

**CS4246 / CS5446**

# Tutorial Week 12

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**First**

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

Wife (W)

Husband (H)

	Ballet	Concert
Ballet		
Concert		

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

Wife (W)

Husband (H)

	Ballet	Concert
Ballet		H=0, W=0
Concert	H=0, W=0	

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- a) Argue that both the strategy profiles  $\langle \text{Ballet}, \text{Ballet} \rangle$  and  $\langle \text{Concert}, \text{Concert} \rangle$  are Nash equilibria.

Question



Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- a) Argue that both the strategy profiles  $\langle \text{Ballet}, \text{Ballet} \rangle$  and  $\langle \text{Concert}, \text{Concert} \rangle$  are Nash equilibria.

**Switch** makes the value **worse**

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- a) Argue that both the strategy profiles  $\langle \text{Ballet}, \text{Ballet} \rangle$  and  $\langle \text{Concert}, \text{Concert} \rangle$  are Nash equilibria.

**Switch** makes the value **worse**

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- (b) Find a Nash equilibrium where both players play mixed strategies.

Question

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- (b) Find a Nash equilibrium where both players play mixed strategies.

Assume H:  $p$  ballet,  $(1-p)$  concert. Find  $p$  that makes W indifference

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- (b) Find a Nash equilibrium where both players play mixed strategies.

Assume H: p ballet, (1-p) concert. Find p that makes W indifference

$$E[W=\text{ballet}] = E[W=\text{concert}]$$

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

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- (b) Find a Nash equilibrium where both players play mixed strategies.

Assume H: p ballet, (1-p) concert. Find p that makes W indifference

$$E[W=\text{ballet}] = E[W=\text{concert}]$$

$$p * U_{W=\text{ballet}, H=\text{ballet}} + (1-p) * U_{W=\text{ballet}, H=\text{concert}}$$

H=concert

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
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Assume H: p ballet, (1-p) concert. Find p that makes W indifference

$$E[W=\text{ballet}] = E[W=\text{concert}]$$

$$p * U_{W=\text{ballet}, H=\text{ballet}} + (1-p) * U_{W=\text{ballet}, H=\text{concert}} = p * U_{W=\text{concert}, H=\text{ballet}} + (1-p) * U_{W=\text{concert}, H=\text{concert}}$$

Wife (W)

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	Ballet	Concert
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$$E[W=\text{ballet}] = E[W=\text{concert}]$$

$$p * U_{W=\text{ballet}, H=\text{ballet}} + (1-p) * U_{W=\text{ballet}, H=\text{concert}} = p * U_{W=\text{concert}, H=\text{ballet}} + (1-p) * U_{W=\text{concert}, H=\text{concert}}$$

$$p * 2 + (1-p) * 0$$



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	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
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$$p * 2 + (1-p) * 0 = p * 0 + (1-p) * 1$$

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$$E[W=\text{ballet}] = E[W=\text{concert}]$$

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H=concert

$$p * 2 + (1-p) * 0 = p * 0 + (1-p) * 1$$

$$p = 1/3$$

Wife (W)

Husband (H)

	Ballet	Concert
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Assume W: q ballet, (1-q) concert. Find q that makes H indifference

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
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Assume W: q ballet, (1-q) concert. Find q that makes H indifference

$$E[H=\text{ballet}] = E[H=\text{concert}]$$

$$q * U_{H=\text{ballet}, W=\text{ballet}} + (1-q) * U_{H=\text{ballet}, W=\text{concert}} = q * U_{H=\text{concert}, W=\text{ballet}} + (1-q) * U_{H=\text{concert}, W=\text{concert}}$$

Wife (W)

Husband (H)

	Ballet	Concert
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Assume W: q ballet, (1-q) concert. Find q that makes H indifference

$$E[H=\text{ballet}] = E[H=\text{concert}]$$

$$q * U_{H=\text{ballet}, W=\text{ballet}} + (1-q) * U_{H=\text{ballet}, W=\text{concert}} = q * U_{H=\text{concert}, W=\text{ballet}} + (1-q) * U_{H=\text{concert}, W=\text{concert}}$$

$$q * 1 + (1-q) * 0$$

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

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Assume W: q ballet, (1-q) concert. Find q that makes H indifference

$$E[H=\text{ballet}] = E[H=\text{concert}]$$

$$q * U_{H=\text{ballet}, W=\text{ballet}} + (1-q) * U_{H=\text{ballet}, W=\text{concert}} = q * U_{H=\text{concert}, W=\text{ballet}} + (1-q) * U_{H=\text{concert}, W=\text{concert}}$$

W=concert

$$q * 1 + (1-q) * 0 = q * 0 + (1-q) * 2$$



Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

(b) Find a Nash equilibrium where both players play mixed strategies.

Assume W: q ballet, (1-q) concert. Find q that makes H indifference

$$E[H=\text{ballet}] = E[H=\text{concert}]$$

$$q * U_{H=\text{ballet}, W=\text{ballet}} + (1-q) * U_{H=\text{ballet}, W=\text{concert}} = q * U_{H=\text{concert}, W=\text{ballet}} + (1-q) * U_{H=\text{concert}, W=\text{concert}}$$

W=concert

$$q * 1 + (1-q) * 0 = q * 0 + (1-q) * 2$$

$$q = 2/3$$

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- c) Compute the expected utility of all three equilibria for the husband. Do the same for the wife.

Question

Wife (W)

Husband (H)

	<b>Ballet</b>	
<b>Ballet</b>	H=1, W=2	

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- c) Compute the expected utility of all three equilibria for the husband. Do the same for the wife.

Wife (W)

Husband (H)

		<b>Concert</b>
<b>Concert</b>		H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- c) Compute the expected utility of all three equilibria for the husband. Do the same for the wife.

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- c) Compute the expected utility of all three equilibria for the husband. Do the same for the wife.

H: p ballet, (1-p) concert

W: q ballet, (1-q) concert

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- c) Compute the expected utility of all three equilibria for the husband. Do the same for the wife.



Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

H: p ballet, (1-p) concert  
W: q ballet, (1-q) concert

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- c) Compute the expected utility of all three equilibria for the husband. Do the same for the wife.

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	pq H=1, W=2	p(1-q) H=0, W=0
Concert	(1-p)q H=0, W=0	(1-p)(1-q) H=2, W=1

H: p ballet, (1-p) concert

W: q ballet, (1-q) concert

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- c) Compute the expected utility of all three equilibria for the husband. Do the same for the wife.

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	pq H=1, W=2	p(1-q) H=0, W=0
Concert	(1-p)q H=0, W=0	(1-p)(1-q) H=2, W=1

H: p ballet, (1-p) concert

W: q ballet, (1-q) concert

$$E[H] = pq * 1 + p(1-q) * 0 + (1-p)q * 0 + (1-p)(1-q) * 2 = \frac{2}{3}$$



Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- c) Compute the expected utility of all three equilibria for the husband. Do the same for the wife.

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	pq H=1, W=2	p(1-q) H=0, W=0
Concert	(1-p)q H=0, W=0	(1-p)(1-q) H=2, W=1

H: p ballet, (1-p) concert

W: q ballet, (1-q) concert

$$E[H] = pq * 1 + p(1-q) * 0 + (1-p)q * 0 + (1-p)(1-q) * 2 = \frac{2}{3}$$

$$E[W] = pq * 2 + p(1-q) * 0 + (1-p)q * 0 + (1-p)(1-q) * 1 = \frac{2}{3}$$

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- c) Compute the expected utility of all three equilibria for the husband. Do the same for the wife.

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	pq H=1, W=2	p(1-q) H=0, W=0
Concert	(1-p)q H=0, W=0	(1-p)(1-q) H=2, W=1

H: p ballet, (1-p) concert

W: q ballet, (1-q) concert

$$E[H] = pq * 1 + p(1-q) * 0 + (1-p)q * 0 + (1-p)(1-q) * 2 = \frac{2}{3}$$

$$E[W] = pq * 2 + p(1-q) * 0 + (1-p)q * 0 + (1-p)(1-q) * 1 = \frac{2}{3}$$

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- (d) Compute the utility of both players going to their preferred activity, and the expected utility of for both players when they both select each activity randomly with equal probability.

Question

Wife (W)

Husband (H)

	Ballet	Concert
Ballet		
Concert	H=0, W=0	

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- (d) Compute the utility of both players going to their preferred activity, and the expected utility of for both players when they both select each activity randomly with equal probability.

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- (d) Compute the utility of both players going to their preferred activity, and the expected utility of for both players when they both select each activity randomly with equal probability.

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	pq H=1, W=2	p(1-q) H=0, W=0
Concert	(1-p)q H=0, W=0	(1-p)(1-q) H=2, W=1

Recall answer from (c)

H: p ballet, (1-p) concert

W: q ballet, (1-q) concert

$$E[H] = pq * 1 + p(1-q) * 0 + (1-p)q * 0 + (1-p)(1-q) * 2 = \frac{2}{3}$$

$$E[W] = pq * 2 + p(1-q) * 0 + (1-p)q * 0 + (1-p)(1-q) * 1 = \frac{2}{3}$$

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	H=1, W=2	H=0, W=0
Concert	H=0, W=0	H=2, W=1

Husband and wife would like to go on a date-night out and there are only two venues for entertainment that night: a Ballet and a K-pop Concert. The wife wants to see the Ballet while the husband wants to see the Concert. But both of them prefer being together than being alone. Out of love for each other, they do not explicitly tell each other their own preferences (Bad idea!). The payoff matrix is shown below where the husband is the row player and the wife is the column player. Please work out the Nash equilibria for them.

- (d) Compute the utility of both players going to their preferred activity, and the expected utility of for both players when they both select each activity randomly with equal probability.

Wife (W)

Husband (H)

	Ballet	Concert
Ballet	pq H=1, W=2	p(1-q) H=0, W=0
Concert	(1-p)q H=0, W=0	(1-p)(1-q) H=2, W=1

Random:  $p = q = \frac{1}{2}$

H: p ballet, (1-p) concert

W: q ballet, (1-q) concert

$$E[H] = pq * 1 + p(1-q) * 0 + (1-p)q * 0 + (1-p)(1-q) * 2 = \frac{3}{4}$$

$$E[W] = pq * 2 + p(1-q) * 0 + (1-p)q * 0 + (1-p)(1-q) * 1 = \frac{3}{4}$$

**Second**

Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

	$t_1$	$t_2$	$t_3$
$s_1$	1	6	0
$s_2$	2	0	3
$s_3$	3	2	4

In this question, we assume that only pure strategies are considered.



Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

	$t_1$	$t_2$	$t_3$
$s_1$	1	6	0
$s_2$	2	0	3
$s_3$	3	2	4



Payoff for the **row** player

In this question, we assume that only pure strategies are considered.

Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

Want to maximize

	$t_1$	$t_2$	$t_3$
$s_1$	1	6	0
$s_2$	2	0	3
$s_3$	3	2	4

Payoff for the row player

In this question, we assume that only pure strategies are considered.

Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

Want to maximize

	$t_1$	$t_2$	$t_3$
$s_1$	1	6	0
$s_2$	2	0	3
$s_3$	3	2	4

Want to minimize

Payoff for the row player

In this question, we assume that only pure strategies are considered.

Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

	$t_1$	$t_2$	$t_3$
$s_1$	1	6	0
$s_2$	2	0	3
$s_3$	3	2	4

In this question, we assume that only pure strategies are considered.

(a) Find the maxmin strategy and value for the row player.

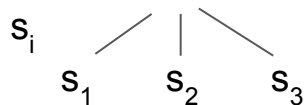
Question

Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

	$t_1$	$t_2$	$t_3$
$s_1$	1	6	0
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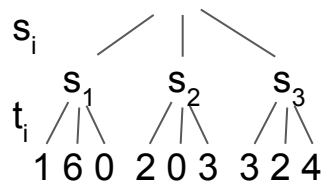


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	$t_1$	$t_2$	$t_3$
$s_1$	1	6	0
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(a) Find the maxmin strategy and value for the row player.

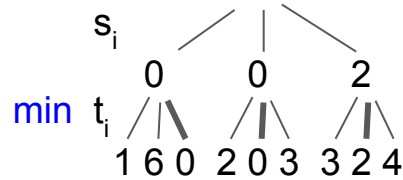


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$s_1$	1	6	0
$s_2$	2	0	3
$s_3$	3	2	4

In this question, we assume that only pure strategies are considered.

(a) Find the maxmin strategy and value for the row player.

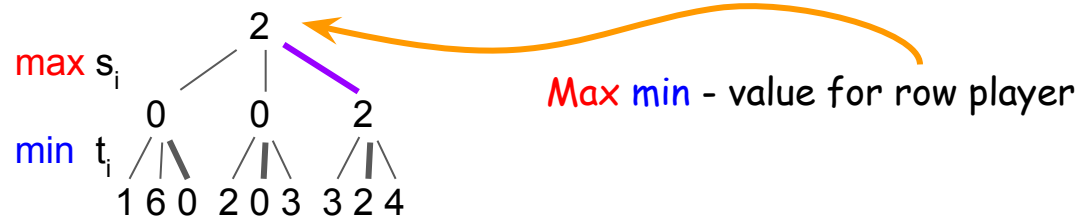


Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

	$t_1$	$t_2$	$t_3$
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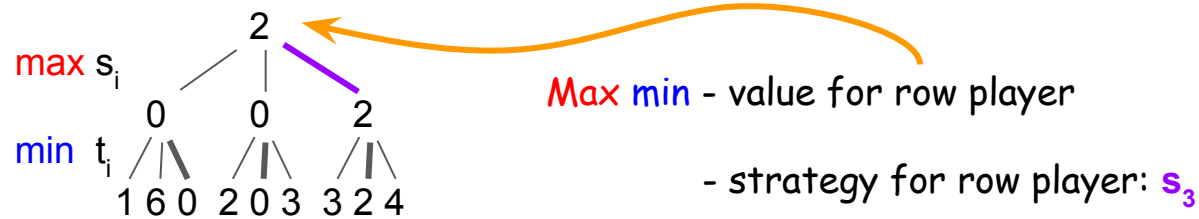


Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

	$t_1$	$t_2$	$t_3$
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$s_3$	3	2	4

In this question, we assume that only pure strategies are considered.

(a) Find the maxmin strategy and value for the row player.



Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

	$t_1$	$t_2$	$t_3$
$s_1$	1	6	0
$s_2$	2	0	3
$s_3$	3	2	4

In this question, we assume that only pure strategies are considered.

- (b) Find the minmax strategy for the column player against the row player and the minmax value for the row player.

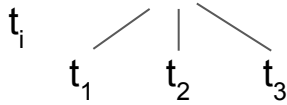
Question

Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

	$t_1$	$t_2$	$t_3$
$s_1$	1	6	0
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$s_3$	3	2	4

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- (b) Find the minmax strategy for the column player against the row player and the minmax value for the row player.

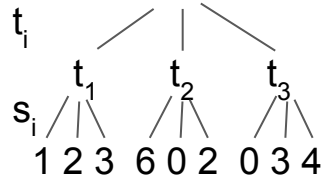


Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

	$t_1$	$t_2$	$t_3$
$s_1$	1	6	0
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$s_3$	3	2	4

In this question, we assume that only pure strategies are considered.

- (b) Find the minmax strategy for the column player against the row player and the minmax value for the row player.

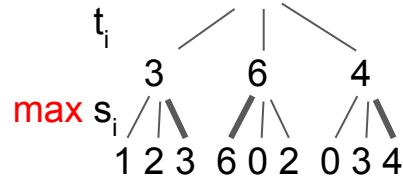


Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

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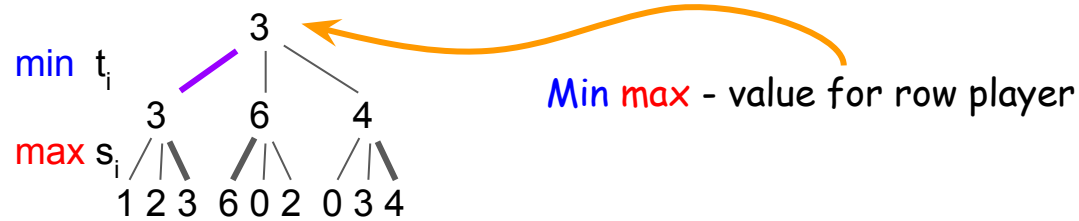


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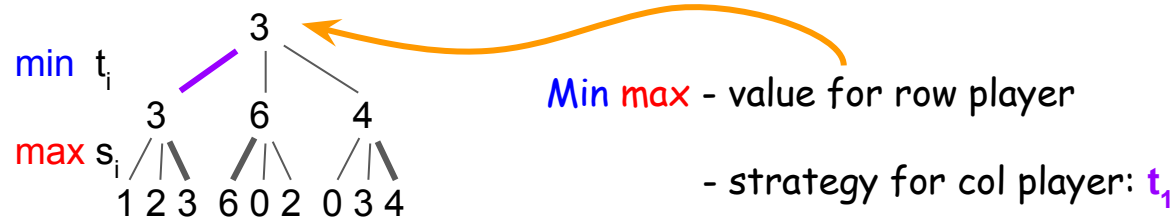


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Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

	$t_1$	$t_2$	$t_3$
$s_1$	1	6	0
$s_2$	2	0	3
$s_3$	3	2	4

In this question, we assume that only pure strategies are considered.

(c) Prove that, in general (i.e., not just for the game above),  $\max_s \min_t u(s, t) \leq \min_t \max_s u(s, t)$ .

Question



Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

	$t_1$	$t_2$	$t_3$
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$$\forall s \quad \forall t \quad \min_{t'} u(s, t') \leq u(s, t)$$

---

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	$t_1$	$t_2$	$t_3$
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$s_2$	2	0	3
$s_3$	3	2	4

$\leq$

	$t_1$	$t_2$	$t_3$
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$\leq$

	$t_1$	$t_2$	$t_3$
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$$\Rightarrow \underbrace{\forall t \max_{s'} \min_t u(s', t)}_{\text{red line}} \leq \underbrace{\max_s u(s, t)}_{\text{blue line}}$$

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$$\Rightarrow \underbrace{\forall t \max_{s'} \min_t u(s', t)}_{\text{red}} \leq \underbrace{\max_s u(s, t)}_{\text{blue}}$$

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$\leq$

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	$t_1$	$t_2$	$t_3$
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	$t_1$	$t_2$	$t_3$
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$\leq$

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Consider the following  $3 \times 3$  two-person, zero-sum matrix game:

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$$\forall s \forall t \min_{t'} u(s, t') \leq u(s, t)$$

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$\leq$

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	$t_1$	$t_2$	$t_3$
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# Any Questions?

Contact the tutors:

- Rizki: [rizki@u.nus.edu](mailto:rizki@u.nus.edu)
- Abhinit: [dcsabh@nus.edu.sg](mailto:dcsabh@nus.edu.sg)
- Evangelos: [esigalas@u.nus.edu](mailto:esigalas@u.nus.edu)