- Show your work.
- This work must be submitted online as a **single** `.pdf` file through gradescope.
- If you worked together with another student in the course, please document who you worked with and on what.
- If you used a numerical program (such as Python, Wolfram Alpha, etc.), all scripts must be submitted in addition to the `.pdf`. You may submit these via email to Sun Myung and myself.

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1. (20 points) Write the one-dimensional, one-speed, steady-state S_2 equations with an isotropic source and isotropic scattering.

   Solution: solution here

2. (20 points) Prove that the S_2 and P_1 equations are equivalent for a steady state, single speed problem with isotropic source and scattering.

   Solution: solution here

3. (10 points) Why are odd order S_N equations avoided?

   Solution: solution here

4. Consider the infinite slab with two regions in Figure 1. There are vacuum boundary conditions on both sides of the slab. Scattering is isotropic in the lab system.

   In region 1:

   - Width is $2cm$.
   - $\Sigma_t = \frac{1}{cm}$.
   - $\Sigma_a = \frac{0.5}{cm}$.
   - There is a uniformly distributed isotropic unit source $\left(1\left[\frac{n}{cm}\right]\right)$.

   In region 2:

   - Width is $4cm$.
   - $\Sigma_t = \frac{1.5}{cm}$.
   - $\Sigma_a = \frac{1.2}{cm}$.

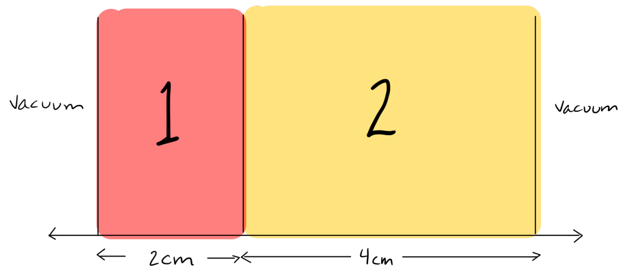

Figure 1: Infinite slab with two regions.

- There is no source.

(a) (40 points) Use Gauss-Legendre quadrature to solve the S_N equations for the slab problem below using a diamond difference approach.

> **Solution:** solution here

(b) (10 points) Refine your solution to part c by refinding the spatial mesh of your solution and angular quadrature until the scalar flux asymptotically converges. What is the spatial order of convergence of the method?

> **Solution:** solution here