Name: **Matric No:**

Tutorial Group: **Seat Number:**

1. [1 marks] Let $S = \{\emptyset, \{a\}, \{\emptyset, \{a\}\}, P(\{a\})\}$. Find $|P(S)|$.

2. [2 marks] Suppose the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{3, 4, 5\}$ and $B = \{2, 3, 4, 7, 8, 9, 10\}$.

(i) Find the bit string representation of $(B - A)$

(ii) Find the set represented by the bit-string 0101111000 (i.e. write all elements in the set)

3. [2 marks] Let A and B be sets. Prove $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$. (Venn Diagram and Membership table cannot be used.)

4. [3 marks] If A, B, C, D are sets, are the following **always true**?

(i) $A \cap (A \cup B) = A$ Y / N

(ii) $(A - C) \cup (C - B) = A - B$ Y / N

(iii) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$ Y / N

5. [2 marks] Determine whether f is a function from $\mathbb{R}$ to $\mathbb{R}$.

(a) $f(x) = \sqrt{\frac{1}{1+x^2}}$ Y / N

(b) $f(x) = \frac{1}{(1-x^2)^2}$ Y / N

6. [3 marks] Let S be the set of all bit strings. Find the domain and range of these functions. (No justification needed.)

(a) The function that assigns to each bit string the number of ones in the string minus the number of zeros in the string.

   Ans: Domain: Range:

(b) The function that assigns to each positive integer the largest integer not exceeding the positive square root of the integer.

   Ans: Domain: Range:

7. [3 marks] Let $f(x) = 3x - 2$ be a function from $\mathbb{R}$ to $\mathbb{R}$. Prove that f is a **bijection** and find the **inverse function** of f.

8. [2 marks] A, B and C are sets. $g : A \to B$, and $f : B \to C$ are functions. Suppose f and $f \circ g$ are onto. Does it follow g is onto? Justify your answer.

Answer:
Y / N

Justification:

9. [2 marks] Prove if x is a **positive** real number, then

$$\lfloor \sqrt{\lfloor x \rfloor} \rfloor = \lfloor \sqrt{x} \rfloor$$

(Hint: Let $\lfloor \sqrt{x} \rfloor = n$.)