CS1231: Discrete Structures

Tutorial 4

Li Wei

Department of Mathematics National University of Singapore

18 February, 2019

Quick Review

- $a \in \{a, b\}$ V.S. $\{a\} \subseteq \{a, b\}$.
- $A \subset B$ definition: $A \subseteq B$ and $A \neq B$.
- A = B definition: $\forall x (x \in A \leftrightarrow x \in B)$.
- ightharpoonup |A| definition: Number of distinct elements in A.
- ▶ Set Operations: \overline{A} , $A \cap B$, $A \cup B$, A B.
- ▶ Truth Set: $T_p = \{x \in D \mid P(x) \text{ is true}\}.$
- ► Cartesian Product: $A \times B = \{(a, b) \mid a \in A, b \in B\}.$
- Power Set: $P(A) = \{X | X \subseteq A\}$
- Ø.

Menu

Question 1	Question 5(a)	Question 9
Overtion 2	Question 5(b)	Question 10(a)-(c)
Question 2	Question 6	Question 10(d)
Question 3	Question 7	Question 11(a)
Question 4	Question 8	Question 11(b)

- 1. Determine whether these are true for false.
- (a) $\emptyset \in \{\emptyset\}$
- (b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
- (c) $\{\emptyset\} \in \{\emptyset\}$
- $(\mathsf{d}) \ \{\varnothing\} \in \{\{\varnothing\}\}$
- (e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}\$ (f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}\$
- (g) $\{\{\emptyset\}\}\subset\{\{\emptyset\},\{\emptyset\}\}\}$

- $x \in A$: $x \in A$: x
- $A \subset B \colon A \text{ is a proper subset of } B \text{, i.e. every member of } A \text{ is a member of } B \text{ and } A \neq B.$

- 1. Determine whether these are true for false.
- (a) $\emptyset \in \{\emptyset\}$
- (b) $\varnothing \in \{\varnothing, \{\varnothing\}\}$
- (c) $\{\emptyset\} \in \{\emptyset\}$
- (d) $\{\emptyset\} \in \{\{\emptyset\}\}\$
- (e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
- (f) $\{\{\emptyset\}\}\subset\{\emptyset,\{\emptyset\}\}$
- (g) $\{\{\emptyset\}\}\subset\{\{\emptyset\},\{\emptyset\}\}\}$

- $x \in A$: $x \in A$: x
- $A \subset B$: A is a proper subset of B, i.e. every member of A is a member of B and $A \neq B$.

Answer.

- 1. Determine whether these are true for false.
- (a) $\emptyset \in \{\emptyset\}$
- (b) $\varnothing \in \{\varnothing, \{\varnothing\}\}$
- (c) $\{\emptyset\} \in \{\emptyset\}$
- (d) $\{\emptyset\} \in \{\{\emptyset\}\}\$
- (e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
- (f) $\{\{\emptyset\}\}\subset\{\emptyset,\{\emptyset\}\}$
- (g) $\{\{\emptyset\}\}\subset\{\{\emptyset\},\{\emptyset\}\}\}$

- $x \in A$: $x \in A$: x
- $A \subset B$: A is a proper subset of B, i.e. every member of A is a member of B and $A \neq B$.

Answer.T,

- 1. Determine whether these are true for false.
- (a) $\emptyset \in \{\emptyset\}$
- (b) $\varnothing \in \{\varnothing, \{\varnothing\}\}$
- (c) $\{\emptyset\} \in \{\emptyset\}$
- (d) $\{\emptyset\} \in \{\{\emptyset\}\}\$
- (e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
- (f) $\{\{\emptyset\}\}\subset\{\emptyset,\{\emptyset\}\}$
- (g) $\{\{\emptyset\}\}\subset\{\{\emptyset\},\{\emptyset\}\}\}$

- $x \in A$: $x \in A$: x
- $A \subset B$: A is a proper subset of B, i.e. every member of A is a member of B and $A \neq B$.

Answer.T,T,

- 1. Determine whether these are true for false.
- (a) $\emptyset \in \{\emptyset\}$
- (b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
- (c) $\{\emptyset\} \in \{\emptyset\}$
- (d) $\{\emptyset\} \in \{\{\emptyset\}\}\$
- (e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
- (f) $\{\{\emptyset\}\}\subset\{\emptyset,\{\emptyset\}\}$
- (g) $\{\{\emptyset\}\}\subset\{\{\emptyset\},\{\emptyset\}\}\}$

- $x \in A$: $x \in A$: x
- $A \subset B$: A is a proper subset of B, i.e. every member of A is a member of B and $A \neq B$.

Answer.T,T,F,

- 1. Determine whether these are true for false.
- (a) $\emptyset \in \{\emptyset\}$
- (b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
- (c) $\{\emptyset\} \in \{\emptyset\}$
- (d) $\{\emptyset\} \in \{\{\emptyset\}\}\$
- (e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
- (f) $\{\{\emptyset\}\}\subset\{\emptyset,\{\emptyset\}\}$
- (g) $\{\{\emptyset\}\}\subset\{\{\emptyset\},\{\emptyset\}\}\}$

- $x \in A$: $x \in A$: x
- $A \subset B$: A is a proper subset of B, i.e. every member of A is a member of B and $A \neq B$.

Answer.T,T,F,T,

- 1. Determine whether these are true for false.
- (a) $\emptyset \in \{\emptyset\}$
- (b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
- (c) $\{\emptyset\} \in \{\emptyset\}$
- (d) $\{\emptyset\} \in \{\{\emptyset\}\}\$
- (e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
- (f) $\{\{\emptyset\}\}\subset\{\emptyset,\{\emptyset\}\}$
- (g) $\{\{\emptyset\}\}\subset\{\{\emptyset\},\{\emptyset\}\}\}$

- $x \in A$: $x \in A$: $x \in A$: $x \in A$:
- $A \subset B$: A is a proper subset of B, i.e. every member of A is a member of B and $A \neq B$.

Answer. T, T, F, T, T,

- 1. Determine whether these are true for false.
- (a) $\emptyset \in \{\emptyset\}$
- (b) $\varnothing \in \{\varnothing, \{\varnothing\}\}$
- (c) $\{\emptyset\} \in \{\emptyset\}$
- (d) $\{\emptyset\} \in \{\{\emptyset\}\}\$
- (e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
- $\text{(f) } \{\{\varnothing\}\} \subset \{\varnothing, \{\varnothing\}\}$
- (g) $\{\{\emptyset\}\}\subset\{\{\emptyset\},\{\emptyset\}\}\}$

- $x \in A$: x is a member of the set A.
- $A \subset B$: A is a proper subset of B, i.e. every member of A is a member of B and $A \neq B$.

Answer. T, T, F, T, T, T,

- 1. Determine whether these are true for false.
- (a) $\emptyset \in \{\emptyset\}$
- (b) $\varnothing \in \{\varnothing, \{\varnothing\}\}$
- (c) $\{\emptyset\} \in \{\emptyset\}$
- (d) $\{\emptyset\} \in \{\{\emptyset\}\}\$
- (e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
- (f) $\{\{\emptyset\}\}\subset\{\emptyset,\{\emptyset\}\}$
- (g) $\{\{\emptyset\}\}\subset\{\{\emptyset\},\{\emptyset\}\}\}$

- $x \in A$: x is a member of the set A.
- $A \subset B$: A is a proper subset of B, i.e. every member of A is a member of B and $A \neq B$.

Answer. T, T, F, T, T, T, F.

Recall

$$\triangle B = D \text{ iff } \forall n (n \in B \leftrightarrow n \in D).$$

$$\begin{array}{ll} n \in B \Rightarrow n \in D, \\ \text{i.e. } B \subseteq D \\ \\ n \in B \\ \\ \Rightarrow n = \\ \\ \Rightarrow n = 3(\\ \\ \Rightarrow n \in D \\ \end{array} \qquad \begin{array}{ll} n \in D \Rightarrow n \in B, \\ \text{i.e. } D \subseteq B \\ \\ n \in D \\ \\ \Rightarrow n = \\ \\ \Rightarrow n = 3(\\ \\ \Rightarrow n \in B \\ \end{array}$$

Recall

$$\triangle B = D \text{ iff } \forall n (n \in B \leftrightarrow n \in D).$$

$$\begin{array}{ll} n \in B \Rightarrow n \in D, \\ \hline \text{i.e. } B \subseteq D \\ \hline n \in B \\ \hline \Rightarrow n = 3j + 2, j \in \mathbb{Z} \\ \hline \Rightarrow n = 3 () - 1 \\ \hline \Rightarrow n \in D \\ \hline \end{array} \quad \begin{array}{ll} n \in D \Rightarrow n \in B, \\ \hline \text{i.e. } D \subseteq B \\ \hline n \in D \\ \hline \Rightarrow n = 3 () + 2 \\ \hline \Rightarrow n \in B \\ \hline \end{array}$$

Recall

$$\triangle B = D \text{ iff } \forall n (n \in B \leftrightarrow n \in D).$$

$$\begin{array}{ll} n \in B \Rightarrow n \in D, \\ \hline \text{i.e. } B \subseteq D \\ \hline n \in B \\ \hline \Rightarrow n = 3j+2, j \in \mathbb{Z} \\ \hline \Rightarrow n = 3(j+1)-1 \\ \hline \Rightarrow n \in D \\ \hline \end{array} \quad \begin{array}{ll} n \in D \Rightarrow n \in B, \\ \hline \text{i.e. } D \subseteq B \\ \hline n \in D \\ \hline \Rightarrow n = 3(\\ \hline \end{pmatrix} + 2$$

Recall

$$\triangle B = D \text{ iff } \forall n (n \in B \leftrightarrow n \in D).$$

$$\begin{array}{ll} n \in B \Rightarrow n \in D, \\ \text{i.e. } B \subseteq D \\ n \in B \\ \Rightarrow n = 3j + 2, j \in \mathbb{Z} \\ \Rightarrow n = 3(j + 1) - 1 \\ \Rightarrow n \in D \\ \end{array} \quad \begin{array}{ll} n \in D \Rightarrow n \in B, \\ \text{i.e. } D \subseteq B \\ n \in D \\ \Rightarrow n = 3j - 1, j \in \mathbb{Z} \\ \Rightarrow n = 3() + 2 \\ \Rightarrow n \in B \end{array}$$

Recall

$$\triangle B = D \text{ iff } \forall n (n \in B \leftrightarrow n \in D).$$

$$\begin{array}{ll} n \in B \Rightarrow n \in D, \\ \text{i.e. } B \subseteq D \\ n \in B \\ \Rightarrow n = 3j + 2, j \in \mathbb{Z} \\ \Rightarrow n = 3(j + 1) - 1 \\ \Rightarrow n \in D \\ \end{array} \quad \begin{array}{ll} n \in D \Rightarrow n \in B, \\ \text{i.e. } D \subseteq B \\ n \in D \\ \Rightarrow n = 3j - 1, j \in \mathbb{Z} \\ \Rightarrow n = 3(j - 1) + 2 \\ \Rightarrow n \in B \end{array}$$

Recall

$$\triangle B = D \text{ iff } \forall n (n \in B \leftrightarrow n \in D).$$

Idea.

$$\begin{array}{ll} n \in B \Rightarrow n \in D, \\ \text{i.e. } B \subseteq D \\ n \in B \\ \Rightarrow n = 3j + 2, j \in \mathbb{Z} \\ \Rightarrow n = 3(j + 1) - 1 \\ \Rightarrow n \in D \\ \end{array} \quad \begin{array}{ll} n \in D \Rightarrow n \in B, \\ \text{i.e. } D \subseteq B \\ n \in D \\ \Rightarrow n = 3j - 1, j \in \mathbb{Z} \\ \Rightarrow n = 3(j - 1) + 2 \\ \Rightarrow n \in B \end{array}$$

Answer. Yes. If n=3j+2, then n=3(j+1)-1, thus $B\subseteq D$. If n=3j-1, then n=3(j-1)+2. Thus $D\subseteq B$. So B=D.

3. Find |A| if $A = \{1, 2, \{2\}, \{4, 5\}, 5, 5\}$.

Recall

- for sets, order, repetition Do Not Matter.
 - Fig. $\{1,3,7\} = \{7,1,3\} = \{7,1,1,1,3,3,1,1\}$.
- A: the cardinality of A, that is, the number of different elements in A.

Idea. $A = \{$ Answer. |A| =.

3. Find |A| if $A = \{1, 2, \{2\}, \{4, 5\}, 5, 5\}$.

Recall

- for sets, order, repetition Do Not Matter.
 - E.g. $\{1,3,7\} = \{7,1,3\} = \{7,1,1,1,3,3,1,1\}.$
- \triangle |A|: the cardinality of A, that is, the number of different elements in A.

Idea. $A = \{1, 2, \{2\}, \{4, 5\}, 5\}.$

Answer. |A| = ...

3. Find |A| if $A = \{1, 2, \{2\}, \{4, 5\}, 5, 5\}$.

Recall

- for sets, order, repetition Do Not Matter.
 - E.g. $\{1,3,7\} = \{7,1,3\} = \{7,1,1,1,3,3,1,1\}.$
- \triangle |A|: the cardinality of A, that is, the number of different elements in A.

Idea. $A = \{1, 2, \{2\}, \{4, 5\}, 5\}.$

Answer. |A| = 5.

Recall

$$\triangle$$
 X and Y are sets, then

$$X - Y = \{x \mid x \in X \text{ and } x \notin Y\}$$

Idea.

$$A = \{a, b, c\}, B = \{b, c, d\}$$

$$\Rightarrow A - B = \{ \},$$

$$C = \{b, c, e\}$$

$$\Rightarrow (A - B) - C = \{ \}$$

$$A = \{a, b, c\}$$

$$\Rightarrow A - (B - C) = \{ \}.$$

Answer.

Recall

$$ightharpoonup X$$
 and Y are sets, then

$$X - Y = \{x \mid x \in X \text{ and } x \notin Y\}$$

Idea.

$$A = \{a, b, c\}, B = \{b, c, d\}$$

$$\Rightarrow A - B = \{a\},$$

$$C = \{b, c, e\}$$

$$\Rightarrow (A - B) - C = \{ \}$$

$$B = \{b, c, d\}, C = \{b, c, e\}$$

$$\Rightarrow B - C = \{ \},$$

$$A = \{a, b, c\}$$

$$\Rightarrow A - (B - C) = \{ \}.$$

Answer.

Recall

$$\triangle$$
 X and Y are sets, then

$$X - Y = \{x \mid x \in X \text{ and } x \notin Y\}$$

Idea.

$$A = \{a, b, c\}, B = \{b, c, d\}$$

$$\Rightarrow A - B = \{a\},$$

$$C = \{b, c, e\}$$

$$\Rightarrow (A - B) - C = \{a\}$$

$$B = \{b, c, d\}, C = \{b, c, e\}$$

$$\Rightarrow B - C = \{ \},$$

$$A = \{a, b, c\}$$

$$\Rightarrow A - (B - C) = \{ \}.$$

Answer.

Recall

$$ightharpoonup X$$
 and Y are sets, then

$$X - Y = \{x \mid x \in X \text{ and } x \notin Y\}$$

Idea.

$$A = \{a, b, c\}, B = \{b, c, d\}$$

$$\Rightarrow A - B = \{a\},$$

$$C = \{b, c, e\}$$

$$\Rightarrow (A - B) - C = \{a\}$$

$$B = \{b, c, d\}, C = \{b, c, e\}$$

$$\Rightarrow B - C = \{d\},$$

$$A = \{a, b, c\}$$

$$\Rightarrow A - (B - C) = \{a\}$$

Answer.

Recall

$$\triangle$$
 X and Y are sets, then

$$X - Y = \{x \mid x \in X \text{ and } x \notin Y\}$$

Idea.

$$\begin{array}{ll} A = \{a,b,c\}, \ B = \{b,c,d\} & B = \{b,c,d\}, \ C = \{b,c,e\} \\ \Rightarrow A - B = \{a\}, & \Rightarrow B - C = \{d\}, \\ C = \{b,c,e\} & A = \{a,b,c\} \\ \Rightarrow (A - B) - C = \{a\} & \Rightarrow A - (B - C) = \{a,b,c\}. \end{array}$$

Answer.

4. Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, $C = \{b, c, e\}$. Find (A - B) - C and A - (B - C). Are they equal?

Recall

$$ightharpoonup X$$
 and Y are sets, then

$$X - Y = \{x \mid x \in X \text{ and } x \notin Y\}$$

Idea.

$$\begin{array}{ll} A = \{a,b,c\}, \ B = \{b,c,d\} & B = \{b,c,d\}, \ C = \{b,c,e\} \\ \Rightarrow A - B = \{a\}, & \Rightarrow B - C = \{d\}, \\ C = \{b,c,e\} & A = \{a,b,c\} \\ \Rightarrow (A - B) - C = \{a\} & \Rightarrow A - (B - C) = \{a,b,c\}. \end{array}$$

Answer. Not equal.

Notice: We do Not have associative laws in set difference.

(a)
$$T_{P\vee Q} = T_P \cup T_Q$$
, $T_{P\wedge Q} = T_p \cap T_Q$,

(b)
$$T_{P\to Q} = \overline{T_p} \cup T_Q$$
.

Recall

Idea. (a) $T_{P\vee Q} = T_P \cup T_Q$

 $\Rightarrow x \in$

(by defintion of \cup)

(a)
$$T_{P\vee Q}=T_P\cup T_Q$$
, $T_{P\wedge Q}=T_p\cap T_Q$,

(b)
$$T_{P\to Q} = \overline{T_p} \cup T_Q$$
.

Recall

$$T_P = \{x \mid P(x) \text{ is true}\}$$

 $A = B \text{ iff } \forall x (x \in A \leftrightarrow x \in B)$

$$\begin{aligned} & \frac{T_{P \vee Q} = T_P \cup T_Q}{x \in T_{P \vee Q}} \\ & \Rightarrow P(x) \vee Q(x) \text{ is true (by definition of } T_{P \vee Q}) \end{aligned}$$

$$\Rightarrow$$
 (by definition of \vee)

$$\Rightarrow x \in x \in (by \text{ definition of } T_P)$$

$$\Rightarrow x \in$$
 (by defintion of \cup)

(a)
$$T_{P\vee Q}=T_P\cup T_Q$$
, $T_{P\wedge Q}=T_p\cap T_Q$,

(b)
$$T_{P\to Q} = \overline{T_p} \cup T_Q$$
.

Recall

$$T_P = \{x \mid P(x) \text{ is true}\}$$

$$A = B \text{ iff } \forall x (x \in A \leftrightarrow x \in B)$$

$$\begin{array}{l} T_{P\vee Q} = T_P \cup T_Q \\ \hline x \in T_{P\vee Q} \\ \Rightarrow P(x) \vee Q(x) \text{ is true } \text{ (by definition of } T_{P\vee Q} \text{)} \\ \Rightarrow P(x) \text{ is true or } Q(x) \text{ is true } \text{ (by definition of } \vee \text{)} \\ \Rightarrow x \in x \in \text{ (by definition of } T_P \text{)} \\ \Rightarrow x \in x \in \text{ (by definition of } \nabla \text{)} \\ \end{array}$$

(a)
$$T_{P\vee Q}=T_P\cup T_Q$$
, $T_{P\wedge Q}=T_p\cap T_Q$,

(b)
$$T_{P\to Q} = \overline{T_p} \cup T_Q$$
.

Recall

$$T_P = \{x \mid P(x) \text{ is true}\}$$

$$A = B \text{ iff } \forall x (x \in A \leftrightarrow x \in B)$$

$$\begin{split} & \frac{T_{P \vee Q} = T_P \cup T_Q}{x \in T_{P \vee Q}} \\ & \Rightarrow P(x) \vee Q(x) \text{ is true } \text{ (by definition of } T_{P \vee Q} \text{)} \\ & \Rightarrow P(x) \text{ is true or } Q(x) \text{ is true } \text{ (by definition of } \vee \text{)} \\ & \Rightarrow x \in T_P \text{ or } x \in T_Q \text{ (by definition of } T_P \text{)} \\ & \Rightarrow x \in \text{ (by definition of } \cup \text{)} \end{split}$$

(a)
$$T_{P\vee Q}=T_P\cup T_Q$$
, $T_{P\wedge Q}=T_p\cap T_Q$,

(b)
$$T_{P\to Q} = \overline{T_p} \cup T_Q$$
.

Recall

$$\triangle A = B \text{ iff } \forall x (x \in A \leftrightarrow x \in B)$$

$$\frac{T_{P\vee Q}=T_P\cup T_Q}{T_{P\vee Q}}$$

$$x \in T_{P \vee Q}$$

$$\Rightarrow P(x) \lor Q(x)$$
 is true (by definition of $T_{P \lor Q}$)

$$\Rightarrow P(x)$$
 is true or $Q(x)$ is true (by definiton of \vee)

$$\Rightarrow x \in T_P \text{ or } x \in T_O \text{ (by definition of } T_P)$$

$$\Rightarrow x \in T_P \cup T_Q$$
 (by defintion of \cup)

```
x \in T_P \cup T_O
                          (by definition of \cup)
\Rightarrow x \in x \in x \in x
                                               (by definition of T_p)
\Rightarrow
                                   (by definition of ∨)
\Rightarrow
                    (by definition of truth set)
\Rightarrow
T_{P \wedge Q} = T_p \cap T_Q
\overline{x \in T_{P \wedge Q}}
                                     (by definition of T_{P \wedge Q})
\Rightarrow
                                                  (by definition of \wedge)
\Rightarrow
                                  (by definition of T_P)
          x \in
\Rightarrow x \in
                         (by defintion of \cap)
\Rightarrow x \in
x \in T_P \cap T_Q
                             (by definition of \cap)
                    x \in
\Rightarrow x \in
                                                  (by definition of T_n)
\Rightarrow
                                   (by definition of \wedge)
\Rightarrow
                    (by definition of truth set)
\Rightarrow
```

```
x \in T_P \cup T_Q
\Rightarrow x \in T_P \text{ or } x \in T_Q \text{ (by definition of } \cup \text{)}
                                                   (by definition of T_n)
\Rightarrow
                                      (by definition of \vee)
\Rightarrow
                     (by definition of truth set)
\Rightarrow
T_{P \wedge Q} = T_p \cap T_Q
\overline{x \in T_{P \wedge O}}
                                        (by definition of T_{P \wedge Q})
\Rightarrow
                                                      (by definition of \wedge)
\Rightarrow
                                     (by definition of T_P)
           x \in
\Rightarrow x \in
                           (by definition of \cap)
\Rightarrow x \in
x \in T_P \cap T_O
                               (by definition of \cap)
                      x \in
\Rightarrow x \in
                                                      (by definition of T_n)
\Rightarrow
                                      (by definition of \wedge)
\Rightarrow
                     (by definition of truth set)
\Rightarrow
```

```
x \in T_P \cup T_Q
\Rightarrow x \in T_P \text{ or } x \in T_O \text{ (by definition of } \cup \text{)}
\Rightarrow P(x) is true or Q(x) is true (by definition of T_n)
                                       (by definition of \vee)
\Rightarrow
                      (by definition of truth set)
\Rightarrow
T_{P \wedge Q} = T_p \cap T_Q
\overline{x \in T_{P \wedge O}}
                                         (by definition of T_{P \wedge Q})
\Rightarrow
```

$$\begin{array}{ll} T_{P \wedge Q} = T_{p} \cap T_{Q} \\ \hline x \in T_{P \wedge Q} \\ \Rightarrow & \text{(by definition of } T_{P \wedge Q}) \\ \Rightarrow & \text{(by definition of } \wedge) \\ \Rightarrow x \in & x \in & \text{(by definition of } T_{P}) \\ \Rightarrow x \in & \text{(by definition of } \cap) \\ \hline x \in T_{P} \cap T_{Q} \\ \Rightarrow x \in & x \in & \text{(by definition of } \cap) \\ \Rightarrow & \text{(by definition of } T_{p}) \\ \hline \end{array}$$

```
x \in T_P \cup T_Q
\Rightarrow x \in T_P \text{ or } x \in T_Q \text{ (by definition of } \cup \text{)}
\Rightarrow P(x) is true or Q(x) is true (by definition of T_n)
\Rightarrow P(x) \lor Q(x) is true (by definition of \lor)
                    (by definition of truth set)
T_{P \wedge Q} = T_p \cap T_Q
\overline{x \in T_{P \wedge Q}}
                                       (by definition of T_{P \wedge Q})
\Rightarrow
```

$$\begin{array}{ll} P \wedge Q = P_p + P_Q \\ \hline x \in T_{P \wedge Q} \\ \Rightarrow & \text{(by definition of } T_{P \wedge Q}) \\ \Rightarrow & \text{(by definition of } \Lambda) \\ \Rightarrow x \in & x \in & \text{(by definition of } T_P) \\ \Rightarrow x \in & \text{(by definition of } \Lambda) \\ \hline x \in T_P \cap T_Q \\ \Rightarrow x \in & x \in & \text{(by definition of } \Lambda) \\ \Rightarrow & \text{(by definition of } \Lambda) \\ \Rightarrow & \text{(by definition of } \Lambda) \\ \Rightarrow & \text{(by definition of truth set)} \\ \end{array}$$

```
\begin{aligned} x &\in T_P \cup T_Q \\ \Rightarrow x &\in T_P \text{ or } x \in T_Q \text{ (by definition of } \cup \text{)} \\ \Rightarrow P(x) \text{ is true or } Q(x) \text{ is true (by definition of } T_p \text{)} \\ \Rightarrow P(x) \vee Q(x) \text{ is true (by definition of } \vee \text{)} \\ \Rightarrow x &\in T_{P \vee Q} \text{ (by definition of truth set)} \end{aligned}
\frac{T_{P \wedge Q} = T_p \cap T_Q}{x \in T_{P \wedge Q}}
```

$$\begin{array}{lll} T_{P \wedge Q} = T_p \cap T_Q \\ \hline x \in T_{P \wedge Q} \\ \Rightarrow & \text{(by definition of } T_{P \wedge Q}) \\ \Rightarrow & \text{(by definition of } \Lambda) \\ \Rightarrow x \in & x \in & \text{(by definition of } T_P) \\ \Rightarrow x \in & \text{(by definition of } \Lambda) \\ \hline x \in T_P \cap T_Q \\ \Rightarrow x \in & x \in & \text{(by definition of } \Lambda) \\ \Rightarrow & \text{(by definition of } \Lambda) \\ \Rightarrow & \text{(by definition of } \Lambda) \\ \Rightarrow & \text{(by definition of truth set)} \end{array}$$

```
x \in T_P \cup T_Q

\Rightarrow x \in T_P \text{ or } x \in T_Q \text{ (by definition of } \cup \text{)}

\Rightarrow P(x) \text{ is true or } Q(x) \text{ is true (by definition of } T_p \text{)}

\Rightarrow P(x) \vee Q(x) \text{ is true (by definition of } \vee \text{)}

\Rightarrow x \in T_{P \vee Q} \text{ (by definition of truth set)}
```

$$\begin{array}{l} T_{P \wedge Q} = T_{p} \cap T_{Q} \\ \hline x \in T_{P \wedge Q} \\ \Rightarrow P(x) \wedge Q(x) \text{ is true } \text{ (by definition of } T_{P \wedge Q} \text{)} \\ \Rightarrow & \text{ (by definition of } \wedge \text{)} \\ \Rightarrow x \in & x \in & \text{ (by definition of } T_{P} \text{)} \\ \Rightarrow x \in & \text{ (by definition of } \cap \text{)} \\ \hline x \in T_{P} \cap T_{Q} \\ \Rightarrow x \in & x \in & \text{ (by definition of } \cap \text{)} \\ \Rightarrow & \text{ (by definition of } T_{p} \text{)} \\ \Rightarrow & \text{ (by definition of } x \cap \text{)} \\ \Rightarrow & \text{ (by definition of } x \cap \text{)} \\ \Rightarrow & \text{ (by definition of truth set)} \end{array}$$

```
x \in T_P \cup T_Q \Rightarrow x \in T_P \text{ or } x \in T_Q \text{ (by definition of } \cup \text{)} \Rightarrow P(x) \text{ is true or } Q(x) \text{ is true (by definition of } T_p\text{)} \Rightarrow P(x) \vee Q(x) \text{ is true (by definition of } \vee \text{)} \Rightarrow x \in T_{P \vee Q} \text{ (by definition of truth set)}
```

$$\begin{array}{l} T_{P \wedge Q} = T_p \cap T_Q \\ \hline x \in T_{P \wedge Q} \\ \Rightarrow P(x) \wedge Q(x) \text{ is true } \text{ (by definition of } T_{P \wedge Q}) \\ \Rightarrow P(x) \text{ is true and } Q(x) \text{ is true } \text{ (by definition of } \wedge) \\ \Rightarrow x \in \qquad x \in \qquad \text{(by definition of } T_P) \\ \Rightarrow x \in \qquad \text{(by definition of } \cap) \\ \hline x \in T_P \cap T_Q \\ \Rightarrow x \in \qquad x \in \qquad \text{(by definition of } \cap) \\ \Rightarrow \qquad \qquad \text{(by definition of } T_p) \\ \Rightarrow \qquad \text{(by definition of } \wedge) \\ \Rightarrow \qquad \text{(by definition of truth set)} \\ \end{array}$$

```
x \in T_P \cup T_Q \Rightarrow x \in T_P \text{ or } x \in T_Q \text{ (by definition of } \cup \text{)} \Rightarrow P(x) \text{ is true or } Q(x) \text{ is true (by definition of } T_p\text{)} \Rightarrow P(x) \vee Q(x) \text{ is true (by definition of } \vee \text{)} \Rightarrow x \in T_{P \vee Q} \text{ (by definition of truth set)}
```

$$\begin{array}{l} T_{P \wedge Q} = T_{p} \cap T_{Q} \\ \hline x \in T_{P \wedge Q} \\ \Rightarrow P(x) \wedge Q(x) \text{ is true } \text{ (by definition of } T_{P \wedge Q} \text{)} \\ \Rightarrow P(x) \text{ is true and } Q(x) \text{ is true } \text{ (by definition of } \wedge \text{)} \\ \Rightarrow x \in T_{P} \text{ and } x \in T_{Q} \text{ (by definition of } T_{P} \text{)} \\ \Rightarrow x \in \text{ (by definition of } \cap \text{)} \\ \hline x \in T_{P} \cap T_{Q} \\ \Rightarrow x \in \text{ (by definition of } \cap \text{)} \\ \Rightarrow \text{ (by definition of } \wedge \text{)} \\ \Rightarrow \text{ (by definition of truth set)} \\ \end{array}$$

```
x \in T_P \cup T_Q \Rightarrow x \in T_P \text{ or } x \in T_Q \text{ (by definition of } \cup \text{)} \Rightarrow P(x) \text{ is true or } Q(x) \text{ is true (by definition of } T_p\text{)} \Rightarrow P(x) \vee Q(x) \text{ is true (by definition of } \vee \text{)} \Rightarrow x \in T_{P \vee Q} \text{ (by definition of truth set)}
```

$$\begin{array}{l} T_{P \wedge Q} = T_{p} \cap T_{Q} \\ \hline x \in T_{P \wedge Q} \\ \Rightarrow P(x) \wedge Q(x) \text{ is true } \text{ (by definition of } T_{P \wedge Q} \text{)} \\ \Rightarrow P(x) \text{ is true and } Q(x) \text{ is true } \text{ (by definition of } \wedge \text{)} \\ \Rightarrow x \in T_{P} \text{ and } x \in T_{Q} \text{ (by definition of } T_{P} \text{)} \\ \Rightarrow x \in T_{P} \cap T_{Q} \text{ (by definition of } \cap \text{)} \\ x \in T_{P} \cap T_{Q} \\ \Rightarrow x \in x \in \text{ (by definition of } \cap \text{)} \\ \Rightarrow \text{ (by definition of } \wedge \text{)} \\ \Rightarrow \text{ (by definition of truth set)} \end{array}$$

```
\begin{aligned} x &\in T_P \cup T_Q \\ \Rightarrow x &\in T_P \text{ or } x \in T_Q \text{ (by definition of } \cup \text{)} \\ \Rightarrow P(x) \text{ is true or } Q(x) \text{ is true (by definition of } T_p \text{)} \\ \Rightarrow P(x) \vee Q(x) \text{ is true (by definition of } \vee \text{)} \\ \Rightarrow x &\in T_{P \vee Q} \text{ (by definition of truth set)} \end{aligned}
```

$$\begin{array}{l} T_{P \wedge Q} = T_{p} \cap T_{Q} \\ \hline x \in T_{P \wedge Q} \\ \Rightarrow P(x) \wedge Q(x) \text{ is true } \text{ (by definition of } T_{P \wedge Q} \text{)} \\ \Rightarrow P(x) \text{ is true and } Q(x) \text{ is true } \text{ (by definition of } \wedge \text{)} \\ \Rightarrow x \in T_{P} \text{ and } x \in T_{Q} \text{ (by definition of } T_{P} \text{)} \\ \Rightarrow x \in T_{P} \cap T_{Q} \text{ (by definition of } \cap \text{)} \\ x \in T_{P} \cap T_{Q} \\ \Rightarrow x \in T_{P} \text{ and } x \in T_{Q} \text{ (by definition of } \cap \text{)} \\ \Rightarrow \text{ (by definition of } \wedge \text{)} \\ \Rightarrow \text{ (by definition of truth set)} \end{array}$$

```
x \in T_P \cup T_Q

\Rightarrow x \in T_P \text{ or } x \in T_Q \text{ (by definition of } \cup \text{)}

\Rightarrow P(x) \text{ is true or } Q(x) \text{ is true (by definition of } T_p \text{)}

\Rightarrow P(x) \vee Q(x) \text{ is true (by definition of } \vee \text{)}

\Rightarrow x \in T_{P \vee Q} \text{ (by definition of truth set)}
```

$$\begin{array}{l} T_{P \wedge Q} = T_p \cap T_Q \\ \hline {x \in T_{P \wedge Q}} \\ \Rightarrow P(x) \wedge Q(x) \text{ is true } \text{ (by definition of } T_{P \wedge Q}) \\ \Rightarrow P(x) \text{ is true and } Q(x) \text{ is true } \text{ (by definition of } \wedge) \\ \Rightarrow x \in T_P \text{ and } x \in T_Q \text{ (by definition of } T_P) \\ \Rightarrow x \in T_P \cap T_Q \text{ (by definition of } \cap) \\ \hline {x \in T_P \cap T_Q} \\ \Rightarrow x \in T_P \text{ and } x \in T_Q \text{ (by definition of } \cap) \\ \Rightarrow P(x) \text{ is true and } Q(x) \text{ is true } \text{ (by definition of } T_p) \\ \Rightarrow \text{ (by definition of } \wedge) \\ \Rightarrow \text{ (by definition of truth set)} \end{array}$$

```
\begin{array}{l} x \in T_P \cup T_Q \\ \Rightarrow x \in T_P \text{ or } x \in T_Q \text{ (by definition of } \cup \text{)} \\ \Rightarrow P(x) \text{ is true or } Q(x) \text{ is true (by definition of } T_p \text{)} \\ \Rightarrow P(x) \vee Q(x) \text{ is true (by definition of } \vee \text{)} \\ \Rightarrow x \in T_{P \vee Q} \text{ (by definition of truth set)} \end{array}
```

$$\begin{array}{l} T_{P \wedge Q} = T_p \cap T_Q \\ \hline {x \in T_{P \wedge Q}} \\ \Rightarrow P(x) \wedge Q(x) \text{ is true } \text{ (by definition of } T_{P \wedge Q}) \\ \Rightarrow P(x) \text{ is true and } Q(x) \text{ is true } \text{ (by definition of } \wedge) \\ \Rightarrow x \in T_P \text{ and } x \in T_Q \text{ (by definition of } T_P) \\ \Rightarrow x \in T_P \cap T_Q \text{ (by definition of } \cap) \\ \hline {x \in T_P \cap T_Q} \\ \Rightarrow x \in T_P \text{ and } x \in T_Q \text{ (by definition of } \cap) \\ \Rightarrow P(x) \text{ is true and } Q(x) \text{ is true (by definition of } T_p) \\ \Rightarrow P(x) \wedge Q(x) \text{ is true (by definition of } \wedge) \\ \Rightarrow \text{ (by definition of truth set)} \end{array}$$

$$x \in T_P \cup T_Q$$

 $\Rightarrow x \in T_P \text{ or } x \in T_Q \text{ (by definition of } \cup \text{)}$
 $\Rightarrow P(x) \text{ is true or } Q(x) \text{ is true (by definition of } T_p \text{)}$
 $\Rightarrow P(x) \vee Q(x) \text{ is true (by definition of } \vee \text{)}$
 $\Rightarrow x \in T_{P \vee Q} \text{ (by definition of truth set)}$

$$\begin{split} & \frac{T_{P \wedge Q} = T_{p} \cap T_{Q}}{x \in T_{P \wedge Q}} \\ \Rightarrow & P(x) \wedge Q(x) \text{ is true } \text{ (by definition of } T_{P \wedge Q} \text{)} \\ \Rightarrow & P(x) \text{ is true and } Q(x) \text{ is true } \text{ (by definition of } \wedge \text{)} \\ \Rightarrow & x \in T_{P} \text{ and } x \in T_{Q} \text{ (by definition of } T_{P} \text{)} \\ \Rightarrow & x \in T_{P} \cap T_{Q} \text{ (by definition of } \cap \text{)} \\ & x \in T_{P} \cap T_{Q} \\ \Rightarrow & x \in T_{P} \text{ and } x \in T_{Q} \text{ (by definition of } \cap \text{)} \\ \Rightarrow & P(x) \text{ is true and } Q(x) \text{ is true } \text{ (by definition of } T_{p} \text{)} \\ \Rightarrow & P(x) \wedge Q(x) \text{ is true (by definition of } \wedge \text{)} \\ \Rightarrow & x \in T_{P \wedge Q} \text{ (by definition of truth set)} \end{split}$$

$$P \to Q \equiv \neg P \lor Q$$

```
x \in T_{P \to Q}
                                         (by definition of truth set)
\Rightarrow
                                          (P \rightarrow Q \equiv \neg P \lor Q)
\Rightarrow
                                                       (by definition of \vee)
\Rightarrow
                                                    (by definition of \neg)
\Rightarrow
                                   (by definition of truth set)
\Rightarrow
                          (by defintion of \cup)
\Rightarrow
x \in \overline{T_P} \cup T_O
                                    (by defintion of \cup)
\Rightarrow
                                                        (by definition of truth set)
\Rightarrow
                                         (by definition of \vee)
\Rightarrow
                                         (P \to Q \equiv \neg P \lor Q)
\Rightarrow
                       (by definition of truth set)
\Rightarrow
```

$$P \to Q \equiv \neg P \lor Q$$

$$\begin{array}{lll} x \in T_{P \to Q} \\ \Rightarrow P(x) \to Q(x) \text{ is true} & \text{(by definition of truth set)} \\ \Rightarrow & (P \to Q \equiv \neg P \lor Q) \\ \Rightarrow & \text{(by definition of } \lor) \\ \Rightarrow & \text{(by definition of } \tau) \\ \Rightarrow & \text{(by definition of truth set)} \\ \Rightarrow & \text{(by definition of } \cup) \\ \hline x \in \overline{T_P} \cup T_Q \\ \Rightarrow & \text{(by definition of } \cup) \\ \Rightarrow & \text{(by definition of } \lor) \\ \Rightarrow & \text{(by defini$$

$$P \to Q \equiv \neg P \vee Q$$

$$\begin{array}{l} x \in T_{P \to Q} \\ \Rightarrow P(x) \to Q(x) \text{ is true } \text{ (by definition of truth set)} \\ \Rightarrow \neg P(x) \lor Q(x) \text{ is true } (P \to Q \equiv \neg P \lor Q) \\ \Rightarrow \qquad \qquad \text{(by definition of } \lor) \\ \Rightarrow \qquad \qquad \text{(by definition of truth set)} \\ \Rightarrow \qquad \qquad \text{(by definition of } \lor) \\ \hline \Rightarrow \qquad \qquad \text{(by definition of } \lor) \\ \Rightarrow \qquad \qquad \text{(by definition$$

$$P \to Q \equiv \neg P \vee Q$$

$$\begin{array}{l} x \in T_{P \to Q} \\ \Rightarrow P(x) \to Q(x) \text{ is true } \text{ (by definition of truth set)} \\ \Rightarrow \neg P(x) \lor Q(x) \text{ is true } \text{ } (P \to Q \equiv \neg P \lor Q) \\ \Rightarrow \neg P(x) \text{ is true or } Q(x) \text{ is true } \text{ (by definition of } \lor) \\ \Rightarrow \qquad \qquad \text{or } Q(x) \text{ is true } \text{ (by definition of } \neg) \\ \Rightarrow \qquad \qquad \text{(by definition of truth set)} \\ \Rightarrow \qquad \qquad \text{(by definition of } \cup) \\ \\ x \in \overline{T_P} \cup T_Q \\ \Rightarrow \qquad \qquad \text{(by definition of } \cup) \\ \Rightarrow \qquad \qquad \text{(by definition of } \lor) \\ \Rightarrow \qquad \qquad \text{(by definition of } \lor) \\ \Rightarrow \qquad \qquad \text{(by definition of } \lor) \\ \Rightarrow \qquad \qquad \text{(by definition of truth set)} \\ \end{array}$$

$$P \to Q \equiv \neg P \vee Q$$

$$\begin{array}{l} x \in T_{P \to Q} \\ \Rightarrow P(x) \to Q(x) \text{ is true } \text{ (by definition of truth set)} \\ \Rightarrow \neg P(x) \lor Q(x) \text{ is true } \text{ } (P \to Q \equiv \neg P \lor Q) \\ \Rightarrow \neg P(x) \text{ is true or } Q(x) \text{ is true } \text{ (by definition of } \lor) \\ \Rightarrow P(x) \text{ is false or } Q(x) \text{ is true } \text{ (by definition of } \neg) \\ \Rightarrow \text{ (by definition of truth set)} \\ \Rightarrow \text{ (by definition of } \cup) \\ \\ x \in \overline{T_P} \cup T_Q \\ \Rightarrow \text{ (by definition of } \lor) \\ \Rightarrow \text{ (by definition of truth set)} \\ \end{array}$$

$$P \to Q \equiv \neg P \vee Q$$

$$\begin{array}{l} x \in T_{P \to Q} \\ \Rightarrow P(x) \to Q(x) \text{ is true } \text{ (by definition of truth set)} \\ \Rightarrow \neg P(x) \lor Q(x) \text{ is true } \text{ } (P \to Q \equiv \neg P \lor Q) \\ \Rightarrow \neg P(x) \text{ is true or } Q(x) \text{ is true } \text{ (by definition of } \lor) \\ \Rightarrow P(x) \text{ is false or } Q(x) \text{ is true } \text{ (by definition of } \neg) \\ \Rightarrow x \in \overline{T_P} \text{ or } x \in T_Q \text{ (by definition of truth set)} \\ \Rightarrow \text{ (by definition of } \cup) \\ \\ x \in \overline{T_P} \cup T_Q \\ \Rightarrow \text{ (by definition of } \lor) \\ \Rightarrow \text{ (by definition of truth set)} \\ \end{array}$$

$$P \to Q \equiv \neg P \vee Q$$

$$\begin{array}{l} x \in T_{P \to Q} \\ \Rightarrow P(x) \to Q(x) \text{ is true } \text{ (by definition of truth set)} \\ \Rightarrow \neg P(x) \lor Q(x) \text{ is true } \text{ } (P \to Q \equiv \neg P \lor Q) \\ \Rightarrow \neg P(x) \text{ is true or } Q(x) \text{ is true } \text{ (by definition of } \lor) \\ \Rightarrow P(x) \text{ is false or } Q(x) \text{ is true } \text{ (by definition of } \neg) \\ \Rightarrow x \in \overline{T_P} \text{ or } x \in T_Q \text{ (by definition of truth set)} \\ \Rightarrow x \in \overline{T_P} \cup T_Q \text{ (by definition of } \cup) \\ \Rightarrow \text{ (by definition of } \lor) \\ \Rightarrow \text{ (by definition of } \lor) \\ \Rightarrow \text{ (by definition of truth set)} \\ \Rightarrow \text{ (by definition of } \lor) \\ \Rightarrow \text{ (by definition of truth set)} \\ \end{array}$$

$$P \to Q \equiv \neg P \vee Q$$

$$\begin{array}{l} x \in T_{P \to Q} \\ \Rightarrow P(x) \to Q(x) \text{ is true } \text{ (by definition of truth set)} \\ \Rightarrow \neg P(x) \lor Q(x) \text{ is true } \text{ } (P \to Q \equiv \neg P \lor Q) \\ \Rightarrow \neg P(x) \text{ is true or } Q(x) \text{ is true } \text{ (by definition of } \lor) \\ \Rightarrow P(x) \text{ is false or } Q(x) \text{ is true } \text{ (by definition of } \neg) \\ \Rightarrow x \in \overline{T_P} \text{ or } x \in T_Q \text{ (by definition of truth set)} \\ \Rightarrow x \in \overline{T_P} \cup T_Q \text{ (by definition of } \cup) \\ x \in \overline{T_P} \cup T_Q \\ \Rightarrow x \in \overline{T_P} \text{ or } x \in T_Q \text{ (by definition of } \cup) \\ \Rightarrow \text{ (by definition of } \lor) \\ \Rightarrow \text{ (by definition of truth set)} \\ \Rightarrow \text{ (by definition of truth set)} \\ \end{array}$$

$$P \to Q \equiv \neg P \vee Q$$

$$x \in T_{P \to Q}$$

$$\Rightarrow P(x) \to Q(x) \text{ is true } \text{ (by definition of truth set)}$$

$$\Rightarrow \neg P(x) \lor Q(x) \text{ is true } \text{ } (P \to Q \equiv \neg P \lor Q)$$

$$\Rightarrow \neg P(x) \text{ is true or } Q(x) \text{ is true } \text{ (by definition of } \lor)$$

$$\Rightarrow P(x) \text{ is false or } Q(x) \text{ is true } \text{ (by definition of } \neg)$$

$$\Rightarrow x \in \overline{T_P} \text{ or } x \in T_Q \text{ (by definition of truth set)}$$

$$\Rightarrow x \in \overline{T_P} \cup T_Q \text{ (by definition of } \cup)$$

$$x \in \overline{T_P} \cup T_Q$$

$$\Rightarrow x \in \overline{T_P} \text{ or } x \in T_Q \text{ (by definition of } \cup)$$

$$\Rightarrow \neg P(x) \text{ is true or } Q(x) \text{ is true } \text{ (by definition of truth set)}$$

$$\Rightarrow \text{ (by definition of } \lor)$$

$$\Rightarrow \text{ (by definition of truth set)}$$

$$P \to Q \equiv \neg P \vee Q$$

$$x \in T_{P \to Q}$$
 $\Rightarrow P(x) \to Q(x)$ is true (by definition of truth set) $\Rightarrow \neg P(x) \lor Q(x)$ is true $(P \to Q \equiv \neg P \lor Q)$ $\Rightarrow \neg P(x)$ is true or $Q(x)$ is true (by definition of \lor) $\Rightarrow P(x)$ is false or $Q(x)$ is true (by definition of \neg) $\Rightarrow x \in \overline{T_P}$ or $x \in T_Q$ (by definition of truth set) $\Rightarrow x \in \overline{T_P} \cup T_Q$ (by definition of \cup)
$$x \in \overline{T_P} \cup T_Q$$
 $\Rightarrow x \in \overline{T_P} \cup T_Q$ (by definition of \cup)
$$\Rightarrow \neg P(x)$$
 is true or $Q(x)$ is true (by definition of truth set) $\Rightarrow \neg P(x) \lor Q(x)$ is true (by definition of \lor)
$$\Rightarrow \qquad (P \to Q \equiv \neg P \lor Q)$$

$$\Rightarrow \qquad (by definition of truth set)$$

(b)
$$T_{P\to Q} = \overline{T_P} \cup T_Q$$
.

$$P \to Q \equiv \neg P \vee Q$$

$$x \in T_{P \to Q}$$
 $\Rightarrow P(x) \to Q(x)$ is true (by definition of truth set) $\Rightarrow \neg P(x) \lor Q(x)$ is true $(P \to Q \equiv \neg P \lor Q)$ $\Rightarrow \neg P(x)$ is true or $Q(x)$ is true (by definition of \lor) $\Rightarrow P(x)$ is false or $Q(x)$ is true (by definition of \neg) $\Rightarrow x \in \overline{T_P}$ or $x \in T_Q$ (by definition of truth set) $\Rightarrow x \in \overline{T_P} \cup T_Q$ (by definition of \cup)
$$x \in \overline{T_P} \cup T_Q$$
 $\Rightarrow x \in \overline{T_P}$ or $x \in T_Q$ (by definition of \cup)
$$\Rightarrow \neg P(x)$$
 is true or $Q(x)$ is true (by definition of truth set)
$$\Rightarrow \neg P(x) \lor Q(x)$$
 is true (by definition of \lor)
$$\Rightarrow P(x) \to Q(x)$$
 is true $(P \to Q \equiv \neg P \lor Q)$
$$\Rightarrow$$
 (by definition of truth set)

$$P \to Q \equiv \neg P \vee Q$$

$$x \in T_{P \to Q}$$
 $\Rightarrow P(x) \to Q(x)$ is true (by definition of truth set) $\Rightarrow \neg P(x) \lor Q(x)$ is true $(P \to Q \equiv \neg P \lor Q)$ $\Rightarrow \neg P(x)$ is true or $Q(x)$ is true (by definition of \lor) $\Rightarrow P(x)$ is false or $Q(x)$ is true (by definition of \neg) $\Rightarrow x \in \overline{T_P}$ or $x \in T_Q$ (by definition of truth set) $\Rightarrow x \in \overline{T_P} \cup T_Q$ (by definition of \cup)
$$x \in \overline{T_P} \cup T_Q$$
 $\Rightarrow x \in \overline{T_P}$ or $x \in T_Q$ (by definition of \cup)
$$\Rightarrow \neg P(x)$$
 is true or $Q(x)$ is true (by definition of truth set)
$$\Rightarrow \neg P(x) \lor Q(x)$$
 is true (by definition of \lor)
$$\Rightarrow P(x) \to Q(x)$$
 is true $(P \to Q \equiv \neg P \lor Q)$
$$\Rightarrow x \in T_{P \to Q}$$
 (by definition of truth set)

6. Let $A=\{1,2,3\}$, $B=\{u,v\}$, $C=\{m,n\}$. List the elements of $(A\times B)\times C$ and $A\times B\times C$. Are the two cartesian products equal?

- $A \times B = \{(x,y) : x \in A, y \in B\}.$
- $A_1 \times A_2 \times \dots A_n = \{(x_1, x_2, \dots, x_n) : x_1 \in A_1, x_2 \in A_2, \dots, x_n \in A_n\}$

6. Let $A=\{1,2,3\}$, $B=\{u,v\}$, $C=\{m,n\}$. List the elements of $(A\times B)\times C$ and $A\times B\times C$. Are the two cartesian products equal?

Recall

$$A \times B = \{(x, y) : x \in A, y \in B\}.$$

$$A_1 \times A_2 \times \dots A_n = \{ (x_1, x_2, \dots, x_n) : x_1 \in A_1, x_2 \in A_2, \dots, x_n \in A_n \}$$

Idea.

$$\begin{split} (A\times B)\times C = &\{((1,u),m),((1,u),n),((1,v),m),((1,v),n),\\ &((2,u),m),((2,u),n),((2,v),m),((2,v),n),\\ &((3,u),m),((3,u),n),((3,v),m),((3,v),n)\} \\ A\times B\times C = &\{(1,u,m),(1,u,n),(1,v,m),(1,v,n),\\ &(2,u,m),(2,u,n),(2,v,m),(2,v,n),\\ &(3,u,m),(3,u,n),(3,v,m),(3,v,n)\} \end{split}$$

6. Let $A=\{1,2,3\},\ B=\{u,v\},\ C=\{m,n\}.$ List the elements of $(A\times B)\times C$ and $A\times B\times C.$ Are the two cartesian products equal?

Recall

$$A \times B = \{(x, y) : x \in A, y \in B\}.$$
 $A_1 \times A_2 \times \dots A_n = \{(x_1, x_2, \dots, x_n) : x_1 \in A_1, x_2 \in A_1, x_2 \in A_2, \dots, x_n\}$

$$A_2, \dots, x_n \in A_n \}$$

Idea.

$$(A \times B) \times C = \{((1, u), m), ((1, u), n), ((1, v), m), ((1, v), n), ((2, u), m), ((2, u), n), ((2, v), m), ((2, v), n), ((3, u), m), ((3, u), n), ((3, v), m), ((3, v), n)\}$$

$$A \times B \times C = \{(1, u, m), (1, u, n), (1, v, m), (1, v, n), (2, u, m), (2, u, n), (2, v, m), (2, v, n), (3, u, m), (3, u, n), (3, v, m), (3, v, n)\}$$

Answer. Not equal.

7. Find the mistake in the following "proof".

Theorem: For all sets A and B, $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$.

Proof. Suppose A and B are sets, and $x \in \overline{A} \cup \overline{B}$. Then $x \in \overline{A}$ or $x \in \overline{B}$. It follows that $x \notin A$ or $x \notin B$ and so $x \notin A \cup B$. Thus

$$x \in \overline{A \cup B}$$
 and hence $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$.

$$\triangle$$
 De Morgen's Law: $\neg P \vee \neg Q \equiv \neg (P \wedge Q),$ so
$$x \notin A \vee x \notin B \leftrightarrow x \notin A \cap B$$

7. Find the mistake in the following "proof".

Theorem: For all sets A and B, $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$.

Proof. Suppose A and B are sets, and $x \in \overline{A} \cup \overline{B}$. Then $x \in \overline{A}$ or $x \in \overline{B}$. It follows that $x \notin A$ or $x \notin B$ and so $x \notin A \cup B$. Thus $x \in \overline{A \cup B}$ and hence $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$.

Recall

 $^{\!\!\!\!\!/}$ De Morgen's Law: $\neg P\vee \neg Q\equiv \neg (P\wedge Q),$ so $x\notin A\vee x\notin B \leftrightarrow x\notin A\cap B$

Answer. $x \notin A$ or $x \notin B$ does not imply $x \notin A \cup B$. Counter example: $A = \{1\}, B = \{2\}, x = 1$.

Then and $(\Rightarrow x \mid B)$ and

 $(\Rightarrow x \quad B).$

 $(\Rightarrow x \quad B).$

Answer. Suppose $\exists x \in (A-C) \cap (B-C) \cap (A-B)$. and $(\Rightarrow x \mid B)$ and Then

 $(\Rightarrow x \quad B).$

Answer. Suppose $\exists x \in (A-C) \cap (B-C) \cap (A-B)$. Then $x \in A - C$ and $(\Rightarrow x \mid B)$ and

 $(\Rightarrow x \quad B).$

Answer. Suppose $\exists x \in (A-C) \cap (B-C) \cap (A-B)$. Then $x \in A - C$ and $x \in B - C$ ($\Rightarrow x \mid B$) and

 $(\Rightarrow x \quad B).$

Answer. Suppose $\exists x \in (A-C) \cap (B-C) \cap (A-B)$. Then $x \in A - C$ and $x \in B - C$ ($\Rightarrow x \mid B$) and $x \in A - B$

Answer. Suppose $\exists x \in (A-C) \cap (B-C) \cap (A-B)$.

 $(\Rightarrow x \quad B).$

Then $x \in A - C$ and $x \in B - C$ ($\Rightarrow x \in B$) and $x \in A - B$

8 Prove that $(A-C) \cap (B-C) \cap (A-B) = C$

8. Prove that $(A-C) \cap (B-C) \cap (A-B) = \emptyset$.

 $(\Rightarrow x \notin B).$

Answer. Suppose $\exists x \in (A-C) \cap (B-C) \cap (A-B)$. Then $x \in A-C$ and $x \in B-C$ ($\Rightarrow x \in B$) and $x \in A-B$

That's a contradiction.

Answer. Suppose $\exists x \in (A-C) \cap (B-C) \cap (A-B)$.

Then $x \in A - C$ and $x \in B - C$ ($\Rightarrow x \in B$) and $x \in A - B$

 $(\Rightarrow x \notin B).$

8. Prove that $(A-C) \cap (B-C) \cap (A-B) = C$

8. Prove that $(A-C) \cap (B-C) \cap (A-B) = \emptyset$.

Answer. Suppose $\exists x \in (A-C) \cap (B-C) \cap (A-B)$. Then $x \in A-C$ and $x \in B-C$ ($\Rightarrow x \in B$) and $x \in A-B$

 $(\Rightarrow x \notin B)$.

That's a contradiction. Thus no such x exists, i.e., $(A-C) \cap (B-C) \cap (A-B) = \emptyset$.

9. Prove that for all sets A, B, C, D,

if
$$A \cap C = \emptyset$$
, then $(A \times B) \cap (C \times D) = \emptyset$.

Hint: in showing some set equals \emptyset , we may try the proof by contradiction

Recall

Prove $P \to Q$.

- By contradiction: assume $\neg(P \to Q) \equiv (P \land \neg Q) \xrightarrow{\text{using various rules of inference}} \text{arrive at a contradiction}.$
- $A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$

Proof by contradiction.

We suppose

- \Rightarrow
- \Rightarrow
- \Rightarrow
- \Rightarrow

if
$$A \cap C = \emptyset$$
, then $(A \times B) \cap (C \times D) = \emptyset$.

Hint: in showing some set equals \emptyset , we may try the proof by contradiction

Recall

 \Rightarrow

Prove $P \rightarrow Q$.

- By contradiction: assume $\neg(P \to Q) \equiv (P \land \neg Q) \xrightarrow{\text{using various rules of inference}} \text{arrive at a contradiction.}$
- $A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$

if
$$A \cap C = \emptyset$$
, then $(A \times B) \cap (C \times D) = \emptyset$.

Hint: in showing some set equals \emptyset , we may try the proof by contradiction

Recall

Prove $P \rightarrow Q$.

- By contradiction: assume $\neg(P \to Q) \equiv (P \land \neg Q) \xrightarrow{\text{using various rules of inference}} \text{arrive at a contradiction.}$
- $A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$

We suppose
$$A \cap C = \emptyset$$
 and $(A \times B) \cap (C \times D) \neq \emptyset$
 $\Rightarrow \exists (x,y) \in (A \times B) \cap (C \times D)$
 $\Rightarrow x \in$
 \Rightarrow

if
$$A \cap C = \emptyset$$
, then $(A \times B) \cap (C \times D) = \emptyset$.

Hint: in showing some set equals \emptyset , we may try the proof by contradiction

Recall

 \Rightarrow

Prove $P \rightarrow Q$.

- By contradiction: assume $\neg(P \to Q) \equiv (P \land \neg Q) \xrightarrow{\text{using various rules of inference}} \text{arrive at a contradiction}.$
- $A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$

We suppose
$$A \cap C = \emptyset$$
 and $(A \times B) \cap (C \times D) \neq \emptyset$ $\Rightarrow \exists (x,y) \in (A \times B) \cap (C \times D)$ $\Rightarrow x \in A \cap C$ $\Rightarrow A \cap C$

if
$$A \cap C = \emptyset$$
, then $(A \times B) \cap (C \times D) = \emptyset$.

Hint: in showing some set equals \emptyset , we may try the proof by contradiction

Recall

Prove $P \rightarrow Q$.

- By contradiction: assume $\neg(P \to Q) \equiv (P \land \neg Q) \xrightarrow{\text{using various rules of inference}} \text{arrive at a contradiction.}$
- $A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$

We suppose
$$A \cap C = \emptyset$$
 and $(A \times B) \cap (C \times D) \neq \emptyset$
 $\Rightarrow \exists (x,y) \in (A \times B) \cap (C \times D)$
 $\Rightarrow x \in A \cap C$
 $\Rightarrow A \cap C \neq \emptyset$

if
$$A \cap C = \emptyset$$
, then $(A \times B) \cap (C \times D) = \emptyset$.

Hint: in showing some set equals \emptyset , we may try the proof by contradiction

Recall

Prove $P \rightarrow Q$.

- $\triangle A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$

Proof by contradiction.

We suppose
$$A \cap C = \emptyset$$
 and $(A \times B) \cap (C \times D) \neq \emptyset$
 $\Rightarrow \exists (x,y) \in (A \times B) \cap (C \times D)$

$$\Rightarrow x \in A \cap C$$

$$\Rightarrow A \cap C \neq \emptyset$$

⇒Contradiction!

10. For each of the following, either prove the statement true or find a counterexample to prove that it is false. (a) If $B \cap C \subseteq A$, then $(A - B) \cap (A - C) = \emptyset$.

- (b) $(A B) \cap (C B) = A (B \cup C)$.
- (c) If $\overline{A} \subseteq B$, then $A \cup B = U$.
- (d) $P(A \cap B) = P(A) \cap P(B)$.

Hint. Use Venn diagram for (a)-(c).

10. For each of the following, either prove the statement true or find a counterexample to prove that it is false. (a) If $B \cap C \subseteq A$, then $(A - B) \cap (A - C) = \emptyset$.

(b)
$$(A - B) \cap (C - B) = A - (B \cup C)$$
.

(b)
$$(A-B) \cap (C-B) = A - (B \cup C)$$
.
(c) If $\overline{A} \subseteq B$, then $A \cup B = U$.

(d)
$$P(A \cap B) = P(A) \cap P(B)$$
.

(a) False. $A = \{1, 2, 3\}, B = \{2\}, C = \{3\}.$

(d)
$$P(A \cap B) = P(A) \cap P(B)$$
.
Hint. Use Venn diagram for (a)-(c).

Idea.

10. For each of the following, either prove the statement true or find a counterexample to prove that it is false.

(a) If
$$B \cap C \subseteq A$$
, then $(A - B) \cap (A - C) = \emptyset$.

(b)
$$(A - B) \cap (C - B) = A - (B \cup C)$$
.

(c) If
$$\overline{A} \subseteq B$$
, then $A \cup B = U$.

(d)
$$P(A \cap B) = P(A) \cap P(B)$$
.
Hint. Use Venn diagram for (a)-(c).

Idea.
(a) False. $A = \{1, 2, 3\}$. $B = \{2\}$. $C = \{3\}$.

(a) False.
$$A = \{1, 2, 3\}, B = \{2\}, C = \{3\}.$$

(b) False. $A = \{1\}, B = \{2\}, C = \{3\}$

$$\Rightarrow x \in (A \subseteq A \cup B)$$

$$\Rightarrow x \in \quad (\overline{A} \subseteq B)$$
$$\Rightarrow x \in \quad (B \subseteq A \cup B)$$

10. For each of the following, either prove the statement true or find a counterexample to prove that it is false. (a) If $B \cap C \subseteq A$, then $(A - B) \cap (A - C) = \emptyset$.

(b)
$$(A - B) \cap (C - B) = A - (B \cup C)$$
.

(c) If
$$\overline{A} \subseteq B$$
, then $A \cup B = U$.

(d) $P(A \cap B) = P(A) \cap P(B)$.

ldea.

(a) False.
$$A = \{1, 2, 3\}, B = \{2\}, C = \{3\}.$$

(b) False. $A = \{1\}, B = \{2\}, C = \{3\}.$

(b) False.
$$A=\{1\},\ B=\{2\},\ C=\{3\}$$

(c) True. As every set is a subset of the universe, $A\cup B\subseteq U$.

(c) True. As every set is a subset of the universe Thus we need to prove $U \subseteq A \cup B$.

So we need to show $x \in U \rightarrow x \in A \cup B$. Case 1. $x \in A$

$$\Rightarrow x \in \qquad (A \subseteq A \cup B)$$
Case 2. $x \in \overline{A}$

$$\Rightarrow x \in \qquad (\overline{A} \subseteq B)$$

 $\Rightarrow x \in (A \subseteq B)$ $\Rightarrow x \in (B \subseteq A \cup B)$

10. For each of the following, either prove the statement true or find a counterexample to prove that it is false. (a) If $B \cap C \subseteq A$, then $(A - B) \cap (A - C) = \emptyset$.

(a) If $B \cap C \subseteq A$, then $(A - B) \cap (A - C) = \emptyset$. (b) $(A - B) \cap (C - B) = A - (B \cup C)$.

(c) If
$$\overline{A} \subseteq B$$
, then $A \cup B = U$.

(d) $P(A \cap B) = P(A) \cap P(B)$.

Hint. Use Venn diagram for (a)-(c).

Idea.

(a) False. $A = \{1, 2, 3\}, B = \{2\}, C = \{3\}.$ (b) False. $A = \{1\}, B = \{2\}, C = \{3\}$

(b) False. A = {1}, B = {2}, C = {3}
(c) True. As every set is a subset of the universe, A ∪ B ⊆ U. Thus we need to prove U ⊆ A ∪ B.

So we need to show $x \in U \to x \in A \cup B$.

Case 1. $x \in A$ $\Rightarrow x \in A \cup B \ (A \subseteq A \cup B)$

Case 2. $x \in \overline{A}$ $\Rightarrow x \in (\overline{A} \subseteq B)$ $\Rightarrow x \in (B \subseteq A \cup B)$

10. For each of the following, either prove the statement true or find a counterexample to prove that it is false. (a) If $B \cap C \subseteq A$, then $(A - B) \cap (A - C) = \emptyset$.

(a) If
$$B \cap C \subseteq A$$
, then $(A - B) \cap (A - C) = \emptyset$.
(b) $(A - B) \cap (C - B) = A - (B \cup C)$.

(c) If
$$\overline{A} \subseteq B$$
, then $A \cup B = U$.

(d)
$$P(A \cap B) = P(A) \cap P(B)$$
.

Hint. Use Venn diagram for (a)-(c).

(a) False.
$$A=\{1,2,3\},\ B=\{2\},\ C=\{3\}.$$
 (b) False. $A=\{1\},\ B=\{2\},\ C=\{3\}$

(c) True. As every set is a subset of the universe, $A \cup B \subseteq U$. Thus we need to prove $U \subseteq A \cup B$.

So we need to show
$$x \in U \rightarrow x \in A \cup B$$
.

Case 1. $x \in A$ $\Rightarrow x \in A \cup B \ (A \subseteq A \cup B)$

Case 2.
$$x \in A$$

 $\Rightarrow x \in B \ (\overline{A} \subseteq B)$
 $\Rightarrow x \in (B \subseteq A \cup B)$

$$A \cup B$$
)

10. For each of the following, either prove the statement true or find a counterexample to prove that it is false. (a) If $B \cap C \subseteq A$, then $(A - B) \cap (A - C) = \emptyset$.

(a) If
$$B \cap C \subseteq A$$
, then $(A - B) \cap (A - C) = \emptyset$.
(b) $(A - B) \cap (C - B) = A - (B \cup C)$.

(c) If
$$\overline{A} \subseteq B$$
, then $A \cup B = U$.

(d)
$$P(A \cap B) = P(A) \cap P(B)$$
.

Hint. Use Venn diagram for (a)-(c).

(a) False. $A = \{1, 2, 3\}, B = \{2\}, C = \{3\}.$

(a) Palse.
$$A = \{1, 2, 3\}, B = \{2\}, C = \{3\}.$$

(b) False. $A = \{1\}, B = \{2\}, C = \{3\}$

(c) True. As every set is a subset of the universe, $A \cup B \subseteq U$.

Thus we need to prove
$$U \subseteq A \cup B$$
.
So we need to show $x \in U \to x \in A \cup B$.

Case 1. $x \in A$ $\Rightarrow x \in A \cup B \ (A \subseteq A \cup B)$

Case 2.
$$x \in A$$

 $\Rightarrow x \in B \ (\overline{A} \subseteq B)$

 $\Rightarrow x \in A \cup B \ (B \subseteq A \cup B)$

(d) $P(A \cap B) = P(A) \cap P(B)$.

Recall

 \triangle P(A): the power set of A, which is the set of all subsets of A.

(d)
$$P(A \cap B) = P(A) \cap P(B)$$
.

 \triangle P(A): the power set of A, which is the set of all subsets of A.

```
Answer. True.
```

$$X \in P(A \cap B)$$

$$\Rightarrow X$$

$$\Rightarrow X$$
 and X

$$\Rightarrow X \in$$

Thus we have proved that

$$X \in P(A) \cap P(B)$$

$$\Rightarrow X$$
 and X

$$\Rightarrow X$$

$$\Rightarrow X$$

(d)
$$P(A \cap B) = P(A) \cap P(B)$$
.

 \triangle P(A): the power set of A, which is the set of all subsets of A.

Answer, True.

$$X \in P(A \cap B)$$

$$\Rightarrow X \subseteq A \cap B$$

$$\Rightarrow X$$
 and X

$$\rightarrow \Lambda$$
 and Λ

$$\Rightarrow X \in$$

Thus we have proved that

$$X \in P(A) \cap P(B)$$

$$\Rightarrow X \qquad \text{ and } X$$

$$\Rightarrow X$$

$$\Rightarrow X$$

(d)
$$P(A \cap B) = P(A) \cap P(B)$$
.

 \triangle P(A): the power set of A, which is the set of all subsets of A.

Answer, True.

$$X \in P(A \cap B)$$

$$\Rightarrow X \subseteq A \cap B$$

$$\Rightarrow X \subseteq A \text{ and } X$$

$$\Rightarrow X \in$$

Thus we have proved that

$$X \in P(A) \cap P(B)$$

$$\Rightarrow X$$
 and X

$$\Rightarrow X$$

$$\Rightarrow X$$

(d)
$$P(A \cap B) = P(A) \cap P(B)$$
.

 \triangle P(A): the power set of A, which is the set of all subsets of A.

Answer, True.

$$X \in P(A \cap B)$$

$$\Rightarrow X \subseteq A \cap B$$

$$\Rightarrow X \subseteq A \text{ and } X \subseteq B$$

$$\Rightarrow X \subseteq A \text{ and } X \subseteq B$$

$$\Rightarrow X \in$$

Thus we have proved that

$$X \in P(A) \cap P(B)$$

$$\Rightarrow X$$
 and X

$$\Rightarrow X$$

$$\Rightarrow X$$

(d)
$$P(A \cap B) = P(A) \cap P(B)$$
.

 \triangle P(A): the power set of A, which is the set of all subsets of A.

Answer, True.

$$X \in P(A \cap B)$$

$$\Rightarrow X \subseteq A \cap B$$

$$\Rightarrow X \subseteq A \text{ and } X \subseteq B$$

$$\Rightarrow X \in P(A) \cap P(B)$$

Thus we have proved that

$$X \in P(A) \cap P(B)$$

$$\Rightarrow X$$
 and X

$$\Rightarrow X$$
 .

$$\Rightarrow X$$

(d)
$$P(A \cap B) = P(A) \cap P(B)$$
.

 \triangle P(A): the power set of A, which is the set of all subsets of A.

Answer, True.

$$X \in P(A \cap B)$$

$$\Rightarrow X \subseteq A \cap B$$

$$\Rightarrow X \subseteq A \text{ and } X \subseteq B$$

$$\Rightarrow X \in P(A) \cap P(B)$$

Thus we have proved that $P(A \cap B) \subseteq P(A) \cap P(B)$.

$$X \in P(A) \cap P(B)$$

$$\Rightarrow X \qquad \text{ and } X$$

$$\Rightarrow X$$

$$\Rightarrow X$$

(d)
$$P(A \cap B) = P(A) \cap P(B)$$
.

 \triangle P(A): the power set of A, which is the set of all subsets of A.

Answer, True.

$$X \in P(A \cap B)$$

$$\Rightarrow X \subseteq A \cap B$$

$$\Rightarrow X \subseteq A \text{ and } X \subseteq B$$

$$\Rightarrow X \in P(A) \cap P(B)$$

Thus we have proved that $P(A \cap B) \subseteq P(A) \cap P(B)$.

$$X \in P(A) \cap P(B)$$

$$\Rightarrow X \subseteq A \text{ and } X \subseteq B$$

$$\Rightarrow X$$

$$\Rightarrow X$$

(d)
$$P(A \cap B) = P(A) \cap P(B)$$
.

 \triangle P(A): the power set of A, which is the set of all subsets of A.

Answer, True.

$$X \in P(A \cap B)$$

$$\Rightarrow X \subseteq A \cap B$$

$$\Rightarrow X \subseteq A \text{ and } X \subseteq B$$

$$\Rightarrow X \in P(A) \cap P(B)$$

Thus we have proved that $P(A \cap B) \subseteq P(A) \cap P(B)$.

$$X \in P(A) \cap P(B)$$

$$\Rightarrow X \subseteq A \text{ and } X \subseteq B$$

$$\Rightarrow X \subseteq A \cap B$$
.

$$\Rightarrow X$$

(d)
$$P(A \cap B) = P(A) \cap P(B)$$
.

 \triangle P(A): the power set of A, which is the set of all subsets of A.

Answer, True.

$$X \in P(A \cap B)$$

$$\Rightarrow X \subseteq A \cap B$$

$$\Rightarrow X \subseteq A \text{ and } X \subseteq B$$

$$\Rightarrow X \in P(A) \cap P(B)$$

Thus we have proved that $P(A \cap B) \subseteq P(A) \cap P(B)$.

$$X \in P(A) \cap P(B)$$

$$\Rightarrow X \subseteq A \text{ and } X \subseteq B$$

$$\Rightarrow X \subseteq A \cap B$$
.

$$\Rightarrow X \in P(A \cap B).$$

(d) $P(A \cap B) = P(A) \cap P(B)$.

Recall

 \triangle P(A): the power set of A, which is the set of all subsets of A.

Answer, True.

$$X \in P(A \cap B)$$

$$\Rightarrow X \subseteq A \cap B$$

$$\Rightarrow X \subseteq A \text{ and } X \subseteq B$$

$$\Rightarrow X \in P(A) \cap P(B)$$

Thus we have proved that $P(A \cap B) \subseteq P(A) \cap P(B)$.

$$X \in P(A) \cap P(B)$$

$$\Rightarrow X \subseteq A \text{ and } X \subseteq B$$

$$\Rightarrow X \subseteq A \cap B$$
.

$$\Rightarrow X \in P(A \cap B).$$

This proves $P(A) \cap P(B) \subseteq P(A \cap B)$

$$A \oplus B = (A - B) \cup (B - A).$$

(a) Let
$$A=\{1,2,3,4\},\ B=\{3,4,5,6\},\ C=\{5,6,7,8\}.$$
 Find $(A\oplus B)\oplus C.$

(b) Prove that if $A \oplus C = B \oplus C$, then A = B.

(a) A - B =, B - A =, so $A \oplus B =$

 $(A \oplus B) \oplus C =$

 $(A \oplus B) - C =$, $C - (A \oplus B) =$, so

(b) Prove that if
$$A \oplus C = B \oplus C$$
, then $A = B$. Idea.

 $A \oplus B = (A - B) \cup (B - A).$

(a) Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$. Find $(A \oplus B) \oplus C$.

(b) Prove that if
$$A \oplus C = B \oplus C$$
, then $A = B$.

Idea.

dea.
(a)
$$A-B=\{1,2\},\ B-A=$$
 , so $A\oplus B=$

(a)
$$A - B = \{1, 2\}, B - A =$$
, so $A \oplus B =$

a)
$$A-B=\{1,2\},\ B-A=$$
 , so $A\oplus B=$ $(A\oplus B)-C=$, $C-(A\oplus B)=$, so

)
$$A-B=\{1,2\},\ B-A=$$
 , so $A\oplus B=$ $(A\oplus B)-C=$, $C-(A\oplus B)=$, so $(A\oplus B)\oplus C=$.

 $A \oplus B = (A - B) \cup (B - A).$

(a) Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$. Find $(A \oplus B) \oplus C$.

(a) $A - B = \{1, 2\}, B - A = \{5, 6\}, \text{ so } A \oplus B = \{1, 2\}, B =$

 $(A \oplus B) \oplus C =$

(b) Prove that if
$$A \oplus C = B \oplus C$$
, then $A = B$. Idea.

 $(A \oplus B) - C =$, $C - (A \oplus B) =$, so

 $A \oplus B = (A - B) \cup (B - A).$

 $(A \oplus B) - C =$, $C - (A \oplus B) =$

 $(A \oplus B) \oplus C =$

(a) Let $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}, C = \{5, 6, 7, 8\}.$ Find $(A \oplus B) \oplus C$.

(b) Prove that if $A \oplus C = B \oplus C$, then A = B.

(a) $A - B = \{1, 2\}, B - A = \{5, 6\}, \text{ so } A \oplus B = \{1, 2, 5, 6\}.$

. so

(b) Prove that if
$$A \oplus C = B \oplus C$$
, then $A = B$. Idea.

 $A \oplus B = (A - B) \cup (B - A).$

 $(A \oplus B) - C = \{1, 2\}, C - (A \oplus B) =$

 $(A \oplus B) \oplus C =$

(a) Let $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}, C = \{5, 6, 7, 8\}.$ Find

. so

 $(A \oplus B) \oplus C$.

(b) Prove that if $A \oplus C = B \oplus C$, then A = B.

(a) $A - B = \{1, 2\}, B - A = \{5, 6\}, \text{ so } A \oplus B = \{1, 2, 5, 6\}.$

Idea.

 $A \oplus B = (A - B) \cup (B - A).$

 $(A \oplus B) \oplus C =$

(a) Let $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}, C = \{5, 6, 7, 8\}.$ Find $(A \oplus B) \oplus C$.

(b) Prove that if $A \oplus C = B \oplus C$, then A = B.

(a) $A - B = \{1, 2\}, B - A = \{5, 6\}, \text{ so } A \oplus B = \{1, 2, 5, 6\}.$ $(A \oplus B) - C = \{1, 2\}, C - (A \oplus B) = \{7, 8\}, \text{ so}$

Idea.

 $A \oplus B = (A - B) \cup (B - A).$

 $(A \oplus B) \oplus C = \{1, 2, 7, 8\}.$

(a) Let $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}, C = \{5, 6, 7, 8\}.$ Find $(A \oplus B) \oplus C$.

(b) Prove that if $A \oplus C = B \oplus C$, then A = B. Idea.

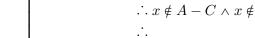
(a) $A - B = \{1, 2\}, B - A = \{5, 6\}, \text{ so } A \oplus B = \{1, 2, 5, 6\}.$ $(A \oplus B) - C = \{1, 2\}, C - (A \oplus B) = \{7, 8\}, \text{ so}$

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.					
ldea.	Case 1.	·:.			
		·:.			
		<i>:</i> .			
		<i>:</i> .			
		<i>:</i> .			
$r \in A$		··.	(as)	
$x \in A \ \langle$	Case 2.	<i>:</i> .			
		<i>:</i> .			
		<i>:</i> .			
		<i>:</i> .			
		(as)	

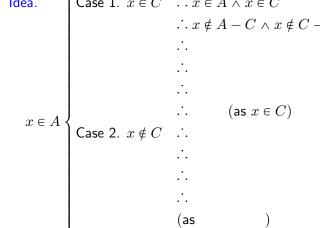
	`	, ,	,		
(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.					
Idea.	Case 1. $x \in C$	<i>:</i> .			
		∴.			
		∴.			
		<i>:</i> .			
$x \in A \ \langle$		<i>:</i> .			
		<i>:</i> .	$(as\ x \in C)$		
	Case 2. $x \notin C$	<i>:</i> .			
		<i>:</i> .			
		·			

Definition: $A \oplus B = (A - B) \cup (B - A)$ (b) Prove that if $A \oplus C = B \oplus C$, then A = B.

Case 1.
$$x \in C$$
 $\therefore x \in A \land x \in C$ $\therefore x \notin A - C \land x \notin C - A$ \therefore



$$\therefore x \notin A - C \land x \notin \vdots$$





Case 2.
$$x \notin C$$
 ...

Case 2.
$$x \notin C$$
 ...





(b) Prove that if
$$A \oplus C = B \oplus C$$
, then $A = B$.

Idea.
$$\begin{cases} \mathsf{Case} \ 1. \ x \in C & \therefore x \in A \land x \in C \\ & \therefore x \notin A - C \land x \notin C - A \\ & \therefore x \notin (A - C) \cup (C - A) = A \oplus C \end{cases}$$

$$\vdots \\ x \in A \begin{cases} \mathsf{Case} \ 2. \ x \notin C & \therefore \\ & \vdots \\ &$$

(b) Prove that if
$$A \oplus C = B \oplus C$$
, then $A = B$. Idea.
$$\begin{cases} \mathsf{Case} \ 1. \ x \in C & \therefore x \in A \land x \in C \\ & \therefore x \notin A - C \land x \notin C \\ & \therefore x \notin (A - C) \cup (C - C) \\ & \therefore x \notin B \oplus C = (B - C) \\ & \therefore x \notin C \end{cases}$$

$$\therefore x \notin (A - C) \cup (C - A) = A \oplus C$$
$$\therefore x \notin B \oplus C = (B - C) \cup (C - B)$$

 $\therefore x \notin B \oplus C = (B - C) \cup (C - B)$

 $x \in A \begin{cases} & \dots \\ & \ddots \\ & \text{Case 2. } x \notin C & \dots \\ & \dots \\ & & \dots \\ \\ & \dots \\ \\ & \dots \\ \\ & \dots \\ \\ &$

 $\therefore x \notin A - C \land x \notin C - A$

(b) Prove that if
$$A \oplus C = B \oplus C$$
, then $A = B$.

dea.
$$\begin{cases} \mathsf{Case} \ 1. \ x \in C & \therefore x \in A \land x \in C \\ & \therefore x \notin A - C \land x \notin C - A \\ & \therefore x \notin (A - C) \cup (C - A) = A \oplus C \\ & \therefore x \notin B \oplus C = (B - C) \cup (C - B) \\ & \therefore x \notin C - B \\ & \therefore x \notin C - B \end{cases}$$

$$x \in A \begin{cases} \mathsf{Case} \ 2. \ x \notin C & \therefore \\ & \vdots \\ &$$

$$\therefore x \notin C - B$$

$$\therefore (as x \in A)$$

$$\left\{\begin{array}{c} A \\ \text{Case 2. } x \notin C \\ \vdots \\ \end{array}\right.$$

$$x \in C$$

$$= (D - C) \cup (C - D)$$

Definition: $A \oplus B = (A - B) \cup (B - A)$

(b) Prove that if
$$A \oplus C = B \oplus C$$
, then $A = B$. Idea.
$$\begin{cases} \mathsf{Case} \ 1. \ x \in C & \therefore x \in A \land x \in C \\ & \therefore x \notin A - C \land x \notin C - A \\ & \therefore x \notin (A - C) \cup (C - A) = A \oplus C \\ & \therefore x \notin B \oplus C = (B - C) \cup (C - B) \\ & \therefore x \notin C - B \\ & \therefore x \in B (\mathsf{as} \ x \in C) \end{cases}$$

$$\mathsf{Case} \ 2. \ x \notin C \quad \therefore \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$\therefore x \notin B \oplus C = (B - C)$$

 $\therefore x \notin B \oplus C = (B - C) \cup (C - B)$

Prove that if
$$A \oplus C = B \oplus C$$
, then $A = B$.

ea.
$$\begin{cases} \mathsf{Case} \ 1. \ x \in C & \therefore x \in A \land x \in C \\ & \therefore x \notin A - C \land x \notin C - A \\ & \therefore x \notin (A - C) \cup (C - A) = A \oplus C \\ & \therefore x \notin B \oplus C = (B - C) \cup (C - B) \\ & \therefore x \notin C - B \\ & \therefore x \in B (\mathsf{as} \ x \in C) \end{cases}$$

$$\mathsf{Case} \ 2. \ x \notin C & \therefore x \in A - C \\ & \therefore x \in A - C \\ & \therefore x \in A \cap C \end{cases}$$

b) Prove that if
$$A \oplus C = B \oplus C$$
, then $A = B$.

Lea.
$$\begin{cases} \mathsf{Case} \ 1. \ x \in C & \therefore x \in A \land x \in C \\ & \therefore x \notin A - C \land x \notin C - A \\ & \therefore x \notin (A - C) \cup (C - A) = A \oplus C \\ & \therefore x \notin B \oplus C = (B - C) \cup (C - B) \\ & \therefore x \notin C - B \\ & \therefore x \in B (\mathsf{as} \ x \in C) \end{cases}$$

$$\therefore x \in B(\text{as } x \in B)$$

$$\begin{array}{c} \therefore x \notin C - B \\ \therefore x \in B(\text{as } x \in C) \end{array}$$

Case 2.
$$x \notin C$$
 $\therefore x \in A - C$

Case 2.
$$x \notin C$$
 . $x \in A - C$

$$\therefore x \in (A - C) \cup (C - A) = A \oplus$$

$$\therefore x \in (A - C) \cup (C - A) = A \oplus$$

$$\therefore$$

$$x \in A \begin{cases} \vdots x \notin C - B \\ \vdots x \in B (\text{as } x \in C) \end{cases}$$
 Case 2. $x \notin C$ $\vdots x \in A - C$ $\vdots x \in (A - C) \cup (C - A) = A \oplus C$

Prove that if
$$A \oplus C = B \oplus C$$
, then $A = B$.

Ea.
$$\begin{cases} \mathsf{Case} \ 1. \ x \in C & \therefore x \in A \land x \in C \\ & \therefore x \notin A - C \land x \notin C - A \\ & \therefore x \notin (A - C) \cup (C - A) = A \oplus C \\ & \therefore x \notin B \oplus C = (B - C) \cup (C - B) \\ & \therefore x \notin C - B \\ & \therefore x \in B (\mathsf{as} \ x \in C) \end{cases}$$

$$\therefore x \notin C - B$$

$$\therefore x \in B(\text{as } x \in C)$$

$$\text{Case 2. } x \notin C \quad \therefore x \in A - C$$

$$\therefore x \in (A - C) \cup (C - A) = A \oplus C$$

$$\therefore x \in (A - C) \cup (C - A) = A \oplus C$$

$$\therefore x \in B \oplus C = (B - C) \cup (C - B)$$

(as $x \notin C - B$)

$$\begin{array}{c} \therefore x \notin C - B \\ \vdots x \in B (\mathsf{as} \ x \in C) \end{array}$$
 Case 2. $x \notin C$ $\therefore x \in A - C$

Prove that if
$$A \oplus C = B \oplus C$$
, then $A = B$.

Ea.
$$\begin{cases}
\mathsf{Case 1.} & x \in C & \therefore x \in A \land x \in C \\
& \therefore x \notin A - C \land x \notin C - A \\
& \therefore x \notin (A - C) \cup (C - A) = A \oplus C \\
& \therefore x \notin B \oplus C = (B - C) \cup (C - B) \\
& \therefore x \notin C - B \\
& \therefore x \in B(\mathsf{as } x \in C)
\end{cases}$$

$$\mathsf{Case 2.} & x \notin C & \therefore x \in A - C \\
& \therefore x \in (A - C) \cup (C - A) = A \oplus C$$

 $\therefore x \in (A - C) \cup (C - A) = A \oplus C$ $\therefore x \in B \oplus C = (B - C) \cup (C - B)$

 $\therefore x \in B - C$ (as $x \notin C - B$)

$$\therefore x \in B(\text{as } x)$$

$$A \left\{ \begin{array}{l} \therefore x \in B (\text{as } x \in C) \\ \text{Case 2. } x \notin C \quad \therefore x \in A - C \\ \therefore x \in (A - C) \cup (C - A) = A \end{array} \right.$$

 $\therefore x \in (A - C) \cup (C - A) = A \oplus C$

$$\therefore x \in (A - C) \cup (C - A) = A \oplus C$$
$$\therefore x \in B \oplus C = (B - C) \cup (C - B)$$
$$\therefore x \in B - C$$

 $\therefore x \in B - C$ (as $x \notin C - B$)

 $\therefore x \in B$

Case 2.
$$x \notin C$$
 $\therefore x \in A - C$
 $\therefore x \in (A - C) \cup (C)$

$$x \in C$$

$$x \in B \begin{cases} \mathsf{Case} \ 1. \ x \in C & \therefore x \in B \land x \in C \\ & \therefore x \notin B - C \land x \notin C - B \end{cases}$$

$$\vdots & \vdots \\ &$$

$$x \in B$$
 Case 1. $x \in C$ $\therefore x \in B \land x \in C$
$$\therefore x \notin B - C \land x \notin C - B$$

$$\therefore x \notin (B - C) \cup (C - B) = B \oplus C$$

$$\therefore x \notin A \oplus C = (A - C) \cup (C - A)$$

$$\vdots$$

$$(as \ x \in C)$$
 Case 2. $x \notin C$
$$\vdots$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$x \in B$$
 Case 1. $x \in C$ $\therefore x \in B \land x \in C$
$$\therefore x \notin B - C \land x \notin C - B$$

$$\therefore x \notin (B - C) \cup (C - B) = B \oplus C$$

$$\therefore x \notin A \oplus C = (A - C) \cup (C - A)$$

$$\therefore x \notin C - A$$

$$\therefore x \in A \text{(as } x \in C)$$
 Case 2. $x \notin C$
$$\therefore$$

$$\therefore$$

$$x \in B$$
 Case 1. $x \in C$ $\therefore x \in B \land x \in C$
$$\therefore x \notin B - C \land x \notin C - B$$

$$\therefore x \notin (B - C) \cup (C - B) = B \oplus C$$

$$\therefore x \notin A \oplus C = (A - C) \cup (C - A)$$

$$\therefore x \notin C - A$$

$$\therefore x \in A \text{(as } x \in C)$$
 Case 2. $x \notin C$
$$\therefore x \in B - C$$

$$\vdots$$

$$\begin{cases} \mathsf{Case}\ 1.\ x \in C & \therefore x \in B \land x \in C \\ & \therefore x \notin B - C \land x \notin C - B \\ & \therefore x \notin (B - C) \cup (C - B) = B \oplus C \\ & \therefore x \notin A \oplus C = (A - C) \cup (C - A) \\ & \therefore x \notin C - A \\ & \therefore x \in A (\mathsf{as}\ x \in C) \\ \mathsf{Case}\ 2.\ x \notin C & \therefore x \in B - C \\ & \therefore x \in (B - C) \cup (C - B) = B \oplus C \end{cases}$$

$$\begin{cases} \mathsf{Case} \ 1. \ x \in C & \therefore x \in B \land x \in C \\ & \therefore x \notin B - C \land x \notin C - B \\ & \therefore x \notin (B - C) \cup (C - B) = B \oplus C \\ & \therefore x \notin A \oplus C = (A - C) \cup (C - A) \\ & \therefore x \notin C - A \\ & \therefore x \in A (\mathsf{as} \ x \in C) \\ \mathsf{Case} \ 2. \ x \notin C & \therefore x \in B - C \\ & \therefore x \in (B - C) \cup (C - B) = B \oplus C \end{cases}$$

 $\therefore x \in A \oplus C = (A - C) \cup (C - A)$

$$\left\{ \begin{array}{ll} \mathsf{Case} \ 1. \ x \in C & \therefore x \in B \land x \in C \\ & \therefore x \notin B - C \land x \notin C - B \\ & \therefore x \notin (B - C) \cup (C - B) = B \oplus C \\ & \therefore x \notin A \oplus C = (A - C) \cup (C - A) \\ & \therefore x \notin C - A \\ & \therefore x \in A (\mathsf{as} \ x \in C) \\ & \\ \mathsf{Case} \ 2. \ x \notin C & \therefore x \in B - C \\ & \therefore x \in (B - C) \cup (C - B) = B \oplus C \\ & \therefore x \in A \oplus C = (A - C) \cup (C - A) \end{array} \right.$$

 $\therefore x \in A - C$ $(\mathsf{as}\ x \notin C - A)$

 $\therefore x \in (B-C) \cup (C-B) = B \oplus C$ $\therefore x \in A \oplus C = (A - C) \cup (C - A)$

$$\begin{cases} \mathsf{Case} \ 1. \ x \in C & \therefore x \in B \land x \in C \\ & \therefore x \notin B - C \land x \notin C - B \\ & \therefore x \notin (B - C) \cup (C - B) = B \oplus C \\ & \therefore x \notin A \oplus C = (A - C) \cup (C - A) \\ & \therefore x \notin C - A \\ & \therefore x \in A (\mathsf{as} \ x \in C) \\ & \therefore x \in (B - C) \cup (C - B) = B \oplus C \\ & \therefore x \in A \oplus C = (A - C) \cup (C - A) \end{cases}$$

 $\therefore x \in A - C$ (as $x \notin C - A$)

 $\therefore x \in A$

$$x \in B \begin{cases} \vdots & x \in A (\mathsf{as} \ x \in C) \\ \mathsf{Case} \ 2. \ x \notin C & \vdots \ x \in B - C \\ \vdots & x \in (B - C) \cup (C - B) = B \oplus C \\ \vdots & x \in A \oplus C = (A - C) \cup (C - A) \end{cases}$$