

**CS4246 / CS5446**

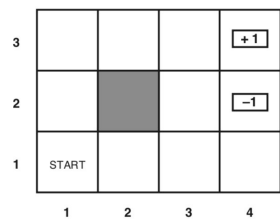
# Tutorial Week 11

Muhammad **Rizki** Maulana  
rizki@u.nus.edu

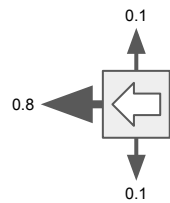
**First**

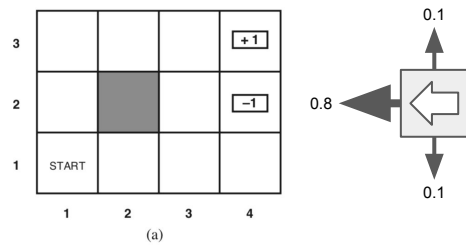
3				<div>+1</div>
2				<div>-1</div>
1	START			
	1	2	3	4

(a)



(a)

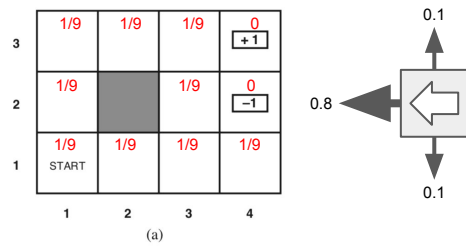




Detect adjacent wall

**Noisy sensor:**

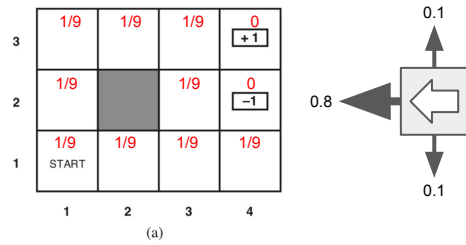
- Correct : 0.9
- Wrong : 0.1



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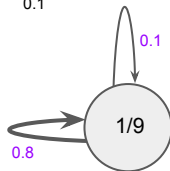
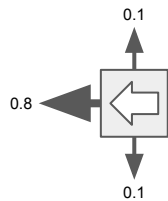
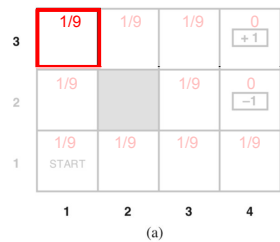
Calculate the exact belief state  $b_1$  (rounded off to 5 decimal places) after the agent moves *Left* and its sensor reports 1 adjacent wall.

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Question



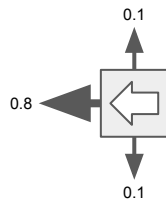
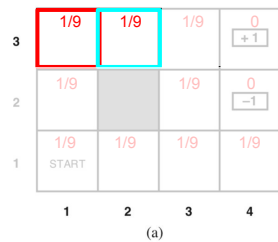
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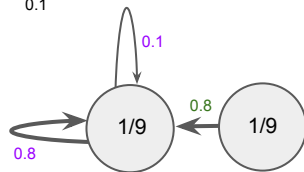
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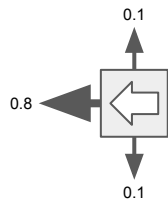
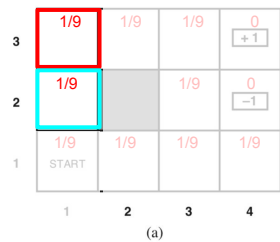
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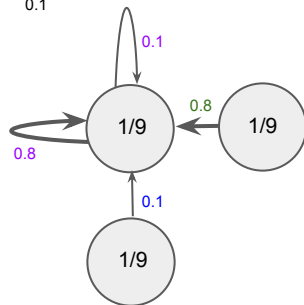


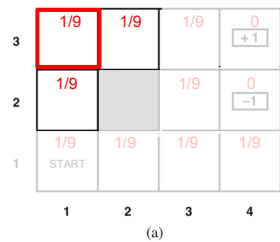
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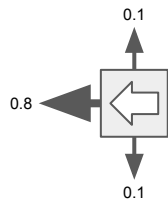




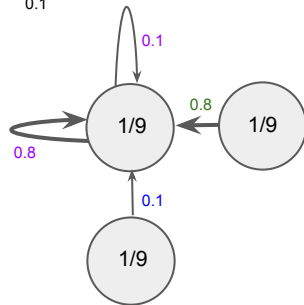
Detect adjacent wall

**Noisy sensor:**

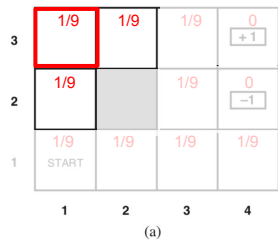
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Calculate the exact belief state  $b_1$  (rounded off to 5 decimal places) after the agent moves *Left* and its sensor reports 1 adjacent wall.



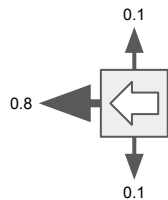
$$P(x'|\hat{Left}, b_0) = \sum_x P(x'|Left, x)b_0(x)$$



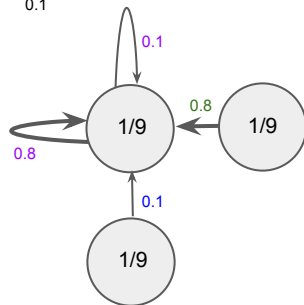
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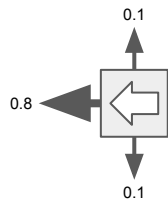
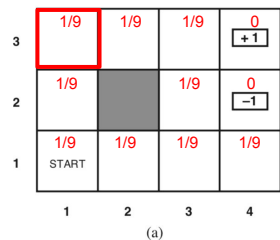


Calculate the exact belief state  $b_1$  (rounded off to 5 decimal places) after the agent moves *Left* and its sensor reports 1 adjacent wall.

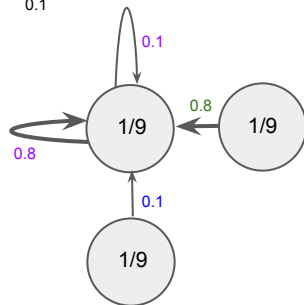


$$P(x'|\hat{Left}, b_0) = \sum_x P(x'|Left, x)b_0(x)$$

$$0.9(1/9) + 0.8(1/9) + 0.1(1/9) = 0.2$$



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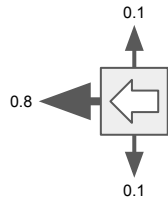
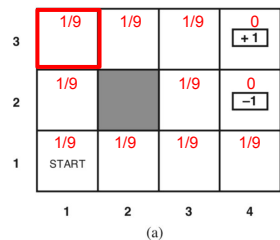
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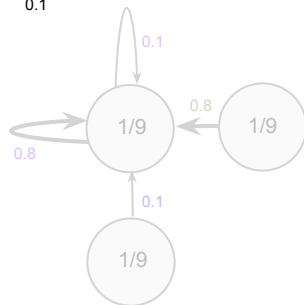
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0.2	$\frac{1}{9}$	$\frac{0.2}{9}$	0
$\frac{1}{9}$	$\times$	$\frac{1}{9}$	$\frac{0.1}{9}$
0.2	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{0.1}{9}$



Calculate the exact belief state  $b_1$  (rounded off to 5 decimal places) after the agent moves *Left* and its sensor reports 1 adjacent wall.



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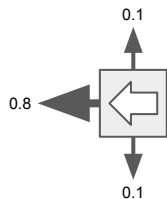
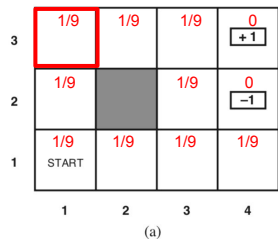
0.2	$\frac{1}{9}$	$\frac{0.2}{9}$	0
$\frac{1}{9}$	$\times$	$\frac{1}{9}$	$\frac{0.1}{9}$
0.2	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{0.1}{9}$

Detect adjacent wall

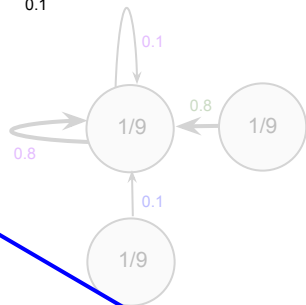
**Noisy sensor:**

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Now, we update these estimates with the sensor data, which says there is one adjacent wall (i.e., multiply by  $P(z = \text{'1 adjacent wall'}|x')$ ):



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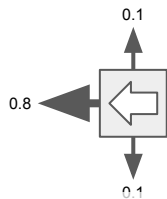
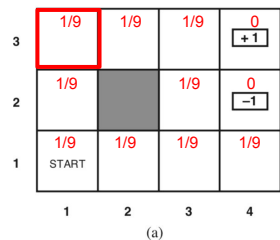
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Now, we update these estimates with the sensor data, which says there is one adjacent wall (i.e., multiply by  $P(z = \text{'1 adjacent wall'} | x')$ ):

$0.1 \times 0.2$	$0.1 \times \frac{1}{9}$	$0.9 \times \frac{0.2}{9}$	0
$0.1 \times \frac{1}{9}$	$\times$	$0.9 \times \frac{1}{9}$	$0.9 \times \frac{0.1}{9}$
$0.1 \times 0.2$	$0.1 \times \frac{1}{9}$	$0.9 \times \frac{1}{9}$	$0.1 \times \frac{0.1}{9}$

1 adj wall

2 adj wall

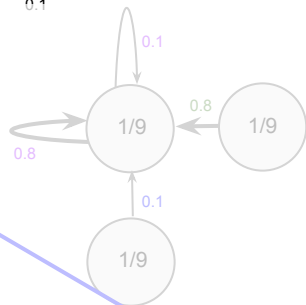


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0.2	$\frac{1}{9}$	$\frac{0.2}{9}$	0
$\frac{1}{9}$	×	$\frac{1}{9}$	$\frac{0.1}{9}$
0.2	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{0.1}{9}$

Now, we update these estimates with the sensor data, which says there is one adjacent wall (i.e., multiply by  $P(z = \text{'1 adjacent wall'} | x')$ ):

$0.1 \times 0.2$	$0.1 \times \frac{1}{9}$	$0.9 \times \frac{0.2}{9}$	0
$0.1 \times \frac{1}{9}$	×	$0.9 \times \frac{1}{9}$	$0.9 \times \frac{0.1}{9}$
$0.1 \times 0.2$	$0.1 \times \frac{1}{9}$	$0.9 \times \frac{1}{9}$	$0.1 \times \frac{0.1}{9}$

2 adj wall

1 adj wall

and renormalize to get  $b_1$ :

0.06569	0.03650	0.06569	0
0.03650	×	0.32847	0.03285
0.06569	0.03650	0.32847	0.00365





**Second**

[Modified from RN 3e 17.14] What is the time complexity of  $d$  steps of POMDP value iteration for a sensorless environment? Give an upper bound on the number of  $\alpha$ -vectors generated in the process.

Question

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sensorless

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↑  
 $p = [a, p']$ ,  $p'$  subplan

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← conditional plans



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## Sensorless Vacuum Cleaner World



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conditional plans

## Sensorless Vacuum Cleaner World



depth=1



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### Sensorless Vacuum Cleaner World

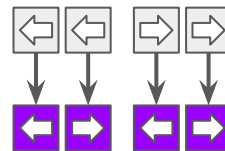


depth=1



2

depth=2



$2^2$

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### Sensorless Vacuum Cleaner World

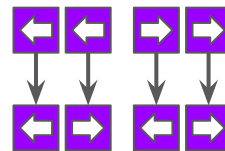


depth=1



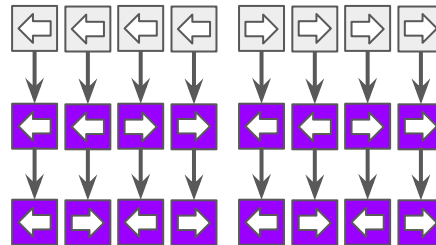
2

depth=2



$2^2$

depth=3



$2^3$

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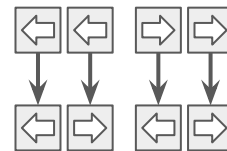


depth=1



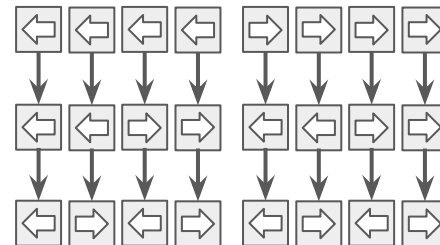
2

depth=2



$2^2$

depth=3



$2^3$

depth=d,  
 $|A|$  actions

$|A|^d$

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 $p = [a, p'], p'$  subplan

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conditional plans

### Sensorless Vacuum Cleaner World



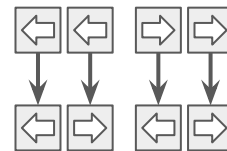
Number of alpha vectors at depth  $d$

depth=1



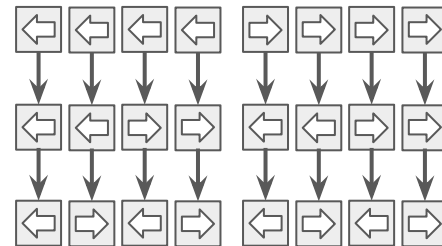
2

depth=2



+2<sup>2</sup>

depth=3



+2<sup>3</sup>

depth= $d$ ,  
 $|A|$  actions

+ $|A|^d$

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### Sensorless Vacuum Cleaner World



### Number of alpha vectors at depth d

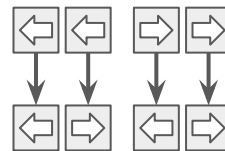
$$\sum_d |A|^d = O(|A|^d)$$

depth=1



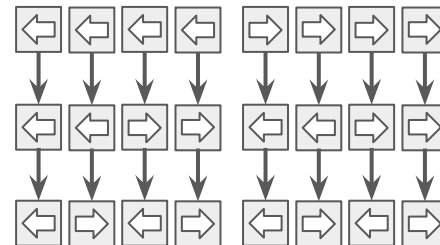
2

depth=2



+2<sup>2</sup>

depth=3



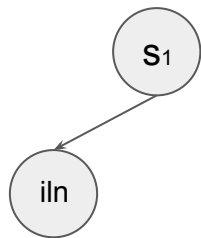
+2<sup>3</sup>

depth=d,  
|A| actions

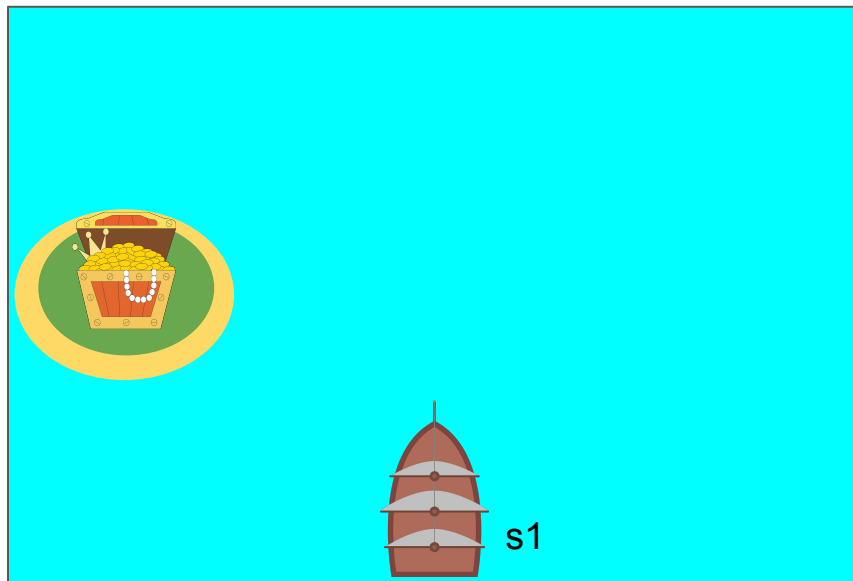
+|A|^d

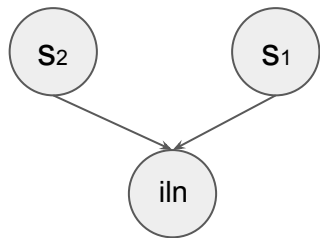
**Third**





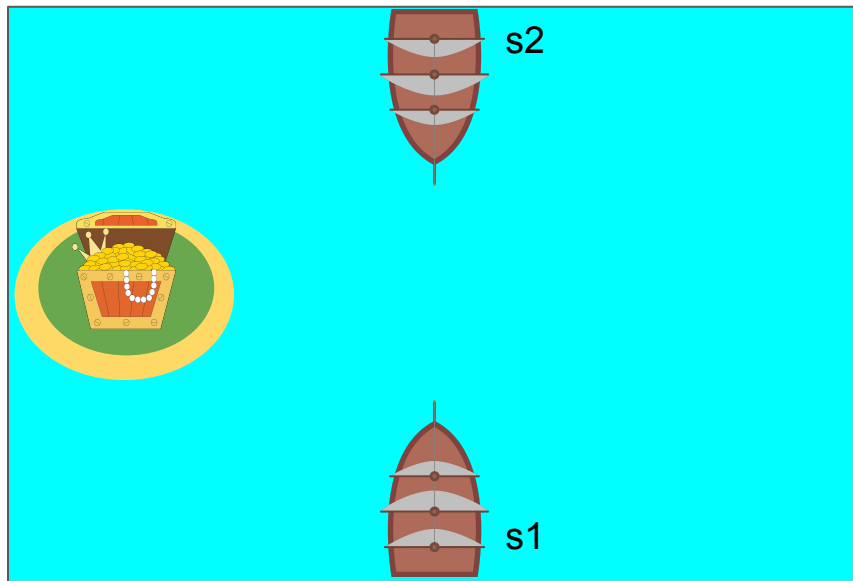
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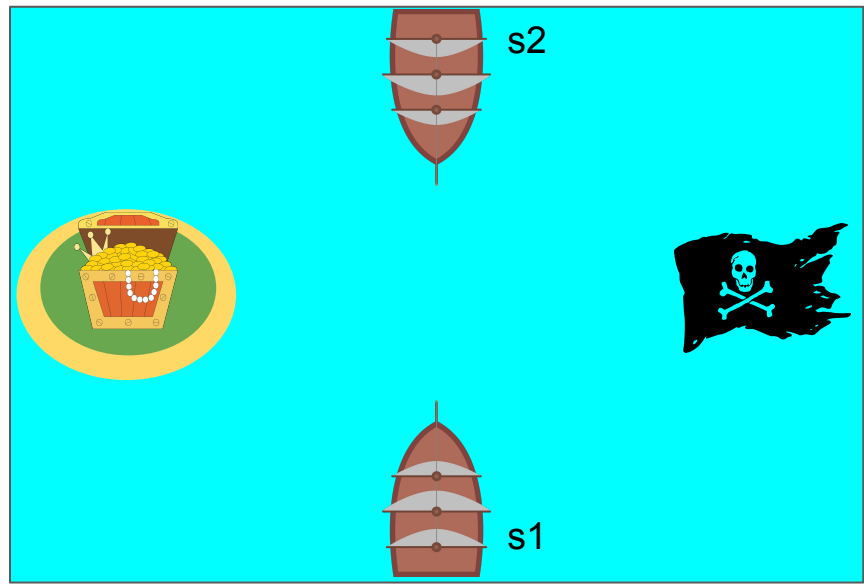
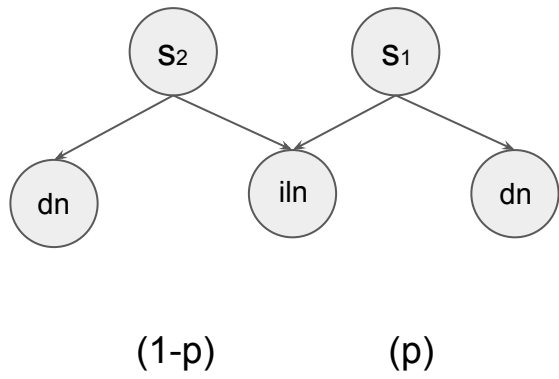


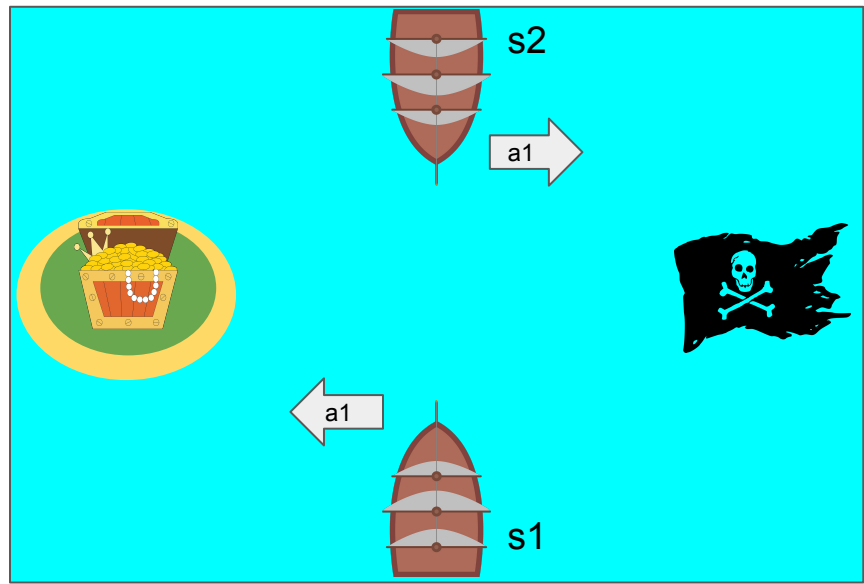
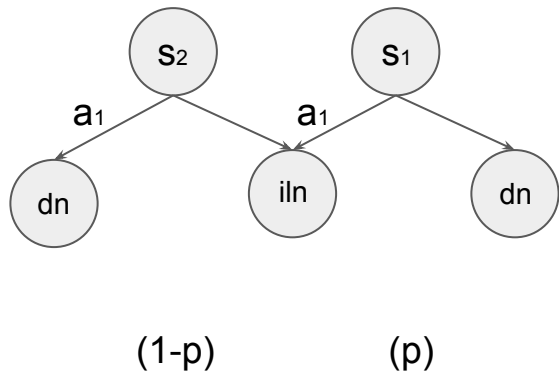


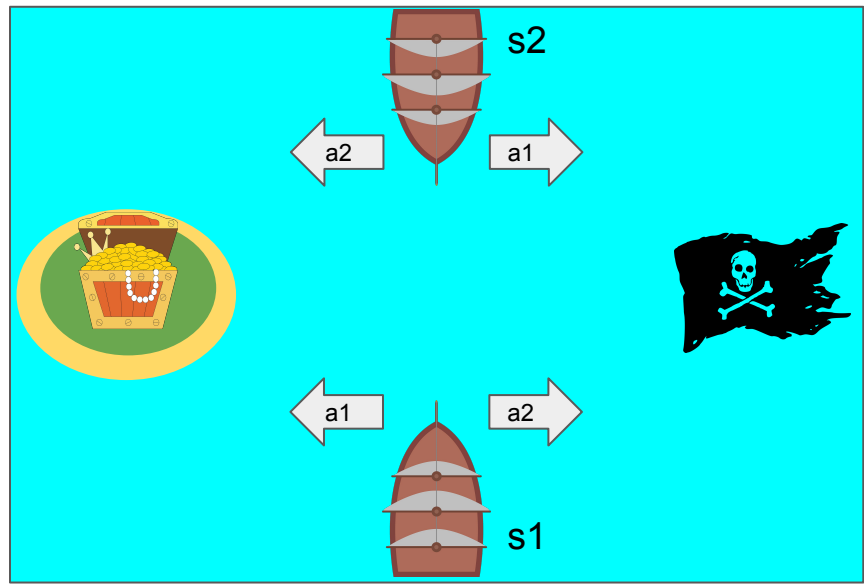
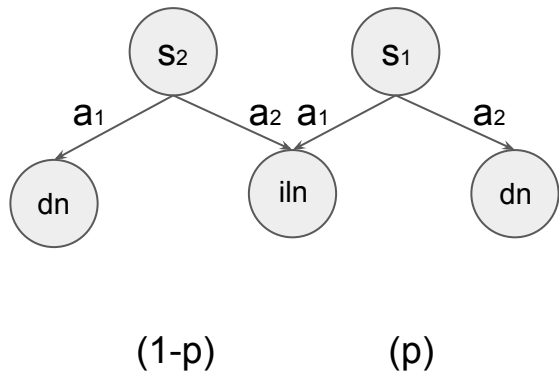
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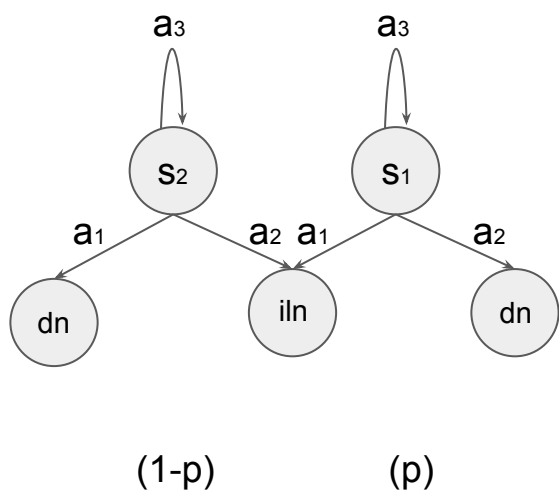
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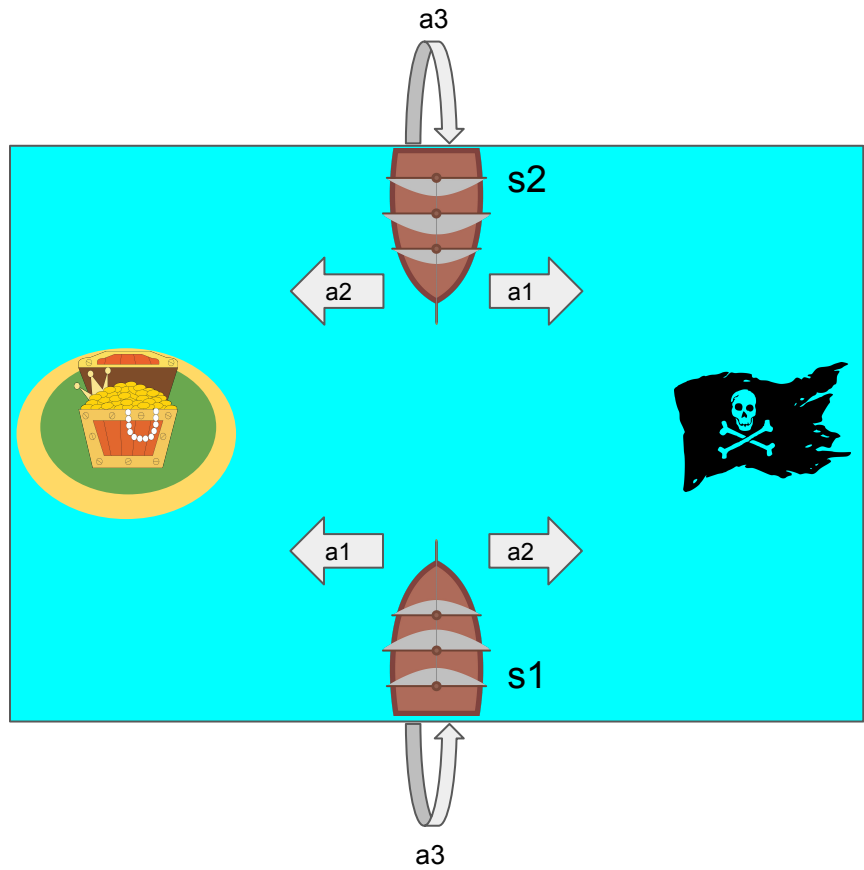


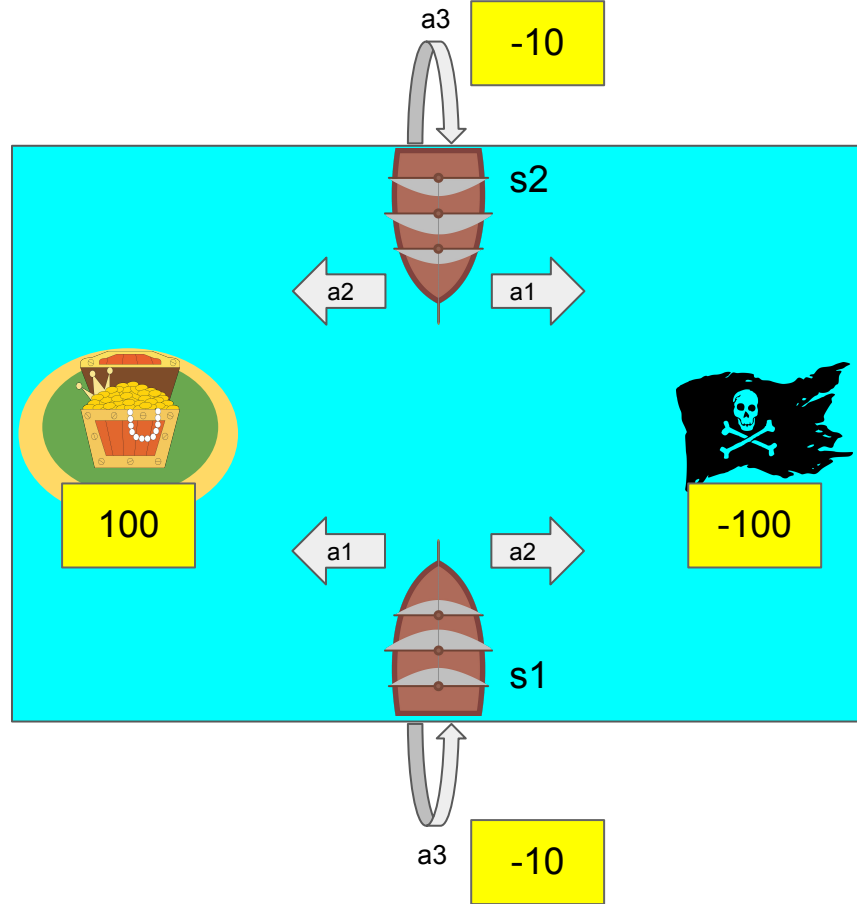
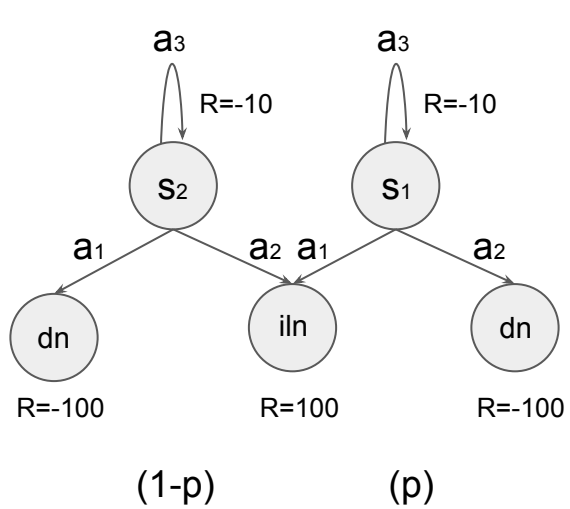


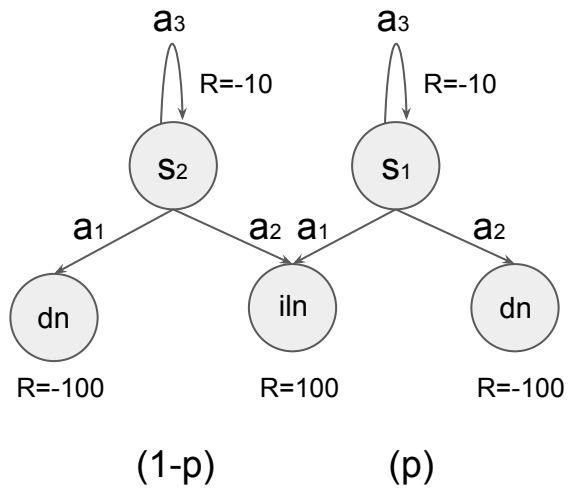




Ask Keeper

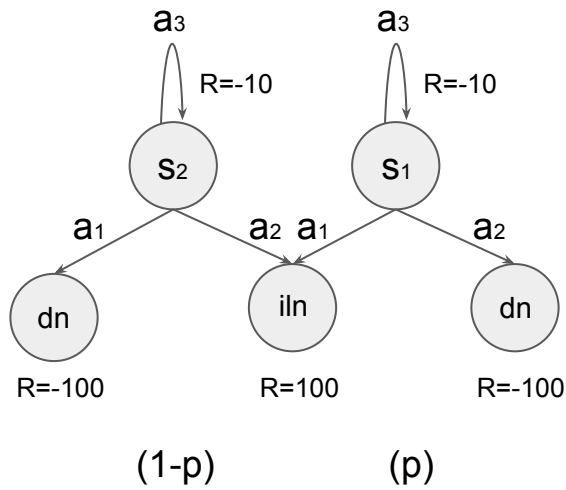






- (a) The value of a one-step plan taken in state  $s$  is simply the reward of taking the action  $a$  in state  $s$ :  $R(s, a)$ . Going left or right are terminal actions while asking the Keeper is non-terminal. Hence, two-step conditional plans can only start with the non-terminal action of asking the Keeper ( $a_3$ ) followed by an observation and ends with taking another action.

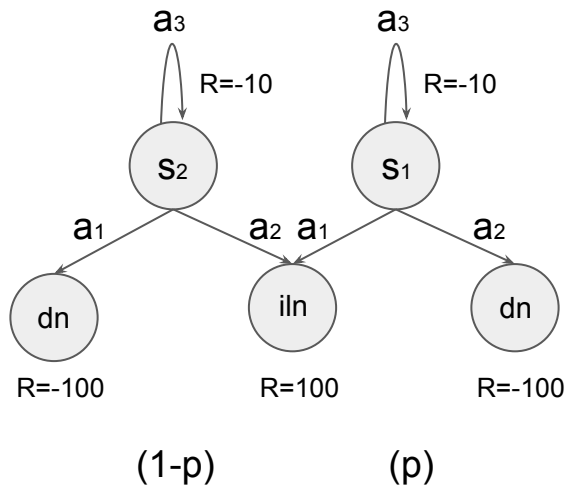




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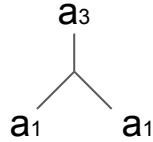
i. How many two-step conditional plans that starts with action  $a_3$  are there?

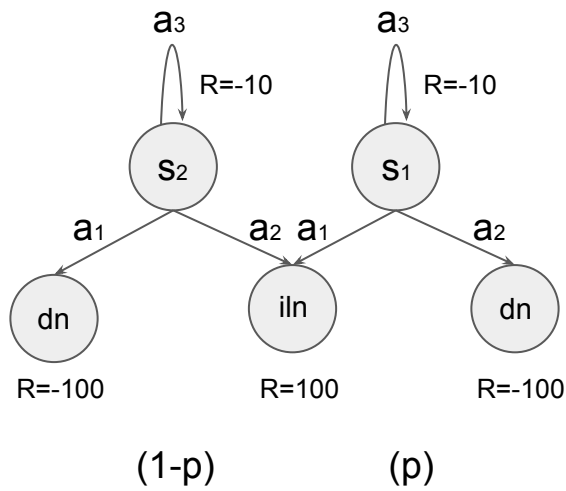
Question



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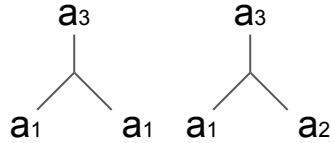
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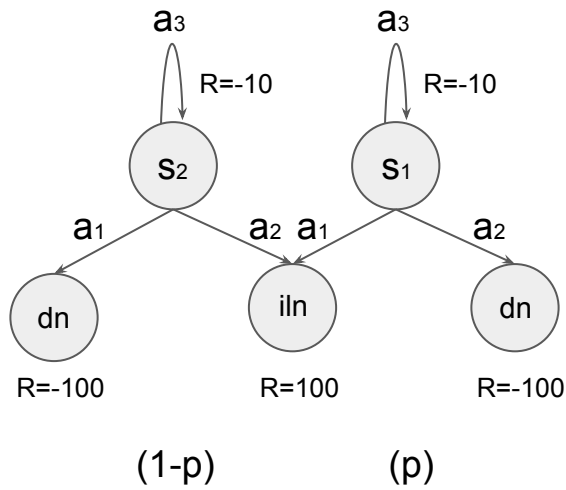




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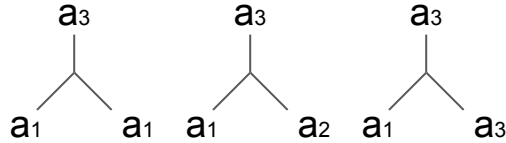
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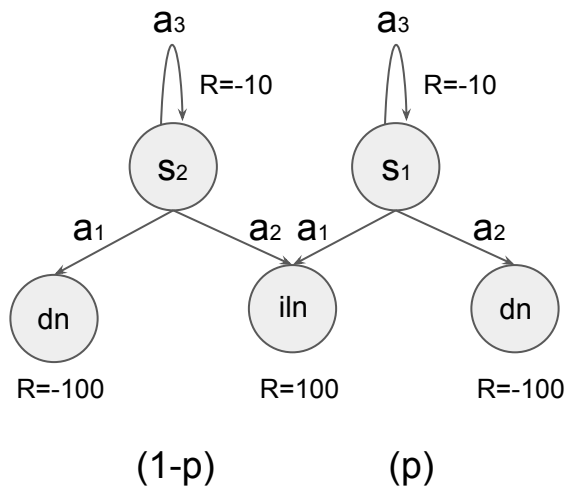




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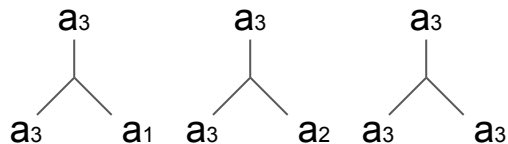
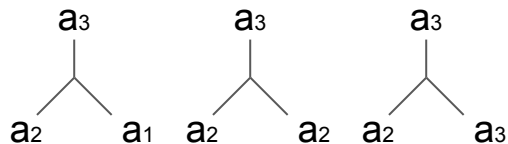
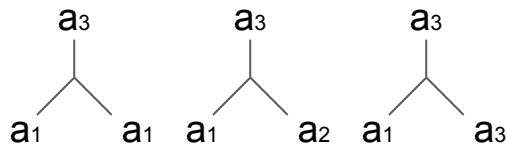
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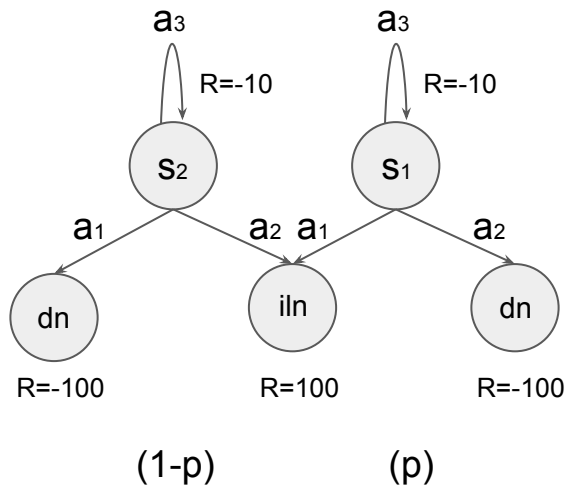




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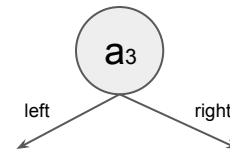
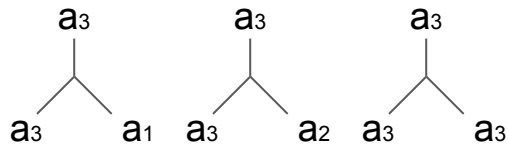
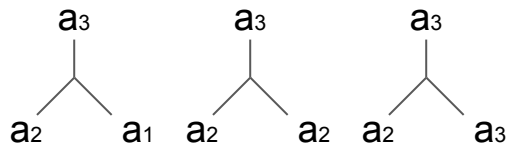
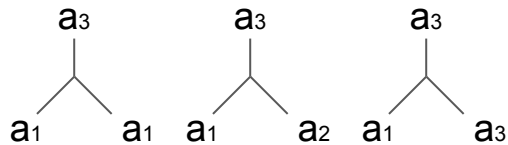
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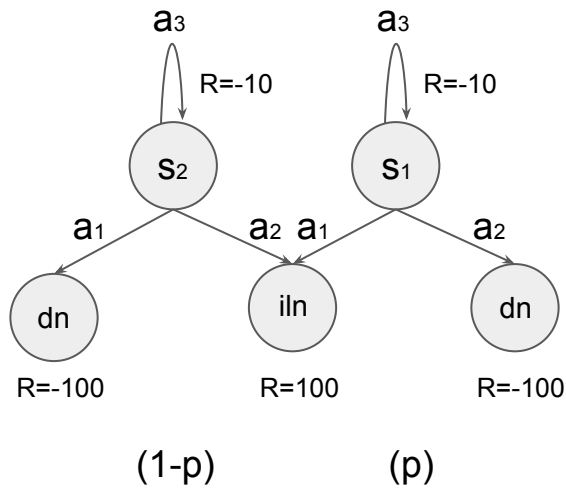




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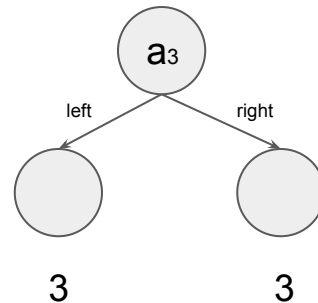
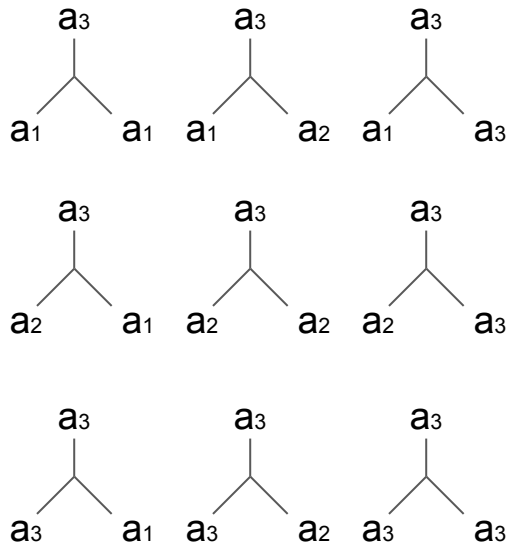
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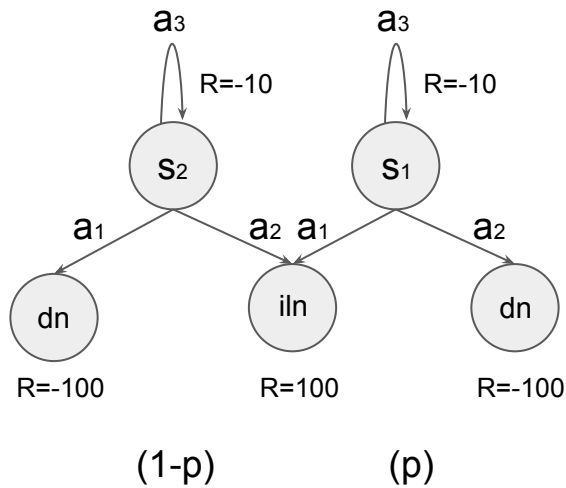




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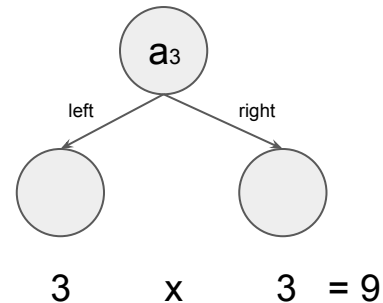
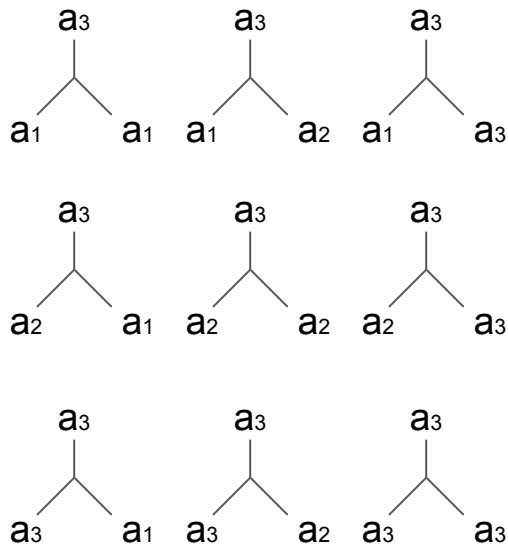
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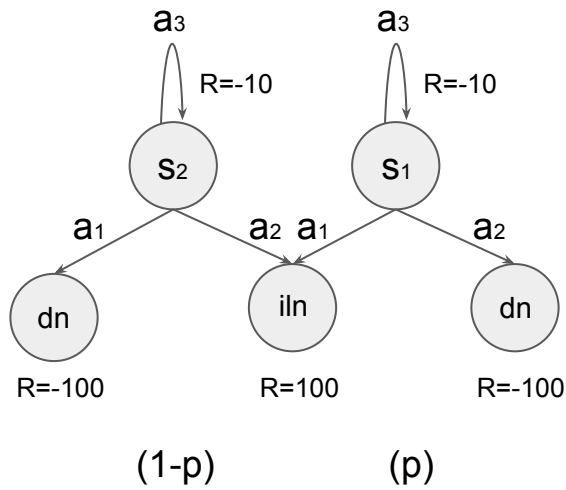


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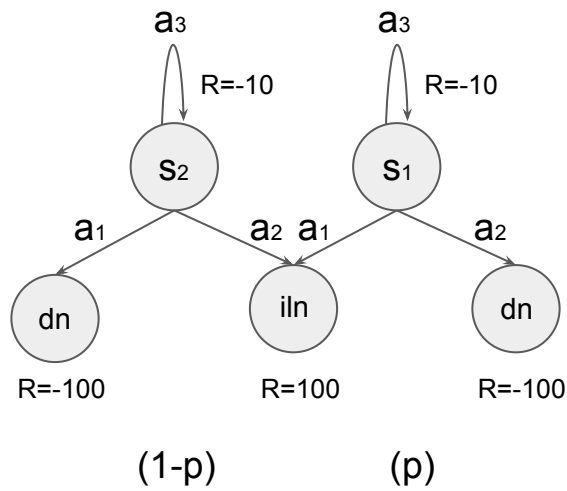






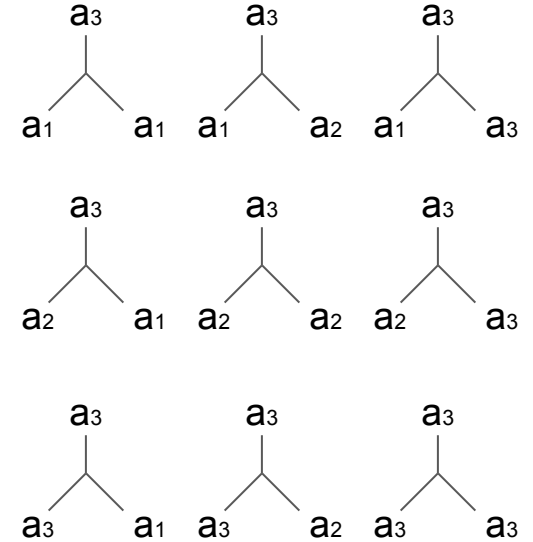
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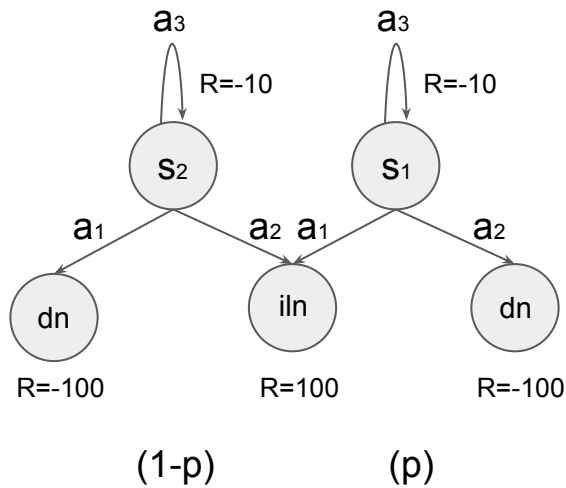


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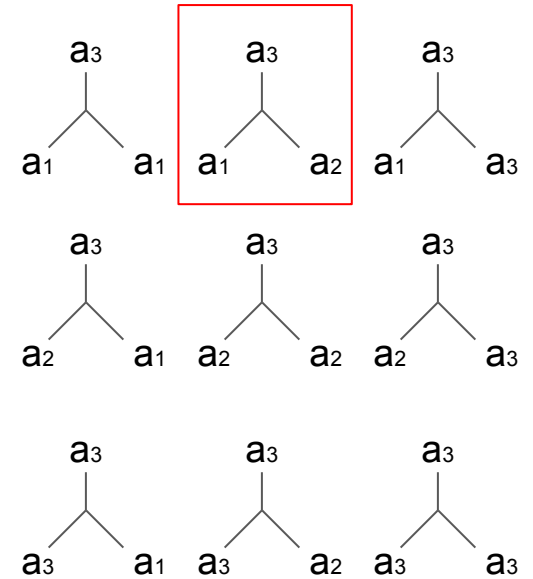


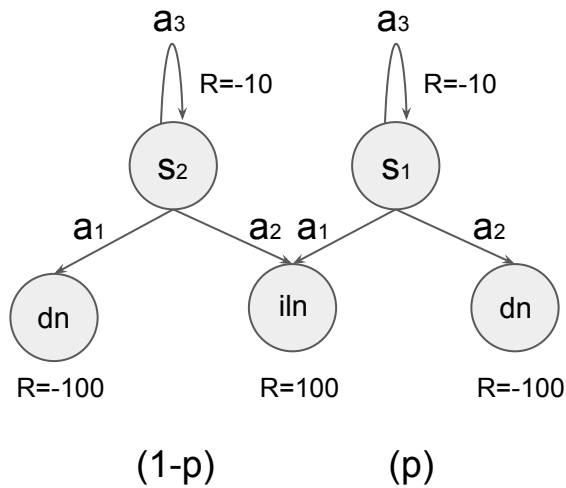
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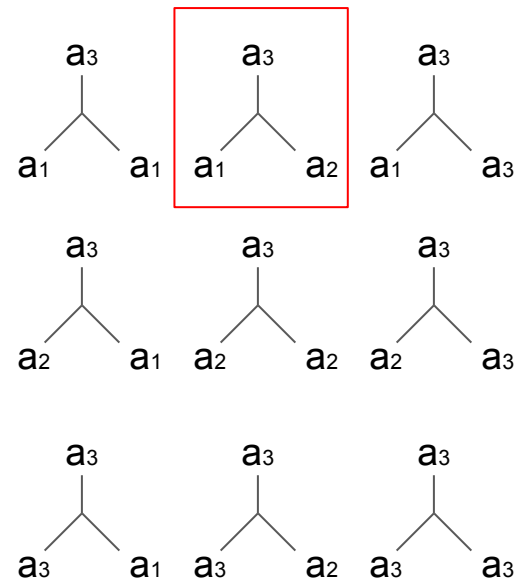
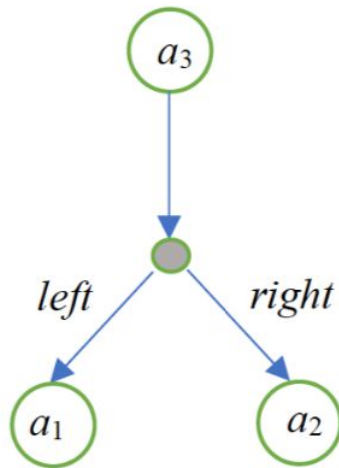
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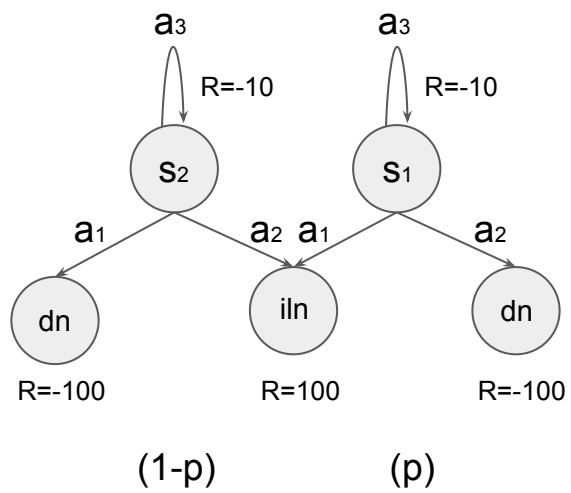


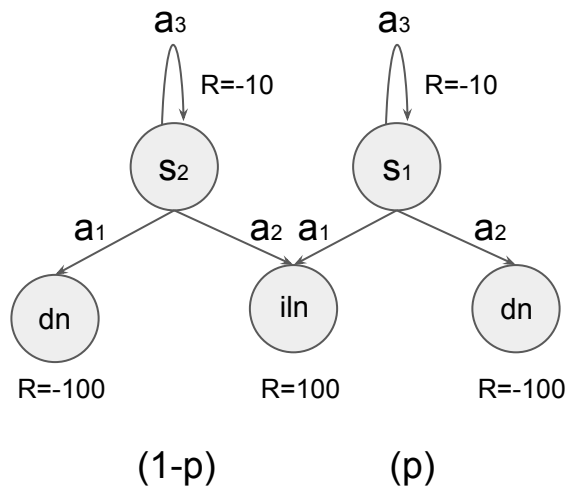


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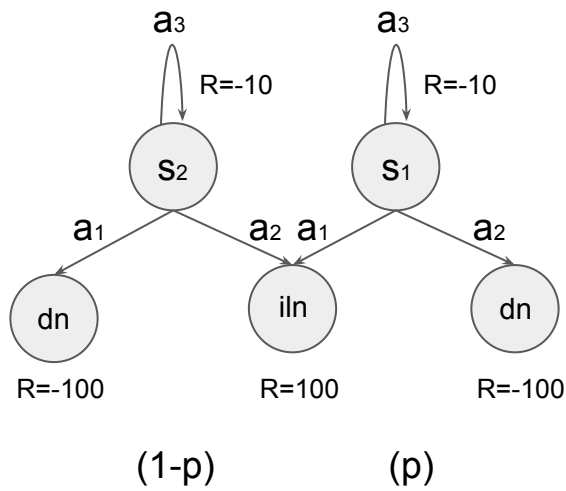
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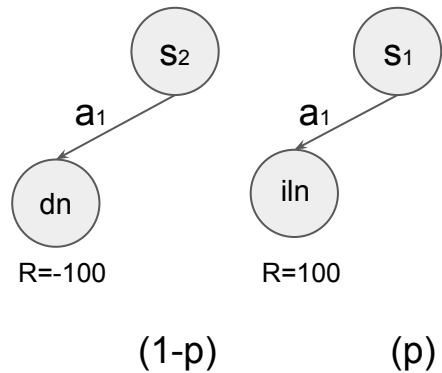
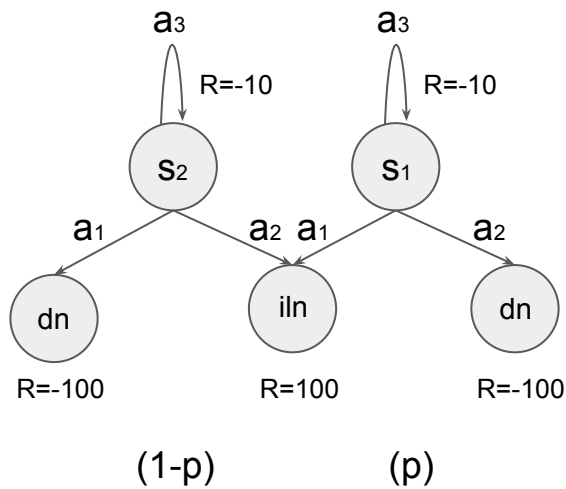
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Question

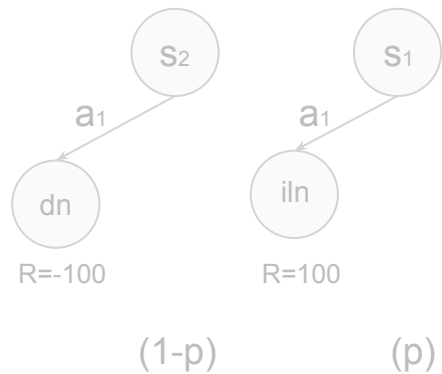
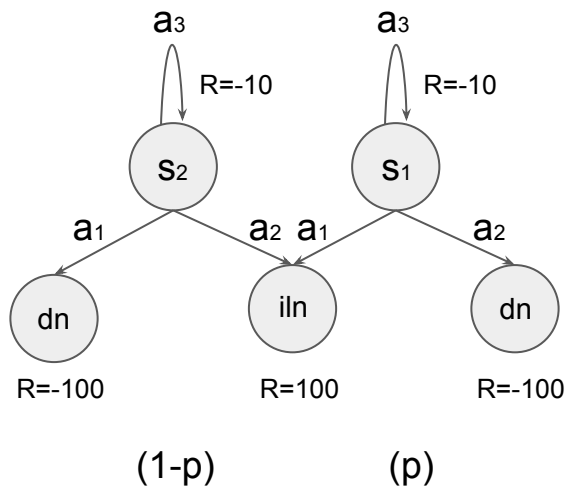


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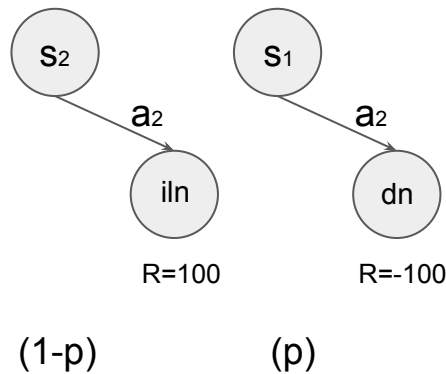




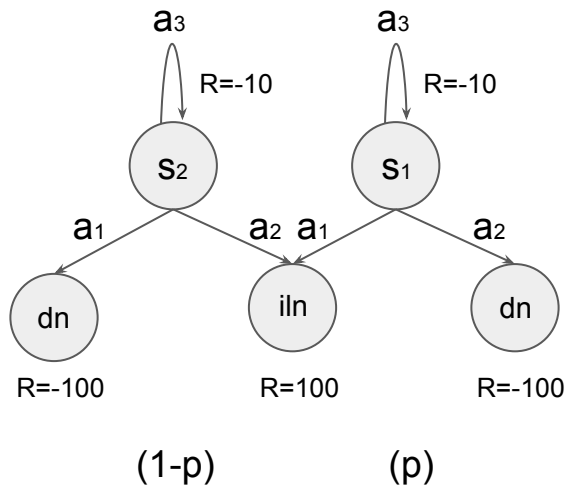
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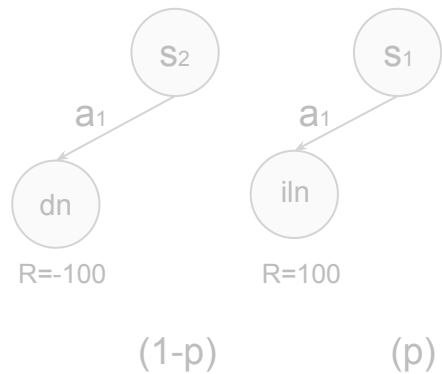


Action right:  $\alpha_r(s_1) = R(s_1, a_2) = -100$   
 $\alpha_l(s_2) = R(s_2, a_2) = 100$

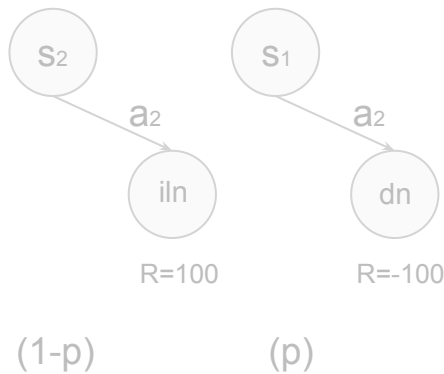


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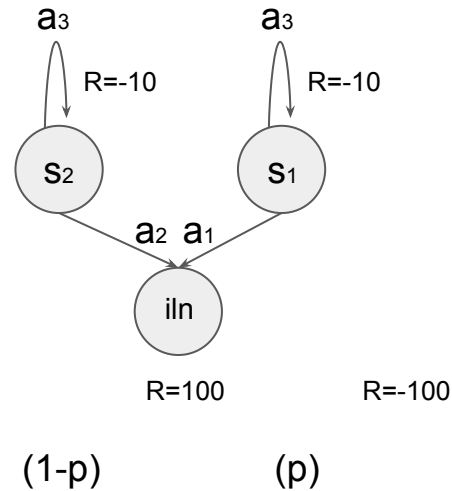
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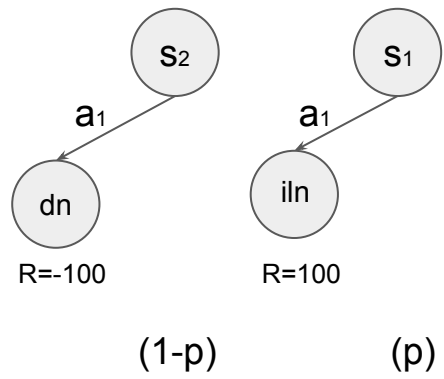
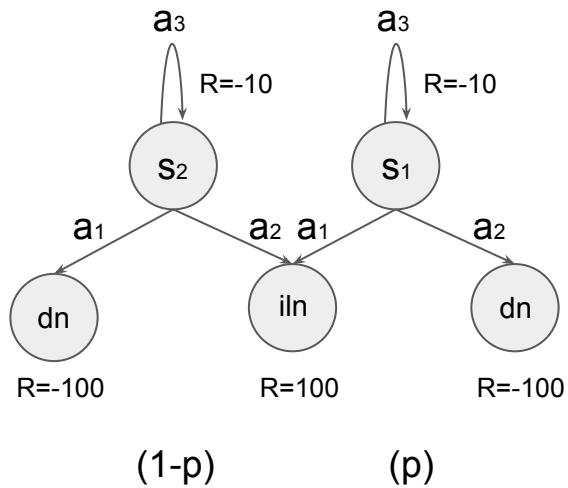
Action left:  $\alpha_l(s_1) = R(s_1, a_1) = 100$ ,  
 $\alpha_l(s_2) = R(s_2, a_1) = -100$



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 $\alpha_l(s_2) = R(s_2, a_2) = 100$



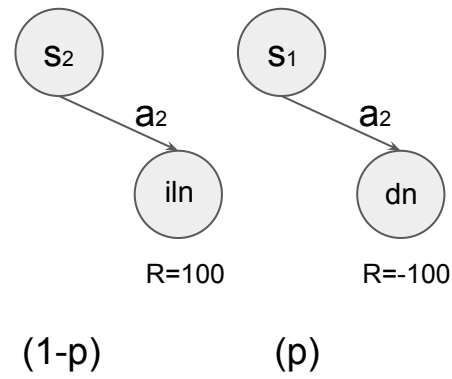
Two-step plan:  
 $\alpha_p(s_1) = \alpha_p(s_2) = -10 + 100 = 90$



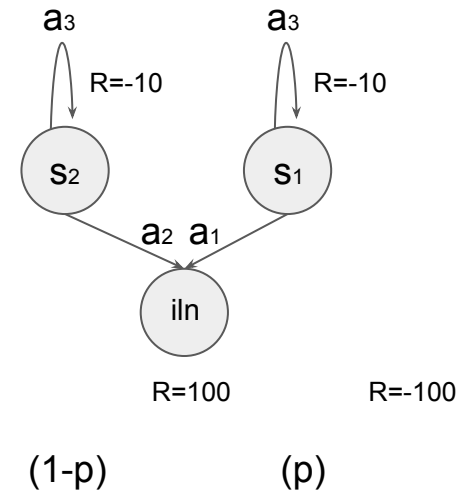
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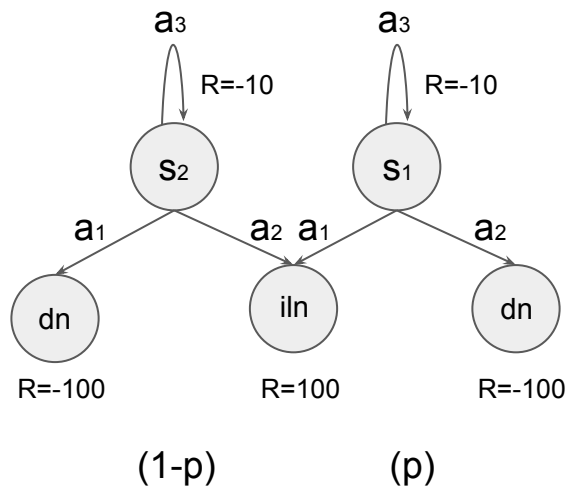
i. Give the three  $\alpha$ -vectors corresponding to the three non-dominated plans. Assume that the discount factor is  $\gamma = 1$  (not discounted).



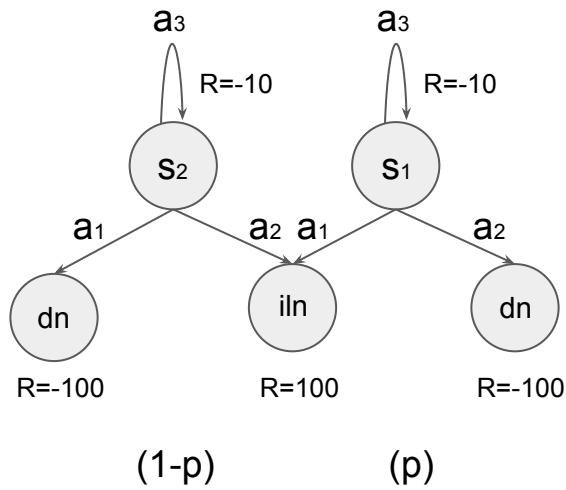
Action right:  $\alpha_r(s_1) = R(s_1, a_2) = -100$   
 $\alpha_l(s_2) = R(s_2, a_2) = 100$



Two-step plan:  
 $\alpha_p(s_1) = \alpha_p(s_2) = -10 + 100 = 90$



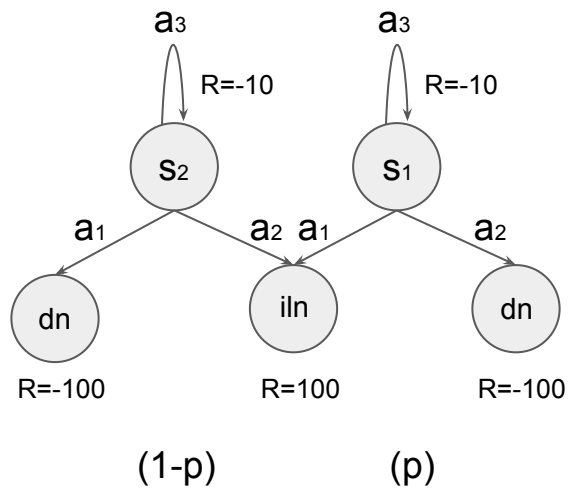
- (b) The one-step plan consisting of asking the Keeper cannot be optimal. Hence there can be at most two non-dominated one-step plans. From part (a) of this question, we know that there is only one non-dominated two-step conditional plan, giving a total of 3 non-dominated one and two step plans.



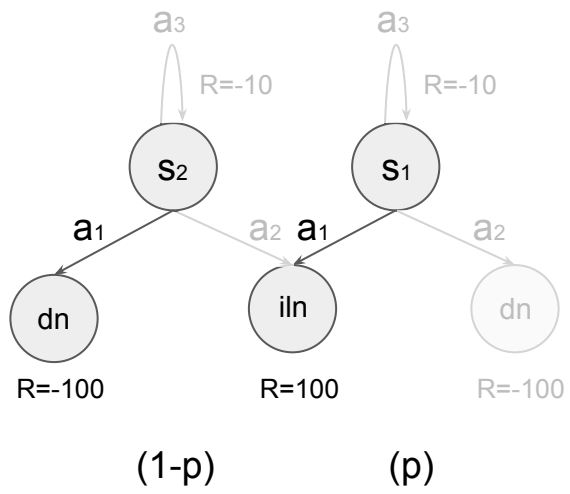
(b) The one-step plan consisting of asking the Keeper cannot be optimal. Hence there can be at most two non-dominated one-step plans. From part (a) of this question, we know that there is only one non-dominated two-step conditional plan, giving a total of 3 non-dominated one and two step plans.

ii. Partition the beliefs into regions where each plan is optimal. Describe the regions.

Question



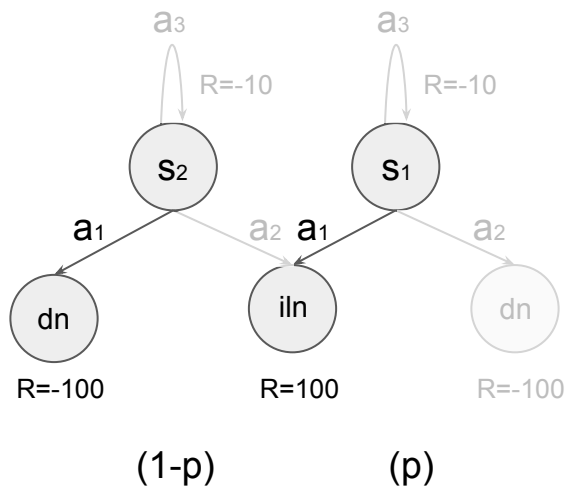
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Left is optimal:

$$E[\alpha_l] \geq E[\alpha_p]$$



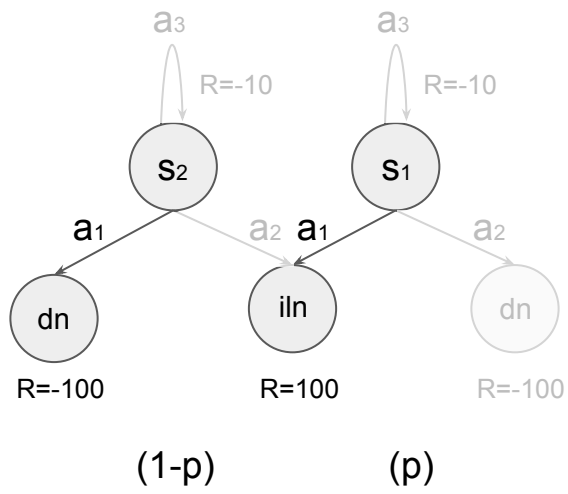
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$$p \times \alpha_l(s_1) + (1 - p) \times \alpha_l(s_2) \geq 90$$





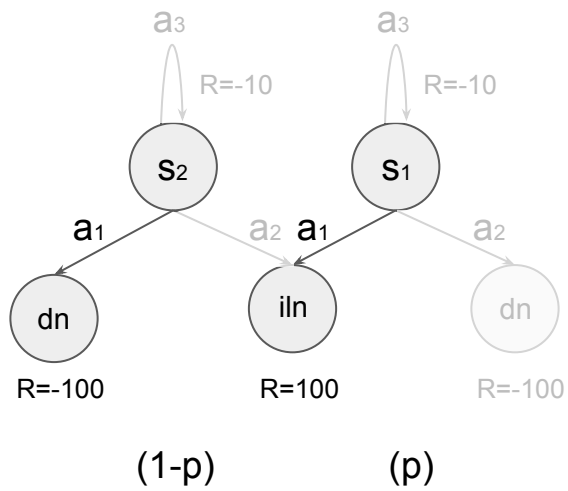
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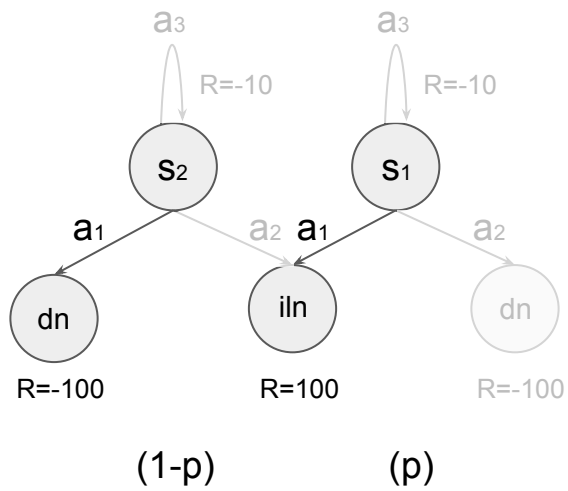
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$$100p - 100 + 100p \geq 90$$



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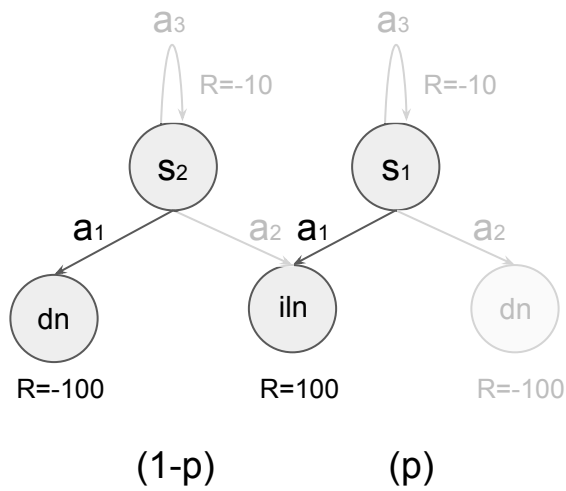
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$$p \times \alpha_l(s_1) + (1 - p) \times \alpha_l(s_2) \geq 90$$

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$$200p \geq 190$$



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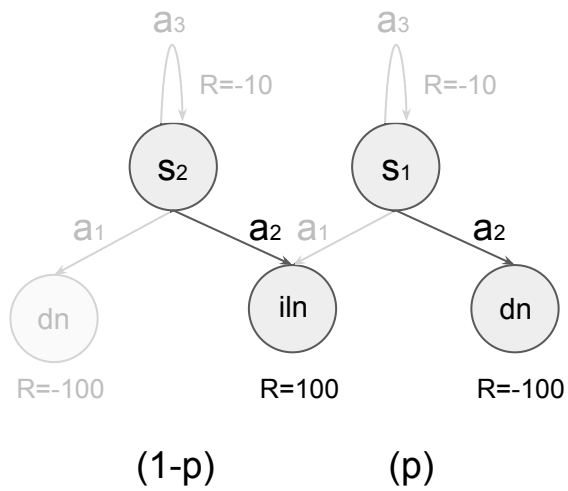
$$p \times \alpha_l(s_1) + (1-p) \times \alpha_l(s_2) \geq 90$$

$$p \times 100 + (1-p) \times -100 \geq 90$$

$$100p - 100 + 100p \geq 90$$

$$200p \geq 190$$

$$p \geq \frac{19}{20}$$

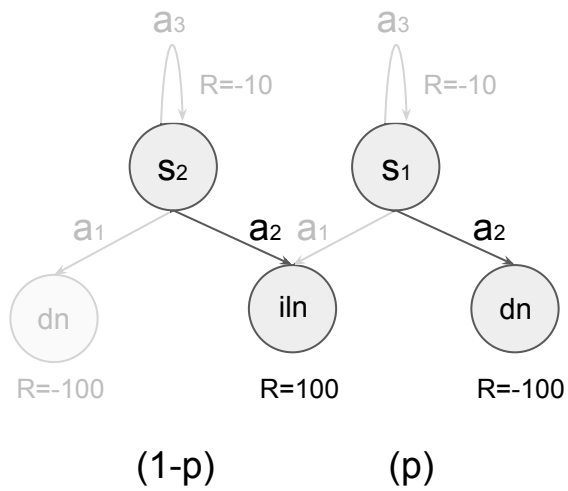


ii. Partition the beliefs into regions where each plan is optimal. Describe the regions.

Left is optimal:

$$\begin{aligned}
 E[\alpha_l] &\geq E[\alpha_p] \\
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 100p - 100 + 100p &\geq 90 \\
 200p &\geq 190 \\
 p &\geq \frac{19}{20}
 \end{aligned}$$

Right is optimal:



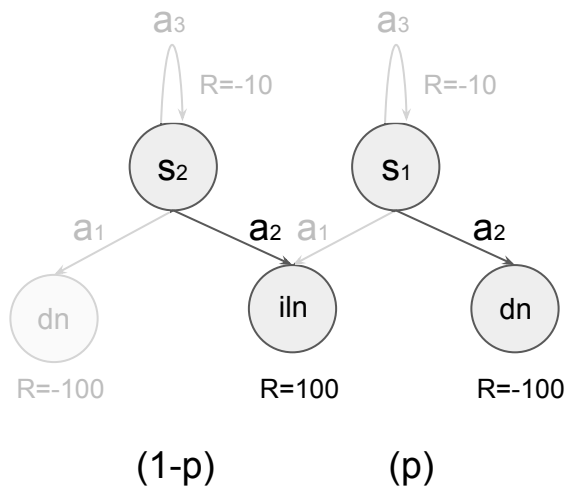
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Right is optimal:

$$E[\alpha_r] \geq E[\alpha_p]$$



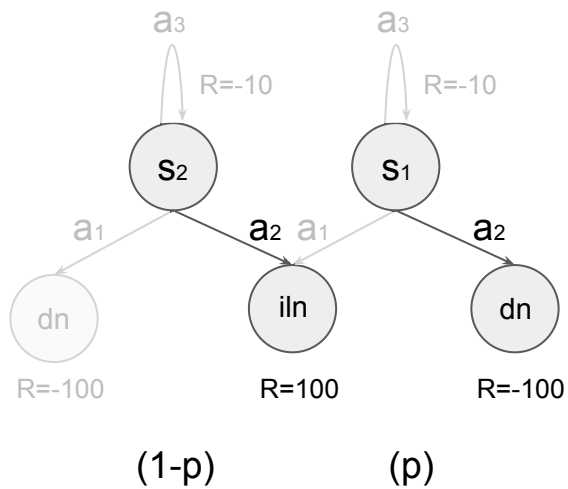
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 \end{aligned}$$

Right is optimal:

$$\begin{aligned}
 E[\alpha_r] &\geq E[\alpha_p] \\
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ii. Partition the beliefs into regions where each plan is optimal. Describe the regions.

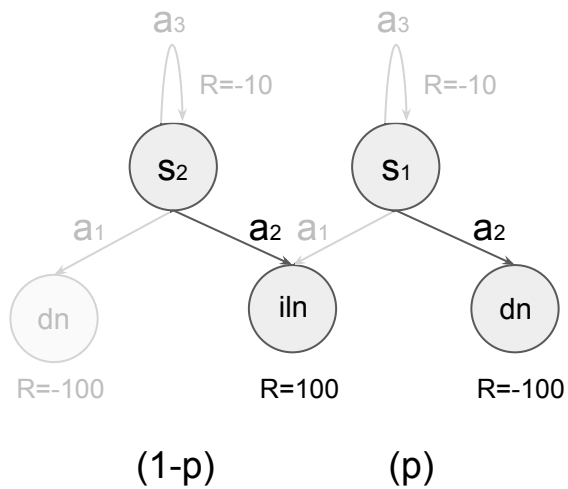
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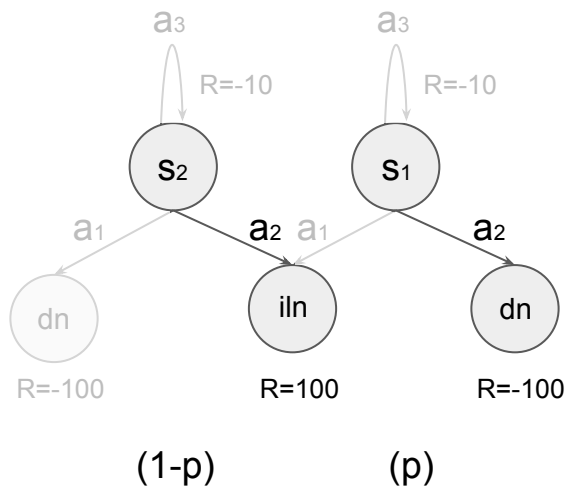
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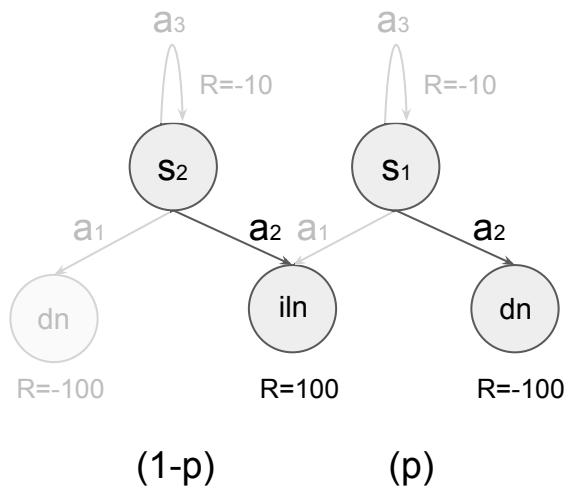
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 -200p &> -10
 \end{aligned}$$



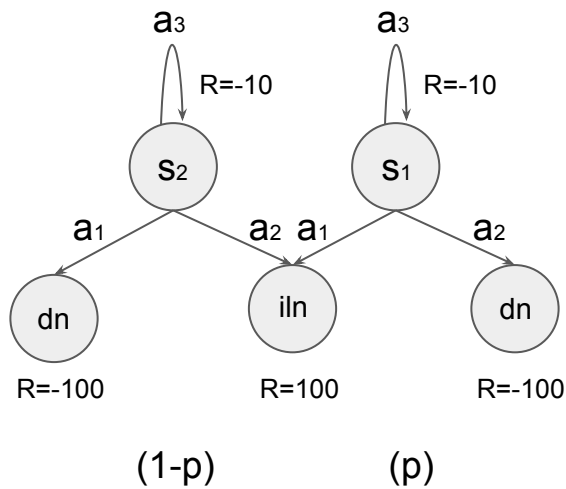
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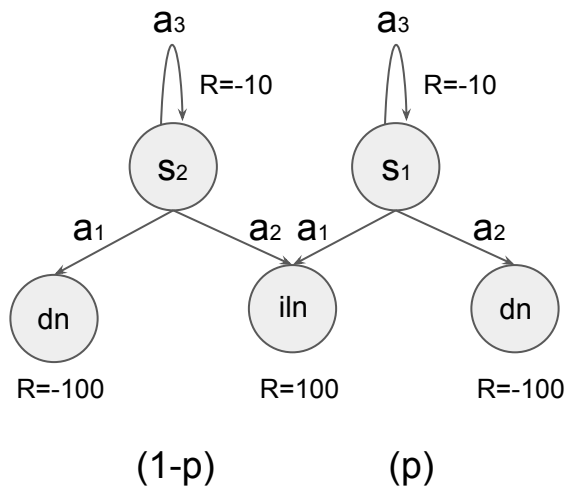
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 \end{aligned}$$

Two-step is optimal:

$$\begin{aligned}
 \frac{1}{20} &\leq p \leq \frac{19}{20} \\
 0.05 &\leq p \leq 0.95
 \end{aligned}$$

Right is optimal:

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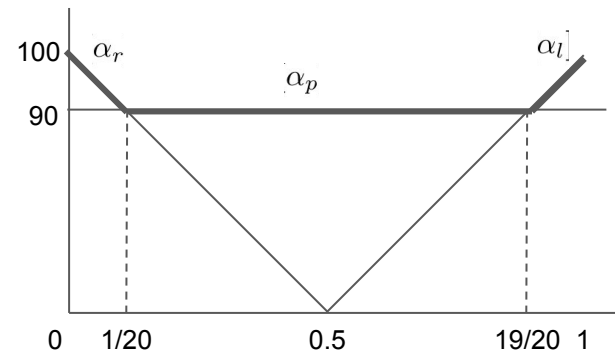
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# Question?

<EOF>

# Credits

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