Neutron Transport Methods

Madicken Munk & Kathryn Huff

Third Computational Project

- Do not merely repeat the methods in the previous Computational Projects or computer-assisted homework assignments.
- This lecture is intended to introduce you to a grab bag of methods to consider implementing instead.
- In addition to the report and code, your project includes a 10 minute presentation, to be presented during finals week.
- Include references to primary sources (journal articles)
- Clearly explain in your report what approximations your method makes (energy, angle, space).

Third Computational Project

- Pick 1:
  - Neutron transport method (if MC, P_N , or S_N must add advanced features)
- Pick 1:
  - Basic problem: Slab
  - Basic problem: Eigenvalue
  - Basic problem: Detector Response
- Pick 2:
  - Challenge problem: Sphere, or other (propose to Munk)
  - Demonstrate spatial convergence (if mesh-based)
  - Demonstrate order of accuracy convergence (angular order of accuracy)

C_N Method

- Quasi analytical, developed at Saclay in the 1970s
- Based on “Placzek lemma”
- An integral equation is provided for the angular flux at the boundary, and its kernel is the infinite medium Green's function.
- Computing time is independent of the size of the domain.
- Knowledge of the angular flux at the boundaries of the media allows the fluxes to be calculated at any point.

Recommended Paper: Kavenoky, A., 1978. TheCN Method of Solving the Transport Equation: Application to Plane Geometry. Nuclear Science and Engineering 65, 209–225. https://doi.org/10.13182/NSE78-A27152

The C_N Method of Solving the Transport Equation: Application to Plane Geometry

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The C_N method of solving the transport equation has been developed at Saclay during the past few years. This method is based on a lemma proved by Placzek; an integral equation is provided for the angular flux at the boundary of the various media, and its kernel is the infinite medium Green's function. Four plane geometry problems are solved in one-velocity theory, with a linearly anisotropic scattering kernel: the albedo for the Milne problem, the extrapolation length for the same problem, albedo and transmission factor for slabs, and the critical thickness for slab reactors. Numerical results are obtained and compared with data computed by reference methods.

F_N Method

- Quasi analytical, modified version of C_N
- Relies on the “Placzek lemma” and the Heaviside step function (H)
- Good results for the half space and the finite slab.

Recommended Paper: Siewert, C.E., Benoist, P., 1979. The FN Method in Neutron-Transport Theory. Part I: Theory and Applications. Nuclear Science and Engineering 69, 156–160. https://doi.org/10.13182/NSE79-1

The F_N Method in Neutron-Transport Theory. Part I: Theory and Applications

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The Placzek lemma is used to establish a system of singular integral equations and constraints that is solved uniquely for a half-space to yield the exact exit distribution. These singular integral equations and constraints are also used to develop a new approximation, the F_N method, that yields concise and accurate results for the half space and the finite slab.

Chebyshev polynomial approximation (T_N)

- Similar to P_N, but with Chebyshev rather than Legendre polynomials.
- T_N for the one dimensional neutron transport equation was first proposed by the authors Aspelund [1] and Conkie [2].

Recommended Paper: Anli, F., Yaşa, F., Güngör, S., Öztürk, H., 2006. TN approximation to neutron transport equation and application to critical slab problem. Journal of Quantitative Spectroscopy and Radiative Transfer 101, 129–134. https://doi.org/10.1016/j.jqsrt.2005.11.010

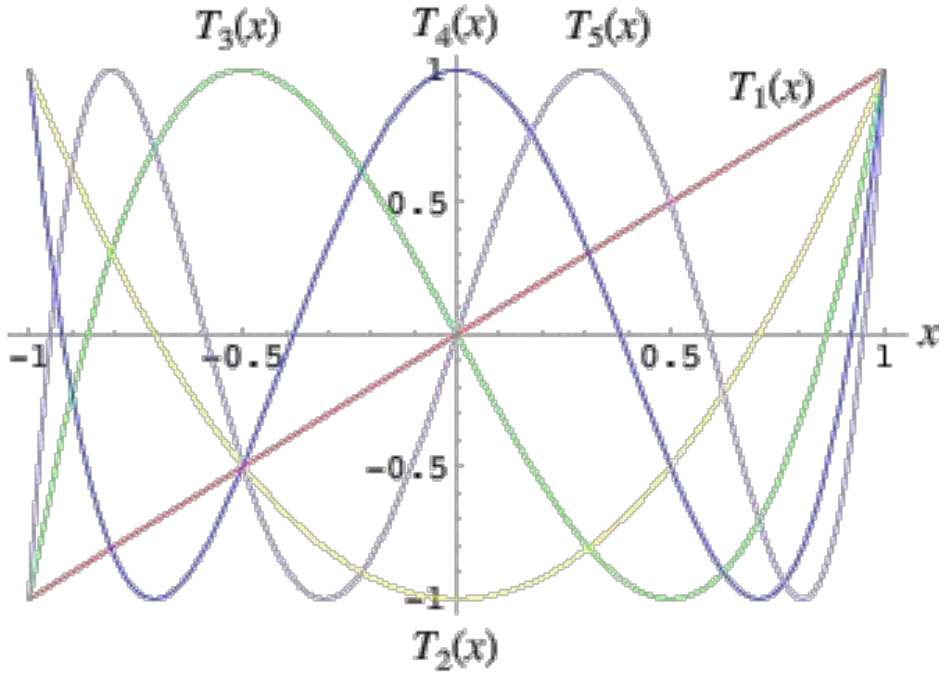

Figure: Chebyshev polynomials.

[1] Aspelund O. PICG 1958;16:530.
[2] Conkie WR. Nucl Sci Eng 1959;6:260.

Journal of Quantitative Spectroscopy & Radiative Transfer 101 (2006) 129–134

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T_N approximation to neutron transport equation and application to critical slab problem

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Abstract

The critical slab problem has been studied in one-speed neutron transport equation with isotropic scattering by using the T_N method. T_N moment criticality solutions are obtained for the uniform finite slab using Mark and Marshak type vacuum boundary conditions. Results obtained by T_N method, using the two type boundary conditions mentioned above, were presented in the Tables and also the Tables included the results obtained by P_N method for the comparisons.
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Collision Probability Method

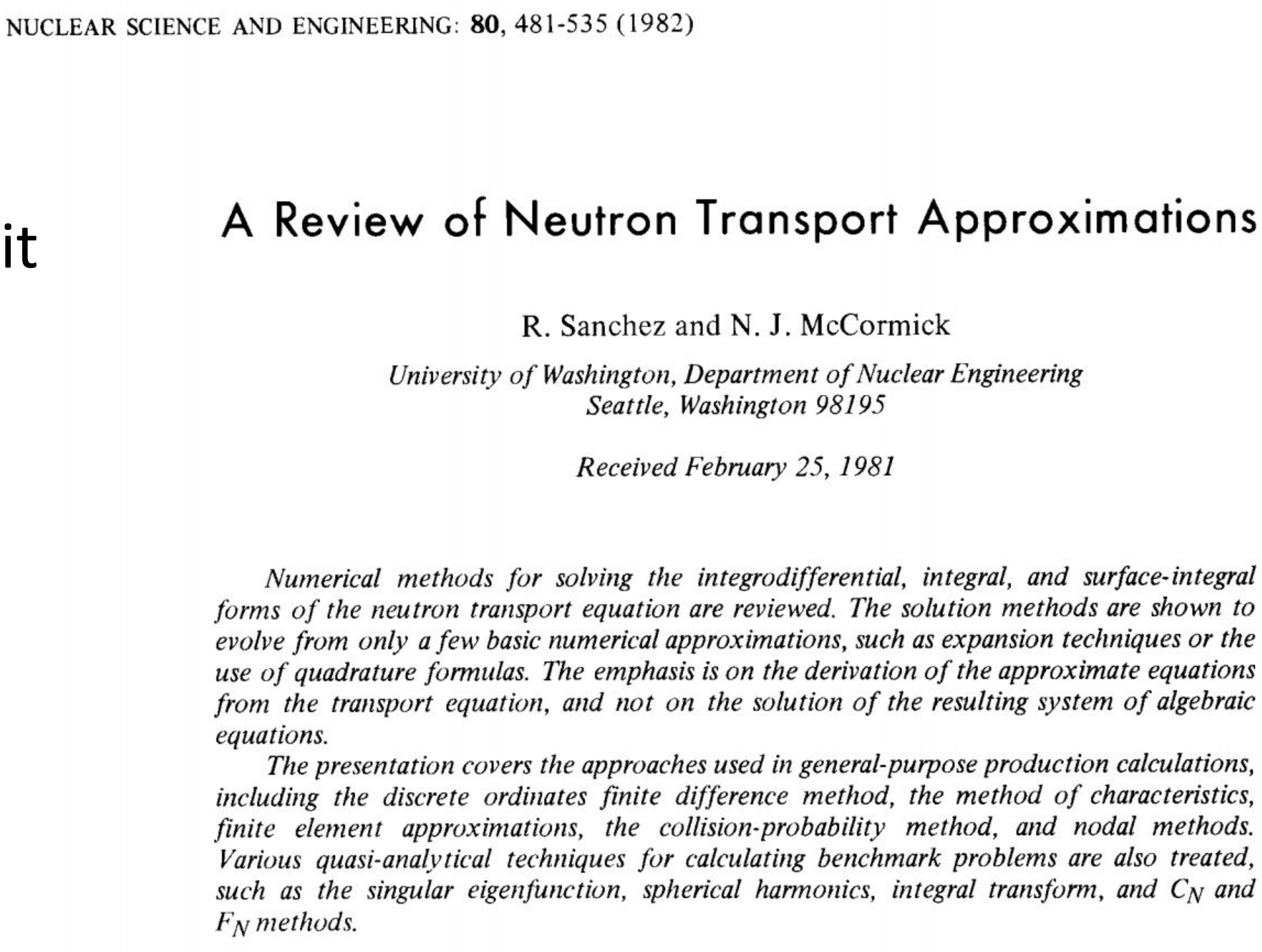
- CPM is a powerful, flexible integral method.
- Upside: No angular approximations of the flux are made.
- Assumption: Sources and scattering assumed isotropic.
- Downside: the resulting matrix H is dense and requires signficant computational memory.
- Downside: flat flux approximation within a region is only first-order accurate: errors proportional to mesh spacing.

Collision Probability Method

1. Subdivide a geometry into cells
2. Assume the flux within the cell is constant (average flux over cell)
3. Solve for reaction rates rather than just flux. Multiply both sides of flux solution by $\Sigma i \Delta i$, where Σi is the total cross-section in region i.
4. Arrive at an equation for $\varphi i \Sigma i \Delta i$, the total collision rate within region i.
5. Result is the "first-flight collision probability," $P_{ii'}$, the probability that a neutron born uniformly and isotropically in a region i′ makes its first collision in region i.
6. Iterate.

Collision Probability Method

- Many sources exist on CPM as it is quite an old method.
- Consider finding an older, straightforward paper concerning a simple geometry and replicating its results.
- Start with Sanchez & McCormick



NUCLEAR SCIENCE AND ENGINEERING: **80**, 481-535 (1982)

A Review of Neutron Transport Approximations

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Numerical methods for solving the integrodifferential, integral, and surface-integral forms of the neutron transport equation are reviewed. The solution methods are shown to evolve from only a few basic numerical approximations, such as expansion techniques or the use of quadrature formulas. The emphasis is on the derivation of the approximate equations from the transport equation, and not on the solution of the resulting system of algebraic equations.

The presentation covers the approaches used in general-purpose production calculations, including the discrete ordinates finite difference method, the method of characteristics, finite element approximations, the collision-probability method, and nodal methods. Various quasi-analytical techniques for calculating benchmark problems are also treated, such as the singular eigenfunction, spherical harmonics, integral transform, and C_N and F_N methods.

Discrete Ordinates (S_N)

If you choose S_N for your project :

- investigate a new geometry (2d slab, sphere, or cylinder)
- or use a new quadrature set.
- Also, rigorously demonstrate spatial order of convergence
- and angular order of accuracy convergence.
- consider demonstrating the use of Coarse Mesh Rebalancing

Quadrature Sets for Discrete Ordinates (S_N)

We covered the notion of quadrature sets in a previous class and stuck with Gauss Legendre which has appealing symmetry and other features. Other quadrature sets can be applied to S_N.

Recall, quadrature sets provide a set of abscissa (x_i) and weights (w_i) that satisfy the following form of integral approximation.

$$\int_a^b f(x)\,dx \approx \sum_{i=0}^{n} w_i\, f(x_i)$$

Quadrature Sets for Discrete Ordinates (S_N)

- Level Symmetric set
  - Fiveland 1991.
  - Two dimensional (mu, nu or x, y)
  - Points on unit sphere are chosen preserve octant symmetry to π/2 rotation.
  - Confines the choice of the direction cosines to subset.

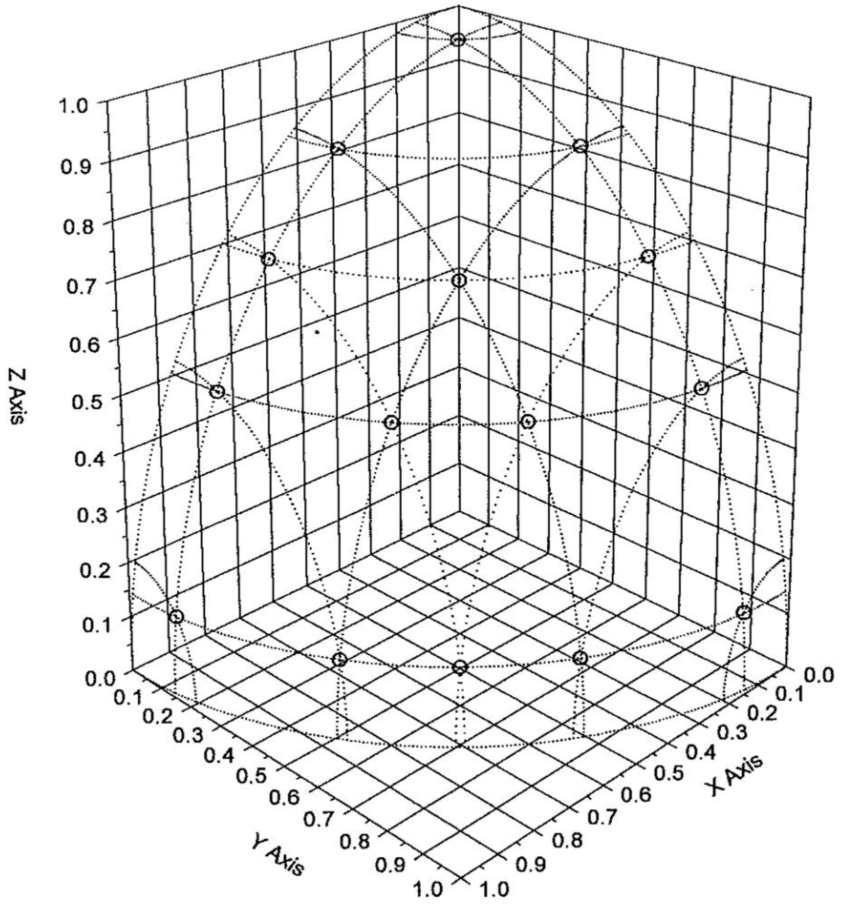
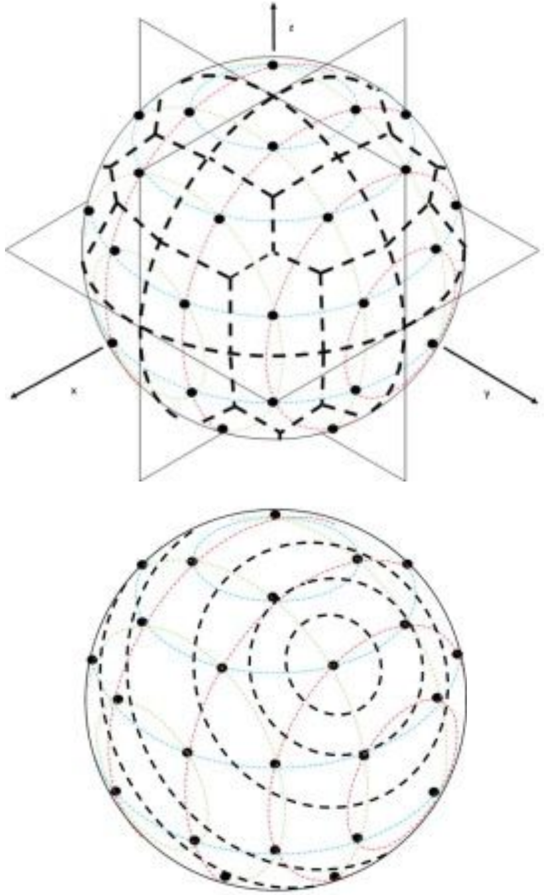


Fig. 1 Arrangement of directions for the LSH_{10} level symmetric set (Fiveland, 1991). The dotted lines are the levels. The intersections of the levels define the directions, denoted by a ○.



Ordinate weighting representations of S6 quadrature set: conventional (left) and proposed rotationally symmetric representation (right). From Roos and Harms 2014

Quadrature Sets for Discrete Ordinates (S_N)

- T_N is based on the (T)Chebyshev Polynomials

Recommended Paper:

The T_N Quadrature Set for the Discrete Ordinates Method

C. P. Thurgood,[1] A. Pollard,[2] and H. A. Becker[1]

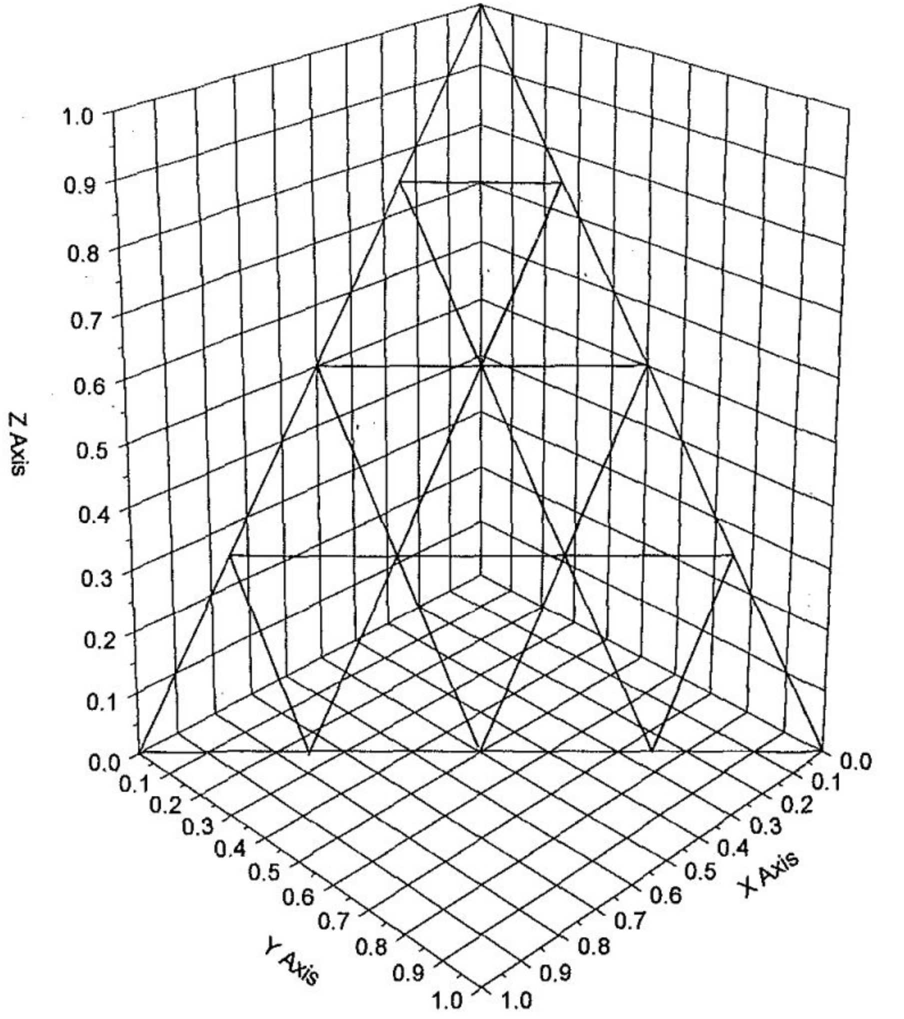
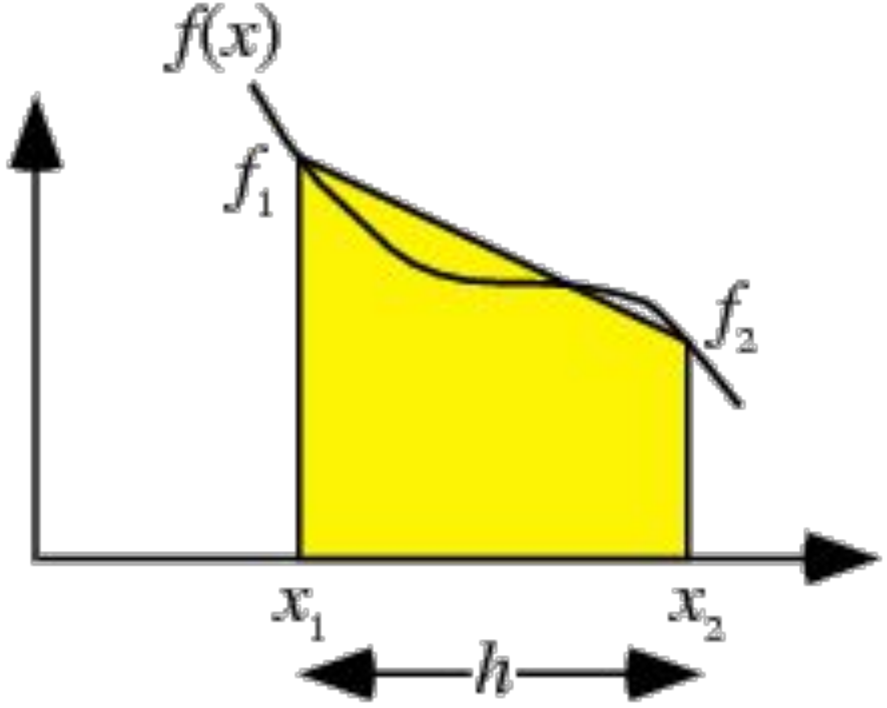


Fig. 2 Tessellation of basal equilateral triangle for the T_4 quadrature set

Quadrature Sets for Discrete Ordinates (S_N)

Newton-Cotes: Simple choice. A constant weight function and a finite interval of integration. Relies on the mid-point rule and the trapezoidal rule.



Adjoint for Acceleration of Monte Carlo

As will be presented in class after fall break one can accelerate monte carlo with the adjoint solution via source biasing and weight windows.

If you do this for your project, start with weight windows, russian roulette and splitting.

And Many More

- Contributon Monte Carlo
- Diffusion Synthetic Method
- Residue Synthetic Method
- Surface Pseudosources Method (with G_N-approximations)
- Discrete elements method (L_N)
- …

Feel free to try any method for which you can find, understand, explain, and replicate a primary journal article. Curiosity encouraged!