1. Let L denote the normal line to the curve $y = x^2$ at the point $(9.11, 9.11^2)$. Find the x-coordinate of the point of the intersection of L and the x-axis. Give your answer correct to the nearest integer.

$$\frac{dy}{dx} = 2x \Rightarrow \left.\frac{dy}{dx}\right|_{x=9.11} = 18.22$$

normal line : $$\frac{y-9.11^2}{x-9.11} = -\frac{1}{18.22}$$

$$y=0 \Rightarrow 18.22 \times 9.11^2 = x - 9.11$$

$$\Rightarrow x = 18.22 \times 9.11^2 + 9.11$$

$$= 1521.22\cdots$$

$$\approx \underline{\underline{1521}}$$

2. Let P denote the point on the ellipse $x^2 + \frac{y^2}{81} = 1$ with coordinates given by $(\cos t, 9\sin t)$ where t is measured in radians and $0 < t < \frac{\pi}{2}$. Let Q denote the reflection of P using the y-axis as a mirror. If the two tangent lines to the ellipse at P and Q respectively are perpendicular to each other, find the value of t. Give your answer correct to two decimal places.

$$Q = (-\cos t, 9\sin t)$$
$$= (\cos(\pi - t), 9\sin(\pi - t))$$

ellipse : $\begin{cases} x = \cos\theta \\ y = 9\sin\theta \end{cases}$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{9\cos\theta}{-\sin\theta} = -9\cot\theta$$

$$\left.\frac{dy}{dx}\right|_{\theta = t} = -9\cot t$$

$$\left.\frac{dy}{dx}\right|_{\theta = \pi - t} = -9\cot(\pi - t) = 9\cot t$$

$$-9\cot t \times 9\cot t = -1$$
$$\Rightarrow \cot^2 t = \frac{1}{81} \Rightarrow \cot t = \frac{1}{9} \quad (\because 0 < t < \tfrac{\pi}{2})$$
$$\Rightarrow t = \cot^{-1}\left(\tfrac{1}{9}\right) = 1.460\cdots$$
$$\approx \underline{\underline{1.46}}$$

3. Let a denote a constant with $a > 1$. If

$$\int_0^{\pi} \frac{\sin\theta}{\sqrt{1 - 2a\cos\theta + a^2}} d\theta = 0.2018,$$

find the value of a. Give your answer correct to two decimal places.

$$\int_0^{\pi} \frac{\sin\theta}{\sqrt{1-2a\cos\theta+a^2}} d\theta = \frac{1}{2a}\int_0^{\pi} \frac{d(1-2a\cos\theta+a^2)}{\sqrt{1-2a\cos\theta+a^2}}$$

$$= \frac{1}{a}\sqrt{1-2a\cos\theta+a^2}\Big|_0^{\pi}$$

$$= \frac{1}{a}\sqrt{(1+a)^2} - \frac{1}{a}\sqrt{(a-1)^2}$$

$$= \frac{a+1}{a} - \frac{a-1}{a} \quad (\because a>1)$$

$$= \frac{2}{a}$$

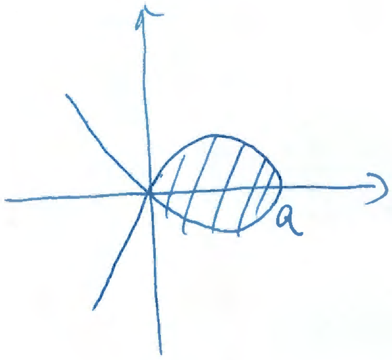
$$\therefore \frac{2}{a} = 0.2018$$

$$a = \frac{2}{0.2018}$$

$$= 9.910\cdots$$

$$\approx \underline{\underline{9.91}}$$

4. Let a denote a positive constant. If the area of the region bounded by the loop of the curve $y^2 = x^2(a-x)$ is equal to 99, find the value of a. Give your answer correct to two decimal places.



$$\text{Area} = 2\int_0^a x\sqrt{a-x}\,dx$$

Let $u = a - x$

$\therefore x = a - u$ and $dx = -du$

$$\therefore \text{Area} = 2\int_a^0 (a-u)\sqrt{u}\,(-du)$$

$$= 2\int_0^a (au^{1/2} - u^{3/2})\,du$$

$$= 2\left[\frac{2}{3}au^{3/2} - \frac{2}{5}u^{5/2}\right]_0^a$$

$$= 2\left\{\frac{2}{3}a^{5/2} - \frac{2}{5}a^{5/2}\right\}$$

$$= \frac{8}{15}a^{5/2}$$

$$\therefore \frac{8}{15}a^{5/2} = 99$$

$$a = \left(\frac{99 \times 15}{8}\right)^{2/5} = 8.080\ldots$$

$$\approx \underline{\underline{8.08}}$$