CS2040S: Data Structures and Algorithms

Tutorial Problems for Week 13

For: Nov 15, 2019, Tutorial Groups 3, 4

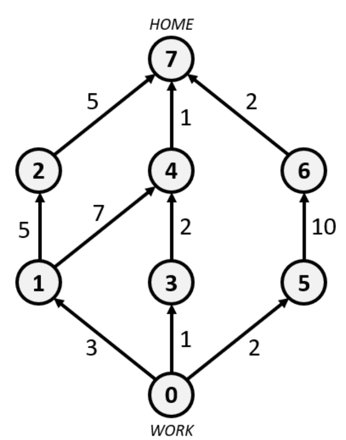
Problem 1. A Random Problem with a dude called Dan

Dan is on his way home from work. The city he lives in is made up of N locations, labelled from 0 to $(N-1)$. His workplace is at location 0 and his home is at location $(N-1)$. These locations are connected by M **directed** roads, each with an associated cost. To go through a road, Dan will need to pay the cost associated with that road. Usually, Dan would try to take the cheapest path home.

The thing is, Dan has just received his salary! For reasons unknown, he wants to flaunt his wealth by going through a *really expensive road.* However, he still needs to be able to make it back home with the money he has. Given that Dan can afford to spend up to D dollars on transportation, help him find **the cost of the most expensive road that he can afford to go through** on his journey back home.

Take note than Dan only cares about the most expensive road in his journey; the rest of the journey can be really cheap, or just as expensive, so long as the entire journey fits within his budget of D dollars. He is also completely focused on this goal and does not mind visiting the same location multiple times, or going through the same road multiple times.

For example, suppose Dan's budget is $D = 13$ dollars. Consider the following city, consisting of $N = 8$ locations and $M = 10$ roads.



The path that Dan will take is $0 \to 1 \to 4 \to 7$. In this journey, the total cost is 11 dollars and the most expensive road has a cost of 7 dollars - the road from locations 1 to 4. Therefore, the expected output for this example would be "7".

Note that this path is neither the cheapest path ($0 \to 3 \to 4 \to 7$), nor is it the most expensive path that fits within his budget of 13 dollars ($0 \to 1 \to 2 \to 7$).

There is also a more expensive road within this city - the road from locations 5 to 6 with a cost of 10 dollars. However, the only path that goes through this road, $0 \to 5 \to 6 \to 7$, has a total cost of 14 dollars which exceeds Dan's budget.

Problem 2. Espionage

You are a spy, currently on a mission to obtain intel on the final examination that will be unleashed upon this world by the dreaded *Dlorah Hos* within the next 19 days. At the moment, you have a message to send to HQ in the form of a **weighted tree** of N vertices, where all of the edge weights are positive integers.

To ensure your message is not at risk of being intercepted, you need to encrypt your message. The way you decide to do this is by inserting additional edges to your tree. The resulting graph, which represents the encrypted message, must satisfy the following properties:

- The resulting graph is a **complete graph** of N vertices (i.e. every pair of vertices in the graph must have exactly 1 edge between them).
- The initial tree is **the unique Minimum Spanning Tree** of the resulting graph.

Inserting edges with large weights will make the encryption process more complicated, so you would like to avoid them wherever possible. The cost of the encryption process is defined as the sum of the weights of the additional edges that are inserted into the graph during the encryption process.

Given the initial message, which consists of the number of vertices in the tree, N, and a list of $N - 1$ edges of the tree, **output the minimum possible cost** of encrypting this message.

For example, let the initial message be the tree on the left, consisting of $N = 4$ vertices. The graph on the right represents the optimal way to encrypt the message with a total cost of $17 + 17 + 13 = 47$. Therefore, the expected output is "47".