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# Problem Set 5 | ML

# Group Submission

# Members:

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# Indranil Pal - (G01235186)

# Date: 11/18/2022

Linear Regression

(a) Use your closed form implementation to fit a model to the data in 1D-no-noise-lin.txt and

2D-noisy-lin.txt. What is the loss of the fit model in both cases? Include plots for both. Note:

for some of these datasets the y-intercept will be non-zero, make sure you handle this case!

(b) What happens when you're using the closed form solution and one of the features (columns of X)

is duplicated? Explain why. Note: you may want to test this out with your code as a useful first

step, but you should think critically about what is happening and why.

(c) Does the same thing happen if one of the training points (rows of X) is duplicated? Explain why.

(d) Does the same thing happen with Gradient Descent? Explain why.

1. Closed form implementation :

1D-no-noise-lin.txt

Theta : [[0.5]] , Mean squared error: 0.00

Chart, scatter chart

Description automatically generated

2D-noisy-lin.txt

Theta : [[4.47467363] [1.9625976 ]] , Mean squared error: 1.20

Chart, scatter chart

Description automatically generated

1. Both MSE and Theta changes .

Chart, scatter chart

Description automatically generated

Theta : [[0.48853868]] , Mean squared error: 0.02

1. When row is repeated, Theta and MSE don’t change

Theta : [[0.5]] , Mean squared error: 0.00

Chart, scatter chart

Description automatically generated

1. With duplicate data(only X is repeated or row is duplicated), the loss is increased :

**Original :**

curr\_theta [array([0.285]), array([0.045])], cur\_loss 121.61812500000002 iteration 1

curr\_theta [array([0.4035]), array([0.06345])], cur\_loss 21.011072625000004 iteration 2

curr\_theta [array([0.4527945]), array([0.070866])], cur\_loss 3.6299309738062417 iteration 3

curr\_theta [array([0.47332369]), array([0.07369717])], cur\_loss 0.6271187447093247 iteration 4

curr\_theta [array([0.48189644]), array([0.0746241])], cur\_loss 0.10834490211962254 iteration 5

curr\_theta [array([0.4854993]), array([0.07476094])], cur\_loss 0.01872013017999265 iteration 6

curr\_theta [array([0.48703622]), array([0.07457078])], cur\_loss 0.0032362919766328116 iteration 7

curr\_theta [array([0.4877142]), array([0.07424611])], cur\_loss 0.0005612387131172915 iteration 8

curr\_theta [array([0.48803496]), array([0.07386691])], cur\_loss 9.906543470552655e-05 iteration 9

curr\_theta [array([0.48820701]), array([0.07346642])], cur\_loss 1.9195450888155244e-05 iteration 10

**in case of only X is repeated in gradient\_descent**

curr\_theta [array([0.31]), array([0.04727273])], cur\_loss 193.14274999999998 iteration 1

curr\_theta [array([0.41873554]), array([0.06372727])], cur\_loss 23.754529531930864 iteration 2

curr\_theta [array([0.45688781]), array([0.06937458])], cur\_loss 2.921558072820383 iteration 3

curr\_theta [array([0.47028654]), array([0.07123244])], cur\_loss 0.35932145792087383 iteration 4

curr\_theta [array([0.47500414]), array([0.071762])], cur\_loss 0.044193268888371676 iteration 5

curr\_theta [array([0.47667717]), array([0.07182636])], cur\_loss 0.005435807226777534 iteration 6

curr\_theta [array([0.47728239]), array([0.07172821])], cur\_loss 0.0006690414198957546 iteration 7

curr\_theta [array([0.47751303]), array([0.07157371])], cur\_loss 8.277413724558271e-05 iteration 8

curr\_theta [array([0.4776122]), array([0.07140007])], cur\_loss 1.0664305081282643e-05 iteration 9

curr\_theta [array([0.47766518]), array([0.07122035])], cur\_loss 1.7904908881373216e-06 iteration 10

**Whole row is repeated :**

curr\_theta [array([0.31727273]), array([0.04818182])], cur\_loss 202.27331818181818 iteration 1

curr\_theta [array([0.42857851]), array([0.06482645])], cur\_loss 24.877495132982713 iteration 2

curr\_theta [array([0.46765178]), array([0.07041235])], cur\_loss 3.0596726704796433 iteration 3

curr\_theta [array([0.48139301]), array([0.0721213])], cur\_loss 0.3763097124016626 iteration 4

curr\_theta [array([0.48625012]), array([0.07247191])], cur\_loss 0.046284238920407506 iteration 5

curr\_theta [array([0.48799139]), array([0.07234746])], cur\_loss 0.005694551122802042 iteration 6

curr\_theta [array([0.48863973]), array([0.0720577])], cur\_loss 0.0007024242597145351 iteration 7

curr\_theta [array([0.4889046]), array([0.07171127])], cur\_loss 8.842361578517254e-05 iteration 8

curr\_theta [array([0.48903478]), array([0.07134623])], cur\_loss 1.2886723164859169e-05 iteration 9

curr\_theta [array([0.48911753]), array([0.07097596])], cur\_loss 3.575437066274882e-06 iteration 10

2. Now use your Gradient Descent implementation to fit a model to 1D-no-noise-lin.txt with

alpha=0.05, num iters=10, and initial Theta set to the zero vector. What is the output (the

full list of (theta, loss) tuples for each of the 10 iterations)?

(b) Using the default parameters for alpha and num iters, and with initial Theta set to 0, do you

get the same model parameters as you did with the closed form solution? the same loss? Report

this for both 1D-no-noise-lin.txt and 2D-noisy-lin.txt.

(c) Find a set of values of the learning rate and iterations where you get the same answers and a set

of values where the answers are noticeably different. Explain what's going on in both cases, and

for both datasets 1D-no-noise-lin.txt and 2D-noisy-lin.txt.

1. 1D-no-noise-lin.txt

curr\_theta [array([1.425]), array([0.225])], cur\_loss 3040.4531249999995 iteration 1

curr\_theta [array([-1.3125]), array([-0.21375])], cur\_loss 11228.926640625003 iteration 2

curr\_theta [array([3.9493125]), array([0.62325])], cur\_loss 41470.39599697267 iteration 3

curr\_theta [array([-6.16169063]), array([-0.99126563])], cur\_loss 153157.4481496576 iteration 4

curr\_theta [array([13.27019719]), array([2.10562172])], cur\_loss 565637.3270448203 iteration 5

curr\_theta [array([-24.07239457]), array([-3.85152919])], cur\_loss 2088997.8886419649 iteration 6

curr\_theta [array([47.69211809]), array([7.59120129])], cur\_loss 7715035.7134086555 iteration 7

curr\_theta [array([-90.22145905]), array([-14.40437198])], cur\_loss 28492980.47816256 iteration 8

curr\_theta [array([174.81666662]), array([27.86072179])], cur\_loss 105229575.94620734 iteration 9

curr\_theta [array([-334.52315806]), array([-53.36785037])], cur\_loss 388631286.3027927 iteration 10

1. 1D-no-noise-lin.txt

With default parameter , the loss increases with each iteration in gradient\_descent, and initial loss is also higher than closed form

linreg\_grad\_desc(init\_theta, data\_X, data\_Y)

Theta : [[0.5]] , Mean squared error: 0.00

curr\_theta [array([1.425]), array([0.225])], cur\_loss 3040.4531249999995 iteration 1

....

curr\_theta [array([1.79617243e+141]), array([2.86444118e+140])], cur\_loss 1.1171417917815495e+286 iteration 499

curr\_theta [array([-3.45181885e+141]), array([-5.50477887e+140])], cur\_loss 4.125800637500844e+286 iteration 500

2D-noisy-lin.txt

Theta : [[4.52484452]

[1.92893988]] , Mean squared error: 1.26

curr\_theta [array([0.22628725, 0.19552166]), array([0.38288361, 0.38288361])], cur\_loss 2549.8183809417246 iteration 1

....

curr\_theta [array([ 2.06429188, -0.62765551]), array([2.70519279, 4.1563301 ])], cur\_loss 0.00014230196655771327 iteration 499

curr\_theta [array([ 2.06438754, -0.62820643]), array([2.70513707, 4.15664106])], cur\_loss 0.00014040878095893393 iteration 500

1. Similar result

1D-no-noise-lin.txt : learning rate : 0.01 , num\_iteration : 8

Theta : [[0.5]] , Mean squared error: 0.00

curr\_theta [array([0.285]), array([0.045])], cur\_loss 121.61812500000002 iteration 1

curr\_theta [array([0.4035]), array([0.06345])], cur\_loss 21.011072625000004 iteration 2

curr\_theta [array([0.4527945]), array([0.070866])], cur\_loss 3.6299309738062417 iteration 3

curr\_theta [array([0.47332369]), array([0.07369717])], cur\_loss 0.6271187447093247 iteration 4

curr\_theta [array([0.48189644]), array([0.0746241])], cur\_loss 0.10834490211962254 iteration 5

curr\_theta [array([0.4854993]), array([0.07476094])], cur\_loss 0.01872013017999265 iteration 6

curr\_theta [array([0.48703622]), array([0.07457078])], cur\_loss 0.0032362919766328116 iteration 7

curr\_theta [array([0.4877142]), array([0.07424611])], cur\_loss 0.0005612387131172915 iteration 8

curr\_theta [array([0.48803496]), array([0.07386691])], cur\_loss 9.906543470552655e-05 iteration 9

curr\_theta [array([0.48820701]), array([0.07346642])], cur\_loss 1.9195450888155244e-05 iteration 10

2D-noisy-lin.txt: learning rate : 0.01 , num\_iteration : 84

Theta : [1.92893988]] , Mean squared error: 1.26

curr\_theta [array([0.04525745, 0.03910433]), array([0.07657672, 0.07657672])], cur\_loss 101.99273523766898 iteration 1

curr\_theta [array([0.08933501, 0.07711721]), array([0.15112897, 0.15120996])], cur\_loss 96.7278732277214 iteration 2

curr\_theta [array([0.13226376, 0.11406704]), array([0.22371009, 0.22395009])], cur\_loss 91.73480923699269 iteration 3

curr\_theta [array([0.17407393, 0.14998153]), array([0.29437201, 0.29484616])], cur\_loss 86.99951090281193 iteration 4

curr\_theta [array([0.21479498, 0.18488762]), array([0.3631653 , 0.36394597])], cur\_loss 82.50867036463995 iteration 5

curr\_theta [array([0.25445558, 0.2188116 ]), array([0.43013919, 0.43129607])], cur\_loss 78.24966685535233 iteration 6

curr\_theta [array([0.29308366, 0.25177904]), array([0.49534163, 0.49694178])], cur\_loss 74.21053122417909 iteration 7

curr\_theta [array([0.3307064 , 0.28381486]), array([0.55881929, 0.56092729])], cur\_loss 70.37991229155382 iteration 8

curr\_theta [array([0.36735026, 0.31494334]), array([0.62061761, 0.6232956 ])], cur\_loss 66.74704494127161 iteration 9

curr\_theta [array([0.40304102, 0.34518812]), array([0.68078085, 0.68408862])], cur\_loss 63.301719860244326 iteration 10

curr\_theta [array([0.43780376, 0.37457223]), array([0.73935208, 0.74334717])], cur\_loss 60.03425484077127 iteration 11

curr\_theta [array([0.47166293, 0.4031181 ]), array([0.79637326, 0.801111 ])], cur\_loss 56.93546756464123 iteration 12

curr\_theta [array([0.5046423 , 0.43084759]), array([0.85188522, 0.85741883])], cur\_loss 53.99664979254289 iteration 13

curr\_theta [array([0.53676505, 0.45778198]), array([0.90592774, 0.91230839])], cur\_loss 51.20954288621784 iteration 14

curr\_theta [array([0.56805371, 0.48394199]), array([0.95853953, 0.96581641])], cur\_loss 48.56631459453661 iteration 15

curr\_theta [array([0.59853025, 0.50934782]), array([1.00975829, 1.01797869])], cur\_loss 46.059537038226296 iteration 16

curr\_theta [array([0.62821604, 0.53401914]), array([1.05962073, 1.06883009])], cur\_loss 43.682165831359804 iteration 17

curr\_theta [array([0.6571319 , 0.55797511]), array([1.10816258, 1.11840455])], cur\_loss 41.42752028090186 iteration 18

curr\_theta [array([0.68529809, 0.58123438]), array([1.15541866, 1.16673516])], cur\_loss 39.289264608646675 iteration 19

curr\_theta [array([0.71273436, 0.60381514]), array([1.20142282, 1.21385414])], cur\_loss 37.26139014275077 iteration 20

curr\_theta [array([0.73945991, 0.62573509]), array([1.24620808, 1.25979285])], cur\_loss 35.338198428795806 iteration 21

curr\_theta [array([0.76549344, 0.64701148]), array([1.28980653, 1.30458187])], cur\_loss 33.51428521289881 iteration 22

curr\_theta [array([0.79085318, 0.6676611 ]), array([1.33224946, 1.34825098])], cur\_loss 31.784525251837945 iteration 23

curr\_theta [array([0.81555686, 0.68770031]), array([1.37356732, 1.39082917])], cur\_loss 30.14405790749261 iteration 24

curr\_theta [array([0.83962174, 0.70714506]), array([1.41378975, 1.43234469])], cur\_loss 28.588273485095343 iteration 25

curr\_theta [array([0.86306464, 0.72601086]), array([1.45294562, 1.47282506])], cur\_loss 27.112800276889455 iteration 26

curr\_theta [array([0.88590193, 0.74431284]), array([1.49106304, 1.51229708])], cur\_loss 25.713492274766942 iteration 27

curr\_theta [array([0.90814954, 0.76206573]), array([1.52816937, 1.55078685])], cur\_loss 24.386417517342828 iteration 28

curr\_theta [array([0.92982299, 0.77928387]), array([1.56429125, 1.58831982])], cur\_loss 23.12784703870735 iteration 29

curr\_theta [array([0.9509374 , 0.79598123]), array([1.59945463, 1.62492073])], cur\_loss 21.934244387785522 iteration 30

curr\_theta [array([0.97150748, 0.81217143]), array([1.63368477, 1.66061373])], cur\_loss 20.80225568884277 iteration 31

curr\_theta [array([0.99154754, 0.82786772]), array([1.66700626, 1.69542231])], cur\_loss 19.728700215192355 iteration 32

curr\_theta [array([1.01107154, 0.84308302]), array([1.69944304, 1.72936937])], cur\_loss 18.71056144960564 iteration 33

curr\_theta [array([1.03009305, 0.8578299 ]), array([1.73101844, 1.7624772 ])], cur\_loss 17.744978606294925 iteration 34

curr\_theta [array([1.04862531, 0.87212062]), array([1.76175515, 1.79476752])], cur\_loss 16.82923859063415 iteration 35

curr\_theta [array([1.06668117, 0.88596711]), array([1.79167527, 1.82626149])], cur\_loss 15.960768374015117 iteration 36

curr\_theta [array([1.08427318, 0.899381 ]), array([1.82080033, 1.85697971])], cur\_loss 15.137127762405807 iteration 37

curr\_theta [array([1.10141354, 0.91237361]), array([1.84915128, 1.88694226])], cur\_loss 14.356002538278128 iteration 38

curr\_theta [array([1.11811412, 0.92495596]), array([1.87674851, 1.91616869])], cur\_loss 13.615197956630203 iteration 39

curr\_theta [array([1.13438652, 0.9371388 ]), array([1.90361191, 1.94467804])], cur\_loss 12.91263257681719 iteration 40

curr\_theta [array([1.15024197, 0.94893259]), array([1.92976079, 1.97248886])], cur\_loss 12.246332412853008 iteration 41

curr\_theta [array([1.16569146, 0.96034752]), array([1.95521401, 1.99961922])], cur\_loss 11.614425385737325 iteration 42

curr\_theta [array([1.18074567, 0.97139353]), array([1.97998988, 2.02608672])], cur\_loss 11.015136062215296 iteration 43

curr\_theta [array([1.19541499, 0.98208027]), array([2.00410627, 2.05190851])], cur\_loss 10.44678066517781 iteration 44

curr\_theta [array([1.20970954, 0.99241717]), array([2.02758056, 2.07710128])], cur\_loss 9.907762341678321 iteration 45

curr\_theta [array([1.22363919, 1.00241341]), array([2.05042966, 2.1016813 ])], cur\_loss 9.396566675265356 iteration 46

curr\_theta [array([1.23721353, 1.01207791]), array([2.07267007, 2.12566442])], cur\_loss 8.91175743001197 iteration 47

curr\_theta [array([1.2504419 , 1.02141938]), array([2.09431781, 2.14906606])], cur\_loss 8.451972514283986 iteration 48

curr\_theta [array([1.26333341, 1.03044631]), array([2.11538852, 2.17190126])], cur\_loss 8.015920152897303 iteration 49

curr\_theta [array([1.27589691, 1.03916694]), array([2.1358974 , 2.19418467])], cur\_loss 7.602375256906629 iteration 50

curr\_theta [array([1.28814103, 1.04758932]), array([2.15585927, 2.21593053])], cur\_loss 7.210175980821147 iteration 51

curr\_theta [array([1.30007417, 1.05572129]), array([2.17528853, 2.23715276])], cur\_loss 6.838220457568839 iteration 52

curr\_theta [array([1.31170451, 1.06357048]), array([2.19419924, 2.25786487])], cur\_loss 6.485463702031657 iteration 53

curr\_theta [array([1.32304 , 1.07114431]), array([2.21260505, 2.27808005])], cur\_loss 6.150914674451372 iteration 54

curr\_theta [array([1.3340884 , 1.07845003]), array([2.23051929, 2.29781114])], cur\_loss 5.83363349544598 iteration 55

curr\_theta [array([1.34485726, 1.0854947 ]), array([2.2479549 , 2.31707065])], cur\_loss 5.532728804813244 iteration 56

curr\_theta [array([1.35535392, 1.09228517]), array([2.26492451, 2.33587076])], cur\_loss 5.24735525669549 iteration 57

curr\_theta [array([1.36558555, 1.09882815]), array([2.28144039, 2.35422332])], cur\_loss 4.9767111440656535 iteration 58

curr\_theta [array([1.37555911, 1.10513015]), array([2.29751452, 2.37213991])], cur\_loss 4.720036145857095 iteration 59

curr\_theta [array([1.38528137, 1.11119751]), array([2.31315853, 2.38963178])], cur\_loss 4.476609190404846 iteration 60

curr\_theta [array([1.39475896, 1.11703642]), array([2.32838377, 2.4067099 ])], cur\_loss 4.245746429194059 iteration 61

curr\_theta [array([1.40399829, 1.1226529 ]), array([2.34320128, 2.42338494])], cur\_loss 4.0267993152190344 iteration 62

curr\_theta [array([1.41300564, 1.12805281]), array([2.3576218 , 2.43966731])], cur\_loss 3.819152780552476 iteration 63

curr\_theta [array([1.42178709, 1.13324188]), array([2.37165581, 2.45556715])], cur\_loss 3.622223508002656 iteration 64

curr\_theta [array([1.43034858, 1.13822566]), array([2.38531348, 2.47109433])], cur\_loss 3.4354582920003085 iteration 65

curr\_theta [array([1.43869589, 1.14300958]), array([2.39860476, 2.48625847])], cur\_loss 3.2583324841086094 iteration 66

curr\_theta [array([1.44683466, 1.14759892]), array([2.41153929, 2.50106892])], cur\_loss 3.090348518786913 iteration 67

curr\_theta [array([1.45477035, 1.15199881]), array([2.42412648, 2.51553482])], cur\_loss 2.931034515265277 iteration 68

curr\_theta [array([1.46250831, 1.15621428]), array([2.4363755 , 2.52966505])], cur\_loss 2.779942951599188 iteration 69

curr\_theta [array([1.47005372, 1.16025019]), array([2.44829525, 2.54346827])], cur\_loss 2.636649407178976 iteration 70

curr\_theta [array([1.47741165, 1.1641113 ]), array([2.45989443, 2.5569529 ])], cur\_loss 2.5007513701585955 iteration 71

curr\_theta [array([1.48458702, 1.16780223]), array([2.47118148, 2.57012717])], cur\_loss 2.3718671064529233 iteration 72

curr\_theta [array([1.49158463, 1.17132751]), array([2.48216463, 2.58299907])], cur\_loss 2.2496345871244503 iteration 73

curr\_theta [array([1.49840913, 1.17469152]), array([2.49285191, 2.5955764 ])], cur\_loss 2.1337104711441466 iteration 74

curr\_theta [array([1.50506509, 1.17789853]), array([2.50325111, 2.60786674])], cur\_loss 2.0237691406688514 iteration 75

curr\_theta [array([1.51155691, 1.18095271]), array([2.51336983, 2.61987749])], cur\_loss 1.919501786123156 iteration 76

curr\_theta [array([1.51788893, 1.18385813]), array([2.52321546, 2.63161585])], cur\_loss 1.8206155385139797 iteration 77

curr\_theta [array([1.52406531, 1.18661873]), array([2.53279522, 2.64308884])], cur\_loss 1.7268326465400679 iteration 78

curr\_theta [array([1.53009017, 1.18923837]), array([2.54211611, 2.65430328])], cur\_loss 1.6378896961833829 iteration 79

curr\_theta [array([1.53596746, 1.19172079]), array([2.55118496, 2.66526584])], cur\_loss 1.553536870589461 iteration 80

curr\_theta [array([1.54170107, 1.19406965]), array([2.56000841, 2.675983 ])], cur\_loss 1.4735372481558349 iteration 81

curr\_theta [array([1.54729476, 1.1962885 ]), array([2.56859295, 2.68646107])], cur\_loss 1.3976661368567855 iteration 82

curr\_theta [array([1.55275221, 1.19838082]), array([2.57694487, 2.6967062 ])], cur\_loss 1.3257104429335291 iteration 83

curr\_theta [array([1.55807698, 1.20034997]), array([2.58507031, 2.70672439])], cur\_loss 1.2574680721749767 iteration 84

curr\_theta [array([1.56327257, 1.20219924]), array([2.59297525, 2.71652147])], cur\_loss 1.1927473621075089 iteration 85

DisSimilar result

1D-no-noise-lin.txt : learning rate : 0.01 , num\_iteration : 8

Theta : [[0.5]] , Mean squared error: 0.00

curr\_theta [array([0.285]), array([0.045])], cur\_loss 121.61812500000002 iteration 1

curr\_theta [array([0.4035]), array([0.06345])], cur\_loss 21.011072625000004 iteration 2

curr\_theta [array([0.4527945]), array([0.070866])], cur\_loss 3.6299309738062417 iteration 3

curr\_theta [array([0.47332369]), array([0.07369717])], cur\_loss 0.6271187447093247 iteration 4

curr\_theta [array([0.48189644]), array([0.0746241])], cur\_loss 0.10834490211962254 iteration 5

curr\_theta [array([0.4854993]), array([0.07476094])], cur\_loss 0.01872013017999265 iteration 6

curr\_theta [array([0.48703622]), array([0.07457078])], cur\_loss 0.0032362919766328116 iteration 7

curr\_theta [array([0.4877142]), array([0.07424611])], cur\_loss 0.0005612387131172915 iteration 8

curr\_theta [array([0.48803496]), array([0.07386691])], cur\_loss 9.906543470552655e-05 iteration 9

curr\_theta [array([0.48820701]), array([0.07346642])], cur\_loss 1.9195450888155244e-05 iteration 10

2D-noisy-lin.txt: learning rate : 0.01 , num\_iteration : 84

Theta : [1.92893988]] , Mean squared error: 1.26

curr\_theta [array([0.04525745, 0.03910433]), array([0.07657672, 0.07657672])], cur\_loss 101.99273523766898 iteration 1

curr\_theta [array([0.08933501, 0.07711721]), array([0.15112897, 0.15120996])], cur\_loss 96.7278732277214 iteration 2

curr\_theta [array([0.13226376, 0.11406704]), array([0.22371009, 0.22395009])], cur\_loss 91.73480923699269 iteration 3

curr\_theta [array([0.17407393, 0.14998153]), array([0.29437201, 0.29484616])], cur\_loss 86.99951090281193 iteration 4

curr\_theta [array([0.21479498, 0.18488762]), array([0.3631653 , 0.36394597])], cur\_loss 82.50867036463995 iteration 5

curr\_theta [array([0.25445558, 0.2188116 ]), array([0.43013919, 0.43129607])], cur\_loss 78.24966685535233 iteration 6

curr\_theta [array([0.29308366, 0.25177904]), array([0.49534163, 0.49694178])], cur\_loss 74.21053122417909 iteration 7

curr\_theta [array([0.3307064 , 0.28381486]), array([0.55881929, 0.56092729])], cur\_loss 70.37991229155382 iteration 8

curr\_theta [array([0.36735026, 0.31494334]), array([0.62061761, 0.6232956 ])], cur\_loss 66.74704494127161 iteration 9

curr\_theta [array([0.40304102, 0.34518812]), array([0.68078085, 0.68408862])], cur\_loss 63.301719860244326 iteration 10

curr\_theta [array([0.43780376, 0.37457223]), array([0.73935208, 0.74334717])], cur\_loss 60.03425484077127 iteration 11

curr\_theta [array([0.47166293, 0.4031181 ]), array([0.79637326, 0.801111 ])], cur\_loss 56.93546756464123 iteration 12

curr\_theta [array([0.5046423 , 0.43084759]), array([0.85188522, 0.85741883])], cur\_loss 53.99664979254289 iteration 13

curr\_theta [array([0.53676505, 0.45778198]), array([0.90592774, 0.91230839])], cur\_loss 51.20954288621784 iteration 14

curr\_theta [array([0.56805371, 0.48394199]), array([0.95853953, 0.96581641])], cur\_loss 48.56631459453661 iteration 15

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curr\_theta [array([0.62821604, 0.53401914]), array([1.05962073, 1.06883009])], cur\_loss 43.682165831359804 iteration 17

curr\_theta [array([0.6571319 , 0.55797511]), array([1.10816258, 1.11840455])], cur\_loss 41.42752028090186 iteration 18

curr\_theta [array([0.68529809, 0.58123438]), array([1.15541866, 1.16673516])], cur\_loss 39.289264608646675 iteration 19

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curr\_theta [array([0.76549344, 0.64701148]), array([1.28980653, 1.30458187])], cur\_loss 33.51428521289881 iteration 22

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curr\_theta [array([0.81555686, 0.68770031]), array([1.37356732, 1.39082917])], cur\_loss 30.14405790749261 iteration 24

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