



School of  
Computing

# Embeddings

CS4248 Natural Language Processing

Week 06

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# Recap of Week 05

Generative vs. Discriminative Classifiers

Classification with Logistic Regression  
and a Runthrough

Cross Entropy

Stochastic Gradient Descent

LR as a Probabilistic ML Classifier

Regularization

XOR

Neural Networks

# Week 06 Agenda

One-hot Representation

Co-occurrence Vectors

Word Embeddings

Word2Vec: CBOW and Skip-gram

Properties of Embeddings

# One-hot Representation



What are various ways to  
represent the meaning of a word?

# Representing words as discrete symbols

In traditional NLP, we regard words as discrete symbols:

*hotel*, *conference*, *b&b* – a localist representation

Words are represented by **one-hot** vectors:

Means one '1'  
and rest '0's

*b&b* = [0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]

*hotel* = [0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]

Where the vector dimension =  $|V|$  # of words in the vocabulary  
(20,000 to 50,000 dictionary lemmas, or 500K inflected tokens)

# Problem with words as discrete symbols

**Example:** in Web search, if the user searches for *singapore hotel*, we would like to match documents containing *singapore b&b*.

*b&b* = [0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]

*hotel* = [0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]

But we can see that the two vectors are orthogonal. Oh no!  
There is no natural notion of similarity for one-hot vectors!

# Distributional Hypothesis

“The meaning of a word is its use in the language”

[Wittgenstein, *Philosophical Investigations*, n.d.]

“You shall know a word by the company it keeps”

[Firth, 1957]

“If A and B have almost identical environments, we say that they are synonyms”

[Harris, 1954]



# ...by the company it keeps...

When a word **w** appears in a text, its context is the set of words that appear nearby (within a fixed-size window).

Use the many contexts of **w** to build up representation for **w**.

*...government debt problems turning into **banking** crises as happened in 2009...*  
*...saying that Europe needs unified **banking** regulation to replace the hodgepodge...*  
*...India has just given its **banking** system a shot in the arm...*

These context words will  
represent banking

# Works for OOV too

Remember the *gompies*?

*All gompies are biff and luff voomly.*

*M'moon is a cramy gompy, she is the biffiest and luffs voomly*

# Co-occurrence Vectors

# Back to the term–document matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
<b>Antony</b>	157	73	0	0	0	0
<b>Brutus</b>	4	157	0	1	0	0
<b>Caesar</b>	232	227	0	2	1	1
<b>Calpurnia</b>	0	10	0	0	0	0
<b>Cleopatra</b>	57	0	0	0	0	0
<b>mercy</b>	2	0	3	5	5	1
<b>worser</b>	2	0	1	1	1	0

# Term–Document Matrix

	<i>As You Like It</i>	<i>Twelfth Night</i>	<i>Julius Caesar</i>	<i>Henry V</i>
<i>battle</i>	1	0	7	17
<i>soldier</i>	2	80	62	89
<i>fool</i>	36	58	1	4
<i>clown</i>	20	15	2	3

How about a **term–context** matrix?

# Term–Context Matrix (Word–Word Matrix)

	<i>knife</i>	<i>dog</i>	<i>sword</i>	<i>love</i>	<i>like</i>
<i>knife</i>	0	1	6	5	5
<i>dog</i>	1	0	5	5	5
<i>sword</i>	6	5	0	5	5
<i>love</i>	5	5	5	0	5
<i>like</i>	5	5	5	5	2

# Word Embeddings

# Sparse versus Dense Vectors

PPMI vectors are

- **Long** (length  $|V| = 20,000$  to  $50,000$ )
- **Sparse** (most elements are zero)

Alternative: learn vectors which are

- **Short** (length 200-1000)
- **Dense** (most elements are non-zero)



# Why Dense Vectors?

**Short** vectors may be easier to use as features (less weights to tune; avoid overfitting)

**Dense** vectors may generalize better than storing explicit counts

They may do better at capturing synonymy

*car* and *automobile* are synonyms but are represented as distinct dimensions;  
This fails to capture similarity between a word with *car* as a neighbor and a word with *automobile* as a neighbor.

In practice, they work better

# Intuition of Word Embeddings

- Hypothesis: Words that are semantically similar often have same surrounding words. (Distributional Hypothesis)
- Goal: We want words that are semantically similar to have close word vectors.
- Intermediate Goal: We need to make the words that have the same surrounding words to have close word vectors.

$$\begin{aligned}
 \text{tuesday} &= \begin{bmatrix} 0.75 \\ -0.18 \\ 0.34 \\ \vdots \\ 0.25 \end{bmatrix} \\
 \text{monday} &= \begin{bmatrix} \vdots \end{bmatrix} \\
 \text{table} &= \begin{bmatrix} \vdots \end{bmatrix}
 \end{aligned}$$

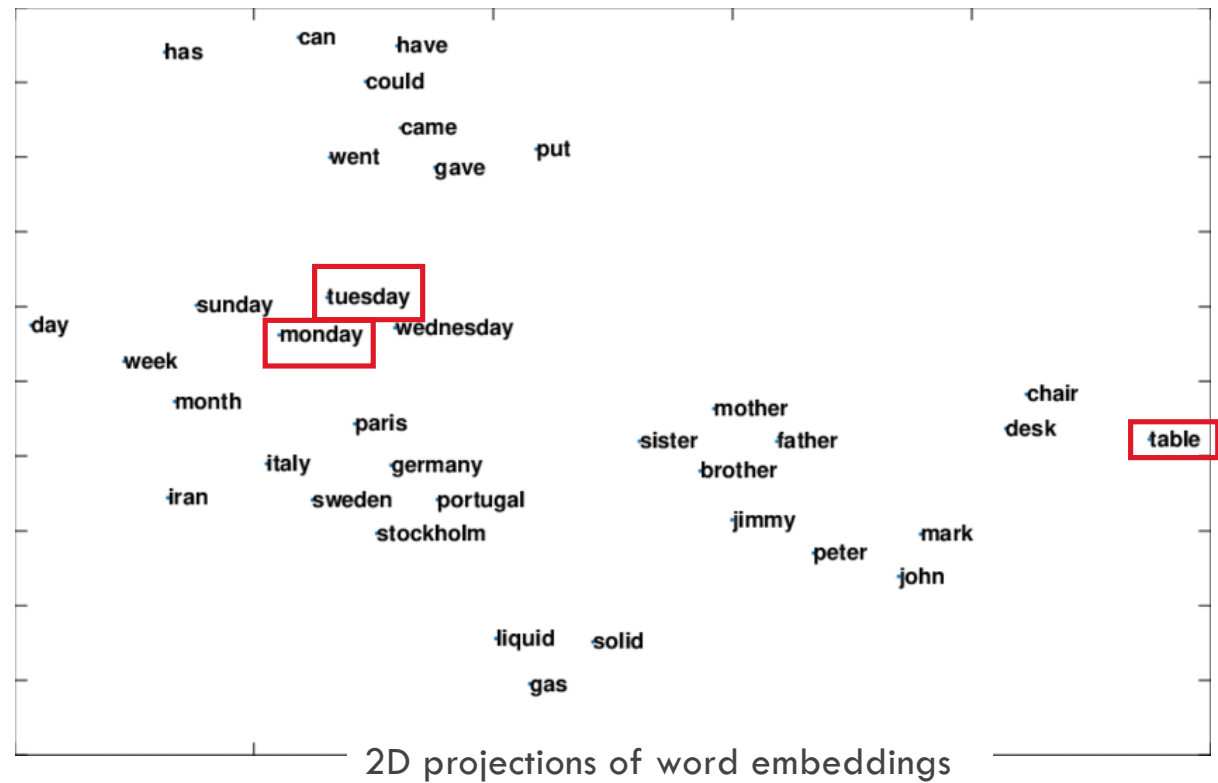
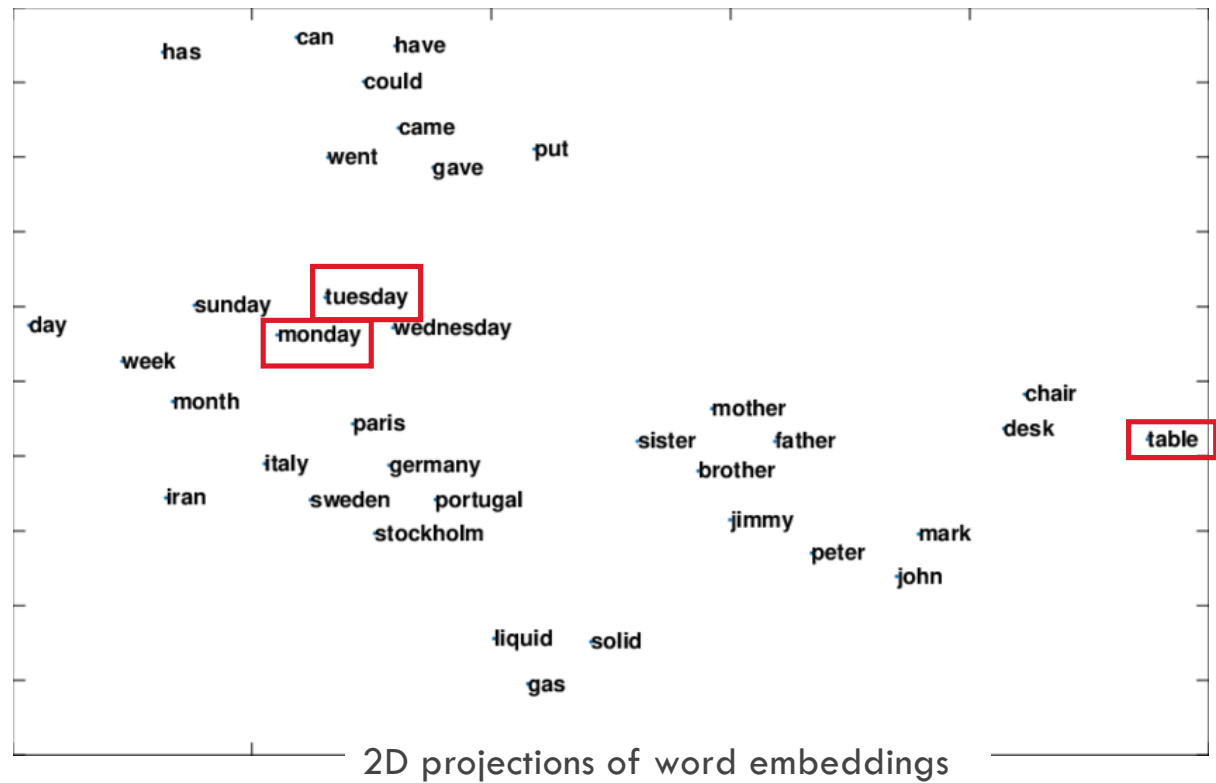


Diagram Credits:  
[https://www.researchgate.net/publication/327074728\\_Principal\\_Word\\_Vectors](https://www.researchgate.net/publication/327074728_Principal_Word_Vectors)

$$\begin{aligned}
 \text{tuesday} &= \begin{bmatrix} 0.75 \\ -0.18 \\ 0.34 \\ \vdots \\ 0.25 \end{bmatrix} \\
 \text{monday} &= \begin{bmatrix} 0.78 \\ -0.2 \\ 0.34 \\ \vdots \\ 0.21 \end{bmatrix} \\
 \text{table} &= \begin{bmatrix} -0.3 \\ 0.12 \\ 0.51 \\ \vdots \\ 0.02 \end{bmatrix}
 \end{aligned}$$



2D projections of word embeddings

Diagram Credits:  
[https://www.researchgate.net/publication/327074728\\_Principal\\_Word\\_Vectors](https://www.researchgate.net/publication/327074728_Principal_Word_Vectors)

# Making Word Embeddings

- Use neural network to train a self-supervised task.
- Use the weight matrix as the word vector representation.
- Common techniques for embedding words: CBOW, Skip-Gram, and GLoVe.
- Do not care about the actual model, we only **take the embedding** weight.

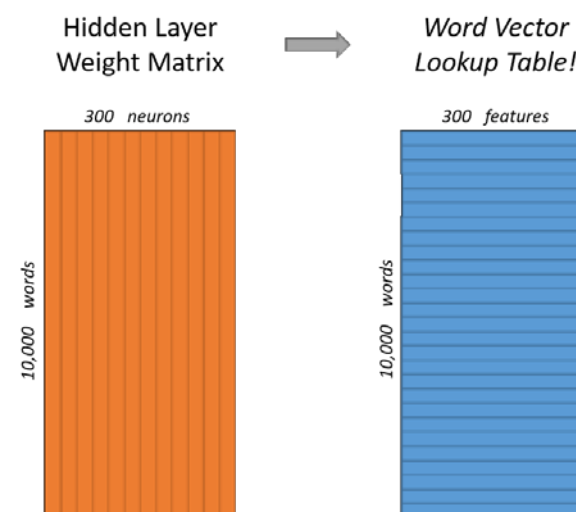


Diagram Credits: <http://mccormickml.com/2016/04/19/word2vec-tutorial-the-skip-gram-model/>



# Word2Vec: CBOW & Skip-Gram

# Intuition

Words that are semantically similar often have same surrounding words. (Distributional Hypothesis)

## Continuous Bag of Words (CBOW):

- Train a model to predict a word from the surrounding words.

## Skip-Gram:

- Train a model to predict the surrounding words from a word.



A man with a mustache and wide eyes is looking down at a game board. The game board has a blue background with white text and yellow borders. The text on the board is as follows:

tablespoon of \_\_\_\_\_ jam

A: traffic B: catchy

C: apricot D: paper

A: traffic

B: catchy

C: apricot

D: paper

# Predicting a word from context

glass                      at                      apricot

cars                      juice                      bottle

honey                      strawberry

table                      bowl

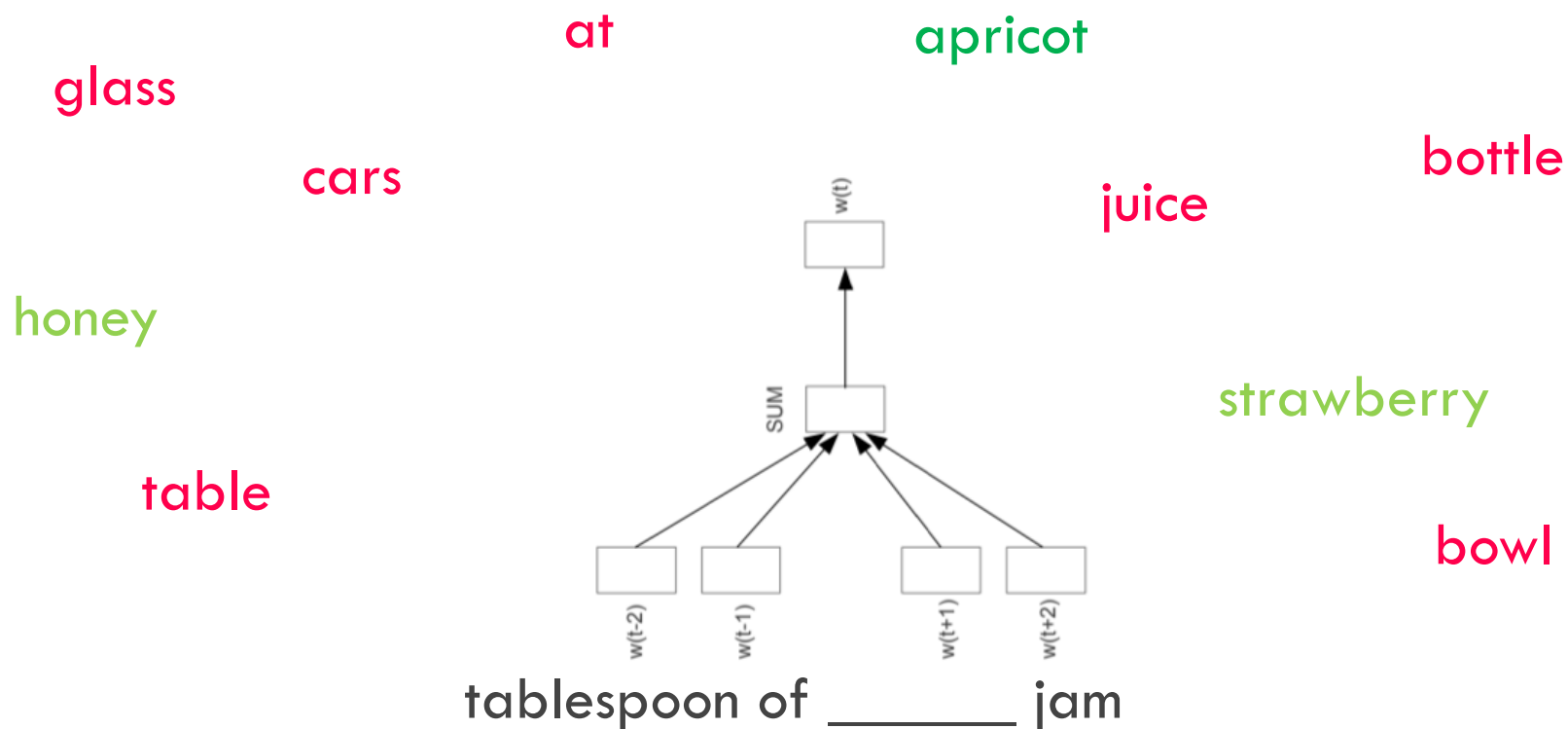
tablespoon of \_\_\_\_\_ jam

# Predicting a word from context

glass at apricot  
cars juice bottle  
honey strawberry  
table bowl

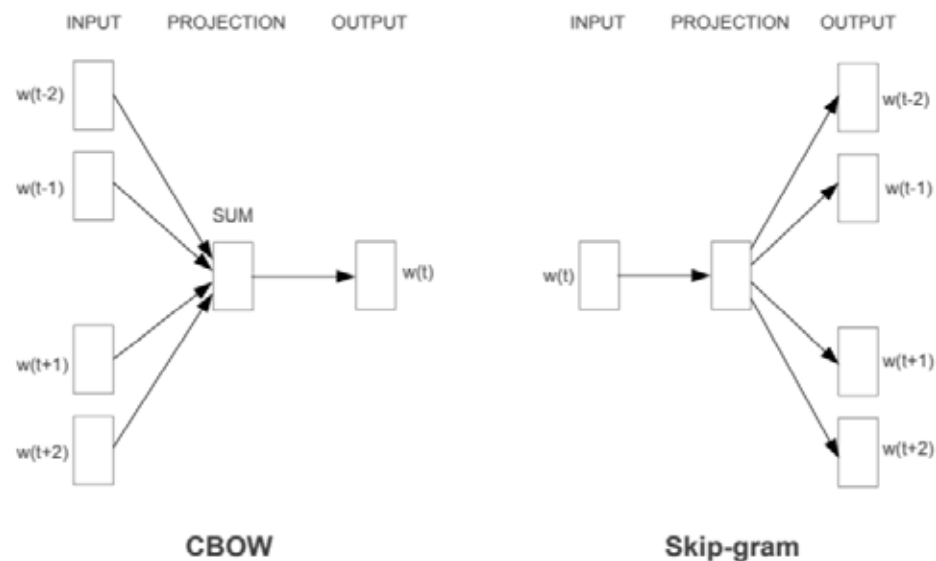
tablespoon of \_\_\_\_\_ jam

# Predicting a word from context



# CBOW and Skip-Gram

- **CBOW** → given a context, predict the word.
- **Skip-gram** → given a word, predict the context.



# Predicting surrounding words

glass at of  
cars wall  
juice  
tablespoon  
table jam  
apron  
\_\_\_\_\_ apricot \_\_\_\_\_

Photo by [Philippe Gonyea](#) on [Unsplash](#)

# Predicting surrounding words


glass                      at                      of  
                          cars                      juice                      wall  
 tablespoon                      jam  
                          table                      apron  

                          \_\_\_\_\_ apricot \_\_\_\_\_

Photo by [Philippe Gonyea](#) on [Unsplash](#)

# Predicting surrounding words


glass                      at                      of  
          cars                      juice                      wall  
 tablespoon                      jam  
          table                      apron  

          apricot

Photo by [Philippe Gonyea](#) on [Unsplash](#)



# Predicting surrounding words

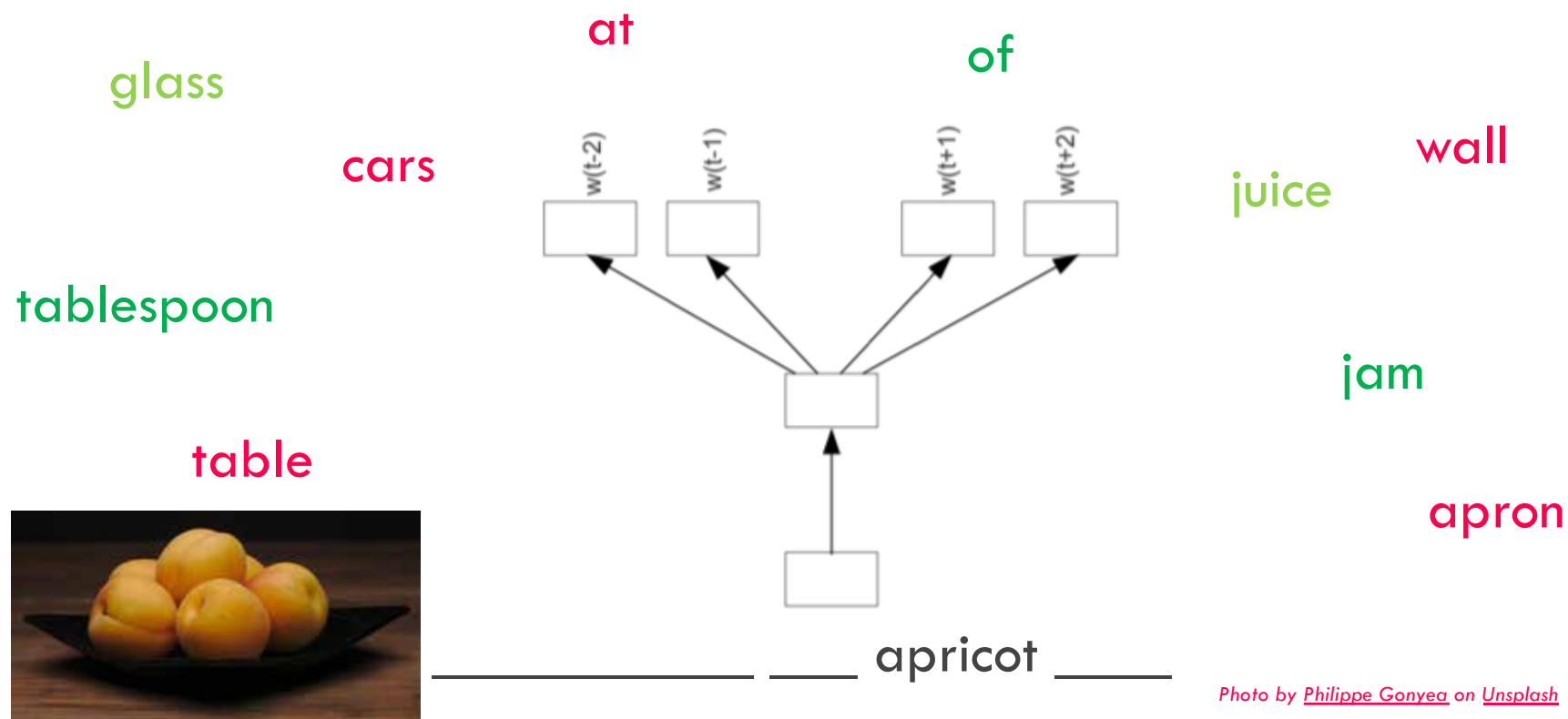


Photo by [Philippe Gonyea](#) on [Unsplash](#)

# Training Objective

Optimize the weight so that word and its context have close vector representation

$$\mathcal{L}(\theta) = -\frac{1}{T} \sum_{t=1}^T \sum_{\substack{-m \leq j \leq m \\ j \neq 0}} \log P(w_{t+j} | w_t)$$

# Training Objective

Optimize the weight so that word and its context have close vector representation

$$\mathcal{L}(\theta) = -\frac{1}{T} \sum_{t=1}^T \sum_{\substack{-m \leq j \leq m \\ j \neq 0}} \log P(w_{t+j} | w_t)$$

$$P(w_{t+j} | w_t) = \frac{\exp(u_{w_{t+j}} \cdot v_{w_t})}{\sum_{w'} \exp(u_{w'} \cdot v_{w_t})}$$

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Optimize the weight so that word and its context have close vector representation

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more similar = higher dot product  
= larger probability

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$$P(w_{t+j}|w_t) = \frac{\exp(u_{w_{t+j}} \cdot v_{w_t})}{\sum_{w'} \exp(u_{w'} \cdot v_{w_t})}$$

more similar = higher dot product  
= larger probability

Normalize over entire vocabulary to  
give probability distribution

# Skip-Gram with Negative Sampling (SGNS)

For each positive sample, create  $k$  negative samples from random word,  $c_{neg}$

*example with  $k = 2$*

## positive examples +

$w$	$c_{pos}$
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

## negative examples -

$w$	$c_{neg}$	$w$	$c_{neg}$
apricot	aardvark	apricot	seven
apricot	my	apricot	forever
apricot	where	apricot	dear
apricot	coaxial	apricot	if

*Slide Credits: Dan Jurafsky (Stanford)*

# SGNS Objective

$$P(w_{t+j}|w_t) = \frac{\exp(u_{w_{t+j}} \cdot v_{w_t})}{\sum_{w'} \exp(u_{w'} \cdot v_{w_t})}$$

$$\mathcal{L}(\theta) = -\frac{1}{T} \sum_{t=1}^T \sum_{\substack{-m \leq j \leq m \\ j \neq 0}} \log p(w_{t+j}|w_t)$$

# SGNS Objective

$$\cancel{P(w_{t+j}|w_t) = \frac{\exp(u_{w_{t+j}} \cdot v_{w_t})}{\sum_{w'} \exp(u_{w'} \cdot v_{w_t})}} \quad \longrightarrow \quad P(+|c_{pos}, w)$$

$$\cancel{\mathcal{L}(\theta) = -\frac{1}{T} \sum_{t=1}^T \sum_{\substack{-m \leq j \leq m \\ j \neq 0}} \log P(w_{t+j}|w_t)} \quad \longrightarrow \quad \text{Contrastive Loss}$$



# SGNS Objective

$$P(+|c_{pos}, w) = \frac{1}{1 + \exp(-c_{pos} \circ w)}$$

$$c_{pos} = \{w_{t-j}, \dots, w_{t-1}, w_{t+1}, \dots, w_{t+j}\}$$

context or surrounding words

# SGNS Objective

$$P(+|c_{pos}, w) = \sigma(c_{pos} \circ w) = \frac{1}{1 + \exp(-c_{pos} \circ w)}$$

$$c_{pos} = \{w_{t-j}, \dots, w_{t-1}, \\ w_{t+1}, \dots, w_{t+j}\}$$

context / surrounding  
words

$$\mathcal{L}(\theta) = -\log \left[ P(+|c_{pos}, w) \prod_{i=1}^k P(-|c_{neg}, w) \right]$$

# SGNS Objective

$$P(+|c_{pos}, w) = \sigma(c_{pos} \circ w) = \frac{1}{1 + \exp(-c_{pos} \circ w)}$$

$c_{pos} = \{w_{t-j}, \dots, w_{t-1}, w_{t+1}, \dots, w_{t+j}\}$

context / surrounding  
words

$$\mathcal{L}(\theta) = -\log \left[ P(+|c_{pos}, w) \prod_{i=1}^k P(-|c_{neg}, w) \right]$$

Remember,  $P(-|c_{neg}, w) = 1 - P(+|c_{neg}, w)$

$$= - \left[ \log P(+|c_{pos}, w) + \sum_{i=1}^k \log (1 - P(+|c_{neg}, w)) \right]$$

# SGNS Objective

$$P(+|c_{pos}, w) = \sigma(c_{pos} \circ w) = \frac{1}{1 + \exp(-c_{pos} \circ w)}$$

$c_{pos} = \{w_{t-j}, \dots, w_{t-1}, w_{t+1}, \dots, w_{t+j}\}$

context / surrounding words

$$\begin{aligned} \mathcal{L}(\theta) &= -\log \left[ P(+|c_{pos}, w) \prod_{i=1}^k P(-|c_{neg}, w) \right] \quad \text{Remember, } P(-|c_{neg}, w) = 1 - P(+|c_{neg}, w) \\ &= - \left[ \log P(+|c_{pos}, w) + \sum_{i=1}^k \log (1 - P(+|c_{neg}, w)) \right] \\ &= - \left[ \log \sigma(c_{pos} \circ w) + \sum_{i=1}^k \log \sigma(-c_{neg} \circ w) \right] \end{aligned}$$

# Negative Sampling

Negative samples chosen according to their ( $\alpha$ -)weighted  
**unigram frequency**

- Common practice  $\alpha < 0.75$
- $\alpha < 1$  gives more chance to rare words

$$P_{\alpha}(w) = \frac{\text{count}(w)^{\alpha}}{\sum_{w'} \text{count}(w')^{\alpha}}$$

# Negative Sampling

Word	Unigram frequency
the	0.99
durian	0.01



Weighted $\alpha = 0.75$
0.97
0.03

$$P_{\alpha}(w) = \frac{\text{count}(w)^{\alpha}}{\sum_{w'} \text{count}(w')^{\alpha}}$$

$$P_{\alpha}(a) = \frac{.99^{.75}}{.99^{.75} + .01^{.75}} = .97$$

$$P_{\alpha}(b) = \frac{.01^{.75}}{.99^{.75} + .01^{.75}} = .03$$

Slide Credits: Dan Jurafsky (Stanford)

# Model training

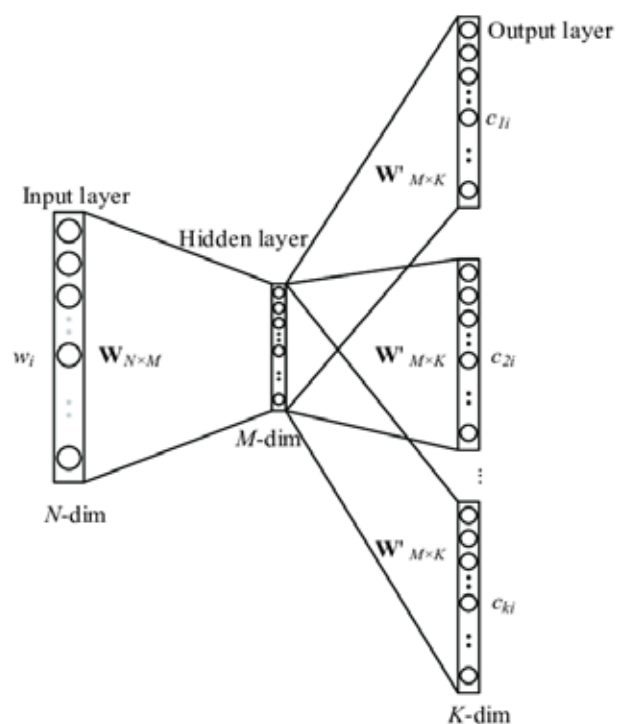
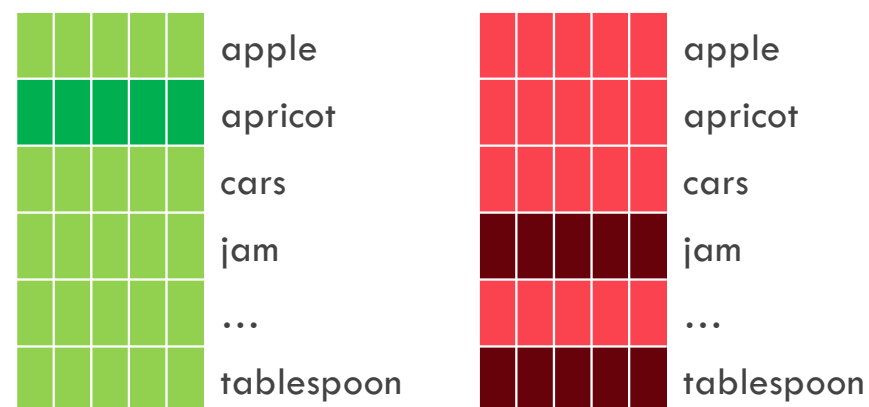


Diagram Credits: Patent Keyword Extraction Algorithm Based on Distributed Representation for Patent Classification (2018)



word	context	target
 apricot	 tablespoon	1
 apricot	 cars	0
 apricot	 jam	1

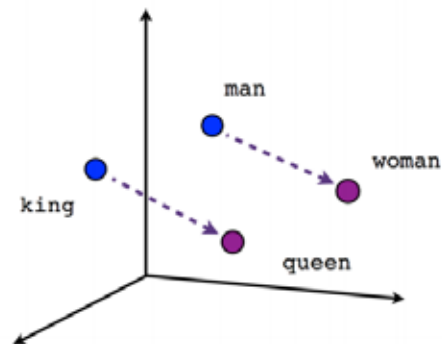
# Properties of Embeddings



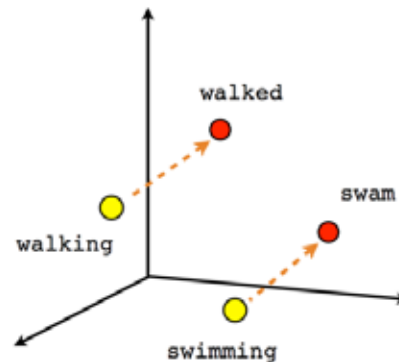
# Vector differences yield semantic relationships!

$$\hat{v}(\text{king}) - \hat{v}(\text{man}) + \hat{v}(\text{woman}) \approx \hat{v}(\text{queen})$$

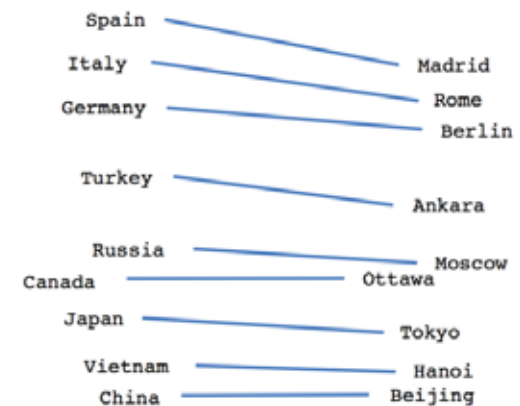
$$\hat{v}(\text{Paris}) - \hat{v}(\text{France}) + \hat{v}(\text{Italy}) \approx \hat{v}(\text{Rome})$$



Male-Female



Verb tense



Country-Capital

# Modeling semantic similarity in GloVe

Global Vectors (GloVe): represent the probabilities as *ratios* of their co-occurrences.

Probability and Ratio	$k = \text{solid}$	$k = \text{gas}$	$k = \text{water}$	$k = \text{fashion}$
$P(k \text{ice})$	$1.9 \times 10^{-4}$	$6.6 \times 10^{-5}$	$3.0 \times 10^{-3}$	$1.7 \times 10^{-5}$
$P(k \text{steam})$	$2.2 \times 10^{-5}$	$7.8 \times 10^{-4}$	$2.2 \times 10^{-3}$	$1.8 \times 10^{-5}$
$P(k \text{ice})/P(k \text{steam})$	8.9	$8.5 \times 10^{-2}$	1.36	0.96

$$F(w_i, w_j, w_k) = \frac{P_{ij}}{P_{jk}}$$

# Nearest Neighbors

- 0. *frog*
- 1. frogs
- 2. toad
- 3. *litoria*
- 4. leptodactylidae
- 5. *rana*
- 6. lizard
- 7. *eleutherodactylus*



3. *litoria*



4. leptodactylidae

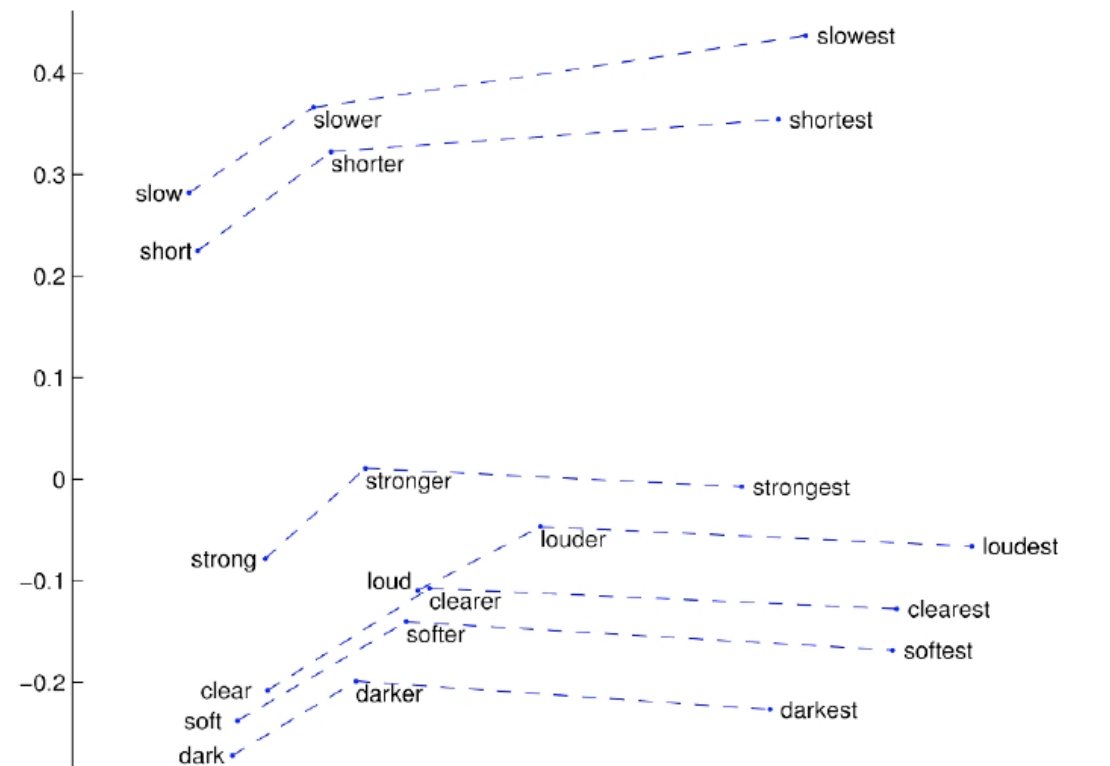


5. *rana*



7. *eleutherodactylus*

# Linear substructures



## Also works for crosslingual cases

Diagram Credits: Sebastian Ruder, Ivan Vulic and Anders Søgaard (JAIR 2019)

# Embeddings Summary

Move from term–document matrix to a term–context matrix

- Solves semantically relatedness

Embed the resultant term–context vectors into a denser space

- The side effect is the objective!
- Solves sparsity problem

Vectorial differences yields semantics relationships

Many extensions, we'll see some later