

CS1231 Review 5

1. Determine whether the following are true or false.

- $\forall x \in \mathbb{R} \forall y \in \mathbb{R} (x + y = 0)$. **F** (e.g. $x = 1, y = 2$)
- $\exists x \in \mathbb{R} \exists y \in \mathbb{R} (x^2 + y^2 = 0)$. **T** ($x = 0, y = 0$)
- T** • $\forall x \in \mathbb{R} \exists y \in \mathbb{R} (x + y = 0)$. For every x , there is $y = -x$ such that $x + y = 0$
- F** • $\exists y \in \mathbb{R} \forall x \in \mathbb{R} (x + y = 0)$. There is y such that $x + y = 0$ for all x
- F** • $\forall x \in \mathbb{R} \exists y \in \mathbb{R} (xy = 1)$. For every x there is y such that $xy = 1$
- T** • $\exists y \in \mathbb{R} \forall x \in \mathbb{R} (xy = 0)$. $y = 0$ $\forall x \in \mathbb{R} (xy = 0)$

2. Let $L(x, y)$ be " x loves y ". Use Quantifiers to Express "Nobody loves everybody".

Nobody loves everybody
 $\equiv \neg$ "everybody loves everybody"
 $\equiv \neg$ "somebody loves everybody"

Nobody gets A
 $\equiv \neg$ "somebody gets A"

Nobody loves everybody
 $\equiv \neg$ "somebody loves everybody"
 $\equiv \neg \exists x$ "x loves everybody"
 $\equiv \neg \exists x \forall y$ "x loves y"
 $\equiv \neg \exists x \forall y L(x, y)$
 $\equiv \forall x \exists y \neg L(x, y)$

For every x , there is y , x does not love y