CS1231 Review 17

1. **Permutation and Combination (without repeat)**

   - A **permutation** of a set of distinct objects is an ordering of the objects.

     The number of permutation of n distinct objects is $n!$ More generally, an r-**permutation** of a set of n distinct objects is an ordering of r elements from the set.

   - The number of r-permutation of a set of n distinct objects is denoted $P(n, r)$. It is equal to $\frac{n!}{(n-r)!}$.

   - Let n, r be integers with $0 \leq r \leq n$. An r-**combination** of a set of n (distinct) objects is a subset of r objects.

   - The number of r-combination of a set of n distinct objects is denoted $\binom{n}{r}$. It is equal to $\frac{n!}{r!(n-r)!}$

2. **Product Rule.** How many positive integers with 3-digits are there such that no digit is repeating?

   $\underset{\neq 0}{\_}\ \_\ \_$ $\quad 9 \times 9 \times 8 = 81 \times 8 = 648$

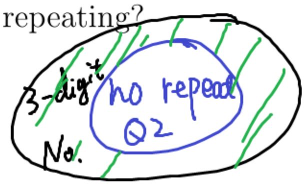
3. **Sum Rule.** How many n-letter passwords are there when $1 \leq n \leq 2$?

   Case 1 $n = 1$. 26

   Case 2 $n = 2$ _ _ 26^2

   Ans. $26 + 26^2$

4. **Difference Rule.** How many positive integers with 3-digits are there such that some digit is repeating?



   $100 \sim 999$

   $\# 900$

   $900 - 648 = 252$

5. **Inclusive/Exclusive Rule.** How many integers from 1 to 100 inclusive are multiples of 2 or 3?

   $A_2 \cup A_3$

   A_2 multiple of 2

   A_3 multiple of 3

   $|A_2 \cup A_3| = |A_2| + |A_3| - |\underbrace{A_2 \cap A_3}_{\text{multiple of } 6}| = \lfloor 100/2 \rfloor + \lfloor 100/3 \rfloor - \lfloor 100/6 \rfloor$

   $= 50 + 33 - 16$

   $= 67$

6. **The Binomial Theorem.** For any positive integer n,

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^{n-i} y^i = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$$

Let $y=1$: $\underbrace{(x+1)^n}_{f(x)} = \sum_{i=0}^{n} \binom{n}{i} x^i 1^{n-i} = \sum_{i=0}^{n} \binom{n}{i} x^i$

$x=y=1$ 7. $\sum_{k=0}^{n} \binom{n}{k} = (1+1)^n = 2^n$

$x=1$, $y=-1$ 8. $\sum_{k=0}^{n} (-1)^k \binom{n}{k} = (1-1)^n = 0$

$x=1$, $y=2$ 9. $\sum_{k=0}^{n} \left(2^k \binom{n}{k} 1^{n-k}\right) = (1+2)^n = 3^n$

$f'(x) = n(x+1)^{n-1} = \sum_{i=1}^{n} i \binom{n}{i} x^{i-1}$

Let $x=1$: $f'(1) = n 2^{n-1} = \sum_{i=1}^{n} i \binom{n}{i}$

10. **Pascal's Identity** $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$

11. What is the coefficient of $a^3b^4c^5$ in the expansion of $(a+2b+3c)^{12}$?

$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$

$(a+2b+3c)^{12} = \sum_{i=0}^{12} \sum_{j=0}^{12-i} \binom{12}{i} \binom{12-i}{j} a^i (2b)^j (3c)^{12-i-j}$

12-many $a+2b+3c$:
- i-many give you a
- j-many give you $2b$ ← left with $12-i$
- $(12-i-j)$-many give you $3c$

$i=3$, $j=4$: $\binom{12}{3}\binom{12-3}{4} a^3 (2b)^4 (3c)^{12-3-4}$

$= \binom{12}{3}\binom{9}{4} a^3 2^4 b^4 3^5 c^5$

$= \underbrace{\binom{12}{3}\binom{9}{4} 2^4 3^5}_{\text{Coeff}} a^3 b^4 c^5$