ST1131 Introduction to Statistics **Tutorial 8**

9.36 Too little or too much wine? Wine-pouring vending machines, previously available in Europe and international airports, have become popular in the last few years in the United States. They are even approved to dispense wine in some Walmart stores. The available pouring options are a 5-ounce glass, a 2.5-ounce half-glass, and a 1-ounce taste. When the machine is in statistical control (see Exercise 7.42), the amount dispensed for a full glass is 5.1 ounces. Four observations are taken each day, to plot a daily mean over time on a control chart to check for irregularities. The most recent day's observations were 5.05, 5.15, 4.95, and 5.11. Could the difference between the sample mean and the target value be due to random variation, or can you conclude that the true mean is now different from 5.1? Answer by showing the five steps of a significance test, making a decision using a 0.05 significance level.

9.24 **How to sell a burger** A fast-food chain wants to compare two ways of promoting a new burger (a turkey burger). One way uses a coupon available in the store. The other way uses a poster display outside the store. Before the promotion, their marketing research group matches 50 pairs of stores. Each pair has two stores with similar sales

volume and customer demographics. The store in a pair that uses coupons is randomly chosen, and after a monthlong promotion, the increases in sales of the turkey burger are compared for the two stores. The increase was higher for 28 stores using coupons and higher for 22 stores using the poster. Is this strong evidence to support the coupon approach, or could this outcome be explained by chance? Answer by performing all five steps of a two-sided significance test about the population proportion of times the sales would be higher with the coupon promotion.

Selling a burger In Exercise 9.24, a fast-food chain 9.37 compared two ways of promoting a turkey burger. In a separate experiment with 10 pairs of stores, the difference in the month's increased sales between the store that used coupons and the store with the outside poster had a mean of \$3000. Does this indicate a true difference between mean sales for the two advertising approaches? Answer by using the output shown to test that the population mean difference is 0, carrying out the five steps of a significance test. Make a decision using a 0.05 significance level.

Test of mu = 0 vs. mu not = 0 Variable Ν Mean St.Dev SE Mean Sales 10 1264.91 3000 4000 95.0% CI Ρ Variable (138.8, 5861.2) 2.372 Sales 0.04177

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Define the null hypothesis of no effect (the patient does not have the disease).

- 9.50 Detecting prostate cancer Refer to the previous exercise about medical diagnoses. A New York Times article (February 17, 1999) about the PSA blood test for detecting prostate cancer stated: "The test fails to detect prostate cancer in 1 in 4 men who have the disease."
 - a. For the PSA test, explain what a Type I error is, and explain the consequence to a man of this type of error.
 - **b.** For the PSA test, what is a Type II error? What is the consequence to a man of this type of error? What is the sample of 5 consequence to a man of this type of error? What is the sample of 5 consequence to a man of this type of error? What is the sample of 5 consequence to a man of this type of error?
 - c. To which type of error does the probability of 1 in 4 refer? type II error
 - d. The article also stated that if you receive a positive result, the probability that you do not actually have prostate cancer is 2/3. Explain the difference between this and the conditional probability of a Type I error, given that you do not actually have prostate cancer.

P(someone receive positive res but no cancer)

9.60 Gender bias in selecting managers Exercise 9.19 tested the claim that female employees were passed over for management training in favor of their male colleagues. Statewide, the large pool of more than 1000 eligible employees who can be tapped for management training is 40% female and 60% male. Let p be the probability of selecting a female for any given selection. For testing H₀: p = 0.40 against H_a: p < 0.40 based on a random sample of 50 selections, using the 0.05 significance level,</p>

a. A Type II error occurs if the sample proportion falls less than 1.645 standard errors below the null hypothesis value, which means that $\hat{p} > 0.286$.

p hat -0.4 / (sqrt. 4x.6/50) > 1.645

b. When p = 0.20, a Type II error has probability 0.06.

P(Z > .286 - .02 / .0566)

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9.52 Misleading summaries? Two researchers conduct separate studies to test H_0 : p = 0.50 against H_a : $p \neq 0.50$, each with n = 400.

- **a.** Researcher A gets 220 observations in the category of interest, and $\hat{p} = 220/400 = 0.550$ and test statistic z = 2.00. Show that the P-value = 0.046 for Researcher A's analysis.
- **b.** Researcher B gets 219 in the category of interest, and $\hat{p} = 219/400 = 0.5475$ and test statistic z = 1.90. Show that the P-value = 0.057 for Researcher B's analysis.
- c. Using α = 0.05, indicate in each case from part a and part b whether the result is "statistically significant." Interpret.
- d. From part a, part b, and part c, explain why important information is lost by reporting the result of a test as "P-value ≤ 0.05" versus "P-value > 0.05," or as "reject H₀" versus "do not reject H₀," instead of reporting the actual P-value.
- e. Show that the 95% confidence interval for p is (0.501, 0.599) for Researcher A and (0.499, 0.596) for Researcher B. Explain how this method shows that, in practical terms, the two studies had very similar results.

- 9.54 Effect of n Example 11 analyzed political conservatism and liberalism in the United States. Suppose that the sample mean of 4.11 and sample standard deviation of 1.43 were from a sample size of only 25, rather than 1933.
 - a. Find the test statistic.
 - **b.** Find the P-value for testing H_0 : $\mu = 4.0$ against H_a : $\mu \neq 4.0$. Interpret.
 - **c.** Show that a 95% confidence interval for μ is (3.5, 4.7).
 - d. Together with the results using n = 1933, explain what this illustrates about the effect of sample size on (i) the size of the P-value (for a given mean and standard deviation) and (ii) the width of the confidence interval.