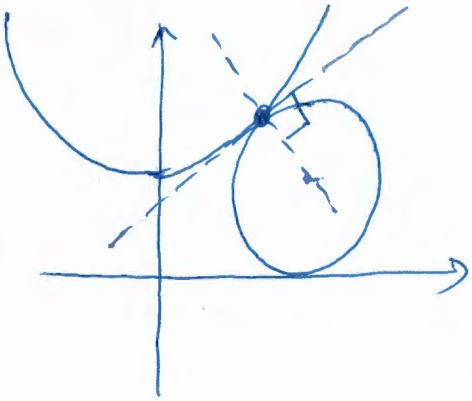
1. Let $f(x) = (x + 1000)^2$. Find the exact value of $f'(9)$.

$f'(x) = 2(x + 1000)$

$f'(9) = 2(9 + 1000) = \underline{\underline{2018}}$

2. Let r denote a positive constant with $r < 57$. Let C denote the circle centred at $(57, r)$ with radius r. It is known that C is tangent to the parabola $y = x^2 + r$ from the outside in the first quadrant. Find the value of r. Give your answer correct to two decimal places.

$y = x^2 + r \Rightarrow \frac{dy}{dx} = 2x$



$\left(\frac{x^2+r-r}{x-57}\right)(2x) = -1$

$\Rightarrow 2x^3 + x - 57 = 0$

$\because 2\times(3)^3 + (3) - 57 = 0$

$\therefore x = 3$ is a root

$$\begin{array}{r} 2x^2+6x+19 \\ x-3\,\overline{)\,2x^3+0x^2+x-57} \\ \underline{2x^3-6x^2} \\ 6x^2+x \\ \underline{6x^2-18x} \\ 19x-57 \\ \underline{19x-57} \end{array}$$

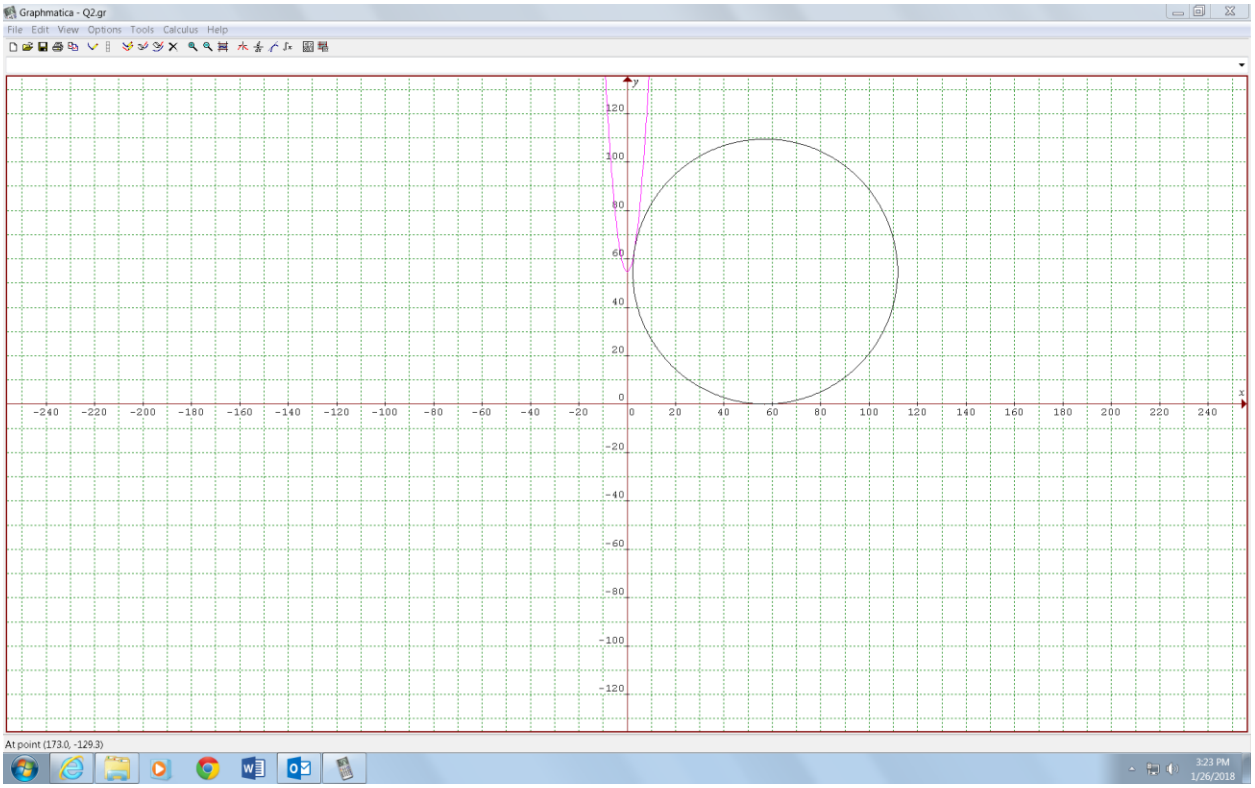
$6^2 - 4\times2\times19 = -ve \Rightarrow 2x^2+6x+19=0$ has no real root.

$\therefore x = 3$ is the only root

$(3-57)^2 + (3^2+r-r)^2 = r^2$

$\Rightarrow r^2 = 2997$

$\Rightarrow r = 54.744\cdots \approx \underline{\underline{54.74}}$


Graphmatica - Q2.gr
File Edit View Options Tools Calculus Help
At point (173.0, -129.3)
3:23 PM
1/26/2018

3. Let a and b denote two positive constants. If

$$\lim_{x\to 0}\left(\frac{\int_0^x \frac{t^2}{\sqrt{a+2t^5}}dt}{bx - e\sin x}\right) = \frac{1}{\pi},$$

find the value of a. Give your answer correct to two decimal places.

$$\lim_{x\to 0} \frac{\int_0^x \frac{t^2}{\sqrt{a+2t^5}}dt}{bx - e\sin x}$$

$$= \lim_{x\to 0} \frac{\frac{x^2}{\sqrt{a+2x^5}}}{b - e\cos x}$$

(note: if $b \neq e$ then this lim $= 0 \neq \frac{1}{\pi}$ so $b = e$)

$$= \lim_{x\to 0} \frac{1}{\sqrt{a+2x^5}}\left(\frac{x^2}{e - e\cos x}\right)$$

$$= \frac{1}{\sqrt{a}} \lim_{x\to 0} \frac{x^2}{e - e\cos x}$$

$$= \frac{1}{\sqrt{a}} \lim_{x\to 0} \frac{2x}{e\sin x} = \frac{2}{e\sqrt{a}}$$

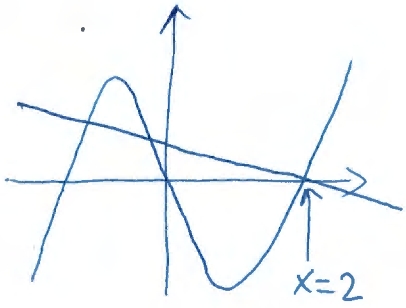
$$\therefore \frac{1}{\pi} = \frac{2}{e\sqrt{a}} \Rightarrow a = \frac{4\pi^2}{e^2}$$

$$= 5.342\ldots$$

$$\approx \underline{\underline{5.34}}$$

4. Find the total area of the finite domains bounded between the curve $y = x^3 - 4x$ and the line $x + 2y = 2$. Give your answer correct to two decimal places.

$y = x^3 - 4x$ and $y = -\frac{1}{2}x + 1$



$\Rightarrow -\frac{1}{2}x + 1 = x^3 - 4x$

$\Rightarrow 2x^3 - 7x - 2 = 0$

note that $x = 2$ is a root

$$\begin{array}{r}
2x^2 + 4x + 1 \\
x-2 \overline{) 2x^3 + 0x^2 - 7x - 2} \\
\underline{2x^3 - 4x^2 \qquad\qquad} \\
4x^2 - 7x \\
\underline{4x^2 - 8x} \\
x - 2 \\
\underline{x - 2}
\end{array}$$

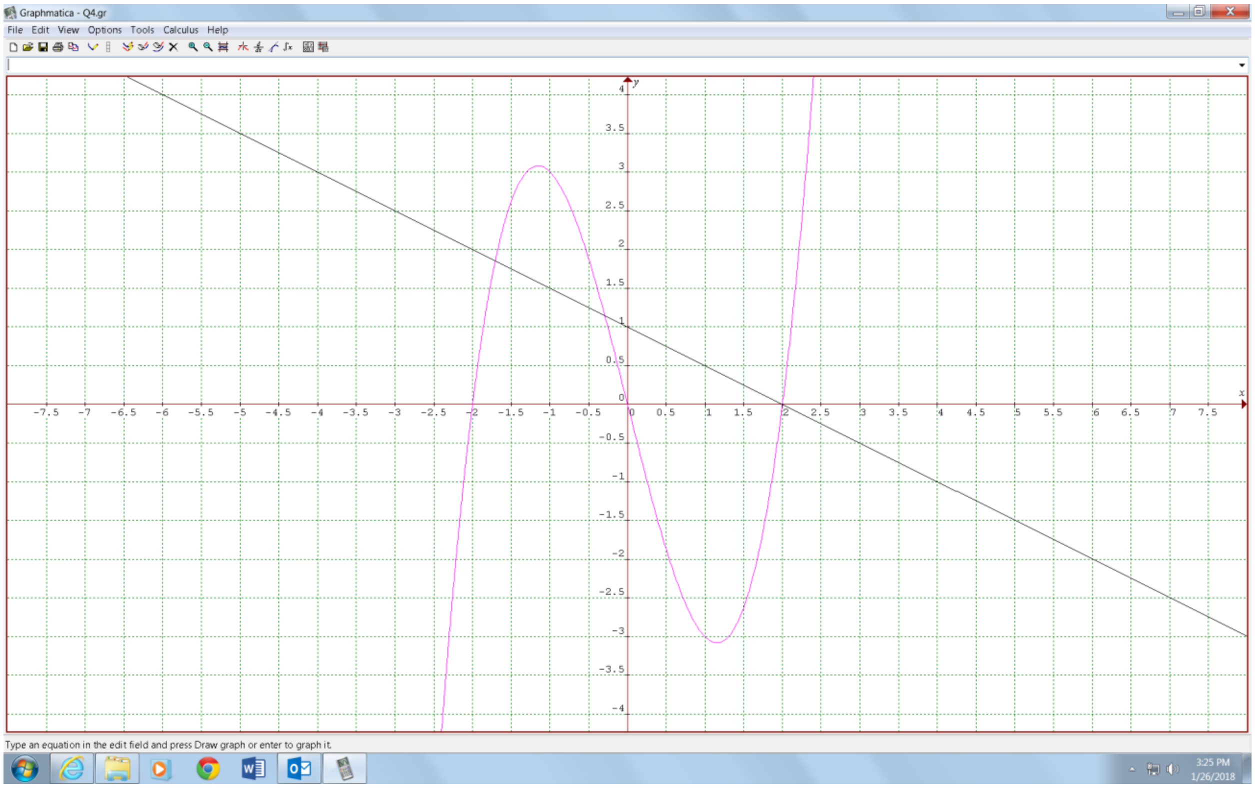
$2x^2 + 4x + 1 = 0 \Rightarrow x = \dfrac{-4 \pm \sqrt{16-8}}{4} = -1 \pm \dfrac{\sqrt{2}}{2}$

$$\text{Area} = \int_{-1-\frac{\sqrt{2}}{2}}^{-1+\frac{\sqrt{2}}{2}} \left\{(x^3 - 4x) - \left(-\tfrac{1}{2}x + 1\right)\right\} dx$$

$$+ \int_{-1+\frac{\sqrt{2}}{2}}^{2} \left\{\left(-\tfrac{1}{2}x + 1\right) - (x^3 - 4x)\right\} dx$$

$= 6.558\ldots$

$\approx \underline{\underline{6.56}}$


Graphmatica - Q4.gr
File Edit View Options Tools Calculus Help
Type an equation in the edit field and press Draw graph or enter to graph it.
3:25 PM
1/26/2018