**NATIONAL UNIVERSITY OF SINGAPORE**

**Department of Statistics and Applied Probability**

(2018/19) Semester 1 ST2334 Probability and Statistics Tutorial 5

1. The random variable , representing the number of errors per 100 lines of software code, has the following probability distribution:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 2 | 3 | 4 | 5 | 6 |
|  | 0.01 | 0.25 | 0.40 | 0.30 | 0.04 |

1. Find the first and second moment of .
2. Find the variance of using (i) the definition of variance and (ii) .
3. Find the mean and variance of the discrete variable .
4. Find the probability function of the random variable . Hence find the mean and variance of from the probability function.
5. Suppose that . Find the mean and variance of in terms of and .
6. Suppose that a grocery store purchases 5 cartons of skim milk at the wholesale price of $1.20 per carton and retails the milk at $1.65 per carton. After the expiration date, the unsold milk is removed from the shelf and the grocer receives a credit from the distributor equal to three-fourths of the wholesale price. Find the expected profit if the probability distribution of the random variable , the number of cartons that are sold from this lot is

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 |
|  | 1/15 | 2/15 | 2/15 | 3/15 | 4/15 | 3/15 |

1. Suppose that 3 fair dice are rolled. Let be the minimum of 3 numbers rolled. Find .

[Hint: Find for all since ]

1. On a laboratory assignment, if the equipment is working, the density function of the observed outcome is , where *X* has the density function
   1. Find the mean and variance of the random variable .
   2. Find the mean and variance of the random variable .
2. The probability density function of random variable is of the form

If , find and .

1. If a random variable is defined such that , , find and .
2. A random variable has a mean and a variance . Using Chebyshev’s theorem, find the bounds for:
3. The value of the constant *c* such that .
4. An electrical firm manufactures a 100-watt light bulb, which, according to specifications written on the package, has a mean life of 900 hours with a standard deviation of 50 hours. At most, what percentage of the bulbs fails to last even 700 hours? Assume that the distribution is symmetric about the mean.
5. Compute , where has the density function:

Compare it with the result given in Chebyshev’s theorem.

1. Determine the value of *c* so that the function represents a joint probability distribution of the random variables *X* and *Y*: , for

**Answers to selected problems**

1. (a) 4.11, 17.63

(b) 0.7379

(c) 10.33, 6.6411

(d)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 4 | 7 | 10 | 13 | 16 |
|  | 0.01 | 0.25 | 0.40 | 0.30 | 0.04 |

(e)

2. Profit =   
(Profit) = $0.80

3.

4. (a) 1/3, 1/18

(b) 1, 1/2

5. (a) ,

6.

7. (a) ,

(b) ,

(c) ,

(d) ,

(e) ,

8. 0.03125

9. 0.9839, at least 0.75

10. 1/15