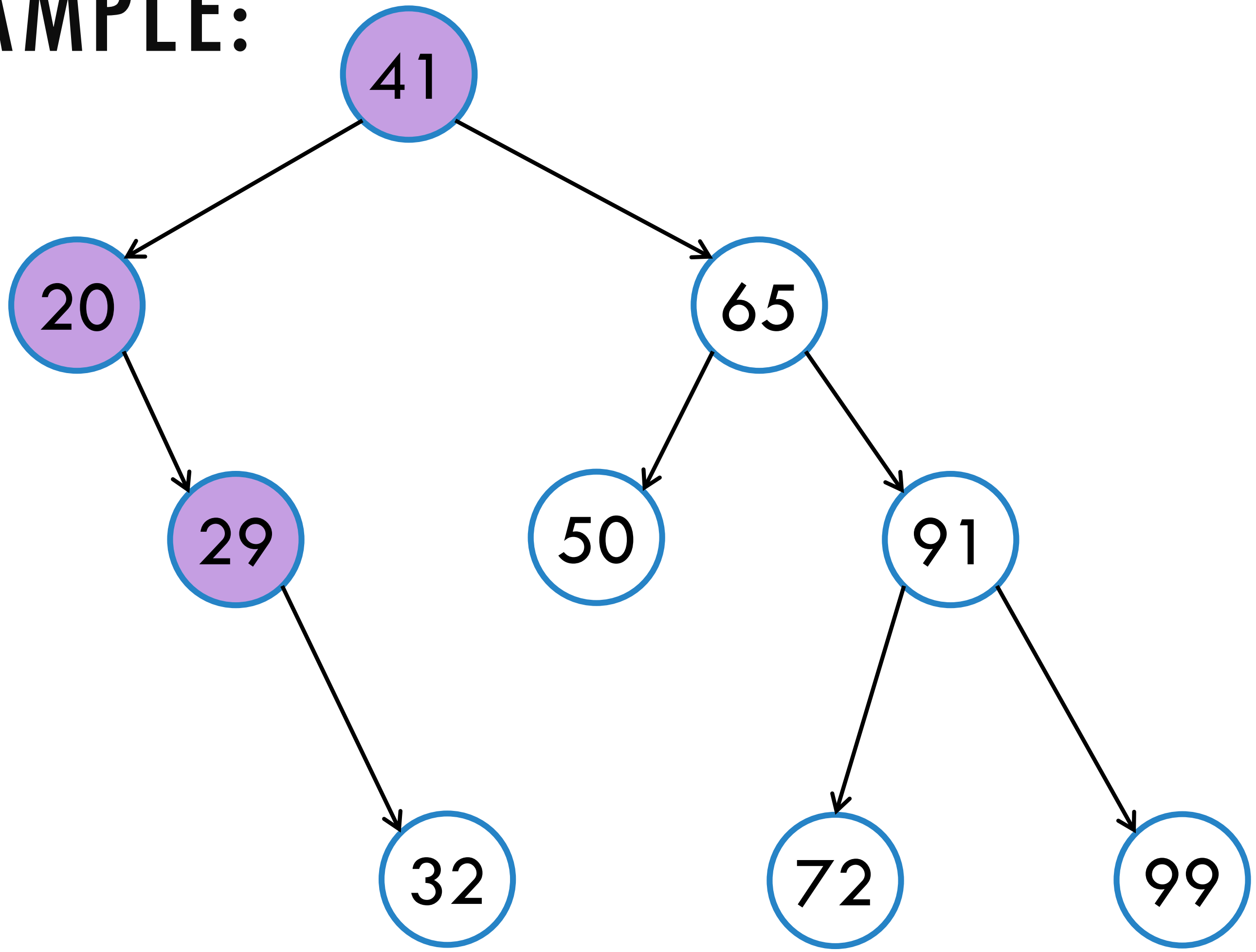

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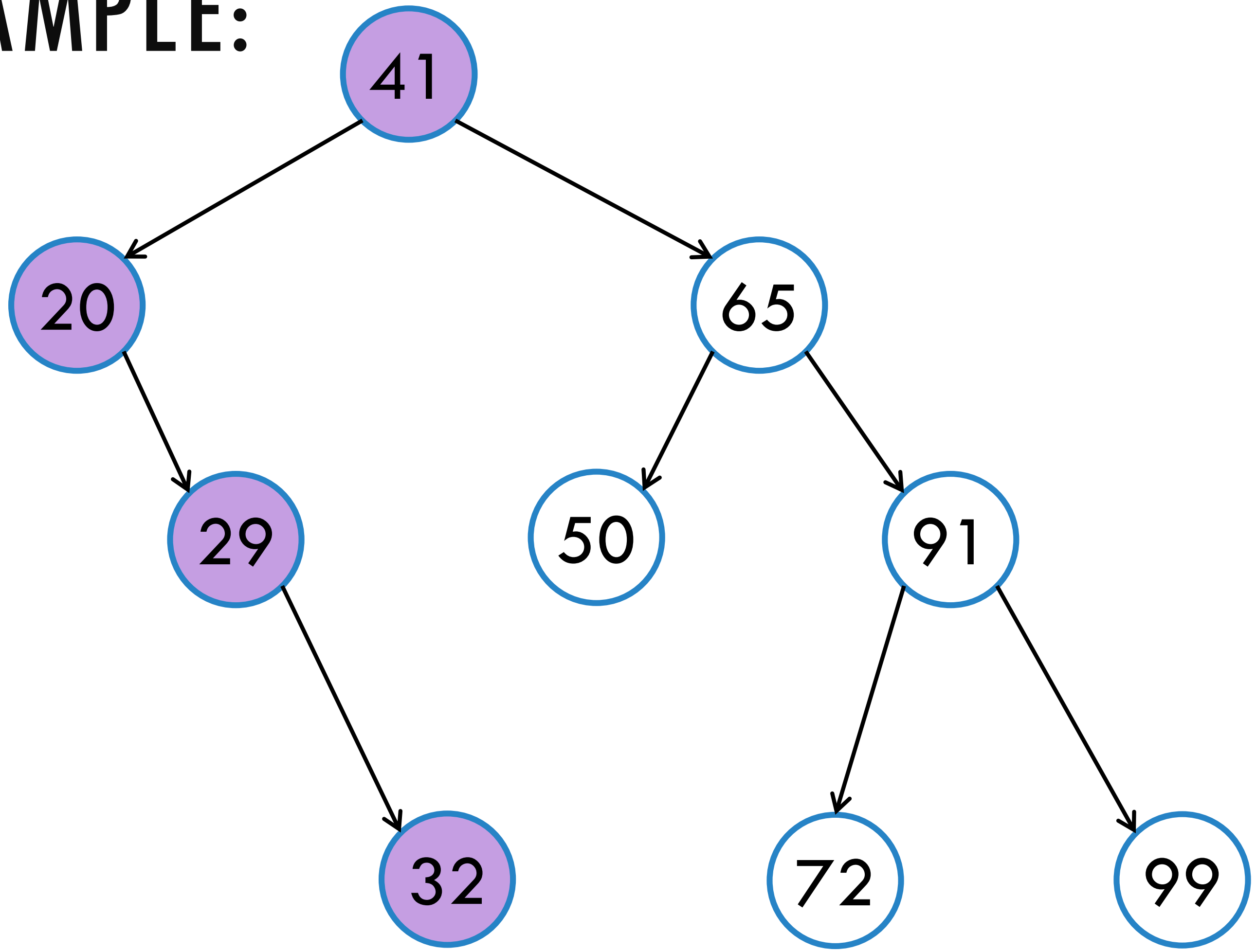

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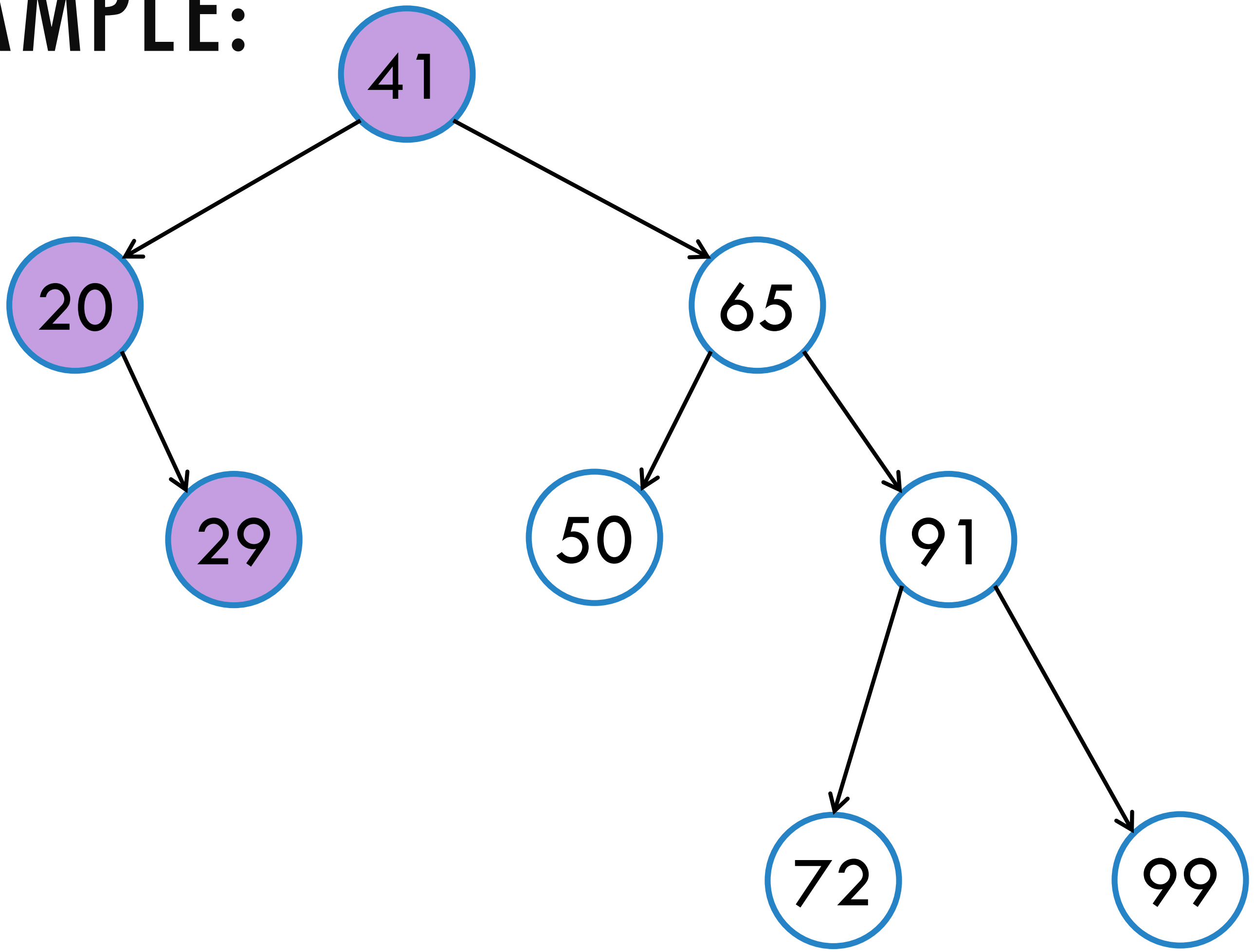

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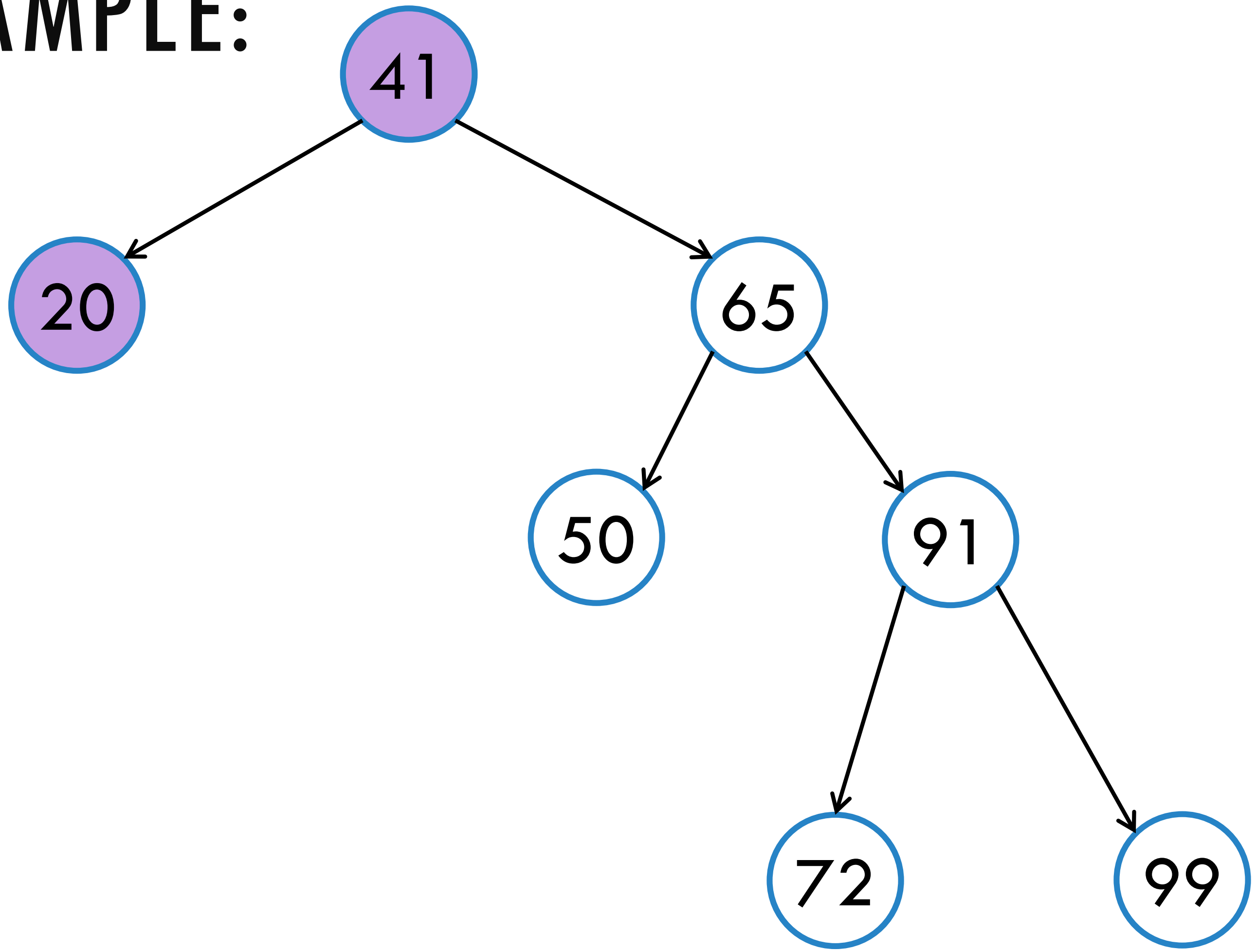

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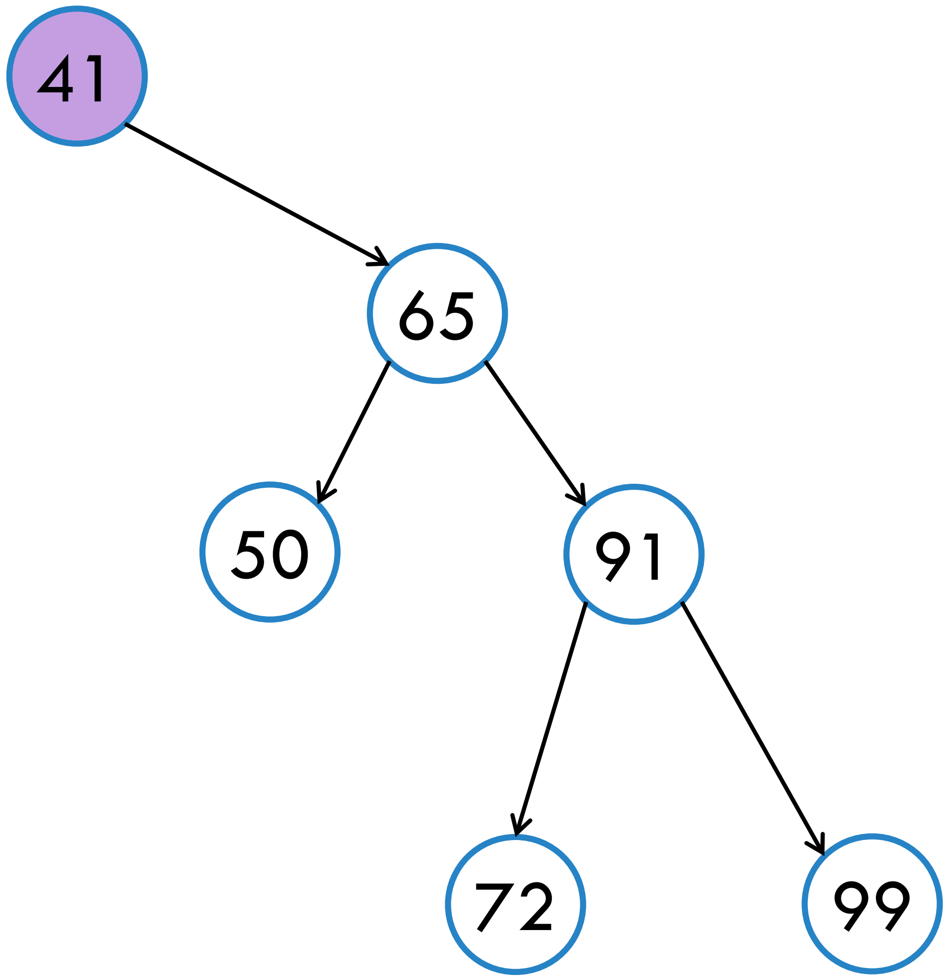

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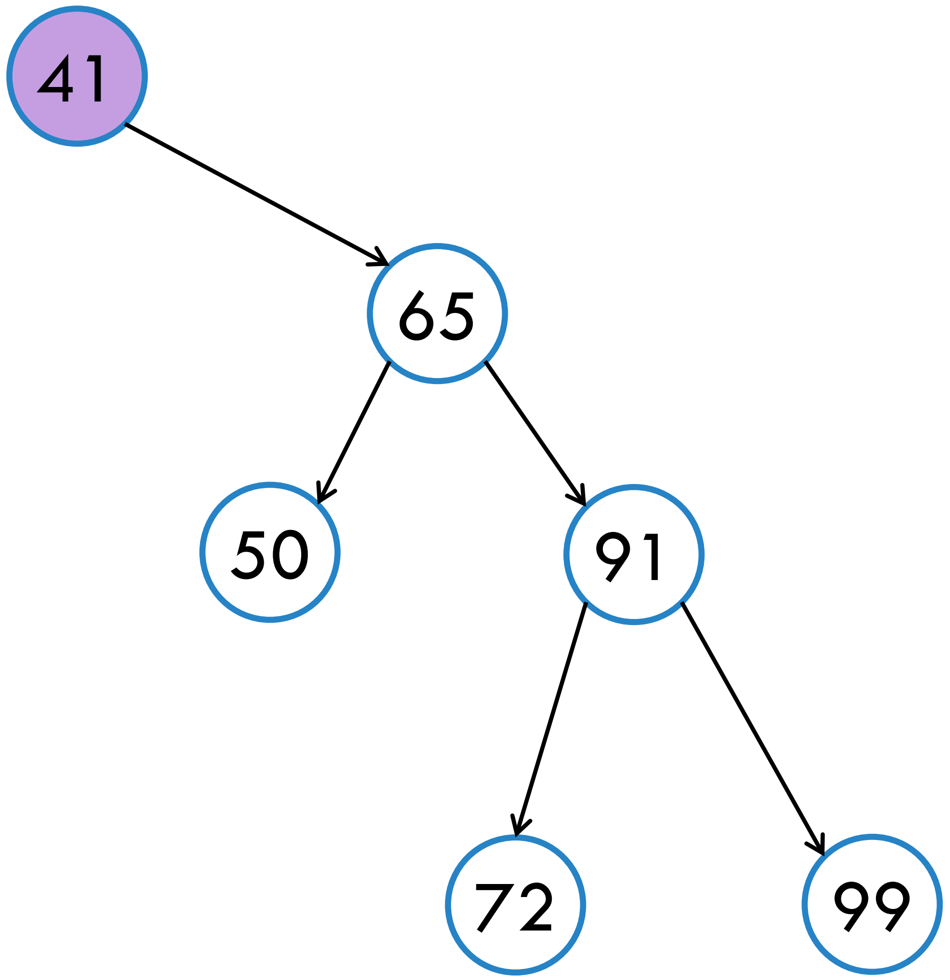

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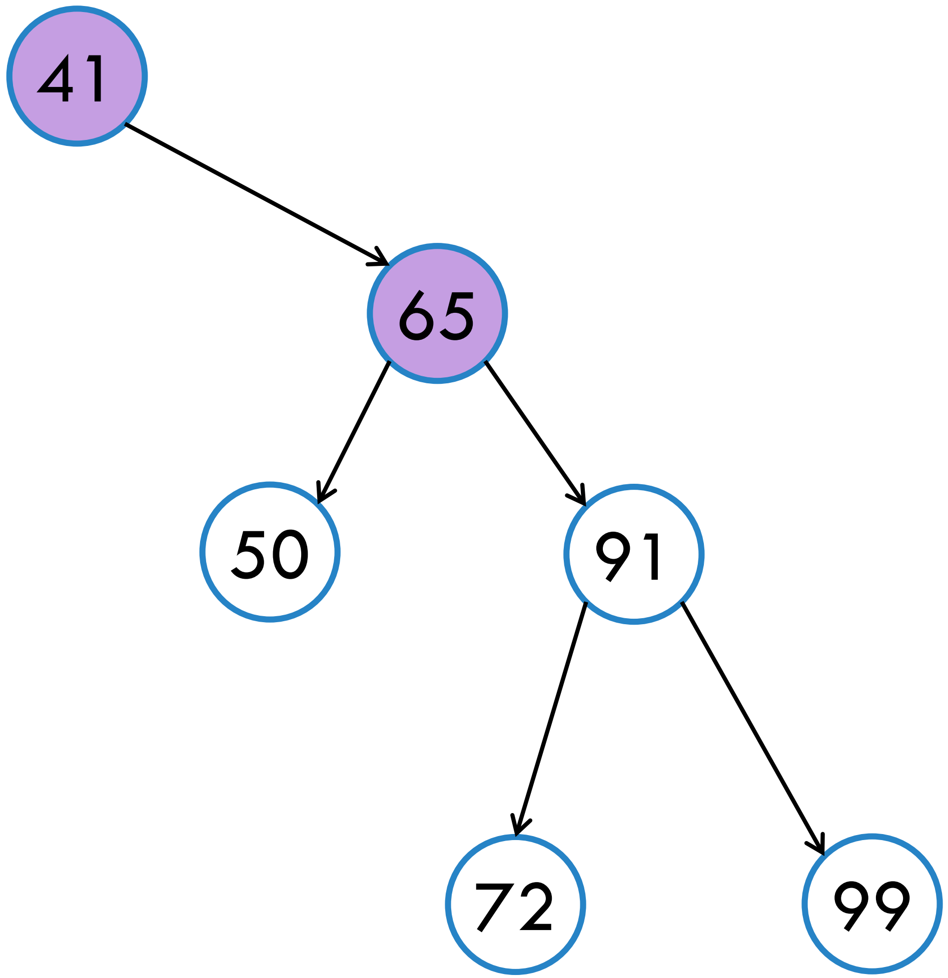

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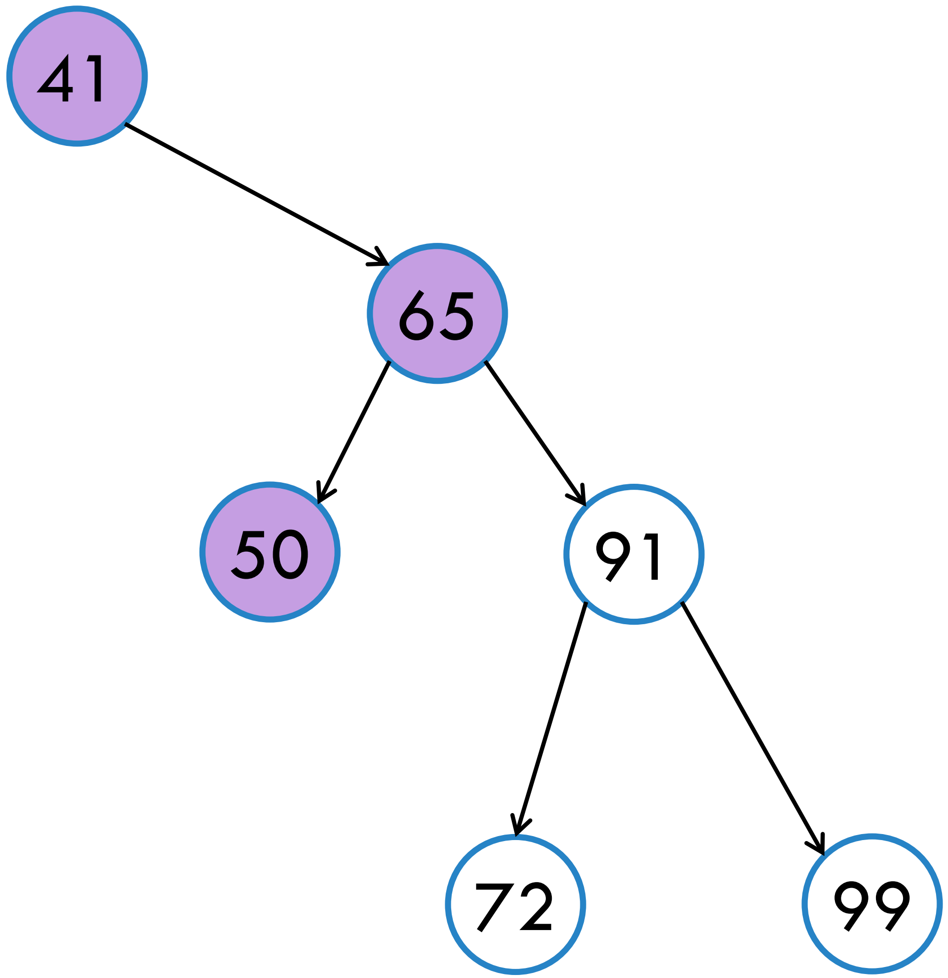

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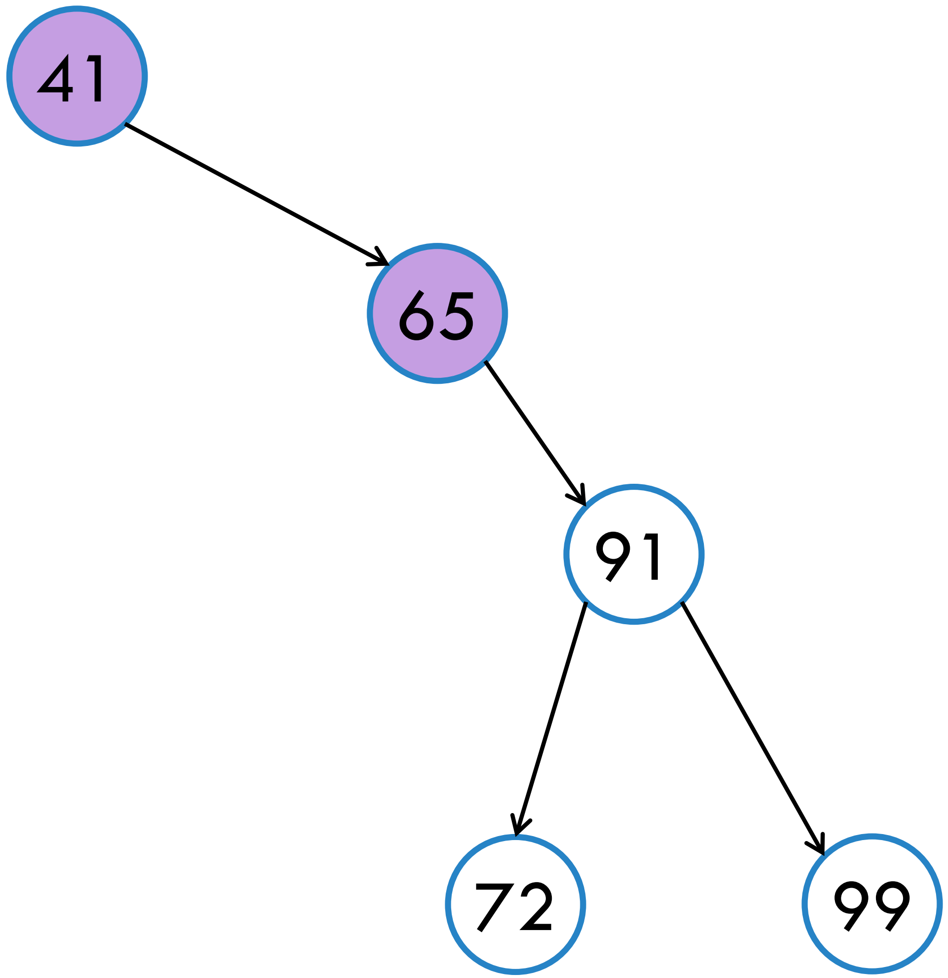

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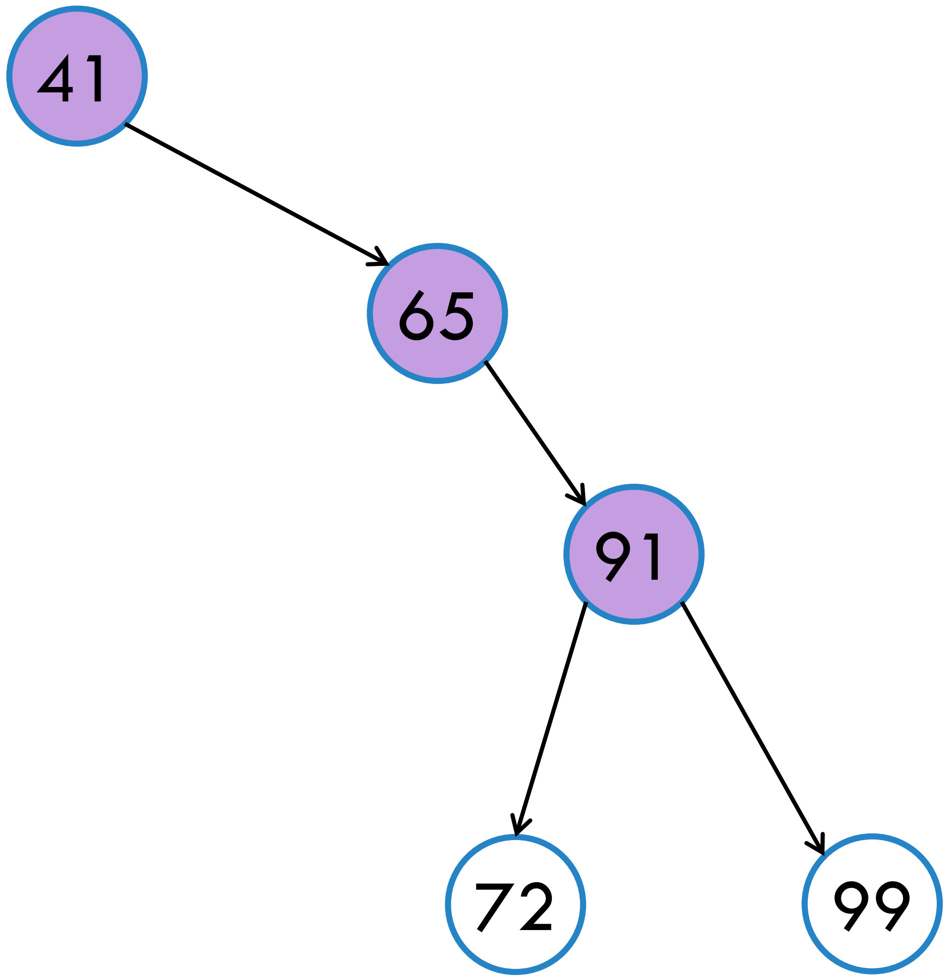

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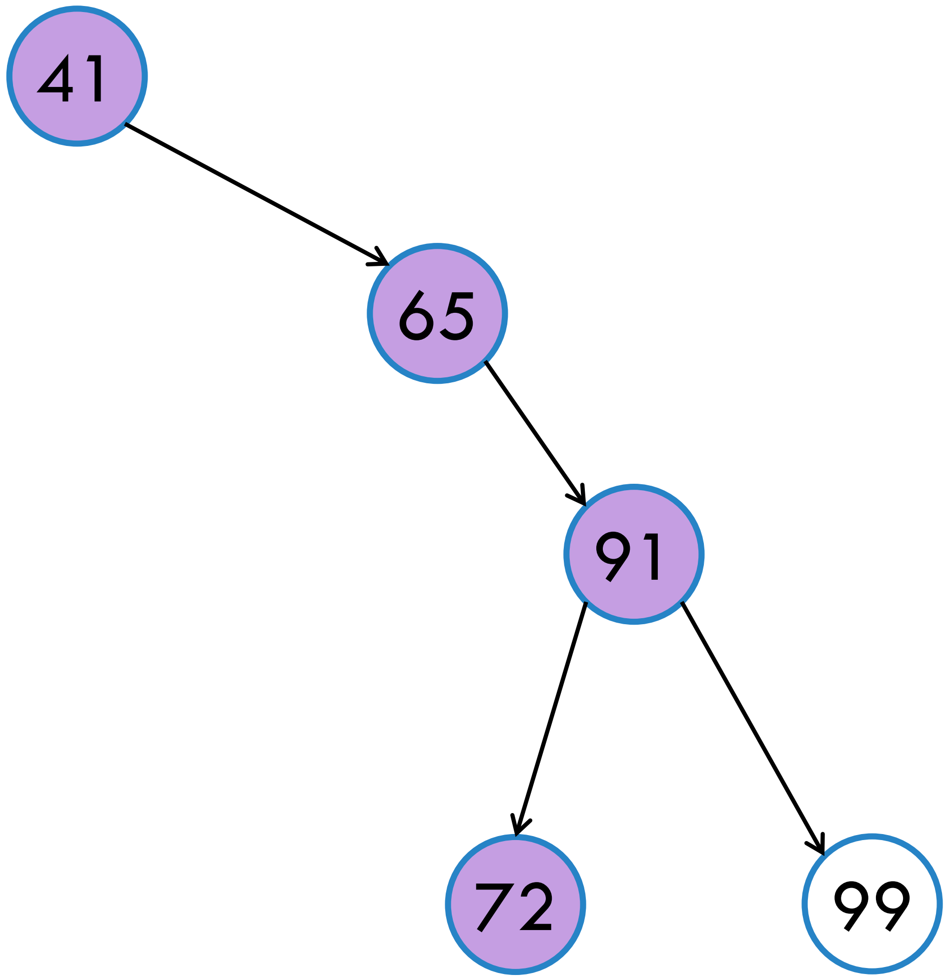
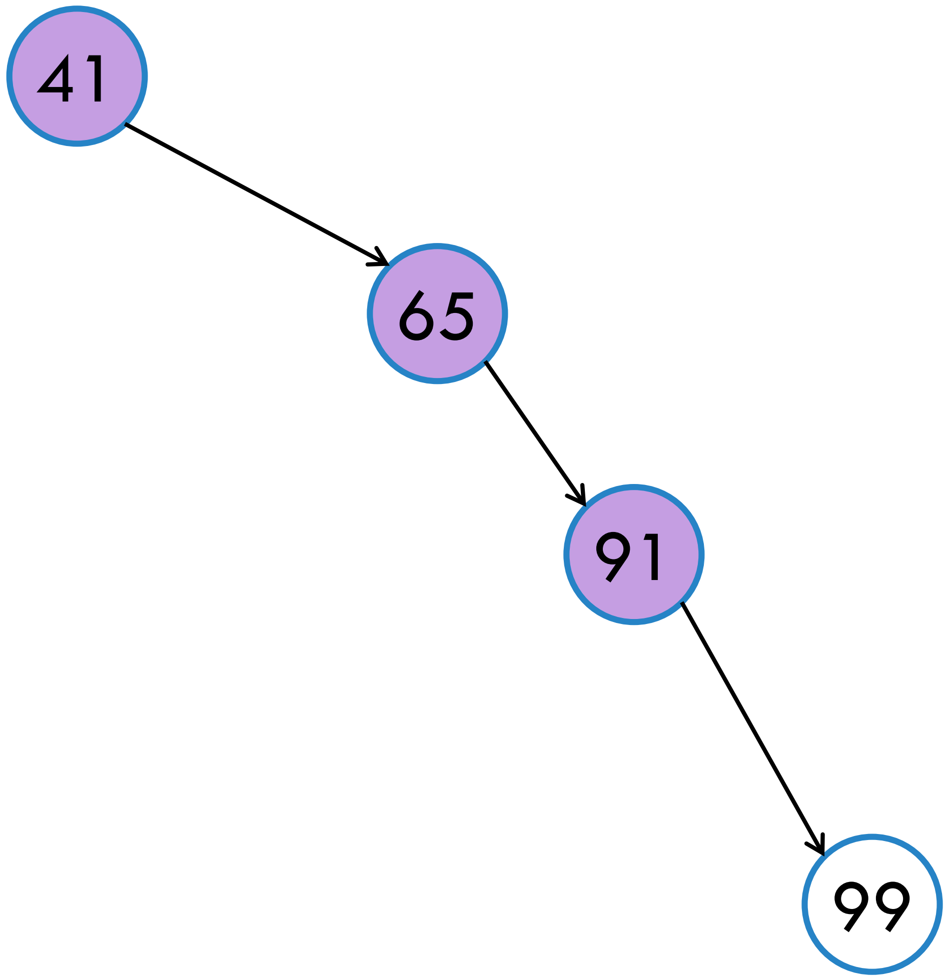
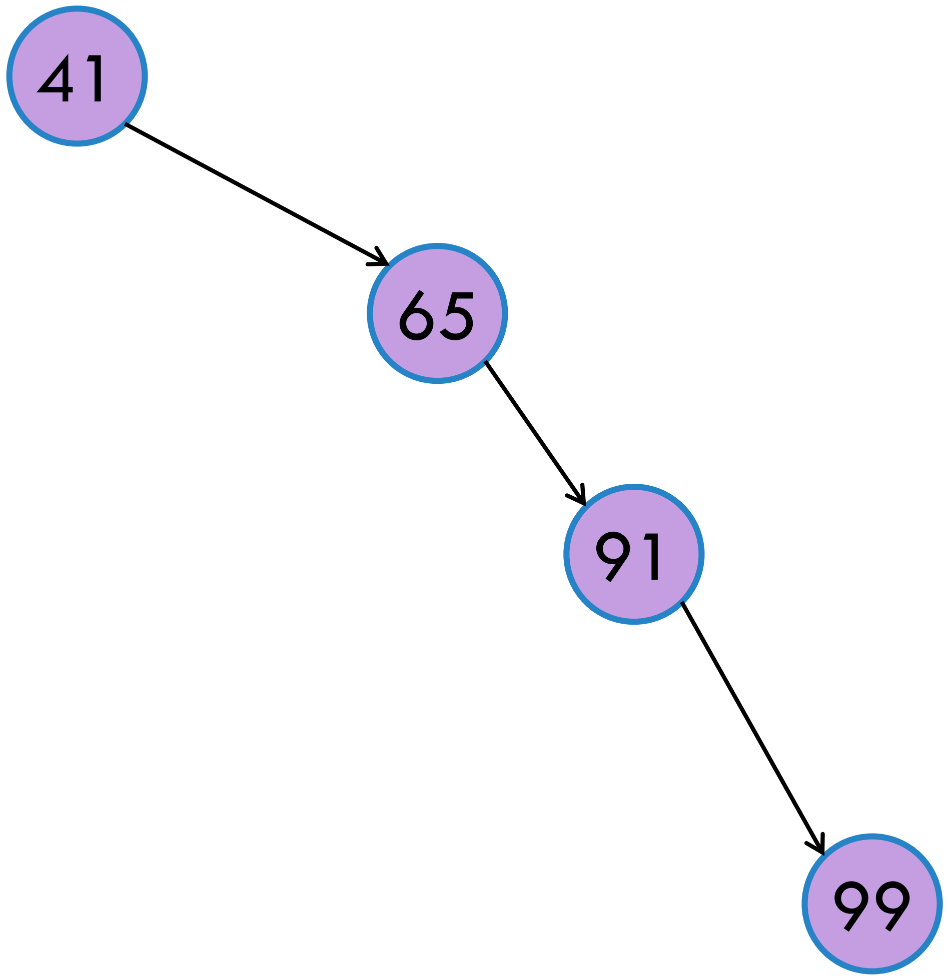


DELETE TREE EXAMPLE:



DELETE TREE EXAMPLE:



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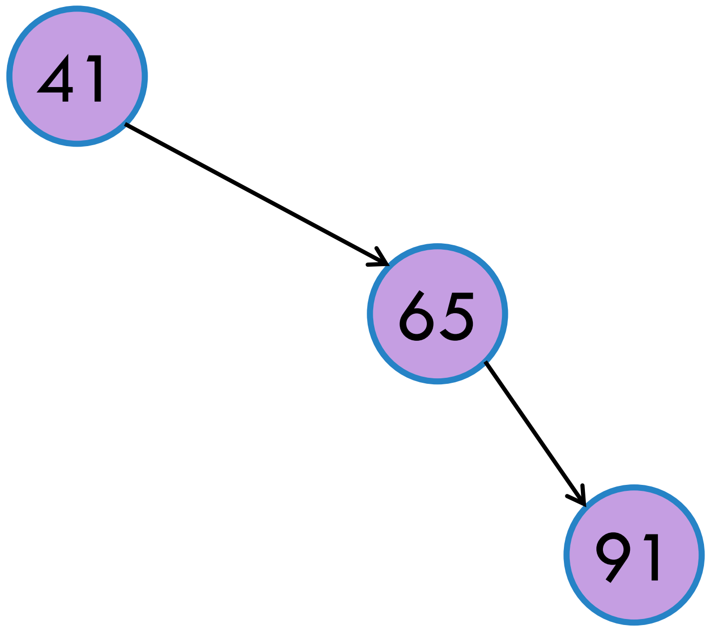

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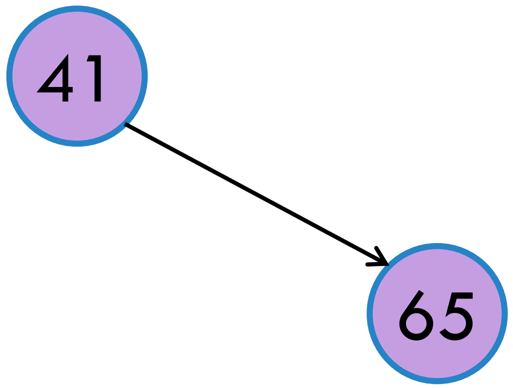


DELETE TREE EXAMPLE:

DELETE TREE EXAMPLE:

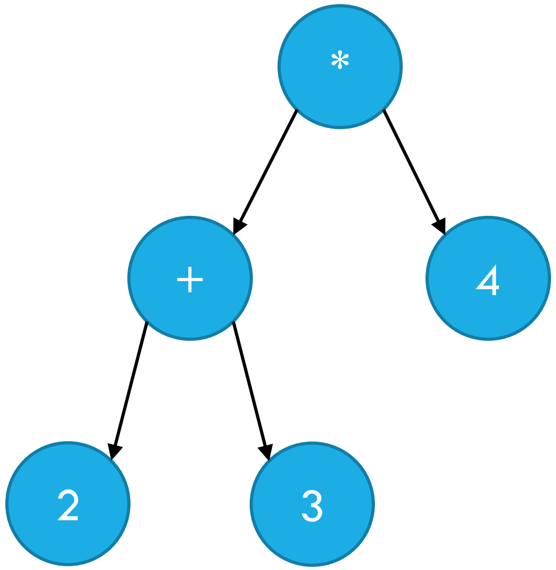
RPN: IN-ORDER, PRE-ORDER, POST-ORDER

Remember Quiz 1 on **Reverse Polish Notation?**

"Normal": $(2+3)*4 \ = \ 20$ **"Infix notation"**

RPN: $2,3,+,4,*$ **"Post-fix notation"**

Which **traversal** do we use to evaluate an expression encoded in a Binary Tree?



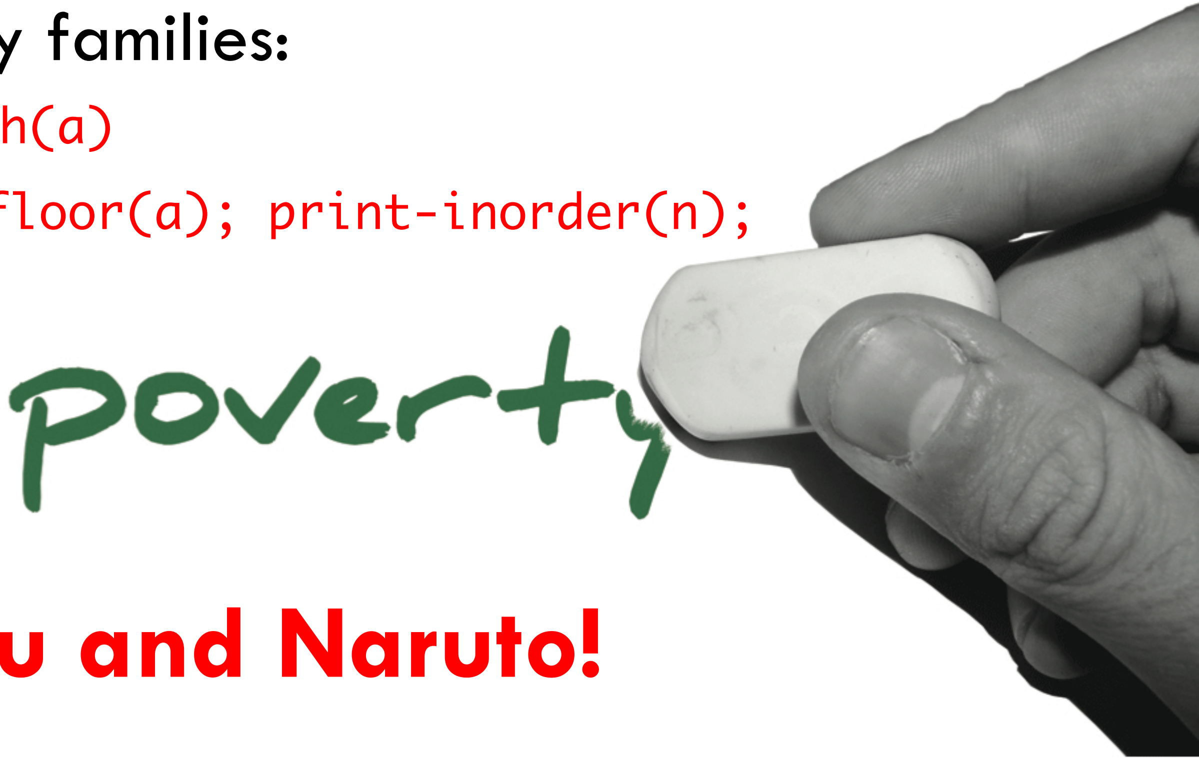
BACK TO OUR PROBLEM

The Stop-Poverty charity calls:

To provide financial aid, Help identify families:

- earning exactly \$a amount `n = search(a)`
- earning less than or equal to \$a `n = floor(a); print-inorder(n);`
- earning more than or equal to \$a:

  `n = ceiling(a); print-inorder(n);`



another win for you and Naruto!

LEARNING OUTCOMES

By the end of this session, students should be able to:

- Describe the **Binary Search Tree (BST)** and its operations
- Analyze **performance of BST operations**
- Relate the **importance of balance in a BST** to enable efficient operations.
- Explain the **pre-order, post-order, and in-order tree traversal** algorithms.

QUESTIONS?