**NATIONAL UNIVERSITY OF SINGAPORE**

**Department of Statistics and Applied Probability**

(2018/19) Semester 1 ST2334 Probability and Statistics Tutorial 7

1. Suppose that and are random variables having the joint probability distribution below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
| 2 | 4 |
|  | 1 | 0.10 | 0.15 |
| 3 | 0.20 | 0.30 |
| 5 | 0.10 | 0.15 |

* 1. Determine whether or not and are independent.
  2. Find
  3. Find
  4. Find
  5. Find
  6. Find and
  7. Find and

1. Consider a ferry that can carry both buses and cars on a trip across a waterway. Each trip costs the owner approximately $10. The fee for cars is $3 and the fee for buses is $8. Let and denote the number of buses and cars, respectively, carried on a given trip. The joint distribution of and is given below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | |  | | |
| 0 | 1 | 2 |
|  | 0 | 0.01 | 0.01 | 0.03 |
| 1 | 0.03 | 0.08 | 0.07 |
| 2 | 0.03 | 0.06 | 0.06 |
|  | 3 | 0.07 | 0.07 | 0.13 |
|  | 4 | 0.12 | 0.04 | 0.03 |
|  | 5 | 0.08 | 0.06 | 0.02 |

Compute the expected value and variance of profit for the ferry trip.

1. A store operates both a drive-in facility and a walk-in facility. On a randomly selected day, let and respectively, be the proportions of times that the drive-in and walk-in facilities are in use, and suppose that the joint density function of these random variables is given below.
   1. Determine whether *X* and are independent.
   2. Find
   3. Find
   4. Find
2. A service facility operates with two service lines. On a randomly selected day, let be the proportion of time that the first line is in use whereas is the proportion of time that the second line is in use. Suppose that the joint probability density function for is given below.
   1. Determine whether and are independent.
   2. Find the mean and variance of .
   3. Find the mean and variance of .
   4. Find the covariance of and .
   5. Find the mean and variance of .
3. The random variables and have the joint probability density function below.

Find

1. Given that and , and is defined as.
   1. Find the variance of if and are independent.
   2. If , find the variance of *Z*.
   3. If , compute the correlation of and *.*
2. An employee is selected from a staff of 10 to supervise a certain project by selecting a tag at random from a box containing 10 tags numbered from 1 to 10.
   1. Find the formula for the probability distribution of *X* representing the number on the tag that is drawn
   2. What is the probability that the number drawn is less than 4?
   3. Find the mean and variance of *X*.

**Answers to selected problems**

1. (a) for all . *X* and are independent

(b) 3

(c) 3.2

(d) 2.6

(e) 9.6

(f) 0.96; 2

(g) 0; 0

1. 6.55; 44.4275
2. (a) , . , and are dependent.

(b) 13/162

(c) 23/324

(d) 1/162

1. (a) , . , . and are dependent.

(b) 5/8; 73/960

(c) 5/8; 73/960

(d) 1/64

(e) ;

1. (a) , for , , for . ,

(b) , for .

(c) , for .

1. (a) 68

(b) 52

(c) 0.2582

1. (a)

(b) 0.3

(c) ;