2016/2017 SEMESTER 2 MID-TERM TEST

MA1521 Calculus for Computing

February/March, 2017

12:30pm to 1:30pm

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

1. This test paper consists of **TEN (10)** multiple choice questions and comprises **THREE (3)** pieces of paper printed on **both sides**.
2. Answer all 10 questions. 1 mark for each correct answer. No penalty for wrong answers. Full mark is 10.
3. All answers (Choices A, B, C, D, E) are to be submitted using the pink form (FORM CC1/10).
4. Use **only 2B pencils** for FORM CC1/10.
5. On FORM CC1/10 (section B), **write** your **matriculation number** and **shade** the corresponding numbered circles **completely**. Your FORM CC1/10 will be graded by a computer and it will record a **ZERO** for your score if your matriculation number is not correct.
6. Write your full name in section A (under Module Code) of FORM CC1/10.
7. Only circles for answers 1 to 10 are to be shaded.
8. For each answer, the circle corresponding to your choice should be **properly** and **completely** shaded. If you change your answer later, you must make sure that the original answer is properly erased.
9. For each answer, **do not shade more than one circle**. The answer for a question with more than one circle shaded will be marked wrong.
10. **Do not fold** FORM CC1/10.
11. Submit FORM CC1/10 before you leave the test hall.

1. Let $y = x^3$. Then $\frac{dy}{dx} =$

   (A) $3x^2$

   (B) x^2

   (**C**) $3x^3$

   (**D**) x

   (**E**) None of the above

2. A conchoid of de Sluze has polar equation $r = \sec\theta + \cos\theta$. Find the slope of its tangent line at the point when $\theta = -\frac{\pi}{3}$. Give your answer correct to two decimal places.

   (A) 4.04

   (B) 3.82

   (C) 3.56

   (D) 4.17

   (E) None of the above

3. Find the slope of the tangent line at the point $(1, 2)$ on the curve $x^4 + y^4 = \frac{17}{2}xy$. Give your answer correct to two decimal places.

(A) 0.54

(B) 0.56

(C) 0.53

(D) 0.55

(E) None of the above

4. Let $y = x^{\cos x}$. Find, correct to two decimal places, the value of $\frac{dy}{dx}$ when $x = \frac{\pi}{6}$.

(A) 1.11

(B) 1.13

(C) 1.15

(D) 1.17

(E) None of the above

5. The function $y = \ln\{-(x+1521)(x+2017)\}$ has a critical point at $x = c$ where $-2017 < c < -1521$. Find the value of c.

(A) -1969

(B) -1767

(C) -1769

(D) -1967

(E) None of the above

6. Let a, b and c denote three positive constants. If

$$\int_0^c (2ax + b)\left(ax^2 + bx\right)^4 dx = 9c^5,$$

find the value of $ac+b$. Give your answer correct to two decimal places.

(A) 1.86

(B) 2.14

(C) 2.32

(D) 1.91

(E) None of the above

7. Find the **exact value** of $\frac{\int_0^{\frac{\pi}{2}} (\sin^{225} x)(\cos^3 x)dx}{\int_0^{\frac{\pi}{2}} (\sin^3 x)(\cos^{223} x)dx}$.

(A) $\frac{54}{55}$

(B) $\frac{55}{56}$

(C) $\frac{56}{57}$

(D) $\frac{57}{58}$

(E) None of the above

8. Find the area of the finite region bounded between the curve $y^2 = x$ and the line $2y = x - 15$.

(A) $\frac{256}{3}$

(B) $\frac{255}{2}$

(C) $\frac{257}{3}$

(D) $\frac{257}{2}$

(E) None of the above

9. A finite region R in the first quadrant is bounded by the curve $y = \sqrt{x^2 + x + 1}$, the x-axis, the y-axis and the line $x = 2$. Find the volume of the solid formed by revolving R one complete round about the x-axis. Give your answer correct to two decimal places.

(A) 19.86

(B) 20.94

(C) 21.57

(D) 22.35

(E) None of the above

10. Find the value of the integral $\int_{-\frac{\pi}{6}}^{\frac{\pi}{4}} |\tan x|\, dx$. Give your answer correct to two decimal places

(A) 0.50

(B) 0.19

(C) 0.20

(D) 0.49

(E) None of the above

END OF PAPER

Answers to mid term test

1. A
2. A
3. D
4. B
5. C
6. B
7. C
8. A
9. B
10. D

1). A

2). A

$$r = \sec\theta + \cos\theta$$

$$\frac{dr}{d\theta} = \sec\theta\tan\theta - \sin\theta$$

$x = r\cos\theta$, $y = r\sin\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$\theta = -\frac{\pi}{3} \Rightarrow \frac{dr}{d\theta} = -2\sqrt{3} + \frac{\sqrt{3}}{2} = -\frac{3\sqrt{3}}{2}$$

$$\frac{dy}{dx} = \frac{\frac{-3\sqrt{3}}{2}\left(-\frac{\sqrt{3}}{2}\right) + \frac{5}{2}\left(\frac{1}{2}\right)}{\frac{-3\sqrt{3}}{2}\left(\frac{1}{2}\right) - \frac{5}{2}\left(-\frac{\sqrt{3}}{2}\right)} = 4.041\ldots$$

$$\approx \underline{\underline{4.04}}$$

3). D

$$4x^3 + 4y^3y' = \frac{17}{2}y + \frac{17}{2}xy'$$

$x=1$, $y=2 \Rightarrow$ $4 + 32y' = 17 + \frac{17}{2}y'$

$$y' = \frac{26}{47} = 0.553\ldots$$

$$\approx \underline{\underline{0.55}}$$

4). B

$$\ln y = \cos x \ln x \Rightarrow \frac{1}{y}y' = -\sin x\ln x + \frac{1}{x}\cos x$$

$$x = \frac{\pi}{6} \Rightarrow y' = \left\{\left(\frac{\pi}{6}\right)^{\cos\frac{\pi}{6}}\right\}\left\{-\sin\frac{\pi}{6}\ln\frac{\pi}{6} + \frac{6}{\pi}\cos\frac{\pi}{6}\right\}$$

$$= 1.129\ldots \approx \underline{\underline{1.13}}$$

5). C

$$y' = \frac{1}{-(x+1521)(x+2017)} \left\{ -(x+2017) - (x+1521) \right\}$$

$$= \frac{2(x+1769)}{(x+1521)(x+2017)}$$

$$y' = 0 \Rightarrow x = \underline{\underline{-1769}}$$

6). B

$$9c^5 = \int_0^c (ax^2+bx)^4 \, d(ax^2+bx)$$

$$= \frac{1}{5}(ax^2+bx)^5 \Big|_0^c = \frac{1}{5}(ac^2+bc)^5$$

$$ac+b = (45)^{1/5} = 2.141\ldots$$

$$\approx \underline{\underline{2.14}}$$

7). C

$$\int_0^{\pi/2} \sin^{225}x \cos^3 x \, dx = \int_0^{\pi/2} (\sin^{225}x)(1-\sin^2 x) \, d(\sin x)$$

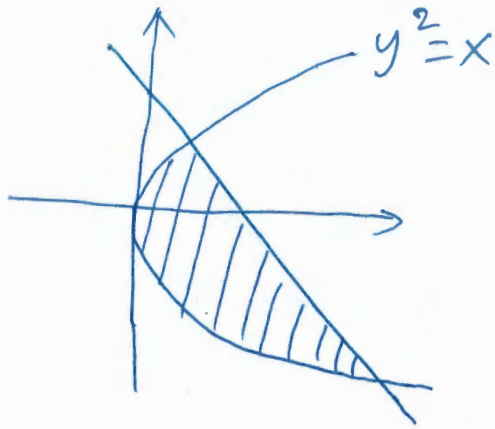
$$= \left[\frac{1}{226}\sin^{226}x - \frac{1}{228}\sin^{228}x \right]_0^{\pi/2} = \frac{1}{226} - \frac{1}{228}$$

$$\int_0^{\pi/2} (\sin^3 x)(\cos^{223}x) \, dx = -\int_0^{\pi/2} (1-\cos^2 x)(\cos^{223}x) \, d(\cos x)$$

$$= \left[-\frac{1}{224}\cos^{224}x + \frac{1}{226}\cos^{226}x \right]_0^{\pi/2} = \frac{1}{224} - \frac{1}{226}$$

$$\frac{\frac{1}{226} - \frac{1}{228}}{\frac{1}{224} - \frac{1}{226}} = \underline{\underline{\frac{56}{57}}}$$

8). A



$y^2=x$ and $2y=x-15$

$\Rightarrow y^2=2y+15$

$\Rightarrow y^2-2y-15=0$

$\Rightarrow (y-5)(y+3)=0$

$\Rightarrow y=-3,\ 5$

$$\text{Area} = \int_{-3}^{5} [(2y+15)-y^2]\,dy = \frac{256}{3}$$

9). B

$$\text{Vol} = \int_0^2 \pi(x^2+x+1)\,dx = 20.943\ldots$$

$$\approx 20.94$$

10). D

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{4}} |\tan x|\,dx$$

$$= \int_{-\frac{\pi}{6}}^{0} -\tan x\,dx + \int_0^{\frac{\pi}{4}} \tan x\,dx$$

$$= 0.490\ldots$$

$$\approx 0.49$$