CS1231: Discrete Structures

Tutorial 2

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Quick Review

- Quantifiers. $\forall$. $\exists$. Their translations.
- Negation of Quantifications.
- Nested Quantifiers.

Menu

1. Translate the following into English where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.

(a) $\forall x(R(x) \rightarrow H(x))$

(b) $\forall x(R(x) \wedge H(x))$

(c) $\exists x(R(x) \rightarrow H(x))$

(d) $\exists x(R(x) \wedge H(x))$

Recall

- ✍ The **domain** is the set of (all) values that may be substituted in place of the variable.
- ✍ $\forall$ (for all); $\exists$ (exists).

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(b) Every animal is a rabbit and hops.

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2. Let $Q(x, y)$ be the predicate "If $x < y$ then $x^2 < y^2$" with domain for both x and y being $\{1, \pm 2\}$.

(a) Why is $Q(x, y)$ false for $(x, y) = (-2, 1)$, and true for $(x, y) = (1, 2)$?

(b) Find all the values of x and y for which $Q(x, y)$ is true.

Recall

✍ $(x, y) = (a, b)$ means $x = a$ and $y = b$.

✍ The truth table for $p \to q$ is

p	q	$p \to q$
T	T	T
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Answer.

(a) $(x, y) = (-2, 1)$: Hypothesis true, conclusion false
$(x, y) = (1, 2)$: Hypothesis true, conclusion true.

$x, y \in \{1, \pm 2\}$.

(x, y)	$x < y$	$x^2 < y^2$	if $x < y$ then $x^2 < y^2$
$(1, 1)$	F	F	T
$(1, 2)$	T	T	T
$(1, -2)$	F	T	T
$(2, 1)$	F	F	T
$(2, 2)$	F	F	T
$(2, -2)$	F	F	T
$(-2, 1)$	T	F	F
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(x,y)	$x < y$	$x^2 < y^2$	if $x < y$ then $x^2 < y^2$
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$(2,-2)$	F	F	T
$(-2,1)$	T	F	F
$(-2,2)$	T	F	F
$(-2,-2)$	F	F	T

Answer.

(b) $\{(1,1),(1,2),(1,-2),(2,1),(2,2),(2,-2),(-2,-2)\}$

3. Rewrite each of the following in the form $\forall$__, if __ then __

(a) All integers having even squares are even.

(b) Given any integer whose square is even, that integer is itself even.

(c) The square of any even integer is even.

(d) All even integers have even squares.

Recall

✍ Expressing $\forall$: "for all", "all", "all of", "for every", "for each", "given any", "any", "for arbitrary", etc.

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(c), (d): $\forall n \in \mathbb{Z}$, if n is even then n^2 is even.

4. Which of the following are true? If false, justify your answers.

(a) $\forall x \in \mathbb{R}, x > 2 \to x > 1.$

(b) $\forall x \in \mathbb{R}, x > 2 \to x^2 > 4.$

(c) $\forall x \in \mathbb{R}, x^2 > 4 \to x > 2.$

(d) $\forall x \in \mathbb{R}, x^2 > 4 \leftrightarrow |x| > 2.$

Recall

- ✍ $\forall x \in D(P(x))$ is true if $P(x)$ is true for all $x \in D$; it is false if we have a counter example in D.
- ✍ the set of real numbers: $\mathbb{R}$.
- ✍ The truth table for $p \to q$ is

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5. Let $D(x)$, $P(x)$, $O(x)$, $W(x)$ be "x is a duck", "x is one of my poultry", "x is an officer", "x is willing to waltz". Express each of (a), (b), (c), (d) using quantifiers, logical connectives and $D(x)$, $P(x)$, $O(x)$, $W(x)$.

(a) No ducks are willing to waltz.

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(c) All my poultry are ducks.

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(e) If (a), (b), (c) are all true, does it follow that (d) is also true.

Recall

- $\forall x \in D(P(x)) \equiv \forall x(x \in D \to P(x))$;
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(c) $\forall x(P(x) \to D(x))$.

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(d) $\forall x(P(x) \to \neg O(x))$.
(e) Yes. Since $P(x) \to D(x) \to \neg W(x) \to \neg O(x)$.

6. Write a negation for each of the following:

(a) $\forall d \in \mathbb{Z}$, if $\frac{6}{d} \in \mathbb{Z}$, then $d = 3$.

(b) If the square of an integer is odd, then the integer is odd.

Recall

✍ $\neg(\forall x \in D(P(x))) \equiv \exists x \in D(\neg P(x))$;
$\neg(\exists x \in D(P(x))) \equiv \forall x \in D(\neg P(x))$.

✍ $\neg(p \to q) \equiv p \wedge \neg q$.

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Answer.

(a) $\exists d \in \mathbb{Z}$ s.t. $\frac{6}{d} \in \mathbb{Z}$ and $d \neq 3$.

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Answer.

(a) $\exists d \in \mathbb{Z}$ s.t. $\frac{6}{d} \in \mathbb{Z}$ and $d \neq 3$.

(b) There is an even integer whose square is odd.

7. Rewrite the following without using the words *necessary* or *sufficient*.

(a) Being a bird is not a necessary condition for an animal being able to fly.

(b) Being a polynomial is not a sufficient condition for a function to have a real root.

Recall

- ✍ $p \to q$: p is sufficient for q; q is necessary for p.
- ✍ $\neg(\forall x \in D(P(x) \to Q(x))) \equiv \exists x \in D(P(x) \wedge \neg Q(x))$.
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Answer.

(a) Some animal is able to fly but not a bird.

(b) There is a function which is a polynomial but has no real roots.

8. Let $D = E = \{0, \pm 1, \pm 2\}$. Write a negation of the following and determine which is true, the given statement or its negation.

$$\exists x \in D \text{ such that } \forall y \in E, x + y = -y.$$

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Idea. Its negation is

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Answer. $\forall x \in D$, $\exists y \in E$ such that $x + y \neq -y$. The negation is true.

9. Write a negation for each of following.

(a) $\forall r \in \mathbb{Q}$, $\exists a \in \mathbb{Z}$ and $\exists b \in \mathbb{Z}$ such that $r = a/b$.

(b) $\exists x \in \mathbb{R}$ such that for all $y \in \mathbb{R}$, $x + y = 0$.

(c) $p \leftrightarrow q$.

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(c) $(p \wedge \neg q) \vee (q \wedge \neg p)$.

10. For any propositions p and q, write a logical expression S involving p, q, using logical connectives so that S is true when exactly one of p, q is true and is false otherwise.

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Answers.Solution 1. $(p \wedge \neg q) \vee (\neg p \wedge q)$ (Show the two cases)

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Answers.Solution 1. $(p \wedge \neg q) \vee (\neg p \wedge q)$ (Show the two cases)
Solution 2. $(p \vee q) \wedge (\neg p \vee \neg q)$ (Show the two requirements)

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Answers. Solution 1. $(p \wedge \neg q) \vee (\neg p \wedge q)$ (Show the two cases)
Solution 2. $(p \vee q) \wedge (\neg p \vee \neg q)$ (Show the two requirements)
Solution 3. $(p \vee q) \wedge \neg(p \wedge q)$ (Show the exception)

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Solution 3. $(p \vee q) \wedge \neg(p \wedge q)$ (Show the exception)
All of them are logically equivalent.