# *Pancake Flipping Problem*
# *(Exploring the Solution Space)*
# *Video 6.2*

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*Experience the fun of problem solving*

# **Quick Review:**

Bill Gates did not *really* flip pancakes (*din* need to)

He flipped pancakes in his head (abstraction)

He devised a very smart algorithm!

His professor regretted not becoming his early employee!
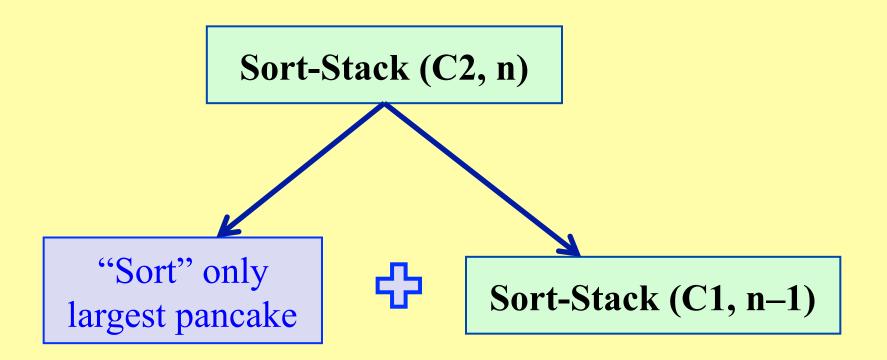
Define the PF problem

Solve a few small instances

Abstraction, Heuristics, Decomposition

Devised a Greedy Algorithms

# Review decomposition process

```
                    ┌─────────────────────────┐
                    │   Sort-Stack (C2, n)    │
                    └─────────────────────────┘
                       ↙                  ↘
   ┌──────────────────┐       ┌───────────────────────┐
   │  "Sort" only     │   ✚   │  Sort-Stack (C1, n–1) │
   │  largest pancake │       │                       │
   └──────────────────┘       └───────────────────────┘
```

# Greedy Algorithm:

Repeatedly "sort" the biggest unsorted pancake;



Unsorted

Largest unsorted

Sorted

**5 flips**

## Greedy Algorithm:

Repeatedly "sort" the biggest unsorted pancake;

**Question:** For a stack of n pancakes, how many flips will Greedy algorithm make?

#flips $\leq 2n$     (2 flips to sort each largest pancake)
#flips $\leq 2(n–1)$   (don't need to sort smallest one)
#flips $\leq 2n–3$    (–1 more for last-2-pancakes)
Greedy algorithm does no more than $(2n–3)$ flips
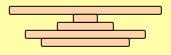
## Theorem:

Greedy algorithm does no more than $(2n–3)$ flips

# Activity: Fun with Pancake Flipping

For this example, first use Greedy Algorithm.
Then try to find a better solution.

**Expected Answer:**
Greedy algorithm used 3 flips.

It is an optimal solution.
(how do we know that?)

___ **Flips (using Greedy Algorithm)**

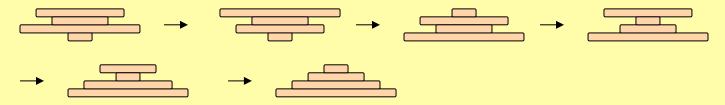___ **Flips (in your best solution)**

# Will Greedy always give optimal soln?

Answer:   NO

**A Counter Example:**
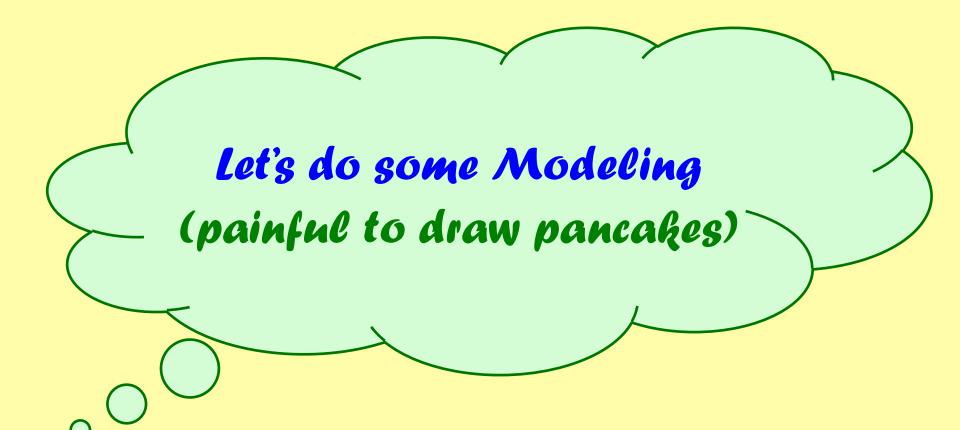
*Greedy method*     **[5 flips]**

*Better (creative) solution*  **[3 flips]**

**Problem:**  Design an algorithm that solve the pancake flipping problems using the *minimum number of flips.*

# Model pancake sizes by numbers

Real world:
   pancake configurations

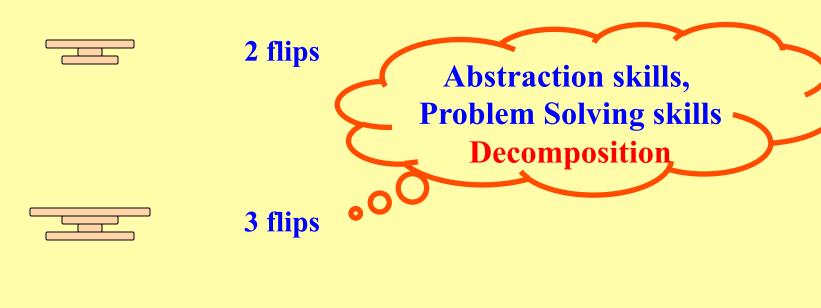3241    4231

Model world:
   permutations of (1234)

324|1   →   4231

Pancake flip

# Real-world pancake-flipping…



2 flips

**Abstraction skills,
Problem Solving skills**
**Decomposition**

3 flips

? flips

# The Model World…

312  **2 flips**

**Abstraction skills,
Problem Solving skills
Decomposition**

4213  **3 flips**
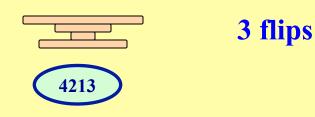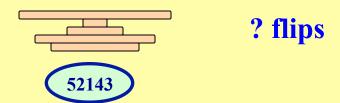
52143  **? flips**

# Let's see both together…

2 flips

312

3 flips

4213

? flips

52143

**Which works better for YOU?**

*Real-World*
*or*
*Model World?*

# Is Greedy "the best" possible? NO

**Model World
(One more slide)**

## A Counter Example:

*Greedy method*  **[5 flips]**

324|1 → 4231| → 13|24 → 312|4 → 21|34 → 1234

*Better (creative) method*  **[3 flips]**

32|41 → 234|1 → 4321| → 1234

13

# Pancake Flipping Problem

PQ: Can we look at this from another perspective!

# A different perspective: the Solution Space…

2314
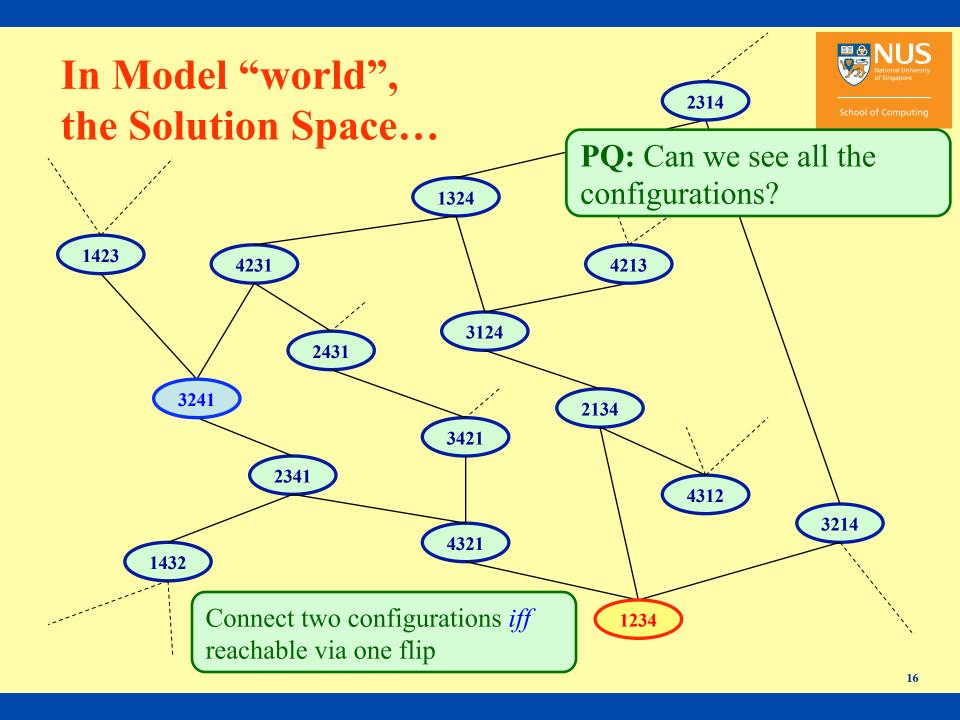
**PQ:** Can we see all the configurations?

**PQ:** How many different configurations are there?

**Answer:**
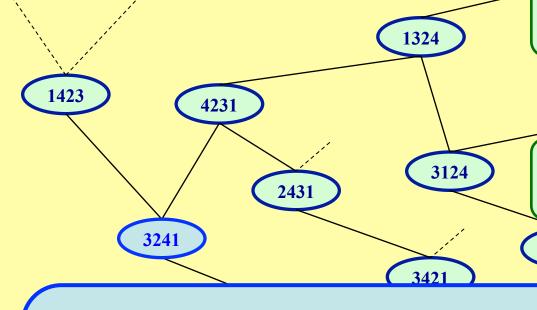
1324

1423

4231

3124

2431

3241

2134

3421

2341

4312

3214

4321

1432

Connect two configurations *iff* reachable via one flip

1234

# In Model "world", the Solution Space…

NUS
National University of Singapore
School of Computing

2314

**PQ:** Can we see all the configurations?

1324

1423

4231

4213

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4312

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4321

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Connect two configurations *iff* reachable via one flip

1234

# In Model "world", the Solution Space…

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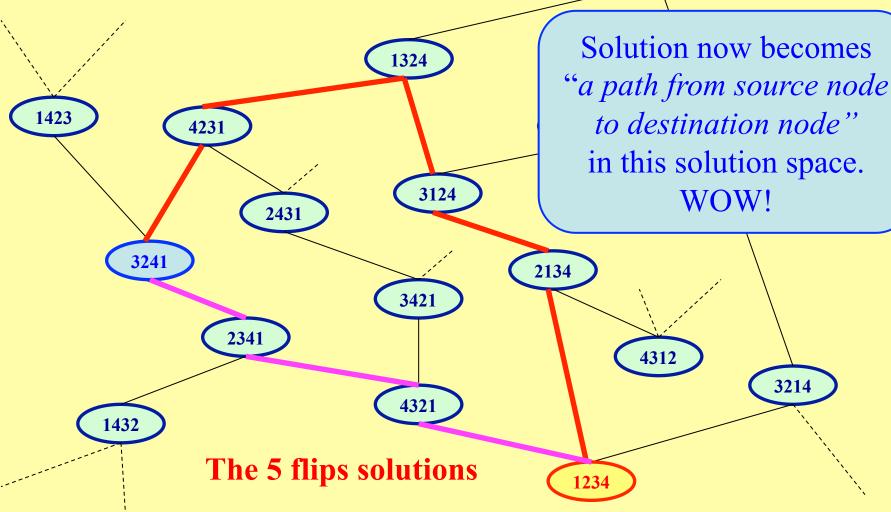3421

Connect two configurations *iff* reachable via one flip

The *Solution Space* is a graph $G = (V, E)$, where the vertices in $V$ are configurations, and the two configurations $u$ and $v$ are connected by an edge in $E$ iff $u$ is reachable from $v$ with one flip.
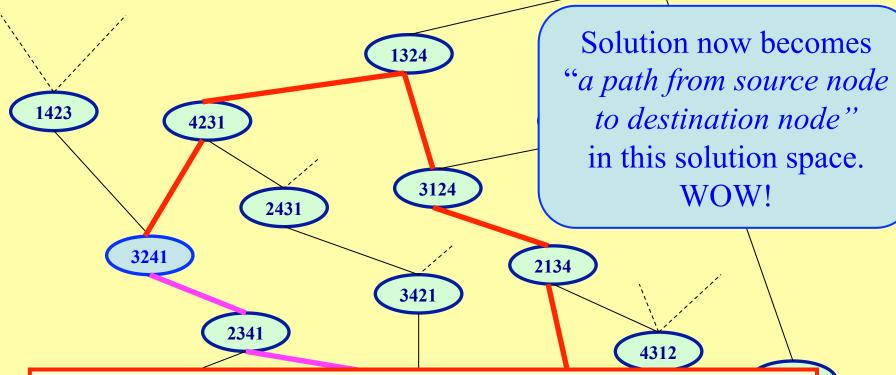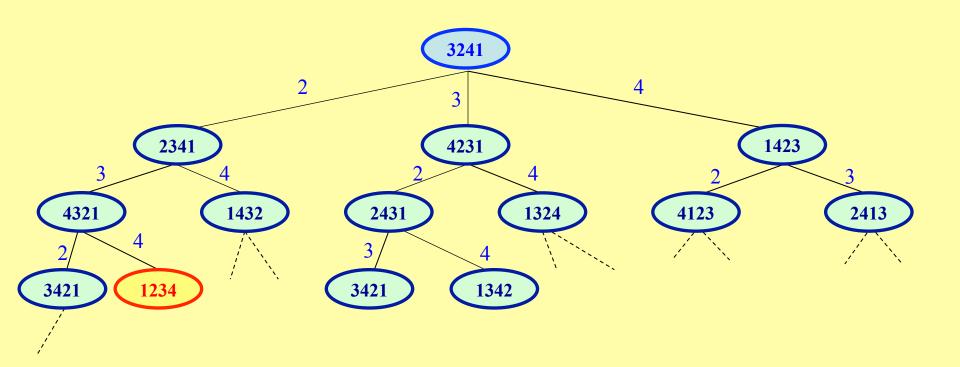
17

# In this Solution Space, what are solutions?

2314

1324

1423

4231

Solution now becomes *"a path from source node to destination node"* in this solution space. WOW!

2431

3124

3241

2134

3421

2341

4312

3214

1432

4321

1234

**The 5 flips solutions**

**The 3 flips solutions**

# In this Solution Space, what are solutions?

2314

1324

1423

4231

Solution now becomes
*"a path from source node
to destination node"*
in this solution space.
WOW!

2431

3124

3241

2134

3421

2341

4312

**New Problem:** (Pancake flipping becomes shortest path)
Want a smart method (algorithm) to search
this space to find the shortest path solution.

# A Search Tree Method:
(systematically search the search space)



**Problem:** Want a smart method (algorithm) to search this space to find a shortest path solution.

# Summary:

Analyzed Greedy Algorithms

    (Greedy takes $\leq (2n–3)$ flips)

Proved Greedy Alg is not optimal

    (Gave counter-example)

Solution Space Graph

    (via Graph Modelling) – yes, graph again!

Pancake Flipping becomes Shortest Path in a Graph

Tree search method to look for shortest path.

    (Works, but is slow, because tree grows exponentially large)

# Pancake Flipping

## Overview:

❑ **When did Bill Gates flip pancakes**

❑ **The Pancake Flipping Problem**

❑ **What we know about Pancake Flipping**

❑ **Why Study Pancake Flipping?**

# Pancake Flipping Problem: Known Results

- Greedy Algorithm uses at most $2n$-3 flips


- For $n$ pancakes, at most $(5n+5)/3$ flips are needed
  [Bill Gates and Papadimitriou, 1979]  ~$1.666n$


- 2008 (about 30 years later), at most $18n/11$ needed
  [a team from UT-Dallas, 2008]  ~$1.6363n$     (*diff*: < 2%)

# Pancake Flipping (Math vs CS)

- **Mathematics – Study its properties**
  - Define $f(n)$ to be the minimum of number of flip for any configuration of $n$ pancakes

  **OPEN PROBLEM:** What is the precise formula for $f(n)$?

- **Computing – Want an algorithm to solve it**
  - Given any $n$ pancake configuration, sort it with the *minimum* number of flips

  **Problem is HARD.** No one has efficient algorithm (Problem is in a class of NP-complete problem)

# Subtle Difference between Mathematics vs Computer Science

**Problem:**

We need to drive from NUS-SOC to Kuala Kubu Bahru in West Malaysia?

**Theorem (from Mathematician)**

There exist a path with total distance no more than 450km.

# An exact route  (from CS person)

# Pancake Flipping

## Overview:

- ❑ **When did Bill Gates flip pancakes**

- ❑ **The Pancake Flipping Problem**

- ❑ **What we know about Pancake Flipping**

- ❑ **Why Study Pancake Flipping?**

# Why study pancake flipping

- **Applications**
  - Study "evolution of species" in biology.    [species]
  - Is human closer to mouse or rat?            [genomes]
  - Compute distances between genomes      [distances]
    (using "sorting by reversals")
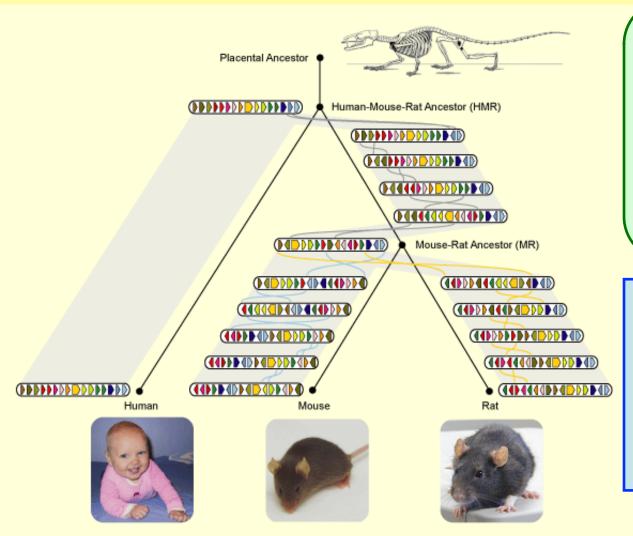  - "reversal" is a generalized pancake flip    [pancake flips]

Study of pancake flipping can help study of evolution of species in biology.

**Curious?**
See details in the extra slides at the end.

# Is human "closer" to mouse or rat?



To answer that, we compare the "distances" between their genomes.

"Distance" is number of flip operations (*or reversals*) to transform one genome to the other.

# Application of Sorting by Reversals

## Genome Rearrangements and Sorting by Reversals

| ABSTRACT | REFERENCES (28) | CITING ARTICLES |

Vineet Bafna and Pavel A. Pevzner

Sequence comparison in molecular biology is in the beginning of a major paradigm shift—a shift from gene comparison based on local mutations (i.e., insertions, deletions, and substitutions of nucleotides) to chromosome comparison based on global rearrangements (i.e., inversions and transpositions of fragments). The classical methods of sequence comparison do not work for global rearrangements, and little is known in computer science about the edit distance between sequences if global rearrangements are allowed. In the simplest form, the problem of gene rearrangements corresponds to sorting by reversals, i.e., sorting of an array using reversals of arbitrary fragments. Recently, Kececioglu and Sankoff gave the first approximation algorithm for sorting by reversals with guaranteed error bound 2 and identified open problems related to chromosome rearrangements. One of these problems is Gollan's conjecture on the reversal diameter of the symmetric group. This paper proves the conjecture. Further, the problem of expected reversal distance between two random permutations is investigated. The reversal distance between two random permutations is shown to be very close to the reversal diameter,

**CS research paper on the use of "sorting by reversals"** to study the evolution from one species to another.

# Link between "evolution of species" and pancake flipping problem

**Is human "closer" to mouse or to rat?**

Compute "distances" between their genomes

"distance" is the *number of flip operations* (or reversal)

"reversal" is a *generalized pancake flip operations*

# More on Pancake Flipping

Have some fun with pancake flipping:

http://www.cut-the-knot.org/SimpleGames/Flipper.shtml

Read about Pancake Flipping:

http://www.amazon.com/Bill-Gates-Speaks-Greatest-Entrepreneur/dp/0471401692

Listen to the "Pancake Puzzle" story on NPR (July-2008):

http://www.npr.org/templates/story/story.php?storyId=92236781

Youtube Video: (Bill Gates and Pancake Flipping)

https://www.youtube.com/watch?v=oDzauRFiWFU

*Q:* How are the following people connected to Bill Gates?

Jacob Goodman, Harry Lewis, Harry Dweighter,

# More on Genome Rearrangments

Genome Rearrangement:

http://dl.acm.org/citation.cfm?id=586673

http://grimm.ucsd.edu/GRIMM/

A video on sorting by reversals: (by Pavel Pevzner)

https://www.youtube.com/watch?v=GVCxeynVE5U

# (End of video 6.2)

If you want to contact me,

Email: leonghw@comp.nus.edu.sg

Additional Slides on

Sorting by Reversals

(only for those interested)

# What are "reversals" in the study of genomics?

**Example:** (2 genomes $G$ and $H$, from different species)

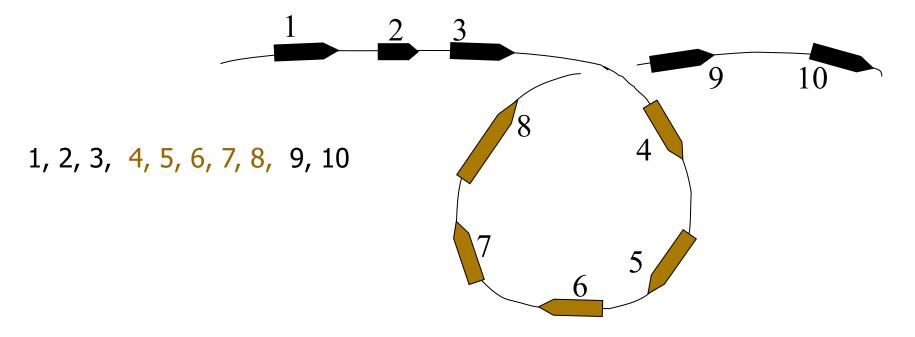$G = \{\ 1, -5, 4, -3, 2\ \}$          $H = \{\ 1, 2, 3, 4, 5\ \}$

A **genome** consists of a sequence of **genes**;

Represent each gene by a integer $\{1, 2, \ldots, n\}$.
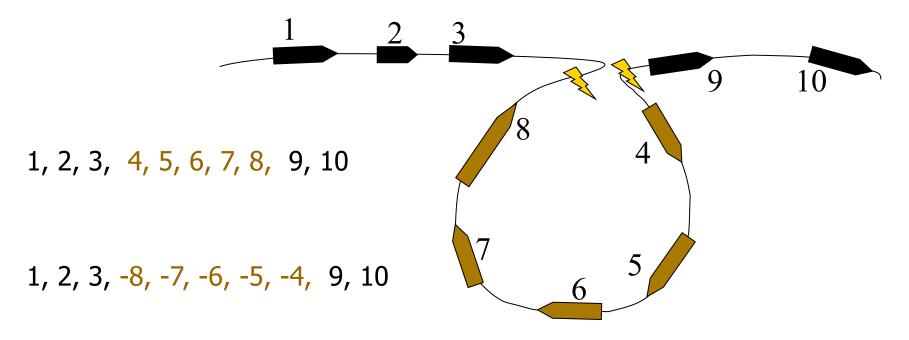
Each gene has a "*direction*" indicated by the sign (like -3).

Then a genome is a "like" a stack of pancakes of diff sizes.
(each pancake is burnt on one side, indicate "direction")
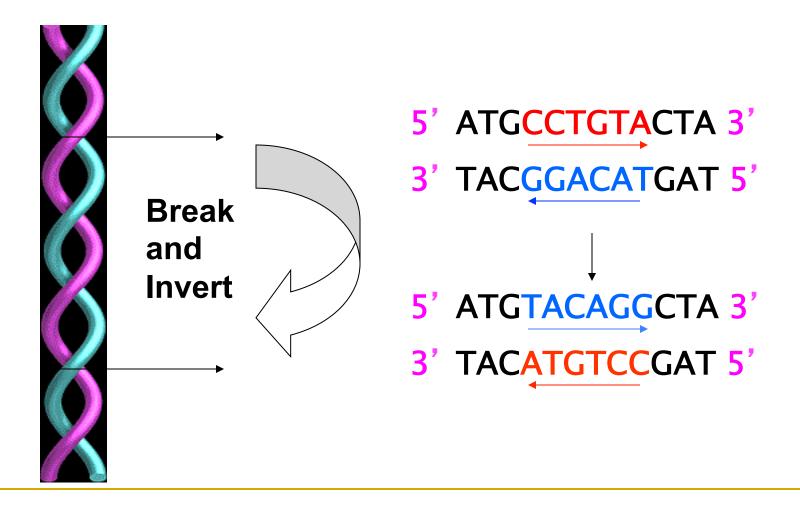
# Reversals in the Gene Sequence



1, 2, 3,  4, 5, 6, 7, 8,  9, 10

\* Consider the block of genes (4, 5, 6, 7, 8,)

# Reversals and Breakpoints

1, 2, 3,  4, 5, 6, 7, 8,  9, 10

1, 2, 3, -8, -7, -6, -5, -4,  9, 10

\* Consider the block of genes (4, 5, 6, 7, 8,)

\* The reversal introduced two *breakpoints*

\* Order of genes reversed, and direction inverted
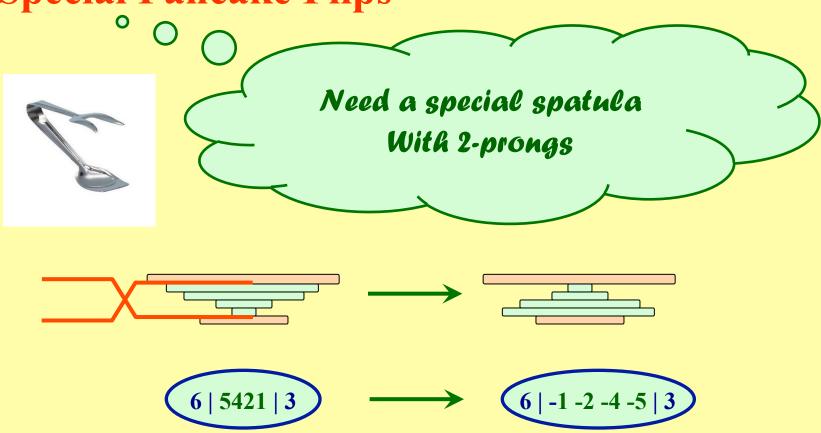   (4, 5, 6, 7, 8,) ➔ (-8, -7, -6, -5, -4,)

# Reversals: Example



5' ATG**CCTGTA**CTA 3'

3' TAC**GGACAT**GAT 5'
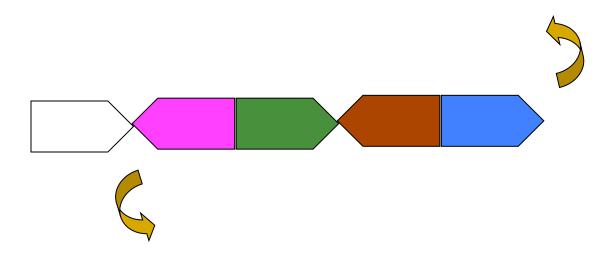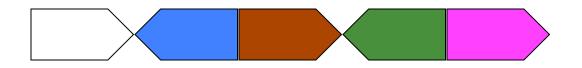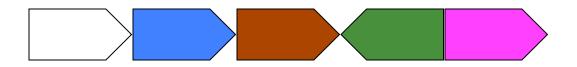
**Break and Invert**

5' ATG**TACAGG**CTA 3'

3' TAC**ATGTCC**GAT 5'

# Reversals are
# Special Pancake Flips



Need a special spatula
With 2-prongs

6 | 5421 | 3  →  6 | -1 -2 -4 -5 | 3

# Evolution of the gene sequence via reversals

- Gene order comparison:

# Evolution of the gene sequence via reversals

- Gene order comparison:

# Evolution of the gene sequence via reversals

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# Evolution of the gene sequence via reversals
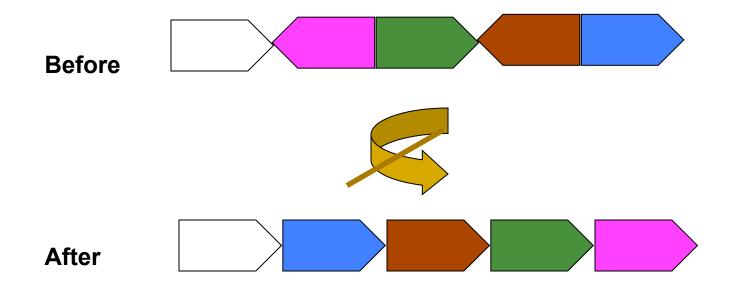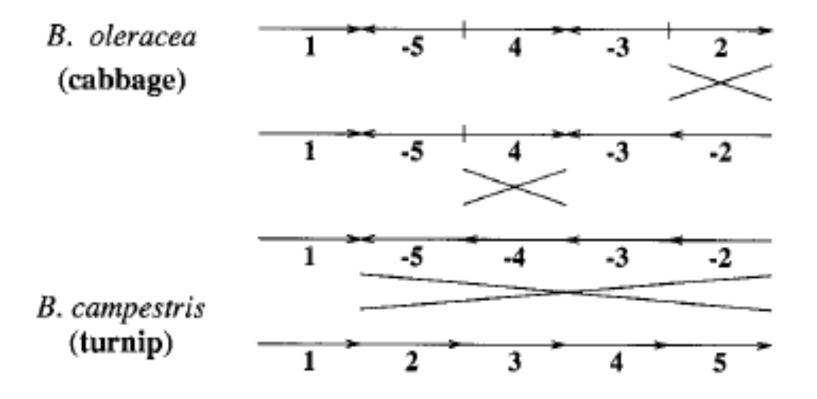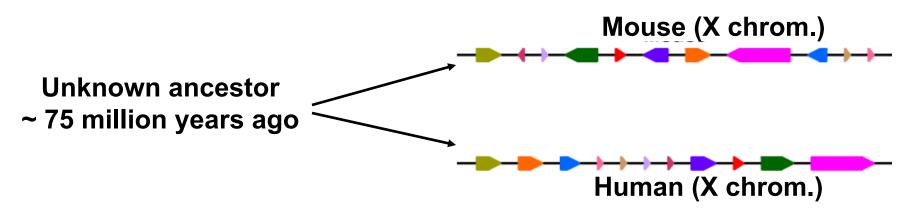
- ## Gene order comparison:



**Before**

**After**

Evolution is manifested as the divergence in gene order

# Transforming Cabbage into Turnip

## (with 3 reversal operations)

# Genome rearrangements



**Mouse (X chrom.)**

**Unknown ancestor ~ 75 million years ago**

**Human (X chrom.)**
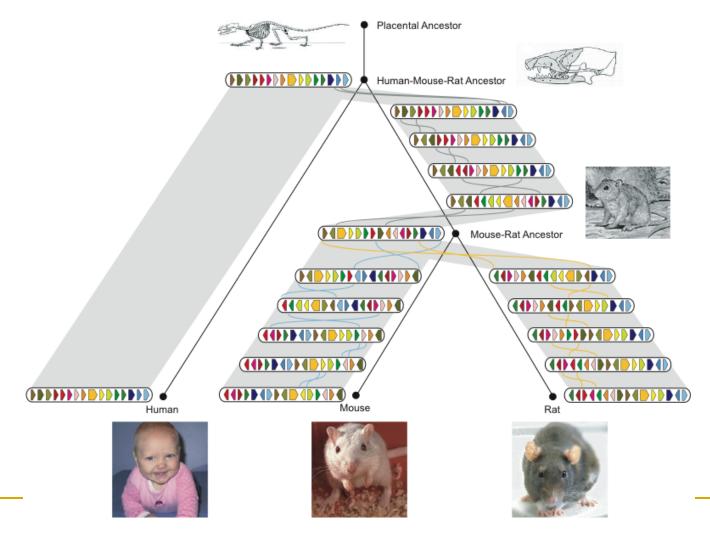
- What are the similarity blocks and how to find them?

- What is the architecture of the ancestral genome?

- What is the evolutionary scenario for transforming one genome into the other?

# History of Chromosome X



Rat Consortium, *Nature*, 2004