CS3243: Introduction to Artificial Intelligence

Semester 2, 2020

First-Order Logic (FOL)

AIMA Chapter 8

Outline

- Why FOL?
- Syntax and semantics of FOL
- Using FOL
- Wumpus world in FOL
- Knowledge engineering in FOL

Propositional Logic

Pros

- Declarative: tells agent what it needs to know to operate in its environment. No need to specify exact behavior
- Allows partial information via disjunction and negation (unlike many other data structures)
- Compositional: meaning of $A \land B$ derived from meanings of A and B.
- Context independent and unambiguous

Cons

- Limited expressive power: cannot concisely say "pits cause breezes in adjacent squares".

First-Order Logic

- Propositional logic assumes that the world contains facts
- First-order logic (like natural language) assumes that the world contains
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  - Relations: unary relations or properties such as red, round, prime, …, or more general n-ary relations such as brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus, …

Syntax of FOL: Basic Elements

Type	**Examples**
Constants	John, 2, NUS,…
Predicates (relations)	$Brother(x, y)$, $x > y$, …
Functions	$\sqrt{x}$, $LeftLeg(x)$,…
Variables	x, y, a, b
Connectives	$\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$
Equality	$=$
Quantifiers	$\forall, \exists$

Atomic Sentences

Term: *constant* or *variable* or $function(x_1, \dots, x_n)$

Functions can be viewed as complex names for constants

Atomic sentence: $predicate(x_1, \dots, x_n)$ or $x_1 = x_2$

E.g.,

- $Brother(John, Richard)$
- $Length\big(LeftLeg(Richard)\big) = Length\big(LeftLeg(John)\big)$

Complex Sentences

Constructed from atomic sentences via connectives

$$\neg\alpha, \alpha_1 \wedge \alpha_2, \alpha_1 \vee \alpha_2, \alpha \Rightarrow \beta, \alpha \Leftrightarrow \beta$$

E.g.,

- $Sibling(John, Richard) \Rightarrow Sibling(Richard, John)$
- $(a \leq b) \vee (a > b)$
- $(1 > 2) \wedge \neg(1 > 2)$

Truth in First-Order Logic

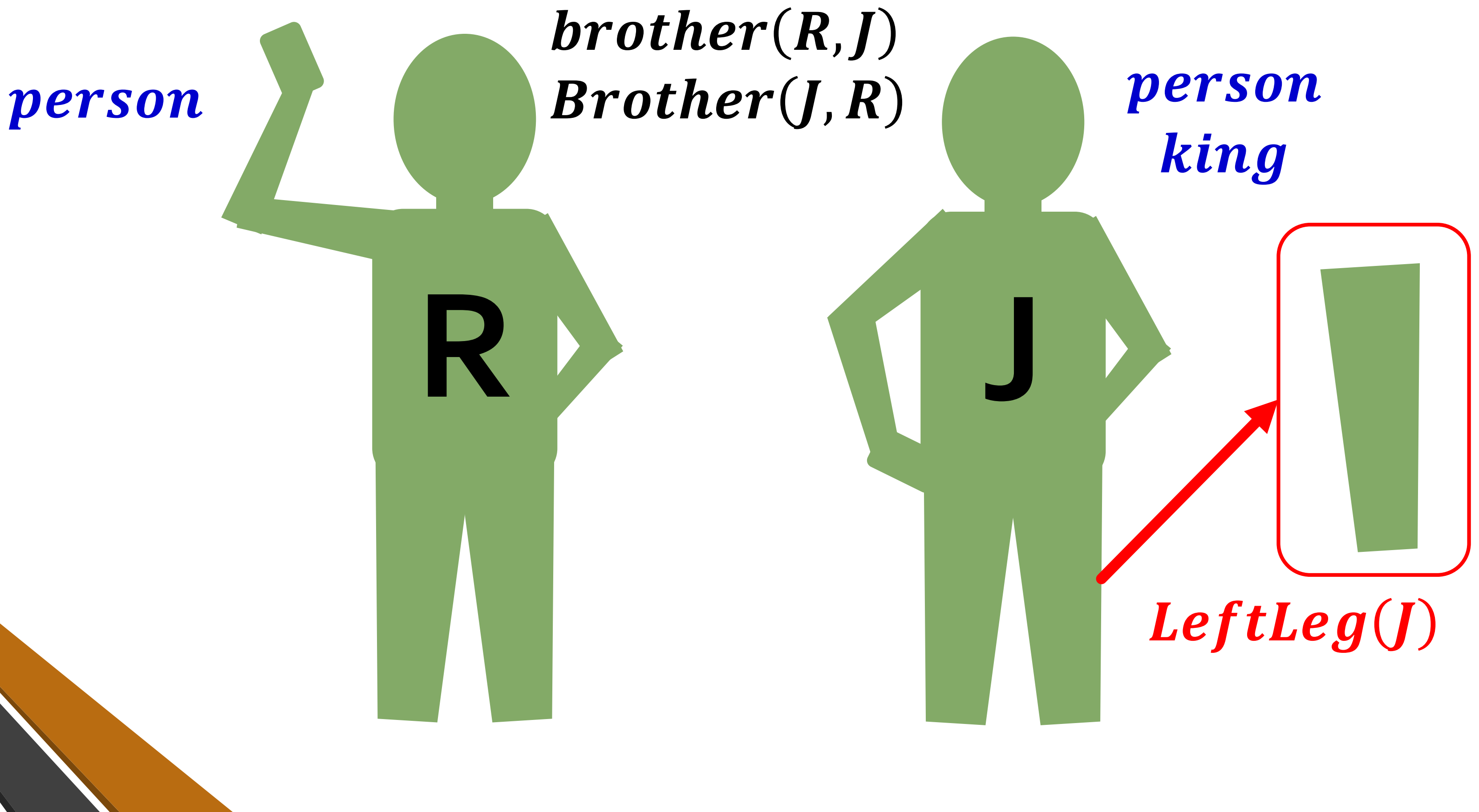
- Sentences are true in a model
- Model comprises a set of objects (domain elements) and an interpretation
- Interpretation specifies referents for

Objects	→	Constants
Relations	→	Predicates
Functional Relations	→	Function Symbols

- An atomic sentence $predicate(x_1, \ldots, x_n)$ is true in a given model if the relation referred to by $predicate$ holds among the objects referred to by $x_1, \ldots, x_n$.

Models for FOL: Example 1

Model contains 5 objects, 2 binary relations (black), 3 unary relations (blue), 1 unary function (red)



Universal Quantification

- $\forall < variables >: < sentence >$
- e.g., everyone at NUS is smart: $\forall x: x \in NUS \Rightarrow Smart(x)$
- $\forall x: P(x)$ is true in a given model if P is true with x referring to each possible object in the model
- Roughly speaking, it is equivalent to the conjunction of instantiations of P

$$\text{Alice} \in NUS \Rightarrow Smart(\text{Alice})$$
$$\wedge\ \text{Bob} \in NUS \Rightarrow Smart(\text{Bob})$$
$$\wedge\ \text{Claire} \in NUS \Rightarrow Smart(\text{Claire})$$
$$\ldots$$

A Common Mistake to Avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$
- Common mistake: using $\wedge$ as the main connective with $\forall$:

$$\forall x: x \in NUS \wedge Smart(x)$$

What does the above mean?

Existential Quantification

- $\exists < vars >: < sentence >$

  e.g., someone at NUS is smart: $\exists x: x \in NUS \wedge Smart(x)$

- $\exists x: P$ is true in a given model if P is true with x referring to at least one object in the model

- Roughly speaking, it is equivalent to the disjunction of instantiations of P

$$
\begin{aligned}
& Alice \in NUS \wedge Smart(Alice) \\
\vee\ & Bob \in NUS \wedge Smart(Bob) \\
\vee\ & Claire \in NUS \wedge Smart(Claire) \\
& \ldots
\end{aligned}
$$

Another Common Mistake to Avoid

- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$$\exists x : x \in NUS \Rightarrow Smart(x)$$

What does this mean?

Negation

- Negation of $\forall x: P(x)$ is $\exists x: \neg P(x)$
- Negation of $\exists x: P(x)$ is $\forall x: \neg P(x)$

$$\forall x: \left(\exists y: P(x,y)\right) \vee \left(\forall z: \exists y: \left(Q(x,y,z) \wedge P(y,z)\right)\right)$$

$$\Downarrow$$

$$\exists x: \left(\forall y: \neg P(x,y)\right) \wedge \left(\exists z: \forall y: \left(\neg Q(x,y,z) \vee \neg P(y,z)\right)\right)$$

Equality

- $x_1 = x_2$ is true under a given interpretation iff x_1 and x_2 refer to the same object
- With function: e.g., $Father(\text{John}) = \text{Henry}$
- With negation: e.g., definition of $Sibling$ in terms of $Parent$:

$$\forall x, y: Sibling(x, y)$$
$$\Leftrightarrow \Big(\neg(x = y)$$
$$\wedge \big(\exists m, f: \neg(m = f) \wedge Parent(m, x)$$
$$\wedge Parent(f, x) \wedge Parent(m, y)$$

Interacting with FOL KBs

- A Wumpus-world agent is using a FOL KB and perceives a smell, a breeze, and glitter at $t = 5$:

TELL(KB, *Percept*([*Smell*, *Breeze*, *Glitter*, *None*, *None*], 5))

ASK(KB, $\exists a$ *BestAction*(a, 5))

  - Quantified query: does the KB entail some best action at $t = 5$? Answer: Yes.

- ASKVARS(KB, S) returns the binding list or substitutions such that $KB \vdash S$
  - e.g., ASKVARS(KB, $\exists a$ *BestAction*(a, 5))
  - Answer: $\{a = Grab\} \leftarrow$ substitution (binding list)

KB for the Wumpus World

- Perception rule
  - Process agent's inputs
  - "If observed a glitter at time t, set $\text{Glitter}(t) = True$"
- Reflex rule
  - Process agent's outputs
  - $\forall t: \text{Glitter}(t) \Rightarrow \text{BestAction}(Grab, t)$
- Above rules yield $\text{BestAction}(\text{Grab}, 5)$

How would we write the above rule in propositional logic?

KB for the Wumpus World

Properties of squares:

- $\forall x, y, a, b\colon \mathrm{Adjacent}([x, y], [a, b]) \Leftrightarrow$
  $$\big(x = a \wedge (y = b - 1 \vee y = b + 1)\big)$$
  $$\vee \big(y = b \wedge (x = a - 1 \vee x = a + 1)\big)$$
- $\forall s, t\colon \mathrm{At}(\mathrm{Agent}, s, t) \wedge \mathrm{Breeze}(t) \Rightarrow \mathrm{Breezy}(s)$

Squares are breezy near a pit:

- $\forall s\colon \mathrm{Breezy}(s) \Leftrightarrow \exists r\colon \mathrm{Adjacent}(r, s) \wedge \mathrm{Pit}(r)$

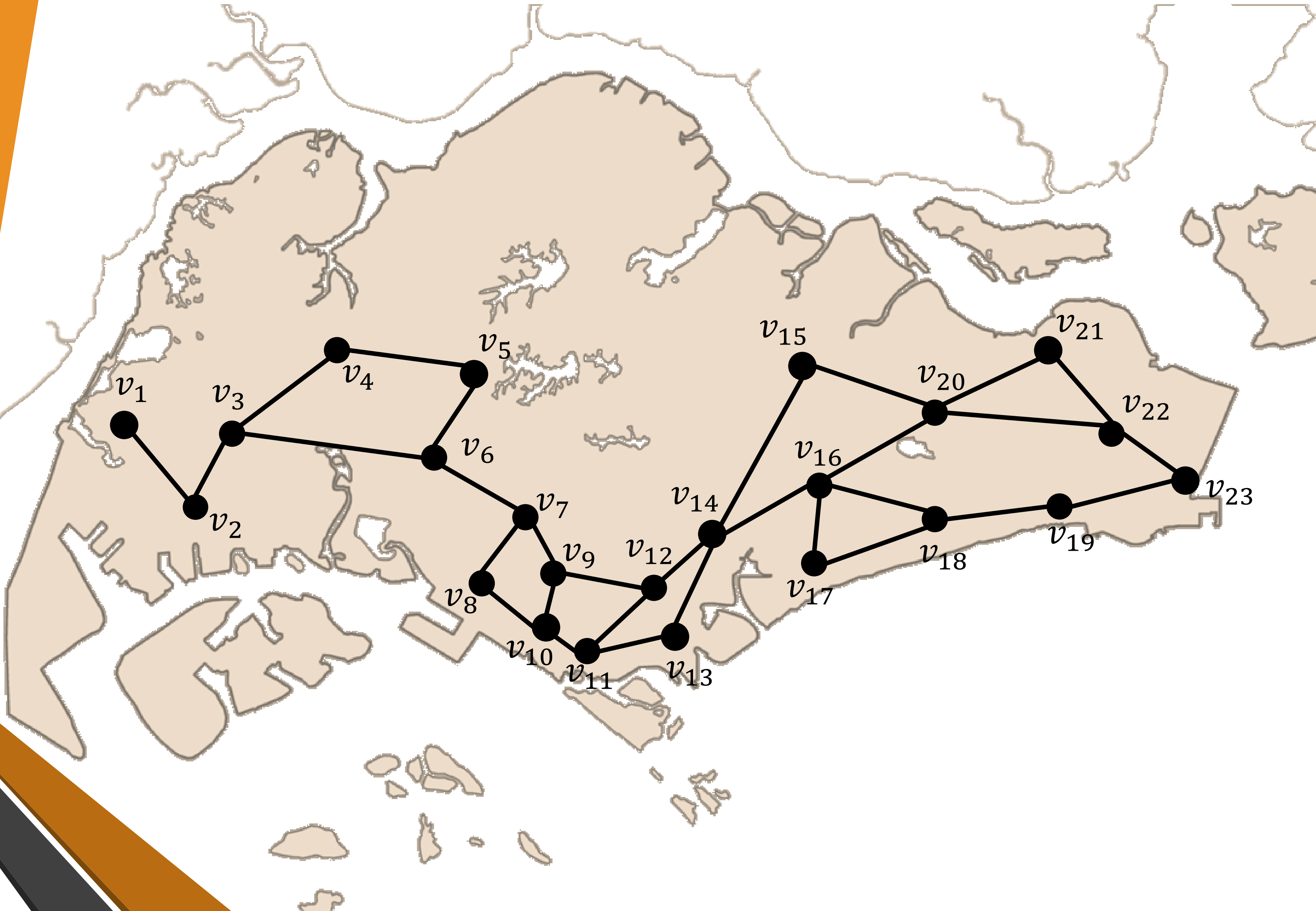
Knowledge Engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

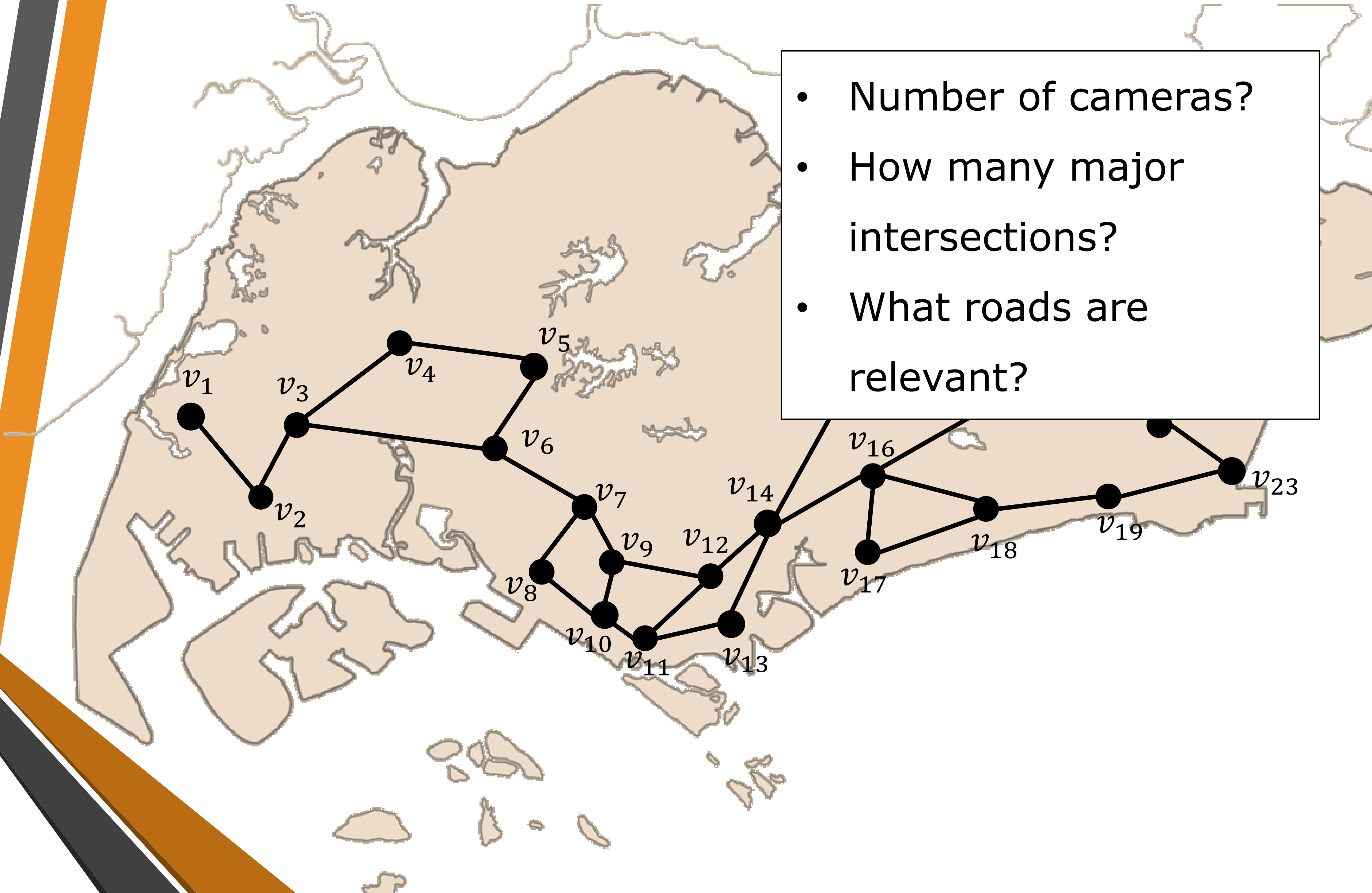
Optimal Traffic Management

- We are approached by the Singapore Police
- Want to optimally position traffic cameras in major intersections so as to cover all relevant roads.
- A camera in an intersection also covers adjacent ones.
- Please help!

Identify the Task



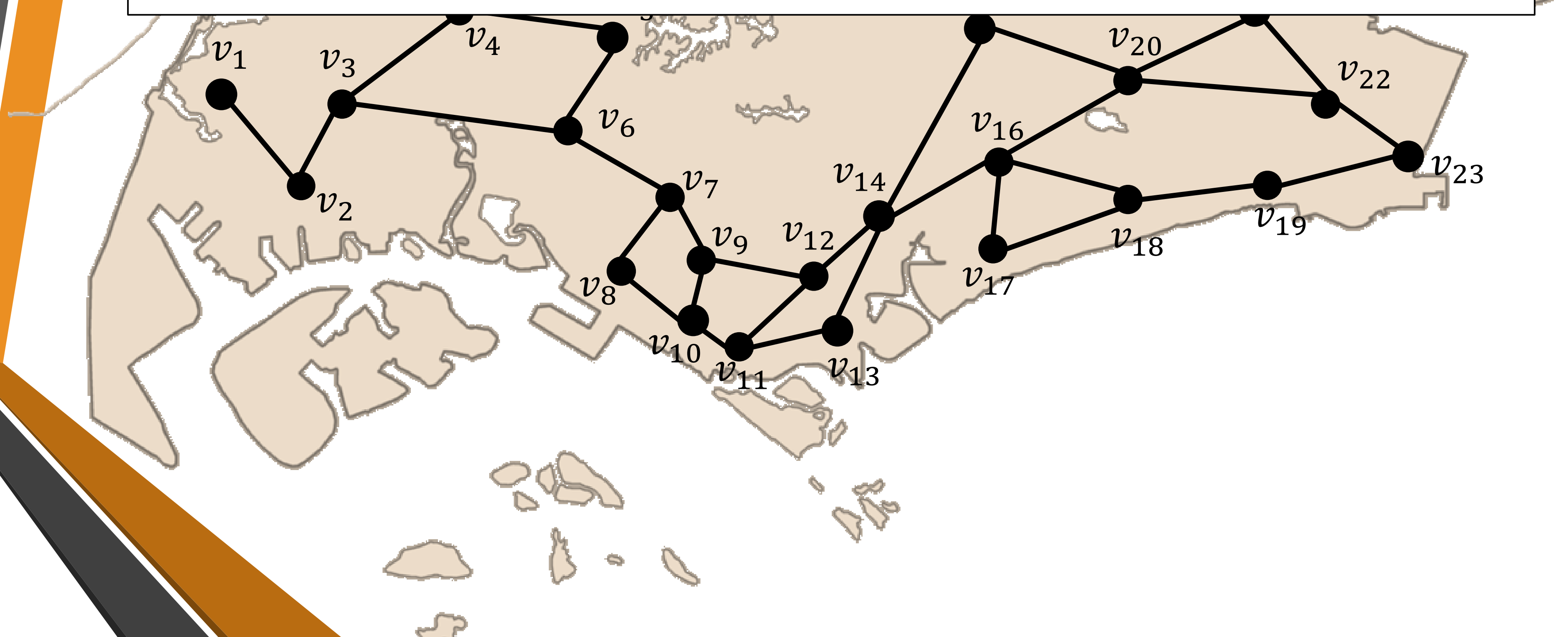
Assemble Relevant Knowledge



- Number of cameras?
- How many major intersections?
- What roads are relevant?

Decide on Vocabulary

- V – set of intersections
- $\text{edge}(u, v) \in \{0,1\}$ – is there a road connecting u and v
- $c(v) \in \{0,1\}$ - there is a camera in location v.
- Maximal number of cameras - $k \in \mathbb{Z}_+$



Encode General Domain Knowledge

- Edges are bidirectional –

$$\forall u, v \colon \mathrm{edge}(u, v) \Leftrightarrow \mathrm{edge}(v, u)$$

- Coverage property –

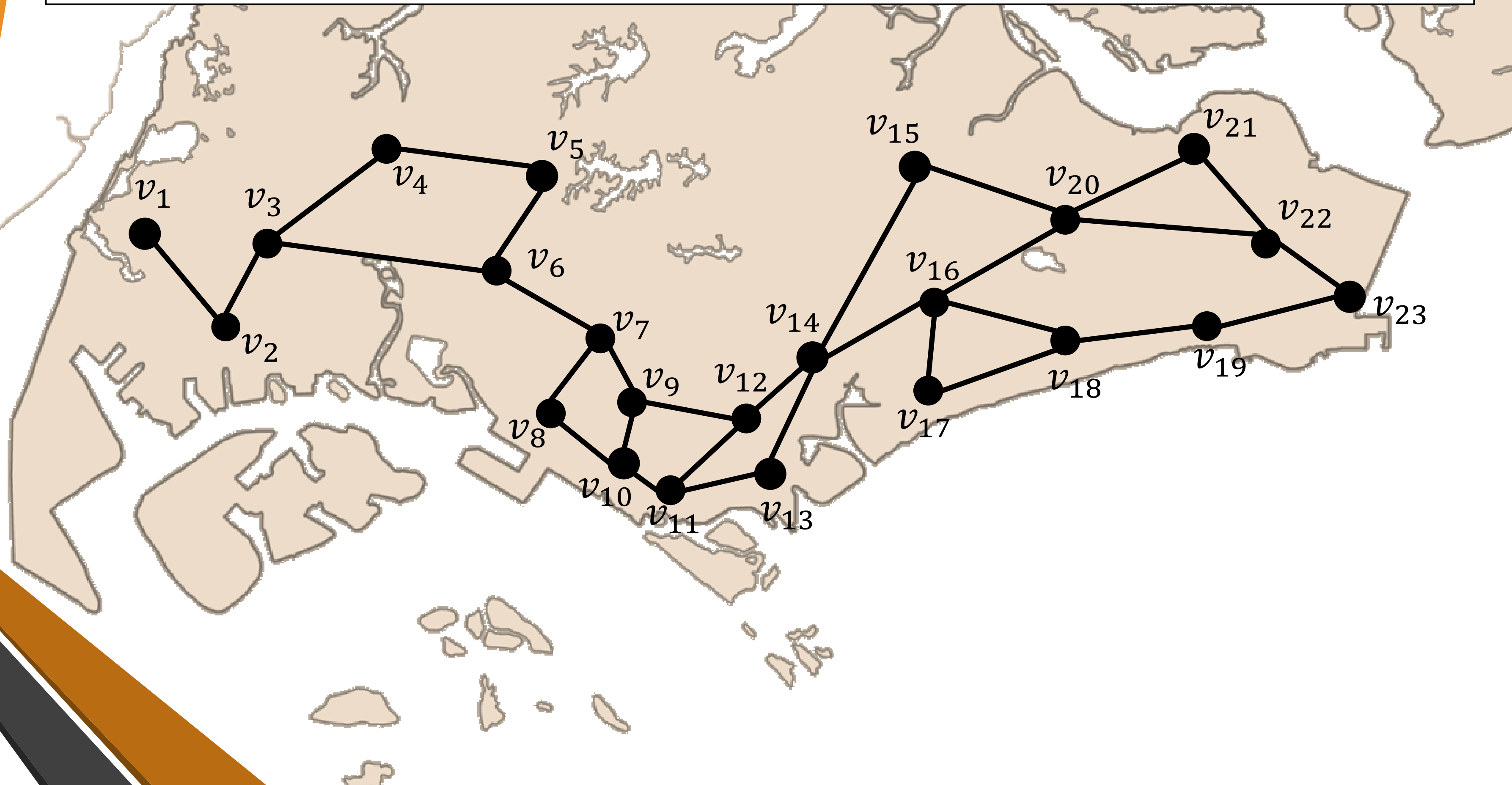
$$\mathrm{Covered}(u, v) \Leftrightarrow c(v) \vee c(u)$$

- Total coverage – $\mathrm{TotalCover}(V) \Leftrightarrow \forall e = \{u, v\} \in E \colon \mathrm{Covered}(e)$
- Is $U \subseteq V$ providing total coverage?

$$\mathrm{IsCovering}(U) \Leftrightarrow \left(\bigwedge_{u \in U} c(u) \right) \wedge \left(\bigwedge_{v \in V \setminus U} \neg c(v) \right) \wedge \mathrm{TotalCover}(V)$$

Encode the Specific Instance

- $V = \{v_1, \ldots, v_{23}\}$
- $\text{edge}(v_1, v_2)$, $\text{edge}(v_2, v_3)$, $\text{edge}(v_3, v_4)$, $\text{edge}(v_3, v_6)$, …



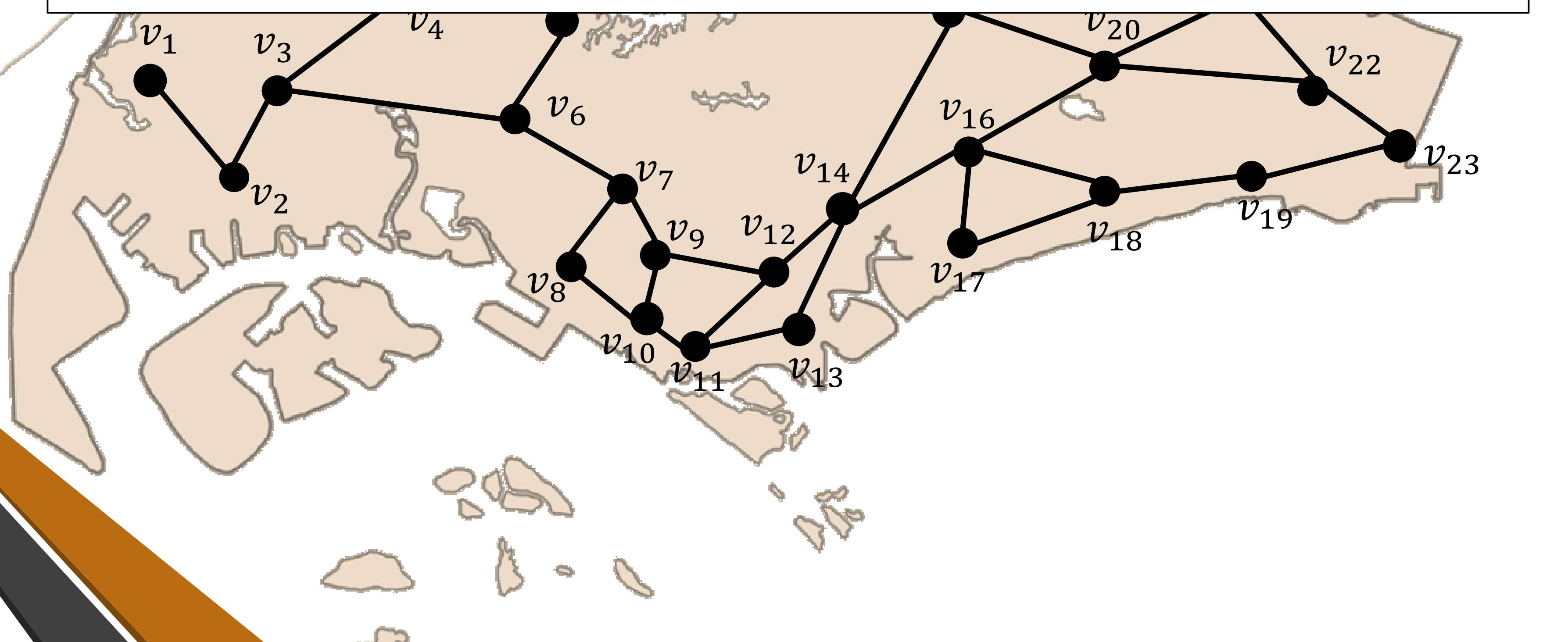
Pose Queries

- Is there a solution using k cameras?

$$\exists u_1, \ldots, u_k : \mathrm{IsCovering}(\{u_1, \ldots, u_k\})$$

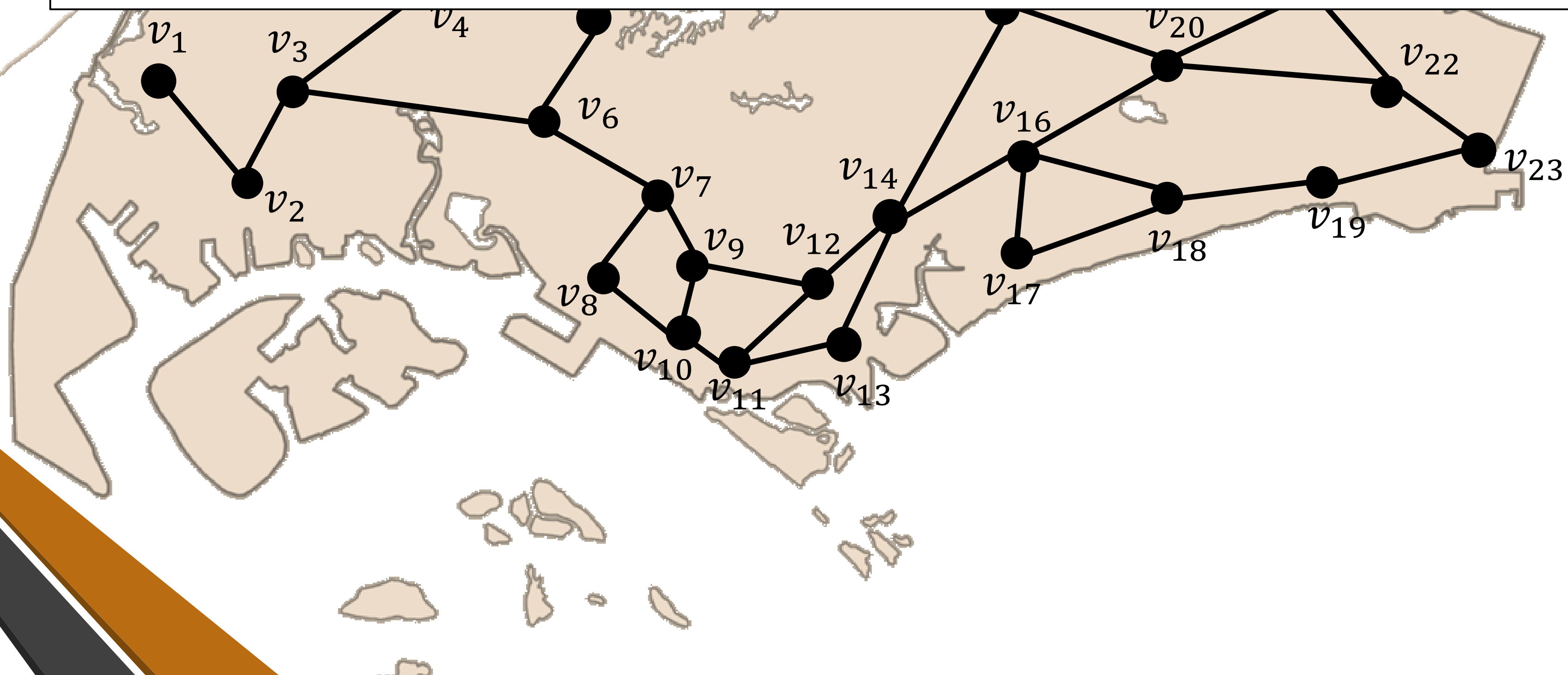
- Will a specific solution work?

$$\mathrm{IsCovering}(\{v_2, v_4, v_6, v_{10}, v_{12}, v_{16}\})$$



Debug Database

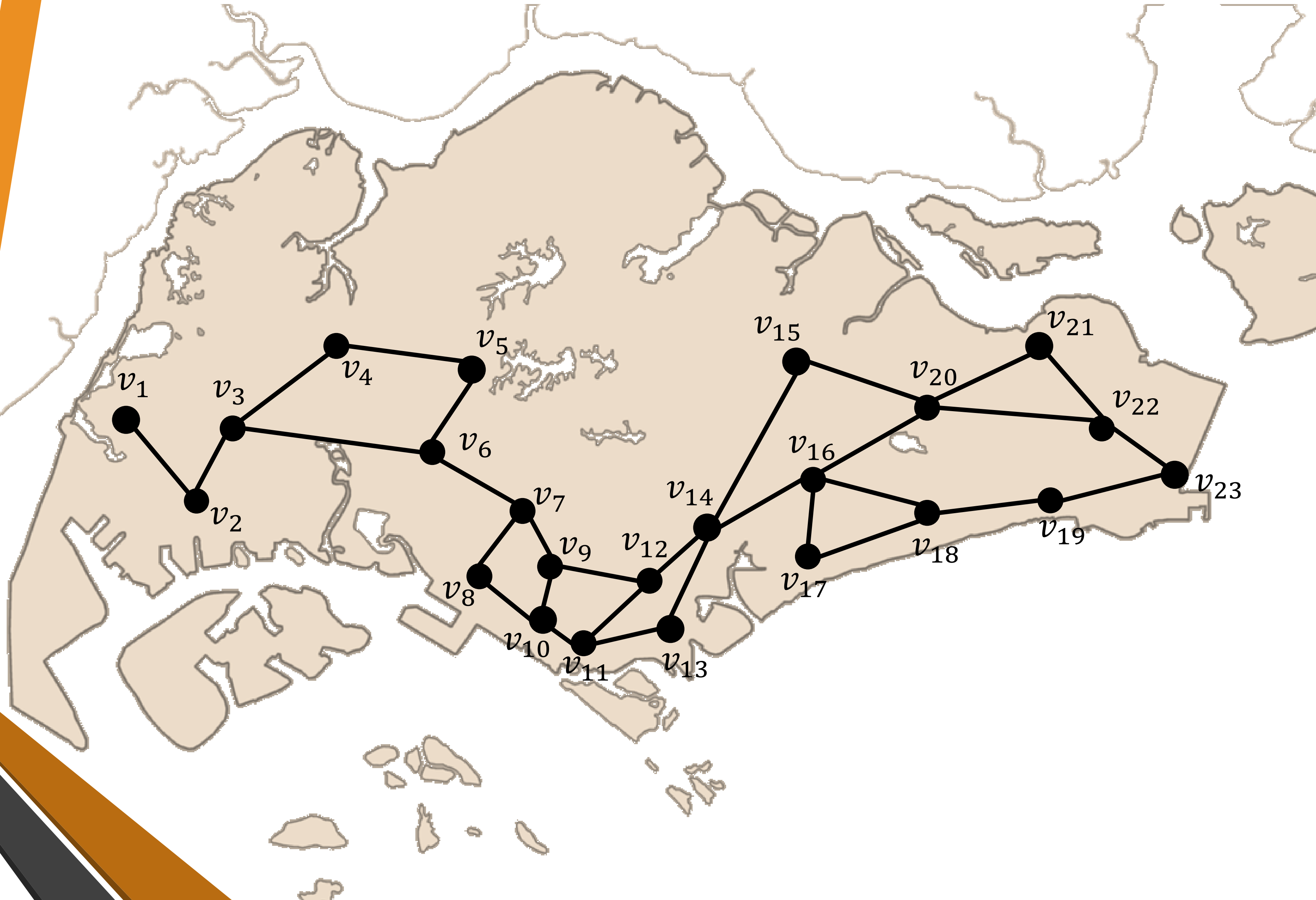
- $\forall u, v\colon \mathrm{edge}(u, v) \Rightarrow u \in V \wedge v \in V$
- $\forall u, v\colon \mathrm{edge}(u, v) \Rightarrow u \neq v$
- $\forall v\colon c(v) \Rightarrow v \in V$
- …



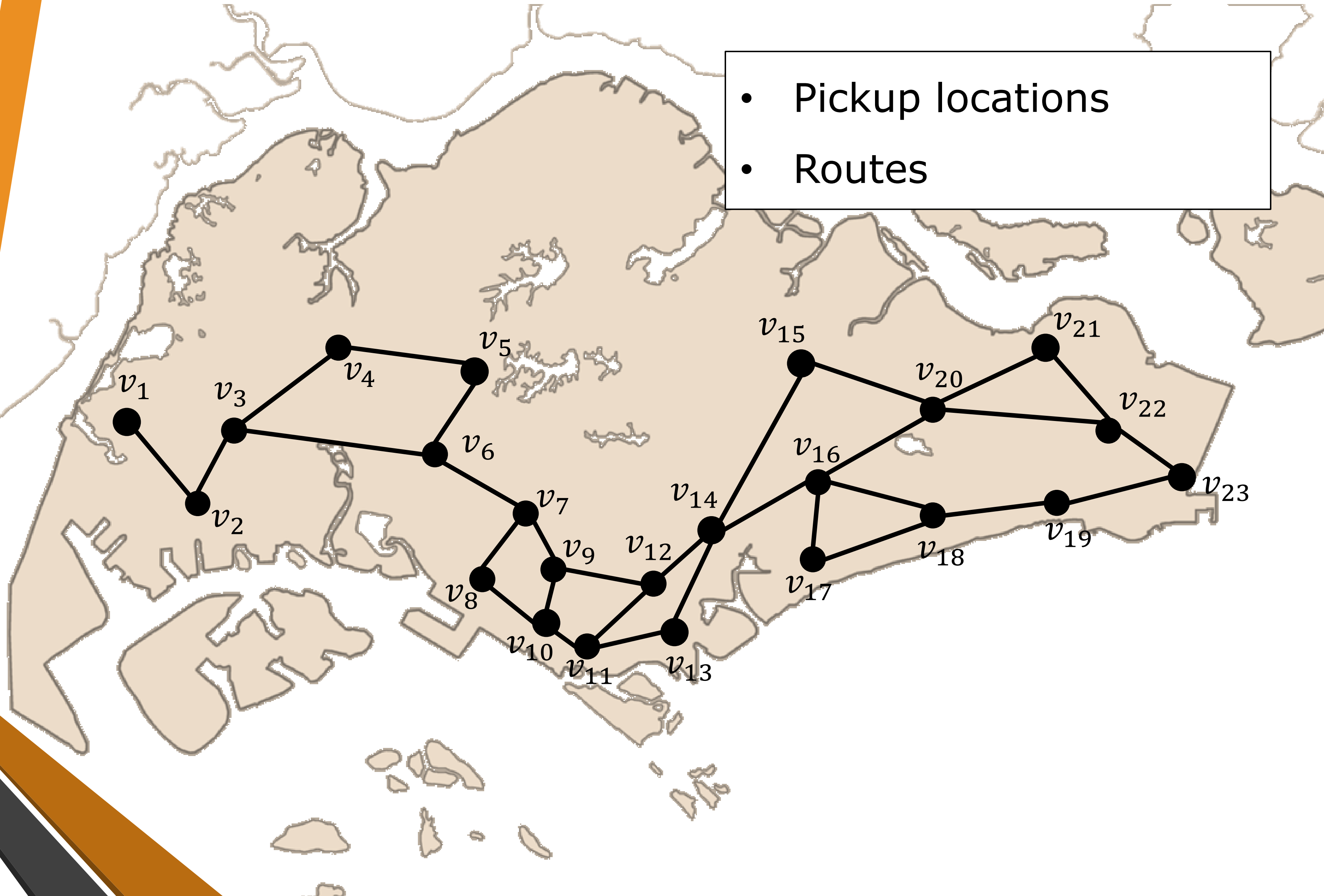
Waste Disposal

- We are approached by a Waste Disposal Service
- Want to optimally collect garbage from various locations.
- Don't want to visit same location twice

Identify the Task

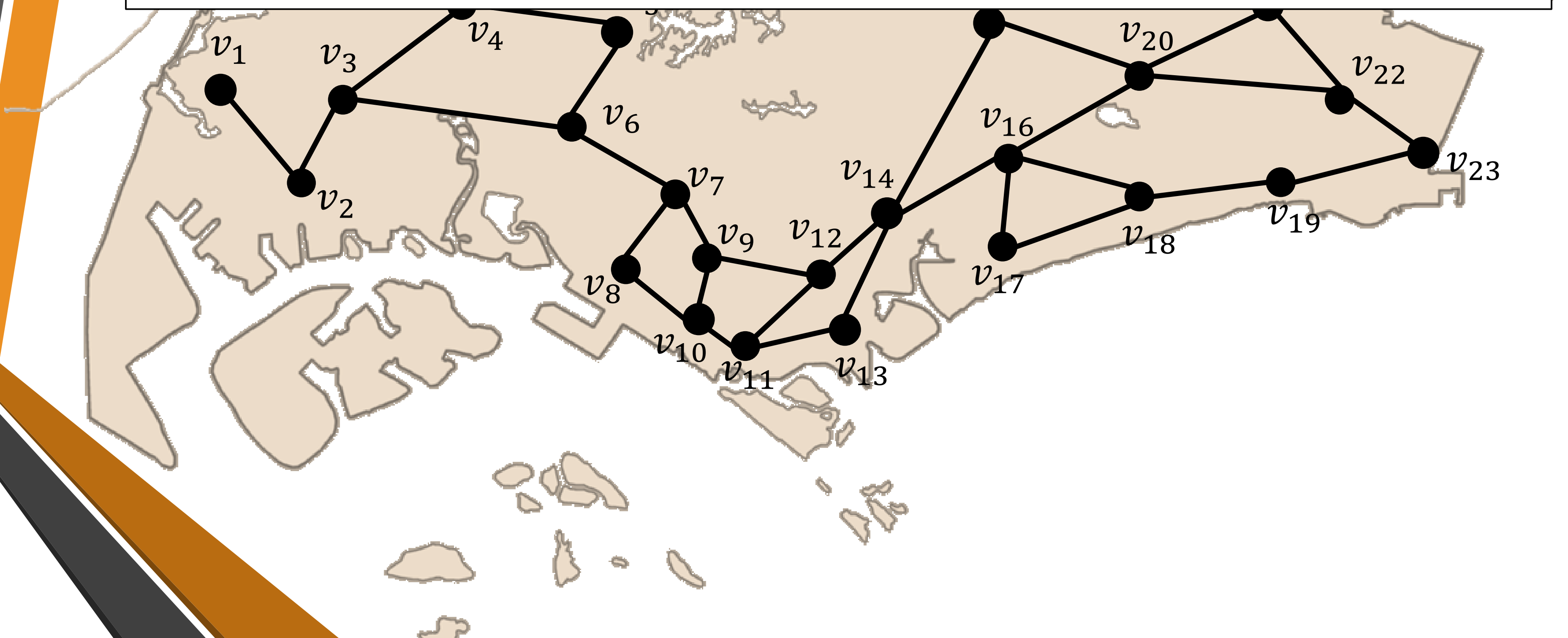


Assemble Relevant Knowledge



Decide on Vocabulary

- V – set of locations
- $\mathrm{edge}(u, v) \in \{0,1\}$ – is there a road connecting u and v
- $\mathrm{next}(u, v) \in \{0,1\}$: we move from u to v.
- Start location: $\mathrm{start}(v)$



Encode General Domain Knowledge

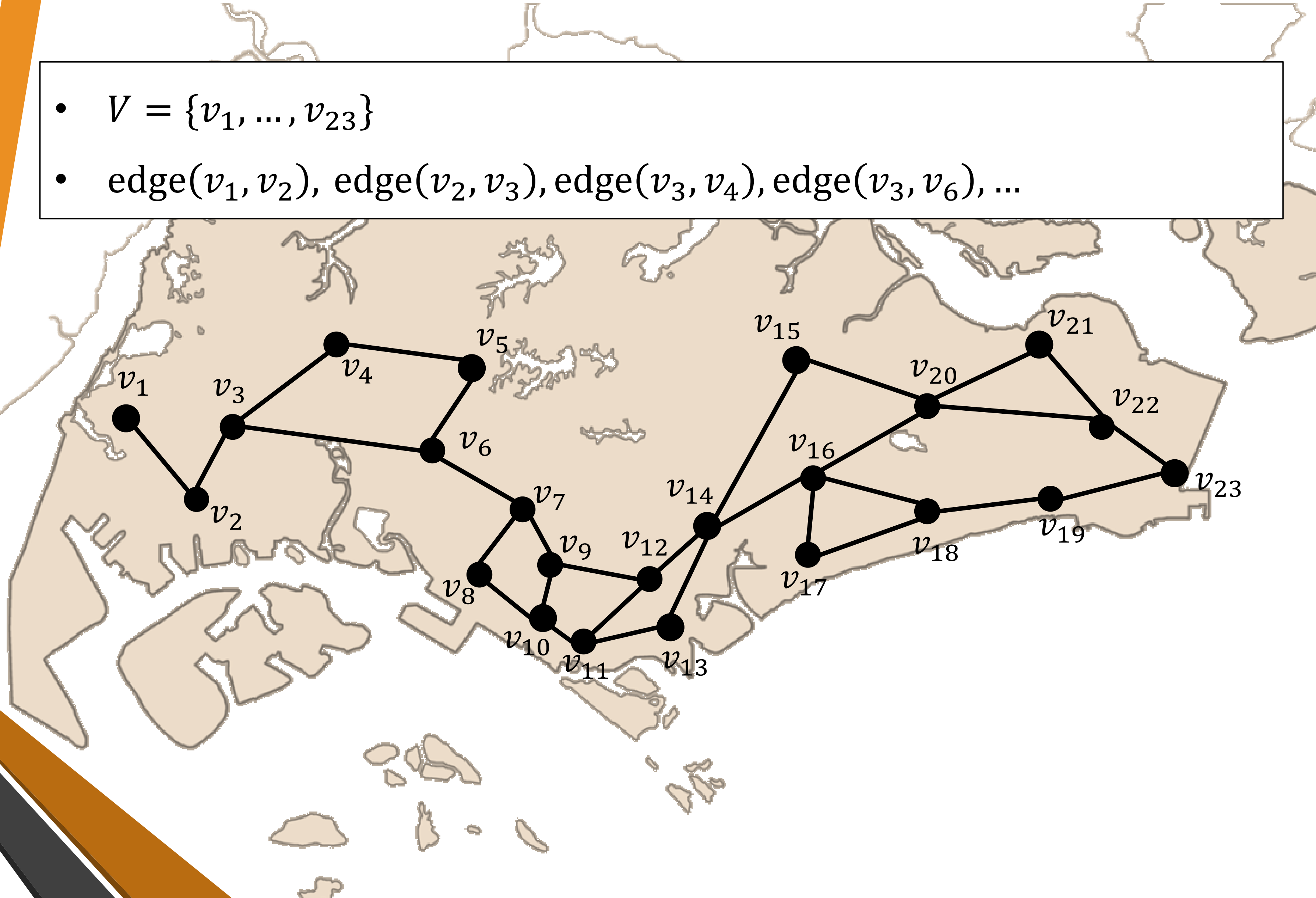
- $\text{edge}(u, v) \in \{0,1\}$: there is an edge between u and v.
- Start location is unique:

$$\exists v_0 : \left(v_0 \in V \wedge \text{start}(v_0)\right) \wedge \left(\forall v : \text{start}(v) \Rightarrow (v = v_0)\right)$$

- Can only travel on edges: $\text{next}(u, v) \Rightarrow \text{edge}(u, v)$
- $\text{Visited}(v) \Leftrightarrow \exists u : \text{next}(u, v) \vee \text{start}(v)$
- $\text{Successor}(u, v) \Leftrightarrow \text{next}(u, v) \vee \exists w : \text{next}(u, w) \wedge \text{Successor}(w, v)$
- $\text{VisitedOnce}(v) \Leftrightarrow \text{Visited}(v) \wedge \neg\text{Successor}(v, v)$

Encode the Specific Instance

- $V = \{v_1, \ldots, v_{23}\}$
- $\text{edge}(v_1, v_2)$, $\text{edge}(v_2, v_3)$, $\text{edge}(v_3, v_4)$, $\text{edge}(v_3, v_6)$, …

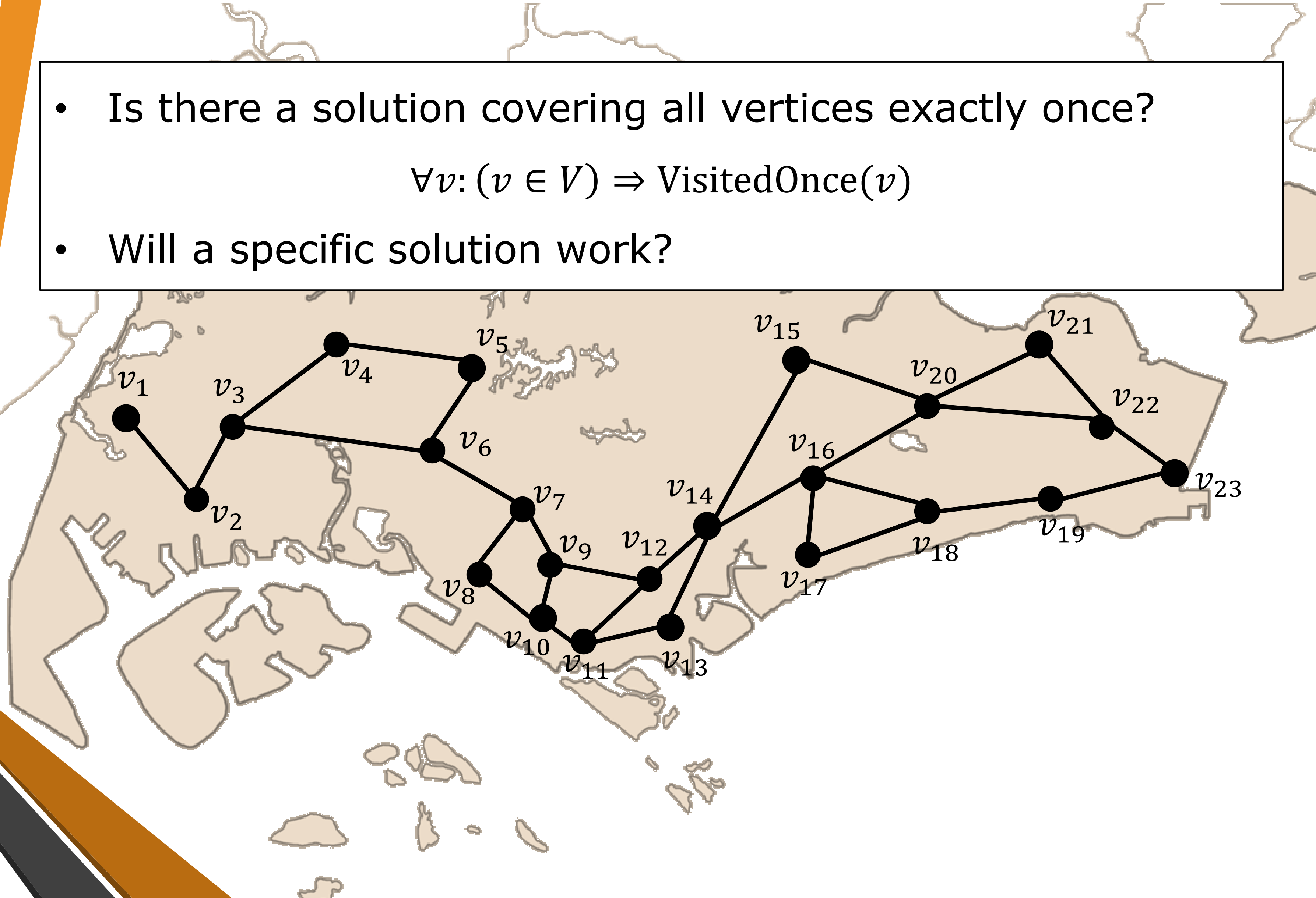


Pose Queries

- Is there a solution covering all vertices exactly once?

$$\forall v: (v \in V) \Rightarrow \mathrm{VisitedOnce}(v)$$

- Will a specific solution work?



Debug Database

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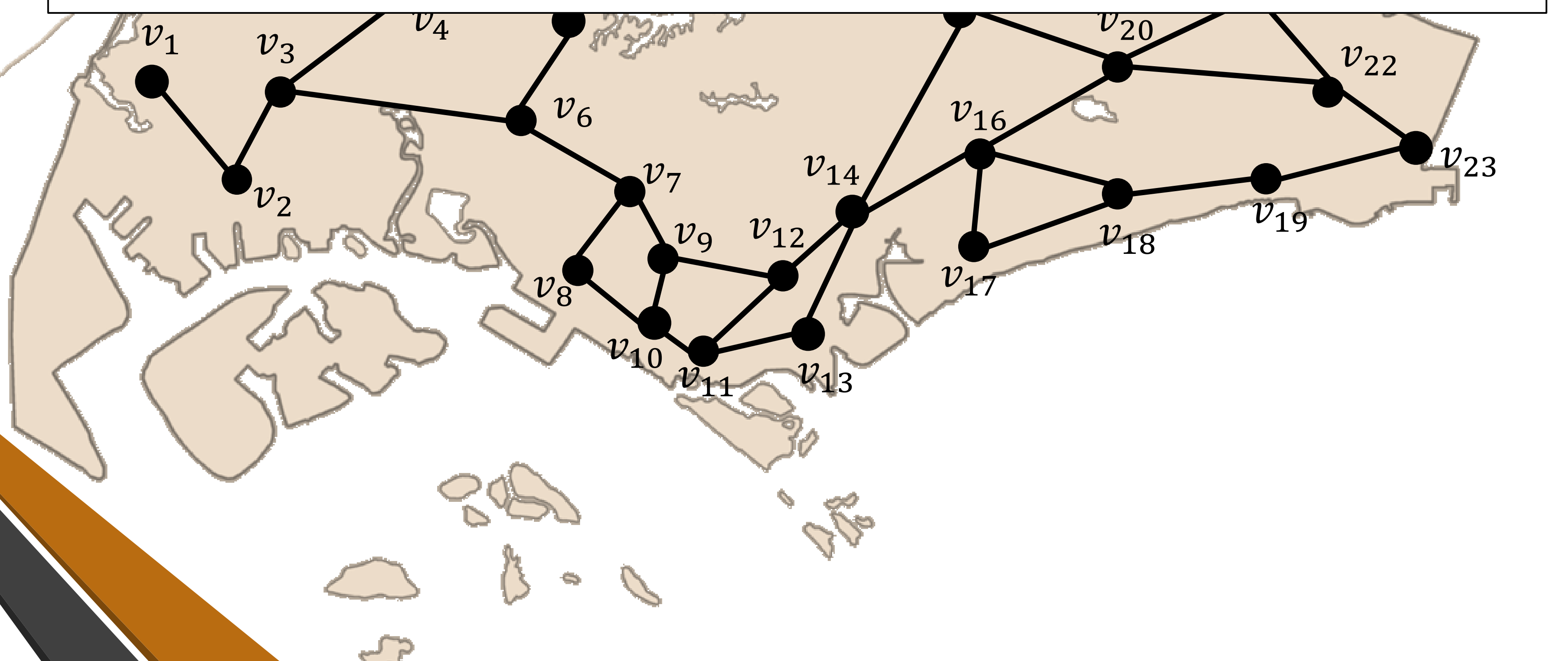

Summary

- First-order logic:
  - objects and relations are semantic primitives
  - syntax: constants, functions, predicates, equality, quantifiers
- Increased expressive power over propositional logic: sufficient to define many non-trivial problems