NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
MA1101R Linear Algebra I

2018-2019 (Semester 1) **Tutorial 5**

1. Determine which of the following sets of vectors span $\mathbb{R}^4$.

   (a) $S = \{(2,3,2,0),(0,2,1,1)\}$.
   (b) $S = \{(2,1,1,0),(1,2,-1,0),(0,3,0,3),(0,1,-1,3)\}$
   (c) $S = \{(3,2,-1,2),(4,0,0,2),(5,6,-3,2),(0,4,-2,-1)\}$
   (d) $S = \{(1,2,-2,1),(4,0,4,0),(1,-1,-1,-1),(1,1,1,1),(0,1,0,1)\}$.

2. Find a set of vectors that spans the solution space of the following homogeneous linear system:
$$\begin{cases} x_1 + x_2 + 2x_4 = 0 \\ -2x_1 - 2x_2 + x_3 - 5x_4 = 0 \\ x_1 + x_2 - x_3 + 3x_4 = 0 \\ 4x_1 + 4x_2 - x_3 + 9x_4 = 0 \end{cases}$$

3. For each of the following sets S_1 and S_2, determine whether

   (i) $\text{span}(S_1) \subseteq \text{span}(S_2)$;
   (ii) $\text{span}(S_2) \subseteq \text{span}(S_1)$;
   (iii) $\text{span}(S_1) = \text{span}(S_2)$.

   (a) $S_1 = \{(2,-2,0),(-1,1,-1),(0,0,9)\}$ and $S_2 = \{(1,1,-1),(-2,-2,1),(1,5,-2)\}$.
   (b) $S_1 = \{\boldsymbol{u},\boldsymbol{v},\boldsymbol{w}\}$, $S_2 = \{\boldsymbol{u},\boldsymbol{u}+\boldsymbol{v},\boldsymbol{u}+\boldsymbol{v}+\boldsymbol{w}\}$ where $\boldsymbol{u},\boldsymbol{v},\boldsymbol{w}$ are vectors in $\mathbb{R}^4$.

4. Let V and W be subspaces of $\mathbb{R}^n$. Define
$$V + W = \{\boldsymbol{v}+\boldsymbol{w} \mid \boldsymbol{v} \in V \text{ and } \boldsymbol{w} \in W\}.$$

   (a) Show that $V + W$ is a subspace of $\mathbb{R}^n$.
   (**Hint:** Since V and W are subspaces, $V = \text{span}(S)$ and $W = \text{span}(T)$ for sets S and T in $\mathbb{R}^n$. Use S and T to find a set R such that $V+W = \text{span}(R)$.)
   (b) Write down the subspace $V + W$ explicitly (that is, find a finite set S such that $V + W = \text{span}(S)$) if
      (i) $V = \{(t,0) \mid t \in \mathbb{R}\}$ and $W = \{(0,t) \mid t \in \mathbb{R}\}$.
      (ii) $V = \{(t,2t,3t) \mid t \in \mathbb{R}\}$ and $W = \{(t,0,-t) \mid t \in \mathbb{R}\}$.
      (iii) V is the line spanned by $(1,1,1)$ in $\mathbb{R}^3$ and W is the plane with equation $x + y - z = 0$ in $\mathbb{R}^3$.

5. For each of the sets $S = \{\boldsymbol{u_1},\boldsymbol{u_2},\cdots,\boldsymbol{u_k}\}$ in Question 1,

   (i) determine if S is a linearly independent set.

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(ii) If S is a linearly dependent set, find a non-trivial solution to the equation

$$c_1\boldsymbol{u_1} + c_2\boldsymbol{u_2} + \cdots + c_k\boldsymbol{u_k} = \mathbf{0}.$$

Hence or otherwise, find a vector $\boldsymbol{x}$ in S such that

$$\operatorname{span}(S) = \operatorname{span}(S - \{\boldsymbol{x}\}).$$