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Bill Gates and the Pancake Flipping Problem Video 6.1

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Experience the fun of problem solving

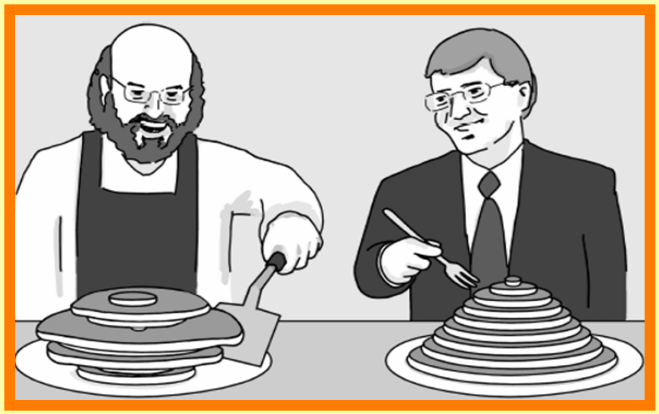
Did you know that …

I used to *flip* pancakes [翻煎饼].

Bill Gates, *founder and former COE* of Microsoft

The Story:

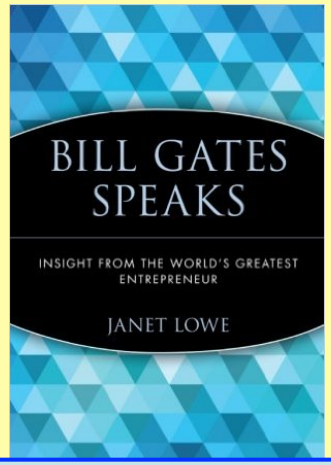
Bill Gates "*flipped pancakes*" when he was at Harvard.



Source: Neil Jones and *Pavel Pevzner*, 2004 "Introduction to BioInformatics Algorithms".

THE PANCAKE PUZZLE

"The chef in our place is sloppy, and when he prepares a stack of pancakes they come out all different sizes. Therefore, when I deliver them to a customer, on the way to the table I rearrange them (so that the smallest winds up on top, and so on, down to the largest at the bottom) by grabbing several from the top and flipping them over, repeating this (varying the number I flip) as many times as necessary. If there are "n" pancakes, what is the maximum number of flips (as a function of "n") that I will ever have to use to rearrange them?"



by Janet Lowe. 2001.

(page 19)

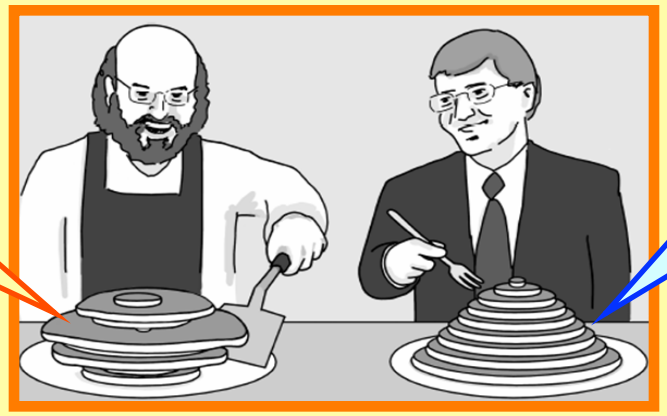
http://www.npr.org/templates/story/story.php?storyId=92236781

The Pancake Puzzle

"The chef in our place is sloppy, and when he prepares a stack of pancakes, they come out all different sizes. Therefore, when I deliver them to a customer, on the way to the table, I rearrange them (so that the smallest winds up on top, above the next smallest, and so on, down to the largest at the bottom) by grabbing several from the top and flipping them over, repeating this (varying the number I flip) as many times as necessary. If there are "n" pancakes, what is the maximum number of flips (as a function of "n") that I will ever have to use to rearrange them?"

What is *pancake flipping*?

Input:
"*Messy*" stack of pancakes



Output: Neat, *sorted* stack of pancakes

Source: Neil Jones and *Pavel Pevzner*, 2004
"Introduction to BioInformatics Algorithms".

pancake flipping

Pancake Flipping

Content:

- When did Bill Gates flip pancakes
- The Pancake Flipping Problem
- What we know about Pancake Flipping
- Why Study Pancake Flipping?

Pancake Flipping Problem

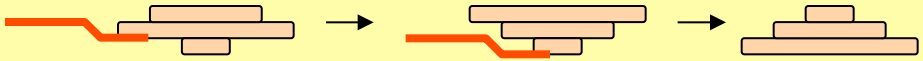
Given an initial pancake configuration...

You want to get a "*sorted*" configuration …

Constraints: can *only* flip …

Source: Neil Jones and Pavel Pevzner, 2004 "Introduction to BioInformatics Algorithms".

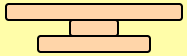
Example …



2 flips

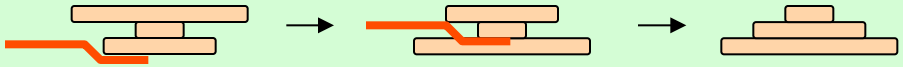
Bill Gates & Christos Papadimitriou:, "Bounds For Sorting By Prefix Reversal." *Discrete Mathematics*, Vol 27, pp 47-57, 1979.

More pancake-flipping examples…

C1  **2 flips**

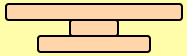
Pause the video here. Try to solve this "in your head".

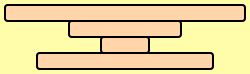
Here's how we do it.
(Actually, you shouldn't need this.)

C1  **2 flips**

Was that what you did mentally?

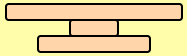
More pancake-flipping examples…

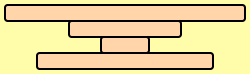
C1  **2 flips**

C2  **3 flips**

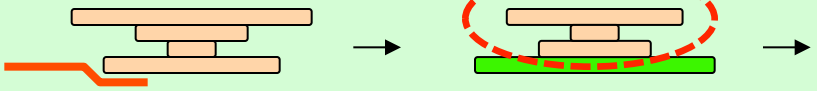
Pause the video here. Try to solve this “in your head”.

Did you do decomposition?

C1  **2 flips**

C2  **3 flips**

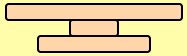
Here's how you can do it…

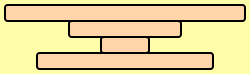
C2 

Largest pancake is sorted;
Next, sort the top-3;

top-3 same as config C1.
Solved before.
This is *Decomposition*.

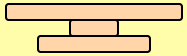
More pancake-flipping examples…

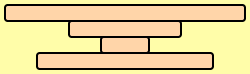
C1  **2 flips**

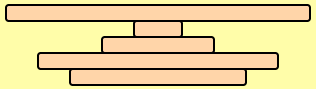
C2  **3 flips**

Abstraction, Heuristics PS, Decomposition

More pancake-flipping examples…

C1  2 flips

C2  3 flips

C3  ? flips

Abstraction,
Heuristics PS,
Decomposition

Need a systematic approach… an algorithm!

We need an Algorithm

Simple Idea:

Q: Can we borrow idea from C2 solution?

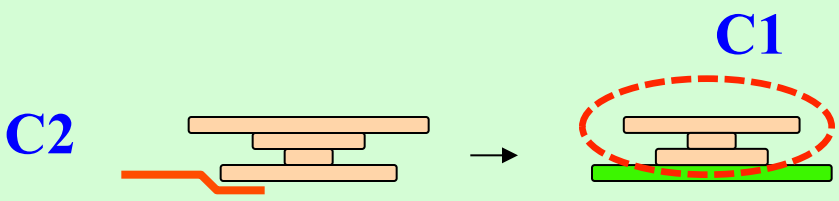
Re-examine decomposition process

Sort-Stack (C2, n)

"Sort" only largest pancake

Sort-Stack (C1, n–1)

Decomposition …in action



"Power" of decomposition process

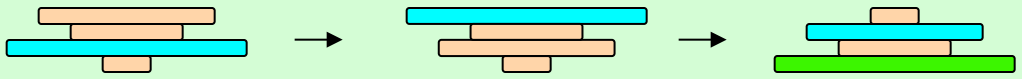
Sort-Stack (C2, n)

"Sort" only largest pancake

1. Flip largest pancake to top;
2. Flip largest pancake to bottom

2 flips

"Sort" only the largest pancake

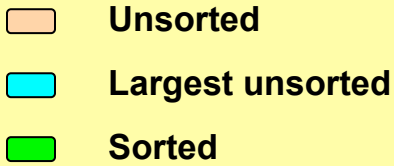


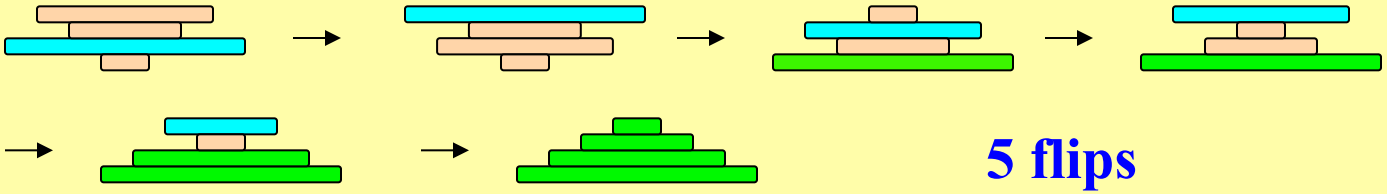
2 flips

An Initial Algorithm (Greedy)

Simple Idea:

"Sort" the biggest ***unsorted*** pancake first…





5 flips

Greedy Algorithm:

Repeatedly "sort" the biggest unsorted pancake;

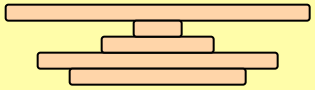
What is a "Greedy" Algorithm

A "greedy" algorithm is one that, when faced with several possible choices, *always* make a *greedy choice* based only on current information.

Such a choice may not always produce an optimal solution. Hence, greedy algorithms do not always give optimal solutions.
(Sometimes, we are lucky and greedy algorithm happens to give optimal solutions.)

Activity: Fun with Pancake Flipping

For this example, first use Greedy Algorithm.
Then try to find a better solution.

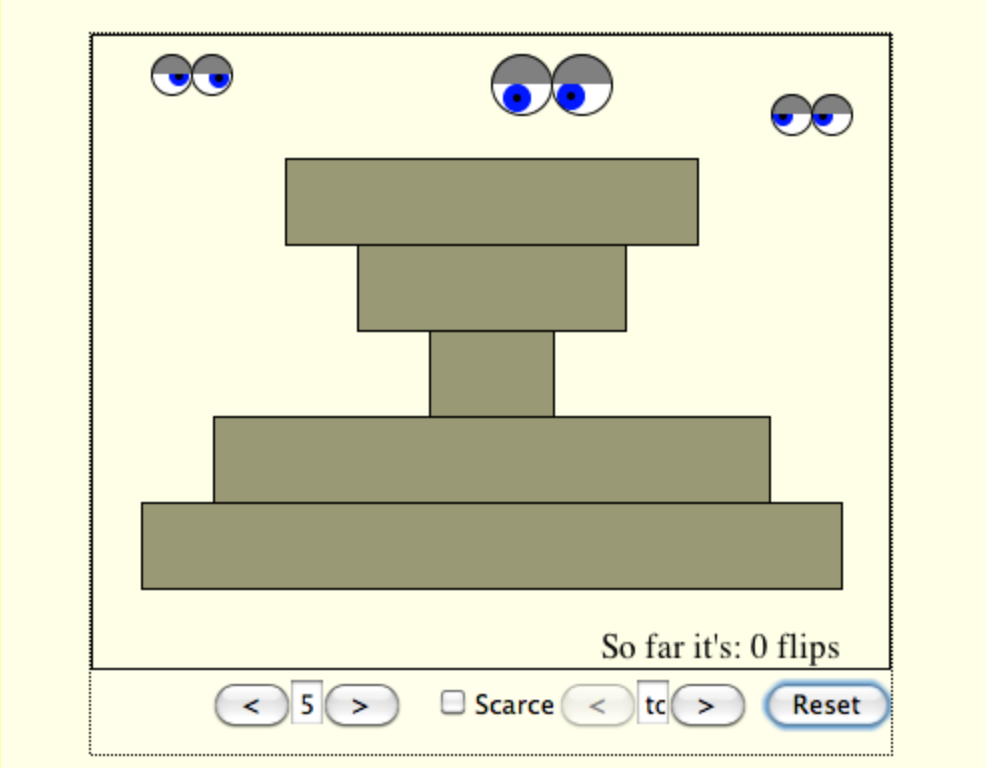


___ Flips (using Greedy Algorithm)

___ Flips (in your best solution)

Try Pancake Flipping Online:

http://www.cut-the-knot.org/SimpleGames/Flipper.shtml



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(End of video 6.1)

If you want to contact me,

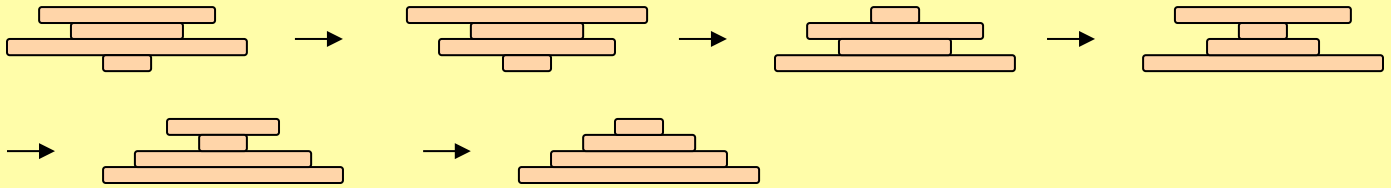
Email: leonghw@comp.nus.edu.sg

Is Greedy "the best" possible?

Answer: NO

A Counter Example:

Greedy method **[5 flips]**



Better way **[3 flips]**

Question: **Design an algorithm that solve the pancake flipping problems using the *minimum number of flips.***

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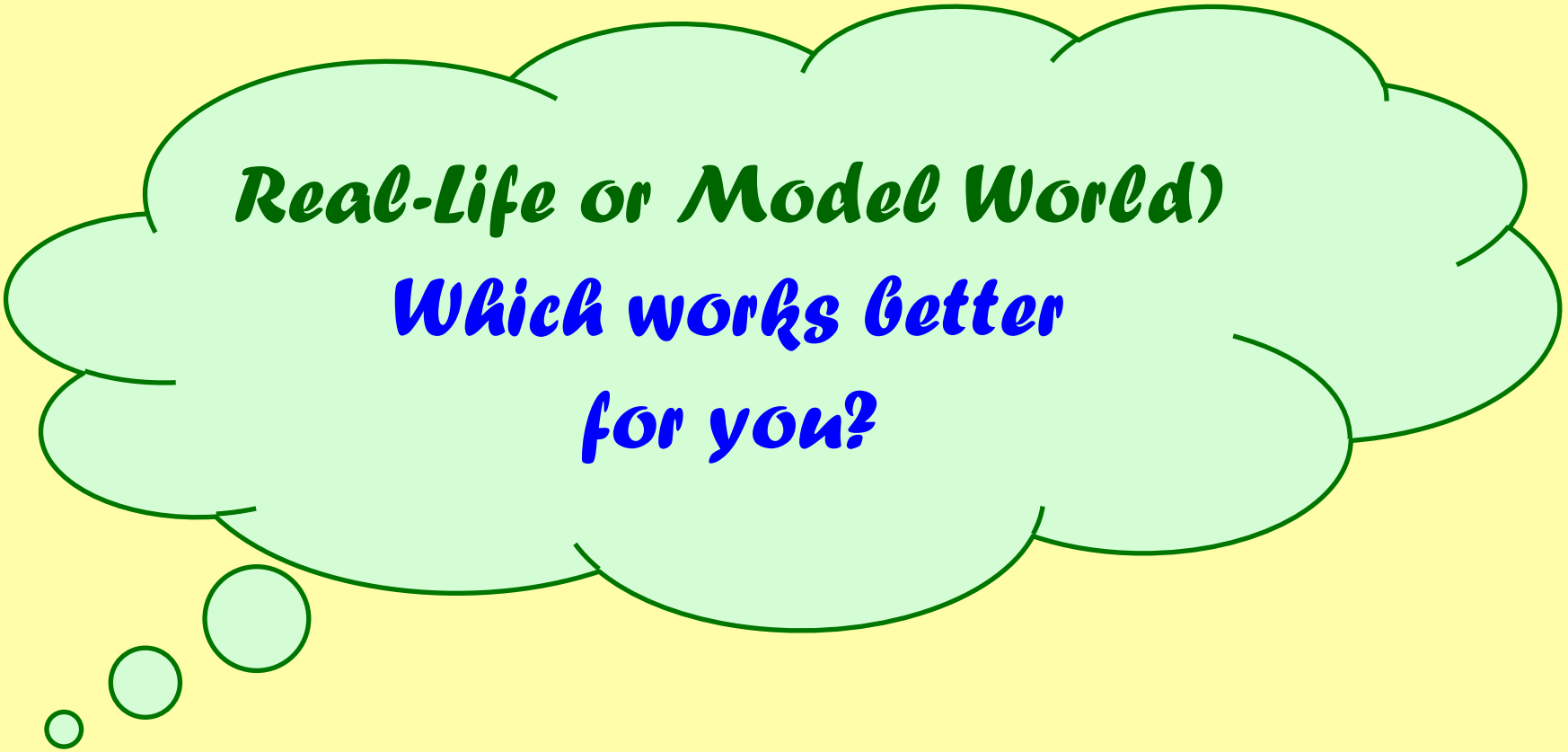
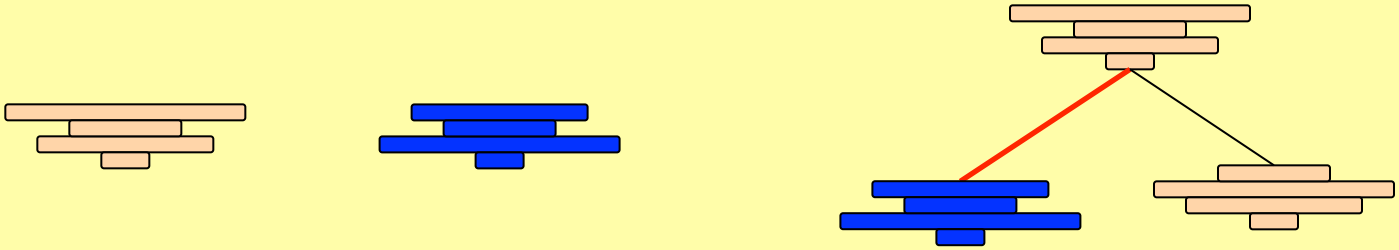
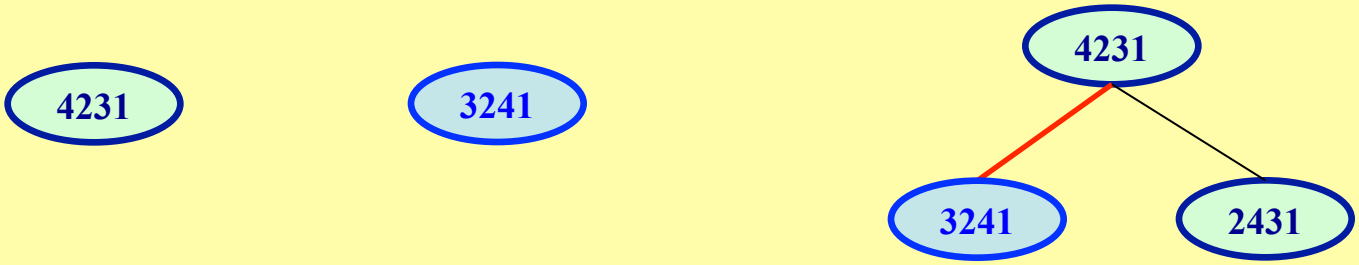
How about Modelling?



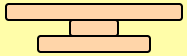
Real World (of Pancakes…) vs Model world (of Graph…)

Represent pancake sizes by numbers.



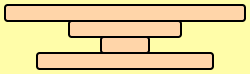


More pancake-flipping examples…



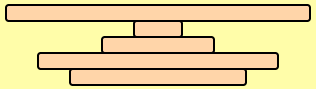
2 flips

Abstraction skills,
Problem Solving skills



3 flips

Need a systematic approach… an algorithm!



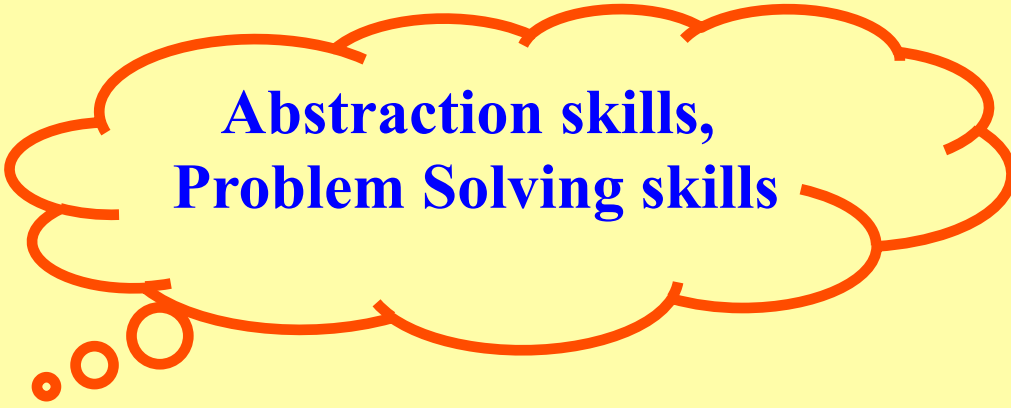
? flips

The Model World…

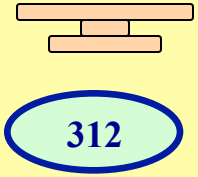
312 — 2 flips

4213 — 3 flips

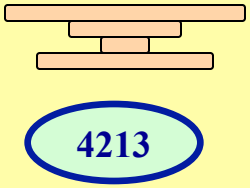
52143 — ? flips



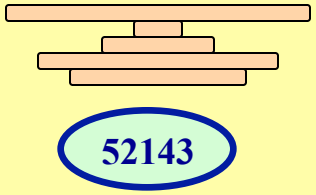
Let's see both together…



2 flips



3 flips



? flips

Which works better for YOU?

Real-World or Model World?

Is Greedy "the best" possible? NO

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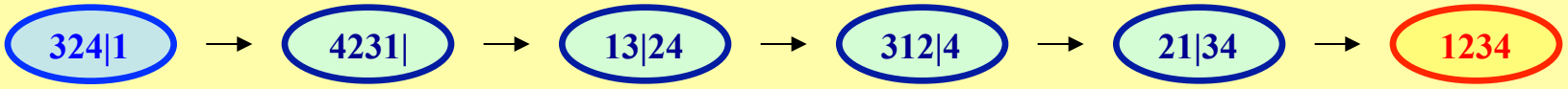
Model World
(One more slide)

A Counter Example:

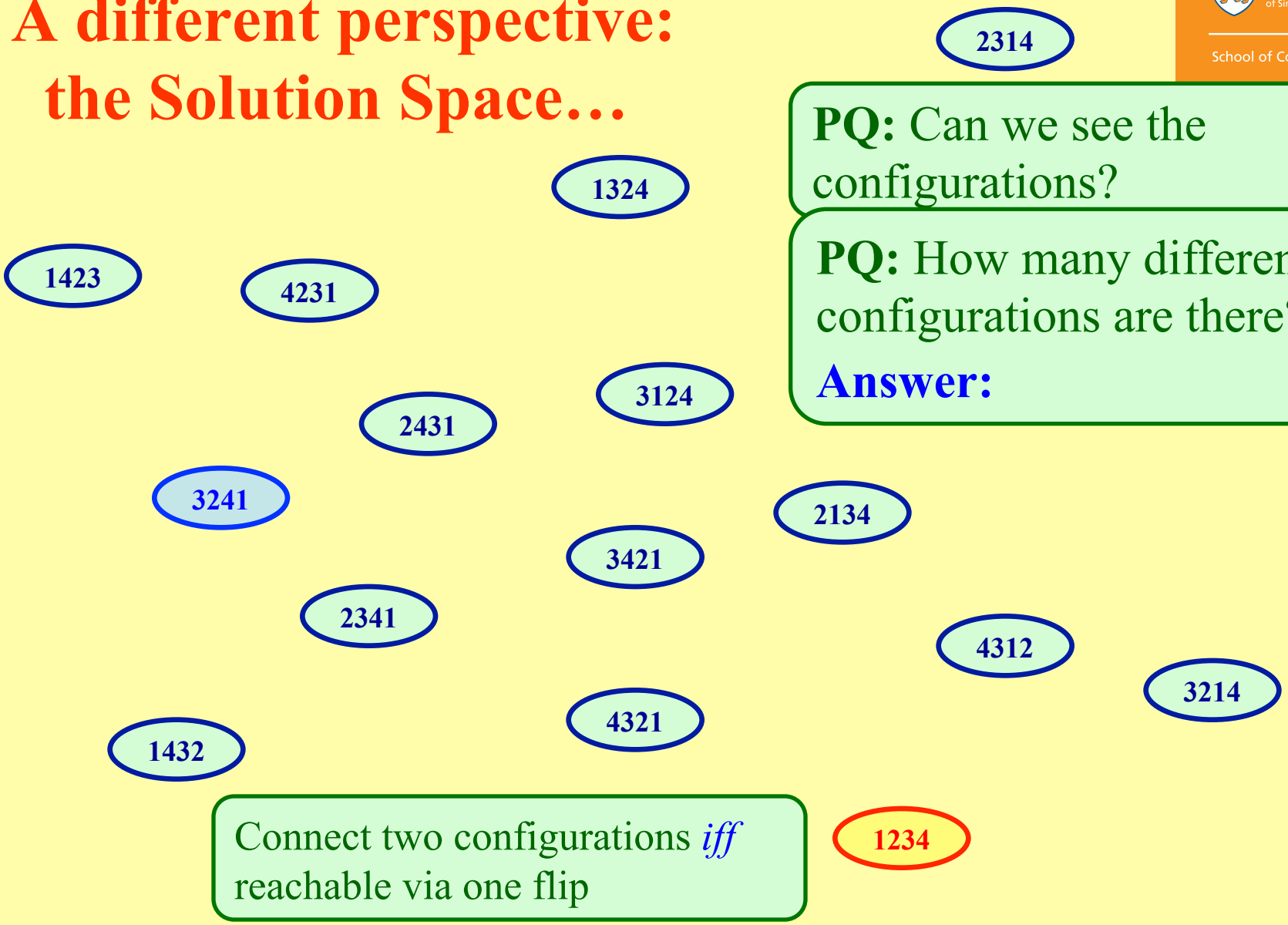
Greedy method **[5 flips]**



Better way **[3 flips]**

Pancake Flipping Problem (An alternative solution)

A different perspective: the Solution Space…



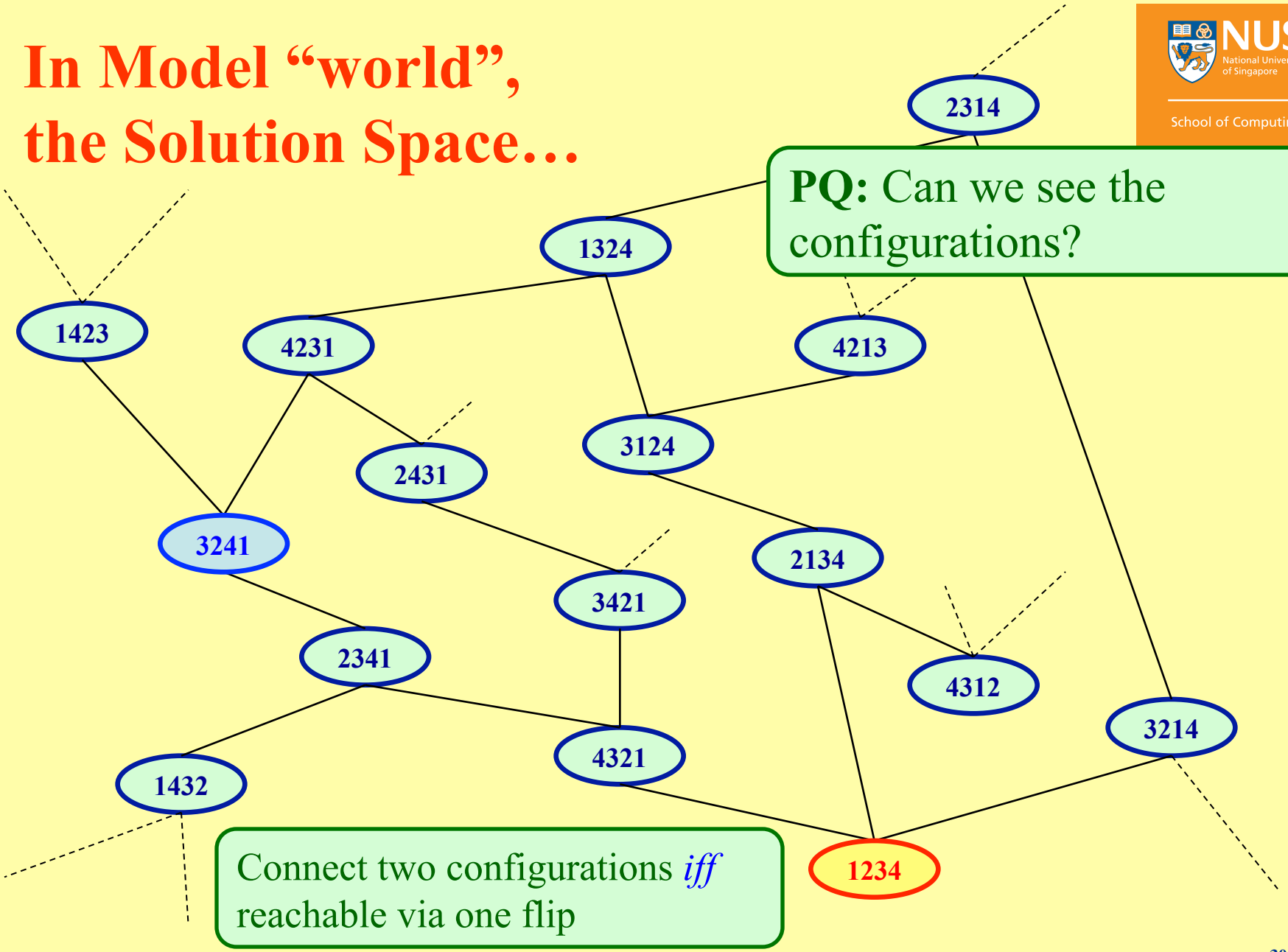
PQ: Can we see the configurations?

PQ: How many different configurations are there?

Answer:

Connect two configurations *iff* reachable via one flip

In Model "world", the Solution Space…



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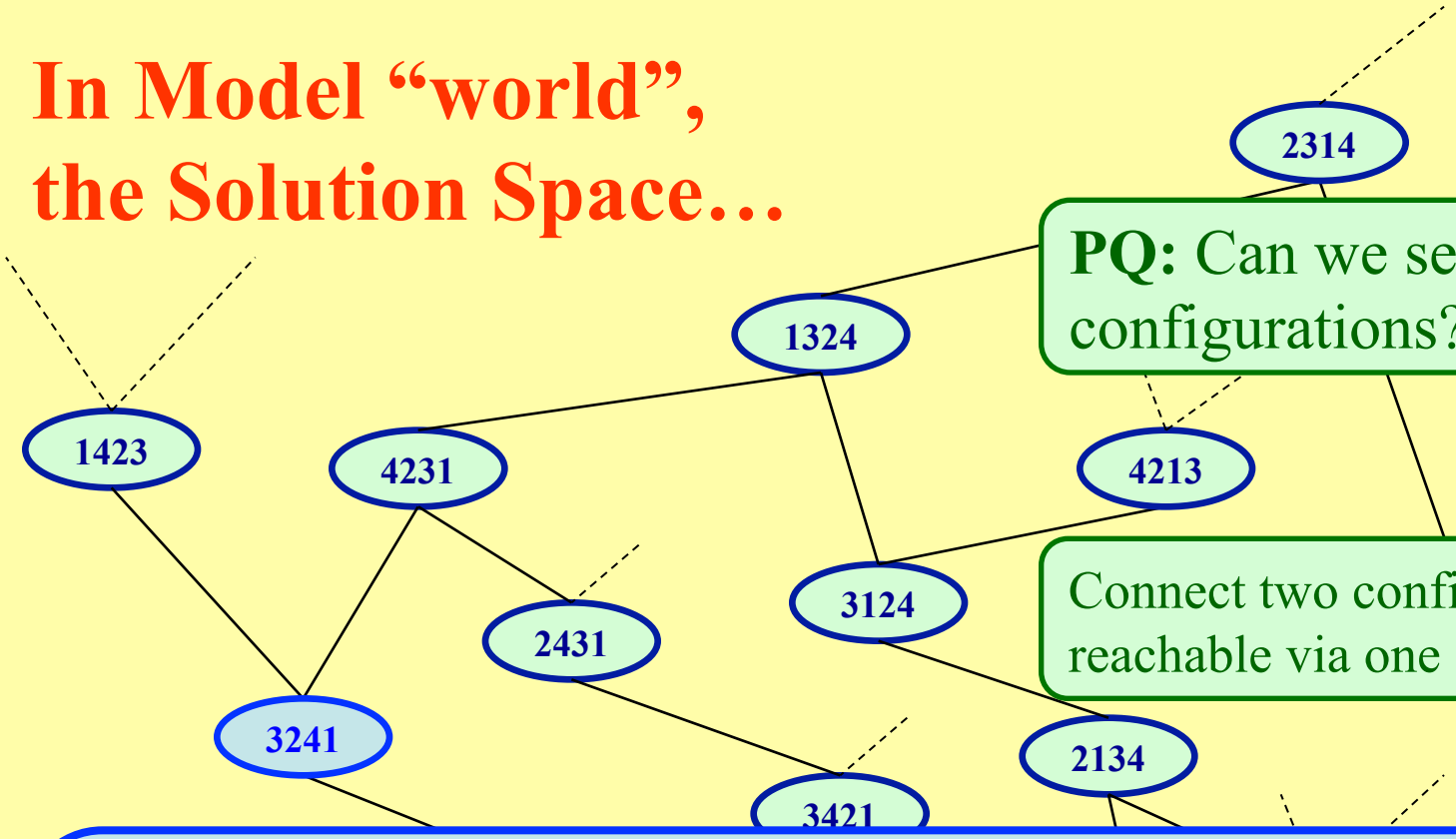
In Model "world", the Solution Space…

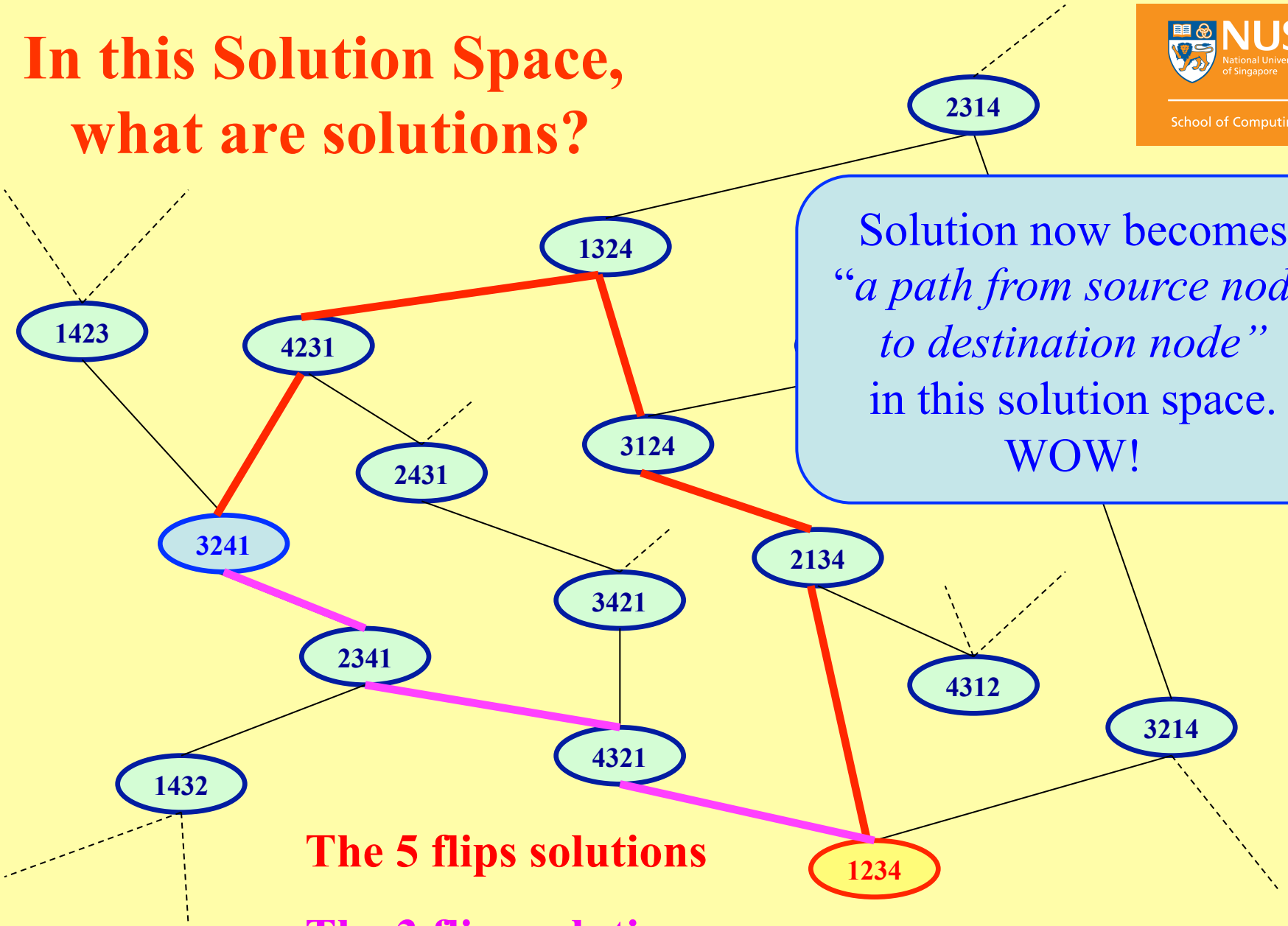


PQ: Can we see the configurations?

Connect two configurations *iff* reachable via one flip

The *Solution Space* is a graph $G = (V, E)$, where the vertices in V are configurations, and the two configurations u and v are connected by an edge in E iff u is reachable from v with one flip.

In this Solution Space, what are solutions?

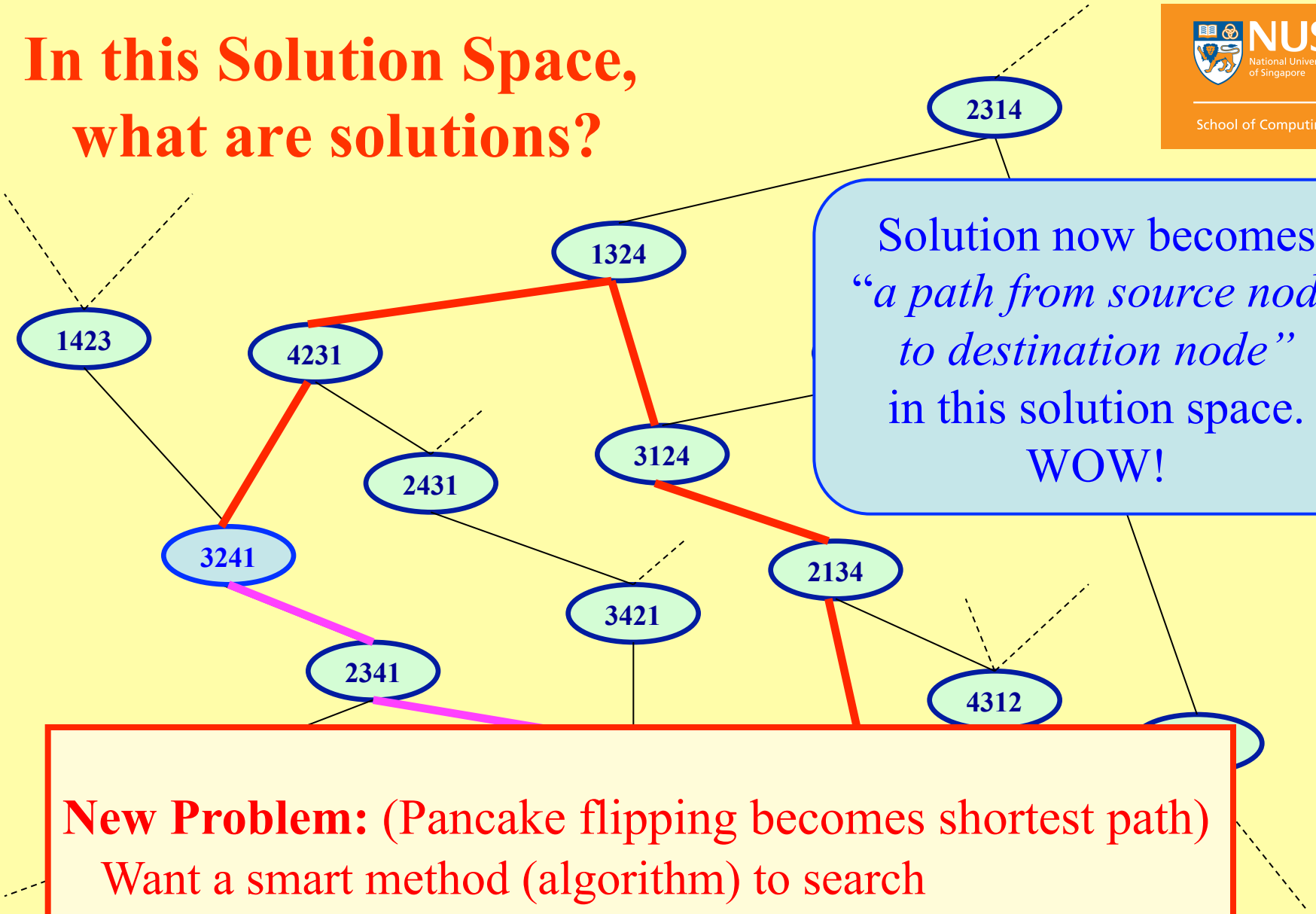


Solution now becomes "*a path from source node to destination node*" in this solution space. WOW!

The 5 flips solutions

The 3 flips solutions

In this Solution Space, what are solutions?

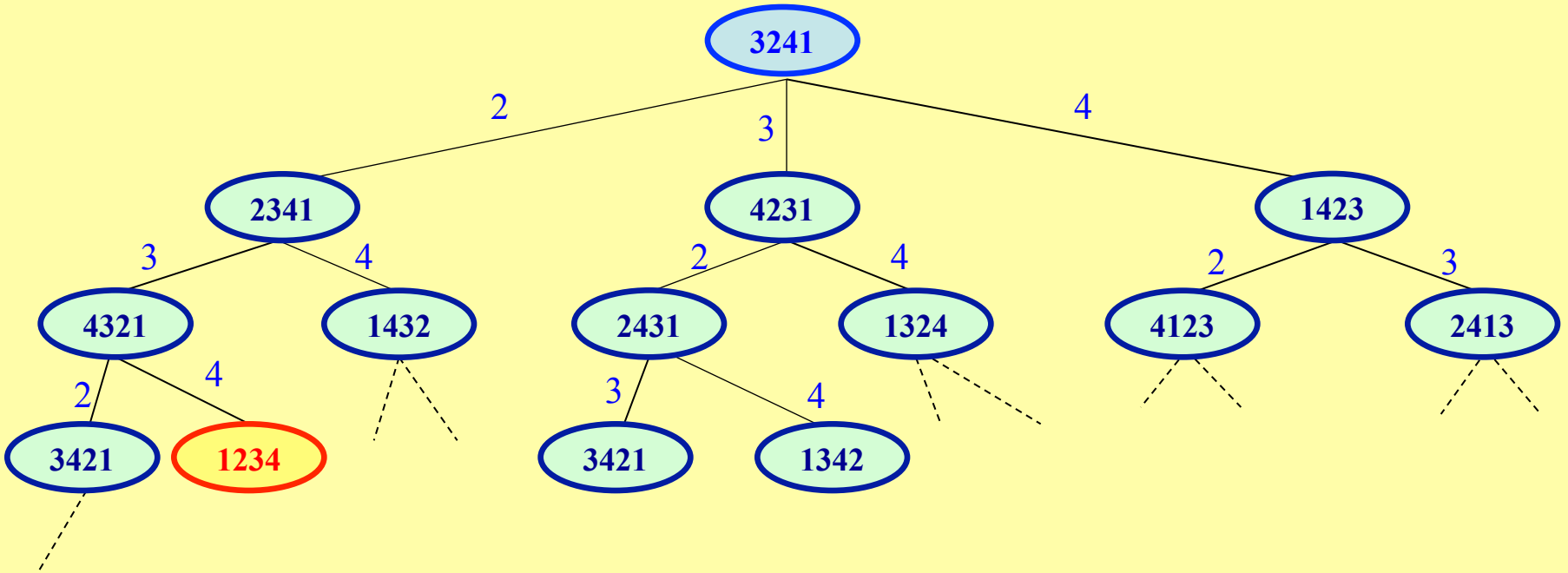


Solution now becomes
"*a path from source node to destination node*"
in this solution space.
WOW!

New Problem: (Pancake flipping becomes shortest path)
Want a smart method (algorithm) to search this space to find the shortest path solution.

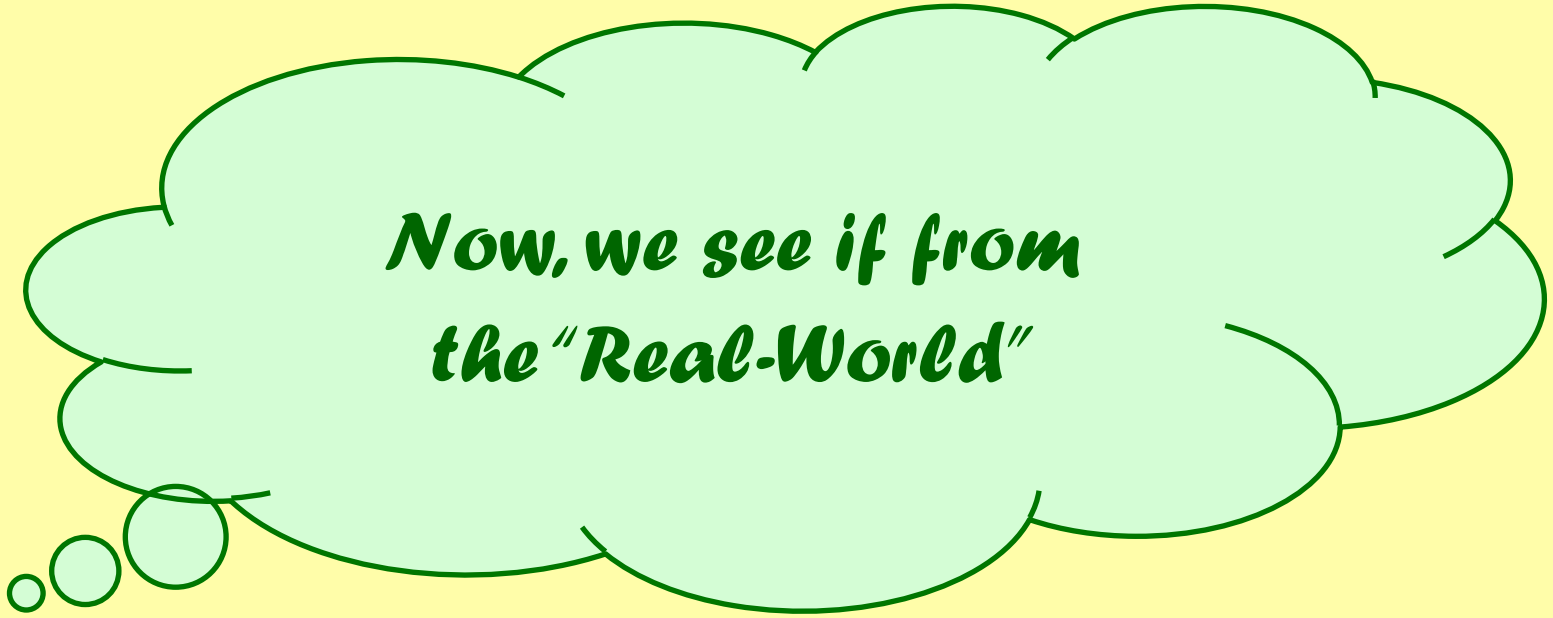
A Search Tree Method:

(systematically search the search space)

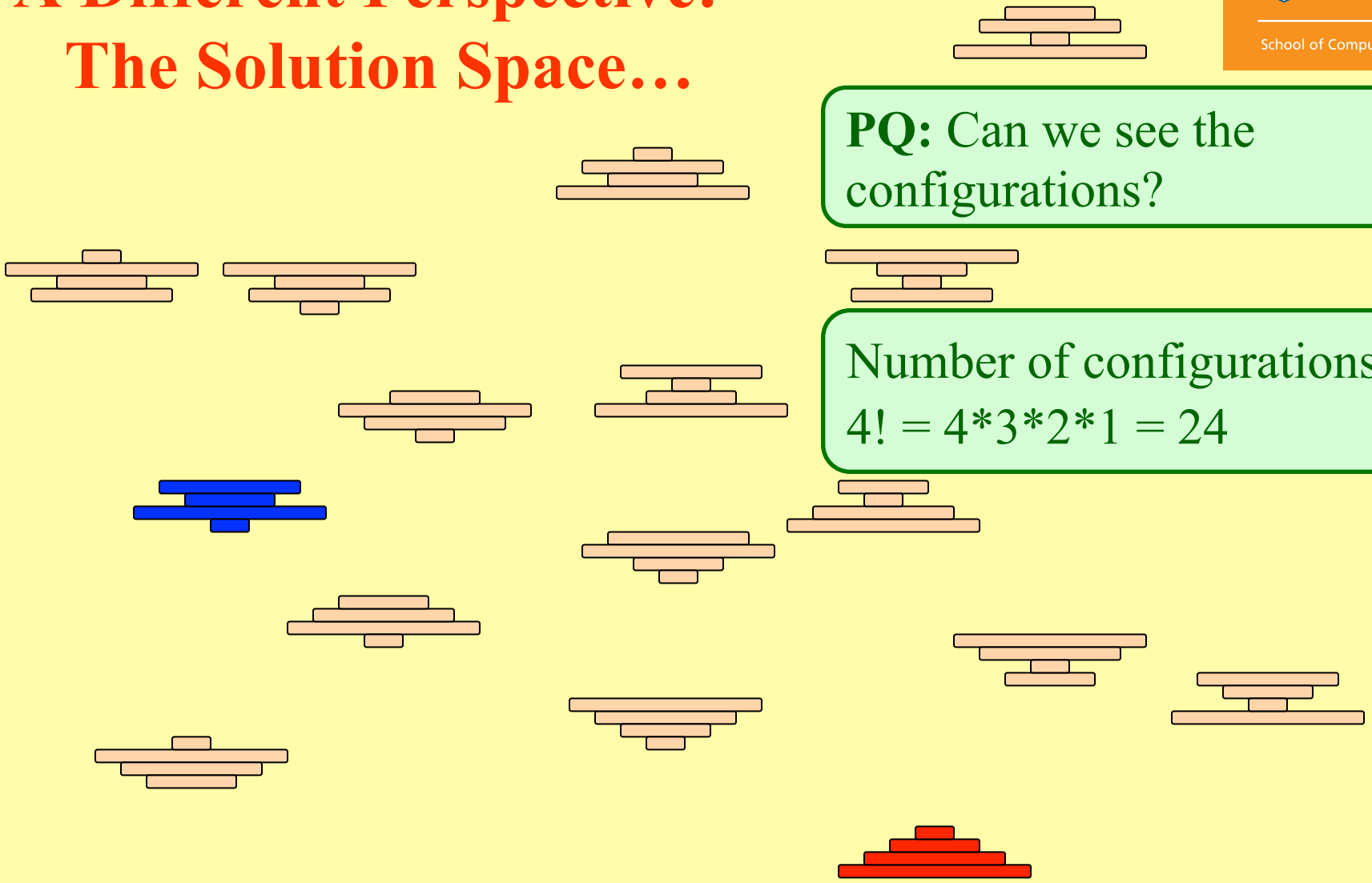


Problem: Want a smart method (algorithm) to search this space to find a shortest path solution.

Pancake Flipping Problem (An alternative solution)



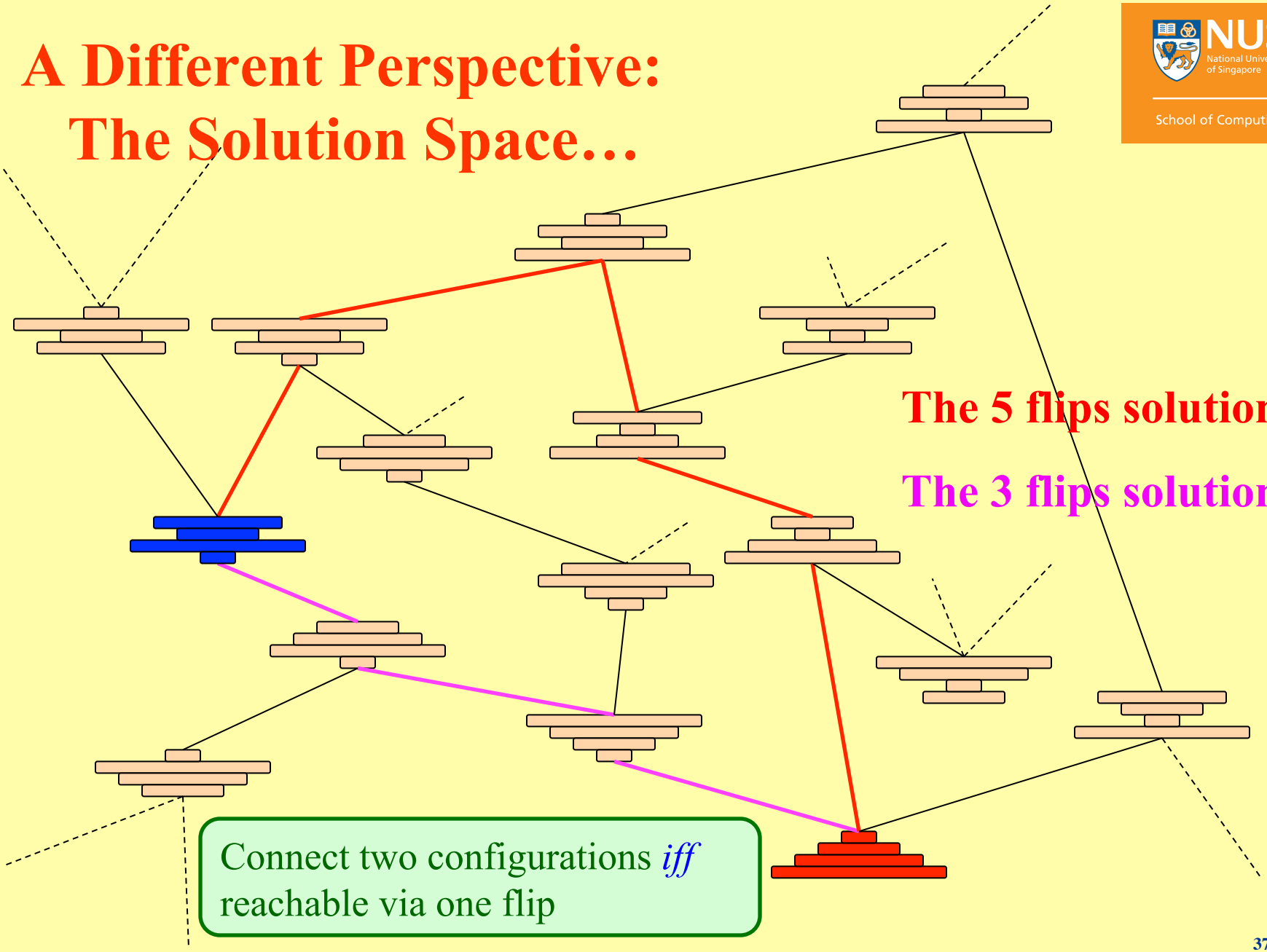
A Different Perspective: The Solution Space…



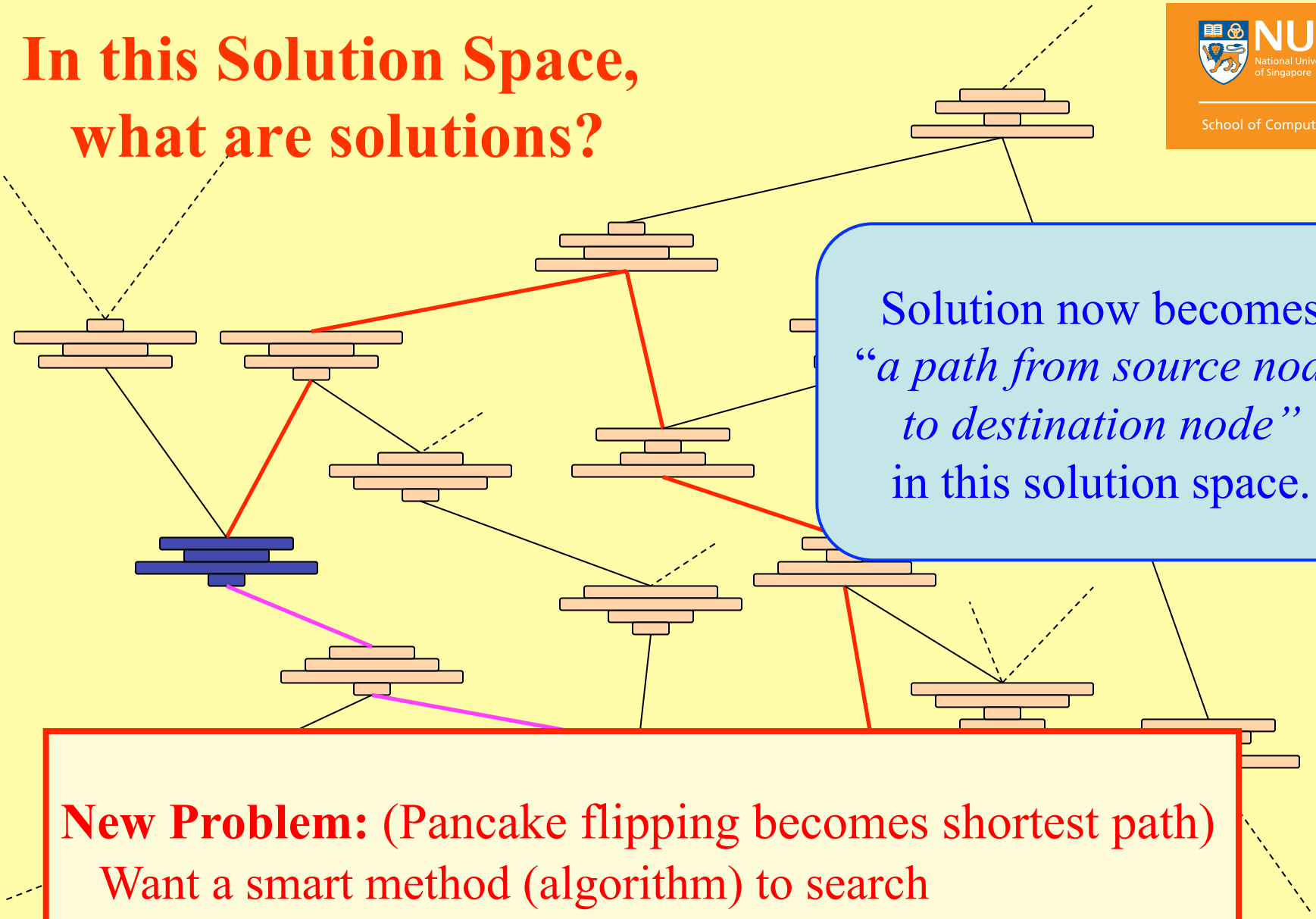
PQ: Can we see the configurations?

Number of configurations:
$4! = 4*3*2*1 = 24$

A Different Perspective: The Solution Space…



In this Solution Space, what are solutions?

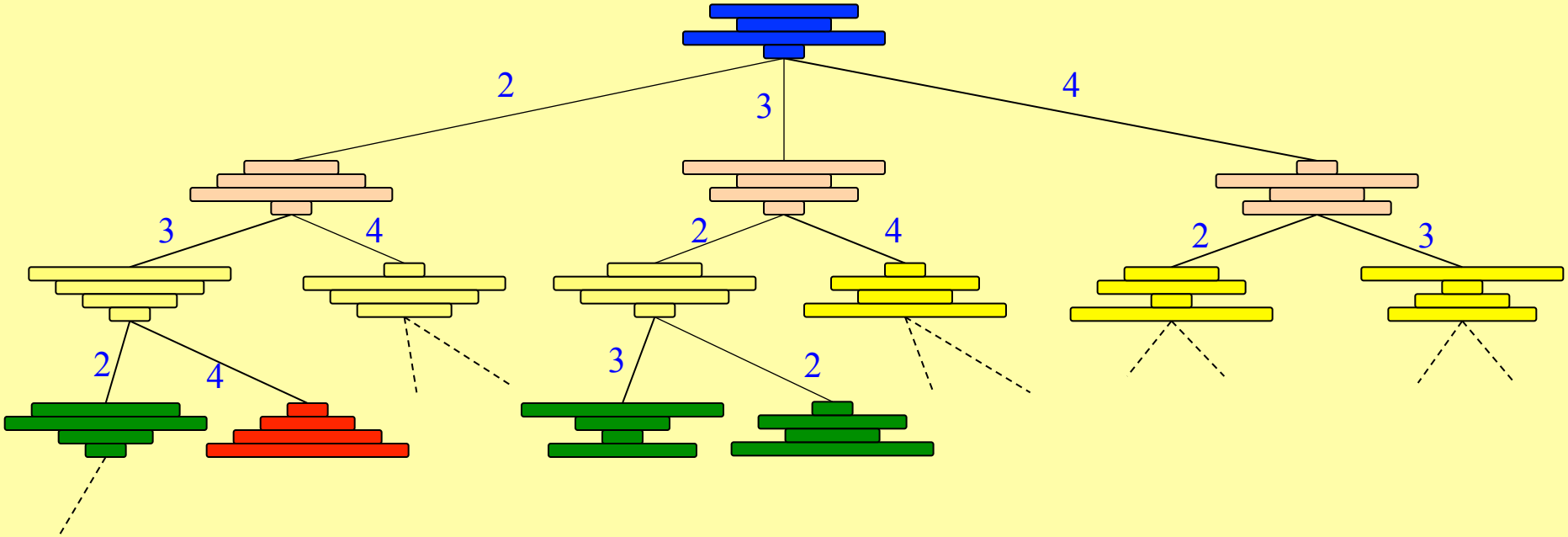


Solution now becomes "*a path from source node to destination node*" in this solution space.

New Problem: (Pancake flipping becomes shortest path)
Want a smart method (algorithm) to search this space to find a shortest path solution.

A Search Tree Method:

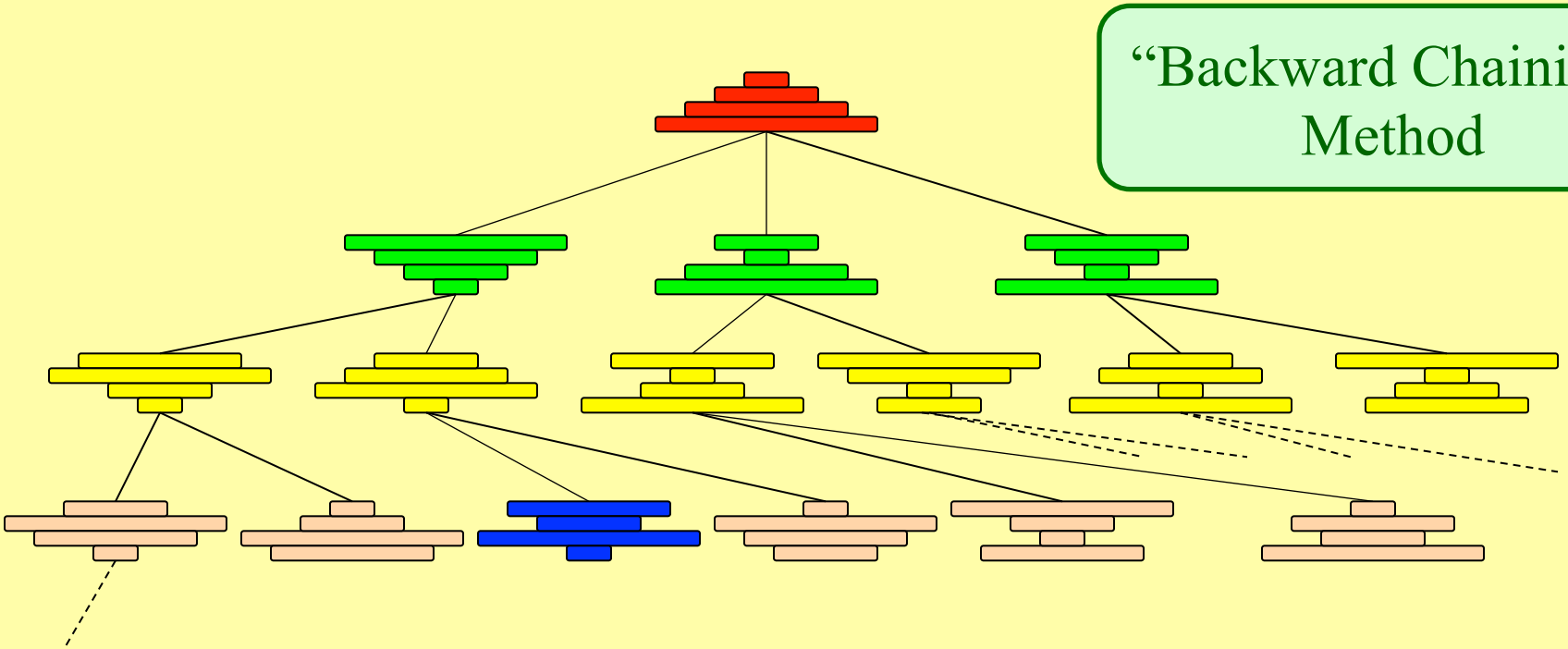
(systematically search the search space)



Problem: Want a smart method (algorithm) to search this space to find the optimal flipping solution.

A Search Tree Method:

(systematically search the search space)



"Backward Chaining" Method

PQ: Can we work backward from the solution to the data? (In AI, this is called "backward chaining"; the previous search [from start towards destination] is called "forward chaining".)

Summary:

NUS National University of Singapore
School of Computing

Bill Gates did not *really* flip pancakes (din need to)

- He flipped pancakes in his head (abstraction)
- He devised a very smart algorithm!
- His professor regretted not becoming his early employee!

Solve small instances (abstraction and decomposition)

Devised a Greedy Algorithms (analyzed it, $\leq (2n-3)$ flips)

Proved Greedy Alg is not optimal (gave counter-example)

Solution Space (via graph modelling, graph again!)

Pancake Flipping becomes Shortest Path (in a graph)

Pancake Flipping

Overview:

- When did Bill Gates flip pancakes
- The Pancake Flipping Problem
- What we know about Pancake Flipping
- Why Study Pancake Flipping?

Pancake Flipping Problem: Known Results

- Greedy Algorithm uses at most $2n$-3 flips
- For n pancakes, at most $(5n+5)/3$ flips are needed
  [Bill Gates and Papadimitriou, 1979] ~1.666n
- 2008 (about 30 years later), at most $18n/11$ needed
  [a team from UT-Dallas, 2008] ~1.6363n (*diff*: < 2%)
- Pancake flipping problem is an open problem in math and an NP-hard problem in computer science

Pancake Flipping

Overview:

- When did Bill Gates flip pancakes
- The Pancake Flipping Problem
- What we know about Pancake Flipping
- Why Study Pancake Flipping?

Why study pancake flipping

- **Mathematics – Study its properties**
  - define $f(n)$ to be the minimum of number of flip for any configuration of n pancakes

- **Computing – Want an algorithm to solve it**
  - Given any n pancake configuration, sort it with the *minimum* number of flips

Math vs Computer Science

Problem:

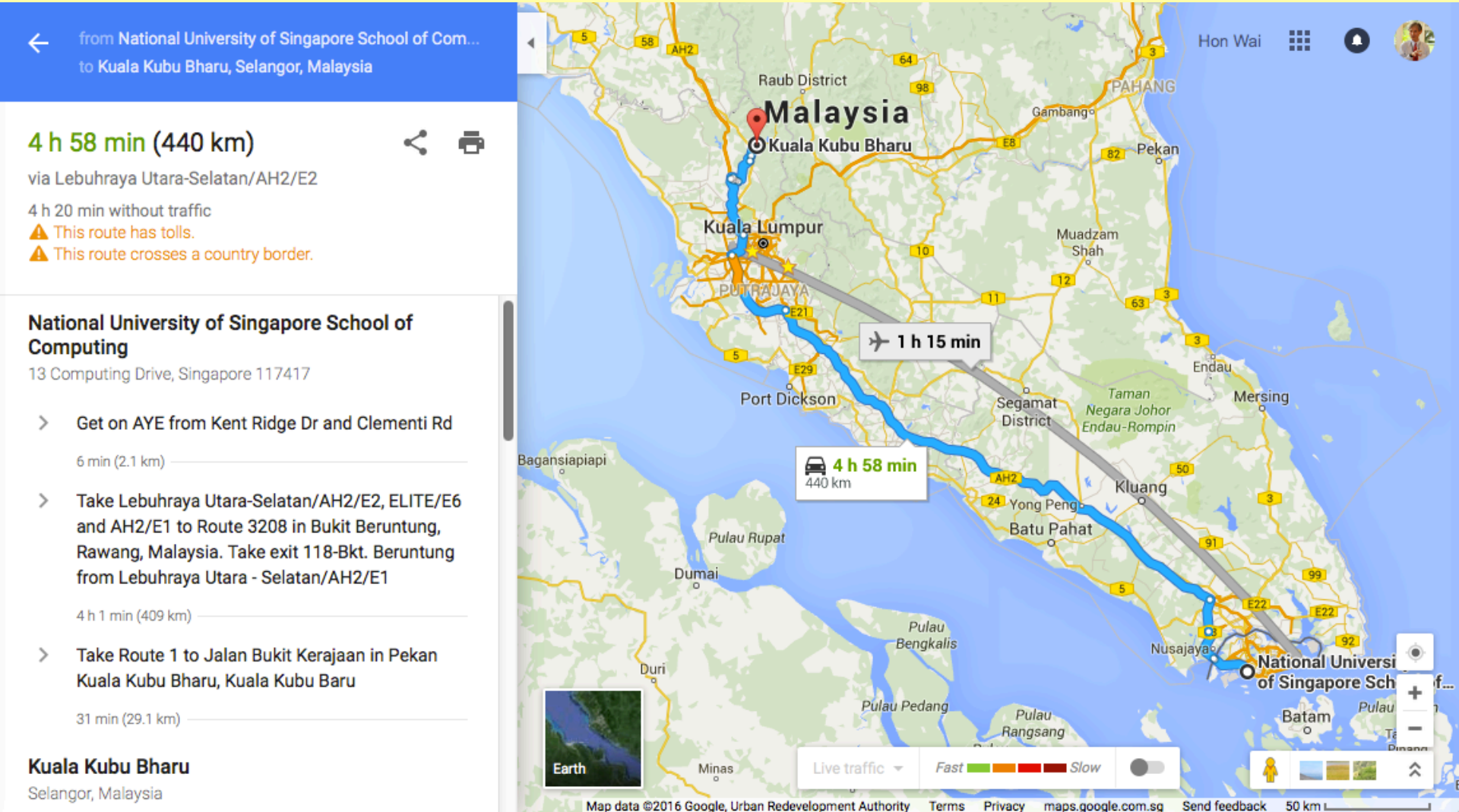
We need to drive from NUS-SOC to Kuala Kubu Bahru in West Malaysia?

Mathematicians:

may give a Theorem

There exist a path with total distance no more than 450km.

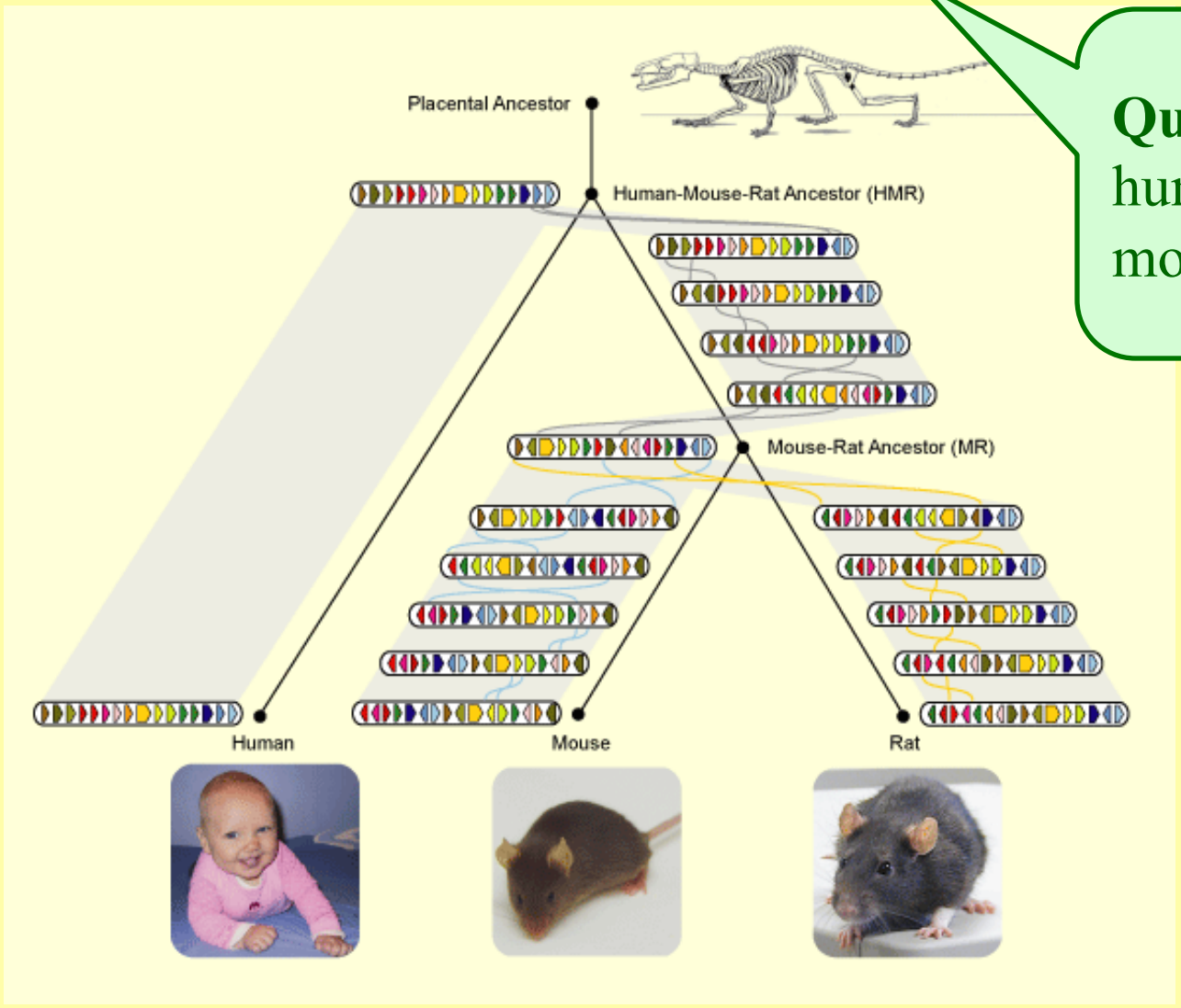
But, a computer scientist wants the exact route (and more)



Why study pancake flipping

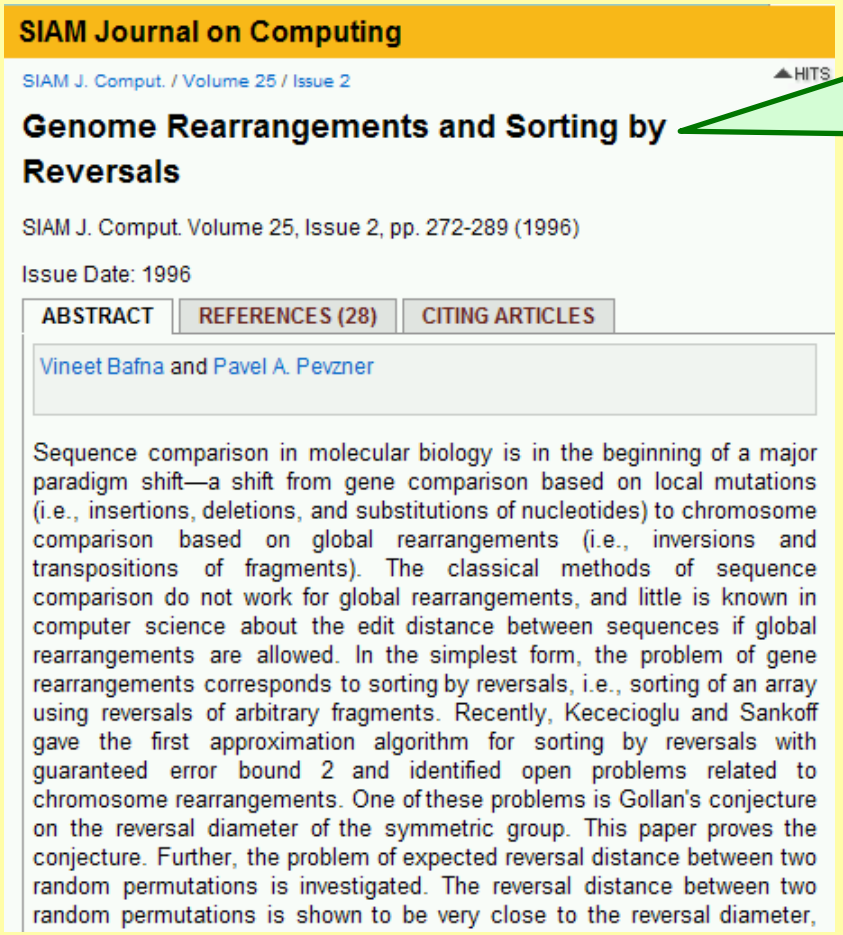
- **Mathematics – Study its properties**
  - define $f(n)$ to be the minimum of number of flip for any configuration of n pancakes
- **Computing – Want an algorithm to solve it**
  - Given any n pancake configuration, sort it with the *minimum* number of flips
- **Applications**
  - sorting by prefix reversal
  - used to study evolution of species in biology

Sorting by Reversals used here…



Question: Is human closer to mouse or rat?

Application of Sorting by Reversals



SIAM Journal on Computing

SIAM J. Comput. / Volume 25 / Issue 2

HITS

Genome Rearrangements and Sorting by Reversals

SIAM J. Comput. Volume 25, Issue 2, pp. 272-289 (1996)

Issue Date: 1996

ABSTRACT | REFERENCES (28) | CITING ARTICLES

Vineet Bafna and Pavel A. Pevzner

Sequence comparison in molecular biology is in the beginning of a major paradigm shift—a shift from gene comparison based on local mutations (i.e., insertions, deletions, and substitutions of nucleotides) to chromosome comparison based on global rearrangements (i.e., inversions and transpositions of fragments). The classical methods of sequence comparison do not work for global rearrangements, and little is known in computer science about the edit distance between sequences if global rearrangements are allowed. In the simplest form, the problem of gene rearrangements corresponds to sorting by reversals, i.e., sorting of an array using reversals of arbitrary fragments. Recently, Kececioglu and Sankoff gave the first approximation algorithm for sorting by reversals with guaranteed error bound 2 and identified open problems related to chromosome rearrangements. One of these problems is Gollan's conjecture on the reversal diameter of the symmetric group. This paper proves the conjecture. Further, the problem of expected reversal distance between two random permutations is investigated. The reversal distance between two random permutations is shown to be very close to the reversal diameter,

Important Application in Computational Biology:
Used to study the evolution from one species to another.

Curious?
See more details in the extra slides at the end.

Relevant Skills and Courses

- **Pancake flipping is a *model* for**
  - sorting by prefix-reversals
- **Many CS problems are model in *similar* ways**
  - sending files over internet (routing problems)
  - time table scheduling (graph colouring, 图着色问题)
- **Courses to learn these things**
  - CS1231 (Discrete Mathematics, 离散数学) [Blogs: 1, 2,]
  - CS3230 (Analysis of Algorithms, 算法设计与分析)
  - new!

More on Pancake Flipping

Have some fun with pancake flipping:

http://www.cut-the-knot.org/SimpleGames/Flipper.shtml

Read about Pancake Flipping:

http://www.amazon.com/Bill-Gates-Speaks-Greatest-Entrepreneur/dp/0471401692

Listen to the story on NPR (July-2008):

http://www.npr.org/templates/story/story.php?storyId=92236781

Youtube Video:

https://www.youtube.com/watch?v=oDzauRFiWFU

How are the following people connected to Bill Gates?

Jacob Goodman, Harry Lewis, Harry Dweighter,

David Cohen, Manuel Blum

More on Genome Rearrangments

Genome Rearrangement:

http://www.cut-the-knot.org/SimpleGames/Flipper.shtml

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NUS National University of Singapore

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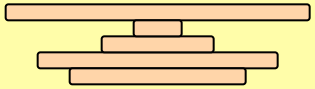
(End of video 6.1)

If you want to contact me,
email, FB: leonghw@comp.nus.edu.sg

School of Computing

Activity: Fun with Pancake Flipping

Try your hand on this example.
How many flips are needed?



? flips

What are Reversals in the study of Genomics?

Example: (2 genomes G and H, from different species)

$G = \{ 1, -5, 4, -3, 2 \}$ $H = \{ 1, 2, 3, 4, 5 \}$

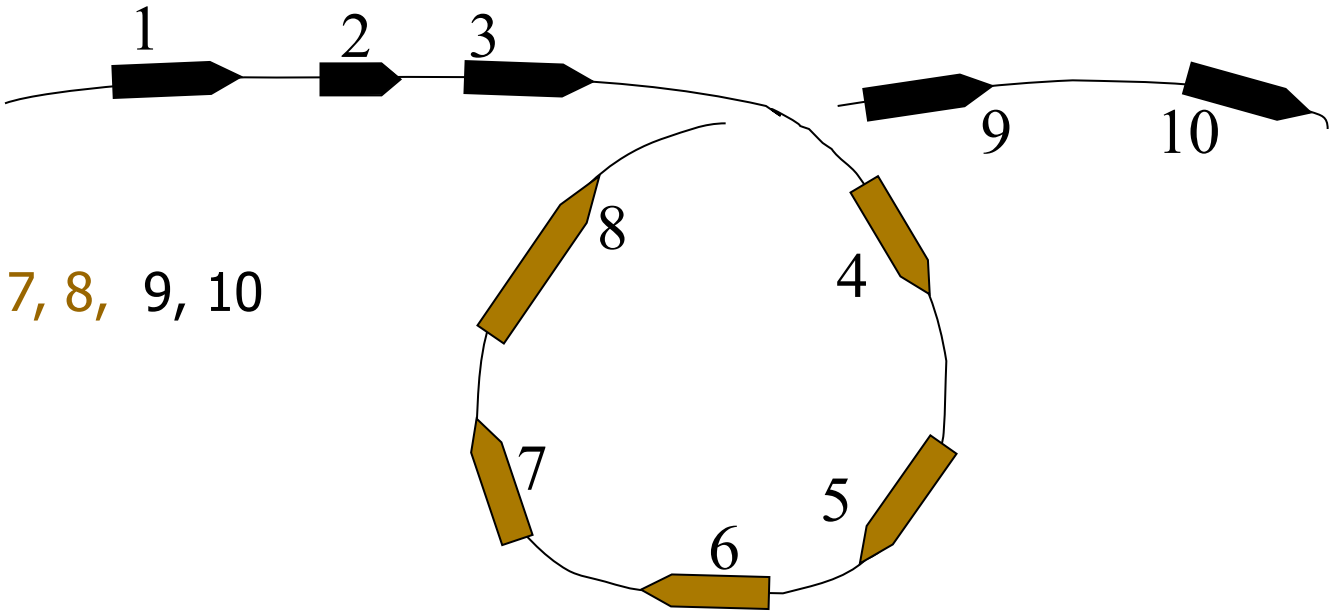
A **genome** consists of a sequence of **genes**;

Represent each gene by a integer $\{1,2, \ldots, n\}$.

Each gene has a "*direction*" indicated by the sign (like -3).

Then a genome is a "like" a stack of pancakes of diff sizes.
(each pancake is burnt on one side, indicate "direction")

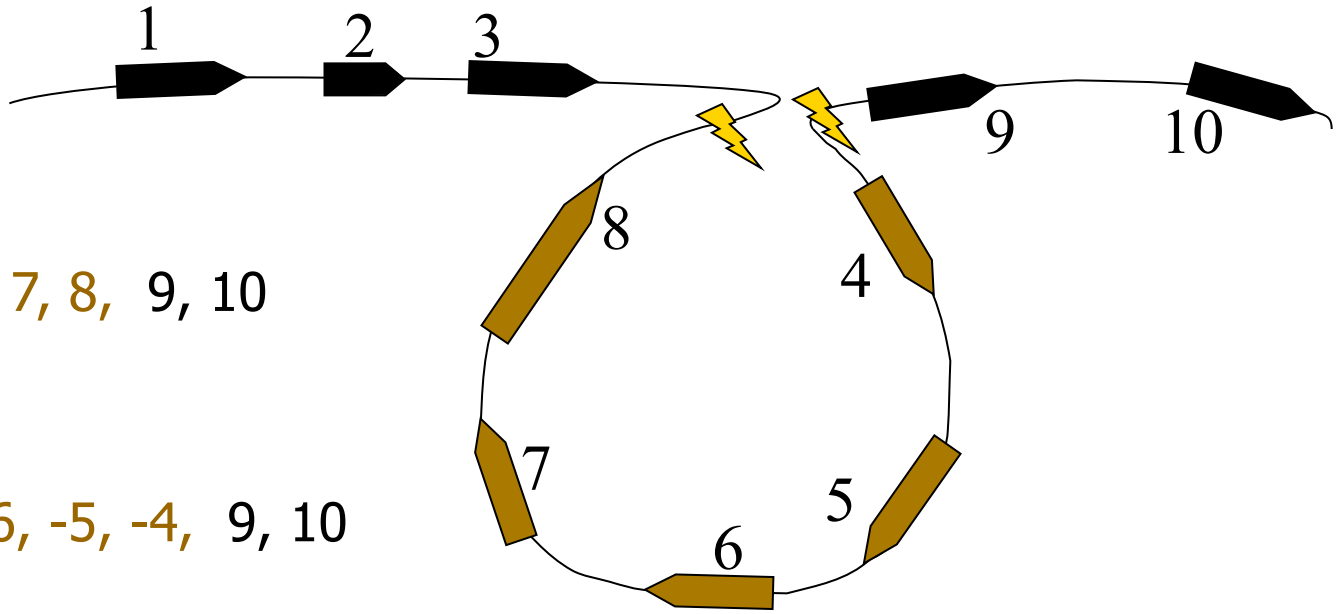
Reversals in the Gene Sequence



1, 2, 3, 4, 5, 6, 7, 8, 9, 10

* Consider the block of genes (4, 5, 6, 7, 8,)

Reversals and Breakpoints

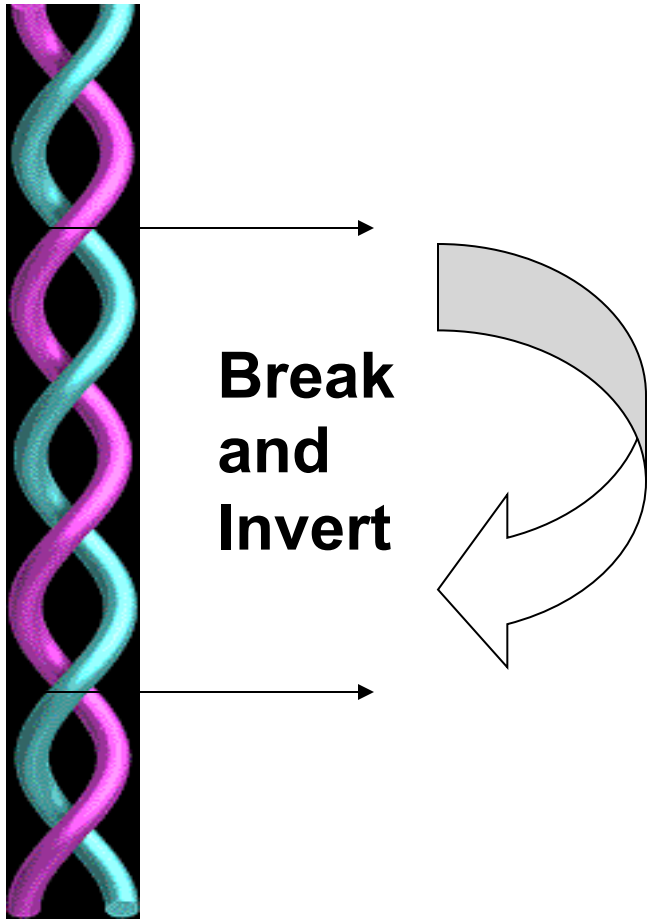


1, 2, 3, 4, 5, 6, 7, 8, 9, 10

1, 2, 3, -8, -7, -6, -5, -4, 9, 10

* Consider the block of genes (4, 5, 6, 7, 8,)
* The reversal introduced two *breakpoints*
* Order of genes reversed, and direction inverted
  (4, 5, 6, 7, 8,) ➔ (-8, -7, -6, -5, -4,)

Reversals: Example



5’ ATGCCTGTACTA 3’

3’ TACGGACATGAT 5’

5’ ATGTACAGGCTA 3’

3’ TACATGTCCGAT 5’

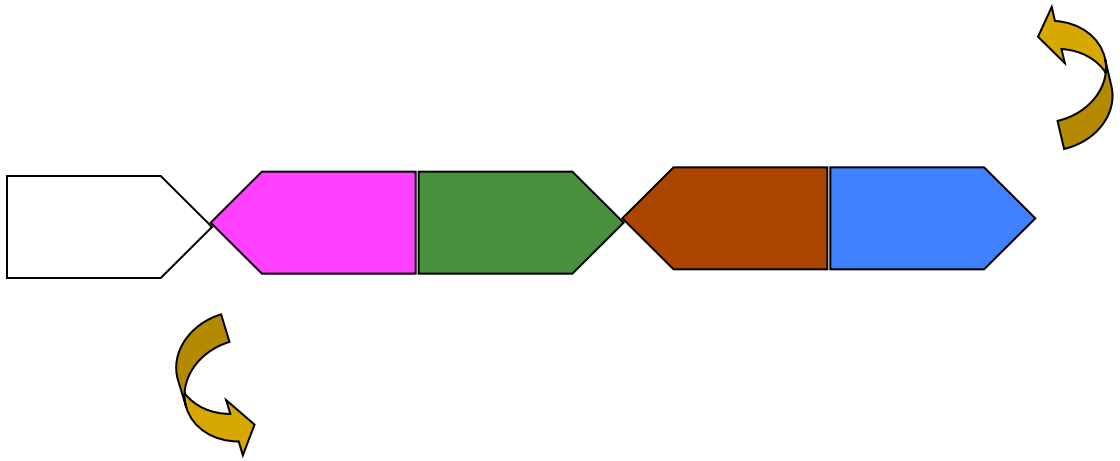
Reversals are Special Pancake Flips

Need a special spatula
With 2-prongs

6 | 5421 | 3 → 6 | -1 -2 -4 -5 | 3

Evolution of the gene sequence via reversals

- Gene order comparison:



Evolution of the gene sequence via reversals

- Gene order comparison:

Evolution of the gene sequence via reversals

- Gene order comparison:

Evolution of the gene sequence via reversals

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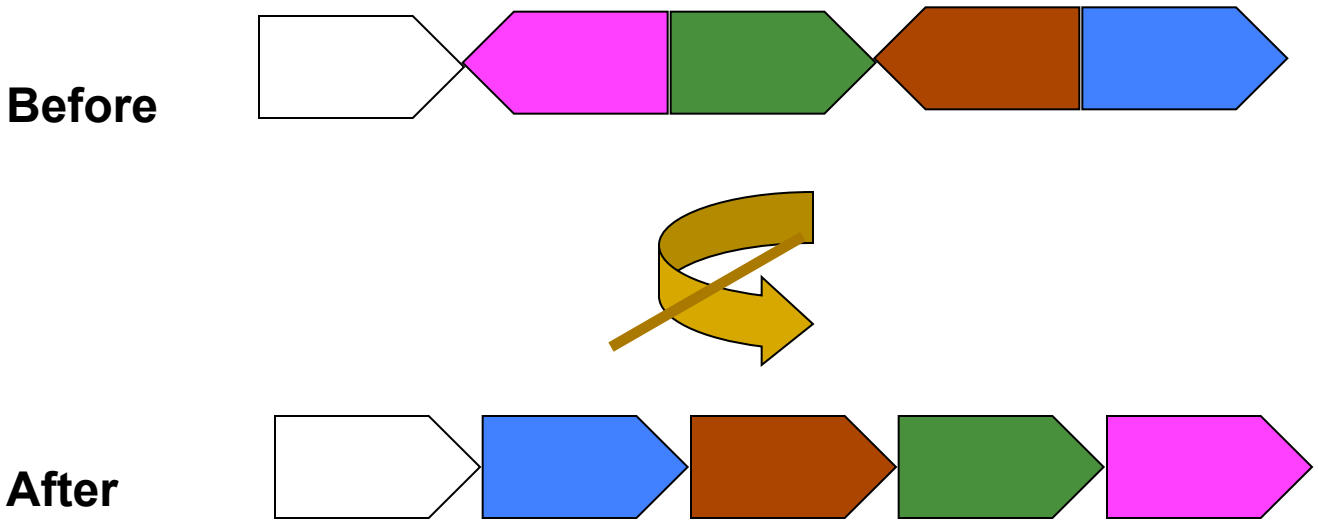
Evolution of the gene sequence via reversals

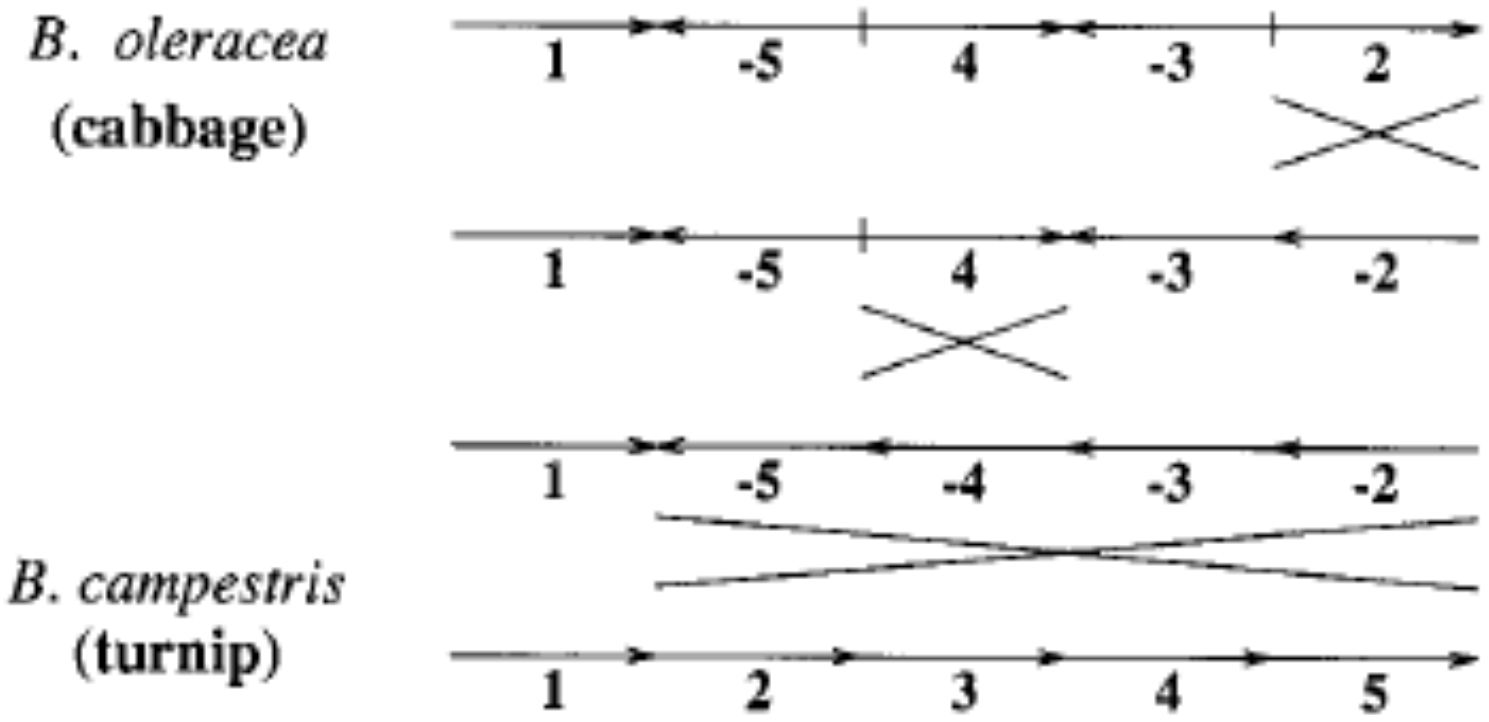
- Gene order comparison:



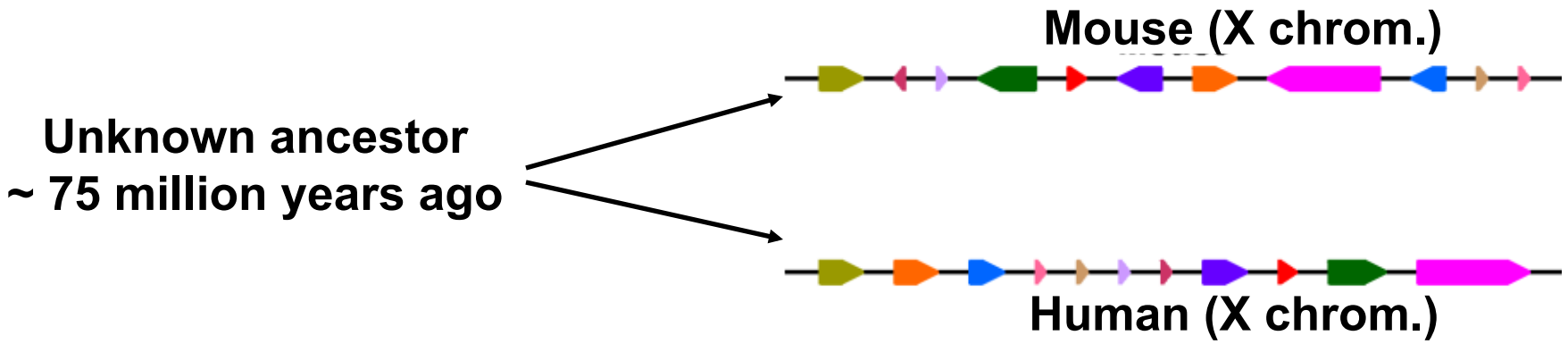
Evolution is manifested as the divergence in gene order

Transforming Cabbage into Turnip

(with 3 reversal operations)

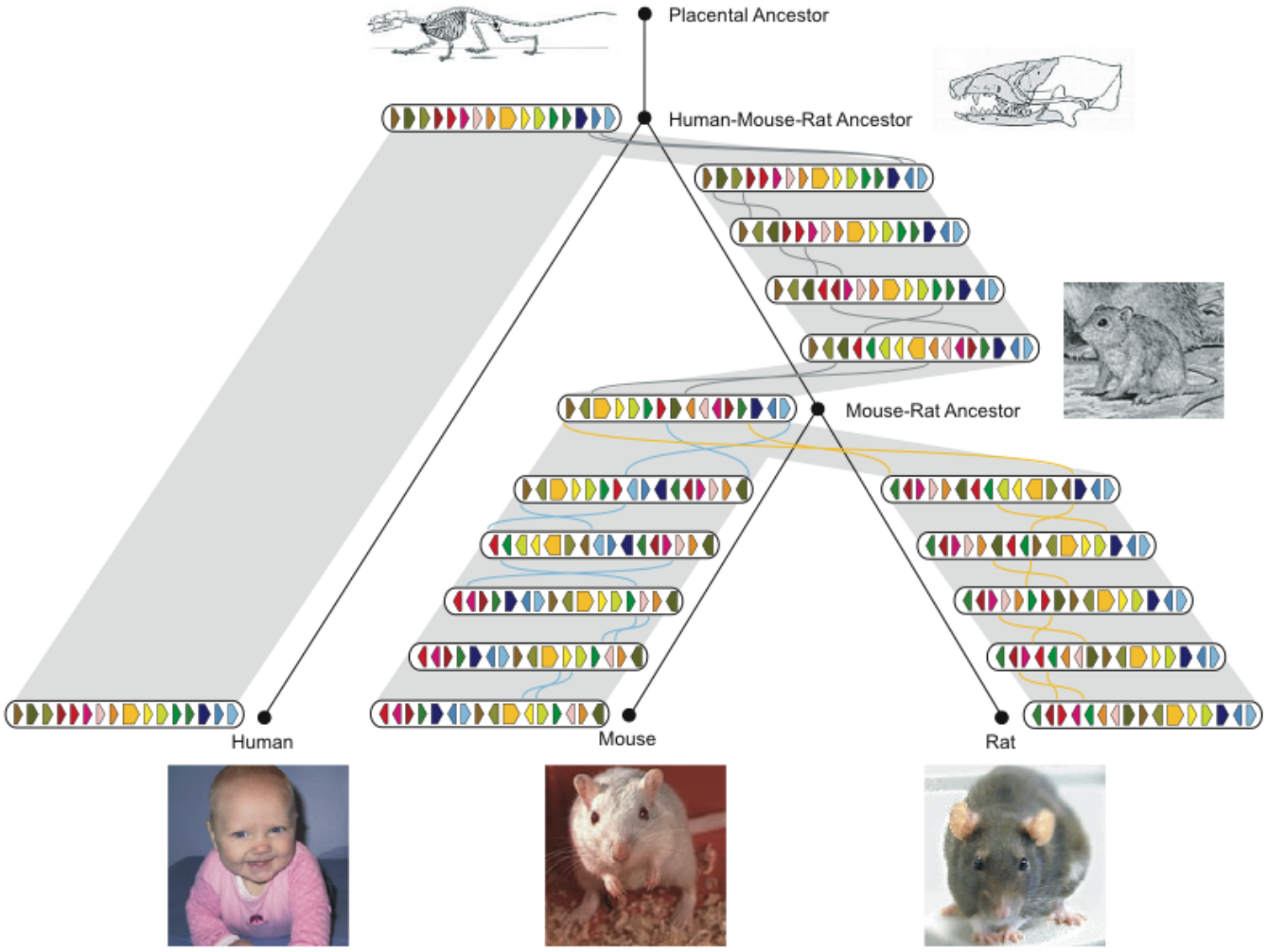


Genome rearrangements



- What are the similarity blocks and how to find them?
- What is the architecture of the ancestral genome?
- What is the evolutionary scenario for transforming one genome into the other?

History of Chromosome X



Rat Consortium, *Nature*, 2004