CS1231: Discrete Structures

Tutorial 4

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Quick Review

- $a \in \{a, b\}$ V.S. $\{a\} \subseteq \{a, b\}$.
- $A \subset B$ definition: $A \subseteq B$ and $A \neq B$.
- $A = B$ definition: $\forall x(x \in A \leftrightarrow x \in B)$.
- $|A|$ definition: Number of distinct elements in A.
- Set Operations: $\overline{A}$, $A \cap B$, $A \cup B$, $A - B$.
- Truth Set: $T_p = \{x \in D \mid P(x) \text{ is true}\}$.
- Cartesian Product: $A \times B = \{(a, b) \mid a \in A, b \in B\}$.
- Power Set: $P(A) = \{X | X \subseteq A\}$
- $\emptyset$.

Menu

1. Determine whether these are true for false.

(a) $\varnothing \in \{\varnothing\}$

(b) $\varnothing \in \{\varnothing, \{\varnothing\}\}$

(c) $\{\varnothing\} \in \{\varnothing\}$

(d) $\{\varnothing\} \in \{\{\varnothing\}\}$

(e) $\{\varnothing\} \subset \{\varnothing, \{\varnothing\}\}$

(f) $\{\{\varnothing\}\} \subset \{\varnothing, \{\varnothing\}\}$

(g) $\{\{\varnothing\}\} \subset \{\{\varnothing\}, \{\varnothing\}\}$

Recall

- ✍ $x \in A$: x is a member of the set A.
- ✍ $A \subset B$: A is a proper subset of B, i.e. every member of A is a member of B and $A \neq B$.

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(d) $\{\emptyset\} \in \{\{\emptyset\}\}$

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Recall

- ✍ $x \in A$: x is a member of the set A.
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Answer. T,T,F,T,T,T,F.

2. Let $B = \{n \in \mathbb{Z} : n = 3j + 2, j \in \mathbb{Z}\}$, $D = \{n \in \mathbb{Z} : n = 3j - 1, j \in \mathbb{Z}\}$. Is $B = D$?

Recall

✍ $B = D$ iff $\forall n (n \in B \leftrightarrow n \in D)$.

Idea.

$\underline{n \in B \Rightarrow n \in D,}$

i.e. $B \subseteq D$

$n \in B$

$\Rightarrow n =$

$\Rightarrow n = 3(\qquad) - 1$

$\Rightarrow n \in D$

$\underline{n \in D \Rightarrow n \in B,}$

i.e. $D \subseteq B$

$n \in D$

$\Rightarrow n =$

$\Rightarrow n = 3(\qquad) + 2$

$\Rightarrow n \in B$

2. Let $B = \{n \in \mathbb{Z} : n = 3j + 2, j \in \mathbb{Z}\}$, $D = \{n \in \mathbb{Z} : n = 3j - 1, j \in \mathbb{Z}\}$. Is $B = D$?

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$\underline{n \in B \Rightarrow n \in D,}$
i.e. $B \subseteq D$
$n \in B$
$\Rightarrow n = 3j + 2, j \in \mathbb{Z}$
$\Rightarrow n = 3(j + 1) - 1$
$\Rightarrow n \in D$

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2. Let $B = \{n \in \mathbb{Z} : n = 3j + 2, j \in \mathbb{Z}\}$, $D = \{n \in \mathbb{Z} : n = 3j - 1, j \in \mathbb{Z}\}$. Is $B = D$?

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✍ $B = D$ iff $\forall n(n \in B \leftrightarrow n \in D)$.

Idea.

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$\underline{n \in D \Rightarrow n \in B,}$
i.e. $D \subseteq B$
$n \in D$
$\Rightarrow n = 3j - 1, j \in \mathbb{Z}$
$\Rightarrow n = 3(j - 1) + 2$
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Answer. Yes. If $n = 3j + 2$, then $n = 3(j + 1) - 1$, thus $B \subseteq D$. If $n = 3j - 1$, then $n = 3(j - 1) + 2$. Thus $D \subseteq B$. So $B = D$.

3. Find $|A|$ if $A = \{1, 2, \{2\}, \{4, 5\}, 5, 5\}$.

Recall

- ✍ for sets, order, repetition Do Not Matter.
  E.g. $\{1, 3, 7\} = \{7, 1, 3\} = \{7, 1, 1, 1, 3, 3, 1, 1\}$.
- ✍ $|A|$: the cardinality of A, that is, the number of different elements in A.

Idea. $A = \{ \qquad\qquad\qquad\qquad \}$.
Answer. $|A| = \quad$.

3. Find $|A|$ if $A = \{1, 2, \{2\}, \{4, 5\}, 5, 5\}$.

Recall

- ✍ for sets, order, repetition Do Not Matter.
  E.g. $\{1, 3, 7\} = \{7, 1, 3\} = \{7, 1, 1, 1, 3, 3, 1, 1\}$.
- ✍ $|A|$: the cardinality of A, that is, the number of different elements in A.

Idea. $A = \{1, 2, \{2\}, \{4, 5\}, 5\}$.
Answer. $|A| =$.

3. Find $|A|$ if $A = \{1, 2, \{2\}, \{4, 5\}, 5, 5\}$.

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- ✍ for sets, order, repetition Do Not Matter.
  E.g. $\{1, 3, 7\} = \{7, 1, 3\} = \{7, 1, 1, 1, 3, 3, 1, 1\}$.
- ✍ $|A|$: the cardinality of A, that is, the number of different elements in A.

Idea. $A = \{1, 2, \{2\}, \{4, 5\}, 5\}$.
Answer. $|A| = 5$.

4. Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, $C = \{b, c, e\}$. Find $(A - B) - C$ and $A - (B - C)$. Are they equal?

Recall

✍ X and Y are sets, then

$$X - Y = \{x \mid x \in X \text{ and } x \notin Y\}$$

Idea.

$A = \{a, b, c\}$, $B = \{b, c, d\}$
$\Rightarrow A - B = \{\quad\}$,
$C = \{b, c, e\}$
$\Rightarrow (A - B) - C = \{\quad\}$

$B = \{b, c, d\}$, $C = \{b, c, e\}$
$\Rightarrow B - C = \{\quad\}$,
$A = \{a, b, c\}$
$\Rightarrow A - (B - C) = \{\qquad\}$.

Answer.

Notice:

4. Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, $C = \{b, c, e\}$. Find $(A - B) - C$ and $A - (B - C)$. Are they equal?

Recall

✍ X and Y are sets, then

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Idea.

$A = \{a, b, c\}$, $B = \{b, c, d\}$
$\Rightarrow A - B = \{a\}$,
$C = \{b, c, e\}$
$\Rightarrow (A - B) - C = \{\quad\}$

$B = \{b, c, d\}$, $C = \{b, c, e\}$
$\Rightarrow B - C = \{\quad\}$,
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Answer.

Notice:

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Idea.

$A = \{a, b, c\}$, $B = \{b, c, d\}$
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Answer.

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Idea.

$A = \{a, b, c\}$, $B = \{b, c, d\}$
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$C = \{b, c, e\}$
$\Rightarrow (A - B) - C = \{a\}$

$B = \{b, c, d\}$, $C = \{b, c, e\}$
$\Rightarrow B - C = \{d\}$,
$A = \{a, b, c\}$
$\Rightarrow A - (B - C) = \{\qquad\}$.

Answer.

Notice:

4. Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, $C = \{b, c, e\}$. Find $(A - B) - C$ and $A - (B - C)$. Are they equal?

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Answer.

Notice:

4. Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, $C = \{b, c, e\}$. Find $(A - B) - C$ and $A - (B - C)$. Are they equal?

Recall

✍ X and Y are sets, then

$$X - Y = \{x \mid x \in X \text{ and } x \notin Y\}$$

Idea.

$A = \{a, b, c\}$, $B = \{b, c, d\}$
$\Rightarrow A - B = \{a\}$,
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$\Rightarrow (A - B) - C = \{a\}$

$B = \{b, c, d\}$, $C = \{b, c, e\}$
$\Rightarrow B - C = \{d\}$,
$A = \{a, b, c\}$
$\Rightarrow A - (B - C) = \{a, b, c\}$.

Answer. Not equal.

Notice: We do **Not** have associative laws in set difference.

5. Let T_P denote the truth set of the predicate $P(x)$. Prove the following:

(a) $T_{P\vee Q} = T_P \cup T_Q$, $T_{P\wedge Q} = T_p \cap T_Q$,

(b) $T_{P\to Q} = \overline{T_p} \cup T_Q$.

Recall

- ✍ $T_P = \{x \mid P(x) \text{ is true}\}$
- ✍ $A = B$ iff $\forall x (x \in A \leftrightarrow x \in B)$

Idea. (a)

$\underline{T_{P\vee Q} = T_P \cup T_Q}$

$x \in T_{P\vee Q}$

$\Rightarrow$ (by definition of $T_{P\vee Q}$)

$\Rightarrow$ (by definiton of $\vee$)

$\Rightarrow x \in$ $x \in$ (by definition of T_P)

$\Rightarrow x \in$ (by defintion of $\cup$)

5. Let T_P denote the truth set of the predicate $P(x)$. Prove the following:

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Idea. (a)

$\underline{T_{P \vee Q} = T_P \cup T_Q}$

$x \in T_{P \vee Q}$

$\Rightarrow P(x) \vee Q(x)$ is true (by definition of $T_{P \vee Q}$)

$\Rightarrow$ (by definiton of $\vee$)

$\Rightarrow x \in \quad x \in$ (by definition of T_P)

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Idea. (a)

$\underline{T_{P \vee Q} = T_P \cup T_Q}$

$x \in T_{P \vee Q}$

$\Rightarrow P(x) \vee Q(x)$ is true (by definition of $T_{P \vee Q}$)

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Recall

- ✍ $T_P = \{x \mid P(x) \text{ is true}\}$
- ✍ $A = B$ iff $\forall x(x \in A \leftrightarrow x \in B)$

Idea. (a)

$\underline{T_{P\vee Q} = T_P \cup T_Q}$

$x \in T_{P\vee Q}$

$\Rightarrow P(x) \vee Q(x)$ is true (by definition of $T_{P\vee Q}$)

$\Rightarrow P(x)$ is true or $Q(x)$ is true (by definiton of $\vee$)

$\Rightarrow x \in T_P$ or $x \in T_Q$ (by definition of T_P)

$\Rightarrow x \in$ (by defintion of $\cup$)

5. Let T_P denote the truth set of the predicate $P(x)$. Prove the following:

(a) $T_{P\vee Q} = T_P \cup T_Q$, $T_{P\wedge Q} = T_p \cap T_Q$,

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Recall

✍ $T_P = \{x \mid P(x) \text{ is true}\}$

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Idea. (a)

$\underline{T_{P\vee Q} = T_P \cup T_Q}$

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$\Rightarrow P(x)$ is true or $Q(x)$ is true (by definiton of $\vee$)

$\Rightarrow x \in T_P$ or $x \in T_Q$ (by definition of T_P)

$\Rightarrow x \in T_P \cup T_Q$ (by defintion of $\cup$)

$x \in T_P \cup T_Q$

$\Rightarrow x \in \qquad x \in \qquad$ (by definition of $\cup$)

$\Rightarrow \qquad$ (by definition of T_p)

$\Rightarrow \qquad$ (by definition of $\vee$)

$\Rightarrow \qquad$ (by definition of truth set)

$\underline{T_{P \wedge Q} = T_p \cap T_Q}$

$x \in T_{P \wedge Q}$

$\Rightarrow \qquad$ (by definition of $T_{P \wedge Q}$)

$\Rightarrow \qquad$ (by definiton of $\wedge$)

$\Rightarrow x \in \qquad x \in \qquad$ (by definition of T_P)

$\Rightarrow x \in \qquad$ (by defintion of $\cap$)

$x \in T_P \cap T_Q$

$\Rightarrow x \in \qquad x \in \qquad$ (by definition of $\cap$)

$\Rightarrow \qquad$ (by definition of T_p)

$\Rightarrow \qquad$ (by definition of $\wedge$)

$\Rightarrow \qquad$ (by definition of truth set)

$x \in T_P \cup T_Q$
$\Rightarrow x \in T_P$ or $x \in T_Q$ (by definition of $\cup$)
$\Rightarrow$ (by definition of T_p)
$\Rightarrow$ (by definition of $\vee$)
$\Rightarrow$ (by definition of truth set)

$\underline{T_{P \wedge Q} = T_p \cap T_Q}$
$x \in T_{P \wedge Q}$
$\Rightarrow$ (by definition of $T_{P \wedge Q}$)
$\Rightarrow$ (by definiton of $\wedge$)
$\Rightarrow x \in$ $x \in$ (by definition of T_P)
$\Rightarrow x \in$ (by defintion of $\cap$)
$x \in T_P \cap T_Q$
$\Rightarrow x \in$ $x \in$ (by definition of $\cap$)
$\Rightarrow$ (by definition of T_p)
$\Rightarrow$ (by definition of $\wedge$)
$\Rightarrow$ (by definition of truth set)

$x \in T_P \cup T_Q$

$\Rightarrow x \in T_P$ or $x \in T_Q$ (by definition of $\cup$)

$\Rightarrow P(x)$ is true or $Q(x)$ is true (by definition of T_p)

$\Rightarrow$ (by definition of $\vee$)

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$x \in T_{P \wedge Q}$

$\Rightarrow$ (by definition of $T_{P \wedge Q}$)

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$\Rightarrow$ (by definition of $\wedge$)

$\Rightarrow$ (by definition of truth set)

$x \in T_P \cup T_Q$

$\Rightarrow x \in T_P$ or $x \in T_Q$ (by definition of $\cup$)

$\Rightarrow P(x)$ is true or $Q(x)$ is true (by definition of T_p)

$\Rightarrow P(x) \vee Q(x)$ is true (by definition of $\vee$)

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$\underline{T_{P \wedge Q} = T_p \cap T_Q}$

$x \in T_{P \wedge Q}$

$\Rightarrow$ (by definition of $T_{P \wedge Q}$)

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$\Rightarrow x \in$ $x \in$ (by definition of T_P)

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$x \in T_P \cap T_Q$

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$\Rightarrow$ (by definition of T_p)

$\Rightarrow$ (by definition of $\wedge$)

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$x \in T_P \cup T_Q$
$\Rightarrow x \in T_P$ or $x \in T_Q$ (by definition of $\cup$)
$\Rightarrow P(x)$ is true or $Q(x)$ is true (by definition of T_p)
$\Rightarrow P(x) \vee Q(x)$ is true (by definition of $\vee$)
$\Rightarrow x \in T_{P \vee Q}$ (by definition of truth set)

$\underline{T_{P \wedge Q} = T_p \cap T_Q}$
$x \in T_{P \wedge Q}$
$\Rightarrow$ (by definition of $T_{P \wedge Q}$)
$\Rightarrow$ (by definiton of $\wedge$)
$\Rightarrow x \in$ $x \in$ (by definition of T_P)
$\Rightarrow x \in$ (by defintion of $\cap$)
$x \in T_P \cap T_Q$
$\Rightarrow x \in$ $x \in$ (by definition of $\cap$)
$\Rightarrow$ (by definition of T_p)
$\Rightarrow$ (by definition of $\wedge$)
$\Rightarrow$ (by definition of truth set)

$x \in T_P \cup T_Q$

$\Rightarrow x \in T_P$ or $x \in T_Q$ (by definition of $\cup$)

$\Rightarrow P(x)$ is true or $Q(x)$ is true (by definition of T_p)

$\Rightarrow P(x) \vee Q(x)$ is true (by definition of $\vee$)

$\Rightarrow x \in T_{P \vee Q}$ (by definition of truth set)

$\underline{T_{P \wedge Q} = T_p \cap T_Q}$

$x \in T_{P \wedge Q}$

$\Rightarrow P(x) \wedge Q(x)$ is true (by definition of $T_{P \wedge Q}$)

$\Rightarrow$ (by definiton of $\wedge$)

$\Rightarrow x \in$ $x \in$ (by definition of T_P)

$\Rightarrow x \in$ (by defintion of $\cap$)

$x \in T_P \cap T_Q$

$\Rightarrow x \in$ $x \in$ (by definition of $\cap$)

$\Rightarrow$ (by definition of T_p)

$\Rightarrow$ (by definition of $\wedge$)

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$x \in T_P \cup T_Q$
$\Rightarrow x \in T_P$ or $x \in T_Q$ (by definition of $\cup$)
$\Rightarrow P(x)$ is true or $Q(x)$ is true (by definition of T_p)
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$\Rightarrow x \in T_{P \vee Q}$ (by definition of truth set)

$\underline{T_{P \wedge Q} = T_p \cap T_Q}$
$x \in T_{P \wedge Q}$
$\Rightarrow P(x) \wedge Q(x)$ is true (by definition of $T_{P \wedge Q}$)
$\Rightarrow P(x)$ is true and $Q(x)$ is true (by definiton of $\wedge$)
$\Rightarrow x \in$ $x \in$ (by definition of T_P)
$\Rightarrow x \in$ (by defintion of $\cap$)
$x \in T_P \cap T_Q$
$\Rightarrow x \in$ $x \in$ (by definition of $\cap$)
$\Rightarrow$ (by definition of T_p)
$\Rightarrow$ (by definition of $\wedge$)
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$x \in T_P \cup T_Q$

$\Rightarrow x \in T_P$ or $x \in T_Q$ (by definition of $\cup$)

$\Rightarrow P(x)$ is true or $Q(x)$ is true (by definition of T_p)

$\Rightarrow P(x) \vee Q(x)$ is true (by definition of $\vee$)

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$\underline{T_{P \wedge Q} = T_p \cap T_Q}$

$x \in T_{P \wedge Q}$

$\Rightarrow P(x) \wedge Q(x)$ is true (by definition of $T_{P \wedge Q}$)

$\Rightarrow P(x)$ is true and $Q(x)$ is true (by definiton of $\wedge$)

$\Rightarrow x \in T_P$ and $x \in T_Q$ (by definition of T_P)

$\Rightarrow x \in$ (by defintion of $\cap$)

$x \in T_P \cap T_Q$

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$\Rightarrow x \in T_{P \vee Q}$ (by definition of truth set)

$\underline{T_{P \wedge Q} = T_p \cap T_Q}$

$x \in T_{P \wedge Q}$

$\Rightarrow P(x) \wedge Q(x)$ is true (by definition of $T_{P \wedge Q}$)

$\Rightarrow P(x)$ is true and $Q(x)$ is true (by definiton of $\wedge$)

$\Rightarrow x \in T_P$ and $x \in T_Q$ (by definition of T_P)

$\Rightarrow x \in T_P \cap T_Q$ (by defintion of $\cap$)

$x \in T_P \cap T_Q$

$\Rightarrow x \in$ $x \in$ (by definition of $\cap$)

$\Rightarrow$ (by definition of T_p)

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$\Rightarrow$ (by definition of truth set)

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$\Rightarrow P(x)$ is true and $Q(x)$ is true (by definiton of $\wedge$)
$\Rightarrow x \in T_P$ and $x \in T_Q$ (by definition of T_P)
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$\Rightarrow x \in T_{P \wedge Q}$ (by definition of truth set)

(b) $T_{P \to Q} = \overline{T_P} \cup T_Q$.

Recall

$P \to Q \equiv \neg P \vee Q$

$x \in T_{P \to Q}$

$\Rightarrow$ (by definition of truth set)

$\Rightarrow$ $(P \to Q \equiv \neg P \vee Q)$

$\Rightarrow$ (by definition of $\vee$)

$\Rightarrow$ (by definition of $\neg$)

$\Rightarrow$ (by definition of truth set)

$\Rightarrow$ (by defintion of $\cup$)

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$\Rightarrow \neg P(x)$ is true or $Q(x)$ is true (by definition of $\vee$)

$\Rightarrow$ or $Q(x)$ is true (by definition of $\neg$)

$\Rightarrow$ (by definition of truth set)

$\Rightarrow$ (by defintion of $\cup$)

$x \in \overline{T_P} \cup T_Q$

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$x \in \overline{T_P} \cup T_Q$
$\Rightarrow$ $x \in \overline{T_P}$ or $x \in T_Q$ (by defintion of $\cup$)
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6. Let $A = \{1, 2, 3\}$, $B = \{u, v\}$, $C = \{m, n\}$. List the elements of $(A \times B) \times C$ and $A \times B \times C$. Are the two cartesian products equal?

Recall

- ✍ $A \times B = \{(x, y) : x \in A, y \in B\}$.
- ✍ $A_1 \times A_2 \times \dots A_n = \{(x_1, x_2, \dots, x_n) : x_1 \in A_1, x_2 \in A_2, \dots, x_n \in A_n\}$

6. Let $A = \{1, 2, 3\}$, $B = \{u, v\}$, $C = \{m, n\}$. List the elements of $(A \times B) \times C$ and $A \times B \times C$. Are the two cartesian products equal?

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Idea.

$$
\begin{aligned}
(A \times B) \times C =& \{((1, u), m), ((1, u), n), ((1, v), m), ((1, v), n), \\
& ((2, u), m), ((2, u), n), ((2, v), m), ((2, v), n), \\
& ((3, u), m), ((3, u), n), ((3, v), m), ((3, v), n)\} \\
A \times B \times C =& \{(1, u, m), (1, u, n), (1, v, m), (1, v, n), \\
& (2, u, m), (2, u, n), (2, v, m), (2, v, n), \\
& (3, u, m), (3, u, n), (3, v, m), (3, v, n)\}
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Idea.

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A \times B \times C =& \{(1, u, m), (1, u, n), (1, v, m), (1, v, n), \\
& (2, u, m), (2, u, n), (2, v, m), (2, v, n), \\
& (3, u, m), (3, u, n), (3, v, m), (3, v, n)\}
\end{aligned}$$

Answer. Not equal.

7. Find the mistake in the following "proof".

Theorem: For all sets A and B, $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$.

Proof. Suppose A and B are sets, and $x \in \overline{A} \cup \overline{B}$. Then $x \in \overline{A}$ or $x \in \overline{B}$. It follows that $x \notin A$ or $x \notin B$ and so $x \notin A \cup B$. Thus $x \in \overline{A \cup B}$ and hence $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$.

Recall

✍ De Morgen's Law: $\neg P \vee \neg Q \equiv \neg(P \wedge Q)$, so

$$x \notin A \vee x \notin B \leftrightarrow x \notin A \cap B$$

7. Find the mistake in the following "proof".

Theorem: For all sets A and B, $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$.

Proof. Suppose A and B are sets, and $x \in \overline{A} \cup \overline{B}$. Then $x \in \overline{A}$ or $x \in \overline{B}$. It follows that $x \notin A$ or $x \notin B$ and so $x \notin A \cup B$. Thus $x \in \overline{A \cup B}$ and hence $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$.

Recall

✍ De Morgen's Law: $\neg P \vee \neg Q \equiv \neg(P \wedge Q)$, so

$$x \notin A \vee x \notin B \leftrightarrow x \notin A \cap B$$

Answer. $x \notin A$ or $x \notin B$ does not imply $x \notin A \cup B$. Counter example: $A = \{1\}$, $B = \{2\}$, $x = 1$.

8. Prove that $(A - C) \cap (B - C) \cap (A - B) = \varnothing$.

Then and ($\Rightarrow x \quad B$) and
($\Rightarrow x \quad B$).

8. Prove that $(A - C) \cap (B - C) \cap (A - B) = \varnothing$.

Answer. Suppose $\exists x \in (A - C) \cap (B - C) \cap (A - B)$.
Then and ($\Rightarrow x \quad B$) and
($\Rightarrow x \quad B$).

8. Prove that $(A - C) \cap (B - C) \cap (A - B) = \varnothing$.

Answer. Suppose $\exists x \in (A - C) \cap (B - C) \cap (A - B)$.
Then $x \in A - C$ and (⇒ $x \quad B$) and
(⇒ $x \quad B$).

8. Prove that $(A - C) \cap (B - C) \cap (A - B) = \emptyset$.

Answer. Suppose $\exists x \in (A - C) \cap (B - C) \cap (A - B)$.
Then $x \in A - C$ and $x \in B - C$ ($\Rightarrow x \quad B$) and
($\Rightarrow x \quad B$).

8. Prove that $(A - C) \cap (B - C) \cap (A - B) = \varnothing$.

Answer. Suppose $\exists x \in (A - C) \cap (B - C) \cap (A - B)$.
Then $x \in A - C$ and $x \in B - C$ ($\Rightarrow x \quad B$) and $x \in A - B$ ($\Rightarrow x \quad B$).

8. Prove that $(A - C) \cap (B - C) \cap (A - B) = \varnothing$.

Answer. Suppose $\exists x \in (A - C) \cap (B - C) \cap (A - B)$.
Then $x \in A - C$ and $x \in B - C$ ($\Rightarrow x \in B$) and $x \in A - B$ ($\Rightarrow x \quad B$).

8. Prove that $(A - C) \cap (B - C) \cap (A - B) = \varnothing$.
Answer. Suppose $\exists x \in (A - C) \cap (B - C) \cap (A - B)$.
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Then $x \in A - C$ and $x \in B - C$ ($\Rightarrow x \in B$) and $x \in A - B$ ($\Rightarrow x \notin B$).
That's a contradiction.

8. Prove that $(A - C) \cap (B - C) \cap (A - B) = \emptyset$.

Answer. Suppose $\exists x \in (A - C) \cap (B - C) \cap (A - B)$.

Then $x \in A - C$ and $x \in B - C$ ($\Rightarrow x \in B$) and $x \in A - B$ ($\Rightarrow x \notin B$).

That's a contradiction.

Thus no such x exists, i.e., $(A - C) \cap (B - C) \cap (A - B) = \emptyset$.

9. Prove that for all sets A, B, C, D,

$$\text{if } A \cap C = \varnothing, \text{ then } (A \times B) \cap (C \times D) = \varnothing.$$

Hint: in showing some set equals $\varnothing$, we may try the proof by contradiction

Recall

Prove $P \to Q$.

- ✍ By contradiction: assume $\neg(P \to Q) \equiv (P \wedge \neg Q) \xrightarrow{\text{using various rules of inference}}$ arrive at a contradiction.
- ✍ $A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$

Proof by contradiction.

We suppose

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

9. Prove that for all sets A, B, C, D,

$$\text{if } A \cap C = \varnothing, \text{ then } (A \times B) \cap (C \times D) = \varnothing.$$

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Recall

Prove $P \rightarrow Q$.

- ✍ By contradiction: assume $\neg(P \rightarrow Q) \equiv (P \wedge \neg Q) \xrightarrow{\text{using various rules of inference}}$ arrive at a contradiction.
- ✍ $A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$

Proof by contradiction.
We suppose $A \cap C = \varnothing$ and $(A \times B) \cap (C \times D) \neq \varnothing$
$\Rightarrow \exists \qquad \in (A \times B) \cap (C \times D)$
$\Rightarrow$
$\Rightarrow$
$\Rightarrow$

9. Prove that for all sets A, B, C, D,

$$\text{if } A \cap C = \varnothing, \text{ then } (A \times B) \cap (C \times D) = \varnothing.$$

Hint: in showing some set equals $\varnothing$, we may try the proof by contradiction

Recall

Prove $P \to Q$.

- ✍ By contradiction: assume $\neg(P \to Q) \equiv (P \wedge \neg Q) \xrightarrow{\text{using various rules of inference}}$ arrive at a contradiction.
- ✍ $A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$

Proof by contradiction.

We suppose $A \cap C = \varnothing$ and $(A \times B) \cap (C \times D) \neq \varnothing$

$\Rightarrow \exists (x, y) \in (A \times B) \cap (C \times D)$

$\Rightarrow x \in$

$\Rightarrow$

$\Rightarrow$

9. Prove that for all sets A, B, C, D,

$$\text{if } A \cap C = \varnothing, \text{ then } (A \times B) \cap (C \times D) = \varnothing.$$

Hint: in showing some set equals $\varnothing$, we may try the proof by contradiction

Recall

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Proof by contradiction.

We suppose $A \cap C = \varnothing$ and $(A \times B) \cap (C \times D) \neq \varnothing$

$\Rightarrow \exists (x, y) \in (A \times B) \cap (C \times D)$

$\Rightarrow x \in A \cap C$

$\Rightarrow A \cap C$

$\Rightarrow$

9. Prove that for all sets A, B, C, D,

$$\text{if } A \cap C = \emptyset, \text{ then } (A \times B) \cap (C \times D) = \emptyset.$$

Hint: in showing some set equals $\emptyset$, we may try the proof by contradiction

Recall

Prove $P \to Q$.

- By contradiction: assume $\neg(P \to Q) \equiv (P \wedge \neg Q) \xrightarrow{\text{using various rules of inference}}$ arrive at a contradiction.
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Proof by contradiction.
We suppose $A \cap C = \emptyset$ and $(A \times B) \cap (C \times D) \neq \emptyset$
$\Rightarrow \exists (x, y) \in (A \times B) \cap (C \times D)$
$\Rightarrow x \in A \cap C$
$\Rightarrow A \cap C \neq \emptyset$
$\Rightarrow$

9. Prove that for all sets A, B, C, D,

$$\text{if } A \cap C = \varnothing, \text{ then } (A \times B) \cap (C \times D) = \varnothing.$$

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Recall

Prove $P \to Q$.

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Proof by contradiction.

We suppose $A \cap C = \varnothing$ and $(A \times B) \cap (C \times D) \neq \varnothing$

$\Rightarrow \exists (x, y) \in (A \times B) \cap (C \times D)$

$\Rightarrow x \in A \cap C$

$\Rightarrow A \cap C \neq \varnothing$

$\Rightarrow$Contradiction!

10. For each of the following, either prove the statement true or find a counterexample to prove that it is false.

(a) If $B \cap C \subseteq A$, then $(A - B) \cap (A - C) = \varnothing$.

(b) $(A - B) \cap (C - B) = A - (B \cup C)$.

(c) If $\overline{A} \subseteq B$, then $A \cup B = U$.

(d) $P(A \cap B) = P(A) \cap P(B)$.

Hint. Use Venn diagram for (a)-(c).

10. For each of the following, either prove the statement true or find a counterexample to prove that it is false.

(a) If $B \cap C \subseteq A$, then $(A - B) \cap (A - C) = \varnothing$.

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(d) $P(A \cap B) = P(A) \cap P(B)$.

Hint. Use Venn diagram for (a)-(c).

Idea.

(a) False. $A = \{1, 2, 3\}$, $B = \{2\}$, $C = \{3\}$.

10. For each of the following, either prove the statement true or find a counterexample to prove that it is false.

(a) If $B \cap C \subseteq A$, then $(A - B) \cap (A - C) = \varnothing$.

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(d) $P(A \cap B) = P(A) \cap P(B)$.

Hint. Use Venn diagram for (a)-(c).

Idea.

(a) False. $A = \{1, 2, 3\}$, $B = \{2\}$, $C = \{3\}$.

(b) False. $A = \{1\}$, $B = \{2\}$, $C = \{3\}$

$\Rightarrow x \in \qquad (A \subseteq A \cup B)$

$\Rightarrow x \in \quad (\overline{A} \subseteq B)$

$\Rightarrow x \in \qquad (B \subseteq A \cup B)$

10. For each of the following, either prove the statement true or find a counterexample to prove that it is false.

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Hint. Use Venn diagram for (a)-(c).

Idea.

(a) False. $A = \{1, 2, 3\}$, $B = \{2\}$, $C = \{3\}$.

(b) False. $A = \{1\}$, $B = \{2\}$, $C = \{3\}$

(c) True. As every set is a subset of the universe, $A \cup B \subseteq U$.
Thus we need to prove $U \subseteq A \cup B$.
So we need to show $x \in U \to x \in A \cup B$.
Case 1. $x \in A$
$\Rightarrow x \in \qquad (A \subseteq A \cup B)$
Case 2. $x \in \overline{A}$
$\Rightarrow x \in \quad (\overline{A} \subseteq B)$
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10. For each of the following, either prove the statement true or find a counterexample to prove that it is false.

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(c) If $\overline{A} \subseteq B$, then $A \cup B = U$.

(d) $P(A \cap B) = P(A) \cap P(B)$.

Hint. Use Venn diagram for (a)-(c).

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(a) False. $A = \{1, 2, 3\}$, $B = \{2\}$, $C = \{3\}$.

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Case 1. $x \in A$
$\Rightarrow x \in A \cup B$ $(A \subseteq A \cup B)$
Case 2. $x \in \overline{A}$
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$\Rightarrow x \in$ $(B \subseteq A \cup B)$

10. For each of the following, either prove the statement true or find a counterexample to prove that it is false.

(a) If $B \cap C \subseteq A$, then $(A - B) \cap (A - C) = \varnothing$.

(b) $(A - B) \cap (C - B) = A - (B \cup C)$.

(c) If $\overline{A} \subseteq B$, then $A \cup B = U$.

(d) $P(A \cap B) = P(A) \cap P(B)$.

Hint. Use Venn diagram for (a)-(c).

Idea.

(a) False. $A = \{1, 2, 3\}$, $B = \{2\}$, $C = \{3\}$.

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Case 1. $x \in A$
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Case 2. $x \in \overline{A}$
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(a) False. $A = \{1, 2, 3\}$, $B = \{2\}$, $C = \{3\}$.

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Case 2. $x \in \overline{A}$
$\Rightarrow x \in B$ $(\overline{A} \subseteq B)$
$\Rightarrow x \in A \cup B$ $(B \subseteq A \cup B)$

(d) $P(A \cap B) = P(A) \cap P(B)$.

Recall

✍ $P(A)$: the power set of A, which is the set of all subsets of A.

(d) $P(A \cap B) = P(A) \cap P(B)$.

Recall

✍ $P(A)$: the power set of A, which is the set of all subsets of A.

Answer. True.

$X \in P(A \cap B)$

$\Rightarrow X$

$\Rightarrow X$ and X

$\Rightarrow X \in$

Thus we have proved that

$X \in P(A) \cap P(B)$

$\Rightarrow X$ and X

$\Rightarrow X$.

$\Rightarrow X$.

This proves

(d) $P(A \cap B) = P(A) \cap P(B)$.

Recall

✍ $P(A)$: the power set of A, which is the set of all subsets of A.

Answer. True.

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Answer. True.

$X \in P(A \cap B)$

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Thus we have proved that

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$\Rightarrow X$ and X

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(d) $P(A \cap B) = P(A) \cap P(B)$.

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✍ $P(A)$: the power set of A, which is the set of all subsets of A.

Answer. True.

$X \in P(A \cap B)$

$\Rightarrow X \subseteq A \cap B$

$\Rightarrow X \subseteq A$ and $X \subseteq B$

$\Rightarrow X \in P(A) \cap P(B)$

Thus we have proved that

$X \in P(A) \cap P(B)$

$\Rightarrow X$ and X

$\Rightarrow X$.

$\Rightarrow X$.

This proves

(d) $P(A \cap B) = P(A) \cap P(B)$.

Recall

✍ $P(A)$: the power set of A, which is the set of all subsets of A.

Answer. True.

$X \in P(A \cap B)$

$\Rightarrow X \subseteq A \cap B$

$\Rightarrow X \subseteq A$ and $X \subseteq B$

$\Rightarrow X \in P(A) \cap P(B)$

Thus we have proved that $P(A \cap B) \subseteq P(A) \cap P(B)$.

$X \in P(A) \cap P(B)$

$\Rightarrow X$ and X

$\Rightarrow X$.

$\Rightarrow X$.

This proves

(d) $P(A \cap B) = P(A) \cap P(B)$.

Recall

✍ $P(A)$: the power set of A, which is the set of all subsets of A.

Answer. True.

$X \in P(A \cap B)$

$\Rightarrow X \subseteq A \cap B$

$\Rightarrow X \subseteq A$ and $X \subseteq B$

$\Rightarrow X \in P(A) \cap P(B)$

Thus we have proved that $P(A \cap B) \subseteq P(A) \cap P(B)$.

$X \in P(A) \cap P(B)$

$\Rightarrow X \subseteq A$ and $X \subseteq B$

$\Rightarrow X$.

$\Rightarrow X$.

This proves

(d) $P(A \cap B) = P(A) \cap P(B)$.

Recall

✍ $P(A)$: the power set of A, which is the set of all subsets of A.

Answer. True.

$X \in P(A \cap B)$

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Thus we have proved that $P(A \cap B) \subseteq P(A) \cap P(B)$.

$X \in P(A) \cap P(B)$

$\Rightarrow X \subseteq A$ and $X \subseteq B$

$\Rightarrow X \subseteq A \cap B$.

$\Rightarrow X$.

This proves

(d) $P(A \cap B) = P(A) \cap P(B)$.

Recall

✍ $P(A)$: the power set of A, which is the set of all subsets of A.

Answer. True.
$X \in P(A \cap B)$
$\Rightarrow X \subseteq A \cap B$
$\Rightarrow X \subseteq A$ and $X \subseteq B$
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Thus we have proved that $P(A \cap B) \subseteq P(A) \cap P(B)$.
$X \in P(A) \cap P(B)$
$\Rightarrow X \subseteq A$ and $X \subseteq B$
$\Rightarrow X \subseteq A \cap B$.
$\Rightarrow X \in P(A \cap B)$.
This proves

(d) $P(A \cap B) = P(A) \cap P(B)$.

Recall

✍ $P(A)$: the power set of A, which is the set of all subsets of A.

Answer. True.

$X \in P(A \cap B)$

$\Rightarrow X \subseteq A \cap B$

$\Rightarrow X \subseteq A$ and $X \subseteq B$

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Thus we have proved that $P(A \cap B) \subseteq P(A) \cap P(B)$.

$X \in P(A) \cap P(B)$

$\Rightarrow X \subseteq A$ and $X \subseteq B$

$\Rightarrow X \subseteq A \cap B$.

$\Rightarrow X \in P(A \cap B)$.

This proves $P(A) \cap P(B) \subseteq P(A \cap B)$

11. Define the **SYMMETRIC DIFFERENCE** as
$A \oplus B = (A - B) \cup (B - A)$.

(a) Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$. Find $(A \oplus B) \oplus C$.

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

(a) $A - B =$, $B - A =$, so $A \oplus B =$.
$(A \oplus B) - C =$, $C - (A \oplus B) =$, so
$(A \oplus B) \oplus C =$.

11. Define the **SYMMETRIC DIFFERENCE** as
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Idea.

(a) $A - B = \{1, 2\}$, $B - A =$, so $A \oplus B =$.
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Idea.

(a) $A - B = \{1,2\}$, $B - A = \{5,6\}$, so $A \oplus B =$.
$(A \oplus B) - C =$, $C - (A \oplus B) =$, so
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Idea.

(a) $A - B = \{1, 2\}$, $B - A = \{5, 6\}$, so $A \oplus B = \{1, 2, 5, 6\}$.
$(A \oplus B) - C =$, $C - (A \oplus B) =$, so
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(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

(a) $A - B = \{1, 2\}$, $B - A = \{5, 6\}$, so $A \oplus B = \{1, 2, 5, 6\}$.
$(A \oplus B) - C = \{1, 2\}$, $C - (A \oplus B) =$, so
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Idea.

(a) $A - B = \{1, 2\}$, $B - A = \{5, 6\}$, so $A \oplus B = \{1, 2, 5, 6\}$.
$(A \oplus B) - C = \{1, 2\}$, $C - (A \oplus B) = \{7, 8\}$, so
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11. Define the **SYMMETRIC DIFFERENCE** as
$A \oplus B = (A - B) \cup (B - A)$.

(a) Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$. Find $(A \oplus B) \oplus C$.

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

(a) $A - B = \{1, 2\}$, $B - A = \{5, 6\}$, so $A \oplus B = \{1, 2, 5, 6\}$.
$(A \oplus B) - C = \{1, 2\}$, $C - (A \oplus B) = \{7, 8\}$, so
$(A \oplus B) \oplus C = \{1, 2, 7, 8\}$.

Definition: $A \oplus B = (A - B) \cup (B - A)$

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

$x \in A$

Case 1. $\therefore$

$\therefore$

$\therefore$

$\therefore$

$\therefore$

$\therefore$ (as)

Case 2. $\therefore$

$\therefore$

$\therefore$

$\therefore$

(as)

$\therefore$

Definition: $A \oplus B = (A - B) \cup (B - A)$

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

$$x \in A \begin{cases} \text{Case 1. } x \in C & \therefore & \\ & \therefore & \\ & \therefore & \\ & \therefore & \\ & \therefore & \\ & \therefore & (\text{as } x \in C) \\ \text{Case 2. } x \notin C & \therefore & \\ & \therefore & \\ & \therefore & \\ & \therefore & \\ & (\text{as} \qquad\qquad\qquad) & \\ & \therefore & \end{cases}$$

Definition: $A \oplus B = (A - B) \cup (B - A)$

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

$$x \in A \begin{cases} \text{Case 1. } x \in C & \therefore x \in A \wedge x \in C & \\ & \therefore & \\ & \therefore & \\ & \therefore & \\ & \therefore & \\ & \therefore & (\text{as } x \in C) \\ \text{Case 2. } x \notin C & \therefore & \\ & \therefore & \\ & \therefore & \\ & \therefore & \\ & (\text{as} \qquad\qquad\qquad) & \\ & \therefore & \end{cases}$$

Definition: $A \oplus B = (A - B) \cup (B - A)$

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

$$x \in A \begin{cases} \text{Case 1. } x \in C & \therefore x \in A \wedge x \in C \\ & \therefore x \notin A - C \wedge x \notin C - A \\ & \therefore \\ & \therefore \\ & \therefore \\ & \therefore \qquad \text{(as } x \in C\text{)} \\ \text{Case 2. } x \notin C & \therefore \\ & \therefore \\ & \therefore \\ & \therefore \\ & \text{(as} \qquad\qquad\qquad \text{)} \\ & \therefore \end{cases}$$

Definition: $A \oplus B = (A - B) \cup (B - A)$

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

$$x \in A \begin{cases} \text{Case 1. } x \in C & \begin{array}{l} \therefore x \in A \wedge x \in C \\ \therefore x \notin A - C \wedge x \notin C - A \\ \therefore x \notin (A - C) \cup (C - A) = A \oplus C \\ \therefore \\ \therefore \\ \therefore \qquad \qquad \text{(as } x \in C\text{)} \end{array} \\ \text{Case 2. } x \notin C & \begin{array}{l} \therefore \\ \therefore \\ \therefore \\ \therefore \\ \text{(as} \qquad \qquad \qquad \text{)} \\ \therefore \end{array} \end{cases}$$

Definition: $A \oplus B = (A - B) \cup (B - A)$

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

$x \in A$

Case 1. $x \in C$

$\therefore x \in A \wedge x \in C$

$\therefore x \notin A - C \wedge x \notin C - A$

$\therefore x \notin (A - C) \cup (C - A) = A \oplus C$

$\therefore x \notin B \oplus C = (B - C) \cup (C - B)$

$\therefore$

$\therefore$ (as $x \in C$)

Case 2. $x \notin C$

$\therefore$

$\therefore$

$\therefore$

$\therefore$

(as)

$\therefore$

Definition: $A \oplus B = (A - B) \cup (B - A)$

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

$$x \in A \begin{cases} \text{Case 1. } x \in C & \therefore x \in A \wedge x \in C \\ & \therefore x \notin A - C \wedge x \notin C - A \\ & \therefore x \notin (A - C) \cup (C - A) = A \oplus C \\ & \therefore x \notin B \oplus C = (B - C) \cup (C - B) \\ & \therefore x \notin C - B \\ & \therefore \qquad\qquad \text{(as } x \in C\text{)} \\ \text{Case 2. } x \notin C & \therefore \\ & \therefore \\ & \therefore \\ & \therefore \\ & \text{(as} \qquad\qquad\qquad \text{)} \\ & \therefore \end{cases}$$

Definition: $A \oplus B = (A - B) \cup (B - A)$

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

$$x \in A \begin{cases} \text{Case 1. } x \in C & \therefore x \in A \wedge x \in C \\ & \therefore x \notin A - C \wedge x \notin C - A \\ & \therefore x \notin (A - C) \cup (C - A) = A \oplus C \\ & \therefore x \notin B \oplus C = (B - C) \cup (C - B) \\ & \therefore x \notin C - B \\ & \therefore x \in B (\text{as } x \in C) \\ \text{Case 2. } x \notin C & \therefore \\ & \therefore \\ & \therefore \\ & \therefore \\ & (\text{as} \qquad\qquad\qquad) \\ & \therefore \end{cases}$$

Definition: $A \oplus B = (A - B) \cup (B - A)$

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

$$
x \in A \begin{cases}
\text{Case 1. } x \in C & \therefore x \in A \wedge x \in C \\
& \therefore x \notin A - C \wedge x \notin C - A \\
& \therefore x \notin (A - C) \cup (C - A) = A \oplus C \\
& \therefore x \notin B \oplus C = (B - C) \cup (C - B) \\
& \therefore x \notin C - B \\
& \therefore x \in B (\text{as } x \in C) \\
\text{Case 2. } x \notin C & \therefore x \in A - C \\
& \therefore \\
& \therefore \\
& \therefore \\
& (\text{as } x \notin C - B) \\
& \therefore
\end{cases}
$$

Definition: $A \oplus B = (A - B) \cup (B - A)$

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

$$x \in A \begin{cases} \text{Case 1. } x \in C & \therefore x \in A \wedge x \in C \\ & \therefore x \notin A - C \wedge x \notin C - A \\ & \therefore x \notin (A - C) \cup (C - A) = A \oplus C \\ & \therefore x \notin B \oplus C = (B - C) \cup (C - B) \\ & \therefore x \notin C - B \\ & \therefore x \in B (\text{as } x \in C) \\ \text{Case 2. } x \notin C & \therefore x \in A - C \\ & \therefore x \in (A - C) \cup (C - A) = A \oplus C \\ & \therefore \\ & \therefore \\ & (\text{as } x \notin C - B) \\ & \therefore \end{cases}$$

Definition: $A \oplus B = (A - B) \cup (B - A)$

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

$$
x \in A \begin{cases}
\text{Case 1. } x \in C & \therefore x \in A \wedge x \in C \\
& \therefore x \notin A - C \wedge x \notin C - A \\
& \therefore x \notin (A - C) \cup (C - A) = A \oplus C \\
& \therefore x \notin B \oplus C = (B - C) \cup (C - B) \\
& \therefore x \notin C - B \\
& \therefore x \in B (\text{as } x \in C) \\
\text{Case 2. } x \notin C & \therefore x \in A - C \\
& \therefore x \in (A - C) \cup (C - A) = A \oplus C \\
& \therefore x \in B \oplus C = (B - C) \cup (C - B) \\
& \therefore \\
& (\text{as } x \notin C - B) \\
& \therefore
\end{cases}
$$

Definition: $A \oplus B = (A - B) \cup (B - A)$

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

$$x \in A \begin{cases} \text{Case 1. } x \in C & \therefore x \in A \wedge x \in C \\ & \therefore x \notin A - C \wedge x \notin C - A \\ & \therefore x \notin (A - C) \cup (C - A) = A \oplus C \\ & \therefore x \notin B \oplus C = (B - C) \cup (C - B) \\ & \therefore x \notin C - B \\ & \therefore x \in B (\text{as } x \in C) \\ \text{Case 2. } x \notin C & \therefore x \in A - C \\ & \therefore x \in (A - C) \cup (C - A) = A \oplus C \\ & \therefore x \in B \oplus C = (B - C) \cup (C - B) \\ & \therefore x \in B - C \\ & (\text{as } x \notin C - B) \\ & \therefore \end{cases}$$

Definition: $A \oplus B = (A - B) \cup (B - A)$

(b) Prove that if $A \oplus C = B \oplus C$, then $A = B$.

Idea.

$$x \in A \begin{cases} \text{Case 1. } x \in C & \therefore x \in A \wedge x \in C \\ & \therefore x \notin A - C \wedge x \notin C - A \\ & \therefore x \notin (A - C) \cup (C - A) = A \oplus C \\ & \therefore x \notin B \oplus C = (B - C) \cup (C - B) \\ & \therefore x \notin C - B \\ & \therefore x \in B (\text{as } x \in C) \\ \text{Case 2. } x \notin C & \therefore x \in A - C \\ & \therefore x \in (A - C) \cup (C - A) = A \oplus C \\ & \therefore x \in B \oplus C = (B - C) \cup (C - B) \\ & \therefore x \in B - C \\ & (\text{as } x \notin C - B) \\ & \therefore x \in B \end{cases}$$

$$x \in B \begin{cases} \text{Case 1.} & \therefore \\ & \therefore \\ & \therefore \\ & \therefore \\ & \therefore \\ & \therefore \qquad \text{(as} \qquad\quad \text{)} \\ \text{Case 2.} & \therefore \\ & \therefore \\ & \therefore \\ & \therefore \\ & \text{(as} \qquad\qquad\qquad \text{)} \\ & \therefore \end{cases}$$

$$x \in B \left\{ \begin{array}{lll} \text{Case 1. } x \in C & \therefore & \\ & \therefore & \\ & \therefore & \\ & \therefore & \\ & \therefore & \\ & \therefore & (\text{as } x \in C) \\ \text{Case 2. } x \notin C & \therefore & \\ & \therefore & \\ & \therefore & \\ & \therefore & \\ & (\text{as} \qquad\qquad\qquad) & \\ & \therefore & \end{array} \right.$$

$$
x \in B \begin{cases}
\text{Case 1. } x \in C & \therefore x \in B \wedge x \in C \\
& \therefore \\
& \therefore \\
& \therefore \\
& \therefore \\
& \therefore \qquad\qquad \text{(as } x \in C\text{)} \\
\text{Case 2. } x \notin C & \therefore \\
& \therefore \\
& \therefore \\
& \therefore \\
& \text{(as} \qquad\qquad\qquad\qquad \text{)} \\
& \therefore
\end{cases}
$$

$$
x \in B \begin{cases}
\text{Case 1. } x \in C & \therefore x \in B \wedge x \in C \\
& \therefore x \notin B - C \wedge x \notin C - B \\
& \therefore \\
& \therefore \\
& \therefore \\
& \therefore \qquad\qquad (\text{as } x \in C) \\
\text{Case 2. } x \notin C & \therefore \\
& \therefore \\
& \therefore \\
& \therefore \\
& (\text{as} \qquad\qquad\qquad\qquad) \\
& \therefore
\end{cases}
$$

$$
x \in B \left\{
\begin{array}{lll}
\textsf{Case 1.} & x \in C & \therefore x \in B \wedge x \in C \\
 & & \therefore x \notin B - C \wedge x \notin C - B \\
 & & \therefore x \notin (B - C) \cup (C - B) = B \oplus C \\
 & & \therefore \\
 & & \therefore \\
 & & \therefore \qquad\qquad (\textsf{as } x \in C) \\
\textsf{Case 2.} & x \notin C & \therefore \\
 & & \therefore \\
 & & \therefore \\
 & & \therefore \\
 & & (\textsf{as} \qquad\qquad\qquad\qquad) \\
 & & \therefore
\end{array}
\right.
$$

$$
x \in B \left\{
\begin{array}{lll}
\text{Case 1.} & x \in C & \therefore x \in B \wedge x \in C \\
 & & \therefore x \notin B - C \wedge x \notin C - B \\
 & & \therefore x \notin (B - C) \cup (C - B) = B \oplus C \\
 & & \therefore x \notin A \oplus C = (A - C) \cup (C - A) \\
 & & \therefore \\
 & & \therefore \qquad\qquad (\text{as } x \in C) \\
\text{Case 2.} & x \notin C & \therefore \\
 & & \therefore \\
 & & \therefore \\
 & & \therefore \\
 & & (\text{as} \qquad\qquad\qquad\qquad) \\
 & & \therefore
\end{array}
\right.
$$

$$
x \in B \begin{cases}
\text{Case 1. } x \in C & \therefore x \in B \wedge x \in C \\
& \therefore x \notin B - C \wedge x \notin C - B \\
& \therefore x \notin (B - C) \cup (C - B) = B \oplus C \\
& \therefore x \notin A \oplus C = (A - C) \cup (C - A) \\
& \therefore x \notin C - A \\
& \therefore \qquad\qquad \text{(as } x \in C\text{)} \\
\text{Case 2. } x \notin C & \therefore \\
& \therefore \\
& \therefore \\
& \therefore \\
& \text{(as} \qquad\qquad\qquad\qquad \text{)} \\
& \therefore
\end{cases}
$$

$$
x \in B \begin{cases}
\text{Case 1. } x \in C & \therefore x \in B \wedge x \in C \\
 & \therefore x \notin B - C \wedge x \notin C - B \\
 & \therefore x \notin (B - C) \cup (C - B) = B \oplus C \\
 & \therefore x \notin A \oplus C = (A - C) \cup (C - A) \\
 & \therefore x \notin C - A \\
 & \therefore x \in A (\text{as } x \in C) \\
\text{Case 2. } x \notin C & \therefore \\
 & \therefore \\
 & \therefore \\
 & \therefore \\
 & (\text{as} \qquad\qquad\qquad) \\
 & \therefore
\end{cases}
$$

$$
x \in B \begin{cases}
\text{Case 1. } x \in C & \therefore x \in B \wedge x \in C \\
& \therefore x \notin B - C \wedge x \notin C - B \\
& \therefore x \notin (B - C) \cup (C - B) = B \oplus C \\
& \therefore x \notin A \oplus C = (A - C) \cup (C - A) \\
& \therefore x \notin C - A \\
& \therefore x \in A (\text{as } x \in C) \\
\text{Case 2. } x \notin C & \therefore x \in B - C \\
& \therefore \\
& \therefore \\
& \therefore \\
& (\text{as } x \notin C - A) \\
& \therefore
\end{cases}
$$

$$
x \in B \begin{cases}
\text{Case 1. } x \in C & \therefore x \in B \wedge x \in C \\
& \therefore x \notin B - C \wedge x \notin C - B \\
& \therefore x \notin (B - C) \cup (C - B) = B \oplus C \\
& \therefore x \notin A \oplus C = (A - C) \cup (C - A) \\
& \therefore x \notin C - A \\
& \therefore x \in A (\text{as } x \in C) \\
\text{Case 2. } x \notin C & \therefore x \in B - C \\
& \therefore x \in (B - C) \cup (C - B) = B \oplus C \\
& \therefore \\
& \therefore \\
& (\text{as } x \notin C - A) \\
& \therefore
\end{cases}
$$

$$
x \in B \begin{cases}
\text{Case 1. } x \in C & \therefore x \in B \wedge x \in C \\
& \therefore x \notin B - C \wedge x \notin C - B \\
& \therefore x \notin (B - C) \cup (C - B) = B \oplus C \\
& \therefore x \notin A \oplus C = (A - C) \cup (C - A) \\
& \therefore x \notin C - A \\
& \therefore x \in A (\text{as } x \in C) \\
\text{Case 2. } x \notin C & \therefore x \in B - C \\
& \therefore x \in (B - C) \cup (C - B) = B \oplus C \\
& \therefore x \in A \oplus C = (A - C) \cup (C - A) \\
& \therefore \\
& (\text{as } x \notin C - A) \\
& \therefore
\end{cases}
$$

$$x \in B \begin{cases} \text{Case 1. } x \in C & \therefore x \in B \wedge x \in C \\ & \therefore x \notin B - C \wedge x \notin C - B \\ & \therefore x \notin (B - C) \cup (C - B) = B \oplus C \\ & \therefore x \notin A \oplus C = (A - C) \cup (C - A) \\ & \therefore x \notin C - A \\ & \therefore x \in A (\text{as } x \in C) \\ \text{Case 2. } x \notin C & \therefore x \in B - C \\ & \therefore x \in (B - C) \cup (C - B) = B \oplus C \\ & \therefore x \in A \oplus C = (A - C) \cup (C - A) \\ & \therefore x \in A - C \\ & (\text{as } x \notin C - A) \\ & \therefore \end{cases}$$

$$
x \in B \begin{cases}
\text{Case 1. } x \in C & \therefore x \in B \land x \in C \\
 & \therefore x \notin B - C \land x \notin C - B \\
 & \therefore x \notin (B - C) \cup (C - B) = B \oplus C \\
 & \therefore x \notin A \oplus C = (A - C) \cup (C - A) \\
 & \therefore x \notin C - A \\
 & \therefore x \in A (\text{as } x \in C) \\
\text{Case 2. } x \notin C & \therefore x \in B - C \\
 & \therefore x \in (B - C) \cup (C - B) = B \oplus C \\
 & \therefore x \in A \oplus C = (A - C) \cup (C - A) \\
 & \therefore x \in A - C \\
 & (\text{as } x \notin C - A) \\
 & \therefore x \in A
\end{cases}
$$