Supplemental Materials for SARA: A Compiler for Scaling Reconfigurable Dataflow Accelerators

Paper #169

1 Pages Body, 1 Pages Total

1 Solver Supplement

1.1 Partitioning

1.1.1 Input Arity Constraint

$$\Sigma_{n_i \in \mathcal{N}} \overbrace{\max\{\underbrace{\mathrm{proj}_{\mathbf{B}}(\Sigma_{n_j \in \mathrm{dest}(n_i)} B_{j,:})}_{\textbf{Output locations of node } i} - B_{i,:}, 0\}}^{\textbf{Nonlocal output partitions of node}} \leq c_i \times \vec{1}$$

1.1.2 Output Arity Constraint

$$\forall p \in [0, P): \\ \Sigma_{n_s \in \mathcal{N}} \mathrm{and}(B_{s,p}, \underbrace{\mathrm{proj}_{\mathbf{B}}(\max\{(\Sigma_{n_d \in \mathrm{dest}(n_s)} B_{d,p}) - K \times B_{s,p}, 0\})}_{\text{Output locations for node}}) \leq c_o$$

1.1.3 Delay Consistency

$$\forall n_i \in \mathrm{N}:$$

$$d_n(i) \geq \underbrace{\max_j (B_{i,j} \times -maxdelay + maxdelay + d_p(j))}_{\text{Convex forumlation for partition delay}}$$

$$d_n(i) \leq \underbrace{\min_j (B_{i,j} \times maxdelay - maxdelay + d_p(j))}_{\text{Concave forumlation for partition delay}}$$

1.2 Merging

1.2.1 Reducible Constraints

$$\forall j \in [0, P).\ \forall \underbrace{(c(\cdot), c_v, r(\cdot))}_{\text{cost function, constraint value, reduction function}} \in C:$$

$$\underbrace{r([c(n_i) \times B_{i,j}]_{n_i \in \mathcal{N}})}_{\text{Total Cost (max / sum / other)}} \leq c_v$$

1.3 Utilities

1.3.1 $\mathbf{proj}_{\mathbf{B}}(\cdot)$

One common construct that the formulations use is a pseudo-projection to boolean; that is, function $\mathrm{proj}_{\mathbf{B}}(v) := \{1 \text{ if } v \geq 0 \text{ otherwise either } 0 \text{ or } 1\}$, where v is convex. Note that the additional constraint to exclude the $b = 1, v = 1$ case is unnecessary for the purposes of the solver.

1.3.2 $\mathrm{and}(\cdot, \cdot)$

Consider boolean variables $a, b \in \{0, 1\}$. Then $\mathrm{and}(a, b) := a \wedge b \equiv \max(a + b - 1, 0)$.

1.3.3 $1[p_i \neq p_j]$

This expression can be expanded to $\Sigma_k \max\{B_{i,k} - B_{j,k}, 0\}$. Note that the row $B_{i,:}$ is the standard basis vector e_{p_i}, and the row $B_{j,:}$ is the standard basis vector e_{p_j}. If $i = j$ then the result is 0. Otherwise, $\max\{B_{i,p_i} - B_{j,p_i}, 0\} = 1$, and $\max\{B_{i,p_j} - B_{j,p_j}, 0\} = \max\{-1, 0\} = 0$.