

CS1231–Midterm 1, 2017

Name:

Matric Number:

Tutorial Group:

Seat Number:

1. [3 marks] Using a truth table, determine if the following is a **contradiction**.

$$\neg(p \vee q \vee \neg r) \wedge ((r \rightarrow p) \vee (r \rightarrow q))$$

$p$	$q$	$r$	$\neg(p \vee q \vee \neg r)$	$r \rightarrow p$	$r \rightarrow q$	$\neg(p \vee q \vee \neg r) \wedge ((r \rightarrow p) \vee (r \rightarrow q))$
$T$	$T$	$T$				
$T$	$T$	$F$				
$T$	$F$	$T$				
$T$	$F$	$F$				
$F$	$T$	$T$				
$F$	$T$	$F$				
$F$	$F$	$T$				
$F$	$F$	$F$				

Ans: 

yes	/	no
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2. [2 marks] Use **theorem of logical equivalence laws** to show that

$$\neg(p \wedge (q \vee r)) \equiv (\neg p \vee \neg q) \wedge (\neg p \vee \neg r).$$

**Proof using theorem:**

3. [2 marks] Simplify  $\neg p \rightarrow q$  to an expression without  $\rightarrow$ .

4. [4 marks] Let  $C(x)$  be “ $x$  is a student in CS1231”,  $G(x)$  be “ $x$  can speak German”,  $F(x)$  be “ $x$  can speak French”. Translate the following into logical expression with domain (i) all students in CS1231; (ii) all students in the university.

(a) Some students in CS1231 can speak both German and French.

Answer: (i)

(ii)

(b) No student in CS1231 can speak both German and French.

Answer: (i)

(ii)

5. [2 marks] Determine, with justification, the truth values of the following expression.

$$\forall x \in \mathbb{R} \exists y \in \mathbb{R}, xy = 1.$$

Truth Value:

Justification:

6. [2 marks] In the specification of a system for booking theatre seats,  $B(p, s)$  denotes the predicate “person  $p$  has booked seat  $s$ ”. Let  $X$  be the domain of all persons  $p$ . Let  $Y$  be the domain of all seats  $s$  in the theatre. Translate the following into logical expressions using quantifiers  $\forall, \exists$ , and logical connectives.

(i) All seats are booked.

Answer:

(ii) No seat is booked by more than one person.

Answer:

7. [2 marks] Translate the following into a logical expression using quantifiers  $\forall, \exists$ , and logical connectives. Use  $C$ , the set of all students in CS1231, and  $H$ , the set of all help sheets as domains. Let  $E(s)$  be “Student  $s$  has a test”. Let  $R(s, k)$  be “Student  $s$  brings help sheet  $k$ ”. Let  $W(k)$  be “Help sheet  $k$  is handwritten”.

“Every CS1231 student having a test brings **exactly one** help sheet which has to be handwritten.”

Answer:

8. [3 marks] Derive the conclusion  $p$  from the following given hypotheses:

(i)  $a \wedge q \rightarrow m$ , (ii)  $(f \rightarrow q) \wedge (\neg p \rightarrow a)$ , (iii)  $\neg m$ , (iv)  $f$ .

Answer: