# CS2102 Tutorial 1: Relational Model    Wk 3, Sem 2, 2020/21

1. **Solution:** In this question (and also Question 2), we are mainly inferring properties of the relation schema from an instance of the schema. Hence the superkeys/candidate keys identified here are generally only possibilities unless we have further information to derive more definite answers.

   (a) Possible superkeys of $R$ are $\{A, C\}$, $\{A, D\}$, $\{A, B, C\}$, $\{A, B, D\}$, $\{A, C, D\}$, and $\{A, B, C, D\}$.

   (b) If $\{A, C\}$ is indeed a superkey of $R$, then $\{A, C\}$ must also be a candidate key of $R$ since neither $\{A\}$ nor $\{C\}$ is a superkey of $R$ (based on $r$). Moreover, any superkey which is a superset of $\{A, C\}$ can't be a candidate key of $R$. The two remaining possible superkeys, $\{A, D\}$ and $\{A, B, D\}$, are possible candidate keys of $R$. Note that at most one of $\{A, D\}$ and $\{A, B, D\}$ could be a candidate key.

2. **Solution:** The possible foreign keys are $W$, $Y$, and $Z$.

3. **Solution:**

   (a) Equivalent.

   (b) Not equivalent. $Q_2$ is an invalid relational algebra expression as the selection condition refers to a non-existent attribute.

   (c) Equivalent.

   (d) Equivalent.

   (e) Equivalent.

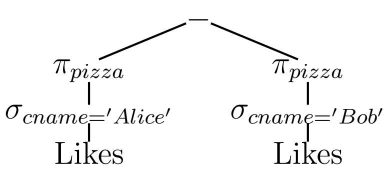
   Note that binary operators in relational algebra are left associative. Hence, in the absence of parenthesis, the expression $R \times S \times T$ is evaluated as $(R \times S) \times T$. As the cross product operator is associative, $(R \times S) \times T$ is equivalent to $R \times (S \times T)$.

   (f) Equivalent.

   (g) Not equivalent. Consider the following database instance $d$: $r = \{(10, 10)\}$ and $s = \{(10, 20)\}$. The result of $Q_1$ on $d$ is $\{(10)\}$ while the result of $Q_2$ on $d$ is $\emptyset$.

4. **Solution:**

   (a)

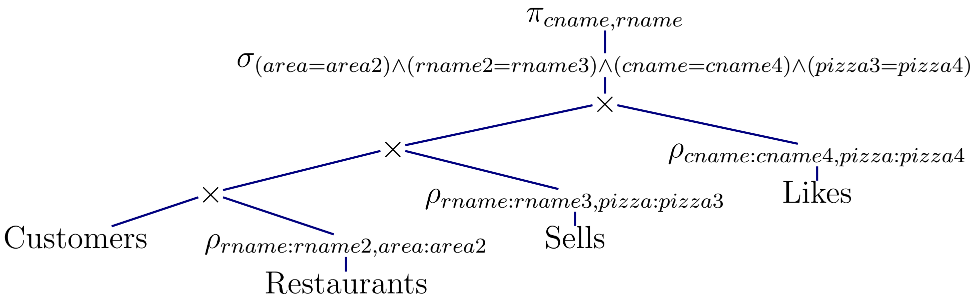



   The following answer is incorrect:

   $\pi_{pizza}(\sigma_{(pizza=pizza2) and (cname='Alice') and (cname2<>'Bob')}(Likes \times \rho_{cname:cname2, pizza:pizza2}(Likes)))$

The above answer will compute exactly the set of pizzas that Alices likes (independent of whether Bob likes them) since for each pizza p that Alice likes, we have $('Alice', p, 'Alice', p) \in Likes \times Likes$.

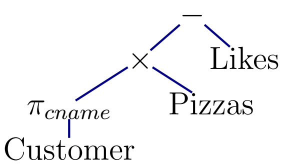
(b)



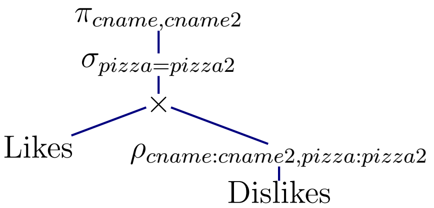

$$\pi_{cname,rname}(Customers \bowtie Restaurants \bowtie Sells \bowtie Likes)$$

$$\pi_{cname,rname}(Customers \bowtie Restaurants) \ \cap \ \pi_{cname,rname}(Sells \bowtie Likes)$$

(c)

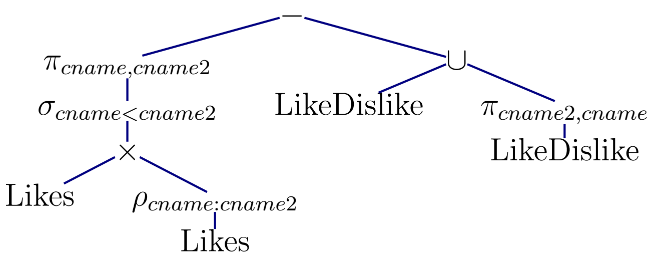



(d) Let `Dislike`(cname, pizza) denote the output computed by part (c).

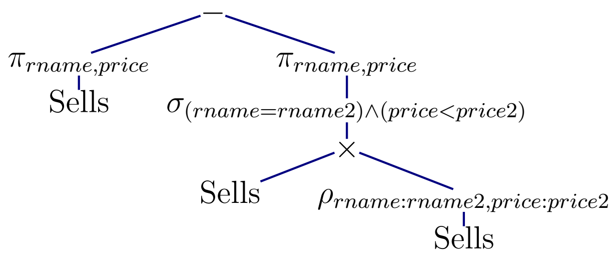



(e) Two customers C and C2 likes exactly the same set of pizzas if there does not exist any pizza that (1) C likes but C2 does not like or (2) C does not like but C2 likes.

Let `LikeDislike`(cname, cname2) denote the output computed by part (d).




(f) The set $\pi_{rname,price}(Sells)$ can be partitioned into two disjoint subsets $\pi_{rname,price}(Sells)$ $= S_1 \cup S_2$ where $S_1$ is the set of restaurant name and price pairs $(r, pr)$ such that $p$ is the price of some pizza sold by $r$ that is not the most expensive pizze sold by $r$, and $S_2$ is the set of restaurant name and price pairs $(r, pr)$ such that $p$ is the price of the most expensive pizza sold by $r$. Thus, the required answer is given by $S_2 = \pi_{rname,price}(Sells) - S_1$. Note that $(r, pr) \in S_1$ if there exists some tuple $(r_2, p_2, pr_2) \in Sells$ where $r_2 = r$ and $pr_2 > pr$.




(g)

$$\pi_{cname,pizza}(Customers \rightarrow (Restaurants \bowtie Sells))$$

The following answer is incorrect:

$$\pi_{cname,pizza}(Customers \rightarrow Restaurants \bowtie Sells)$$

The above answer is equivalent to

$$\pi_{cname,pizza}((Customers \rightarrow Restaurants) \bowtie Sells)$$

However, outer joins are generally not associative with inner joins. In particular,

$$(Customers \rightarrow Restaurants) \bowtie Sells \not\equiv Customers \rightarrow (Restaurants \bowtie Sells)$$

To see this, consider a customer tuple $c \in Customers$ who is not co-located with any restaurant. Therefore, $c$ will appear exactly once in the result of $Customers \rightarrow Restaurants$ with a null value for attributes `rname`. Due to the null value for `rname`, this tuple with $c$ will be a dangling tuple w.r.t. the natural join with $Sells$. Thus, $c$ will not be preserved in the result of $(Customers \rightarrow Restaurants) \bowtie Sells$. In contrast, by performing the outer join after the natural join, $c$ will be preserved in the result of $Customers \rightarrow (Restaurants \bowtie Sells)$.

⚠ You should pay attention to the join ordering when using outer joins!

The following is another incorrect answer:

$$\pi_{cname,pizza}(Customers \leftrightarrow (Restaurants \leftrightarrow Sells))$$

By using full outer joins for both joins, this answer will also preserve dangling tuples from `Sells` and `Restaurants` relations; thus, the query result could have tuples with a null value for `cname`, but such tuples are not required by the question.

To see this, consider a tuple $(r, p, pr) \in Sells$ where the restaurant $r$ is not co-located with any customer. Thus, $(r, p, pr)$ will join with exactly one tuple from `Restaurants` (say $(r, a)$), but the resultant tuple $(r, a, p, pr)$ from the first join computation will not join with any tuple from `Customers`. This will produce $(null, p)$ in the query result, which is incorrect.

Moreover, if there is some tuple $r \in Restaurants$ where $r$ is not co-located with any customer and $r$ does not sell any pizza, then this will produce $(null, null)$ in the query result, which is also incorrect.

⚠ The type of joins matters: you should use appropriate outer joins to preserve the required dangling tuples only when necessary!

5. **Solution:**

- $R_1$ is the set of pizzas that Maggie likes.
- $R_2$ is the set of restaurant-pizza pairs $(r, p)$ where $r$ is a restaurant that sells some pizza and $p$ is some pizza that Maggie likes.
- $R_3$ is the set of restaurants that don't sell some pizza that Maggie likes.
- $R_4$ is the set of restaurants that sell all the pizzas that Maggie likes.
- $R_5$ is the set of pizzas that Ralph likes.
- $R_6$ is the set of restaurants that sell some pizza that Ralph likes.
- $R_7$ is the set of restaurants that sell all the pizzas that Maggie likes and don't sell any pizza that Ralph likes.