

# Introduction

## **Basic terminology**

- Population vs. Sample
- Parameter vs. Statistic
- Descriptive vs. Inferential Statistics
- Types of variables

# What is Statistics

- Statistics is the art of learning from data.  
~Sheldon M. Ross
- Statistics is the science of learning from data.  
~Moore, McCabe & Craig
- Statistics is the art and science of learning from data.  
~Alan Agresti
- Statistics is the science whereby inferences are made about specific random phenomena on the basis of relatively limited sample materials.  
~ Bernard Rosner

## The Population

- The collection of **all** subjects of interest



## The Sample

- A subset of the population

## Parameters

- Numerical measures computed using population data

## Statistics

- Numerical measures computed using sample data

## **Descriptive statistics**

Collecting, summarizing, and presenting data

## **Inferential statistics**

Drawing conclusions about a population based only on sample data

Some questions of interest:

- What is the average height of this class?
- What proportion of NUS students are female?
- What is the total sugar consumption (in a year) of all Singaporeans?

### Descriptive Statistics

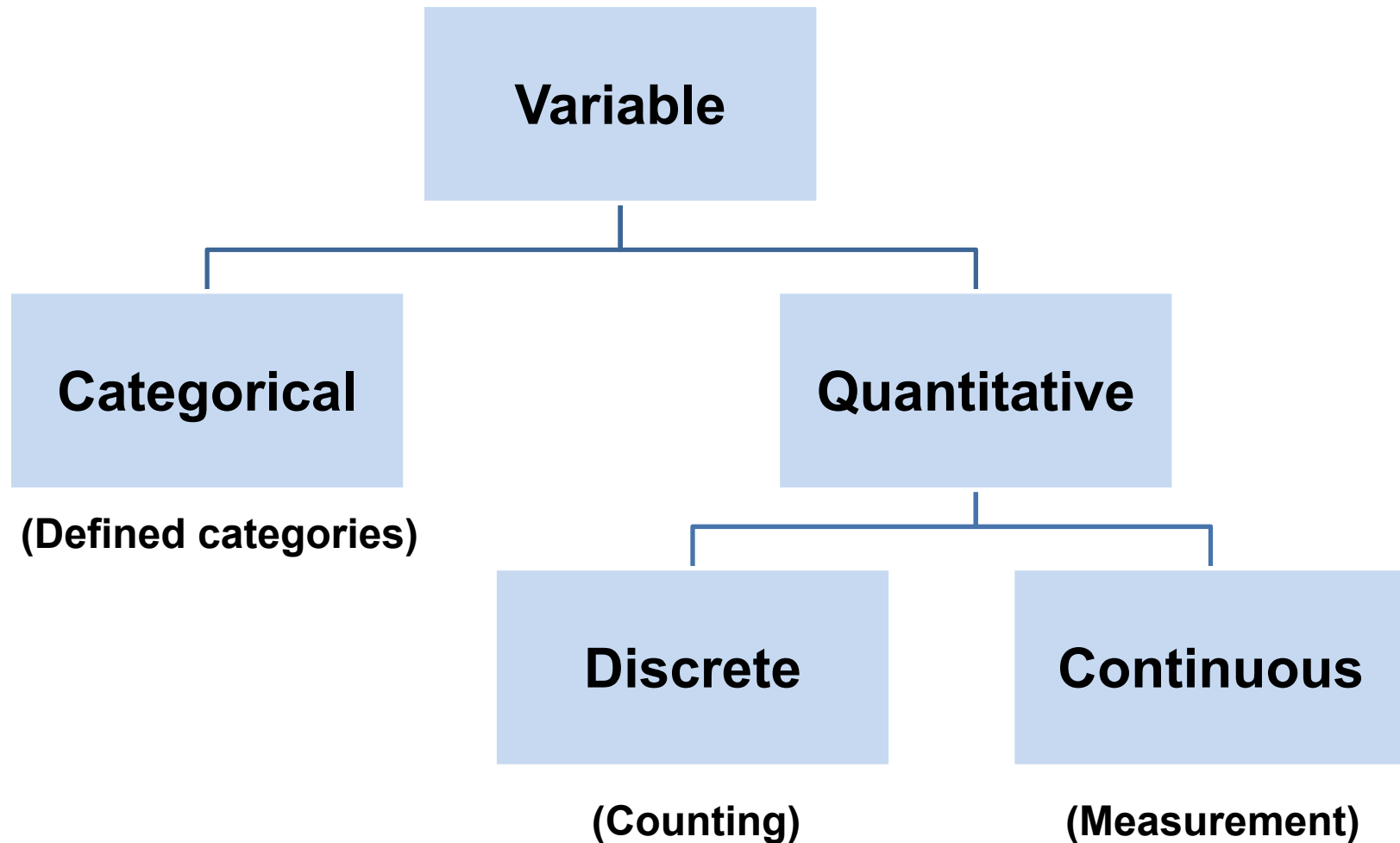
- Organize / graph
- Numerical summaries

### Inferential Statistics

- Estimate / predict
- Decide /conclude

A **variable** is any characteristics that is recorded for subjects in the study.

The terminology **variable** highlights the fact that data vary.

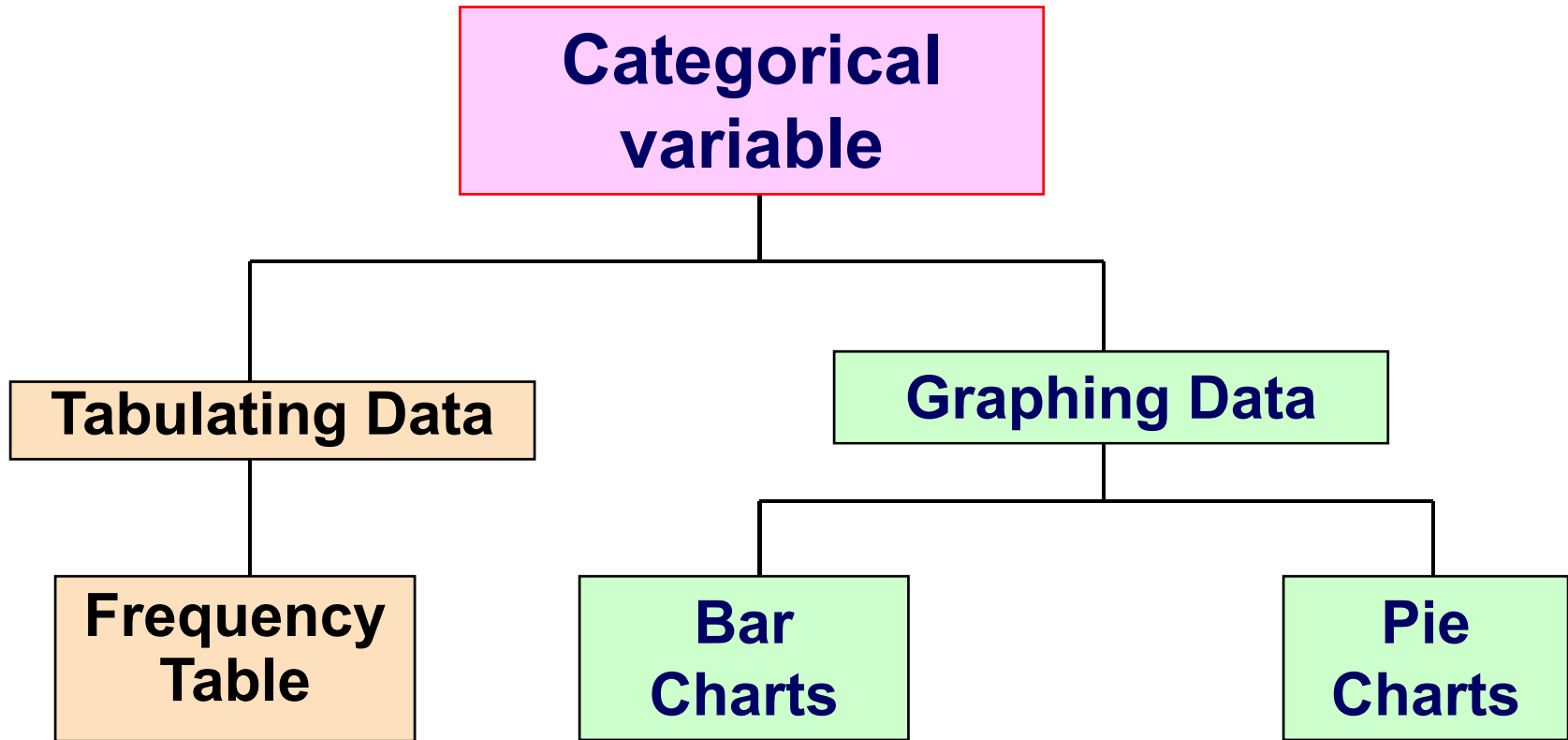


# Descriptive Statistics

- Graphical presentation
- Numerical Summary

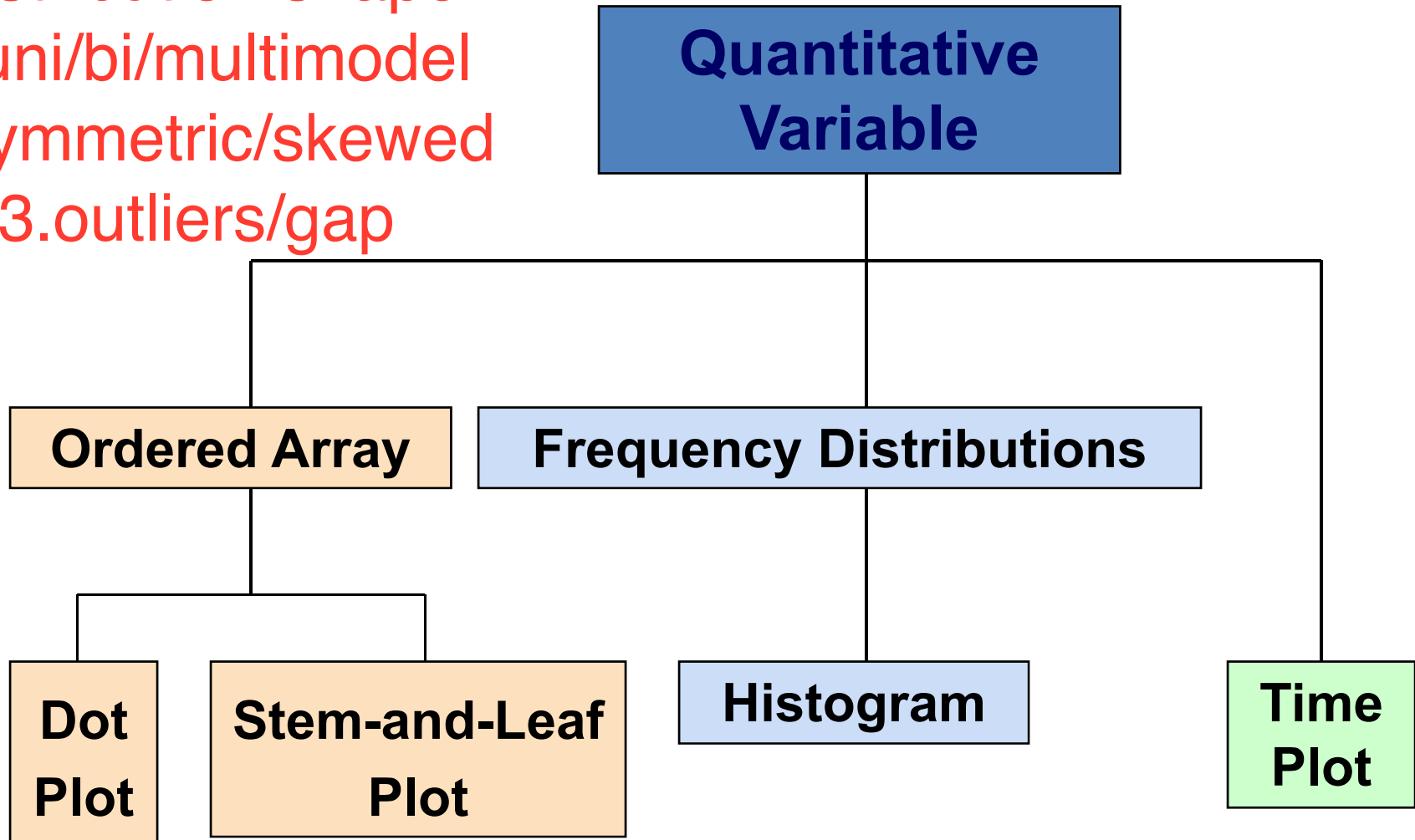
Categorical

Quantitative

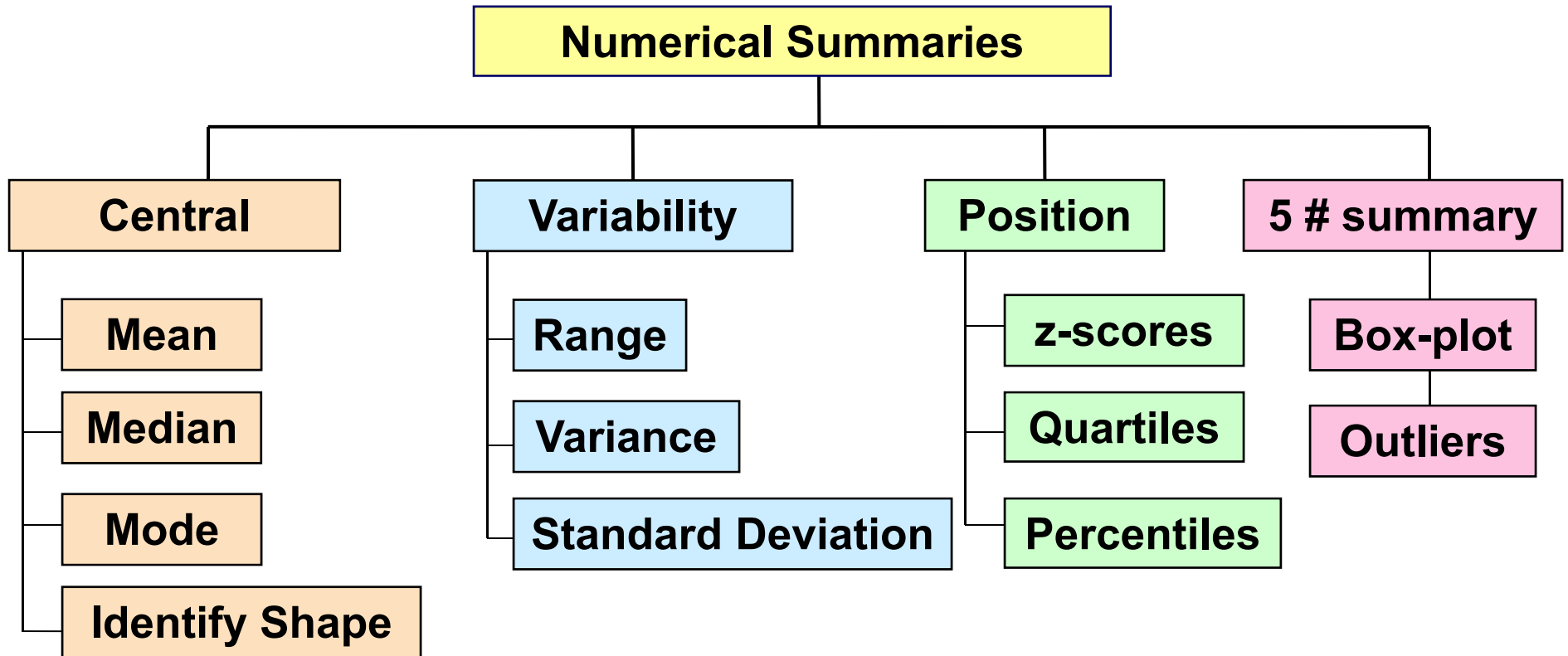




Distribution shape  
1.uni/bi/multimodel  
2.symmetric/skewed  
3.outliers/gap

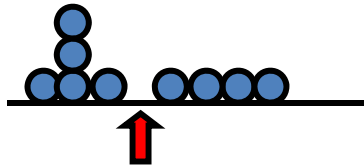


# Quantitative



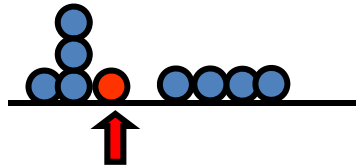
# Central Tendency

Mean



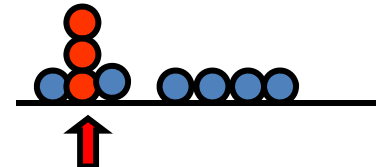
The center of gravity or  
the balance point

Median



Midpoint of  
ranked values

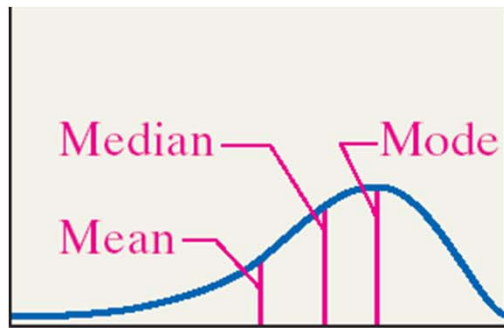
Mode



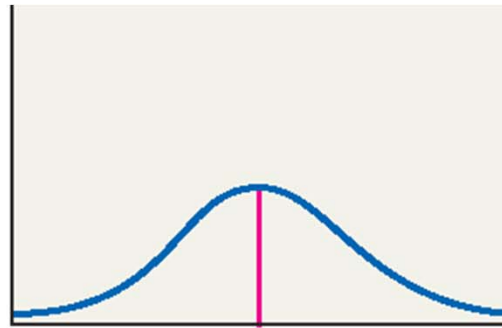
Most frequently  
observed value

affected by  
outliers but  
utilize all info

# Distribution Shape

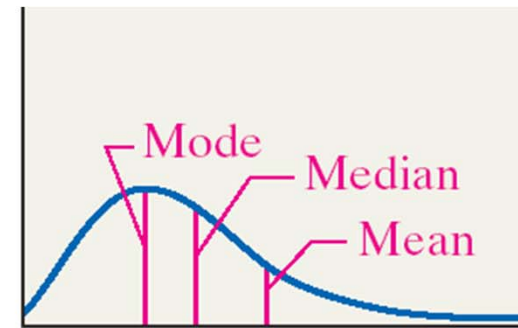


**(a) Skewed Left**  
 $\text{Mean} < \text{Median}$

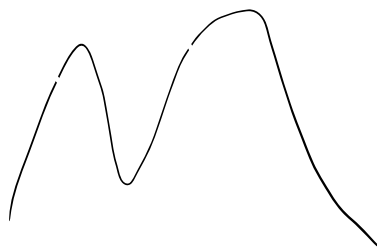


$\text{Mean} = \text{Median} = \text{Mode}$

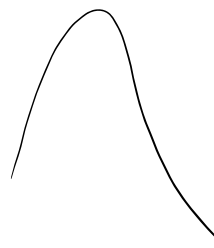
**(b) Symmetric**  
 $\text{Mean} = \text{Median}$



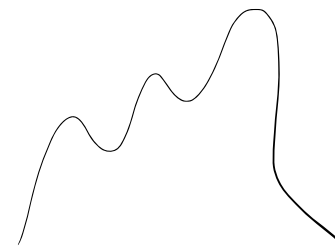
**(c) Skewed Right**  
 $\text{Mean} > \text{Median}$



**bimodel**

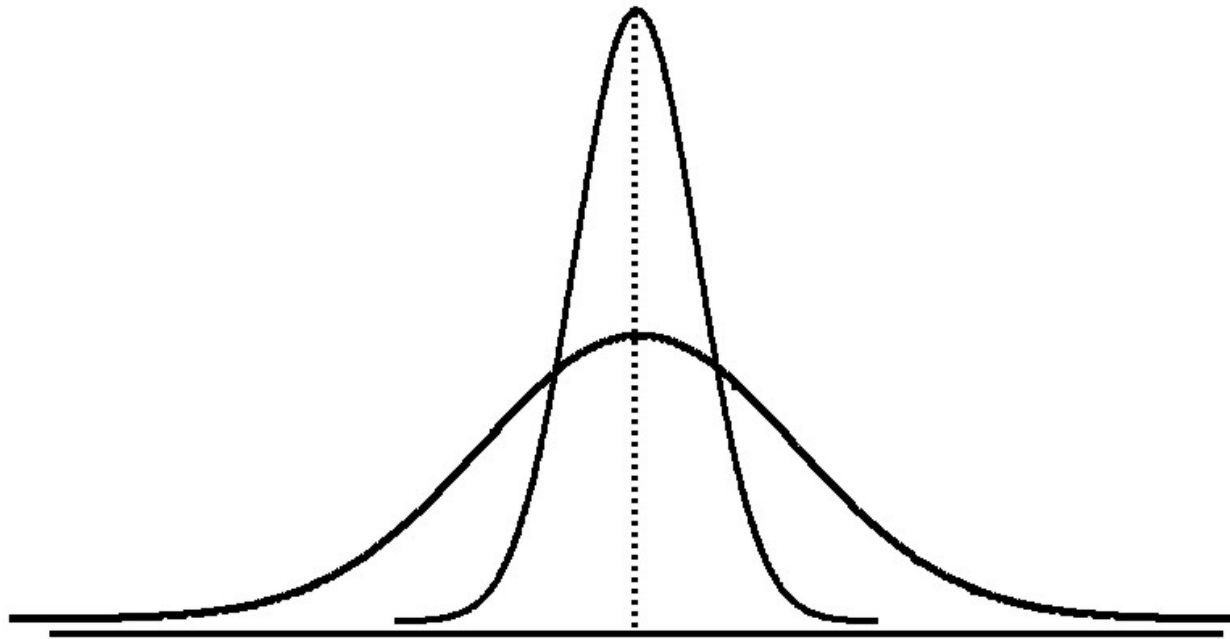


**unimodel**



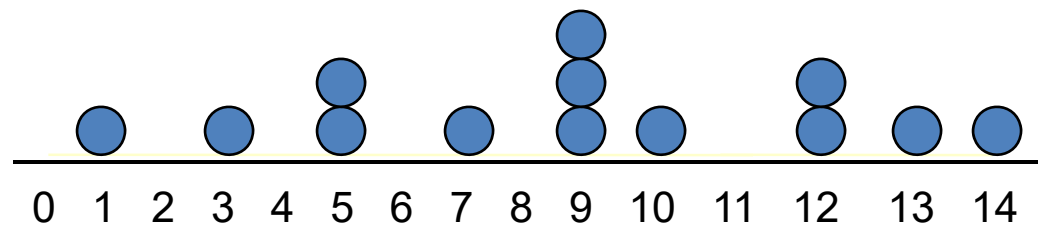
**multimodel**

# Variability

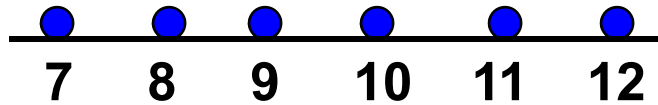


**Same center,  
different variation**

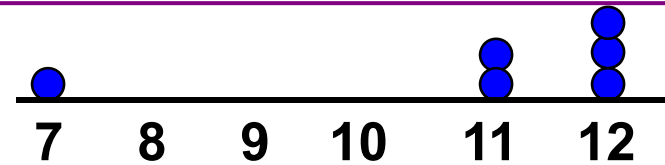
$$\text{Range} = X_{\text{largest}} - X_{\text{smallest}}$$



**Range =**



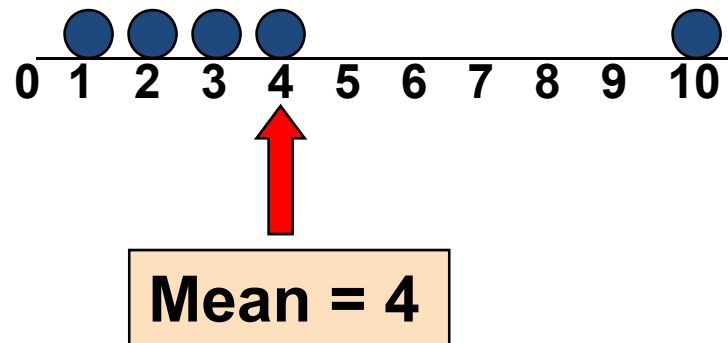
**Range =**



**Range =**

# Deviation

$$\text{Deviation} = X - \text{mean}$$



$$\text{av. dev.} = 0$$

# Variance

- The population variance,  $\sigma^2$  of a variable is the sum of squared deviations divided by the number in the population

$$\frac{\sum (x_i - \mu)^2}{N} = \frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \dots + (x_N - \mu)^2}{N}$$

mean=u(para), x bar (stat)

- The sample variance,  $s^2$  of a variable is the sum of squared deviations divided by one less than the number in the sample

$$\frac{\sum (x_i - \bar{x})^2}{n - 1} = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n - 1}$$

proportion = p(para), p^ (stat) size= N (para), n (stat)

variance o^2(para), s^2 (stat)

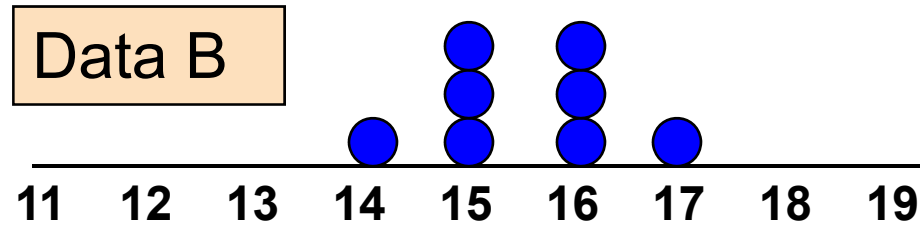
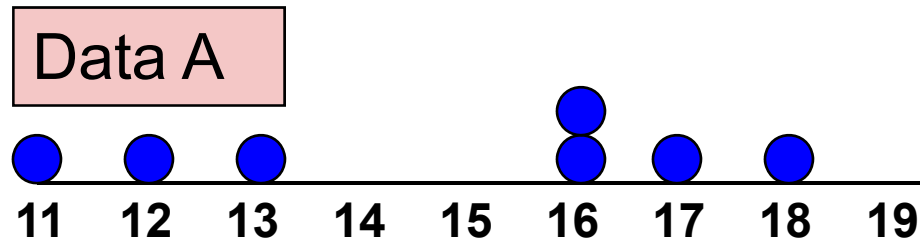


# Standard Deviation

- The standard deviation is the square root of the variance.
- $\sigma$  is the population standard deviation.
- $s$  is the sample standard deviation.

# Standard Deviation

## Comparing Standard Deviations



# z-scores

- z-scores can be used to compare the relative positions of data values in different samples

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{X - \bar{X}}{s}$$

Pat received:

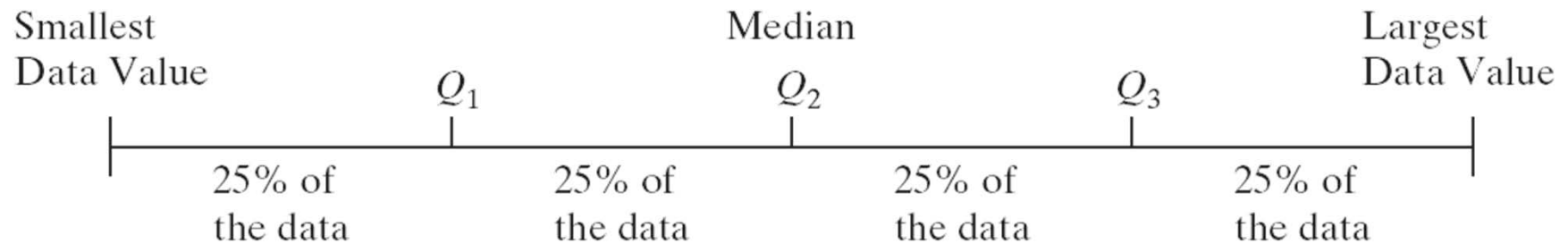
- A grade of 82 on her statistics exam where the mean grade was 74 and the standard deviation was 12
- A grade of 72 on her biology exam where the mean grade was 65 and the standard deviation was 10
- A grade of 91 on her kayaking exam where the mean grade was 88 and the standard deviation was 6

# z-scores

- Statistics
  - Grade of 82
  - $Z =$
- Biology
  - Grade of 72
  - $Z =$
- Kayaking
  - Grade of 91
  - $Z =$
- \_\_\_\_\_ was the highest relative grade

# Quartiles

- Quartiles divide the data set into four equal parts



# Percentiles

- Percentiles divide the data set into 100 equal parts

# Five-number Summary

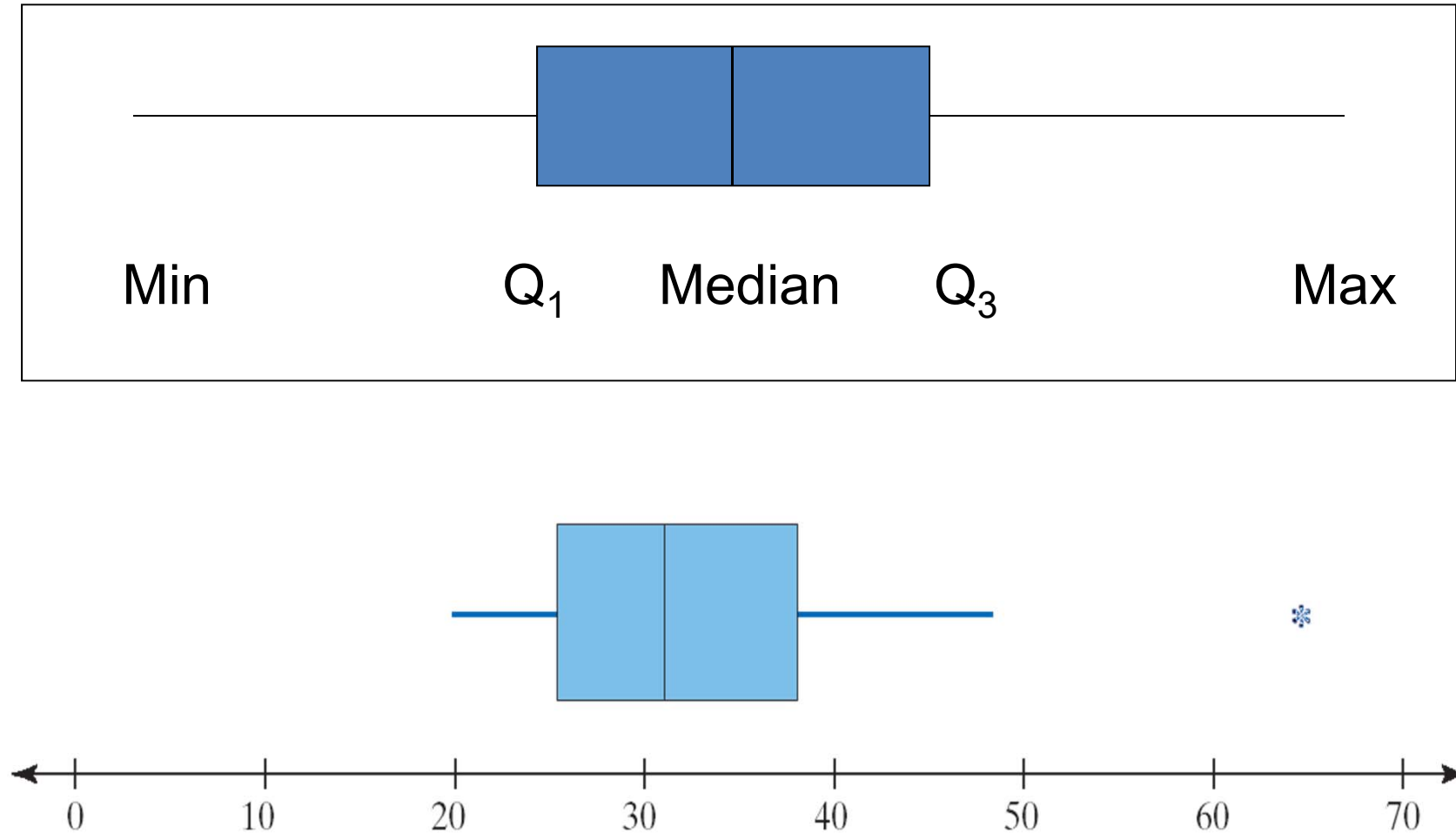
- The five-number summary is the collection of
  - The smallest value
  - The first quartile ( $Q_1$  or  $P_{25}$ )
  - The median ( $M$  or  $Q_2$  or  $P_{50}$ )
  - The third quartile ( $Q_3$  or  $P_{75}$ )
  - The largest value
- These five numbers give a concise description of the distribution of a variable

median, IQR= OUTLIERS  
mean, SD = NO OUTLIERS

# Five-number Summary

- Compute the five-number summary for  
1, 3, 4, 7, 8, 15, 16, 19, 23, 24, 27, 31, 33, 54
- Calculations
  - The minimum = 1
  - $Q_1 = 7$
  - $M = 17.5$
  - $Q_3 = 27$
  - The maximum = 54
- The five-number summary is

# Boxplot





# Outliers

- Extreme observations in the data are referred to as outliers
- One way to check for outliers uses the inter-quartiles range,  $IQR = Q_3 - Q_1$
- The fences used to identify outliers are
  - Lower fence =  $LF = Q_1 - 1.5 \times IQR$
  - Upper fence =  $UF = Q_3 + 1.5 \times IQR$
- Values less than the lower fence or more than the upper fence could be considered outliers

# Outliers

- Is the value 54 an outlier?

1, 3, 4, 7, 8, 15, 16, 19, 23, 24, 27, 31, 33, 54

- Calculations

- $Q_1 = 7$

- $Q_3 = 27$

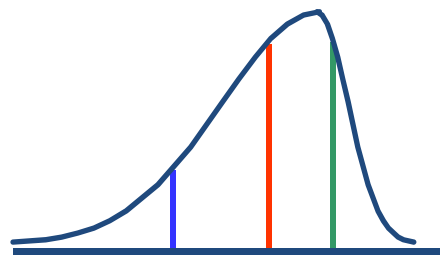
- $IQR = 20$

- $UF = 57$

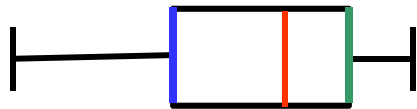
- Using the fence rule, the value 54 is / is not an outlier

# Boxplot

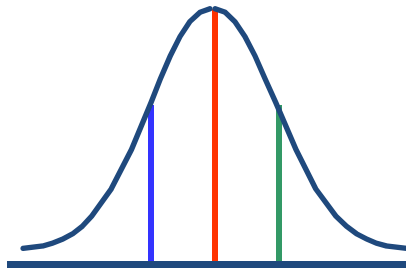
Left-Skewed



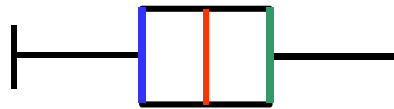
Q1 Q2 Q3



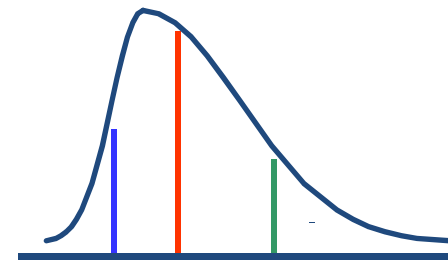
Symmetric



Q1 Q2 Q3



Right-Skewed



Q1 Q2 Q3

