ST2334 (2018/2019 Semester 2) Solutions to Questions in Tutorial 10

Question 1

(a) $\chi^2(4; 0.99) = 0.297$
(b) $\chi^2(19; 0.025) = 32.852$
(c) $\Pr(5.226 < \chi^2_{12} < 21.026) = 0.95 - 0.05 = 0.90$
(d) $t_{15;0.025} = 2.131$
(e) $t_{7;0.95} = -1.895$
(f) $\Pr(T_7 < 2.365) = 0.975$
(g) $\Pr(-1.356 < T_{12} < 2.179) = 1 - 0.10 - 0.025 = 0.875$
(h) $F(7, 5; 0.05) = 4.88$
(i) $F(5, 7; 0.05) = 3.97$
(j) $F(24, 18; 0.01) = 3.00$
(k) $F(18, 24; 0.95) = 1/2.15 = 0.4651$
(l) $F(12, 24; 0.99) = 1/3.78 = 0.2646$

Question 2

If $\mu = 20$, then $\Pr(\bar{X} > 24) = \Pr\left(\frac{\bar{X}-20}{4.1/\sqrt{9}} > \frac{24-20}{4.1/\sqrt{9}}\right) = \Pr(T_8 > 2.9268) < 0.01$ since $\Pr(T_8 > 2.8965) = 0.01$. Note $\Pr(T_8 > 2.9268) = 0.00955$.
We conclude that $\mu > 20$. $\Pr(\bar{X} > 24|\mu = 20)$ being small shows that it is very unlikely to get a mean of 24 if the population mean is really 20.

Question 3

(a) $\Pr(\bar{X}_B - \bar{X}_A \geq 0.2) = \Pr\left(Z > \frac{0.2}{\sqrt{1/36+1/36}}\right) = \Pr(Z > 0.85) = 0.1977$ (from the normal table). Note: $\Pr\left(Z > \frac{0.2}{\sqrt{1/36+1/36}}\right) = 0.1981$ (from Excel: "1-norm.dist(0.2/sqrt(2/36);0;1;TRUE)")
(b) Since the probability in part (a) is not small, therefore it is not unlikely to observe $\bar{X}_B - \bar{X}_A \geq 0.2$ when $\mu_A = \mu_B$. Hence, the conjecture that $\mu_A \neq \mu_B$ is likely not true.

Question 4

(a) $\Pr(S^2 > 9.1) = \Pr\left(\frac{(n-1)S^2}{\sigma^2} > \frac{(24)(9.1)}{6}\right) = \Pr(\chi^2_{24} > 36.4) \approx 0.05$ (from the χ^2-table)
(b) $\Pr(3.462 < S^2 < 10.745) = \Pr\left(\frac{(24)(3.462)}{6} < \frac{(n-1)S^2}{\sigma^2} < \frac{(24)(10.745)}{6}\right) =$
$\Pr(13.848 < \chi^2_{24} < 42.98) = 0.95 - 0.01 = 0.94$

Question 5

Since σ_1^2 and σ_2^2 are equal, therefore S_1^2/S_2^2 follows an F distribution with (7, 11) degrees of freedom. Hence $\Pr(S_1^2/S_2^2 < 4.89) = 0.99$ (From F-table). Remark: $\Pr(S_1^2/S_2^2 < 4.89) = 0.99003$ (From Excel: "=f.dist(4.89;7;11;TRUE)")

Question 6

Mine 1:	8260	8130	8350	8070	8340		$S_1^2 = 15750$
Mine 2:	7950	7890	7900	8140	7920	7840	$S_2^2 = 10920$

$$\Pr\left(\frac{S_1^2}{S_2^2} > \frac{15750}{10920}\right) = \Pr\left(\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} > \frac{15750}{10920}\right) = \Pr\left(F_{4,5} > 1.44\right) > 0.05$$

[From F-table, $\Pr\left(F_{4,5} > 5.19\right) = 0.05$. Since $1.44 < 5.19$, therefore $\Pr\left(F_{4,5} > 1.44\right) > \Pr\left(F_{4,5} > 5.19\right) = 0.05$.] Remark: $\Pr\left(F_{4,5} > 1.4423\right) = 0.3436$ [From Excel: "1-f.dist(15750/10920;4;5;TRUE)"].
Since the probability is not small, the two variances may be considered equal.

Question 7

(a) $E(U) = E(X)/n = np/n = p$. Since $E(U) = p$, therefore U is an unbiased estimator of p.

(b) $E(V) = \frac{E(X+n/2)}{3n/2} = \frac{np+n/2}{3n/2} = \frac{p+1/2}{3/2} = \frac{2p+1}{3} \neq p$ unless $p = 1$. Since $E(V) \neq p$, therefore V is a biased estimator of p.

Question 8

Y = helium porosity of a coal sample. $Y \sim N(\mu, \sigma^2)$.

(a) It is given that $\sigma = 0.75, n = 20$ and $\bar{y} = 4.85$. Hence a 95% confidence interval for μ is given by $\bar{Y} \pm z_{0.025}\frac{\sigma}{\sqrt{n}} = 4.85 \pm 1.96\frac{0.75}{\sqrt{20}} = 4.85 \pm 0.3287 = (4.5213, 5.7187)$.

(b) The length of a 95% confidence interval is $2\, z_{0.025}\frac{\sigma}{\sqrt{n}}$. Hence the length of 95% CI being 0.4 implies that $2(1.96)\frac{0.75}{\sqrt{n}} = 0.4$. Therefore $n = 54$.

(c) It is given that $S = 0.75,\ n = 20$ and $\bar{y} = 4.85$. Hence a 95% confidence interval for μ is given by $\bar{Y} \pm t_{19;\,0.025}\frac{S}{\sqrt{n}} = 4.85 \pm 2.093\frac{0.75}{\sqrt{20}} = 4.85 \pm 0.351 = (4.499, 5.201)$.

Question 9

(a) 95% confidence interval for μ is given by $\bar{X} \pm z_{0.025}\frac{\sigma}{\sqrt{n}} = 0.310 \pm (1.96)\frac{0.0015}{\sqrt{75}} = 0.310 \pm 0.00034 = (0.3097, 0.3103)$

(b) $n \geq \left(\frac{z_{0.025}\,\sigma}{e}\right)^2 = \left(\frac{1.96\ x\ 0.0015}{0.0005}\right)^2 = (5.88)^2 = 34.6$. Take $n = 35$.

Question 10

A 90% confidence interval for μ is given by $\bar{X} \pm t_{11;0.05}\frac{s}{\sqrt{n}} = 48.50 \pm (1.796)\frac{1.5}{\sqrt{12}} = 48.50 \pm 0.7777 = (47.722, 49.278)$

Question 11

A 94% confidence interval for $\mu_1 - \mu_2$ is given by $(\bar{X}_1 - \bar{X}_2) \pm z_{0.03}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = (80 - 75) \pm (1.88)\sqrt{\frac{5^2}{25} + \frac{3^2}{36}} = 5 \pm 2.102 = (2.898, 7.102)$

Question 12

98% confidence interval for $\mu_1 - \mu_2$ is given by $(\bar{X}_1 - \bar{X}_2) \pm z_{0.01}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = (12.2 - 9.1) \pm (2.33)\sqrt{\frac{1.1^2}{100} + \frac{0.9^2}{200}} = 3.1 \pm 0.296 = (2.804, 3.396)$
Since the 98% confidence interval does not cover 0 and is in the positive range, the treatment appears to reduce the mean amount of metal removed.