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Problem Set 3 | Bayes

Group Submission

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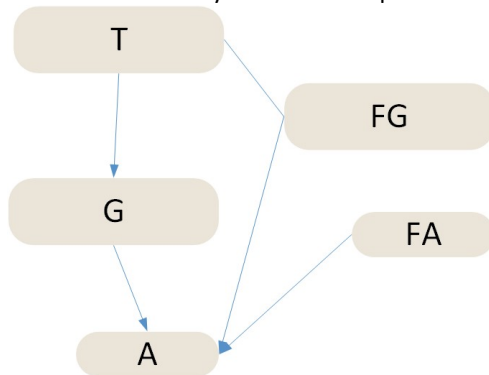
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In your friendly neighborhood nuclear power plant, there is an alarm that senses whether a particular temperature gauge reports a temperature that is too high. The gauge itself reads either "high" or "normal". The recently-laid-off technician estimates that on any given day the actual temperature is too high about 2% of the time. Please work with the following Boolean random variables,

- A - Alarm sounds
- FA - Alarm is faulty
- G - Gauge reports high temperature
- FG - Gauge is faulty
- T - Actual temperature is too high

and answer the following questions,

1. Draw a Bayes net for this problem.



2. Suppose that the probability that the gauge reports the temperature accurately is x when it is working, and y when it is faulty. Write the conditional probability table for G , conditioned on all of its parent(s) in your Bayes net.

	FG(False Gauge – does not report correctly)		-FG (False gauge – reports temperature accurately)	
	-T(normal)	T(High)	-T(normal)	T(High)
G (High temp, reports correctly)	$1-x$	x	y	$1-y$
-G(normal temp, reports correctly)	x	$1-x$	$1-y$	y

3. Suppose that the alarm works correctly at all times except when it is faulty, in which case it never sounds. Write the conditional probability table for A , when conditioned on all of its parent(s) in your Bayes net.

	G (High temp, reports correctly)		-G(normal temp, reports correctly)	
	FA (false alarm is true)	-FA(false alarm is false)	FA (false alarm is true)	-FA(false alarm is false)
A (Alarm sounds)	0	1	1	0
-A(Alarm does not sound)	1	0	0	1

4. Suppose we know the alarm and gauge are working properly, and the alarm sounds! Write an expression for the probability that the actual temperature is too high. Please show your steps.

Alarm working correctly \rightarrow -FA

Gauge working correctly \rightarrow -FG

Alarm sounds – A

Temp High – T

So we have to find $p(T | A, G, -FA, -FG)$

$$\text{Bayes Theorem : } P(B | A) = \frac{P(A, B)}{P(B)}$$

Here B is T

And A is A, G, -FA, -FG

so

$$\text{So, } P(T | A, G, -FA, -FG) = \frac{P(T, A, G, -FA, -FG)}{P(A, G, -FA, -FG)}$$

Now, applying chain rule

$$P(T, A, G, -FA, -FG) = P(A | -FA, -FG, G) * P(-FA) * P(G | T) * P(-FG | T) * P(T)$$

$$P(A, G, -FA, -FG) = P(A | -FA, -FG, G) * P(-FA)$$

So,

$$\frac{P(T, A, G, -FA, -FG)}{P(A, G, -FA, -FG)} = \frac{P(A | -FA, -FG, G) * P(-FA) * P(G | T) * P(-FG | T) * P(T)}{P(A | -FA, -FG, G) * P(-FA)}$$

$$\begin{aligned} \frac{P(T, A, G, -FA, -FG)}{P(A, G, -FA, -FG)} &= \frac{P(G | T) * P(-FG | T) * P(T)}{P(A | -FA, -FG, G) * P(-FA)} \\ &= P(G | T) * P(-FG | T) * P(T) \end{aligned}$$