National University of Singapore
School of Computing

Semester 1, AY2021-22

CS4246/CS5446 AI Planning and Decision Making

Issued: 03 Sep, 2021

Tutorial Week 4: Rational Decision Making

Guidelines

You may discuss the content of the questions with your classmates. But to everyone should work on and be ready to present ALL the solutions.

Problem 1: Allais Paradox

The Allais paradox (Allais, 1953) is a well-known problem potentially suggesting that humans are "predictably irrational" (Ariely, 2009) [1]. People are given a choice between lotteries A and B and then between C and D, which have the following prizes:

A: 80% chance of \$4000	C: 20% chance of \$4000
B: 100% chance of \$3000	D: 25% chance of \$3000

Most people consistently prefer B over A (i.e., taking the sure payoff), and C over D (taking the higher EMV).

a) Show that the normative analysis (i.e., describing how a rational agent should act) disagrees. [**Hint**: Set U(\$0) = 0; show that the preferences between A, B and C, D are opposites, hence a contradiction.]

Solution:
Inequality from preferring B to A:

$$0.8U(\$4000) < U(\$3000)$$

Inequality from preferring C to D:

$$0.2U(\$4000) > 0.25U(\$3000)$$

Now multiply by 4:

$$0.8U(\$4000) > U(\$3000)$$

There is a contradiction between the 2 preferences.

In that case, then B$\succ$A implies that $U(\$3000) > 0.8U(\$4000)$, whereas C$\succ$D implies exactly the reverse. In other words, there is no utility function that is consistent with these choices.

[1] For a possible explanation of such a paradox, refer to page 620 of AIMA textbook.

b) Prove that the judgments B $\succ$ A and C $\succ$ D in the above Allais paradox violate the axiom of substitutability. [**Hint**: You may wish to consider using the axiom of decomposability.]

Solution:
We know A$\prec$B and C$\succ$D. By the axiom of decomposability,

$$C \sim [0.25, A; 0 : 75; \$0] \text{ and } D \sim [0.25, B; 0.75, \$0]. \quad (1)$$

Using the above and C$\succ$D, we have $[0.25, A; 0.75; \$0] \succ [0.25, B; 0.75, \$0]$. This implies that EU(A)>EU(B) $\implies$ A $\succ$B.

From A$\prec$B and substitutability, we would have $[0.25, A; 0.75, \$0] \prec [0.25, B; 0.75, \$0]$.

This is a contradiction, hence substitutability is violated.

Problem 2: Preference Modelling

Alex is given the choice between two games:

- **Game 1**: a fair coin is flipped and if it comes up heads, Alex receives \$100. If the coin comes up tails, Alex receives nothing.
- **Game 2**: a fair coin is flipped twice. Each time the coin comes up heads, Alex receives \$50, and Alex receives nothing for each coin flip that comes up tails.

Alex prefers Game 2 to Game 1. Argue that Alex would prefer to receive \$50 compared to being allowed to participate in Game 1.

Solution:
Since Alex prefers Game 2 to Game 1:

$$\begin{aligned}
&0.5U(\$0) + 0.5U(\$100) < 0.25U(\$0) + 0.5U(\$50) + 0.25U(\$100)\\
&\implies 0.25U(\$0) + 0.25U(\$100) < 0.5U(\$50)\\
&\implies 0.5U(\$0) + 0.5U(\$100) < U(\$50)
\end{aligned}$$

The left hand side is the expected utility of participating in Game 1 while the right hand side is the expected utility of receiving \$50. Since the LHS is less than the RHS, Alex would prefer \$50 over participating in game 1.

Problem 3: Risk Tolerance

Economists often make use of an exponential utility function for money: $U(x) = -exp(-x/R)$ where R is a positive constant representing an individual's risk tolerance.

Risk tolerance reflects how likely an individual is to accept a lottery with a particular EMV versus some certain payoff. As R (which is measured in the same units as x) becomes larger, the individual becomes less risk-averse. Mary is risk adverse.

a) Assume that Mary has an exponential utility function with R = \$500. Mary is given the choice between receiving \$500 with certainty (probability 1) or participating in a lottery which has a 60% probability of winning \$5000 and a 40% probability of winning nothing. Assuming Mary acts rationally, which option would she choose? Show how you derived your answer.

Solution:
Getting \$500 as sure payoff has expected utility of

$$-\exp(-500/500) = -1/e \approx 0.3679$$

while getting \$5000 with probability 0.6 and \$0 otherwise has expected utility of

$$0.6 \times -\exp(-5000/500) + 0.4 \times -\exp(-0/500) = -(0.6\exp(-10) + 0.4) \approx -0.4000$$

So, Mary would prefer the sure payoff.

b) Consider the choice between receiving \$100 with certainty (probability 1) or participating in a lottery which has a 50% probability of winning \$500 and a 50% probability of winning nothing. What value of R in the exponential utility function specified above that would cause an individual to be indifferent to these two alternatives? (Writing the equation will do, no need to solve.)

Solution:
We want to find R such that

$$\exp(-100/R) = 0.5\exp(-500/R) + 0.5$$

The solution is R $\approx$152.