Multigroup treatment of P_N:

Recall our multigroup transport equation that we derived several lectures ago:

$$\frac{d}{dt}\left[\frac{1}{\bar{v}_g}\varphi_g(\vec{r},\hat{\Omega},t)\right] = -\Omega\cdot\nabla\varphi_g(\vec{r},\hat{\Omega},t)$$
$$-\Sigma_{t,g}(\vec{r},t)\,\varphi_g(\vec{r},\hat{\Omega},t)$$
$$+\sum_{g'=1}^{G}\int_{4\pi}\Sigma_{s,g'\to g}(\vec{r},\hat{\Omega}'\to\hat{\Omega},t)\,\varphi_{g'}(\vec{r},\Omega',t)\,d\hat{\Omega}'$$
$$+\frac{\chi_g}{4\pi}\sum_{g'=1}^{G}\overline{\nu\Sigma_f}(\vec{r},t)\int_{4\pi}\varphi_{g'}(\vec{r},\Omega',t)\,d\hat{\Omega}'$$

Recall as we derived this we defined several terms in terms of their discretized energies.

$$\varphi_g(\vec{r},\hat{\Omega},t) = \int_{E_g}^{E_{g-1}}\varphi(\vec{r},\hat{\Omega},E,t)\,dE$$

$$\chi_g = \int_{E_g}^{E_{g-1}}\chi(E)\,dE$$

$$\overline{\Sigma}_{x,g}(\vec{r},t) = \frac{\int_{E_g}^{E_{g-1}}\Sigma_x(\vec{r},E,t)\,\varphi(\vec{r},\hat{\Omega},E,t)\,dE}{\int_{E_g}^{E_{g-1}}\varphi(\vec{r},\hat{\Omega},E,t)\,dE}$$

$\varphi_g(\vec{r},\hat{\Omega},t)$

Let us return to our 1D planar geometry

$$\left(\mu \frac{d}{dx} + \Sigma(x)\right) \varphi(x, E, \mu)$$

$$= \int_0^\infty \int_{4\pi} \Sigma_s(x, E' \to E, \mu' \to \mu)\, \varphi(x, E', \mu')\, dE'\, d\mu'\,,$$

$$+ \; Q(x, E, \mu)$$

To expand $\Sigma_s(x, E' \to E, \mu' \to \mu)$ we proceed as we previously did by using

$\mu_0 = \mu' - \mu$

$$\Sigma_s(x, E' \to E, \mu_0) = \sum_{\ell=0}^{\infty} (2\ell+1)\, \Sigma_{s\ell}(x, E' \to E)\, P_\ell(\mu_0)$$

where

$$\Sigma_{s\ell}(x, E' \to E) = \frac{1}{2} \int_{-1}^{1} \Sigma_s(x, E' \to E, \mu_0)\, P_\ell(\mu_0)\, d\mu_0$$

The integral equation becomes:

$$\left(\mu \frac{d}{dx} + \Sigma_t(x, E)\right) \varphi(x, E, \mu) =$$

$$\underbrace{\int_0^\infty \int_{-1}^{1} \sum_{\ell=0}^{\infty} (2\ell+1)\, \Sigma_{s\ell}(x, E' \to E)\, P_\ell(\mu)\, P_\ell(\mu')\, \varphi(x, E', \mu')\, d\mu\, dE'}_{\text{rearranging} \;\downarrow} + Q$$

$$= \sum_{\ell=0}^{\infty} (2\ell+1)\, P_\ell(\mu) \int_0^\infty \Sigma_{s\ell}(x, E' \to E) \underbrace{\int_{-1}^{1} P_\ell(\mu')\, \varphi(x, E', \mu')\, d\mu'}_{\Phi_\ell(x, E')}\, dE'$$

Just as we did for the multigroup transport equations we can integrate the LHS + RHS over a particular group bounds to get that group's equation

generally:

$$\int_g dE = \int_{E_g}^{E_{g-1}} dE \Rightarrow \int_{E_g \text{ (low)}}^{E_{g-1} \text{ (high)}} dE$$

and then we sum all the energy group equations to get the totals

$$\sum_{g=0}^{G}$$

doing so on our 1D equation, the group equation is

$$\mu \frac{d}{dx} \psi_g(x,\mu) + \bar{\Sigma}_{tg}(x,\mu)\, \psi_g(x,\mu)$$
$$= \underbrace{\sum_{l=0}^{\infty} (2l+1) P_l(\mu) \sum_{g'=1}^{G} \bar{\Sigma}_{lgg'}(x)\, \phi_{lg'}(x)}_{\text{new form of the scattering term}} + Q_g$$

which is comprised of the following definitions:

$$\psi_g(x,\mu) = \int_g \psi(x,E,\mu)\, dE$$

$$\overline{\Sigma}_{t,g}(x,\mu) = \frac{\int_g \Sigma_t(x,E)\,\psi(x,E,\mu)\,dE}{\int_g \psi(x,E,\mu)\,dE}$$

$$\phi_{\ell g}(x) = \frac{1}{2}\int_{-1}^{1} P_\ell(\mu)\,\psi_g(x,\mu)\,d\mu$$

note that this is dependent on the angular flux, which is undesirable. We are trying to solve for the flux.

we expand the angular flux:

$$\sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\mu)\,\Sigma_{\ell g}(x)\,\phi_{\ell g}(x) = \psi(x,E,\mu)$$

where

$$\overline{\Sigma}_{\ell g}(x) = \frac{\int_g \Sigma(x,E)\ \phi_\ell(x,E)\,dE}{\phi_{\ell g}(x)}$$

and

$$\phi_{\ell g}(x) = \int_g \phi_\ell(x,E)\,dE$$

The fission term:

So far we've had an arbitrary source in these 1D equations, but the group wise fission source is also important!

$$Q_g = \int_{E_g}^{E_{g-1}} \chi(E) \int_0^\infty \frac{\nu}{k} \Sigma_f(E') \phi(x, E') dE' dE$$

$$= \frac{1}{k} \chi_g \sum_{g'=1}^{G} \overline{(\nu \Sigma_f)}_{g'} \phi_g$$

where

$$\phi_g(x) = \int_{E_g}^{E_{g-1}} \left(\int_{-1}^{1} \varphi(x, E, \mu) d\mu \right) dE$$

$$\bar{\nu}_{g'} \bar{\Sigma}_{fg'} = \overline{(\nu \Sigma_f)}_{g'} = \frac{1}{\phi_{g'}} \int_{E_g}^{E_{g-1}} \Sigma_f(E') \nu(E') \phi(E') dE'$$

$$\chi_g = \int_{E_g}^{E_{g-1}} \chi(E) dE$$

where k = multiplication constant

Typically, a multigroup solution requires group constants calculated the following way:

$\phi'(E)$

0 | Decide energy group structure
1 | Guess / Assume flux spectrum
2 | Calculate group constants
3 | Conduct a transport calculation
4 | Compare calculated flux spectrum to your guess
5 | If they're different, return to step 0 or 1
6 | If they're similar, end

The input to typical deterministic codes
(Denovo, Partisn, Pares, Dantsys) is
a cross section matrix [XSTYPE]×[GROUPS]
and a scattering matrix that captures
scattering XS [GROUPS]×[GROUPS]