Analysis and Design of Algorithms

NUS
National University of Singapore

Algorithms
CS3230

Tutorial

Week 4

Question 4

FIB’(*n*)

1. **if** *n* == 0
2. **return** 0
3. **elseif** *n* == 1
4. **return** 1
5. *sum* = 1
6. **for** *k* = 1 to *n* - 2
7. *sum* = *sum* + FIB’(*k*)
8. **return** *sum*

Your clever classmate suggests the following way to compute the Fibonacci number.

It is not obvious that FIB'(n) correctly returns the n-th Fibonacci number. To understand the algorithm, you do a correctness proof of the algorithm.

The proof will be done by induction. For the base cases of n = 0 and n = 1, the algorithm simply returns the value defined, hence is correct. As inductive hypothesis, we assume that FIB'(k) correctly return the k-th Fibonacci number for all k < n.

Assuming the inductive hypothesis, we need show that FIB'(n) works correctly. Within FIB'(n) is a loop (line 6 and 7), and we need to show that the loop correctly returns the n-th Fibonacci number assuming that FIB'(k) correctly return the k-th Fibonacci number for all k < n (from the inductive hypothesis).

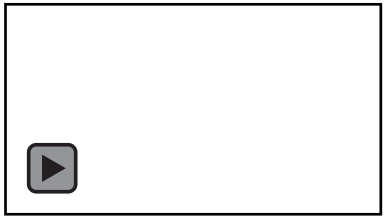
State a suitable loop invariant (line 6 and 7). Then, show that FIB'(n) works correctly

Question 5

FIB'(*n*)

```
1. if n == 0
2.     then return 0
3. elseif n == 1
4.     then return 1
5. sum = 1
6. for k = 1 to n-2
7.     do sum = sum + FIB'(k)
8. return sum
```

Can you give the recursive formula for the runtime of this algorithm?



Question 6

For the recursive formula in the previous question, can you analyze its time complexity?

(Hint: You can check $T(n) - T(n-1)$.)