National University of Singapore
School of Computing

Semester 1, AY2021-22

CS4246/CS5446 AI Planning and Decision Making

Issued: 27 Aug 2021

Tutorial Week 4: Rational Decision Making

Guidelines

You may discuss the content of the questions with your classmates. But to everyone should work on and be ready to present ALL the solutions.

Problem 1: Allais Paradox

The Allais paradox (Allais, 1953) is a well-known problem potentially suggesting that humans are "predictably irrational" (Ariely, 2009). People are given a choice between lotteries A and B and then between C and D, which have the following prizes:

A: 80% chance of $4000	C: 20% chance of $4000
B: 100% chance of $3000	D: 25% chance of $3000

Most people consistently prefer B over A (i.e., taking the sure payoff), and C over D (taking the higher EMV).

a Show that the normative analysis (i.e., describing how a rational agent should act) disagrees. [**Hint**: Set U($0) = 0; show that the preferences between A, B and C, D are opposites, hence a contradiction]

b Prove that the judgments $B \succ A$ and $C \succ D$ in the above Allais paradox violate the axiom of substitutability. [**Hint**: You may wish to consider using the axiom of decomposability.]

Problem 2: Preference Modelling

Alex is given the choice between two games:

- **Game 1**: a fair coin is flipped and if it comes up heads, Alex receives $100. If the coin comes up tails, Alex receives nothing.

- **Game 2**: a fair coin is flipped twice. Each time the coin comes up heads, Alex receives $50, and Alex receives nothing for each coin flip that comes up tails.

Alex prefers Game 2 to Game 1. Argue that Alex would prefer to receive $50 compared to being allowed to participate in Game 1.

Problem 3: Risk Tolerance

Economists often make use of an exponential utility function for money: $U(x) = -exp(-x/R)$ where R is a positive constant representing an individual's risk tolerance.

Risk tolerance reflects how likely an individual is to accept a lottery with a particular EMV versus some certain payoff. As R (which is measured in the same units as x) becomes larger, the individual becomes less risk-averse. Mary is risk adverse.

a Assume that Mary has an exponential utility function with R = \$500. Mary is given the choice between receiving \$500 with certainty (probability 1) or participating in a lottery which has a 60% probability of winning \$5000 and a 40% probability of winning nothing. Assuming Mary acts rationally, which option would she choose? Show how you derived your answer.

b Consider the choice between receiving \$100 with certainty (probability 1) or participating in a lottery which has a 50% probability of winning \$500 and a 50% probability of winning nothing. What value of R in the exponential utility function specified above that would cause an individual to be indifferent to these two alternatives? (Writing the equation will do, no need to solve.)
