CS2102
Database Systems

Question 1(a): Does the decomposition preserve all FDs?

- Schema R(A, B, C, D), with $A \rightarrow BCD$, $C \rightarrow D$
- Decomposition: R1(A, B, C), R2(C, D)
- Closures on R2:
  - $\{C\}^+ = \{CD\}$, $\{D\}^+ = \{D\}$
- So we have $C \rightarrow D$ on R2
- Closures on R1:
  - $\{A\}^+ = \{ABCD\}$, $\{B\}^+ = \{B\}$, $\{C\}^+ = \{C\}$
- So we have $A \rightarrow BC$ on R1
- Given $C \rightarrow D$ and $A \rightarrow BC$, we have
  - $\{A\}^+ = \{ABCD\}$, $\{C\}^+ = \{CD\}$
- So all FDs on R (i.e., $A \rightarrow BCD$ and $C \rightarrow D$) are preserved

Question 1(b): Does the decomposition preserve all FDs?

- Schema R(A, B, C, D), with $A \rightarrow BCD$, $C \rightarrow D$
- Decomposition: R1(A, C), R2(A, B, D)
- Closures on R1:
  - $\{A\}^+ = \{ABCD\}$, $\{C\}^+ = \{CD\}$
- So we have $A \rightarrow C$ on R1
- Closures on R2:
  - $\{A\}^+ = \{ABCD\}$, $\{B\}^+ = \{B\}$, $\{D\}^+ = \{D\}$
- So we have $A \rightarrow BD$ on R2
- Given $A \rightarrow C$ and $A \rightarrow BD$, we have
  - $\{A\}^+ = \{ABCD\}$, $\{C\}^+ = \{C\}$
- So $C \rightarrow D$ is not preserved by the decomposition

Question 1(c): Does the decomposition preserve all FDs?

- Schema R(A, B, C, D, E), with AB→C, AC→D, E→ABCD
- Decomposition: R1(A, B, C), R2(A, B, E), R2(A, C, D)
- Closures on R1:
  - $\{A\}^+ = \{A\}$, $\{B\}^+ = \{B\}$, $\{C\}^+ = \{C\}$
  - $\{AB\}^+ = \{ABCD\}$, $\{AC\}^+ = \{ACD\}$, $\{BC\}^+ = \{BC\}$
- So we have AB→C on R1
- Closures on R2:
  - $\{A\}^+ = \{A\}$, $\{B\}^+ = \{B\}$, $\{E\}^+ = \{EABCD\}$
  - $\{AB\}^+ = \{ABCD\}$
- So we have E→AB on R2
- Closures on R3:
  - $\{A\}^+ = \{A\}$, $\{C\}^+ = \{C\}$, $\{D\}^+ = \{D\}$
  - $\{AC\}^+ = \{ACD\}$, $\{AD\}^+ = \{AD\}$, $\{CD\}^+ = \{CD\}$
- So we have AC→D on R3
- Given AB→C, E→AB, and AC→D, we have
  - $\{AB\}^+ = \{ABCD\}$, $\{AC\}^+ = \{ACD\}$, $\{E\}^+ = \{EABCD\}$
- So all FDs on R (i.e., AB→C, AC→D, E→ABCD) are preserved

Question 2(a): Is R in BCNF?

- Schema R(A, B, C, D), with ABC→D, D→A
- Let's check the closures on R
  - $\{A\}^+ = \{A\}$, $\{B\}^+ = \{B\}$, $\{C\}^+ = \{C\}$, $\{D\}^+ = \{DA\}$
- $\{D\}^+ = \{DA\}$ indicates a violation of BCNF
- Therefore, R is not in BCNF

Question 2(b): Is R in 3NF?

- Schema R(A, B, C, D), with ABC→D, D→A
- First, let's derive the key(s) of R
- Observe that B and C do not appear on the right hand side of any FD
- So BC must be in every key
- Let's check the attribute subsets containing BC:
  - $\{BC\}^+ = \{BC\}$
  - $\{ABC\}^+ = \{ABCD\}$, $\{BCD\}^+ = \{ABCD\}$
- So ABC and BCD are the only keys of R

Question 2(b): Is R in 3NF?

- Schema R(A, B, C, D), with ABC→D, D→A
- Keys of R: ABC, BCD
- So all attributes in R are prime attributes
- Therefore, there won't be any violation of 3NF
- So R is in 3NF

Question 3(a): Is R in 3NF?

- Schema R(A, B, C, D, E), with A→E, CD→A, E→B, E→D, A→BD
- First, let's derive the key(s) of R
- Observe that C does not appear on the right hand side of any FD
- So C must be in every key
- Let's check the attribute subsets containing BC:
  - $\{C\}^+ = \{C\}$
  - $\{AC\}^+ = \{ACEBD\}$, $\{BC\}^+ = \{BC\}$, $\{CD\}^+ = \{CDAEB\}$, $\{CE\}^+ = \{CEBDA\}$
- So AC, CD, and CE are the only keys of R

Question 3(a): Is R in 3NF?

- Schema R(A, B, C, D, E), with A→E, CD→A, E→B, E→D, A→BD
- Keys of R: AC, CD, CE
- So A, C, D, E are prime attributes, but B is not
- Let's check the FDs one by one:
  - First, consider E→B:
    - The left hand side is not a superkey
    - The right hand side is not a prime attribute
- So E→B violates 3NF
- Therefore, R is not in 3NF

Question 3(b): 3NF Decomposition of R

- Schema R(A, B, C, D, E), with F = {A→E, CD→A, E→B, E→D, A→BD}
- First, let's derive a minimal basis of F
  - Step 1: Make the FDs non-trivial and decomposed:
    - F = {A→E, CD→A, E→B, E→D, A→B, A→D}
  - Step 2: Remove redundant attributes on the left hand side:
    - Only CD→A has more than one attribute on the left
    - Can we simplify it to C→A?
      - Given F, we have $\{C\}^+ = \{C\}$, so C→A is not legit
    - Can we simplify it to D→A?
      - Given F, we have $\{D\}^+ = \{D\}$, so D→A is not legit
    - F = {A→E, CD→A, E→B, E→D, A→B, A→D}

Question 3(b): 3NF Decomposition of R

- Schema R(A, B, C, D, E), with F = {A→E, CD→A, E→B, E→D, A→BD}
- First, let's derive a minimal basis of F
  - Result of Step 2: F = {A→E, CD→A, E→B, E→D, A→B, A→D}
  - Step 3: Remove redundant FDs
    - Let check the FDs one by one
    - First, consider A→E
      - If we remove A→E from F, we have $\{A\}^+$ = {ABD}, which does not contain E
      - So A→E is not redundant and cannot be removed
    - Second, consider CD→A
      - If we remove CD→A from F, we have $\{CD\}^+$ = {CD}, which does not contain A
      - So CD→A is not redundant and cannot be removed

Question 3(b): 3NF Decomposition of R

- Schema R(A, B, C, D, E), with F = {A→E, CD→A, E→B, E→D, A→BD}
- First, let's derive a minimal basis of F
  - Result of Step 2: F = {A→E, CD→A, E→B, E→D, A→B, A→D}
  - Step 3: Remove redundant FDs
    - Let check the FDs one by one
    - Third, consider E→B
      - If we remove E→B from F, we have $\{E\}^+ = \{ED\}$, which does not contain B
      - So E→B is not redundant and cannot be removed
    - Fourth, consider E→D
      - If we remove E→D from F, we have $\{E\}^+ = \{EB\}$, which does not contain D
      - So E→D is not redundant and cannot be removed

Question 3(b): 3NF Decomposition of R

- Schema R(A, B, C, D, E), with F = {A→E, CD→A, E→B, E→D, A→BD}
- First, let's derive a minimal basis of F
  - Result of Step 2: F = {A→E, CD→A, E→B, E→D, A→B, A→D}
  - Step 3: Remove redundant FDs
    - Let check the FDs one by one
    - Fifth, consider A→B
      - If we remove A→B from F, we have $\{A\}^+$ = {AEBD}, which contains B
      - So A→B is redundant and can be removed
    - F = {A→E, CD→A, E→B, E→D, A→D}
    - Sixth, consider A→D
      - If we remove A→D from F, we have $\{A\}^+$ = {AEBD}, which contains D
      - So A→D is redundant and can be removed
    - F= {A→E, CD→A, E→B, E→D}

Question 3(b): 3NF Decomposition of R

- Schema R(A, B, C, D, E), with F = {A→E, CD→A, E→B, E→D, A→BD}
- First, let's derive a minimal basis of F
  - Minimal basis: F= {A→E, CD→A, E→B, E→D}
- Second, combine the FDs whose left hand sides are the same:
  - Result: F= {A→E, CD→A, E→BD}
- Third, construct a table for each FD
  - Result: R1(A, E), R2(A, C, D), R3(B, D, E)
- Fourth, check whether R1, R2, or R3 contains one of the keys of R
  - Keys of R: AC, CD, CE
  - AC and CD are contained in R2
- Final decomposition: R1(A, E), R2(A, C, D), R3(B, D, E)

Question 3(c): Is the Decomposition in BCNF?

- Schema R(A, B, C, D, E), with F = {A→E, CD→A, E→B, E→D, A→BD}
- 3NF decomposition: R1(A, E), R2(A, C, D), R3(B, D, E)
- R1 has only two attributes, so it is in BCNF
- Let's check R2
  - First, check the closures on R2
  - $\{A\}^+ = \{AEBD\}$, $\{C\}^+ = \{C\}$, $\{D\}^+ = \{D\}$
  - $\{A\}^+ = \{AD\}$ indicates a violation of BCNF
  - So R2 is not in BCNF
- So the decomposition does not satisfy BCNF

Question 4(a): Is R in 3NF?

- Schema R(A, B, C, D, E), with AB→CDE, AC→BDE, B→C, C→B, C→D, B→E
- First, let's derive the key(s) of R
- Observe that A does not appear on the right hand side of any FD
- So A must be in every key
- Let's check the attribute subsets containing BC:
  - $\{A\}^+ = \{A\}$
  - $\{AB\}^+ = \{ABCDE\}$, $\{AC\}^+ = \{ACBDE\}$, $\{AD\}^+ = \{AD\}$, $\{AE\}^+ = \{AE\}$
  - $\{ADE\}^+ = \{ADE\}$
- So AB and AC are the only keys of R

Question 4(a): Is R in 3NF?

- Schema R(A, B, C, D, E), with AB→CDE, AC→BDE, B→C, C→B, C→D, B→E
- Keys: AB, AC
- So A, B, C are prime attributes, but D, E are not
- Let's check the FDs one by one:
  - AB→CDE: the left hand side is a superkey, so it is OK
  - AC→BDE: the left hand side is a superkey, so it is OK
  - B→C: the right hand side is a prime attribute, so it is OK
  - C→B: the right hand side is a prime attribute, so it is OK
  - C→D:
    - The left hand side is not a superkey
    - The right hand side is not a prime attribute
  - So C→D violates 3NF
- Therefore, R is not in 3NF

Question 4(b): 3NF Decomposition of R

- Schema R(A, B, C, D, E), with F = {AB→CDE, AC→BDE, B→C, C→B, C→D, B→E}
- First, let's derive a minimal basis of F
  - Step 1: Make the FDs non-trivial and decomposed:
    - F = {AB→C, AB→D, AB→E, AC→B, AC→D, AC→E, B→C, C→B, C→D, B→E}
  - Step 2: Remove redundant attributes on the left hand side:
    - First, consider AB→C
    - Can we simplify it to B→C?
      - Given F, we have $\{B\}^+$ = {BCDE}, so B→C is legit
    - F = {B→C, AB→D, AB→E, AC→B, AC→D, AC→E, C→B, C→D, B→E}

Question 4(b): 3NF Decomposition of R

- Schema R(A, B, C, D, E), with F = {AB→CDE, AC→BDE, B→C, C→B, C→D, B→E}
- First, let's derive a minimal basis of F
  - Step 2: Remove redundant attributes on the left hand side:
    - Previous result: F = {B→C, AB→D, AB→E, AC→B, AC→D, AC→E, C→B, C→D, B→E}
    - Consider AB→D
      - Can we simplify it to B→D?
      - Given F, we have $\{B\}^+$ = {BCDE}, so B→D is legit
    - F = {B→C, B→D, AB→E, AC→B, AC→D, AC→E, C→B, C→D, B→E}

Question 4(b): 3NF Decomposition of R

- Schema R(A, B, C, D, E), with F = {AB→CDE, AC→BDE, B→C, C→B, C→D, B→E}
- First, let's derive a minimal basis of F
  - Step 2: Remove redundant attributes on the left hand side:
    - Previous result: F = {B→C, B→D, AB→E, AC→B, AC→D, AC→E, C→B, C→D, B→E}
    - Consider AB→E
      - Can we simplify it to B→E?
      - Given F, we have $\{B\}^+$ = {BCDE}, so B→E is legit
    - F = {B→C, B→D, B→E, AC→B, AC→D, AC→E, C→B, C→D}

Question 4(b): 3NF Decomposition of R

- Schema R(A, B, C, D, E), with F = {AB→CDE, AC→BDE, B→C, C→B, C→D, B→E}
- First, let's derive a minimal basis of F
  - Step 2: Remove redundant attributes on the left hand side:
    - Previous result: F = {B→C, B→D, B→E, AC→B, AC→D, AC→E, C→B, C→D}
    - Consider AC→B
      - Can we simplify it to C→B?
      - Given F, we have $\{C\}^+$ = {CBDE}, so C→B is legit
    - F = {B→C, B→D, B→E, C→B, AC→D, AC→E, C→D}

Question 4(b): 3NF Decomposition of R

- Schema R(A, B, C, D, E), with F = {AB→CDE, AC→BDE, B→C, C→B, C→D, B→E}
- First, let's derive a minimal basis of F
  - Step 2: Remove redundant attributes on the left hand side:
    - Previous result: F = {B→C, B→D, B→E, C→B, AC→D, AC→E, C→D}
    - Consider AC→D
      - Can we simplify it to C→D?
      - Given F, we have $\{C\}^+$ = {CBDE}, so C→D is legit
    - F = {B→C, B→D, B→E, C→B, C→D, AC→E}
    - Consider AC→E
      - Can we simplify to C→E
      - Given F, we have $\{C\}^+$ = {CBDE}, so C→E is legit
    - F = {B→C, B→D, B→E, C→B, C→D, C→E}

Question 4(b): 3NF Decomposition of R

- Schema R(A, B, C, D, E), with F = {AB→CDE, AC→BDE, B→C, C→B, C→D, B→E}
- First, let's derive a minimal basis of F
  - Step 3: Remove redundant FDs:
    - Previous result: F = {B→C, B→D, B→E, C→B, C→D, C→E}
    - Let's check the FDs one by one
    - First, consider B→C
      - If we remove B→C from F, we have $\{B\}^+$ = {BDE}, which does not contain C
      - So B→C is not redundant and cannot be removed
    - Second, consider B→D
      - f we remove B→D from F, we have $\{B\}^+$ = {BCED}, which contains D
      - So B→D is redundant and can be removed
    - F = {B→C, B→E, C→B, C→D, C→E}

Question 4(b): 3NF Decomposition of R

- Schema R(A, B, C, D, E), with F = {AB→CDE, AC→BDE, B→C, C→B, C→D, B→E}
- First, let's derive a minimal basis of F
  - Step 3: Remove redundant FDs:
    - Previous result: F = {B→C, B→E, C→B, C→D, C→E}
    - Third, consider B→E
      - If we remove B→E from F, we have $\{B\}^+$ = {BCDE}, which contain E
      - So B→E is redundant and can be removed
    - F = {B→C, C→B, C→D, C→E}
    - Fourth, consider C→B
      - If we remove C→B from F, we have $\{C\}^+$ = {CD}, which does not contain B
      - So C→B is not redundant and cannot be removed

Question 4(b): 3NF Decomposition of R

- Schema R(A, B, C, D, E), with F = {AB→CDE, AC→BDE, B→C, C→B, C→D, B→E}
- First, let's derive a minimal basis of F
  - Step 3: Remove redundant FDs:
    - Previous result: F = {B→C, C→B, C→D, C→E}
    - Fifth, consider C→D
      - If we remove C→D from F, we have $\{C\}^+$ = {CBE}, which does not contain D
      - So C→D is not redundant and cannot be removed
    - Sixth, consider C→E
      - If we remove C→E from F, we have $\{C\}^+$ = {CBDE}, which does not contain D
      - So C→E is not redundant and cannot be removed
    - Final result: F = {B→C, C→B, C→D, C→E}

Question 4(b): 3NF Decomposition of R

- Schema R(A, B, C, D, E), with F = {AB→CDE, AC→BDE, B→C, C→B, C→D, B→E}
- First, let's derive a minimal basis of F
  - Minimal basis: F = {B→C, C→B, C→D, C→E}
- Second, combine the FDs whose left hand sides are the same:
  - Result: F= {B→C, C→BDE}
- Third, construct a table for each FD
  - Result: R1(B, C), R2(B, C, D, E)
- Fourth, check whether R1 or R2 contains one of the keys of R
  - Keys of R: AB, AC
  - Neither R1 nor R2 contains a key of R
  - So we need to add a table R3(A, B) or R3(A, C)
  - Suppose that choose R3(A, B)
- Final decomposition: R1(B, C), R2(B, C, D, E), R3(A, B)

Question 4(c): Is the Decomposition in BCNF?

- Schema R(A, B, C, D, E), with F = {B→C, C→B, C→D, C→E}
- 3NF decomposition: R1(B, C), R2(B, C, D, E), R3(A, B)
- R1 and R3 have only two attributes, so they must be in BCNF
- Let's check R2
  - First, check the closures on R2
  - $\{B\}^+ = \{BCDE\}$, $\{C\}^+ = \{BCDE\}$, $\{D\}^+ = \{D\}$, $\{E\}^+ = \{E\}$
  - $\{DE\}^+ = \{DE\}$
  - Other attributes subsets contain B or C, so they are superkeys
  - There is no violation of BCNF
- So the decomposition satisfies BCNF