CS1231 TUTORIAL 3

1.

p	q	r	P_1	P_2	C_1	Q_1	Q_2	C_2
T	T	T	T	T	T	T	F	F
T	T	F	F	F	F	T	T	T
T	F	T	T	T	T	T	T	F
T	F	F	F	T	F	F	T	T
F	T	T	T	T	T	T	F	T
F	T	F	T	F	F	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

Note that P_1, P_2 are the two premises and C_1 the conclusion for (a). In all cases where P_1 and $P2$ are T, C_1 is also true. Thus the argument form is valid.

Note that Q_1, Q_2 are the two premises and C_2 the conclusion for (b). There is one instance when Q_1 and Q_2 are T, but C_2 is F. Thus the argument form is invalid.

For part (c), as table has 16 rows, I'll list out only the rows in which the premises are all T. The symbol $*$ means can be either true or false. The table shows that the form is valid.

p	q	r	s	$p \vee q$	$p \to r$	$q \to s$	$r \vee s$
T	T	T	T	T	T	T	T
T	F	T	$*$	T	T	T	T
F	T	$*$	T	T	T	T	T

2.

1.	$p \to t$	
2.	$\neg t$	
3.	$\therefore \neg p$	From 1, 2 (modus tollens)
4.	$\therefore \neg p \vee q$	From 3(generalization)
5.	$\neg p \vee q \to r$	
6.	$\therefore r$	From 4,5 (modus ponens)
7.	$\therefore \neg p \wedge r$	From 3, 6(conjunction)
8.	$\neg p \wedge r \to \neg s$	
9.	$\therefore \neg s$	From 7, 8 (modus ponens)
10.	$s \vee \neg q$	
11.	$\therefore \neg q$	From 9, 10 (elimination)

3. As in T1, we have(i) $\neg K \to H$, (ii) $R \to \neg V$ and $\neg R \to V$, (iii) $A \to R$, (iv) $V \leftrightarrow K$, (v) $H \to A \wedge K$. We conclude that only V and K are chatting as shown below.

1.	$\neg K \to H$	i
2.	$H \to A \wedge K$	v
3.	$\therefore \neg K \to A \wedge K$	From 1, 2 (transitivity)
4.	$\therefore K \vee (A \wedge K)$	From 3
	$\equiv (K \wedge K) \vee (K \wedge A) \equiv K$	
5.	$K \to V$	iv
6.	$\therefore V$	From 4,5 (modus ponens)
7.	$R \to \neg V$	ii
8.	$\therefore \neg R$	From 6, 7(modus tollens)
9.	$A \to R$	iii
10.	$\therefore \neg A$	From 8, 9 (modus tollens)
11.	$\therefore \neg(A \wedge K)$	From 10
12.	$\therefore \neg H$	From 2, 11 (modus tollens)

4. Hypotheses :

(i) $a \wedge w \to p$, (ii) $\neg a \to i$, (iii) $\neg w \to m$, (iv) $\neg p$, (v) $e \to (\neg i \wedge \neg m) \equiv e \to \neg(i \vee m)$

Proof:

1. $\neg p$, (iv)
2. $a \wedge w \to p$, (i)
3. $\neg(a \wedge w) \equiv \neg a \vee \neg w$, from 1,2 (modus tollen)
4. $\neg a \to i$, (ii)
5. $\neg w \to m$, (iii)
6. $i \vee m$, from 3, 4, 5. (modus ponen)
7. $e \to \neg(i \vee m)$, (iv)
8. $\neg e$, from 6, 7 (modus tollen)