**CS1231 TUTORIAL 10**

**1.** Let $A_i$ be multiples of $i$ that are between 1 and 2019 inclusive. Now

$$\begin{aligned}|A_6 \cup A_7 \cup A_9| &= |A_6| + |A_7| + |A_9| - |A_{42}| - |A_{18}| - |A_{63}| + |A_{126}| \\ &= 336 + 288 + 224 - 48 - 112 - 32 + 16 = 672.\end{aligned}$$

Note that multiples of 12 in $A_6 \cup A_7 \cup A_9$ are just multiples of 12 between 1 and 2019 inclusive. Thus the answer is $672 - \lfloor 2019/12 \rfloor = 672 - 168 = 504$.

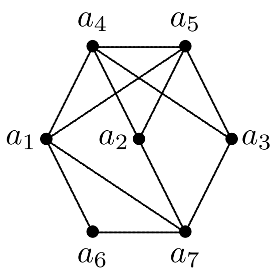
**2.** Sample Space: all outcomes of the six time roll. Event: An even number never comes up. The number of outcomes of the six time roll is $6^6$. The number of outcome in the event is $3^6$. Therefore, the answer is $3^6/6^6 = 1/2^6 = 1/64$.

**3.** $\binom{12}{5}(-2)^7$

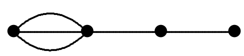
**4.** $a$ can come from any 5 of the factors $(a+b+c)$. $b$ comes from any 2 of the remanding 10 factors and $c$ from the remaining 8. Thus the ans. is $\binom{15}{5}\binom{10}{2}\binom{8}{8}$

**5.** $(1+4)^n = 5^n$,
$\sum_{k=0}^{n}(-1)^k\binom{n}{k}3^{2n-2k}2^{2k} = \sum_{k=0}^{n}\binom{n}{k}(3^2)^{n-k}(-2^2)^k = (3^2-2^2)^n = 5^n$.

**6.** Let $G$ be the graph whose vertices are the 7 students and two vertices are adjacent iff they are not one of the pairs. Any group of students must correspond to subgraph that is complete. The largest complete subgraph is a $K_3$. Thus the largest possible group size is 3 and so we need at least 3 groups. This is indeed possible. For example: $\{a_2, a_4, a_5\}, \{a_1, a_6\}, \{a_3, a_7\}$.




**7.** (i) $adfbgcdea$ (ii) 7: $adfbgcea$ (iii) $f, g, d, e$ (iv) $cdce$, $cece$.

**8.** (i) No. Degree sum must be even. (ii) Yes. 

**9.** Let $a$ and $b$ be the number of vertices of degrees 3 and 5, respectively. Then

$$9 \times 2 + 3a + 5b + 10 \times 6 = 100.$$

Therefore $3a + 5b = 22$. The only solution is $a = 4$ and $b = 2$. Thus the answer is 25.