Date No.

3. To show the problem is NP-complete, I will first show a reduction from Partition to Subset-Sum, and after showing Subset-Sum is NP-Complete, this problem can be shown to be NP-complete.

Let k to be the maximum of n, number of people who owe money, and m, number of people who are owed. To show that subset-sum is NP-complete, for a given target sum X and an input set $S = \{a_1, a_2, a_3, \ldots, a_k\}$ simply partition S into two sets S_1 and S_2 and set X to be $\frac{1}{2}\sum_{i=1}^{k} a_i$. It is clear that no pre-processing for S is needed, therefore the reduction runs in polynomial time, and shown that subset-sum is NP-complete.

To prove the problem is NP-complete, I will now show a reduction from subset sum. Set k as the max of (n,m). Select the set of smaller size between A and B, which is the people who owe money and the people who are owed. For each value in the set, select the target sum as the value, and determine if there exist a subset sum in the other set. This is done at most k times, and a yes-instance of the problem is returned if all the subset sums are yes instance. The preprocessing runs in poly time, therefore the reduction is complete, and the problem is NP-complete.

(1) Choose smaller set out of A, B

(2) For each value out of chosen set, select subset-sum target as the value, find if there exist a subset-sum in the set not chosen.

(3) Runs at most |Size of chosen set| times.

Shown that Partition $\leq_p$ subset-sum $\leq_p$ pay money, therefore this problem is NP-complete.