

Problem 1. Easy Money

Oizne Mak went bankrupt after his previous party. He just got a new job, and wants to impress his new boss, Nayr Wehc to get a raise. The company is always constantly exchanging currencies to pay their suppliers from all over the world, so Nayr Wehc wants Oizne Mak to find a way to minimise their loss from exchanging different currencies. From a quick search on the search engine Elgoog, Oizne Mak now knows the exchange rates for all pairs of currencies that he needs to know. Being lazy as usual, Oizne Mak wants you to help him do this work, so he can get paid for free!

Problem 1.a. Give an algorithm to help determine the best sequence of currencies to be exchanged for all possible pairs of currencies A to B , such that each unit of A results in the maximum number of units of B .

Oizne Mak realises that it is possible to have currencies that have a greater value that it started with. For example, currency A with one unit could go through a series of exchanges that ends at A again, but results in more than one unit of A . Oizne Mak wants to exploit these currencies so he can earn more easy money, and shares his idea with you so that you can help him do it.

Problem 1.b. Give an algorithm to help determine if there are any currencies that can be exploited. **You should use your algorithm in Problem 1.a. to solve this problem.**

Problem 2. Comfy Taxi

Wehttam drives a really comfortable taxi, and Oizne Mak wants to get a comfy ride around Singapore. Oizne Mak has a premium ticket for Wehttam's taxi, which allows him to travel in the taxi for free!

Wehttam needs to plan out his journey before picking up Oizne Mak. He does this by creating a map of all the roads and road junctions in Singapore, and remembers the routes by remembering the sequence of road junctions and roads he has to go through.

Problem 2.a. Oizne Mak realises that his ticket is not a premium ticket! His ticket is a special ticket that only allows him to travel using exactly k roads in a single trip. Can Wehttam work out beforehand whether it is **possible** to travel from any pickup point, to any other place in Singapore, for a given value of k ?

(**Note:** Unlike regular paths, Wehttam is allowed to revisit the same location.)

(**Hint:** First consider the cases $k = 0, 1, 2$. Then, how can we extend that to longer trips?)

Problem 2.b. Oh no! Wehttam has fallen sick! He has asked Mahri to take over for him for the rest of the day. Mahri does not have much experience and is not familiar with the roads in Singapore. He will need to drive many other passengers, each with a even more special ticket that allows them to travel only using not more than k roads.

Mahri is paranoid and wants to know all such shortest routes from any pickup point, so he is prepared for any contingency. Give an algorithm to find for all locations in Singapore, the shortest distance to any other place in Singapore that uses not more than k roads.