CS1231 TUTORIAL 5

1. Determine whether $f : \mathbb{Z} \to \mathbb{R}$ is function:

(a) $f(n) = \pm n$. (b) $f(n) = \sqrt{n^2+1}$. (c) $f(n) = 1/(n^2-4)$. (d) $f(n) = \sin n$.

2. Find the domain and range of these functions.

(a) The function that assigns to each nonnegative integer its last digit.

(b) The function that assigns the next largest integer to a positive integer.

(c) The function that assigns to a bit string the number of one bits in the string.

(d) The function that assigns to a bit string the number of bits in the string.

3. Find these values:

(a) $\lfloor 1.1 \rfloor$, (b) $\lceil 1.1 \rceil$ (c) $\lfloor -.1 \rfloor$ (d) $\lceil -.1 \rceil$ (e) $\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \rfloor$ (f) $\lceil \frac{1}{2} + \lfloor \frac{3}{2} \rfloor \rceil$

4. Determine whether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto.

(a) $f(m,n) = 2m - n$ (b) $f(m,n) = m^2 - n^2$.

5. Determine which of the following are bijections. For those that are bijections, find the inverse functions.

(a) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = -3x^2 + 7$.

(b) $f : \mathbb{R} - \{-2\} \to \mathbb{R} - \{1\}$ where $f(x) = (x+1)/(x+2)$.

6. Given a set S, the characteristic function of $A \subseteq S$, $k_A : S \to \mathbb{Z}$ is defined as

$$k_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

It is easy to see that $|A| = \sum_{x \in S} k_A(x)$. Prove that $\forall A, B \subseteq S$ and $\forall x \in S$:

(a) $k_{A \cap B}(x) = k_A(x) \cdot k_B(x)$.

(b) $k_{\overline{A \cup B}}(x) = (1 - k_A(x))(1 - k_B(x))$.

(c) $k_{A-B}(x) = (1 - k_B(x))k_A(x)$.

(d) Deduce from (b) that $|A \cup B| = |A| + |B| - |A \cap B|$.

(Example: $S = \{1, \ldots, 20\}$, $A = \{1, 5, 7\}$. Then $k_A(1) = 1$, $k_A(2) = 0$, $k_A(3) = 0$, $k_A(4) = 0$, $k_A(5) = 1$, etc. $\sum_{x \in S} k_A(x) = 3 = |A|$. $k_{\overline{A}}(1) = 0 = 1 - k_A(1)$, $k_{\overline{A}}(2) = 1 = 1 - k_A(2)$, etc. In general $k_{\overline{A}}(x) = 1 - k_A(x)$.)

7. Prove that $\lfloor 3x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{3} \rfloor + \lfloor x + \frac{2}{3} \rfloor$ (Hint: If $\lfloor x \rfloor = n$, what are the possible values of $\lfloor 3x \rfloor$? This will give an indication of the various cases.)