CS2040S: Data Structures and Algorithms

Discussion Group Problems for Week 8

For: Oct. 10, 2019

Problem 1. Wizarding School Hijinks

Parry Otter and his friends are up to create more mischief at Warthogs School again! This time, you will be helping them to execute their pranks successfully! Parry Otter and his friends want to create some Lopyjuice Potion. All the ingredients can be requested from their Potions master. However, in order to not raise the suspicion of the potions master, they have to ensure that when they request the ingredients, the potions master does not guess they are making Lopyjuice potion!

Problem 1.a. The potions master will know that the students are making Lopyjuice potion if people request a certain pair of ingredients at the same time (for example if someone requests Soomblang Skin and Vampire Fangs, then the potions master would strongly suspect them for illicit activities). As Parry Otter's friends are very smart, they have figured out every single pair of ingredients which cannot be bought together. To reduce the number of visits to the potions master, they decide that all the ingredients should be bought in at most 2 trips.

Given a list of ingredients as well as a list of pair of ingredients that cannot be bought together, can you determine whether there is a way to buy the ingredients in at most 2 trips so as to do so without arising suspicion?

After successfully making some Lopyjuice potion, Parry Otter needs to find a good delivery method to spread the potion such as to maximise mayhem! He plans to release it via the school's plumbing system.

The plumbing system is made up of many junctions which are interconnected with pipes. Parry Otter has the map of the entire school network, and he wants to flood the entire school through this network. From a release point at junction u, it is only able to flood another junction v if there are a sequence of pipes that go from junction u to junction v.

Problem 1.b. Unfortunately, some of the pipes are faulty. These faulty pipes have been marked out within the plumbing system network Parry Otter has. Since Parry Otter wants to flood the entire school, he may need to release the Lopyjuice potion from more than one location. Given this map, determine the minimum number of release points Parry Otter and his friends need so that they can flood the entire network.

Problem 1.c. Parry Otter also wants to minimise the time it takes for a network to be flooded. Given a single connected pipe network, you wish to determine the best release point such that the entire network is flooded. Assume the following when flooding the network:

- The release point is immediately flooded at $t = 0$

- Given a set of junctions S that are flooded at time t, at time $t+1$, any junction j that is adjacent to some junction i in set S will be flooded as well.

Determine what will be the minimum time to flood the entire network.
Hint: *At a particular release point, how can you calculate how long it will take for the entire network to be flooded?*

Problem 2. 10 Kinds of People

Next, we will attempt the **10 Kinds Of People** problem on Kattis

`https://nus.kattis.com/problems/10kindsofpeople`

Problem 3. Hiking Trails

You are the manager of a popular hiking area, which has a nice, picturesque summits and many trails in between. However, as there are many trails and most hikers have little to no sense of direction, you decide to put in place signs so as to ensure hikers do not get lost.

The park consists of many checkpoints with trails in between that connect them. You may assume that each pair of checkpoints has a single path connecting them. Since this task is something of a large scale, you ask your subordinates to come up with a suitable orientation for each path such that paths in the direction of the arrow lead towards any of the summits and if you traverse it in the opposite manner you will eventually reach back to the entrance of the park.

Problem 3.a. For a start, you want to ensure that hikers following the arrows do not end up travelling in circles. Given an orientation of all the paths in the park, determine whether there exists a directed cycle in the graph.

After many months you want to open up a new hike within your park. It just so happens that on this hike, there are paths between every two checkpoints (without passing through others). As certain paths are narrow, you decide to fix orientations of certain paths within this network (based on some criteria). Now in this mixed graph with unidirectional and bidirectional edges, you wish to find out whether there is a path that can pass through **all** the checkpoints while respecting the directions of the unidirectional paths. In addition, if there are multiple possible paths that achieve this, you want the *lexicographically smallest one*. (Since that is more attractive to tourists and brings in more money)

Problem 3.b. Given a graph with some directed and some undirected edges, how would you go about printing the lexicographically smallest valid path?