CS1231 Review 7

1. Determine whether the following are true or false.
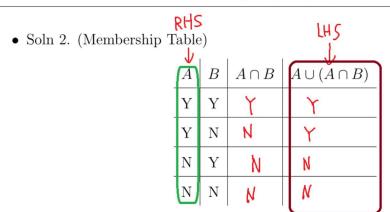
- $\emptyset = \{\emptyset\}$ **F**
- $\emptyset \in \{\emptyset\}$ **T**
- $\emptyset \subseteq \{\emptyset\}$ **T** $\leftarrow$ $\boxed{\phi \subseteq S}$
- $\emptyset \in \{\{1\}, 2\}$ **F**
- $\emptyset \subseteq \{\{1\}, 2\}$ **T**
- $\emptyset \subsetneq \{\{1\}, 2\}$ **T**
  $\leq$ and $\neq$

- $\{\emptyset\} \in \{\emptyset\}$ **F** $\phi \in \{\phi\}$
- $\{\emptyset\} \subseteq \{\emptyset\}$ **T**
- $\{\emptyset\} \subsetneq \{\emptyset\}$ **F** $\subsetneq$ means $\subseteq$ and $\neq$
- $\{\emptyset\} \subseteq \{\{1\}, 2\}$ **F**
- $\{1, 2\} = \{\{1\}, 2\}$ **F**
  $\underset{=\{1,2\}}{} \qquad \overset{\text{diff}}{}$

2. Cartesian Product: $\{\{1\}, 2\} \times \{1, 1, 2\} = \{ (\{1\}, 1), (\{1\}, 2), (2, 1), (2, 2) \}$
   $\qquad \qquad \qquad \qquad = \{1, 2\}$

3. Power Set: $P(\{\{1\}, 2\}) = \{ \phi, \{\{1\}\}, \{2\}, \{\{1\}, 2\} \}$

4. Prove Absorption Law: $A \cup (A \cap B) = A$.

- Soln 1. Consider
  $$\underset{\text{LHS}}{x \in A \cup (A \cap B)}.$$

  We have two cases. Case 1: $x \in A$

  $x \in \text{RHS}$

  Case 2: $x \in A \cap B$

  Then $x \in A$ and $x \in B$
  $\therefore x \in \text{RHS}$

  Since both cases lead to $x \in \text{RHS}$, we conclude $\text{LHS} \subseteq \text{RHS}$
  $\qquad \qquad \qquad \qquad \qquad \text{RHS} \qquad \qquad \text{LHS} = A \cup (A \cap B)$

  Conversely, consider $x \in A$, then $x \in A \cup (A \cap B)$ $\Rightarrow$ $\text{RHS} \subseteq \text{LHS}$.
  $\qquad \qquad \qquad \qquad \qquad \qquad \underset{\text{another set}}{}$

  Thus the proof is complete

  $\text{LHS} = \text{RHS}$

- Soln 2. (Membership Table)

RHS   LHS

A	B	$A \cap B$	$A \cup (A \cap B)$
Y	Y	Y	Y
Y	N	N	Y
N	Y	N	N
N	N	N	N

- Soln 3. (Venn Diagram) (up to 3 sets)

$A = 1+2$
$B = 1+3$

$A \cap B = \underline{1}$ $A \cup (A \cap B) = \underline{1+2} = A$