CS1231 TUTORIAL 2

1. Translate the following into English where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.

(a) $\forall x(R(x) \to H(x))$ (b) $\forall x(R(x) \wedge H(x))$

(c) $\exists x(R(x) \to H(x))$ (d) $\exists x(R(x) \wedge H(x))$

2. Let $Q(x, y)$ be the predicate "If $x < y$ then $x^2 < y^2$" with domain for both x and y being $\{1, \pm 2\}$..

(a) Why is $Q(x, y)$ false for $(x, y) = (-2, 1)$, and true for $(x, y) = (1, 2)$?

(b) Find all the values of x and y for which $Q(x, y)$ is true.

3. Rewrite each of the following in the form $\forall$___, if ___ then ___

(a) All integers having even squares are even.

(b) Given any integer whose square is even, that integer is itself even.

(c) The square of any even integer is even.

(d) All even integers have even squares.

4. Which of the following are true? If false, justify your answers.

(a) $\forall x \in \mathbb{R}, x > 2 \ \to \ x > 1$.

(b) $\forall x \in \mathbb{R}, x > 2 \ \to \ x^2 > 4$.

(c) $\forall x \in \mathbb{R}, x^2 > 4 \ \to \ x > 2$.

(d) $\forall x \in \mathbb{R}, x^2 > 4 \ \leftrightarrow \ |x| > 2$.

5. Let $D(x)$, $P(x)$, $O(x)$, $W(x)$ be "x is a duck", "x is one of my poultry", "x is an officer", "x is willing to waltz". Express each of (a), (b), (c), (d) using quantifiers, logical connectives and $D(x)$, $P(x)$, $O(x)$, $W(x)$.

(a) No ducks are willing to waltz. (b) No officers ever decline to waltz.

(c) All my poultry are ducks. (d) My poultry are not officers.

(e) If (a), (b), (c) are all true, does it follow that (d) is also true?

6. Write a negation for each of the following:

(a) $\forall d \in \mathbb{Z}$, if $\frac{6}{d} \in \mathbb{Z}$, then $d = 3$.

(b) If the square of an integer is odd, then the integer is odd.

7. Rewrite the following without using the words *necessary* or *sufficient*.

(a) Being a bird is not a necessary condition for an animal being able to fly.

(b) Being a polynomial is not a sufficient condition for a function to have a real root.

8. Let $D = E = \{0, \pm 1, \pm 2\}$. Write a negation of the following and determine which is true, the given statement or its negation.

$\exists x \in D$ such that $\forall y \in E$, $x + y = -y$.

9. Write a negation for each of following.

(a) $\forall r \in \mathbb{Q}$, $\exists a \in \mathbb{Z}$ and $\exists b \in \mathbb{Z}$ such that $r = a/b$.

(b) $\exists x \in \mathbb{R}$ such that for all $y \in \mathbb{R}$, $x + y = 0$.

(c) $p \leftrightarrow q$

10. For any propositions p and q, write a logical expression S involving p, q, using logical connectives so that S is true when exactly one of p, q is true and is false otherwise.