Economic Dispatch Problem (Hw 2)

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Economic dispatch through QP method

The main aim of economic dispatch problem is to determine the generation dispatch that minimizes the operating cost function subject to operational constraints.Basically, to formulate the above definition mathematically we can write it as follows:

Objective Function

$$min \sum_{i=1}^{N_g} C_i(X_i) \tag{1}$$

Here, Decision vector

$$X_i = \begin{bmatrix} P_{g1} \\ . \\ . \\ P_{gi} \\ . \\ . \\ P_{gN_g} \end{bmatrix}$$

Here, If we look into the objective function in detail, that is if we plot the graph for fuel cost function(C_i) and power generated(P_{gi}) for i number of plants. Then we obtain curve most likely to be parabola.

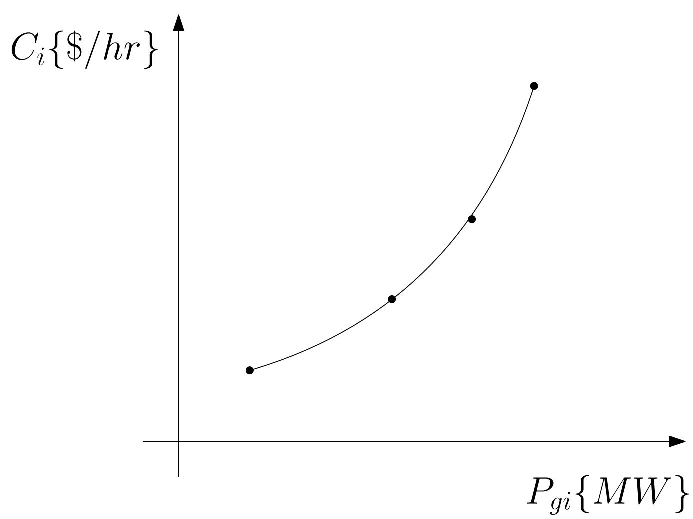

Figure 1: Fuel-Cost curve

Assuming one condition i.e. Network is going to be a Loss less network. So that we can write our objective function along with operational constraints in more meaningful way as follows:

Objective Function

$$min \sum_{i=1}^{N_g} \alpha_i P_{gi}^2 + \beta_i P_{gi} + \gamma_i \tag{2}$$

subject to:

$$\sum_{i=1}^{N_g} P_{gi} - D = 0 \tag{3}$$

$$P_{gi_{min}} \leqslant P_{gi} \leqslant P_{gi_{max}} \tag{4}$$

The operational constraints are Eq: (3) and Eq: (4) which basically tells us that:

(1).The total power generated must be equal to demand if it is a loss less network.

(2). Each plant has limits (minimum and maximum) on generating power that shouldn't exceed.

Characterizing the Minimization Problem

Mathematically, It is a constrained optimization problem. To understand this type of problem we use a calculus method that involves Lagrange function. In order to obtain the necessary conditions for the objective function, we should add the constraint functions to the objective function after the constraint function has been multiplied by lagrangian multiplier.

$$\begin{aligned}\mathcal{L}(P_{gi}, \lambda, \widehat{\mu}_i, \widecheck{\mu}_i) = &\sum_{i=1}^{N_g} \alpha_i P_{gi}^2 + \beta_i P_{gi} \\ &+ \lambda(\sum_{i=1}^{N_g} P_{gi} - D) \\ &+ \sum_{i=1}^{N_g} \widehat{\mu}_i (P_{gi} - \widehat{P_{gi}}) \\ &+ \sum_{i=1}^{N_g} \widecheck{\mu}_i (-P_{gi} + \widecheck{P_{gi}})\end{aligned} \tag{5}$$

KKT – conditions

The conditions for optimum for the point P^0_{gi}, λ^0, $\widehat{\mu}^0_i$, $\widecheck{\mu}^0_i$ are:

$$\frac{\partial \mathcal{L}}{\partial P_{gi}}(P^0_{gi}, \lambda^0, \widehat{\mu}^0_i, \widecheck{\mu}^0_i) = 0 \quad \forall i \in 1...\mathcal{N}_g \tag{6}$$

$$\sum_{i=1}^{N_g} P^0_{gi} - D = 0 \quad \forall i \in 1...\mathcal{N}_g \tag{7}$$

$$\begin{aligned} &(P^0_{gi} - \widehat{P_{gi}}) \leqslant 0 \\ &(-P^0_{gi} + \widecheck{P_{gi}}) \leqslant 0 \quad \forall i \in 1...\mathcal{N}_g \end{aligned} \tag{8}$$

$$\begin{aligned} &\widehat{\mu^0_i}(P^0_{gi} - \widehat{P_{gi}}) = 0 \\ &\widecheck{\mu^0_i}(-P^0_{gi} + \widecheck{P_{gi}}) = 0 \\ &\widehat{\mu^0_i} \geqslant 0 \\ &\widecheck{\mu^0_i} \geqslant 0 \quad \forall i \in 1...\mathcal{N}_g \end{aligned} \tag{9}$$

Loosely speaking, What lagrangian function tells us is, It took the constrained optimization problem and turned into a unconstrained optimization problem. Basically, Lagrange is characterization of minimum value of given objective function but not how to solve the problem.

Numerical Solution approach

Given data for our problem is, case1:D=250MW ,cas2:D=450MW and also,

Generator Data		
Bus	$P_{gi_{min}}$	$P_{gi_{max}}$
1	50	200
2	37.5	150
3	45	180

Generator Cost Function Data		
α_i	β_i	γ_i
0.00533	11.669	213.1
0.00889	10.333	200
0.00741	10.833	240

In order to make the generation costs more realistic, we need to multiply all coefficients $(\alpha_i, \beta_i, \gamma_i) \times 3$.

To proceed further with numerical solution for this minimization problem, we need to remove the cost function coefficient γ from our objective function because, γ is fixed cost, which is an offset. It will play a role in finding primary costs but doesn't effect the dispatch of the generators.So, objective function becomes:

Objective Function

$$min \sum_{i=1}^{N_g} \alpha_i P_{gi}^2 + \beta_i P_{gi} \tag{10}$$

subject to:

$$\sum_{i=1}^{N_g} P_{gi} - D = 0 \tag{11}$$

$$P_{gi_{min}} \leqslant P_{gi} \leqslant P_{gi_{max}} \tag{12}$$

Eq: (10), (11), (12) are general human understanding equation, to implement this equation in machine understanding form(For MATLAB using Quadprog command) we need to convert it into following form:

$$\begin{aligned} \min_x \frac{1}{2} x^T H x + f^T x \\ A * x = b, \\ A_e q * x = b_e q, \\ lb \leqslant x \leqslant ub \end{aligned}$$

Here,

$$A_{eq} = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}$$

$$b_{eq} = \begin{bmatrix} D \end{bmatrix}$$

$$H = \begin{bmatrix} 2\alpha_1 & \cdots & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & 2\alpha_i & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & \cdots & 2\alpha_{Ng} \end{bmatrix}$$

$$f^T = \begin{bmatrix} \beta_1 & \cdots & \beta_i & \cdots & \beta_{Ng} \end{bmatrix}$$

$$x = \begin{bmatrix} P_{g1} \\ \vdots \\ P_{gi} \\ \vdots \\ P_{gNG} \end{bmatrix}$$

$$lb = \begin{bmatrix} P_{gi_{min}} \end{bmatrix}$$

$$ub = \begin{bmatrix} P_{gi_{max}} \end{bmatrix}$$

Numerical Results

1.1

case1: When D=250MW	
Dispatch of each plant P(MW)	Marginal Cost(Inc cost)
51.0810	36.6406
105.7662	36.6406
93.1527	36.6406

The total generation cost is 10586$/hr and the shadow price for the power balance constraint is 36.6406$/MWhr.

Numerically verifying the KKT conditions:

Condition 1 :

$$\frac{\partial \mathcal{L}}{\partial P_{gi}}(P_{gi}^0, \lambda^0, \widehat{\mu}_i^0, \widecheck{\mu}_i^0) = 0 \quad \forall i \in 1...\mathcal{N}_g \tag{13}$$

i.e.

$$\frac{\partial \mathcal{L}}{\partial P_{gi}} = 0 \tag{14}$$

When i=1

$$\frac{\partial \mathcal{L}}{\partial P_{g1}} = 2\alpha_1 P_{g1} + \beta_1 + \lambda + \widehat{\mu_1} - \widecheck{\mu_1} = 2*3*(0.00533)(51.0810) + 3*11.669 + 36.6406 + 0 - 0 = 0 \tag{15}$$

We know that, Marginal cost$(MC) = \frac{\partial C_i}{\partial P_{g1}} = 2\alpha_1 P_{gi} + \beta_i = 2*3*(0.00533)(51.0810) + 3*11.669 =$ 36.6406 and which is also equal to $(MC) = \frac{\partial C_i}{\partial P_{g1}} = -\lambda$. So, the first condition is a necessary condition to obtain local optimum.

Condition 2 :

$$\sum_{i=1}^{N_g} P_{gi}^0 - D = 0 \quad \forall i \in 1...\mathcal{N}_g \tag{16}$$

for i=1:3

$$\sum_{i=1}^{N_g} P_{gi}^0 - D = 51.0810 + 105.7662 + 93.1527 - 250 = 0 \tag{17}$$

This condition is based on equality constraint, we know that equality constraints are always active. Therefore, they always have a value i.e. Lambda will always have a value different than zero. This Lambda is the shadow price associated with that constraint.

More precisely, If we relax the equality constraint by 1 unit, we can observe the change in the objective function but we cannot tell that change in objective function can be positive or negative. So that lambda can also be positive or negative, this is the main reason for not being an Lagrangian condition.

The value of lambda corresponds to the change in the objective function, when the corresponding constraint is relaxed by "1" unit.

To explain that numerically, I am going to increase the load demand by 1 unit, Therefore we can observe the changes as follows:

case1: When D=251MW	
Dispatch of each plant P(MW)	Marginal Cost(Inc cost)
51.5123	36.6544
105.0248	36.6544
93.4629	36.6544

The total generation cost is 10622.6\$/hr.

So, If we see the change in the total generation cost, Change=$10622.6 - 10586 = 36.6$, which is nothing but the value of lambda. Therefore, lambda is the value, of the change observed in total generation cost when the constraint is relaxed by 1 unit.

Condition 3 :

$$\begin{aligned} &(P_{gi}^0 - \widehat{P_{gi}}) \leqslant 0 \\ &(-P_{gi}^0 + \widecheck{P_{gi}}) \leqslant 0 \quad \forall i \in 1...\mathcal{N}_g \end{aligned} \tag{18}$$

for i=1

$$\begin{aligned} &(P_{gi}^0 - \widehat{P_{gi}}) = 51.0810 - 200 = -148.919 \leqslant 0 \\ &(-P_{gi}^0 + \widecheck{P_{gi}}) = -51.0810 + 50 = -1.0810 \leqslant 0 \end{aligned} \tag{19}$$

So, these inequality constraints are necessary conditions and they are satisfied with optimum value.

Condition 4 :

$$\begin{aligned} &\widehat{\mu_i^0}(P_{gi}^0 - \widehat{P_{gi}}) = 0 \\ &\widecheck{\mu_i^0}(-P_{gi}^0 + \widecheck{P_{gi}}) = 0 \\ &\widehat{\mu_i^0} \geqslant 0 \\ &\widecheck{\mu_i^0} \geqslant 0 \quad \forall i \in 1...\mathcal{N}_g \end{aligned} \tag{20}$$

case1: If optimum value of dispatch of generators is with in the limits, i.e.$\widecheck{P_{gi}} < P_{gi} < \widehat{P_{gi}}$

case1: When D=250MW		
$P_{gi_{min}}$	Dispatch of each plant P(MW)	$P_{gi_{max}}$
50	51.5123	200
37.5	105.0248	150
45	93.4629	180

Here, $(P_{gi}^0 - \widehat{P_{gi}}) = -148.919 \leqslant 0$ i.e inequality constraints are inactive, According to KKT condition 4, $\widehat{\mu}_i$ has to be zero i.e. $\widehat{\mu}_i = 0$ this is same for the $\widecheck{\mu}_i$.

Therefore, $\widehat{\mu_i} = \widecheck{\mu_i} = 0$ from KKT condition(1) we can also write that,

$$\frac{\partial \mathcal{L}}{\partial P_{g1}} = 2\alpha_1 P_{g1} + \beta_1 + \lambda + \widehat{\mu_1} - \widecheck{\mu_1} = 2*3*(0.00533)(51.0810) + 3*11.669 + 36.6406 + 0 - 0 = 0 \quad (21)$$

Eq: (21) implies that $MC_i = MC_j = -\lambda \quad \forall i, j$

case2: If we make one of the inequality constraint active i.e. $P_{gi} = \widehat{P_{gi}}$

According to KKT condition(4), $\widehat{\mu_i} \geqslant 0$, which implies that $MC_i + \lambda + \widehat{\mu_i} = 0$

Then, (1) $MC_j = MC_k = -\lambda \quad \forall j, k \neq i$

(2) $MC_i = -\lambda - \widehat{\mu_i} \leqslant \lambda$ because $\widehat{\mu_i} \geqslant 0$.

For i=2 and I changed the Demand value,D=450MW

(1). $MC_1 = MC_3 = -\lambda = -39.5379$

(2).$MC_2 = -\lambda - \widehat{\mu_2} = -39 - 0.5379 \leqslant \lambda$

From the above verification we can say that "If i^{th} generator is touching the upper limit, then the marginal cost of that generator will be less than the system marginal cost".

case3: If we make one of the inequality constraint active i.e. $P_{gi} = \widecheck{P_{gi}}$

According to KKT condition(4), $\widecheck{\mu_i} \geqslant 0$, which implies that $MC_i + \lambda - \widecheck{\mu_i} = 0$

Then, (1) $MC_j = MC_k = -\lambda \quad \forall j, k \neq i$

(2) $MC_i = -\lambda + \widecheck{\mu_i} \geqslant \lambda$ because $\widecheck{\mu_i} \geqslant 0$.

If we do the same verification from case 3, by taking D=200 ,i=1

(1). $MC_2 = MC_3 = -\lambda = -34.4544$

(2).$MC_1 = -\lambda + \widecheck{\mu_1} = -36.6060 + 0.5379 \geqslant \lambda$

From the above verification we can say that "If i^{th} generator is touching the lower limit, then the marginal cost of that generator will be greater than the system marginal cost", that means that particular dispatching unit has to pay the penalty factor.

1.4

case2: When D=450MW	
Dispatch of each plant P(MW)	Marginal Cost(Inc cost)
141.6797	39.5379
150.0000	39.0000
158.3203	39.5379

The total generation cost is 18192$/hr and the shadow price for the power balance constraint is 39.5379$/MWhr.Analysis is same as the first case (1.1).

Economic dispatch through LP method

Economic dispatch through LP method is done by transforming the quadratic cost functions to piecewise linear functions considering "n" segments , splitting the power range into "n" segments. For our problem n is 2. So we are going to divide the cost curve into 2 segments as shown in Figure: (2).

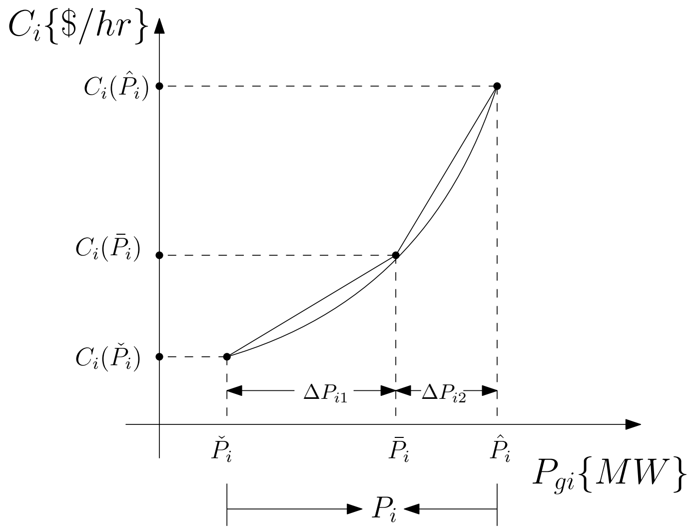

Figure 2: Fuel-Cost curve approximated by straight line segments

Here, $\bar{P} = \frac{\hat{P}+\check{P}}{2}$ is the power at which the quadratic function is divide into two line segments. Now $\Delta P_{i1} = \bar{P}_i - \check{P}_i$ and $\Delta P_{i2} = \hat{P}_i - \bar{P}_i$.

As we said earlier we are going to transform the quadratic cost function into linear cost function as follows:

$$C_i(P_i) = \sum_{i=1}^{3} C_i(\check{P}_i) + m_{i1}P_{i1} + m_{i2}P_{i2} \tag{22}$$

Here,

$$m_{i1} = \frac{C_i(\bar{P}_i) - C_i(\check{P}_i)}{\bar{P}_i - \check{P}_i} \quad \forall i \in 1,.,3. \tag{23}$$

and

$$m_{i2} = \frac{C_i(\hat{P}_i) - C_i(\bar{P}_i)}{\hat{P}_i - \bar{P}_i} \quad \forall i \in 1,.,3. \tag{24}$$

We know that the main aim of economic dispatch problem is to determine the generation dispatch that minimizes the operating cost function subject to operational constraints. In this case our minimization problem becomes:

Objective Function

$$min \sum_{i=1}^{3} C_i(P_i) \tag{25}$$

subject to:

$$\sum_{i=1}^{3} P_i - D = 0 \tag{26}$$

$$\check{P}_i \leqslant P_{gi} \leqslant \hat{P}_i \tag{27}$$

From the figure: (2), The dispatch of each plant is the sum of minimum value each individual generators and their power in each segments, which is written as follows:

$$\sum_{i=1}^{3} P_i = \sum_{i=1}^{3} \check{P}_i + P_{i1} + P_{i2}$$

We can also write the objective function from Eq: (22) as follows:

Objective Function

$$min\ \ C1(\check{P}_1) + m_{11}P_{11} + m_{12}P_{12} + C_2(\check{P}_2) + m_{21}P_{21} + m_{22}P_{22} + C_3(\check{P}_3) + m_{31}P_{31} + m_{32}P_{32} \tag{28}$$

subject to:

$$P_{11} + P_{12} + P_{21} + P_{22} + P_{31} + P_{32} = D - \check{P}_1 - \check{P}_2 - \check{P}_3 \tag{29}$$

$$\begin{aligned} 0 &\leqslant P_{11} \leqslant \Delta P_{11} \\ 0 &\leqslant P_{12} \leqslant \Delta P_{12} \\ 0 &\leqslant P_{21} \leqslant \Delta P_{21} \\ 0 &\leqslant P_{22} \leqslant \Delta P_{22} \\ 0 &\leqslant P_{31} \leqslant \Delta P_{31} \\ 0 &\leqslant P_{32} \leqslant \Delta P_{32} \end{aligned} \tag{30}$$

Numerical Solution approach

Given data for our problem is, Demand, D=250MW and also,

Generator Data		
Bus	$P_{gi_{min}}$	$P_{gi_{max}}$
1	50	200
2	37.5	150
3	45	180

To proceed further with numerical solution for this minimization problem, we need to remove the cost function $C_i(\check{P}_i) \quad \forall i \in 1,.,3$ from our objective function because,$C_i(\check{P}_i)$, which is an offset. It will play a role in finding total generating costs but doesn't effect the dispatch of the generators.So, objective function becomes:

Objective Function

$$min \;\; m_{11}P_{11} + m_{12}P_{12} + m_{21}P_{21} + m_{22}P_{22} + m_{31}P_{31} + m_{32}P_{32} \tag{31}$$

subject to:

$$P_{11} + P_{12} + P_{21} + P_{22} + P_{31} + P_{32} = D - \check{P}_1 - \check{P}_2 - \check{P}_3 \tag{32}$$

$$\begin{aligned} 0 \leqslant P_{11} \leqslant \Delta P_{11} \\ 0 \leqslant P_{12} \leqslant \Delta P_{12} \\ 0 \leqslant P_{21} \leqslant \Delta P_{21} \\ 0 \leqslant P_{22} \leqslant \Delta P_{22} \\ 0 \leqslant P_{31} \leqslant \Delta P_{31} \\ 0 \leqslant P_{32} \leqslant \Delta P_{32} \end{aligned} \tag{33}$$

Eq: (31), (32), (33) are general human understanding equation, to implement this equation in machine understanding form(For MATLAB using linprog command) we need to convert it into following form:

$$\begin{aligned} \min_x f^T x \\ A * x = b, \\ A_e q * x = b_e q, \\ lb \leqslant x \leqslant ub \end{aligned}$$

Here,

$$A_{eq} = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}$$

$$b_{eq} = \begin{bmatrix} D - \check{P}_1 - \check{P}_2 - \check{P}_3 \end{bmatrix}$$

$$f^T = \begin{bmatrix} \beta_1 & \cdots & \beta_3 \end{bmatrix}$$

$$x = \begin{bmatrix} P_{g1} \\ \vdots \\ P_{g3} \end{bmatrix}$$

$$lb = \begin{bmatrix} P_{gi_{min}} \end{bmatrix}$$

$$ub = \begin{bmatrix} P_{gi_{max}} \end{bmatrix}$$

Numerical Results

2.1

When D=250MW
Dispatch of each plant P(MW)
50
93.75
106.25

The total generation cost is 10843$/hr and the shadow price(Lambda) for the power balance constraint is 37.5254$/MWhr.

Numerically verifying the KKT 4 conditions:

Lagrangian

$$\begin{aligned}\mathcal{L}(P_i, \lambda, \widehat{\mu}_i, \breve{\mu}_i) = &\sum_{i=1}^{3} C_i(\check{P}_i) + m_{i1}P_{i1} + m_{i2}P_{i2} \\ &+ \lambda((\sum_{i=1}^{3} P_i) - D) \\ &+ \sum_{i=1}^{3} \widehat{\mu}_i(P_{i1} - \Delta P_{i1}) \\ &+ \sum_{i=1}^{3} \widehat{\mu}_i(P_{i2} - \Delta P_{i2}) \\ &+ \sum_{i=1}^{3} \breve{\mu}_i(-P_{i1}) \\ &+ \sum_{i=1}^{3} \breve{\mu}_i(-P_{i2})\end{aligned} \tag{34}$$

Condition 1 :

$$\frac{\partial \mathcal{L}}{\partial P_i}(P_i^0, \lambda^0, \widehat{\mu}_i^0, \breve{\mu}_i^0) = 0 \quad \forall i \in 1...3 \tag{35}$$

i.e. Derivative of lagrangian with respective to i_{th} generator dispatch should be equal to zero at local optimum point

$$\frac{\partial \mathcal{L}}{\partial P_i} = 0 \tag{36}$$

Condition 2 :

$$\sum_{i=1}^{3} P_i^0 - D = 0 \tag{37}$$

for i=1:3

$$\sum_{i=1}^{3} P_i^0 - D = 50 + 93.75 + 106.25 - 250 = 0 \tag{38}$$

This condition is based on equality constraint, we know that equality constraints are always active. Therefore, they always have a value i.e. Lambda will always have a value different than zero. This Lambda which is a lagrangian multiplier of equality constraint, it is the shadow price associated with that constraint.

More precisely, If we relax the equality constraint by 1 unit, we can observe the change in the objective function but we cannot tell that change in objective function can be positive or negative. So that lambda can also be positive or negative, this is the main reason for not being an Lagrangian condition.

The value of lambda corresponds to the change in the objective function, when the corresponding constraint is relaxed by "1" unit.

To explain that numerically, I am going to increase the load demand by 1 unit, Therefore we can observe the changes as follows:

When D=251MW
Dispatch of each plant P(MW)
50
93.75
106.25

The total generation cost is 10880\$/hr.

So, If we see the change in the total generation cost, Change=$10880 - 10843 = 37$, which is nothing but the value of lambda. Therefore, lambda is the value, of the change observed in total generation cost when the constraint is relaxed by 1 unit.

Condition 3 :

case1: When D=250MW			
Lower Bound(LB)	X	values of X	Upper Bound(UB)ΔP
0	P_{11}	0	75
0	P_{12}	0	75
0	P_{21}	56.25	56.25
0	P_{22}	0	56.25
0	P_{31}	61.25	67.5
0	P_{32}	0	67.5

If we observe the above table, we can say that $P_{i1} - \Delta P_i 1 \leqslant 0$, $P_{i2} - \Delta P_i 2 \leqslant 0$, $-P_{i1} \leqslant 0$, $-P_{i2} \leqslant 0$.For example take i=1, then, $P_{i1} - \Delta P_i 1 = 0 - 75 = -75 \leqslant 0$, $P_{i2} - \Delta P_i 2 = 0 - 75 = 75 \leqslant 0$, $-P_{i1} = 0 \leqslant 0$, $-P_{i2} = 0 \leqslant 0$.

So, these inequality constraints are necessary conditions and they are satisfied with optimum value.

Condition 4 :

For Generator 1 dispatch is $P_1 = 50$ MW, i.e. $P_{g1} = \check{P}_{g1}$ generator 1 dispatch is touching it's lower limit.According to KKT condition (4) $\check{\mu}_i \geqslant 0$ which implies that $MC_i + \lambda - \check{\mu}_i = 0$

Then, (1)$MC_j = MC_k = -\lambda \quad \forall j, k \neq i$

(2) $MC_i = -\lambda + \check{\mu}_i \geqslant \lambda$ becuase $\check{\mu}_i \geqslant 0$

For D=250 ,i=1

(1). $MC_2 = MC_3 = -\lambda = -37.5254$

(1).$MC_1 = -\lambda + \widecheck{\mu_{i1}} + \widecheck{\mu_{i2}} = -37.5254 + 0.2798 + 2.6783 \geqslant \lambda$

From the above verification we can say that "If i^{th} generator is touching the lower limit, then the marginal cost of that generator will be greater than the system marginal cost", that means that particular dispatching unit has to pay the penalty factor.

Observations

If we observe the lagrangian multipliers associated with inequality constraints of both methods, namely through QP method and LP method.

In QP method, multipliers are written in the table below. Both the $(\check{\mu}_i)$ and $(\hat{\mu}_i)$ values are zero,i.e. because, none of the dispatches (P_i) of individual generators are touching the limits (i.e.$\check{P}_i < P_i < \hat{P}_i$).

case1: When D=250MW		
Lagrangian multipliers	LAMBDA.lower($\check{\mu}_i$)	LAMBDA.upper($\hat{\mu}_i$)
μ_1	0	0
μ_2	0	0
μ_3	0	0

case1: When D=250MW		
$P_{gi_{min}}$	Dispatch of each plant P(MW)	$P_{gi_{max}}$
50	51.5123	200
37.5	105.0248	150
45	93.4629	180

In LP method, multipliers are written in the table below. For generator 1, multipliers $\breve{\mu}_{i1} \geqslant 0$ and $\breve{\mu}_{i2} \geqslant 0$that is the inequality constraint associated with that multiplier is active, that is reason for $P_{11} = 0$ and $P_{12} = 0$. Here, If we observe the final dispatch of generators by LP method, generator 1 is touching it's lower limit which explains the reason mentioned above.

case1: When D=250MW		
Lagrangian multipliers	LAMBDA.lower($\breve{\mu}_i$)	LAMBDA.upper($\hat{\mu}_i$)
μ_{11}	0.2798	0
μ_{12}	2.6783	0
μ_{21}	0	0.4197
μ_{22}	1.3791	0
μ_{31}	0	0
μ_{32}	2.1587	0

case1: When D=250MW			
Lower Bound(LB)	X	values of X	Upper Bound(UB)ΔP
0	P_{11}	0	75
0	P_{12}	0	75
0	P_{21}	56.25	56.25
0	P_{22}	0	56.25
0	P_{31}	61.25	67.5
0	P_{32}	0	67.5

case1: When D=250MW		
$P_{gi_{min}}$	Dispatch of each plant P(MW)	$P_{gi_{max}}$
50	50	200
37.5	93.75	150
45	106.25	180

2.2

The penalty cost for the power system, when we use LP method instead of QP method to solve economic dispatch problem is calculated as follows:

The total generation cost calculated from QP method is 10586\$/hr and the total generation cost calculated from LP method is 10843\$/hr. Difference between them is given by the penalty cost, i.e.

$$Penalty\ cost = 10843 - 10586 = 257\$/hr \tag{39}$$

Environmental dispatch through QP method

In recent studies about optimum economic dispatch has raised many concerns regarding it's performance related to environment friendliness. And also, due to other policies regarding the protection of environment, environmental dispatch had came into the picture. In environmental dispatch, basic constraints of the economic dispatch problem remains same but the model is optimized to minimize pollutant emissions (fly ash, green house gases etc.) in addition to minimizing fuel costs. Due to this considerations, concerning reduction of pollution further complicate the power dispatch problem.

In our problem we are asked to minimize the fly ash content going out into the air in (lb/hr).

Problem Setup

In this problem we need to convert the quadratic cost function ($\$/hr$) from economic dispatch into cost function(lb/hr) related to fly ash, which is a environmental dispatch problem. This can be done using the given data:

1.Cost of coal used in all three power plants is $42\$/st$, Here, st is short tonne, which is nearly equal to 2000lb.

2. The combustion process in each unit results in 11.75% of the coal by weight going up the stack as fly ash. Units 1,2 and 3 have precipitators installed that remove, respectively, 95%,90% and 92% of the fly ash. Therefore fly ash content that needs to be multiply with each generator is given by these following values:

Generator1: Flyash content that needs to be multiplied with cost function is $FC_1 = \frac{2000lb}{42\$} * \frac{11.75}{100} * \frac{5}{100} = 0.28$.

Generator2: Flyash content that needs to be multiplied with cost function is $FC_2 = \frac{2000lb}{42\$} * \frac{11.75}{100} * \frac{10}{100} = 0.56$.

Generator3: Flyash content that needs to be multiplied with cost function is $FC_3 = \frac{2000lb}{42\$} * \frac{11.75}{100} * \frac{8}{100} = 0.448$.

3.Total demand is D=250MW.

4.

Generator Data		
Bus	$P_{gi_{min}}$	$P_{gi_{max}}$
1	50	200
2	37.5	150
3	45	180

5.

Generator Cost Function Data		
α_i	β_i	γ_i
0.00533	11.669	213.1
0.00889	10.333	200
0.00741	10.833	240

In order to make the generation costs more realistic, we need to multiply all coefficients $(\alpha_i, \beta_i, \gamma_i) \times 3$.

So, objective function for our minimization problem becomes

$$min \sum_{i=1}^{3} \alpha_i P_{gi}^2 + \beta_i P_{gi} + \gamma_i * FC_i \tag{40}$$

subject to:

$$\sum_{i=1}^{3} P_{gi} - D = 0 \tag{41}$$

$$P_{gi_{min}} \leqslant P_{gi} \leqslant P_{gi_{max}} \quad \forall i \in 1...3. \tag{42}$$

To proceed further with numerical solution for this minimization problem, we need to remove the cost function coefficient γ from our objective function because, γ is fixed cost, which is an offset. It will play a role in finding primary costs but doesn't effect the dispatch of the generators.So, objective function becomes:

Objective Function

$$min \sum_{i=1}^{3} \alpha_i P_{gi}^2 + \beta_i P_{gi} * FC_i \tag{43}$$

subject to:

$$\sum_{i=1}^{3} P_{gi} - D = 0 \tag{44}$$

$$P_{gi_{min}} \leqslant P_{gi} \leqslant P_{gi_{max}} \quad \forall i \in 1...3 \tag{45}$$

To implement this minimization problem in machine understanding form(For MATLAB using Quadprog command) we need to convert it into following form:

$$\begin{aligned} \min_x \frac{1}{2} x^T H x + f^T x \\ A * x = b, \\ A_e q * x = b_e q, \\ lb \leqslant x \leqslant ub \end{aligned}$$

Here,

$$A_{eq} = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}$$

$$b_{eq} = \begin{bmatrix} D \end{bmatrix}$$

$$H = \begin{bmatrix} 2\alpha_1 * 0.28 & 0 & 0 \\ 0 & 2\alpha_2 * 0.56 & 0 \\ 0 & 0 & 2\alpha_3 * 0.448 \end{bmatrix}$$

$$f^T = \begin{bmatrix} \beta_1 * 0.28 & \beta_2 * 0.56 & \beta_3 * 0.448 \end{bmatrix}$$

$$x = \begin{bmatrix} P_{g1} \\ P_{g2} \\ P_{g3} \end{bmatrix}$$

$$lb = \begin{bmatrix} P_{gi_{min}} \end{bmatrix}$$

$$ub = \begin{bmatrix} P_{gi_{max}} \end{bmatrix}$$

Numerical Results

3.1

case1: When D=250MW	
Dispatch of each plant P(MW)	Emissions in lb/MWhr
167.5	11.3018
37.5	18.4796
45	15.4559

The cost function represents the total amount of fly ash going out in air is 3952.3lb/hr and the emissions of 11.3018lbs of fly ash per MWhr(5.126Kgs of fly ash per MWhr).

3.2

The economic cost in (\$/hr) for environmental dispatch is given by following equations:

$$\begin{aligned} C1 &= 3 * 213.1 + 3 * 11.669 * 167.5 + 3 * 0.00533 * 167.5^2 \\ C2 &= 3 * 200 + 3 * 10.333 * 37.5 + 3 * 0.00889 * 37.5^2 \\ C3 &= 3 * 240 + 3 * 10.833 * 45 + 3 * 0.00741 * 45^2 \end{aligned} \tag{46}$$

$$\begin{aligned} C1 &= 6952 \\ C2 &= 1800 \\ C3 &= 2227 \end{aligned} \tag{47}$$

Therefore, the total economic cost is 10980 $\$/hr$ for environmental dispatch of generators.
Whereas,

case1: When D=250MW	
Dispatch of each plant P(MW)	Marginal Cost(Inc cost)
51.0810	36.6406
105.7662	36.6406
93.1527	36.6406

The total generation cost for economic dispatch of generators is 10586$/hr.

Therefore, the penalty cost associated with environmental dispatch when compared to economic dispatch is, penalty cost is 394$/hr.

The environmental cost(lb/hr) for economic dispatch is calculated using following equations:

$$\begin{aligned}
C1 &= 0.28(3*213.1 + 3*11.669*51.0810 + 3*0.00533*51.0810^2) \\
C2 &= 0.56(3*200 + 3*10.333*105.7662 + 3*0.00889*105.7662^2) \\
C3 &= 0.448(3*240 + 3*10.833*93.1527 + 3*0.00741*93.1527^2)
\end{aligned} \tag{48}$$

The total amount of fly ash going out in air through economic dispatch of generators is 4796lb/hr.
Whereas,

case1: When D=250MW	
Dispatch of each plant P(MW)	Marginal Cost(Inc cost)
167.5	11.3018
37.5	18.4796
45	15.4559

The total amount of fly ash going out in air through environmental dispatch is 3952.3lb/hr.

Therefore, the penalty cost(lb/hr) associated with economic dispatch when compared to environmental dispatch is, penalty cost of economic dispatch or the amount of fly ash that is going out into atmosphere more than the environmental dispatch is 843.7 lb/hr.

Conclusion

Though there are different methods to solve dispatch problem, each has its own significance in their own way. If one wants to use the economic dispatch they have to compromise about environmental friendliness or one wants to take benefits from environmental dispatch, then they have to compromise about the cost of that dispatch. This is explained clearly in above results in numbers, which say that if you use environmental dispatch for your power system operation then you have to pay an extra cost of 394$/hr more than the economic dispatch, which is not good for your cost of operation of power system. And also, if you want to use economic dispatch then you will release fly ash of 843.7lb/hr more into atmosphere than the environmental dispatch which is not good for environment.