NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

EXAMINATION FOR

SEMESTER 2 AY 2013/2014

CS1231 - DISCRETE STRUCTURES

Apr/May 2014 Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE** questions and comprises **EIGHT** printed pages, including this page.
2. Answer **ALL** questions within the space in this booklet.
3. This is a Closed Book examination. Candidates are allowed to bring in an A4-sized help sheet, written on both sides.
4. Calculators are allowed.
5. Please write your Matriculation Number below.

Matriculation NO: ______________________________

Page	Marks	Remarks
2		
3		
4		
5		
6		
7		
8		
Total		

Question A [36 marks]. For each of the following, just write down the answers in the spaces provided. Detailed workings are not required. Also numerical answers are to be written as integers or powers of a single integer. For example, you can write 2300 or 3^27 but not $\binom{5}{1}\binom{3}{1}$.

(1) Find -532 **Div** 9.

(2) Find -532 **Mod** 9.

(3) Is (a)667, (b) 839 a prime number? Answer Yes or No. (a) (b)

(4) The integers x_n, $n \in \mathbb{Z}^+$, $0 \leq x_n < 7$, are defined by $x_0 = 2$ and for $n \geq 1$, $x_n \equiv 3x_{n-1} + 4 \pmod 7$. Find the value of x_4.

(5) Find the value of $\sum_{k=0}^{100} \binom{100}{k} 3^{100-k}$.

(6) *For this question express you answers in terms of factorials, such as* 2!9!/6!.
Five chemistry majors, 6 mathematics majors and 7 physics majors are to be arranged in a row. Find the number of arrangements if

(a) the chemistry majors are to occupy the first 5 positions;

(b) the chemistry majors cannot occupy the first 5 positions;

(c) students with the same major must be together in a block.

(7) Find the coefficient of a^3b^9 in the expansion of $(a - 2b)^{12}$.

(8) How many bit strings of length 8 are there where the '1' bits are not next to each other?

(9) Consider positive integers ≤ 1000.

(a) How many are divisible by either 9 or 15?

(b) How many are divisible by 8 but not by 15?

(10) Suppose in an ordered rooted tree the universal address of a vertex v is 4.3.2.5.1.6. Find (a) the level of v;

(b) the address of the parent of v;

(c) the minimum number of vertices that the tree can have.

(11) Mr Brown wants to paint a wall in his living room. The wall is made up of 4 rectangular panels arranged in the form of a 2×2 grid. He wants to paint each panel with one colour so that adjacent panels are painted with different colours. (Two panels are adjacent if the share a common side.) If he has 10 colours, how many choices does he have?

(12) Let G be a graph with 14 vertices and 30 edges in which every vertex is of degree 4 or 5. How many vertices are there of degree 5?

(13) Consider the 7-cube Q_7.

(a) How many vertices are there?

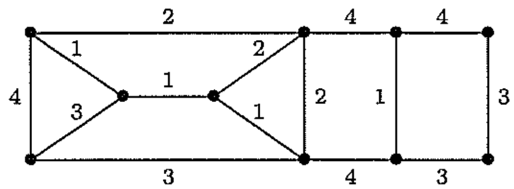
(b) How many edges are there?

(c) Which of the following is adjacent to 0101010? *(Just write (i), (ii), etc, in the box.)*

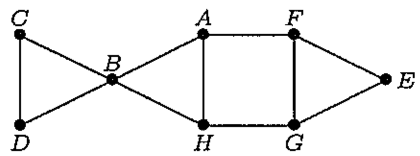
(i) 0101011, (ii) 0101111, (iii) 0111110, (iv) 1101010.

(c) What is the length of a shortest path from 0101010 to 1010101?

(14) Find the weight of a minimum spanning tree in the following graph.



(15)



Let G be the graph above. Using the alphabetical ordering, find a spanning tree by depth first search and by breadth first search. Draw the trees below.

Depth First Search **Breadth First Search**

Question B [6 marks]. (a) Find $\gcd(555, 252)$.

(b) Find s with $1 \le s \le 184$ so that $84s \equiv 1 \pmod{185}$.

(c) Find 5^{1317} Mod 97.

Question C [6 marks]. Use mathematical induction to prove that for $n \in \mathbb{Z}^+$,

$$1^3 + 3^3 + \cdots + (2n+1)^3 = (n+1)^2(2n^2 + 4n + 1).$$

Question D [6 marks]. Does there exist a full m-ary tree with 188 leaves and height 5 where m is some positive integer? If the answer is no, give your reasons. If the answer is yes, give a value of m and describe how the tree can be constructed. (*Hint: The formula* $i + \ell = mi + 1$ *is useful.*)

Question E [6 marks]. You are to colour four squares, with 2 red and 2 blue, in a 5×5 grid so that squares of the same colour do not lie in the same row or same column. How many ways are there to do it?

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