

**10.2 Sampling sleep** The 2009 Sleep in America poll of a random sample of 1000 adults reported that respondents slept an average of 6.7 hours on weekdays and 7.1 hours on weekends, and that 28% of respondents got eight or more hours of sleep on weekdays whereas 44% got eight or more hours of sleep on weekends (www.sleepfoundation.org).

**TRY**

- To compare the means or the percentages using inferential methods, should you treat the samples on weekdays and weekends as independent samples, or as dependent samples? Explain.
- To compare these results to polls of other people taken in previous years, should you treat the samples in the two years as independent samples, or as dependent samples? Explain.

- Dependent samples. Every respondent is in both samples.**
- Independent samples. No one person is in both samples.**

**10.6 Aspirin and heart attacks in Sweden** A Swedish study used 1360 patients who had suffered a stroke. The study randomly assigned each subject to an aspirin treatment or a placebo treatment.<sup>4</sup> The table shows MINITAB output, where X is the number of deaths due to heart attack during a follow-up period of about 3 years. Sample 1 received the placebo and sample 2 received aspirin.

**TRY**

- Each “Sample p” is obtained by taking the proportion of the people in that sample who had a heart attack.
- The “estimate for  $p(1)-p(2)$ ” is obtained by subtracting the “Sample p” for the second sample from the “Sample p” from the first sample. There is a difference of 0.014.
- The confidence interval tells us that we can be 95% confident that the population difference in proportions is between -0.005 and 0.033. Because zero is in this interval, it is plausible that there is no difference between proportions. There may be no difference in proportions of heart attacks between the aspirin and placebo groups.
- The estimate for the difference would change in sign; it would be negative instead of positive. The endpoints of the confidence interval also would change in signs. They would be (-0.033, 0.005). The confidence interval still includes zero; there may be no difference in proportions of heart attacks between those who take aspirin and those who take placebo.

- Explain how to obtain the values labeled “Sample p.”
- Explain how to interpret the value given for “estimate for difference.”
- Explain how to interpret the confidence interval, indicating the relevance of 0 falling in the interval.
- If we instead let sample 1 refer to the aspirin treatment and sample 2 the placebo treatment, explain how the estimate of the difference and the 95% confidence interval would change. Explain how then to interpret the confidence interval. (Note that the output below would change for the analysis of this difference.)

#### Deaths due to heart attacks in Swedish study

Sample	X	N	Sample p
1	28	684	0.040936
2	18	676	0.026627

Difference =  $p(1) - p(2)$   
 Estimate for difference: 0.0143085  
 95% CI for difference: (-0.00486898, 0.0334859)  
 Test for difference = 0 (vs not = 0):  
 $Z = 1.46$  P-Value = 0.144

**10.8 Significance test for aspirin and cancer deaths study**

In the study for cancer death rates, consider the null hypothesis that the population proportion of cancer deaths  $p_1$  for placebo is the same as the population proportion  $p_2$  for aspirin. The sample proportions were  $\hat{p}_1 = 347/11,535 = 0.030$  and  $\hat{p}_2 = 327/14,035 = 0.023$ .

- For testing  $H_0: p_1 = p_2$  against  $H_a: p_1 \neq p_2$ , show that the pooled estimate of the common value  $p$  under  $H_0$  is  $\hat{p} = 0.027$  and the standard error is 0.002.
- Show that the test statistic is  $z = 3.5$ .
- Find and interpret the P-value in context.

$$a) \quad \hat{p} = (347+327)/(11,535+14,035) = 674/25,570 = 0.026$$

$$se_0 = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{0.026(1-0.026)\left(\frac{1}{11,535} + \frac{1}{14,035}\right)} = 0.002$$

$$b) \quad z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{se_0} = \frac{0.030 - 0.023}{0.002} = 3.5$$

$$c) \quad \text{P-value} = 2 * P(Z > 3.5) = 0.0005.$$

If  $H_0$  were true, there is 0.0005 probability of getting a result at least as extreme as the value observed.

We have strong evidence that there is a difference in the population of cancer deaths between those taking placebo and those taking aspirin.

Since the test statistic is positive, we can conclude that  $p_1 > p_2$ .

Remarks:

When  $H_0$  is not rejected, we are at risk of a Type I error.

The probability of a Type I error here is the P-value, 0.0005.

**10.12 TV watching** A researcher predicts that the percentage of people who do not watch TV is higher now than before the advent of the Internet. Let  $p_1$  denote the population proportion of American adults in 1975 who reported watching no TV. Let  $p_2$  denote the corresponding population proportion in 2008.

- Set up null and alternative hypotheses to test the researcher's prediction.
- According to General Social Surveys, 57 of the 1483 subjects sampled in 1975 and 87 of the 1324 subjects sampled in 2008 reported watching no TV. Find the sample estimates of  $p_1$  and  $p_2$ .
- Show steps of a significance test. Explain whether the results support the researcher's claim.

$$a) \quad H_0: p_1 = p_2, H_a: p_1 < p_2.$$

$$b) \quad \hat{p}_1 = \frac{57}{1483} = 0.0384, \quad \hat{p}_2 = \frac{87}{1324} = 0.0657.$$

c) 1. Categorical, random and independent, large enough sample to ensure at least 10 successes and 10 failures.

$$2. \quad H_0: p_1 = p_2, H_a: p_1 < p_2.$$

$$3. \quad \hat{p} = \frac{57 + 87}{1483 + 1324} = 0.0513$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.0384 - 0.0657}{\sqrt{0.0513(1-0.0513)\left(\frac{1}{1483} + \frac{1}{1324}\right)}} = -3.27.$$

4. P

5. If  $H_0$  were true, there is 0.0005 probability of getting a result at least as extreme as the value observed.

Since the P-value is very small, we reject  $H_0$  and conclude that the proportion of American adults in 1975 who reported watching no TV is less than the proportion of American adults in 2008 who reported watching no TV.

Remarks:

When  $H_0$  is rejected, we are at risk of a Type II error.

However, we are unable to compute the probability of a Type II error here unless we are provided with more information.