Oct 09

CP01 extended to oct 13th (Friday @ 11:59pm) ☺

- Doc strings ⇒ python (in code for important functions)
- in-line comments on code
  - Read me ⇒ Run code ⇒ instructions

Notes:

- lets return to our scattering form:

$$\int_{4\pi}\int_0^\infty \sum_{n=0}^{\infty} \frac{2n+1}{4\pi} \Sigma_{s,n}(\vec{r},E',E,t)\, P_n(\mu_0)\, \varphi(\vec{r},\hat{\Omega}',E',t)\, dE'\, d\hat{\Omega}'$$

- $P_n(\mu_0) = P_n(\hat{\Omega}\cdot\hat{\Omega}')$

- Expand (addition Theorem):

$$P_n(\mu_0) = \frac{4\pi}{2n+1} \sum_{m=-n}^{n} Y^*_{nm}(\theta',\gamma')\, Y(\theta,\gamma)$$

- let us put this expression back into our scattering terms:

$$\int_{4\pi}\int_0^\infty \left[\sum_{n=0}^{\infty} \frac{2n+1}{4\pi} \Sigma_{s,n}(\vec{r},E',E,t)\, \frac{4\pi}{2n+1} \sum_{m=-n}^{n} Y^*_{nm}(\theta',\gamma')\, Y(\theta,\gamma)\right] \varphi(\vec{r},\hat{\Omega}',E',t)\, dE'\, d\hat{\Omega}'$$

- This simplified (after a few steps):

Stwich into θ & γ

$$\int_{4\pi}\int_0^\infty \left[\sum_{n=0}^{\infty} \Sigma_{s,n}(\vec{r},E',E,t) \sum_{m=-n}^{n} Y^*_{nm}(\theta',\gamma')\, Y_{nm}(\theta,\gamma)\right] \varphi(\vec{r},\hat{\Omega}',E',t)\, dE'\, d\hat{\Omega}'$$

- our φ term is in terms of $\hat{\Omega}'$, but our spherical harmonics are in terms of θ,γ. lets modify the flux term
- Also before, re-arrange (primes together)

$$\Rightarrow \int_0^\infty \left[\sum_{n=0}^{\infty} \Sigma_{s,n}(\vec{r},E',E,t) \sum_{m=-n}^{n} Y_{nm}(\theta,\gamma) \int_0^{4\pi} Y^*_{n,m}(\theta',\gamma')\, \varphi(\vec{r},\hat{\Omega}',E',t)\, d\hat{\Omega}'\right] dE'$$

change of var. ⇒

$$\int_0^\infty \left[\sum_{n=0}^{\infty} \Sigma_{s,n}(\vec{r},E',E,t) \sum_{m=-n}^{n} Y_{n,m}(\theta,\gamma) \underbrace{\int_0^{2\pi}\int_0^{\pi} Y^*_{nm}(\theta',\gamma')\, \varphi(\vec{r},\theta',\gamma',E',t) \sin(\theta')\, d\theta'\, d\gamma'}_{\text{Spherical harmonics expansion for angular flux}}\right] dE'$$

Generic definition: (from last time)

$$f_{n,m}(y) = \int_0^{2\pi}\int_0^{\pi} Y^*_{n,m}(\theta,\gamma)\, f(\theta,\gamma,y)\sin\theta\, d\theta\, d\gamma$$

↳ $f_{n,m}(y)$ in this case is $\varphi_{n,m}(r, E', t)$

↳ our scattering term is now:

$$\int_0^{\infty}\left[\sum_{n=0}^{\infty} \Sigma_{s,n}(\vec{r}, E', E, t) \sum_{m=-n}^{n} Y_{n,m}(\theta,\gamma)\, \varphi_{n,m}(\vec{r}, E', t)\right] dE'$$

↳ we successfully removed $\hat{\Omega}'$ (OR (θ', γ')), but we still have ∞ series.

- n, must be truncated
- our approximations will be more accurate as $N\uparrow$ ($\sim$ 8,10)
- In practice, we usually go to $N \leq 8$, and $N=1$ is most common

→ Why do we care about PN? ⇒ 2g - Diffusion (good)
↳ need: group cross-sections

} Big material change →
- Burnable Abs
- Control Rods

P1 Scattering term approximation

for $n=0$, $m=0$

$n=1$, $m=-1,0,1$

↳ for a given PN order, we need $(N+1)^2$ moments to integrate scattering operator.

↳ for P_1, we need 4.

⇒ First we start with $n=0, m=0$

$$Y_{0,0}(\theta,\gamma) = \sqrt{\frac{1}{4\pi}}\, \underbrace{P_0(\cos(\theta))}_{1}$$

$P_0(\cos\theta) = 1$

$$= \sqrt{\frac{1}{4\pi}}$$

$\hookrightarrow$ $f_{n,m}(y) = \int_0^{2\pi}\int_0^{\pi} Y_{nm}^*(\theta,\gamma) f(\theta,\gamma,y) \sin\theta \, d\theta \, d\gamma$

$\Rightarrow$ Using definition above we calculate $\varphi_{n,m}(\vec{r},E,t)$

$$\varphi_{0,0}(\vec{r},E',t) = \sqrt{\frac{1}{4\pi}} \int_0^{2\pi}\int_0^{\pi} \varphi(\vec{r},\theta',\gamma',E',t) \sin\theta' \, d\theta' \, d\gamma'$$

$$= \sqrt{\frac{1}{4\pi}} \underbrace{\int_0^{2\pi}\int_0^{\pi} \varphi(\vec{r},\hat{\Omega}',E',t) \, d\hat{\Omega}'}_{\substack{\text{scalar flux!} \\ \phi(\vec{r},E',t)!}}$$

* The zeroth spherical harmonic over all angular flux is the scalar flux!

$$\varphi_{0,0} = \sqrt{\frac{1}{4\pi}} \, \phi(\vec{r},E',t)$$

$\hookrightarrow$ Scattering term looks like:

$$\int_0^{\infty} \Big[\underbrace{\Sigma_{s,0}(\vec{r},E',E,t) \, Y_{0,0}(\theta,\gamma) \, \varphi_{0,0}(\vec{r},E',t)}_{\Sigma_{s,0}(\vec{r},E',E,t)\left(\sqrt{\frac{1}{4\pi}}\right)\left(\sqrt{\frac{1}{4\pi}}\,\phi(\vec{r},E',t)\right)} + \sum_{n=1}^{\infty} \Sigma_{s,n}(\vec{r},E',E,t) \sum_{m=-n}^{n} Y_{n,m}(\theta,\gamma) \, \varphi_{n,m}(\vec{r},E',t) \Big] dE'$$

zeroth-term

$\hookrightarrow$ first term reduces further:

$$= \frac{\Sigma_{s,0}(\vec{r},E',E,t)}{4\pi} \, \phi(\vec{r},E',t)$$

Continue w/ $n=1, m=-1,0,1$

this is $P_{1,0}$ $\Rightarrow$ same thing and P_n

$\hookrightarrow$ $Y_{1,0}(\theta,\gamma) = \sqrt{\frac{3}{4\pi}} \, P_1(\cos(\theta))$

$$Y_{1,1}(\theta,\gamma) = \sqrt{\left(\frac{3}{2\pi}\right)\left(\frac{1}{2}\right)}\, P_{1,1}(\cos(\theta))\cos(\gamma)$$

inside polynomial

not inside polynomial

$$Y_{1,-1}(\theta,\gamma) = -\sqrt{\left(\frac{3}{2\pi}\right)\left(\frac{1}{2}\right)}\, P_{1,1}(\cos(\theta))\sin(-\gamma)$$

easier

check if rigth

Now we need $P_{1,0}$ and $P_{1,1}$ for our expansions

$$P_1(\cos(\theta)) = \cos\theta$$

$$P_{1,1}(\cos(\theta)) = -\sqrt{1-\cos^2(\theta)}$$

check:

we know $\sin(-\gamma) = -\sin(\gamma)$

$$Y_{1,0}(\theta,\gamma) = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_{1,1}(\theta,\gamma) = -\sqrt{\frac{3}{4\pi}}\sqrt{1-\cos^2\theta}\,\cos(\gamma)$$

$$Y_{1,-1}(\theta,\gamma) = -\sqrt{\frac{3}{4\pi}}\sqrt{1-\cos^2\theta}\,\sin(\gamma)$$

These correspond to our direction vectors

$$Y_{1,0}(\theta,\gamma) = \sqrt{\frac{3}{4\pi}}\,\Omega_x$$

$$Y_{1,1}(\theta,\gamma) = -\sqrt{\frac{3}{4\pi}}\,\Omega_y$$

$$Y_{1,-1}(\theta,\gamma) = -\sqrt{\frac{3}{4\pi}}\,\Omega_z$$

lets consider the m summation:

$$\sum_{m=-1}^{1} Y_{1,m}(\theta,\gamma)\,\varphi_{1,m}(\vec{r},E,t)$$

$$= \sum_{m=-1}^{1} Y_{1,m}(\theta,\gamma)\int_{4\pi} Y_{1,m}(\hat{\Omega}')\,\varphi(\vec{r},E',\hat{\Omega}',t)\,d\Omega'$$

expand summation & evaluate spherical harmonics terms:

$$\sum_{m=-1}^{1} Y_{1,m}(\theta,\gamma)\,\varphi_{1,m}(\vec{r},E',t)$$

$$= \sqrt{\frac{3}{4\pi}}\,\Omega_x \int_{4\pi} \sqrt{\frac{3}{4\pi}}\,\Omega'_x\,\varphi(\vec{r},E,\Omega',t)\,d\Omega' + \left(-\sqrt{\frac{3}{4\pi}}\,\Omega_y\right)\int_{4\pi} -\sqrt{\frac{3}{4\pi}}\,\Omega'_y\,\varphi(\vec{r},E,\Omega',t)\,d\Omega' + \left(-\sqrt{\frac{3}{4\pi}}\,\Omega_z\right)\int_{4\pi} -\sqrt{\frac{3}{4\pi}}\,\Omega'_z\,\varphi(\vec{r},E,\Omega',t)\,d\Omega'$$

factor out $\sqrt{\frac{3}{4\pi}}$ terms and collecting x, y and z terms:

$$= \frac{3}{4\pi}\int_{4\pi} (\Omega_x\Omega'_x + \Omega_y\Omega'_y + \Omega_z\Omega'_z)\,\varphi(\vec{r},E',\Omega',t)\,d\Omega'$$

$$= \frac{3}{4\pi}\int_{4\pi} (\hat{\Omega}\cdot\hat{\Omega}')\,\varphi(\vec{r},E',\Omega',t)\,d\hat{\Omega}'$$

pull out $\hat{\Omega}$ from integral

$$= \frac{3}{4\pi}\,\hat{\Omega}\cdot\underbrace{\int_{4\pi} \hat{\Omega}'\,\varphi(\vec{r},E',\Omega',t)\,d\hat{\Omega}'}_{\text{The current } J!!}$$

- so,

$$\sum_{m=-1}^{1} Y_{1,m}(\theta,\gamma)\,\varphi_{1,m}(\vec{r},E',t) = \frac{3}{4\pi}\,\hat{\Omega}\cdot\vec{J}(\vec{r},E',t)$$

→ So our scattering term is now:

$$\int_0^\infty \left[\frac{\Sigma_{s,0}(\vec{r},E',E,t)}{4\pi}\,\phi(\vec{r},E',t) + \frac{3}{4\pi}\,\Sigma_{s,1}(\vec{r},E',E,t)\,\hat{\Omega}\cdot\vec{J}(\vec{r},E',t) + \sum_{n=2}^{\infty}\Sigma_{s,n}\ldots\right]dE'$$

$$\boxed{\int_{4\pi}\int_0^\infty \Sigma_s(\vec{r},E'\to E,\hat{\Omega}'\to\hat{\Omega},t)\,\varphi(\vec{r},E',\Omega',t)\,dE'\,d\hat{\Omega}' \approx \int_0^\infty \frac{\Sigma_{s,0}(\vec{r},E',E,t)}{4\pi}\,\phi(\vec{r},E',t) + \frac{3}{4\pi}\,\Sigma_{s,1}(\vec{r},E',E,t)\,\hat{\Omega}\cdot\vec{J}(\vec{r},E',t)\,dE'}$$

physical interpretations only for first two terms