CS1231 Review 19

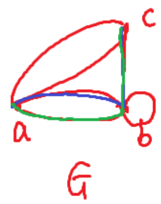
1. Let G be an undirected graph.

   Two distinct vertices a, b in G are **adjacent** or **neighbours** if ab is an edge.

   An edge e and a vertex a in G are **incident** if $e = ax$ for some x

2. Draw graph G if its adjacency matrix is as follows.

$$\begin{array}{cc} & \begin{array}{ccc} a & b & c \end{array} \\ A = \begin{array}{c} a \\ b \\ c \end{array} & \begin{pmatrix} 0 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix} \end{array}$$



   Then we compute

$$\begin{array}{cc} & \begin{array}{ccc} a & b & c \end{array} \\ A^2 = \begin{array}{c} a \\ b \\ c \end{array} & \begin{pmatrix} 8 & 4 & 2 \\ 4 & 6 & 5 \\ 2 & 5 & 5 \end{pmatrix} \end{array}$$

   ? # of length 2 paths from a to c

   How many paths are there in G connecting a and c of length 2?

3. The **degree** of a vertex u in G, denoted by $\deg(u)$, is the number of edges incident with u, with each loop counted as 2.

4. A vertex is **isolated** if its degree is 0.

5. **Handshaking Theorem.** Let $G = (V, E)$ be a graph. Then $\sum_{v \in G} \deg(v) = 2|E|$.

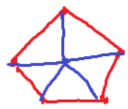
6. The **complete graph** on n vertices, denoted by K_n, is the simple graph s.t 2 distinct vertices are adjacent. How many edges are there? $\binom{n}{2}$.

   K_4 

   C_3 

7. The **cycle**, denoted by C_n, $n \geq 1$, consists of n vertices $v_1 v_2 \cdots v_n$ and $v_1v_2, v_2v_3, \ldots v_{n-1}v_n$ and v_nv_1 edges.

8. The **wheel**, denoted by W_n, $n \geq 3$, is the simple graph obtained from C_n by adding a new vertex and connect it to all others. e.g W_5



   C_5

Q_3 $\boxed{010}$ – $\boxed{011}$ 110

9. The n-**dimensional hypercube** or n-**cube**, denoted by Q_n, is the simple graph whose vertices represent bit string of length n. Two vertices are adjacent iff the bit string they represent diff at exactly one bit

10. A simple graph $G = (V, E)$ is called **bipartite** if V can be divided into 2 groups such that All edges are connecting V_1 and V_2. When this condition holds, we call (V_1, V_2) a biparfition of V.

11. A graph is bipartite iff it contains no odd cycle.

V_1 has m vertices

V_2 has n vertices

12. Let $m, n \in \mathbb{Z}^+$. A $\boxed{\textbf{complete}}$ **bipartite graph** on (m, n) vertices, denoted $K_{m,n}$, is a simple graph with vertices vertices $v_1, v_2, \ldots, v_m, w_1, w_2, \ldots, w_n$ and edges $v_i w_j$, $i = 1, \cdots, m$, $j = 1, \cdots, n$. How many edges are there? mn.

13. A graph $H = (W, F)$ is a **subgraph** of a graph $G = (V, E)$ if ________.

$W \subseteq V$ and $F \subseteq E$