NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
**MA1101R Linear Algebra I**

**2018-2019 (Semester 1)** **Tutorial 1**

1. For each of the following, determine if such a linear system exists that has the following as its general solution. If such a linear system exist, find an example. If it does not, explain why.

$$\begin{cases} x &=& 5-\frac{3s}{2} \\ y &=& 2+\frac{s}{2} \\ z &=& s, \quad s\in\mathbb{R} \end{cases}$$

   (a) A linear system with 1 equation and 3 unknowns.

   (b) A linear system with 2 equations and 3 unknowns.

   (c) A linear system with 3 equations and 3 unknowns.

2. Consider the system of linear equations

$$\begin{cases} 3x_1 &+& 4x_2 &-& 5x_3 &=& -8 \\ x_1 &-& 2x_2 &+& x_3 &=& 2 \end{cases}$$

   (a) For any real number $t$, verify that $x_1=\frac{1}{5}(-4+3t), x_2=\frac{1}{5}(-7+4t), x_3=t$ is a solution to the linear system.

   (b) Write down two particular solutions to the system.

3. (a) Give a geometrical interpretation for the linear equation $x+y-z=2$.

   (b) Give a geometrical interpretatino for the linear equation $x-y=1$ in (i) the $xy$-plane; and (ii) the $xyz$-space.

   (c) Solve the linear system

$$\begin{cases} x &+& y &-& z &=& 2 \\ x &-& y && &=& 1 \end{cases}$$

   (d) Give a geometrical interpretation for the solution obtained in (c).

4. Solve the following linear systems first by using Gaussian Elimination, and then again by using Gauss-Jordan Elimination.

   (a)

$$\begin{cases} x_1 &+& x_2 &=& 7 \\ 2x_1 &+& 4x_2 &=& 18 \end{cases}$$

   (b)

$$\begin{cases} x &+& y &-& 2z &=& 1 \\ 2x &-& 3y &+& z &=& -8 \\ 3x &+& y &+& 4z &=& 7 \end{cases}$$

(c)

$$\left\{\begin{array}{rcrcrcrcl} u & + & 3v & + & x & + & 5y & = & 2 \\ 2u & + & 7v & + & 9x & + & 2y & = & 4 \\ 4u & + & 13v & + & 11x & + & 12y & = & 8 \end{array}\right.$$

5. Consider a linear system with $m$ equations and $n$ unknowns $x_1, x_2, \cdots, x_n$. Denote this linear system by (1).

   Suppose one of the equations in (1) is multiplied by a nonzero constant $k$. Denote the resulting linear system by (2). Show that

$$x_1 = c_1, x_2 = c_2, \cdots, x_n = c_n$$

   is a solution to (1) if and only if it is a solution to (2).

   Repeat the question when (2) is obtained from (1) by adding $k$ times of one equation in (1) to another equation in (1).

   **Remark:** By completing this question, we have essentially proven **Theorem 1.2.7** introduced during lecture.