NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
MA1101R Linear Algebra I

2018-2019 (Semester 1) **Tutorial 9**

1. (a) In $\mathbb{R}^2$, find the distance from the point $(1,5)$ to the line $x-y=0$.
   (b) In $\mathbb{R}^3$, find the distance from the point $(1,0,-2)$ to the plane $2x+y-2z=0$.
   (c) In $\mathbb{R}^3$, find the distance from the point $(1,0,-2)$ to the line
   $$L = \{(t, 2t, 2t) \mid t \in \mathbb{R}\}.$$

2. Let $V = \text{span}\{\boldsymbol{v_1} = (1,0,1), \boldsymbol{v_2} = (0,1,-2)\}$.
   (a) Is $\{\boldsymbol{v_1}, \boldsymbol{v_2}\}$ a basis for V? Justify your answer.
   (b) Use Gram-Schmidt Process to find an orthonormal basis for V.
   (c) Compute the projection of $\boldsymbol{w} = (1,1,1)$ onto V using
      (i) Theorem 5.2.15 (Orthogonal projection); and
      (ii) Theorem 5.3.8 together with Theorem 5.3.10 (Least Squares solution).

3. A series of experiments were performed to investigate the relationship between two physical quantities x and y. The results of the experiments are shown in the table below.

x	0	1	2	3
y	3	2	4	4

   (a) Find a least squares solution $\boldsymbol{x} = (\hat{a}, \hat{b})$ if it is believed that x and y are related linearly, that is, $y = ax + b$.
   (b) Find a least squares solution $\boldsymbol{x} = (\hat{a}, \hat{b}, \hat{c})$ if it is believed that x and y are related by the quadratic polynomial $y = ax^2 + bx + c$.

4. (All vectors in this question are written as column vectors.) Let $\boldsymbol{A}$ be an orthogonal matrix of order n and $S = \{\boldsymbol{u_1}, \boldsymbol{u_2}, \cdots, \boldsymbol{u_n}\}$ be a basis for $\mathbb{R}^n$.
   (a) For any vector $\boldsymbol{x} \in \mathbb{R}^n$, show that $||\boldsymbol{x}|| = ||\boldsymbol{A}\boldsymbol{x}||$.
   (b) For any two vectors $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$, show that $d(\boldsymbol{x}, \boldsymbol{y}) = d(\boldsymbol{A}\boldsymbol{x}, \boldsymbol{A}\boldsymbol{y})$.
   (c) For any two vectors $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$, show that the angle between $\boldsymbol{x}$ and $\boldsymbol{y}$ is the same as the angle between $\boldsymbol{A}\boldsymbol{x}$ and $\boldsymbol{A}\boldsymbol{y}$.
   (d) Show that $T = \{\boldsymbol{A}\boldsymbol{u_1}, \boldsymbol{A}\boldsymbol{u_2}, \cdots, \boldsymbol{A}\boldsymbol{u_n}\}$ is also a basis for $\mathbb{R}^n$.
   (e) If S is an orthogonal basis, show that T is also an orthogonal basis.
   (f) If S is orthonormal, is T orthonormal?

5. Let
   $$\boldsymbol{v_1} = (1,1,1,-1), \quad \boldsymbol{v_2} = (1,1,3,5).$$
   It is easy to see that $S = \{\boldsymbol{v_1}, \boldsymbol{v_2}\}$ is an orthogonal set. Extend this set to an orthogonal basis for $\mathbb{R}^4$.