CS1231 TUTORIAL 9

1. (a) Pick Thousands, Hundreds, Tens, Units digits in that order: $9 \times 9 \times 8 \times 7 = 4536$

(b) Pick units, thousands. hundred, tens digits in that order: $5 \times 8 \times 8 \times 7 = 2240$

(c) Pick the thousands, units, hundreds and tens digit in that order. There are two cases: 1st digit is 5, 7 or 9 and 1st digit is 6 or 8: Ans $3 \times 4 \times 8 \times 7 + 2 \times 5 \times 8 \times 7 = 1232$.

Alt: Pick the units, thousands, hundreds and tens digits in that order. There are 2 cases: (i) unit digit is 1, 3 and (ii) unit digit is 5, 7, 9. Thus ans: $2 \times 5 \times 8 \times 7 + 3 \times 4 \times 8 \times 7$.

2. First arrange the women (10! ways) and then insert the men ($(\binom{11}{6}6!$ ways). Answer: $10! \times \times 6! \times \binom{11}{6}$.

3. By the multiplication rule, the number of strings with no adjacent letters the same is $4 \times 3 \times \cdots \times 3 = 4 \times 3^{n-1}$. Thus the ans is $4^n - 4 \times 3^{n-1}$.

4. The answer is not affected by the inclusion of 0 in the universe. Let the universe be integers from 0 through 999999. $|A_i| = 9^6$, $|A_i \cap A_j| = 8^6$ and $|A_1 \cap A_2 \cap A_3| = 7^6$.

Thus $|A_1 \cup A_2 \cup A_3| = 3 \times 9^6 - 3 \times 8^6 + 7^6$. So the answer is $10^6 - (3 \times 9^6 - 3 \times 8^6 + 7^6) = 74460$.

5. (a) Either both are odd or both are even: $\binom{50}{2} + \binom{50}{2} = 2450$ (b) one odd and one even: $\binom{50}{1} \times \binom{50}{1} = 2500$. Alt: $\binom{100}{2} - \left(\binom{50}{2} + \binom{50}{2}\right) = 2500$.

6. (a) Case 1: 3 even. First method: (i) 3 identical numbers: $\binom{50}{1}$; (ii) 2 identical numbers: first choose 2 numbers say a, b and there are 2 ways to form triples a, a, b or a, b, b. So the number of ways is $2\binom{50}{2}$. (iii) 3 distinct numbers: $\binom{50}{3}$. so the answer is $\binom{50}{1} + 2\binom{50}{2} + \binom{50}{3} = 22100$.

Second method: This is equivalent to choosing 3 numbers, with repetitions allowed, from 50 even numbers. The answer is $\binom{50+2}{3}$.

Case 2: 2 odd 1 even. $\binom{50}{1}\left(\binom{50}{1} + \binom{50}{2}\right) = 63750$. Thus $ans = 22100 + 63750 = 85850$

(b) 3 odd or 1 odd 2 even. Same answer.

7. One of the elements, say x, in Y has 2 preimages while the rest have 1 preimage each. There are $\binom{5}{2} = 10$ ways to choose the two images for x and 4 ways to choose x. Thereafter, there are 3! ways to assign the preiamges for the other elements of Y. Thus the answer is $10 \times 4 \times 6 = 240$.

8. Represent each number chosen by 1 and not chosen by 0. Then each choice is represented by a bit string of length 100 with exactly five 1 bits that are not consecutive. Such a bit string can be formed by placing 95 0 bits in a row and inserting the 1 bits into the 96 spaces created by the 0s. Thus the required ans is $\binom{96}{5}$.

9. A: multiples of 2, B: multiples of 9. (a) $|A \cup B| = |A| + |B| - |A \cap B| = 500 + 111 - 55 = 556$.

(b) $|\overline{(A \cup B)}| = 1000 - 556 = 444$.