CS1231 Review 21

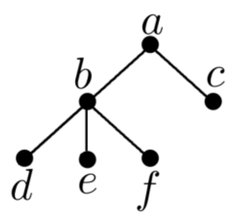
1. A **tree** is a <u>connected</u> (undirected) graph with no <u>cycles</u>.

2. A graph is a FOREST if its connected components are trees. no cycles

3. A graph is a tree iff there is a <u>unique simple</u> path between any two of its vertices.

4. A ROOTED TREE is a tree in which one vertex has been designated as the ROOT and all edges are directed away from the root.

5. In the following tree



- <u>a</u> is the root.
- <u>c d e f</u> are leaves.
- <u>a b</u> are internal vertices.
- <u>a</u> is the parent of b.
- <u>d e f</u> are children of b.
- <u>b a</u> are ancestors of e.
- <u>b c d e f</u> are descendants of a.

6. Suppose T is a rooted tree with root a. If $\overline{uv}$ is an edge, then u is the <u>parent</u> of v and

   u v

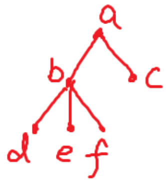
   v is a <u>child</u> of u.

   Vertices with the same parent are called <u>siblings</u>.

   If $u \neq v$, and there is a simple path from u to v (with u above v), then u is an <u>ancestor</u> of v and v a <u>descendant</u> of u.

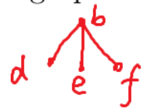
   u v

A vertex if called a leaf if it has no children.



A vertex that is not a leaf is called an internal vertex.

7. If u is a vertex of T, the SUBTREE rooted at u is the subgraph consisting of u and all its descendants



8. A rooted tree is called an m**-ary tree** if each of its internal vertices has $\leq m$ children.

An m-ary is **full** if every internal vertex has exactly m children.

When $m=2$, such a tree is called a **binary tree**.

9. Let v be a vertex of degree 1 in a tree T with $n \geq 2$ vertices. Then $T - v$ is also a tree.

Hand-shaking $\sum_{v \in G} \deg(v) = 2|E|$

10. A tree with n vertices has $n-1$ edges.

11. A full m-ary tree with i internal vertices contains $n = mi+1$ vertices.

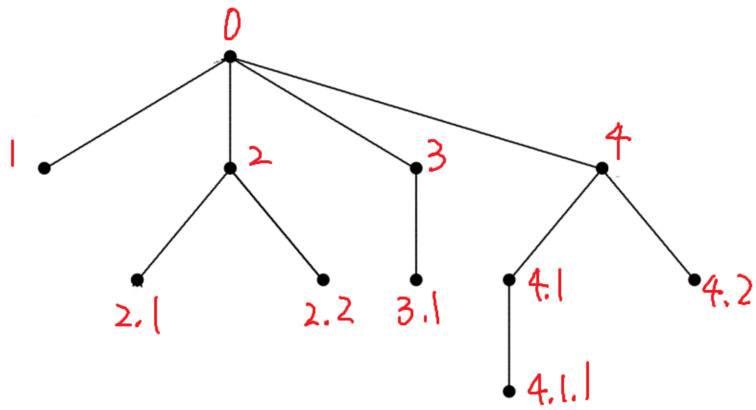
12. A tree with i internal vertices and ℓ leaves contains $n = i + \ell$ vertices.

13. Let T be a tree rooted at a. (level 0) The level of a vertex v is the length of the simple path from a to v. The level of a is defined to be 0. The height of T is the maximum of the levels of vertices. A rooted m-ary tree of height h is **balanced** if all leaves are at level h or $h-1$

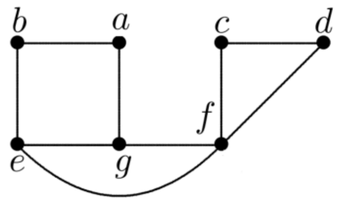
14. In an m-ary tree there are at most m^k vertices at level k. If the height is h, there are at most m^h leaves.

15. If an m-ary tree of height h has ℓ leaves, then $h \geq \lceil \log_m \ell \rceil$. If the tree is full and balanced, then $h = \lceil \log_m \ell \rceil$.

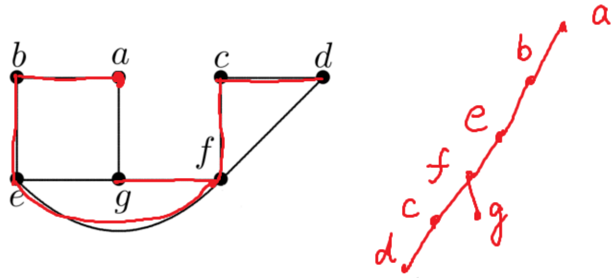
16. Universal Address System



17. Spanning Tree



DEPTH FIRST SEARCH (Alphabetical Ordering)



BREADTH FIRST SEARCH (Alphabetical Ordering)