Economic Dispatch Problem (Hw 2)

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The main aim of economic dispatch problem is to determine the generation dispatch that minimizes the operating cost function subject to operational constraints.Basically, to formulate the above definition mathematically we can write it as follows:

Objective Function

$$min \sum_{i=1}^{N_g} C_i(X_i) \tag{1}$$

Here, Decision vector

$$X_i = \begin{bmatrix} P_{g1} \\ . \\ . \\ P_{gi} \\ . \\ . \\ P_{gN_g} \end{bmatrix}$$

Here, If we look into the objective function in detail, that is if we plot the graph for fuel cost function(C_i) and power generated(P_{gi}) for i number of plants. Then we obtain curve most likely to be parabola.

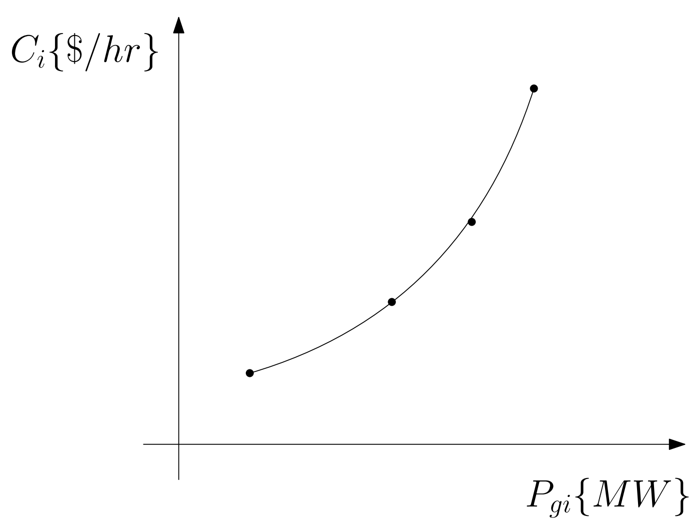

Figure 1: Fuel-Cost curve

Assuming one condition i.e. Network is going to be a Loss less network. So that we can write our objective function along with operational constraints in more meaningful way as follows:

Objective Function

$$min \sum_{i=1}^{N_g} \alpha_i P_{gi}^2 + \beta_i P_{gi} + \gamma_i \tag{2}$$

subject to:

$$\sum_{i=1}^{N_g} P_{gi} - D = 0 \tag{3}$$

$$P_{gi_{min}} \leqslant P_{gi} \leqslant P_{gi_{max}} \tag{4}$$

The operational constraints are Eq: (3) and Eq: (4) which basically tells us that:

(1).The total power generated must be equal to demand if it is a loss less network.

(2). Each plant has limits (minimum and maximum) on generating power that shouldn't exceed.

Characterizing the Minimization Problem

Mathematically, It is a constrained optimization problem. To understand this type of problem we use a calculus method that involves Lagrange function. In order to obtain the necessary conditions for the objective function, we should add the constraint functions to the objective function after the constraint function has been multiplied by lagrangian multiplier.

$$\begin{aligned} \mathcal{L}(P_{gi}, \lambda, \widehat{\mu}_i, \breve{\mu}_i) = & \sum_{i=1}^{N_g} \alpha_i P_{gi}^2 + \beta_i P_{gi} \\ & + \lambda(\sum_{i=1}^{N_g} P_{gi} - D) \\ & + \sum_{i=1}^{N_g} \widehat{\mu}_i (P_{gi} - \widehat{P_{gi}}) \\ & + \sum_{i=1}^{N_g} \breve{\mu}_i (-P_{gi} + \overset{\smile}{P_{gi}}) \end{aligned} \tag{5}$$

KKT – conditions

The conditions for optimum for the point P_{gi}^0, λ^0, $\widehat{\mu}_i^0$, $\widecheck{\mu}_i^0$ are:

$$\frac{\partial \mathcal{L}}{\partial P_{gi}}(P_{gi}^0, \lambda^0, \widehat{\mu}_i^0, \widecheck{\mu}_i^0) = 0 \quad \forall i \in 1...\mathcal{N}_g \tag{6}$$

$$\sum_{i=1}^{N_g} P_{gi}^0 - D = 0 \quad \forall i \in 1...\mathcal{N}_g \tag{7}$$

$$\begin{aligned} &(P_{gi}^0 - \widehat{P_{gi}^0}) \leqslant 0 \\ &(-P_{gi}^0 + \widecheck{P_{gi}^0}) \leqslant 0 \quad \forall i \in 1...\mathcal{N}_g \end{aligned} \tag{8}$$

$$\begin{aligned} &\widehat{\mu_i^0}(P_{gi}^0 - \widehat{P_{gi}^0}) = 0 \\ &\widecheck{\mu_i^0}(-P_{gi}^0 + \widecheck{P_{gi}^0}) = 0 \\ &\widehat{\mu_i^0} \geqslant 0 \\ &\widecheck{\mu_i^0} \geqslant 0 \quad \forall i \in 1...\mathcal{N}_g \end{aligned} \tag{9}$$

Loosely speaking, What lagrangian function tells us is, It took the constrained optimization problem and turned into a unconstrained optimization problem. Basically, Lagrange is characterization of minimum value of given objective function but not how to solve the problem.

Numerical Solution approach

Given data for our problem is, case1:D=250MW ,cas2:D=450MW and also,

Generator Data		
Bus	$P_{gi_{min}}$	$P_{gi_{max}}$
1	50	200
2	37.5	150
3	45	180

Generator Cost Function Data		
α_i	β_i	γ_i
0.00533	11.669	213.1
0.00889	10.333	200
0.00741	10.833	240

In order to make the generation costs more realistic, we need to multiply all coefficients $(\alpha_i, \beta_i, \gamma_i) \times 3$.

To proceed further with numerical solution for this minimization problem, we need to remove the cost function coefficient γ from our objective function because, γ is fixed cost, which is an offset. It will play a role in finding primary costs but doesn't effect the dispatch of the generators.So, objective function becomes:

Objective Function

$$min \sum_{i=1}^{N_g} \alpha_i P_{gi}^2 + \beta_i P_{gi} \tag{10}$$

subject to:

$$\sum_{i=1}^{N_g} P_{gi} - D = 0 \tag{11}$$

$$P_{gi_{min}} \leqslant P_{gi} \leqslant P_{gi_{max}} \tag{12}$$

Eq: (10), (11), (12) is general human understanding equation, to implement this equation in machine understanding form(For MATLAB) we need to convert it into following form:

$$\begin{aligned} \min_x \frac{1}{2} x^T H x + f^T x \\ A * x = b, \\ A_e q * x = b_e q, \\ lb \leqslant x \leqslant ub \end{aligned}$$

Here,

$$A_{eq} = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}$$

$$b_{eq} = \begin{bmatrix} D \end{bmatrix}$$

$$H = \begin{bmatrix} 2\alpha_1 & \cdots & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & 2\alpha_i & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & \cdots & 2\alpha_{Ng} \end{bmatrix}$$

$$f = \begin{bmatrix} \beta_1 & \cdots & \beta_i & \cdots & \beta_{Ng} \end{bmatrix}$$

$$x = \begin{bmatrix} P_{g1} \\ \vdots \\ P_{gi} \\ \vdots \\ P_{gNG} \end{bmatrix}$$

$$lb = \begin{bmatrix} P_{gi_{min}} \end{bmatrix}$$

$$ub = \begin{bmatrix} P_{gi_{max}} \end{bmatrix}$$

Numerical Results

1.1

case1: When D=250MW	
Dispatch of each plant P(MW)	Marginal Cost(Inc cost)
51.0810	36.6406
105.7662	36.6406
93.1527	36.6406

The total generation cost is 8627.2\$/hr and the shadow price for the power balance constraint is 36.6406\$/MWhr.

Numerically verifying the KKT conditions:

Condition 1 :

$$\frac{\partial \mathcal{L}}{\partial P_{gi}}(P^0_{gi}, \lambda^0, \widehat{\mu}^0_i, \breve{\mu}^0_i) = 0 \quad \forall i \in 1 ... \mathcal{N}_g \tag{13}$$

i.e.

$$\frac{\partial \mathcal{L}}{\partial P_{gi}} = 0 \tag{14}$$

When i=1

$$\frac{\partial \mathcal{L}}{\partial P_{g1}} = 2\alpha_1 P_{g1} + \beta_1 + \lambda + \widehat{\mu_1} - \widecheck{\mu_1} = 2*3*(0.00533)(51.0810) + 3*11.669 + 36.6406 + 0 - 0 = 0 \tag{15}$$

We know that, Marginal cost$(MC) = \frac{\partial C_i}{\partial P_{g1}} = 2\alpha_1 P_{gi} + \beta_i = 2*3*(0.00533)(51.0810)+3*11.669 =$ 36.6406 and which is also equal to $(MC) = \frac{\partial C_i}{\partial P_{g1}} = -\lambda$. So, the first condition is a necessary condition to obtain local optimum.

Condition 2 :

$$\sum_{i=1}^{N_g} P^0_{gi} - D = 0 \quad \forall i \in 1 ... \mathcal{N}_g \tag{16}$$

for i=1:3

$$\sum_{i=1}^{N_g} P^0_{gi} - D = 51.0810 + 105.7662 + 93.1527 - 250 = 0 \tag{17}$$

This condition is based on equality constraint, we know that equality constraints are always active. Therefore, they always have a value i.e. Lambda will always have a value different than zero. This Lambda is the shadow price associated with that constraint.

More precisely, If we relax the equality constraint by 1 unit, we can observe the change in the objective function but we cannot tell that change in objective function can be positive or negative. So that lambda can also be positive or negative, this is the main reason for not being an Lagrangian condition.

The value of lambda corresponds to the change in the objective function, when the corresponding constraint is relaxed by "1" unit.

To explain that numerically, I am going to increase the load demand by 1 unit, Therefore we can observe the changes as follows:

case1: When D=251MW	
Dispatch of each plant P(MW)	Marginal Cost(Inc cost)
51.5123	36.6544
105.0248	36.6544
93.4629	36.6544

The total generation cost is 8663.8\$/hr.

So, If we see the change in the total generation cost, Change=8663.8-8627.2=36.6, which is nothing but the value of lambda.

Condition 3 :

$$\begin{aligned} &(P_{gi}^0 - \widehat{P_{gi}^0}) \leqslant 0 \\ &(-P_{gi}^0 + \widecheck{P_{gi}^0}) \leqslant 0 \quad \forall i \in 1 \ldots \mathcal{N}_g \end{aligned} \tag{18}$$

for i=1

$$\begin{aligned} &(P_{gi}^0 - \widehat{P_{gi}^0}) = 51.0810 - 200 = -148.919 \leqslant 0 \\ &(-P_{gi}^0 + \widecheck{P_{gi}^0}) = -51.0810 + 50 = -1.0810 \leqslant 0 \end{aligned} \tag{19}$$

So, this inequality constraint is a necessary condition and it is satisfied with optimum value.

Condition 4 :

$$\begin{aligned} &\widehat{\mu_i^0}(P_{gi}^0 - \widehat{P_{gi}^0}) = 0 \\ &\widecheck{\mu_i^0}(-P_{gi}^0 + \widecheck{P_{gi}^0}) = 0 \\ &\widehat{\mu_i^0} \geqslant 0 \\ &\widecheck{\mu_i^0} \geqslant 0 \quad \forall i \in 1 \ldots \mathcal{N}_g \end{aligned} \tag{20}$$

case1: If optimum value of dispatch of generators is with in the limits, i.e.$\widecheck{P_{gi}} < P_{gi} < \widehat{P_{gi}}$

case1: When D=250MW		
$P_{gi_{min}}$	Dispatch of each plant P(MW)	$P_{gi_{max}}$
50	51.5123	200
37.5	105.0248	150
45	93.4629	180

Here, $(P_{gi}^0 - \widehat{P_{gi}^0}) = -148.919 \leqslant 0$ i.e inequality constraints are inactive, According to KKT condition 4, $\widehat{\mu}_i$ has to be zero i.e. $\widehat{\mu}_i = 0$ this is same for the $\breve{\mu}_i$.

Therefore, $\widehat{\mu}_i = \breve{\mu}_i = 0$ from KKT condition(1) we can also write that,

$$\frac{\partial \mathcal{L}}{\partial P_{g1}} = 2\alpha_1 P_{g1} + \beta_1 + \lambda + \widehat{\mu_1} - \widecheck{\mu_1} = 2*3*(0.00533)(51.0810) + 3*11.669 + 36.6406 + 0 - 0 = 0 \tag{21}$$

Eq: (21) implies that $MC_i = MC_j = -\lambda \quad \forall i, j$

case2: If we make one of the inequality constraint active i.e. $P_{gi} = \widehat{P_{gi}}$

According to KKT condition(4), $\widehat{\mu_i} \geqslant 0$, which implies that $MC_i + \lambda + \widehat{\mu_i} = 0$

Then, (1) $MC_j = MC_k = -\lambda \quad \forall j, k$

(2) $MC_i = -\lambda - \widehat{\mu_i} \leqslant \lambda$ because $\widehat{\mu_i} \geqslant 0$.

For i=2 and I changed the Demand value,D=450MW

(1). $MC_1 = MC_3 = -\lambda = -39.5379$

(2).$MC_2 = -\lambda - \widehat{\mu_2} = -39 - 0.5379 \leqslant \lambda$

From the above verification we can say that "If i^{th} generator is touching the upper limit, then the marginal cost of that generator will be less than the system marginal cost".

case3: If we make one of the inequality constraint active i.e. $P_{gi} = \widecheck{P_{gi}}$

According to KKT condition(4), $\check{\mu_i} \geqslant 0$, which implies that $MC_i + \lambda - \check{\mu_i} = 0$

Then, (1) $MC_j = MC_k = -\lambda \quad \forall j, k$

(2) $MC_i = -\lambda + \check{\mu_i} \geqslant \lambda$ because $\check{\mu_i} \geqslant 0$.

If we do the same verification from case 3, by taking D=200 ,i=1

(1). $MC_2 = MC_3 = -\lambda = -34.4544$

(2).$MC_1 = -\lambda + \widecheck{\mu_1} = -36.6060 + 0.5379 \geqslant \lambda$

From the above verification we can say that "If i^{th} generator is touching the lower limit, then the marginal cost of that generator will be greater than the system marginal cost", that means that particular dispatching unit has to pay the penalty factor.

1.4

case1: When D=450MW	
Dispatch of each plant P(MW)	Marginal Cost(Inc cost)
141.6797	39.5379
150.0000	39.0000
158.3203	39.5379

The total generation cost is 16233$/hr and the shadow price for the power balance constraint is 39.5379$/MWhr.Analysis is same as the first case (1.1).