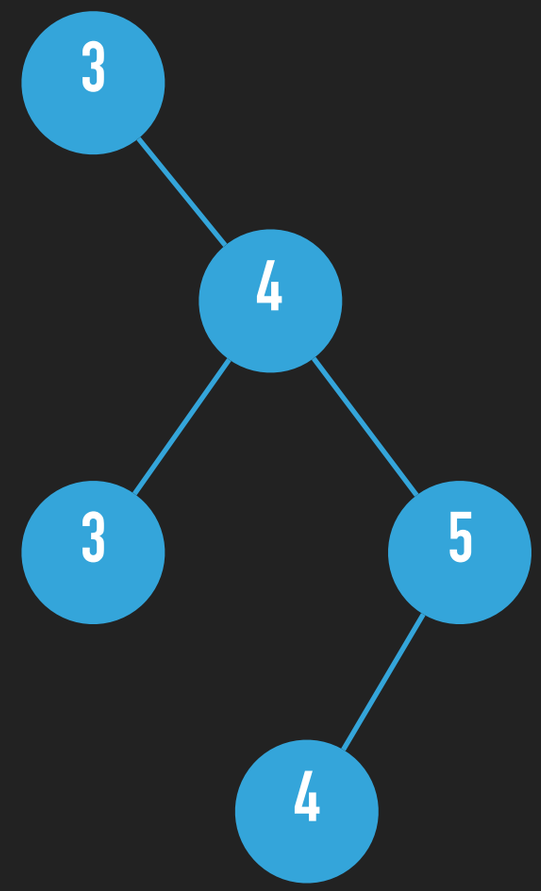
DG WEEK 6

TREE INSERTION

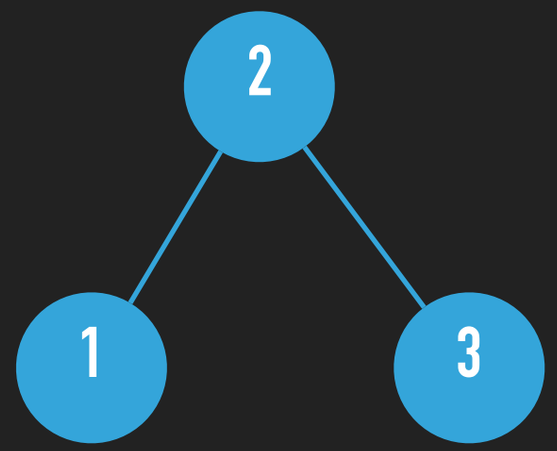
PROBLEM

- Given a Binary Search Tree.
  - Find the number of insertion sequences that result in this tree.
- Example:
  - 3, 4, 3, 5, 4
  - 3, 4, 5, 4, 3
  - 3, 4, 5, 3, 4



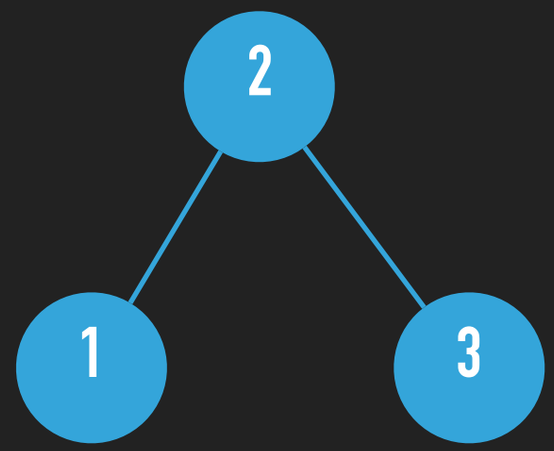
A SIMPLE EXAMPLE

- **Question**: How many sequences can result in this BST?



A SIMPLE EXAMPLE

- **Question**: How many sequences can result in this BST?
  - 2, 1, 3
  - 2, 3, 1



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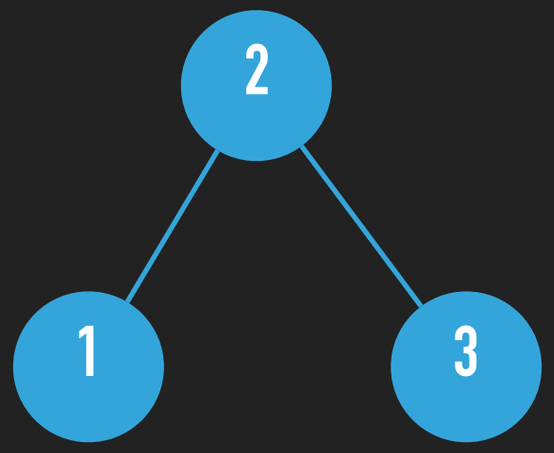

  - 2, 1, 3
  - 2, 3, 1

- *Notice something?*
  -

A SIMPLE EXAMPLE

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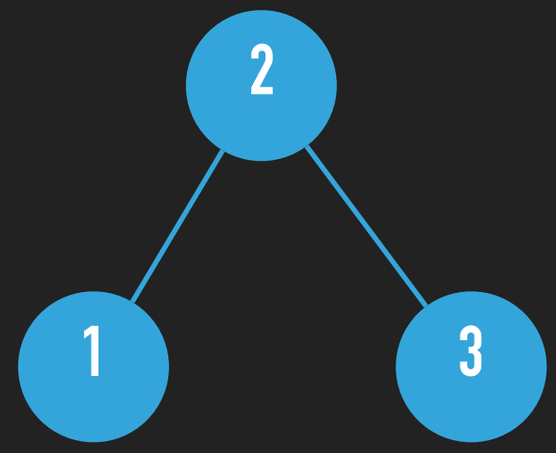
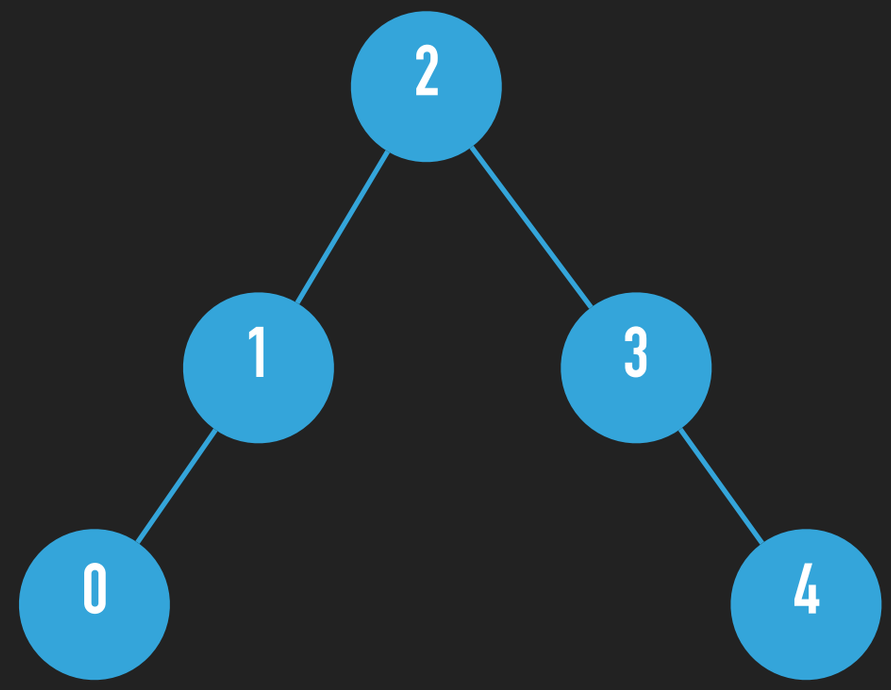


- *Notice something?*
  - The position of root element is fixed.

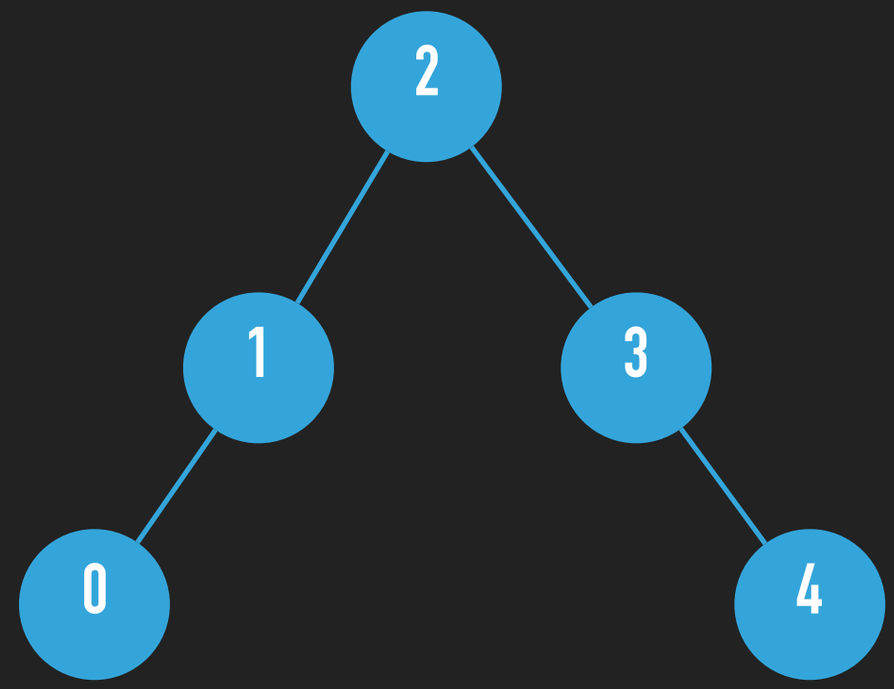
ANOTHER EXAMPLE

- **Question**: How many sequences can result in this BST?
  - 2, …
  - 2, …
  - …



ANOTHER EXAMPLE

- **Question**: How many sequences can result in this BST?
  - 2, 1, 0, 3, 4
  - 2, 1, 3, 4, 0
  - 2, 1, 3, 0, 4
  - 2, 3, 4, 1, 0
  - 2, 3, 1, 0, 4
  - 2, 3, 1, 4, 0



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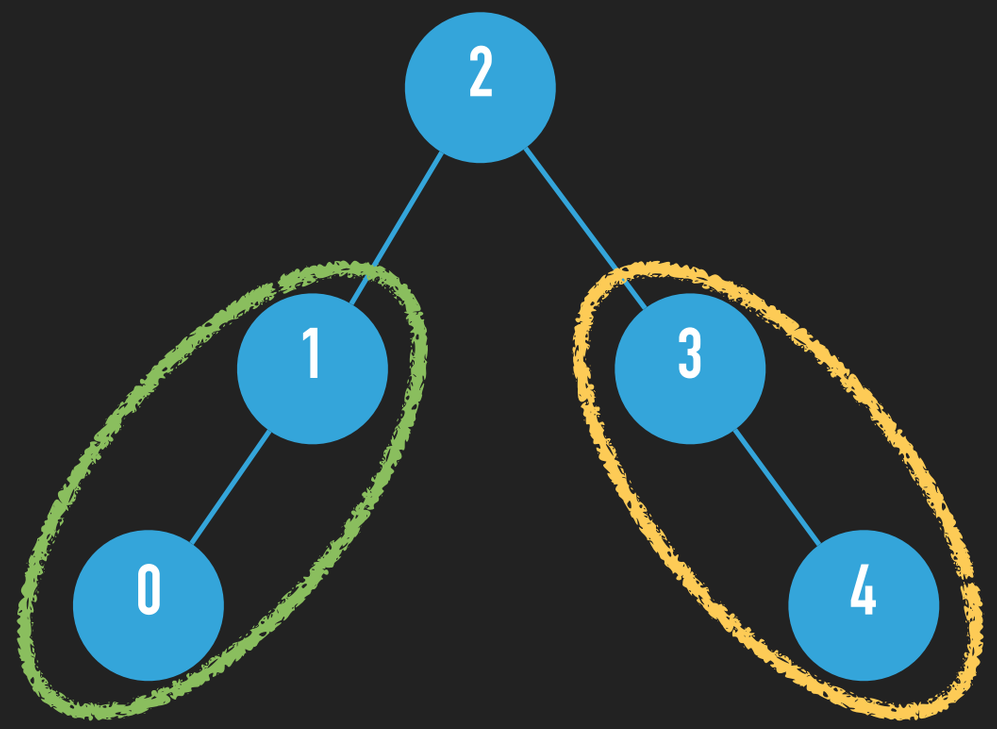

How many ways to arrange these subtrees?

- 2, 1, 0, 3, 4
- 2, 1, 3, 4, 0
- 2, 1, 3, 0, 4
- 2, 3, 4, 1, 0
- 2, 3, 1, 0, 4
- 2, 3, 1, 4, 0

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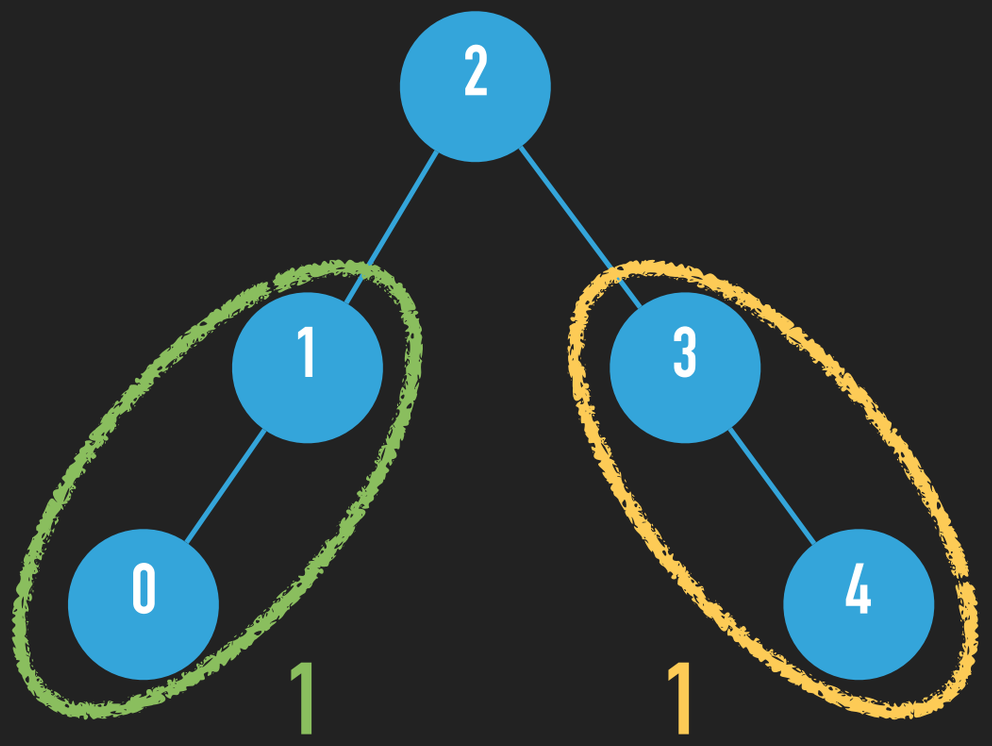
- **Question**: How many sequences can result in this BST?
  - 2, 1, 0, 3, 4
  - 2, 1, 3, 4, 0
  - 2, 1, 3, 0, 4
  - 2, 3, 4, 1, 0
  - 2, 3, 1, 0, 4
  - 2, 3, 1, 4, 0



How many ways to arrange these subtrees?

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  - 2, 1, 0, 3, 4
  - 2, 1, 3, 4, 0
  - 2, 1, 3, 0, 4
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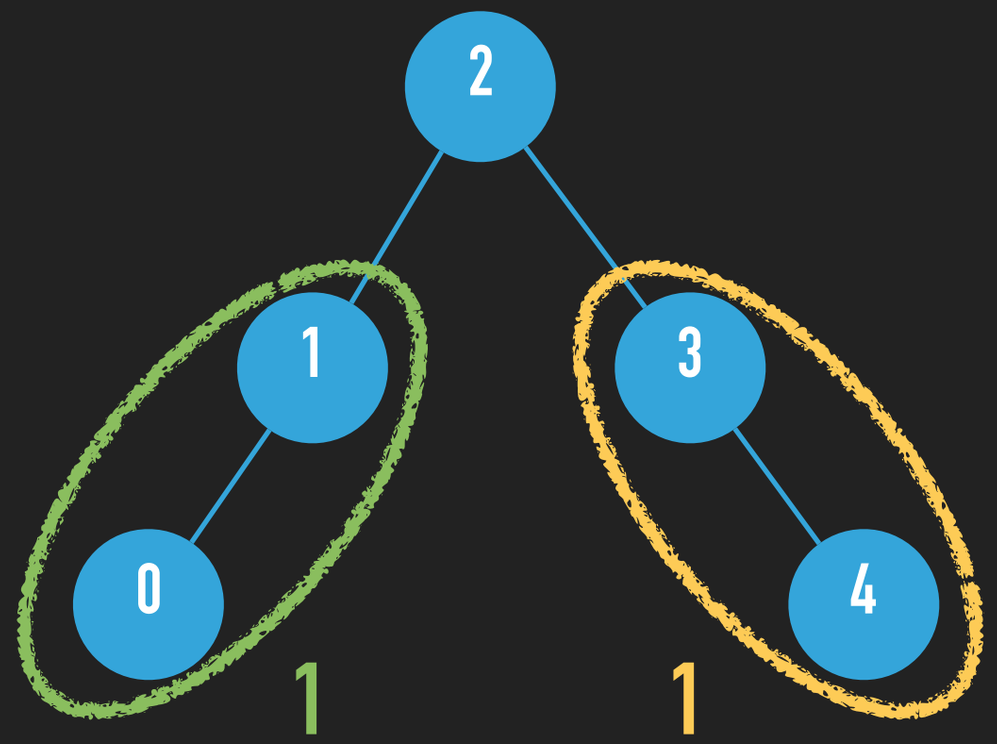
  - 2, 3, 1, 0, 4
  - 2, 3, 1, 4, 0



With 2 fixed in the front, how many ways to interleave the two subsequences?

ANOTHER EXAMPLE

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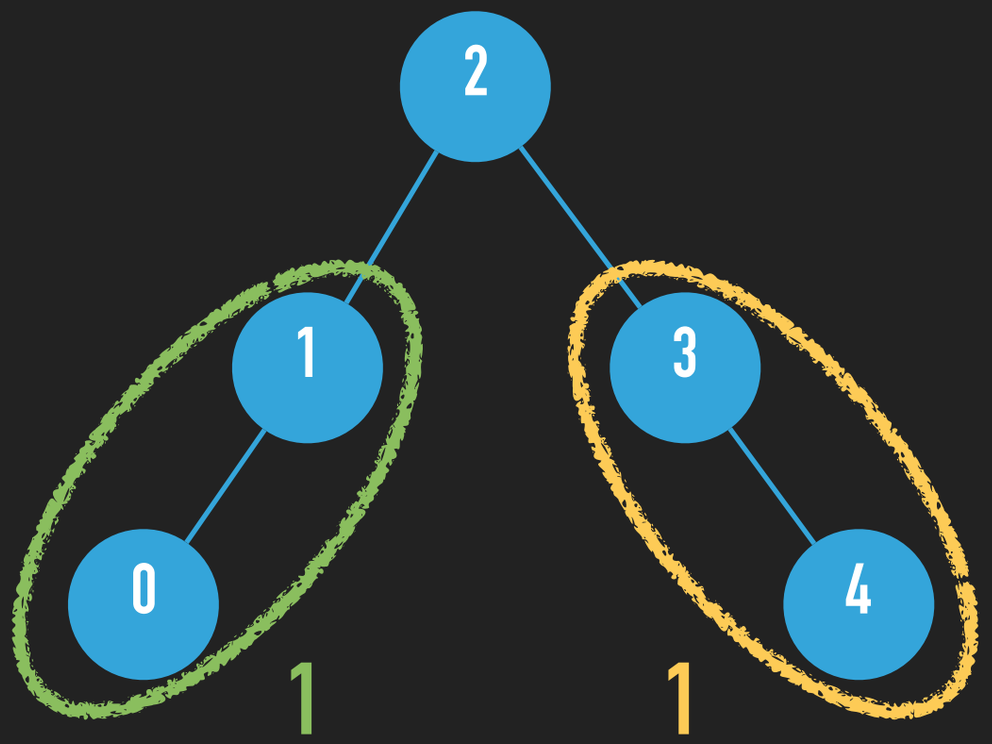
  - 2, 1, 3, 4, 0
  - 2, 1, 3, 0, 4
  - 2, 3, 4, 1, 0
  - 2, 3, 1, 0, 4
  - 2, 3, 1, 4, 0



With 2 fixed in the front, how many ways to interleave the two subsequences?

m = 2 and n = 2

ANOTHER EXAMPLE

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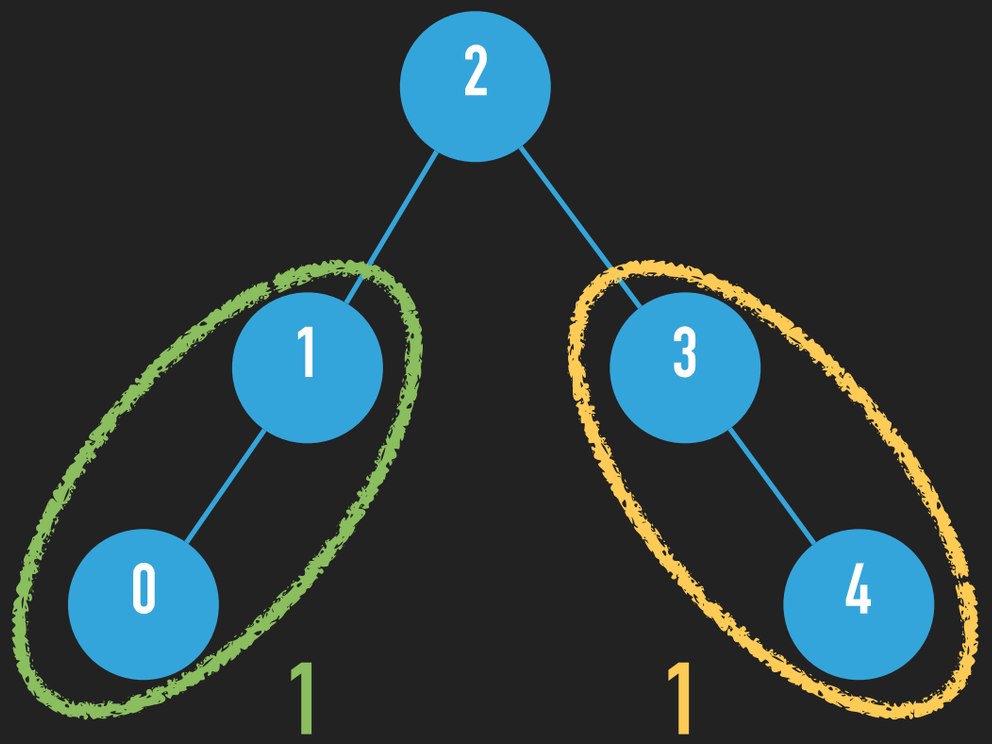
  - 2, 3, 1, 0, 4
  - 2, 3, 1, 4, 0



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m = 2 and n = 2 $\quad {}^{(m+n)}C_n$

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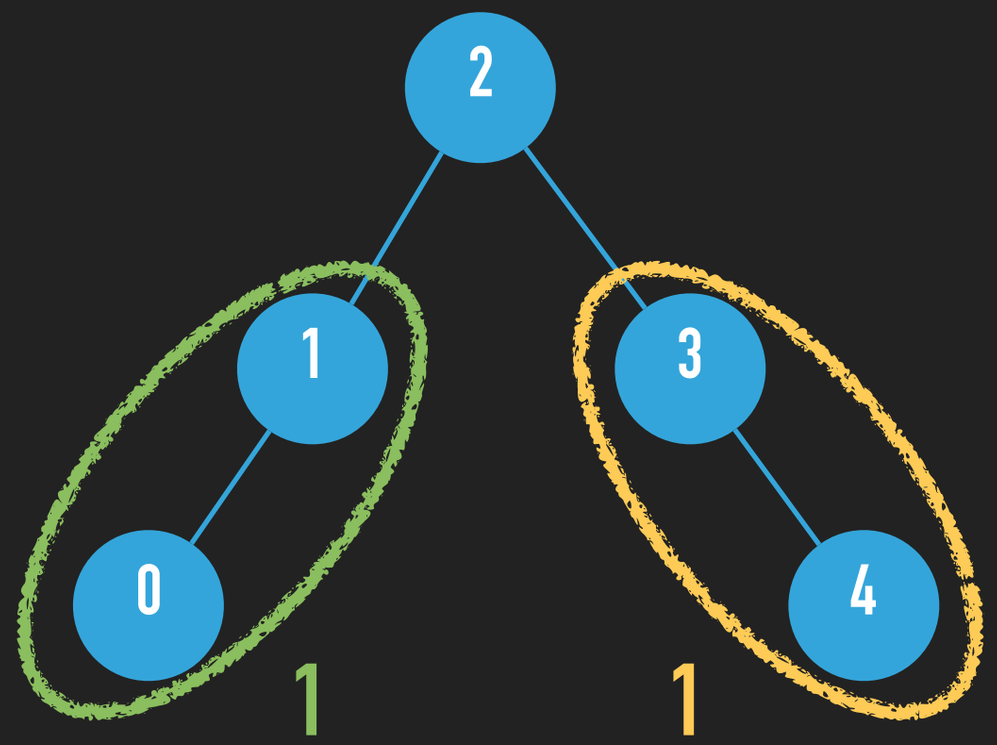

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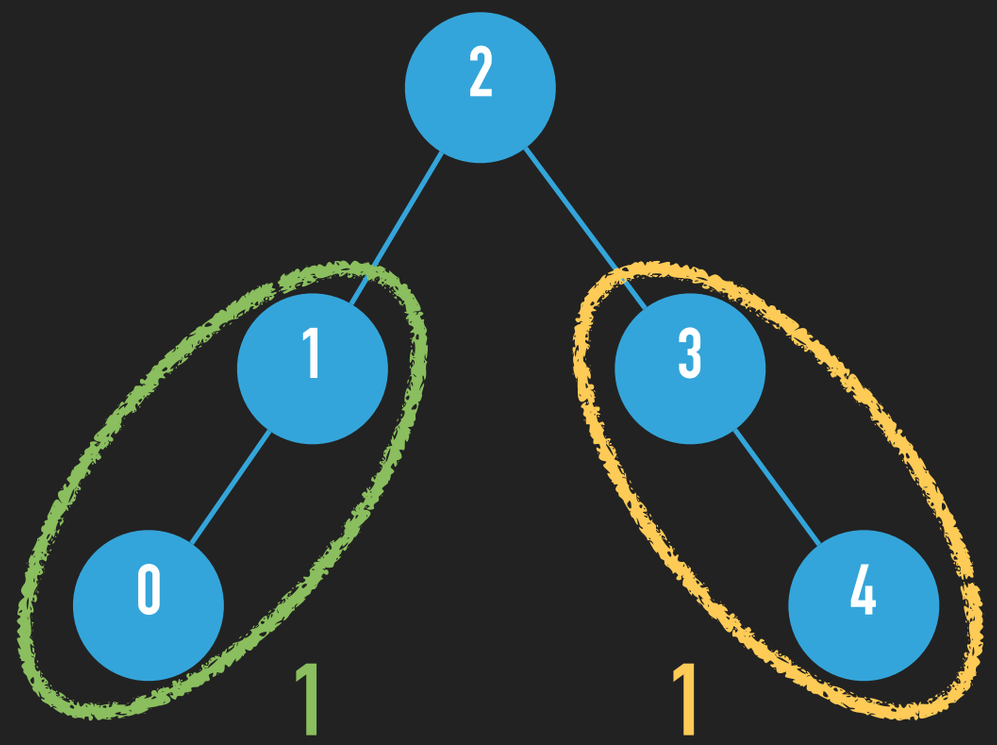
  - 2, 3, 4, 1, 0
  - 2, 3, 1, 0, 4
  - 2, 3, 1, 4, 0
  - **Answer**: 1 x 1 x 6 = 6



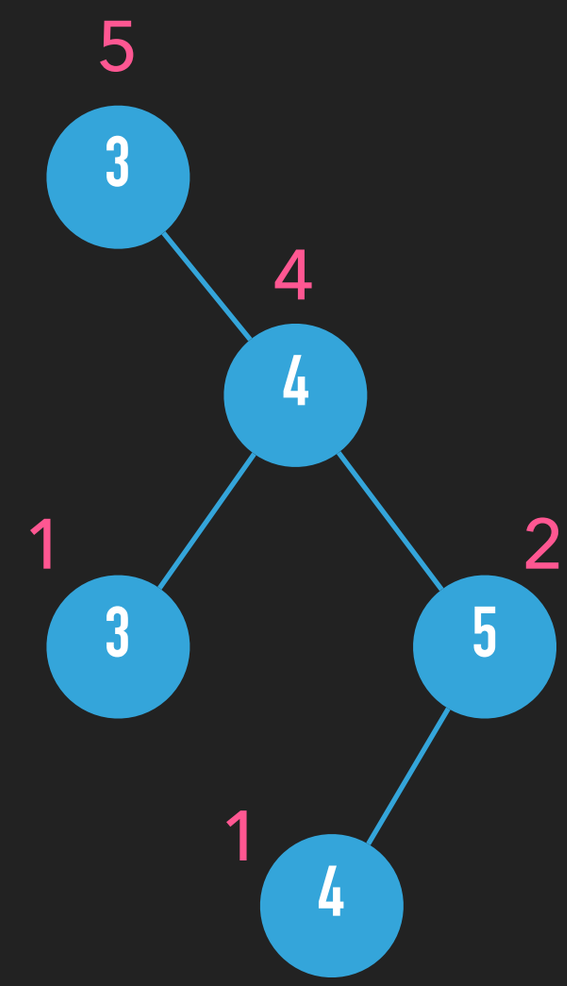
With 2 fixed in the front, how many ways to interleave the two subsequences?

m = 2 and n = 2

$$^{(m+n)}C_n = 6$$

GENERAL EXAMPLE

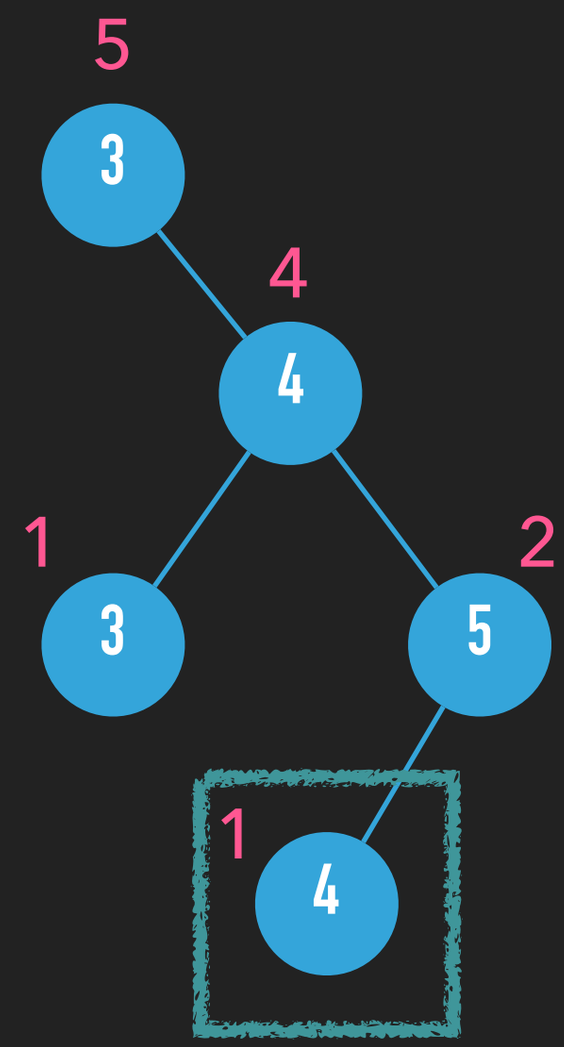
- Let’s keep the subtree size at each node.



GENERAL EXAMPLE

- Let’s keep the subtree size at each node.
- Let’s solve in recursively!

What’s the number of ways to arrange subtree at the current node?



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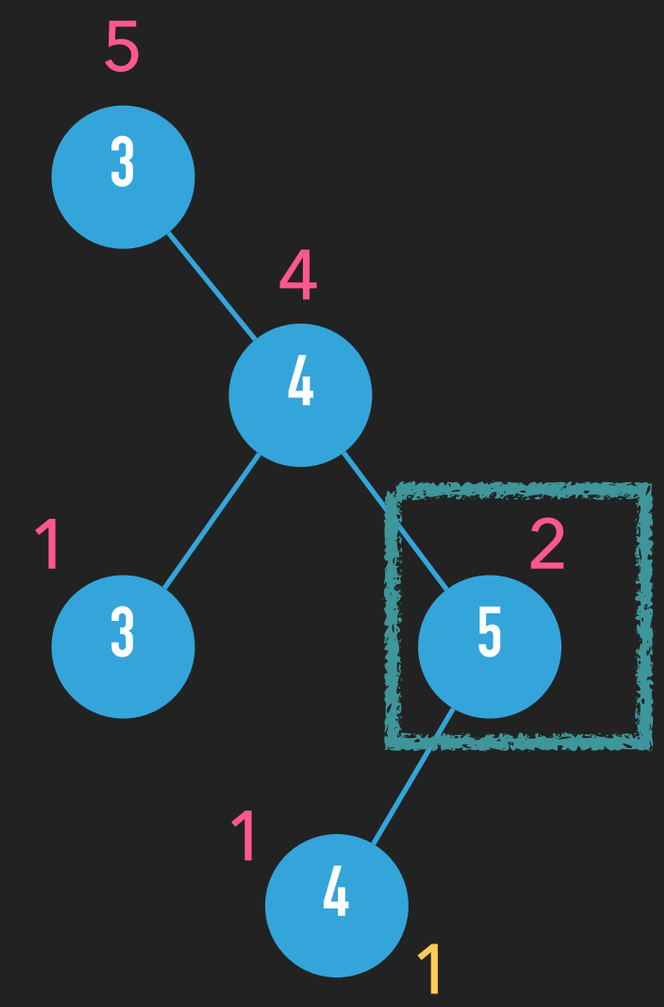

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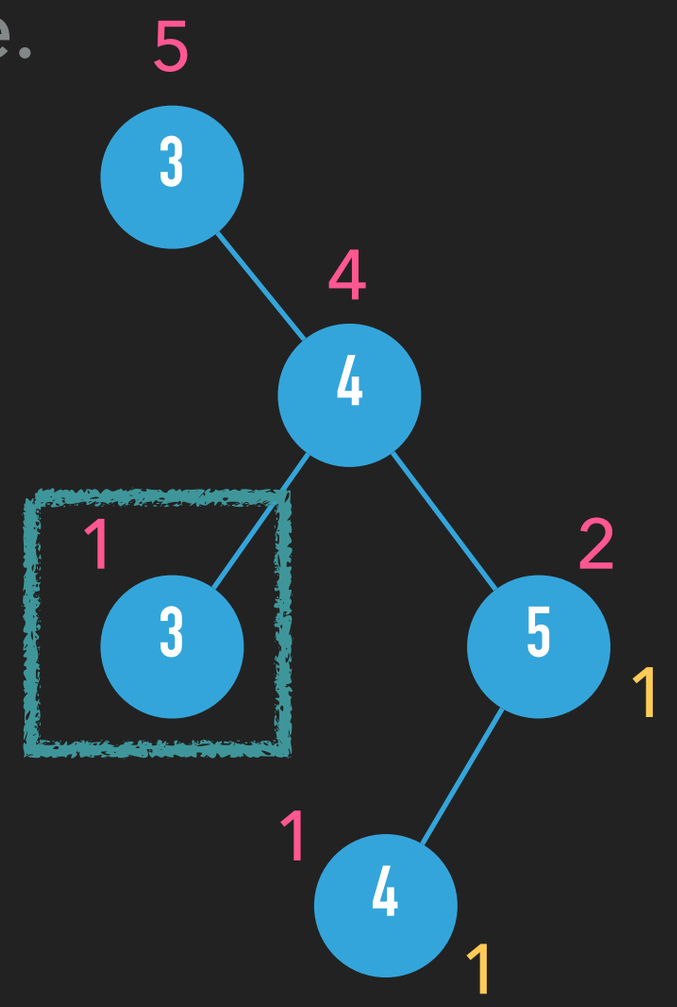

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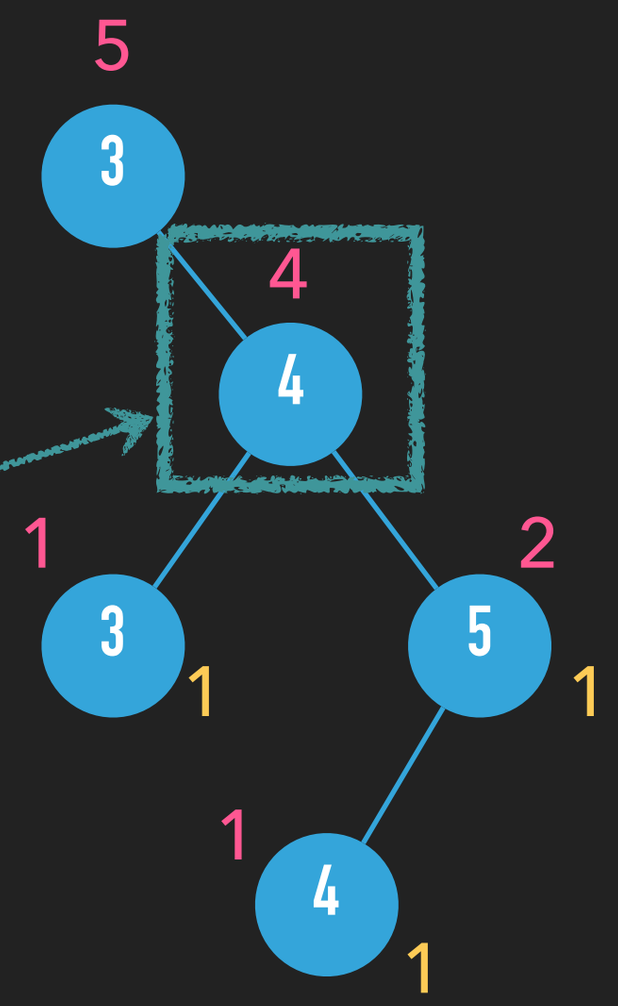
Interesting case! Both left and right child present.

What’s the number of ways to arrange subtree at the current node?

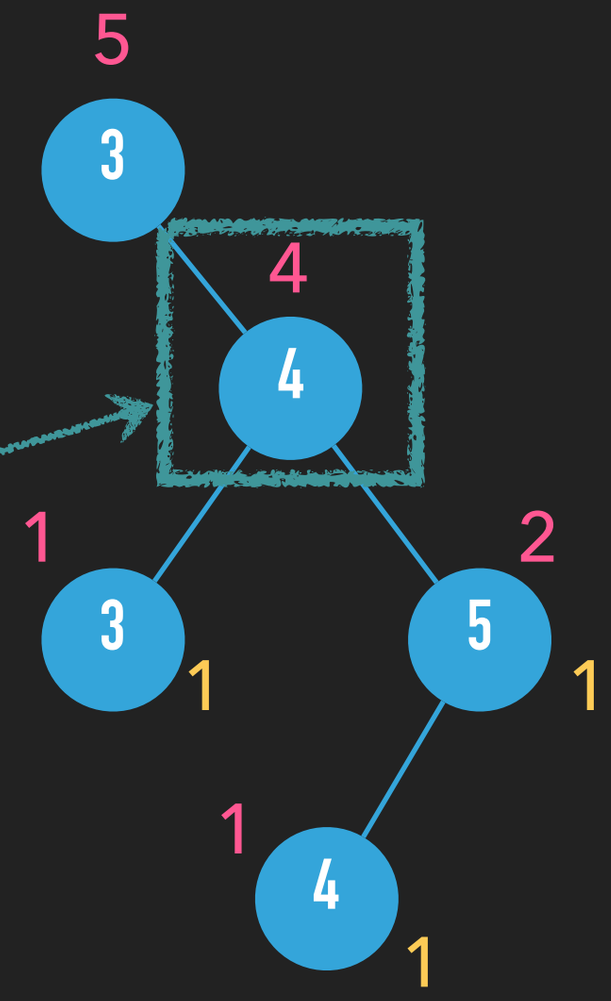


GENERAL EXAMPLE

- Let's keep the subtree size at each node.
- Let's solve in recursively!

Answer:
# of ways to arrange left
x # of ways to arrange right
x number of ways to interleave

What's the number of ways to arrange subtree at the current node?

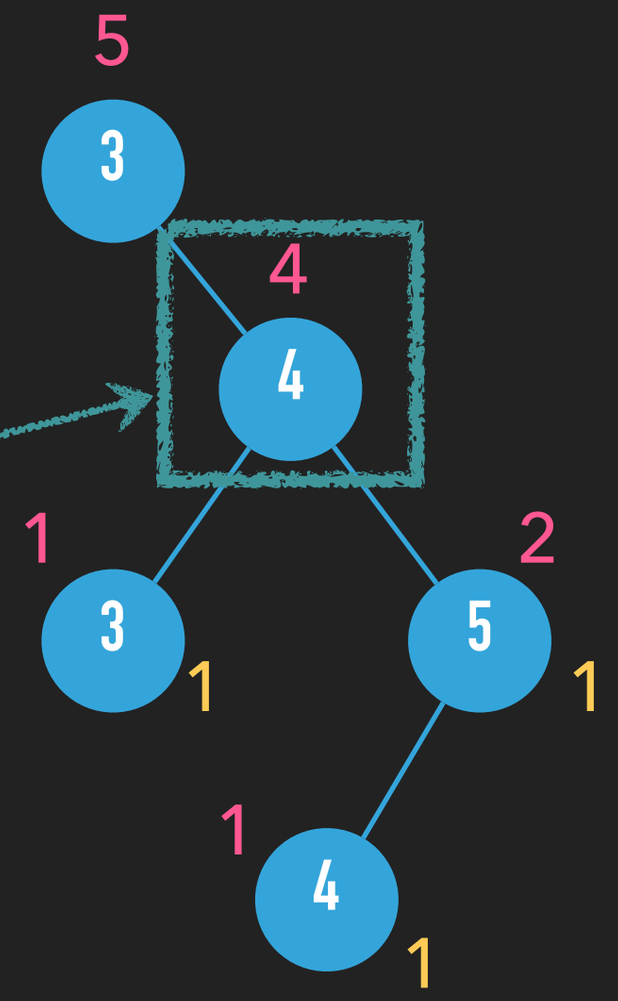


GENERAL EXAMPLE

- Let's keep the subtree size at each node.
- Let's solve in recursively!

Answer:
1
x 1
x $^{(1+2)}C_2$

What's the number of ways to arrange subtree at the current node?



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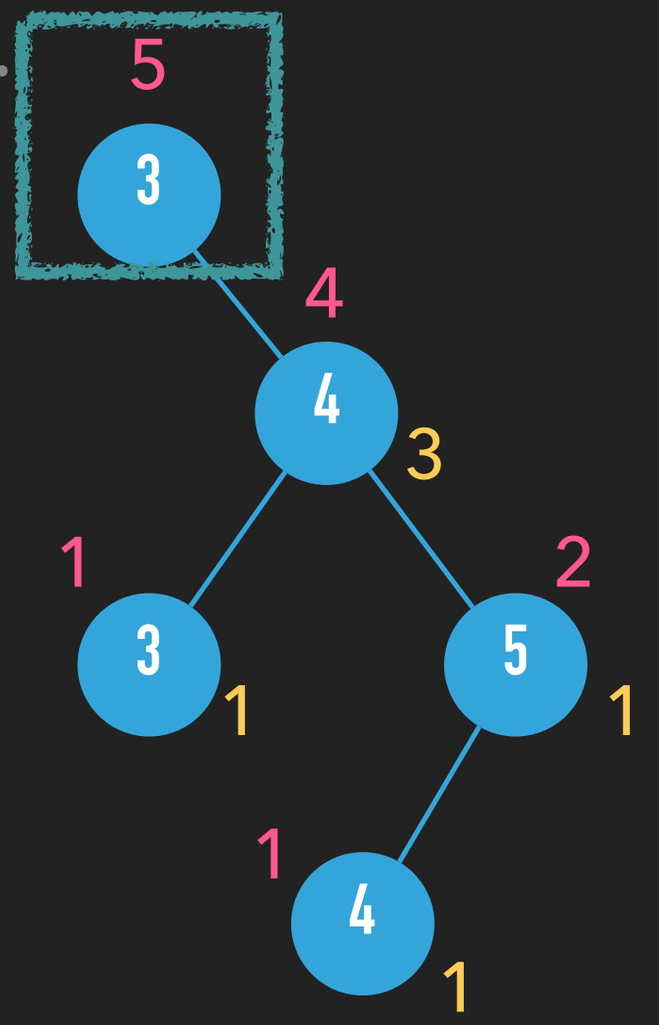

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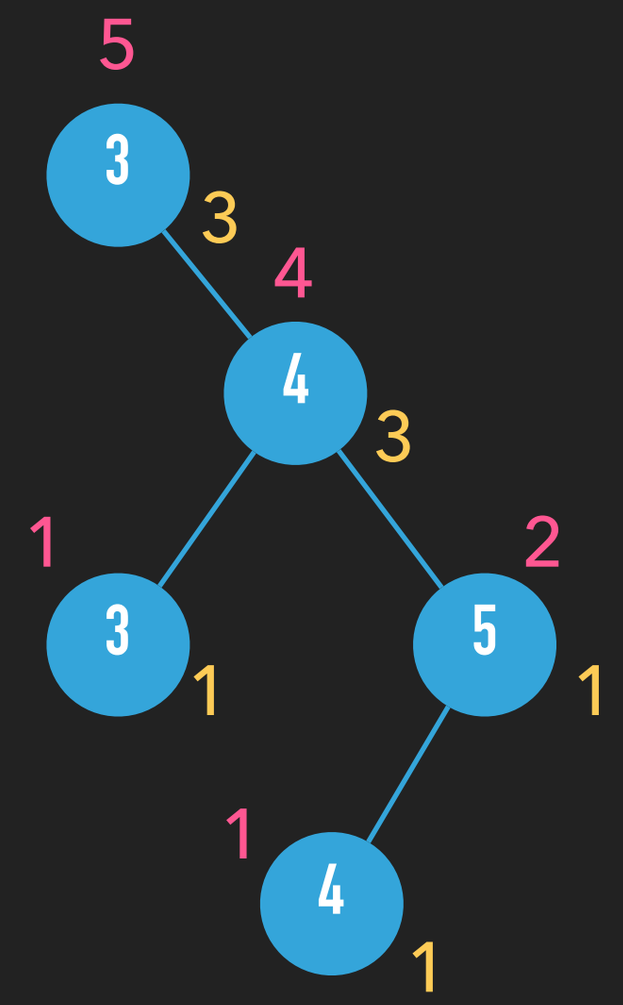

Reached root –> Done!