CS1231–Midterm 1, 2018

Name: **Matric Number:**

Tutorial Group: **Seat Number:**

1. [1 marks] Which of these sentences are propositions? Ans: (c)

(a) Can you answer this question? (b) $x+2 = 11$ (c) Boston is the capital of Massachusetts. (d) Do not pass go.

2. [4 marks] Show that $\neg p \to (q \to r) \equiv q \to (p \vee r)$ by two different methods.

Using truth table:

p	q	r	$\neg p$	$q \to r$	$\neg p \to (q \to r)$	$p \vee r$	$q \to (p \vee r)$
T	T	T	F	T	T	T	T
T	T	F	F	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

Using theorem:

$\neg p \to (q \to r) \equiv \neg(\neg p) \vee (q \to r) \equiv p \vee (\neg q \vee r) \equiv \neg q \vee (p \vee r) \equiv q \to (p \vee r)$.

3. [4 marks] Let D be the set of all animals. Let $P(x)$ be "x can fly" and $B(x)$ be "x is a bird". For each of the following, translate into a logical expression with domain D.

(i) Every bird can fly.

$\forall x \in D, B(x) \to P(x)$.

(ii) Being a bird is not a necessary condition for an animal being able to fly. (Hint: Use quantifier(s). Simplify your answer as much as possible.)

$\exists x \in D, P(x) \wedge \neg B(x)$.

4. [2 marks] Determine, with justification, the truth values of the following expression.

$$\exists x \in \mathbb{Z} \forall y \in \mathbb{Z}, xy = x.$$

Truth Value: T.

Justification: For $x = 0$, $\forall y \in \mathbb{Z}, xy = x$ becomes $\forall y \in \mathbb{Z}, 0 \times y = 0$, which is true.

5. [4 marks] Let D consist of all people in the world. Let $L(x, y)$ be the statement "x loves y" and $Q(x, y)$ be the statement "x and y are the same person", where the domain for both x and y is D. Translate the following into logical expressions using quantifiers $\forall, \exists$, and logical connectives.

(i) Everybody loves Jerry.

Answer: $\forall x \in D, L(x, Jerry)$.

(ii) There is somebody whom no one loves.

Answer: $\exists x \in D \forall y \in D, \neg L(y, x)$.

(iii) There are exactly two people whom Lynn loves.

Answer: $\exists x, y \in D \forall z \in D, \neg Q(x, y) \wedge (L(Lynn, z) \leftrightarrow Q(x, z) \vee Q(y, z))$.

6. [2 marks] Translate the following into a logical expression using quantifiers $\forall, \exists$, and logical connectives. Use U, D and M, where U is a set which consists of all students in the university, D is a set which consists of all departments in the university, and M is a set which consists of all courses in the university. Let $C(x)$ be the statement "x is a student in this class", $T(x, y)$ be "the student x has taken the course y" and $O(y, z)$ be "the course y is offered by the department z."

" There is a student in this class who has taken every course offered by one of the departments in the university."

Answer: $\exists x \in U \exists z \in D \forall y \in M, C(x) \wedge (O(y, z) \rightarrow T(x, y))$.

7. [3 marks] Using valid arguments forms, derive the conclusion r from the following given hypotheses:

(i) $(p \wedge t) \rightarrow (r \vee s)$, (ii) $q \rightarrow (u \wedge t)$, (iii) $u \rightarrow p$, (iv) $\neg s$, (v) q.

Answer:

1. q
2. $q \rightarrow (u \wedge t)$
3. $u \wedge t$ (From 1, 2)
4. u (From 3)
5. $u \rightarrow p$
6. p (From 4, 5)
7. t (From 3)
8. $p \wedge t$ (From 6,7)
9. $(p \wedge t) \rightarrow (r \vee s)$
10. $r \vee s$ (From 8,9)
11. $\neg s$
12. r (From 10,11)