ST2334 (2018/2019 Semester 2) Solutions to Questions in Tutorial 3

Question 1

(a) $\Pr(A \cap B \cap C) = \Pr(A)\Pr(B \mid A)\Pr(C \mid A \cap B) = 0.75(0.9)(0.8) = 0.54$.

(b) $\Pr(B) = \Pr(A \cap B) + \Pr(A' \cap B) = \Pr(A)\Pr(B \mid A) + \Pr(A')\Pr(B \mid A') = (0.75)(0.9) + (0.25)(0.8) = 0.875$.

(c) $\Pr(A \mid B) = \Pr(A \cap B)/\Pr(B) = [(0.75)(0.9)]/0.875 = 0.7714$.

(d) $\Pr(B \cap C) = \Pr(A \cap (B \cap C)) + \Pr(A' \cap (B \cap C))$. But $\Pr(A' \cap B \cap C) = \Pr(A') \Pr(B \mid A')\Pr(C \mid A' \cap B) = 0.25(0.8)(0.7) = 0.14$. Therefore $\Pr(B \cap C) = 0.54 + 0.14 = 0.68$.

(e) $\Pr(A \mid B \cap C) = \Pr(A \cap B \cap C) / \Pr(B \cap C) = 0.54/0.68 = 0.7941$.

Question 2

Let A = {product A profitable}, B = {product B profitable} and $C = A \cup B$.

$\Pr(A) = \Pr(B) = 0.18$. $\Pr(A \cap B) = 0.05$. So $\Pr(C) = \Pr(A \cup B) = 0.18 + 0.18 - 0.05 = 0.31$.

(a) $\Pr(A \mid B) = \Pr(A \cap B) / \Pr(B) = 0.05/0.18 = 0.2777$.

(b) $\Pr(A \mid C) = \Pr(A \cap C) / \Pr(C) = \Pr(A)/\Pr(C) = 0.18/0.31 = 0.5806$.

Question 3

Let A = {TQM implemented} and B = {sales increased}.

(a) $\Pr(A) = 0.3$. $\Pr(B) = 0.6$.

(b) Since $\Pr(A \mid B) = 20/60$, therefore $\Pr(A \cap B) = \Pr(A \mid B) \Pr(B) = (1/3)0.6 = 0.2$. As $\Pr(A \cap B) \neq \Pr(A)\Pr(B) = 0.18$, therefore A and B are not independent events.

(c) Since $\Pr(A \mid B) = 18/60$, therefore $\Pr(A \cap B) = \Pr(A \mid B) \Pr(B) = (0.3)0.6 = 0.18$. As $\Pr(A \cap B) = \Pr(A)\Pr(B)$, therefore A and B are independent events.

Question 4

Let B be the event that a component needs rework. Then

$\Pr(B) = \Pr(A_1)\Pr(B \mid A_1) + \Pr(A_2)\Pr(B \mid A_2) + \Pr(A_3)\Pr(B \mid A_3) = (0.5)(0.05) + (0.3)(0.08) + (0.2)(0.1) = 0.069$.

(a) $\Pr(A_1 \mid B) = [(0.5)(0.05)]/0.069 = 0.3623$.

(b) $\Pr(A_2 \mid B) = [(0.3)(0.08)]/0.069 = 0.3478$.

(c) $\Pr(A_3 \mid B) = [(0.2)(0.1)]/0.069 = 0.2899$.

Notice that $\Pr(A_1 \mid B) + \Pr(A_2 \mid B) + \Pr(A_3 \mid B) = 1$.

Question 5

(a) $\Pr(A_1) = \Pr(\text{draw slip 1 or 4}) = 1/2$. Similarly, $\Pr(A_2) = 1/2$ and $\Pr(A_3) = 1/2$.
$\Pr(A_1 \cap A_2) = \Pr(\text{draw slip 4}) = 1/4$, Similarly $\Pr(A_1 \cap A_3) = 1/4$ and $\Pr(A_2 \cap A_3) = 1/4$.
Since $\Pr(A_1 \cap A_2) = \Pr(A_1) \Pr(A_2)$, $\Pr(A_1 \cap A_3) = \Pr(A_1) \Pr(A_3)$ and $\Pr(A_2 \cap A_3) = \Pr(A_2) \Pr(A_3)$, therefore the events A_1, A_2 and A_3 are pairwise independent.

(b) $\Pr(A_1 \cap A_2 \cap A_3) = \Pr(\text{draw slip 4}) = ¼$. But $\Pr(A_1)\Pr(A_2)\Pr(A_3) = 1/8 \neq 1/4$, therefore the events A_1, A_2 and A_3 are not mutually independent.

Question 6

(a) Since all the four components work independently, $\Pr(\text{system works}) = \Pr(A \cap (B \cup C) \cap D) = \Pr(A)\Pr(B \cup C)\Pr(D) = (0.95)[0.7 + 0.8 - (0.7)(0.8)](0.9) = 0.8037$.

(b) $\Pr(C \text{ does not work} \mid \text{system works}) = \Pr(\text{system works but } C \text{ does not work})/\Pr(\text{System works}) = \Pr(A \cap B \cap C' \cap D)/\Pr(\text{system works}) = [(0.95)(0.7)(0.2)(0.9)]/0.8037 = 0.1489$.

Question 7

Let A_i = {ith vehicle passes the inspection}. $\Pr(A_1) = \Pr(A_2) = \Pr(A_3) = 0.6$

(a) $\Pr(A_1 \cap A_2 \cap A_3) = \Pr(A_1)\Pr(A_2)\Pr(A_3) = (0.6)^3 = 0.216$ since A_i's are independent.

(b) Pr(At least one failures) = 1 – Pr(All pass) = 1 – 0.216 = 0.784.
Or $\Pr(A_1' \cup A_2' \cup A_3') = \Pr((A_1 \cap A_2 \cap A_3)') = 1 - \Pr(A_1 \cap A_2 \cap A_3) = 1 - 0.216 = 0.784$.

(c) $\Pr(A_1 \cap A_2' \cap A_3') + \Pr(A_1' \cap A_2 \cap A_3') + \Pr(A_1' \cap A_2' \cap A_3) = (0.6)(0.4)(0.4) + (0.4)(0.6)(0.4) + (0.4)(0.4)(0.6) = 0.288$.

(d) Pr(At least one pass) = 1 – Pr(All fail) = $1 - (0.4)^3 = 0.936$.
Pr(#pass =3 | #pass ≥ 1) = Pr(#pass = 3 ∩ #pass ≥ 1)/Pr(#pass ≥ 1) = Pr(#pass = 3) / Pr(#pass ≥ 1) = 0.216/ 0.936 = 0.2308.

Question 8

Let A = {Get into a house}, B = {the house is unlocked} and C = {Agent gets the correct key}

It is given that $\Pr(B) = 0.4$.

$\Pr(C) = 1/8 + (7/8)(1/7) + (7/8)(6/7)(1/6) = 3/8$.

Alternatively, $\Pr(C) = ({}_1C_1)({}_7C_2)/{}_8C_3 = 3/8$.

$\Pr(A) = \Pr(B)\Pr(A \mid B) + \Pr(B')\Pr(A \mid B') = 0.4(1) + 0.6(3/8) = 5/8$.