**NATIONAL UNIVERSITY OF SINGAPORE**

**Department of Statistics and Applied Probability**

(2018/19) Semester 1 ST2334 Probability and Statistics Tutorial 6

1. Let denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let denote the number of times a technician is called on an emergency call. Their joint probability distribution is given below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | |  | | |
| 1 | 2 | 3 |
|  | 1 | 0.05 | 0.05 | 0.1 |
| 2 | 0.05 | 0.10 | 0.35 |
| 3 | 0 | 0.2 | 0.1 |

1. Evaluate the marginal distribution of .
2. Evaluate the marginal distribution of .
3. Find
4. Find the conditional distribution of given .
5. Determine whether and are dependent or independent.
6. From a sack of fruit containing 3 oranges, 2 apples, and 3 bananas, a random sample of 4 pieces of fruit is selected. If is the number of oranges and is the number of apples in the sample, find
7. The joint probability distribution of and .
8. .
9. .
10. .
11. and hence .
12. Consider an experiment that consists of two rolls of a balanced die. If is the number of 4s and is the number of 5s obtained in the two rolls of the die, find
    1. The joint probability distribution of and .
    2. .
    3. Determine whether and are dependent or independent.
13. Each rear tire on an experimental airplane is supposed to be filled to a pressure of 40 pound per square inch (psi). Let denote the actual air pressure (in 10 pound per square inch) for the right tire and denote the actual air pressure (in 10 pound per square inch) for the left tire. Suppose that and are random variables with the joint density
    1. Find *.*
    2. Find .
    3. Find and hence .
14. A candy company distributes boxes of chocolates with a mixture of creams, toffees, and cordials. Suppose that the weight of each box is 1 kilogram, but the individual weights of the creams, toffees, and cordials vary from box to box. For a randomly selected box, let and represent the weights of the creams and the toffees, respectively, and suppose that the joint density function of these variables is

Find the probability that in a given box the cordials account for more than of the weight.

* 1. Find the marginal density for the weight of the creams and the marginal density for the weight of the toffees.
  2. Determine if and are independent.
  3. Find the probability that the weight of the toffees in a box is less than of a kilogram if it is known that creams constitute of the weight.

1. The amount of kerosene, in thousands of liters, in a tank at the beginning of any day is a random amount from which a random amount is sold during that day. Suppose that the tank is not resupplied during the day so that , and assume that the joint density function of these variables is
   1. Determine if and are independent.
   2. Find .
   3. Find the average waiting time between successive speeders.

**Answers to selected problems**

1. (a) (b)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 |  |  |  |  |  |  |  |  |  |  |  | 1 | 2 | 3 |
|  | 0.10 | 0.35 | 0.55 |  |  |  |  |  |  |  |  |  |  |  | 0.20 | 0.50 | 0.30 |

(c) 4/7

|  |  |  |  |
| --- | --- | --- | --- |
|  | 1 | 2 | 3 |
|  | 1/7 | 2/7 | 4/7 |

(d)

(e) and are dependent

2. (a)

(b) 0.2571 (c) 0.5

(d)

(e) , 0.3

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | |  | | |
| 0 | 1 | 2 |
|  | 0 | 4/9 | 2/9 | 1/36 |
| 1 | 2/9 | 1/18 | 0 |
| 2 | 1/36 | 0 | 0 |

3. (a) (b) 11/12

(c) and are dependent

4. (a) 3/392 (b) 0.25

(c) 0.2328

5. (a) , for ; , for ;

(b) and are not independent (c) 1/4

6. (a) , for and , for , and are not independent (b) 1/3