

Sampling Distribution

- The *sampling distribution* of a *statistic* is the probability distribution that specifies probabilities for the possible values the statistic can take.
- To distinguish the standard deviation of a *sampling distribution* from the standard deviation of an ordinary probability distribution, we refer to it as a *standard error*.

probability distribution
random variable

mean
standard deviation, SD

sampling distribution
sample statistic

mean
standard error, SE

Categorical Data: Proportion

Quantitative Data: Mean

Sampling Distribution

Sample Statistic

Sample Proportion

$$\hat{p} = \frac{X}{n}$$

$$\mu_{\hat{p}} = E(\hat{p}) = p$$

$$\sigma_{\hat{p}} = SE(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$$

When n is large,
 np & $n(1-p) \geq 15$
 $\rightarrow \hat{p}$ is approx normal

Sample Mean

$$\bar{Y} = \frac{\sum Y}{n}$$

$$\mu_{\bar{y}} = E(\bar{Y}) = \mu$$

$$\sigma_{\bar{y}} = SE(\bar{Y}) = \frac{\sigma}{\sqrt{n}}$$

When n is large ($n \geq 30$),
 $\rightarrow \bar{Y}$ is approx normal
Or
When Y is normal,
 $\rightarrow \bar{Y}$ is approx normal

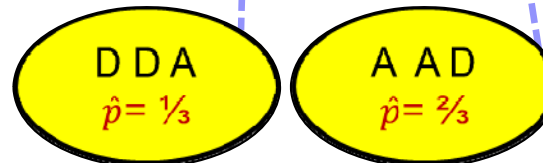
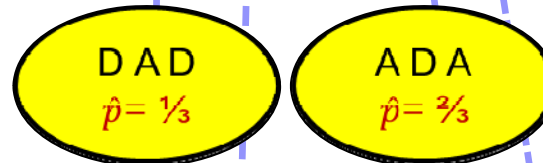
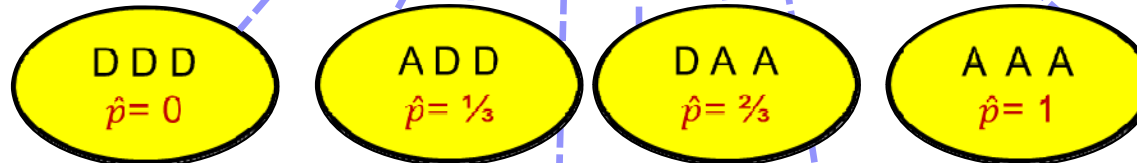
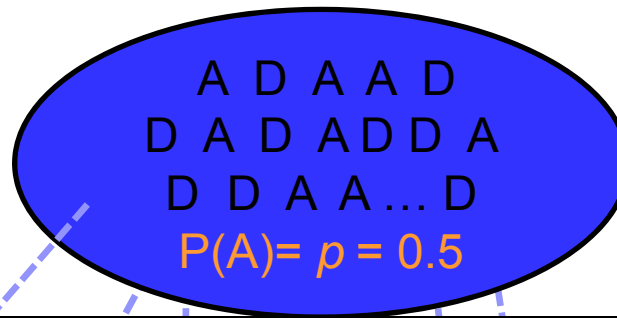
Mean

Standard
Error (SE)

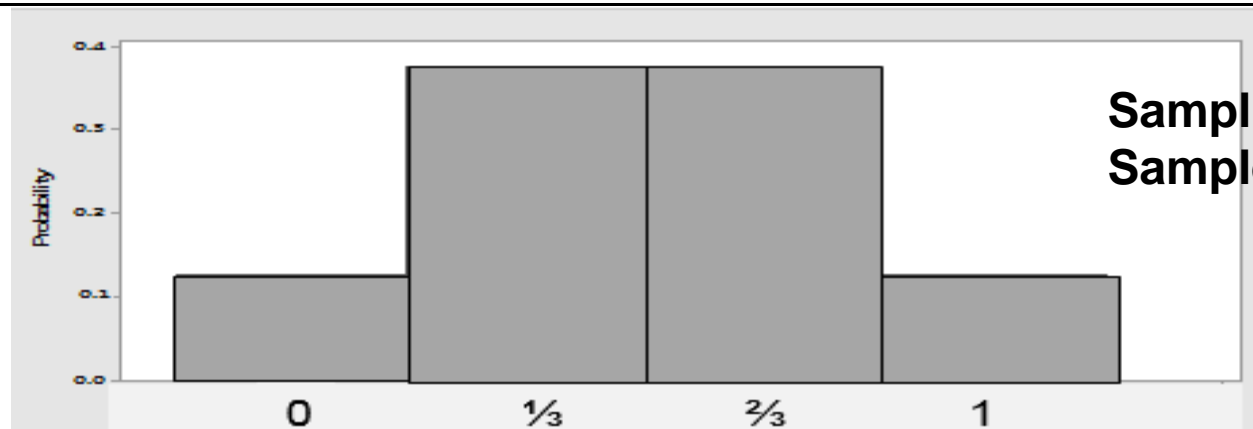
Shape

Categorical Data: Proportion

Population Distribution



(Sample) Data Distribution
Sample size $n = 3$



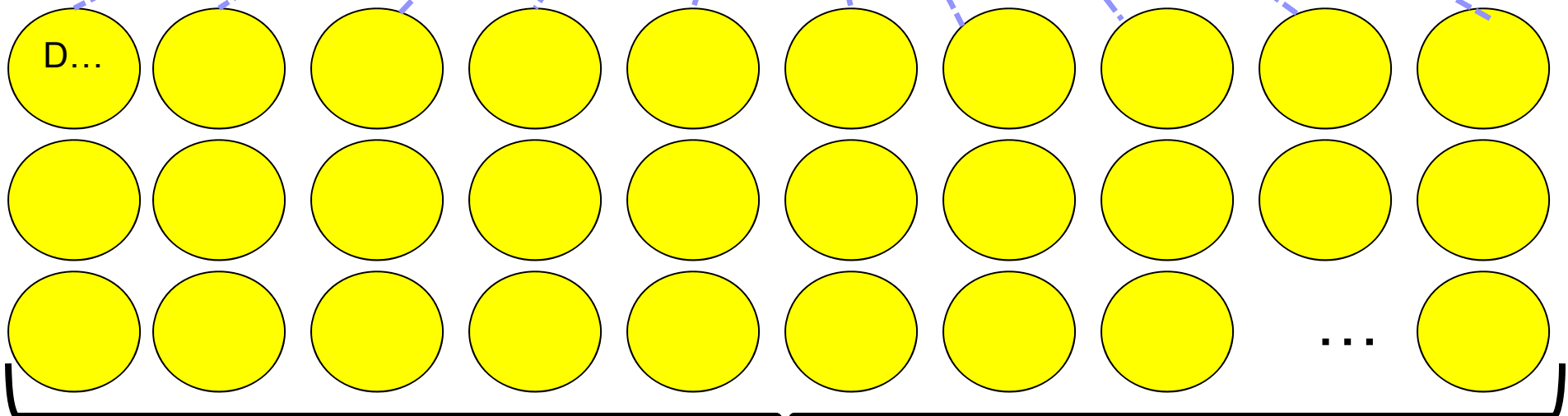
Sampling Distribution
Sample size $n = 3$

Categorical Data: Proportion

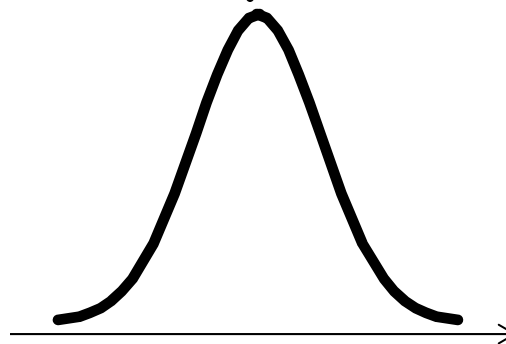
Population Distribution

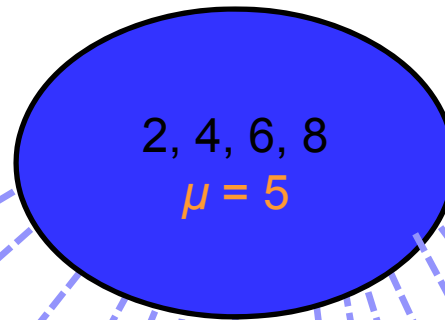
A D A A D
D A D A D D A
D D A A ... D
 $P(A) = p = 0.5$

(Sample) Data Distribution
Sample size n

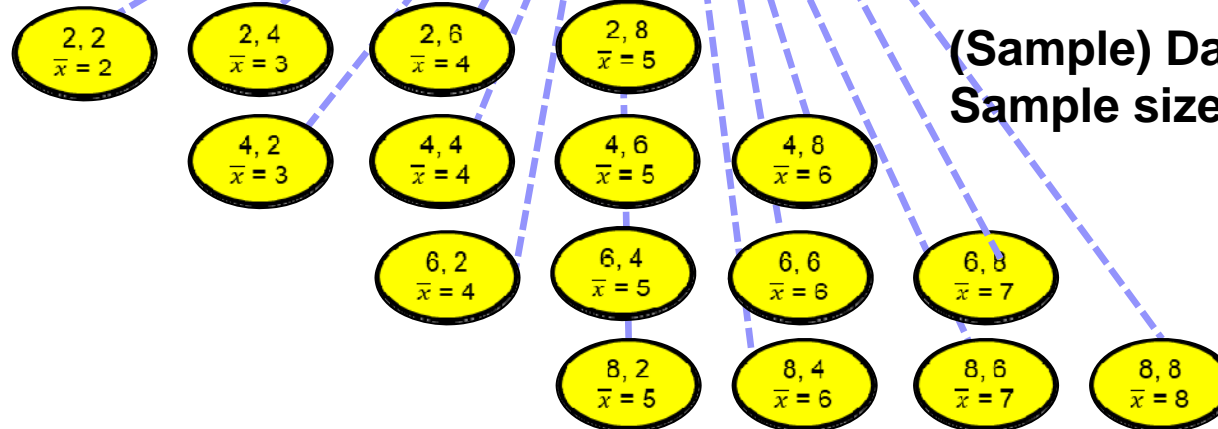


Sampling Distribution
Sample size n

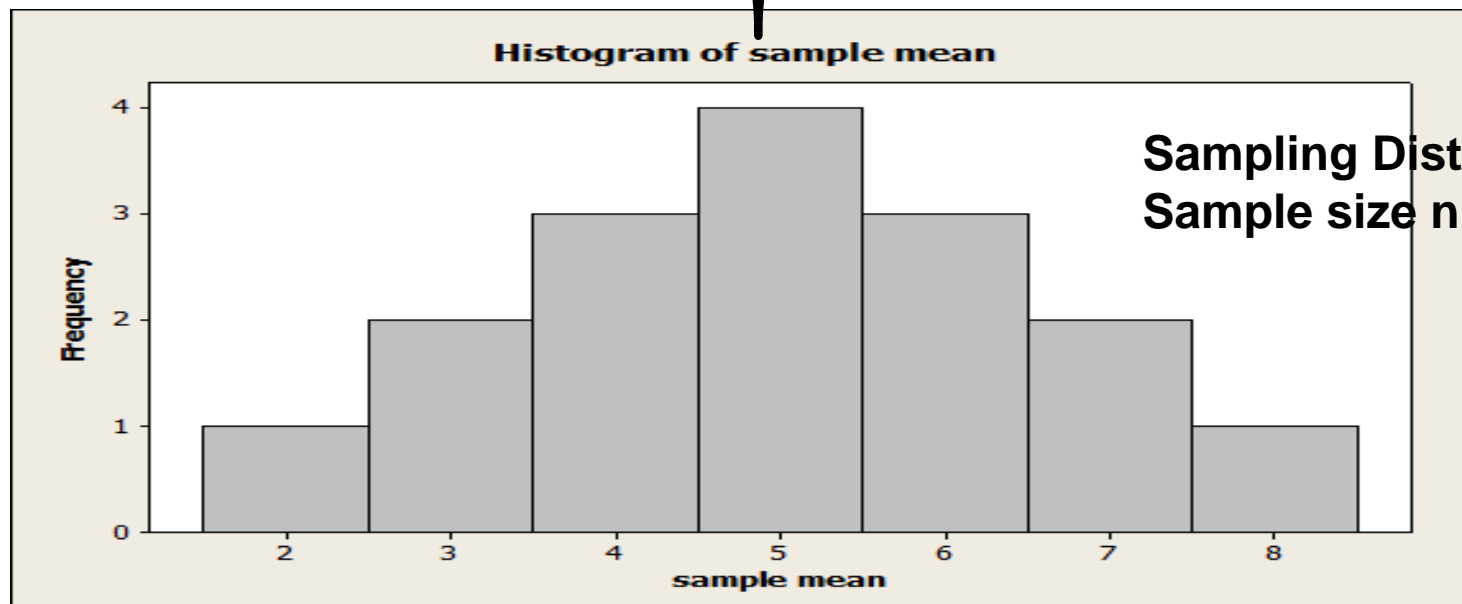




Population Distribution

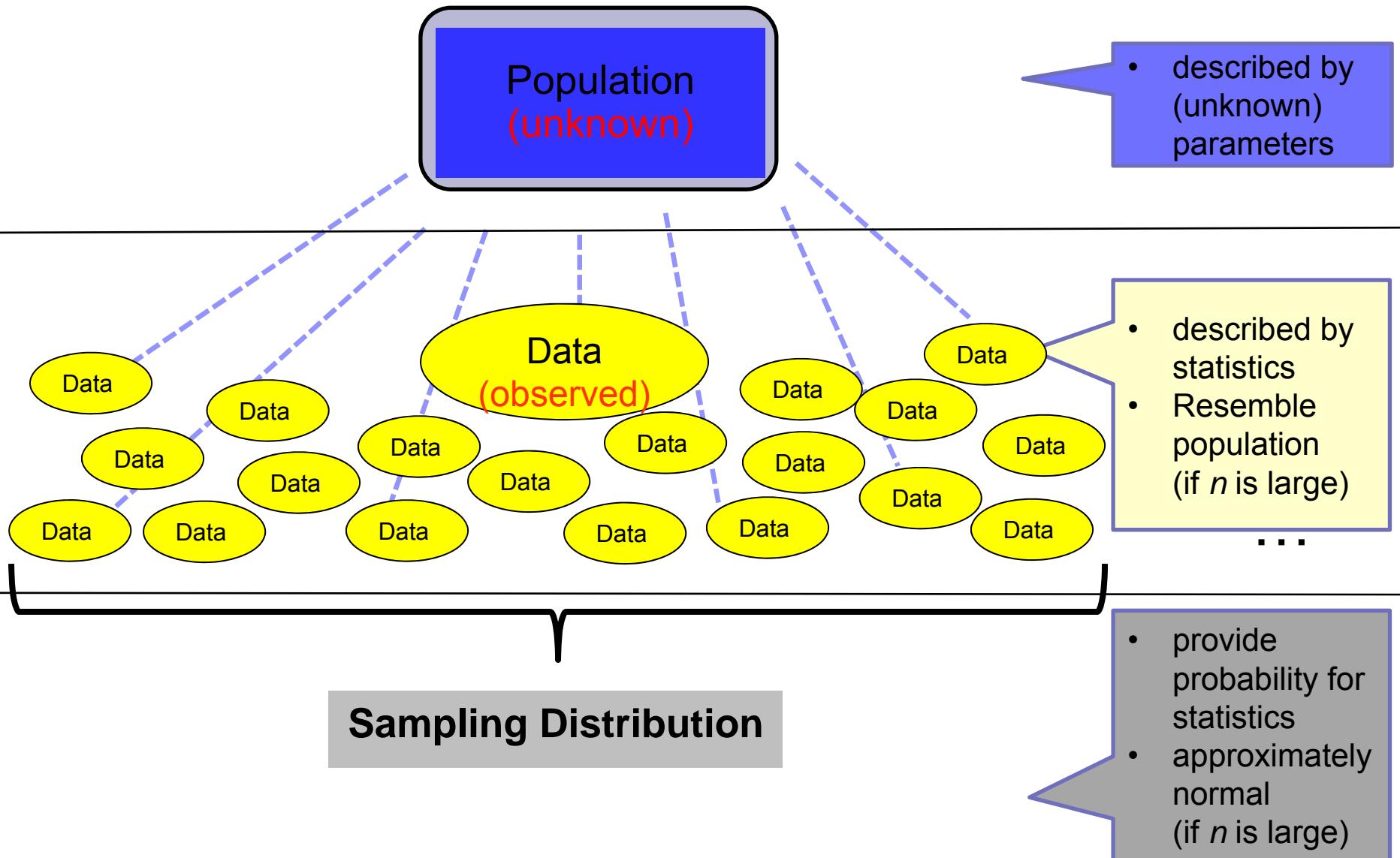


(Sample) Data Distribution
Sample size $n = 2$



Sampling Distribution
Sample size $n = 2$

Three Distinct Types of Distributions



The Sampling Distribution of The Sample Proportion (\hat{p})

The population proportion of students who are myopic is 0.70.

A random sample of 81 is taken from the population and the sample proportion to be computed.

(a) What is the shape of the sampling distribution of the sample proportion?

normal distributed, np and $n(1-p) \geq 15$

(b) What is the probability that the sample proportion is below 0.70?

mean = 0.7, probability = 0.5

(c) What is the probability that the sample proportion is above 0.6?

$P(Z > -1.96)$

(d) What is the probability that the sample proportion is between 0.60 to 0.80?

$P(-1.96 < Z < 1.96)$

(e) Without computation, explain whether the probability in part (d) will increase / decrease / remain unchanged if the sample size is increased to 100.

The Sampling Distribution of The Sample Mean (\bar{Y})

The height of students in a large class is normally distributed. The mean is 176cm and the standard deviation is 8cm.

A random sample of 16 is taken from the population and the sample mean is to be computed.

- (a) What is the median height of students in the class?
- (b) What is the shape of the sampling distribution of the sample mean?
- (c) What is the probability that the sample mean is above 178cm?
- (d) What is the probability that the sample mean is between 172cm to 180cm?
- (e) Without computation, explain whether the probability in part (d) will increase / decrease / remain unchanged if the sample size is increased to 100.