Physics : Part 1: Video 5 : Theoretical Enquiry

Phil Chan
Department of Physics

Contents:

Polya’s theoretical enquiry ?

Does one need to learn new Mathematics ?

Are there any underlying Principles ?

Are there Symmetries ? Beautiful ?

Is it possible to construct Gedanken (s)?

Can one construct an Analogy ?

Polya’s Method of Theoretical Enquiry

- Understanding the Problem
- Devising Plan
- Carrying out the plan
- Looking back

G. Polya, *How to Solve it*, 2014, New Princeton Science Library Edition.

Understanding the Problem

What is the unknown ?

Are there observations or experimental data ?

What are the condition or constraints or assumptions

G. Polya, *How to Solve it*, 2014, New Princeton Science Library Edition.

Devising Plan 1

Have you seen the problem before ?
Have you seen the same problem in a different form or context ?
Do you know a related problem ?
Is there a familiar problem having a similar unknown ?
Here is a problem related to yours and solved before. Could you use its results or method ?

G. Polya, *How to Solve it*, 2014, New Princeton Science Library Edition.

Devising Plan 2

If you cannot solve the proposed problem, can one try solve first some related problem?
Could you imagine a more accessible related problem ?
Is there an analogous problem or a more special problem ?
Could one solve a part of the problem?

G. Polya, *How to Solve it*, 2014, New Princeton Science Library Edition.

Carrying out the Plan

When carrying out the plan to the solution, did one check each step.

Can one see the steps clearly ?

Can one prove it ?

Can one arrive at the same conclusion from an alternative way ?

G. Polya, *How to Solve it*, 2014, New Princeton Science Library Edition.

Looking Back

Can you check the results and the arguments ?
Can you derive the results differently ?
Can you use the result or the method for some other problems?

G. Polya, *How to Solve it*, 2014, New Princeton Science Library Edition.

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The Language : Mathematics

Physical phenomena is normally translated into mathematical Equations using the elegant and versatile tool of calculus … differential equations.

Example : $$\frac{dy}{dx} = g(x)$$

In words : What is the function $y(x)$ whose derivatives is $g(x)$?

Arising in the study of the motion of the pendulum.

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = d\cos\omega t$$

R. Bellman and K. Cooke, *Modern Elementary Differential Equations*, Dover, 1971

Solution of a Differential Equation

Consider this $\dfrac{d^2y}{dt^2} + y = 0$ what is the solution $y\,(t)$?

There are many : $y = \cos t$ $y = \sin t$ $y = 3\cos t - \sin t$

Generally solution :

$$y = a\cos t + b\sin t \; where\; a\; and\; b\; are\; cons\tan ts$$

R. Bellman and K. Cooke, *Modern Elementary Differential Equations*, Dover, 1971

Babylonian vs Greek traditions

In physics we need the Babylonian method, and not the Euclidian or Greek method (of doing mathematics). The method of always starting from the axioms is not very efficient …
Mathematicians are only dealing with the structure of reasoning, and they do not really care what they are talking about. They do not even need to know what they are talking about, or as they themselves say, whether what they say is true. Mathematicians like to make their reasoning as general as possible.

The physicist is always interested in the special case; he is never interested in the general case. If I say to them, I want to talk about ordinary 3 dimensional space, thy say 'if you have a space of n dimensions, then here are the theorems, but I only want the case of 3.

R. P. Feynman (on seeking new laws), *The Character of Physical Law*, 1967, MIT Press.

What good is the Solution ?

The equation given by the solution describe the phenomenon or throw light into the phenomenon.

Predict other behavior or other parameter that you may calculate.

This solution may be “used at a Law” by the Applied Physicist or Engineers.

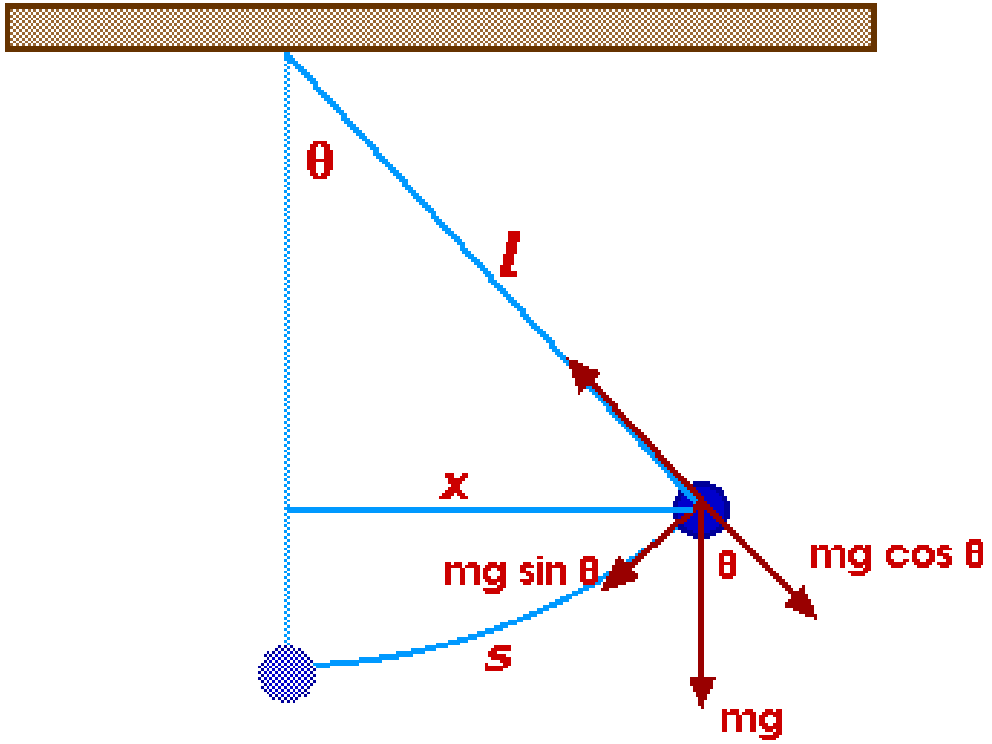
An Example : The Simple Pendulum

$$m\frac{d^2x}{dt^2} + mg\frac{x}{L} = 0$$

$$mg\sin\theta \approx mg\theta \approx mg\frac{x}{l}$$

$$ml\ddot{\theta} + mg\theta = 0$$

The "Law" : $T = 2\pi\sqrt{\frac{L}{g}}$

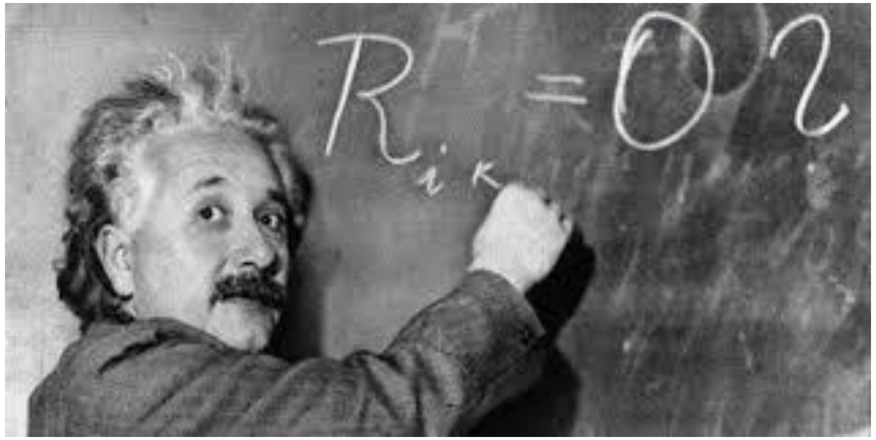


H. Pain, *The Physics of Vibrations and Waves*, Wiley, *1988*

Does one need to learn new language ?

When Einstein needed the mathematics to express his Principle of General Covariance which eventually led to the idea that Gravity is not a force, he seek his former school mate, Marcel Grossman to teach him Riemannian Geometry.

$$R_{ik} - \frac{1}{2} R g_{ik} = \frac{8\pi G T_{ik}}{c^4}$$



I. Egdall, *Einstein*, 2014, World Scientific Pub.

Physics is not Mathematics

Mathematicians are only dealing with structure of reasoning, and they do not really care what they are talking about. They do not even need to know what they are talking about, or as they themselves say, whether what they say is true … In other words, mathematicians prepare abstract reasoning ready to be used if you have a set of axioms about the real world. But the physicist has meaning to all his phrases.

Physics is not mathematics, and mathematics is not physics. One helps the other. But in physics you have to have an understanding of the connection of words with the real world.

R. P. Feynman (on Relation of Mathematics to Physics), *The Character of Physical Law,* 1967, MIT Press.

Example :
About this cartoon

$$E = m_0 c^2 \quad \text{but} \quad E^2 \neq \left(m_0 c^2\right)^2$$

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Why call Principle ?

So, rather grand word !

Principles (Scientific) are introduced in order to allow some progress to be made when one has no experimental data to go by.

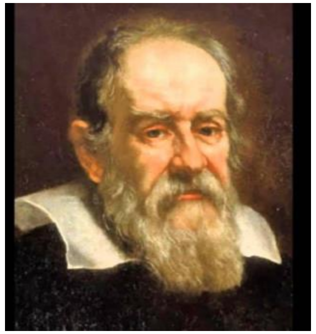
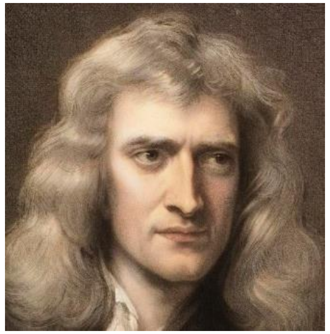
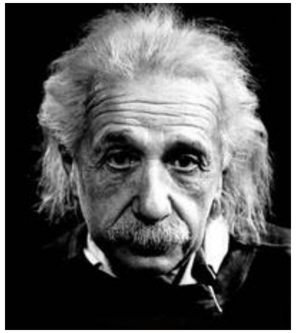
Peter Coles

P. Coles, *Cosmology*, 2001, Oxford U. Press.

Example : Einstein's Special Relativity

1) Principle of Relativity.

2) Principle of the constancy of the speed of light.

Paraphrase :

1) The laws of physics are the same.

2) The velocity of light in vacuo is the same.

Sander Bais

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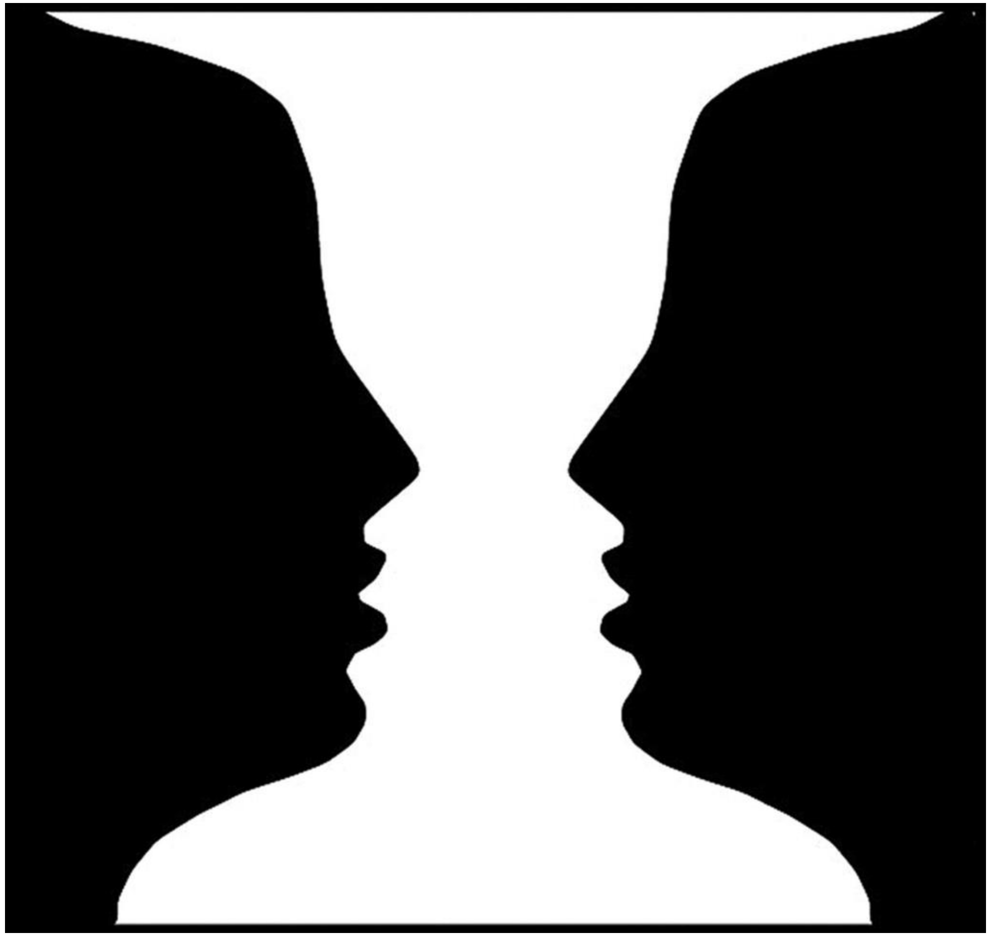
Symmetries in Physics



They are not tricks of the mind and eyes.

Symmetries

Pauli’s style of theoretical thinking and research influenced physics all over the world. It emphasizes the essential roots and the symmetries of the laws of nature in their mathematical form and accomplishes this without much extraneous talk of hand-waving.

His economical way thinking and working remains for many of us an ideal to be emulated. Those who worked with him often ask, “What would Pauli say to this?” Sometimes we have to admit, “Pauli would never accept this.” … “This is not even wrong !”

S. Weinberg, Facing UP, Harvard U Press, 2003

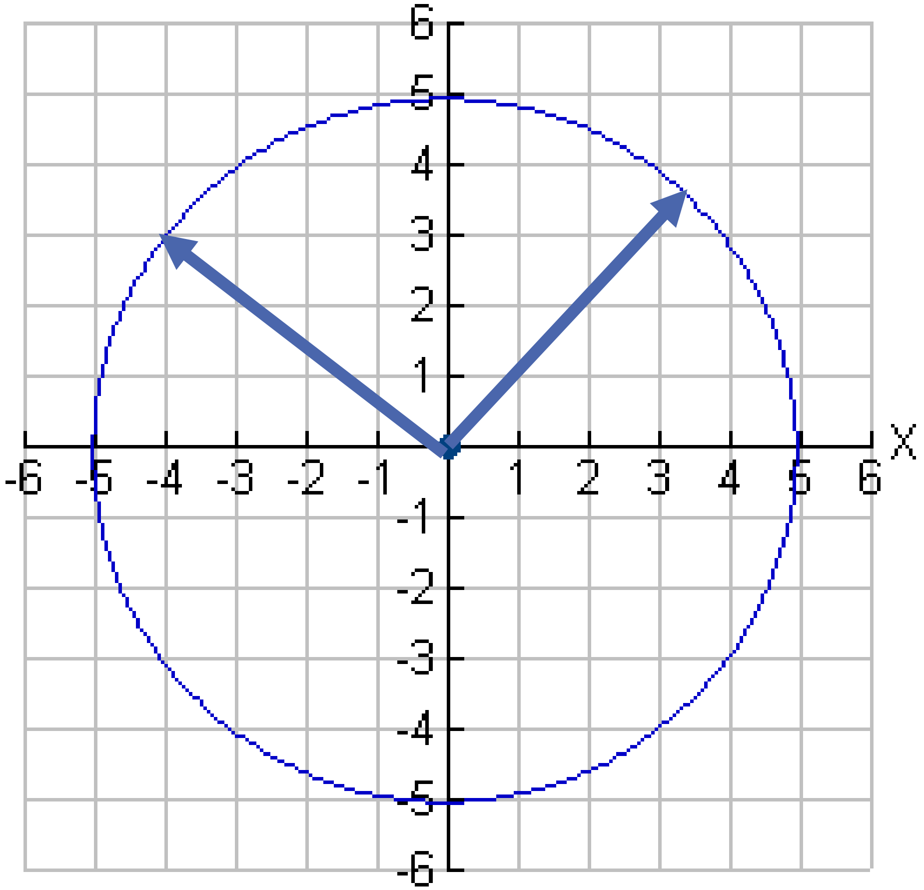
Symmetries in Physics

Transformation: Invariant vs Covariant

Theoretically physicists normally say that a quantity is invariant if it is unchanged after a transformation.

Whereas covariant refers to an equation if it is unchanged after a transformation.

Illustration : Symmetries in Physics



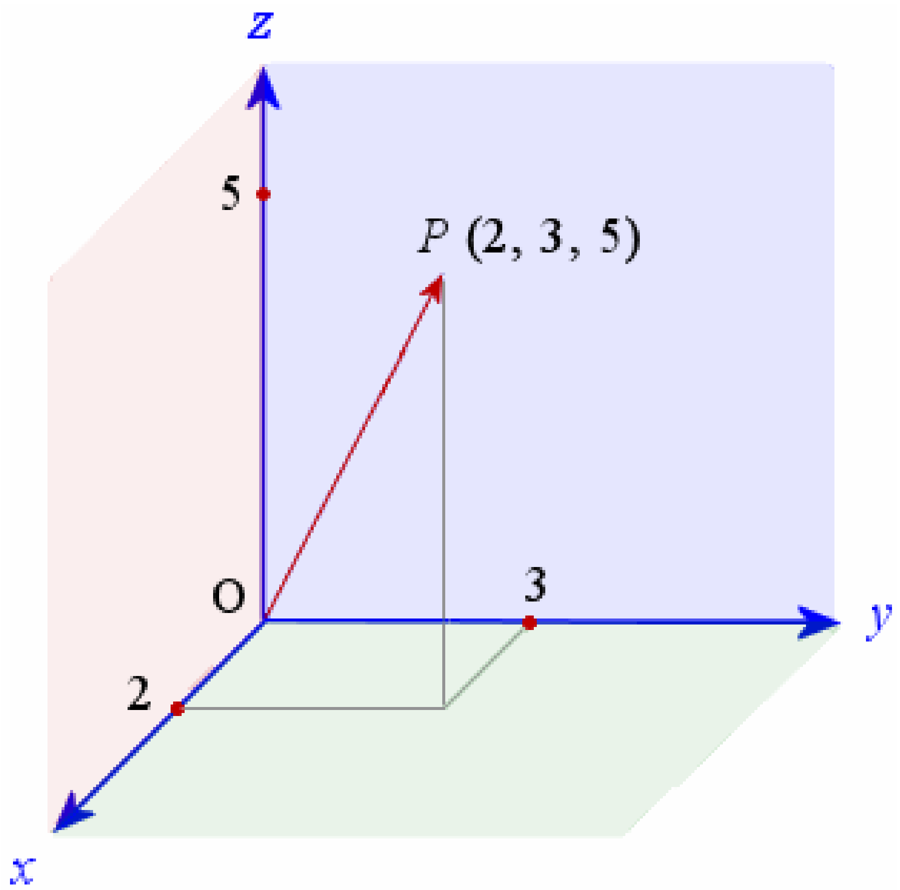
Consider 2 variables, namely the co-ordinate (x, y).

Is there an invariant quantity after a rotational transformation ?

Answer : The radius

$$x^2 + y^2 = r^2$$

Illustration : Symmetries in Physics



Consider 3 variables, namely the co-ordinate (x, y, z).

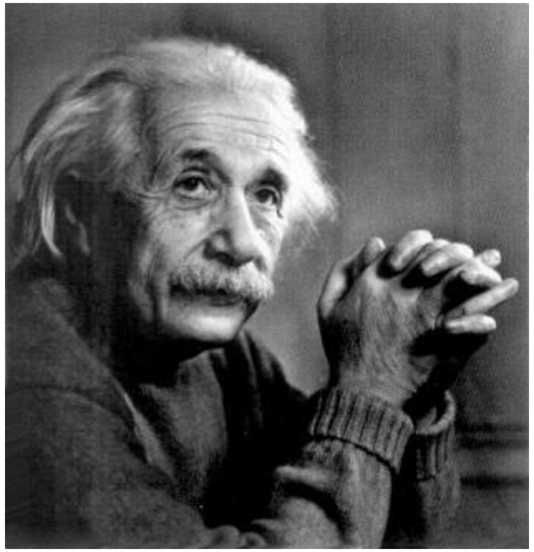
Is there an invariant quantity ?

Answer : The radius

$$x^2 + y^2 + z^2 = r^2$$

Illustration : Symmetries in Physics

I rarely think … in words at all.
A. Einstein (*1879 - 1955*)



This pseudo-radius is called the Metric.

Consider 4 variables, namely the co-ordinate (x, y, z, t).

Is there an invariant quantity ?

Answer :
The pseudo-radius(pseudo- rotation)

$$x^2 + y^2 + z^2 - t^2 = r^2$$

This should be a surprise to us ?

Why … interested in Symmetries of Nature ?

Examples of symmetries and the associated conservation laws

Noether's Theorem		
Symmetry		**Conservation Law**
Translation in time	$\longleftrightarrow$	Energy
Translation in space	$\longleftrightarrow$	Momentum
Rotation	$\longleftrightarrow$	Angular Momentum
Gauge transformation	$\longleftrightarrow$	Charge

D. Griffiths, *Intro. to Particle Physics*, 1987 John-Wiley & Sons.

Symmetries in Nature

Examples of symmetries and the associated conservation laws

Symmetry		Conservation Law
Translation in time	$\longleftrightarrow$	Energy

In words :

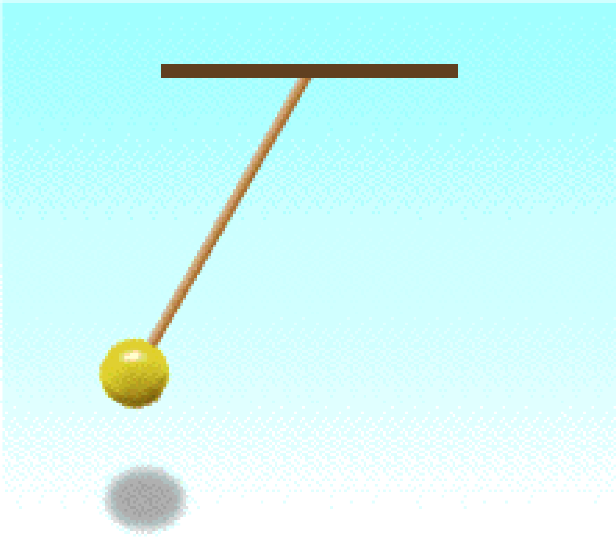
In this time transformation, the same laws of physics that apply today have been applied in the past, and will apply in the future.

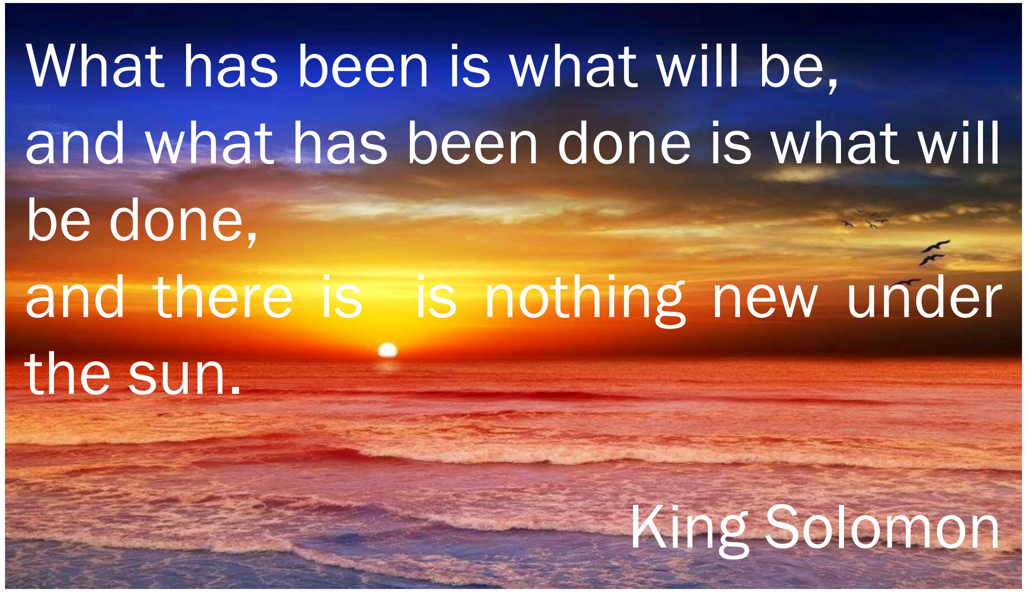
F. Wilczek, *A beautiful Equation*, 2015 Penguin Book.

Symmetries in Nature

Symmetry and the Time Transformation

So you now know why Energy is conserved.





F. Wilczek, *A beautiful Equation*, 2015 Penguin Book.

Examples : Supersymmetry

Ordinary Matter (Bosons Fermions)	Super-matter partner Introduce more Matter
Quarks	Squarks
Leptons	Sleptons
W Particles	Winos, Zinos
Higgs particle	Higgsino
Gluon	Gluino
Photon	Photino
Graviton	Gravitino

G. Kane, *Supersymmetry*, 2000, Perseus Pub.
Zhang YY and A.H. Chan, *Stochastic model for Supersymmetric Particle …* , to appear Modern Physics Letters **A** (2017)

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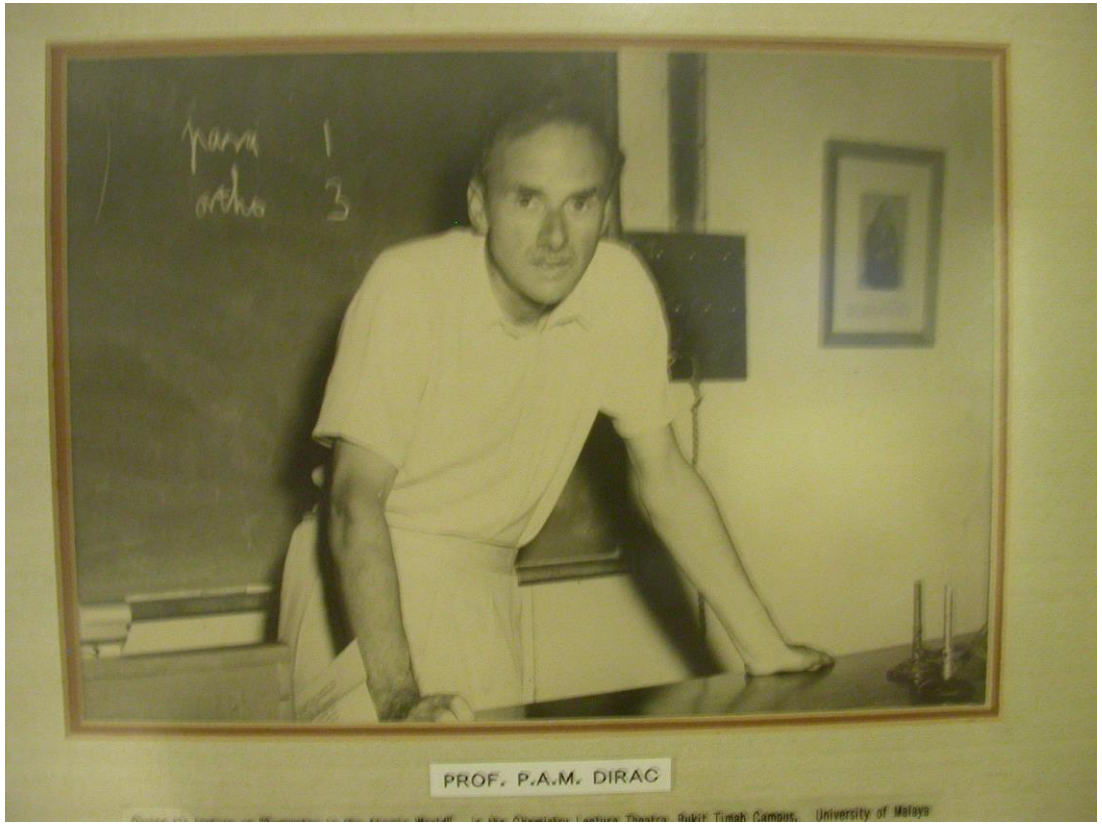
Is there Beauty in Equation ?

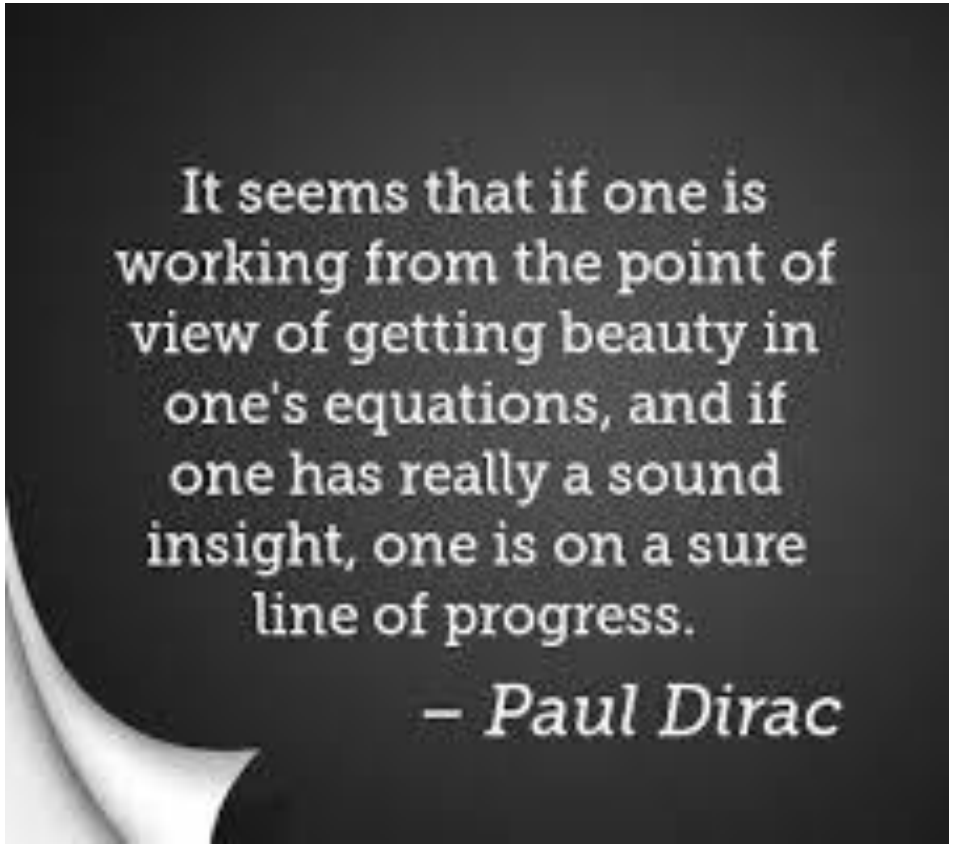


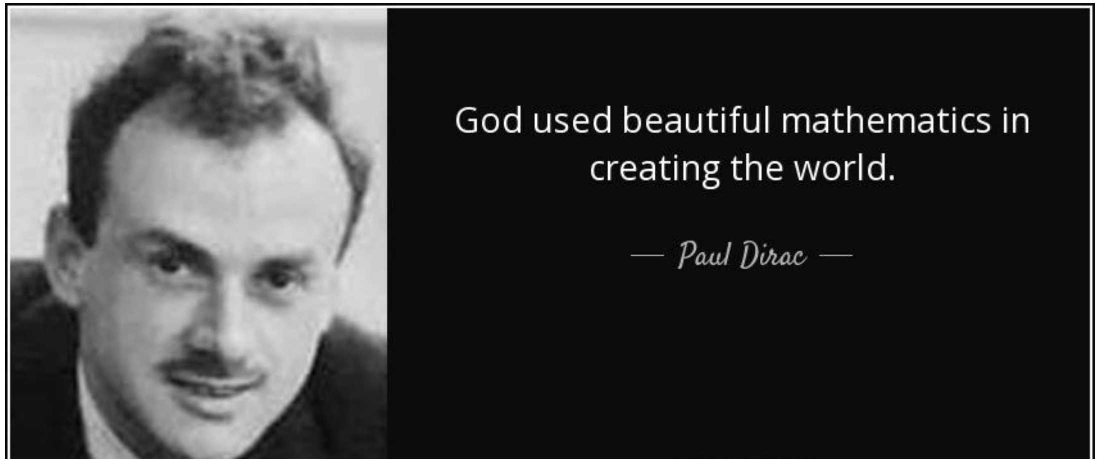
It is more important to have beauty in one's equations than to have them fit experiment.

Paul Dirac

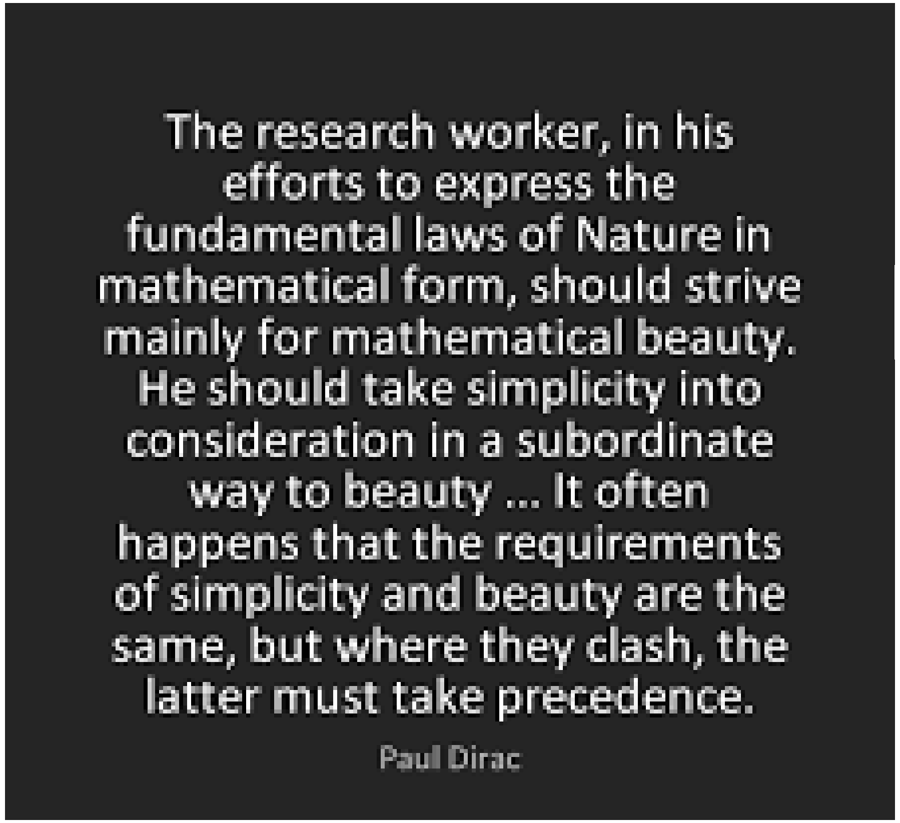
This picture was taken at the physics department in the Singapore University at Bukit Timah campus

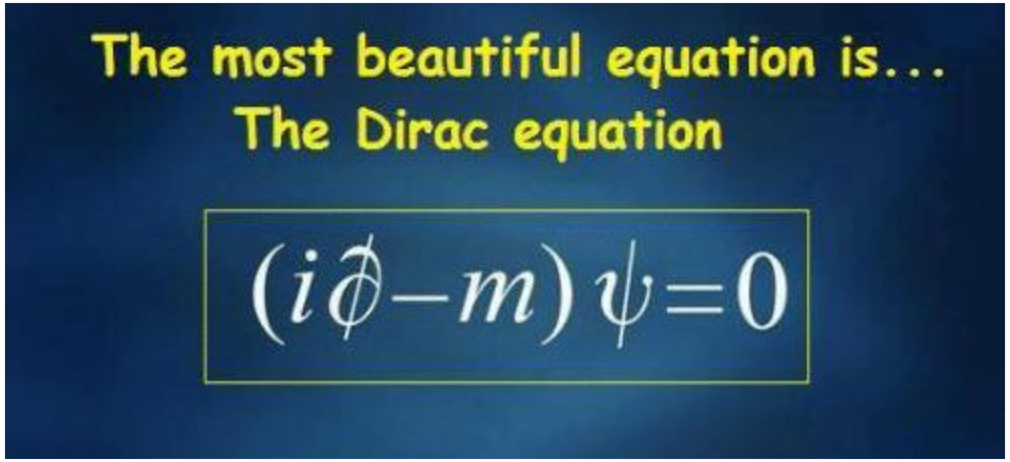
Is there Beauty in Equation ?

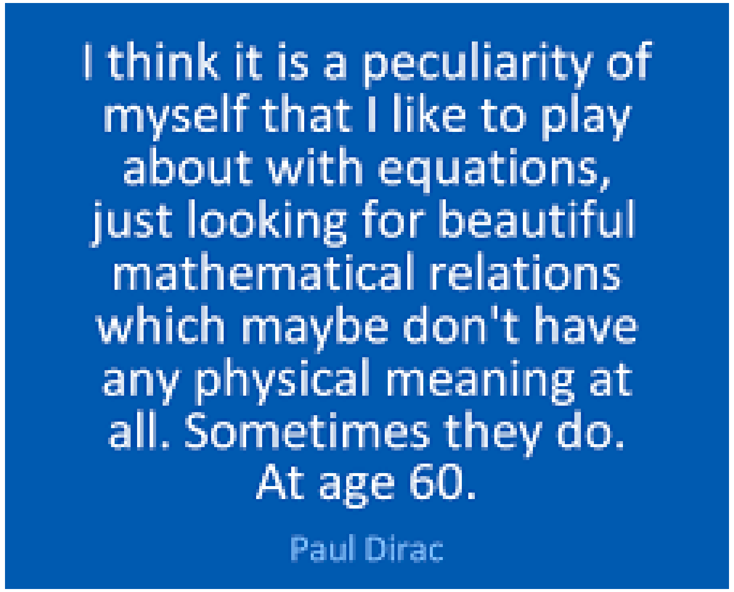




Dirac shared the 1933 Nobel Prize in Physics.







Among other discoveries, he formulated the Dirac equation which describes the behaviour of fermions and predicted the existence of antimatter. Dirac shared the 1933 Nobel Prize in Physics with Erwin Schrödinger "for the discovery of new productive forms of atomic theory

Schrodinger vs Dirac

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} = i\hbar\frac{\partial\Psi}{\partial t}$$

Schrodinger equation for free particle in one dimension.

$$i\left(\frac{\partial}{\partial t}+\alpha\boldsymbol{\sigma}\cdot\frac{\partial}{\partial \mathbf{x}}\right)\psi\left(\boldsymbol{x},t\right)=m\beta\psi\left(\boldsymbol{x},t\right)$$

A great deal of my work is just playing with equations and seeing what they give.

Paul Dirac

Euler Equation is Beautiful !

Caveat :

Reminder :

Mathematics is not Physics

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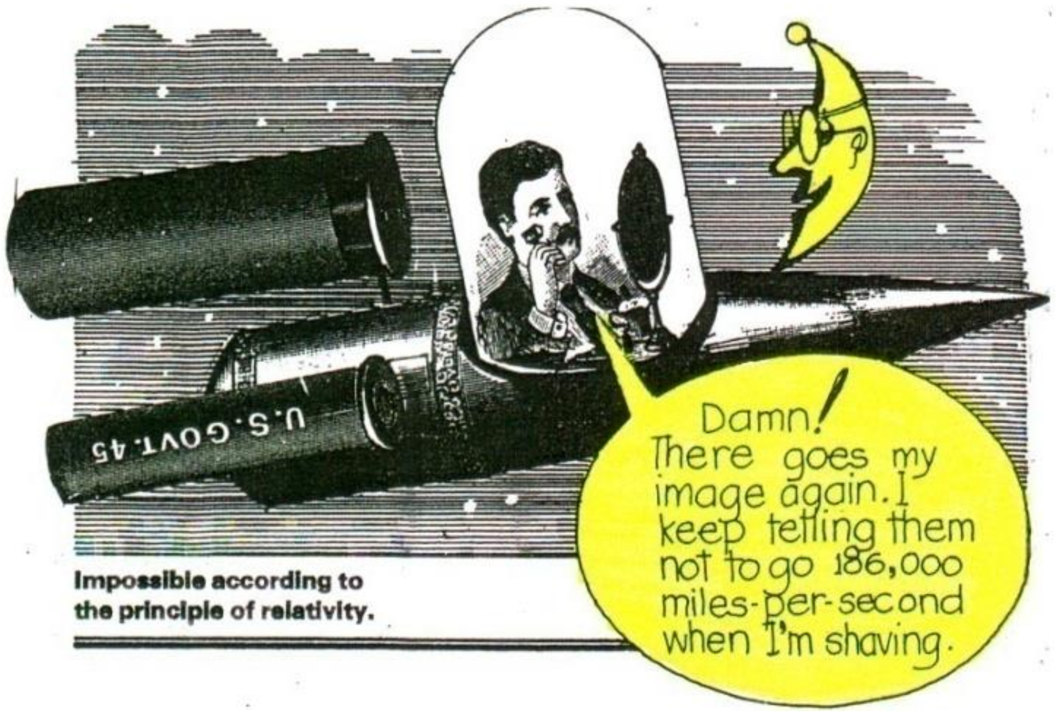
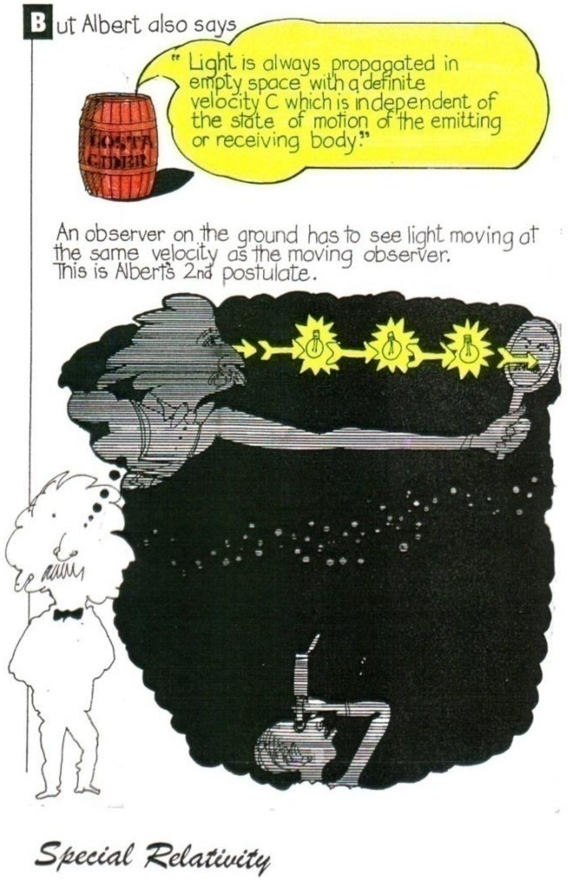
Example : Einstein's Mirror

Einstein also asked if he can see himself in the mirror when he rides on a beam of light ?

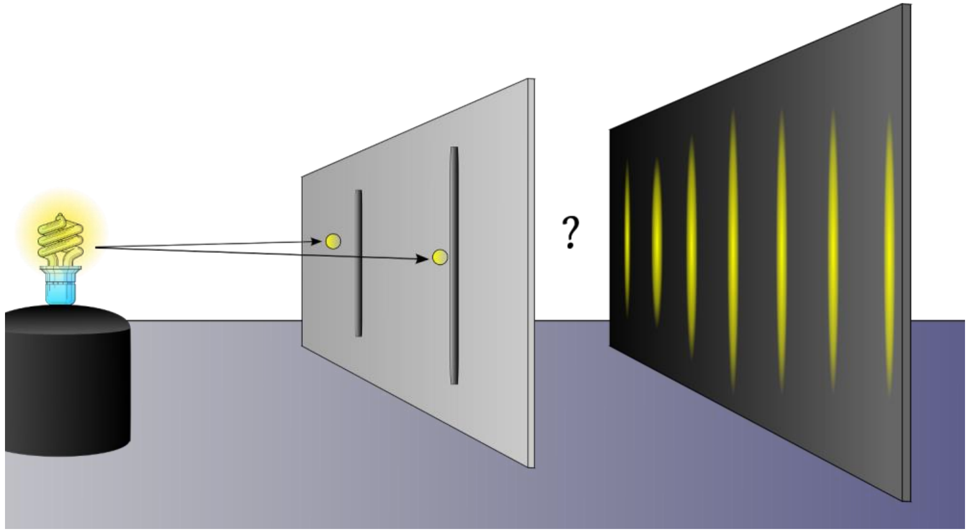


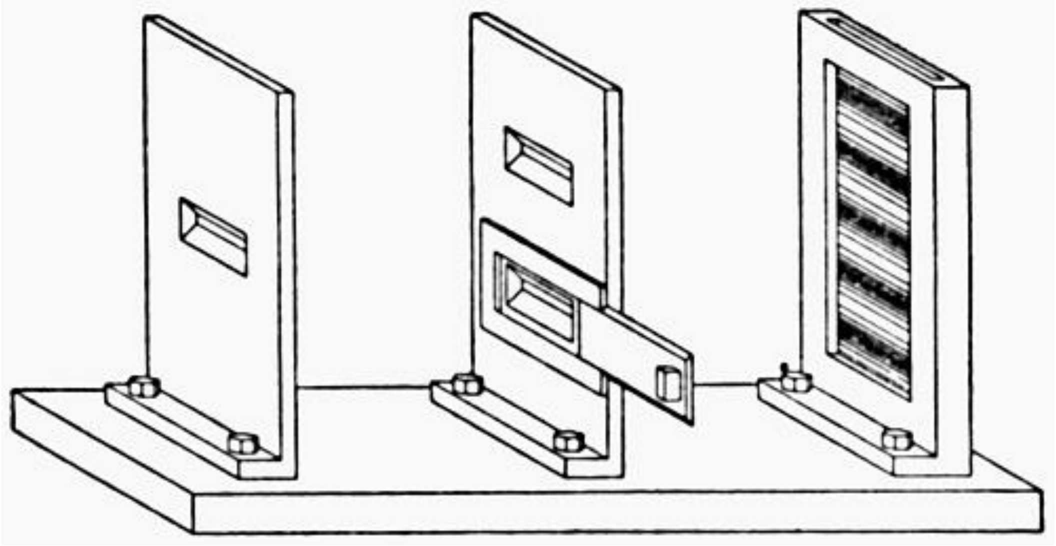


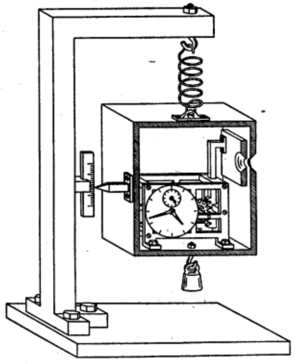
Is it possible construct Gedanken (s)?

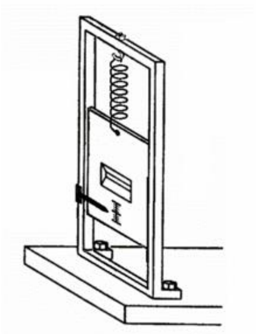
At the 5th Solvay conference in 1927 (see picture below), Einstein raised doubts regarding Bohr's Quantum interpretation of the 2 slits experiment.

Is it possible construct Gedanken (s)?









Bohr

https://en.wikipedia.org/wiki/Niels_Bohr

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Are there Broken Symmetries ?

2013 Nobel Prize in Physics:

Discovery of the Higgs Boson and the Symmetry Breaking Theory that predicted it.

François Englert (Universite Libre de Bruxelles) and Peter Higgs (University of Edinburgh)were instrumental in developing the theory that helps explain the origin of mass of elementary particles and predicts the existence of the Higgs particles.

Analogy : (Higgs Field) particle give mass

Higg's particle gives mass to the other particles like the electrons and quarks.

"Playing football on a wet field"

Higg's particle is the manifestation of the Higgs field.

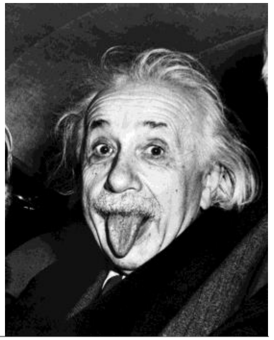
What is a good Physics Theory ?

Any sound scientific theory, whether of time or of any other concept, should in my opinion be based on the most workable philosophy of science : the positivist approach put forward by Karl Popper and others. According to this way of thinking, a scientific theory is a mathematical model that describes and codifies the observations we make. A good theory will describe a large range of phenomena on the basis of a few simple postulates and will make definite predictions that can be tested.

If the predictions agree with the observations, the theory survives that test, though it can never be proved to be correct. On the other hand, if the observations disagree with the predictions, one has to discard or modify the theory. If one takes the positives position, as I do, one cannot say what "*time*" actually is. All one can do is to describe what has been found to be a very good mathematical model for time and say what predictions it makes.

S. Hawking, *The Universe in a NutShell*, 2001, Bantam Book.
S. Fuller, Kuhn vs Popper, 2004, Columbia U. Press

Conclusion : A Caveat !

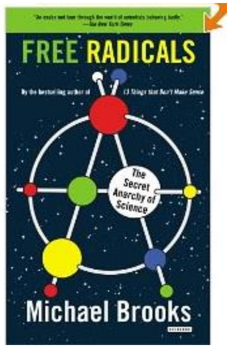


Einstein once advised that if you want to know how theoretical physics gets done, the last person you should ask is a theorist.

I advised you to stick closely to one principle; don't listen to their words, fix your attention on their deeds', he said.

M. Brooks, *The Secret Anarchy of Science*, 2012 Profile Books

Surprise : Einstein did not manage to prove $E = mc^2$!



"Einstein was entirely cavalier about the sacred process of science …

Einstein once advised that if you want to know how theoretical physics gets done, the last person *you should ask is a theorist …*

The final attempt to prove $E = mc^2$ came in 1934 … four hundred scientists were given the treat of watching him remodel his universe … But the proof was still wrong … The error had been pointed out years by no less an authority than Max Planck." Michael Brooks

M. Brooks, *The Secret Anarchy of Science*, 2012 Profile Books

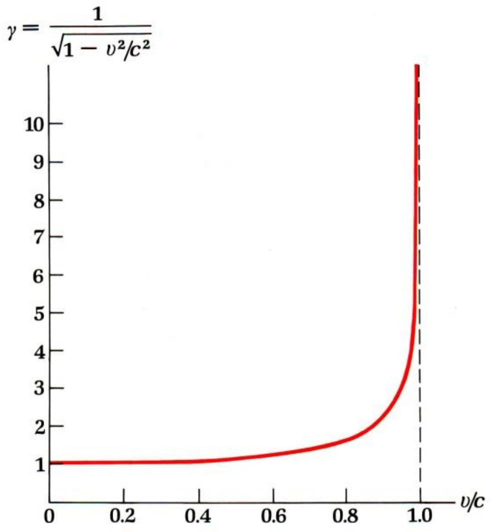
Appendix : Relativistic Mass

Does this mean that momentum can increase without any limit?

Does this mean that speed can also increase without any limit?

Caveat Emptor

In Newtonian ideas, the particle behaves as if its mass increases with speed. Einstein initially favoured this interpretation but later changed his mind to keep mass as a constant, an intrinsic property of matter that is the same in all frames of reference. So it is gamma that changes with speed, not mass.



$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma\, m_0$$