09/11

- To solve NTE you need:
  1) DFQ of system
  2) B.C.
  3) I.C

Boundary conditions:

1) Vacuum BC: $\varphi(\vec{r},E,\hat{\Omega},t)=0$ for $\hat{\Omega}\cdot\hat{e}_s<0$ → All angles pointing into Region, for all $\vec{r}$ on S.

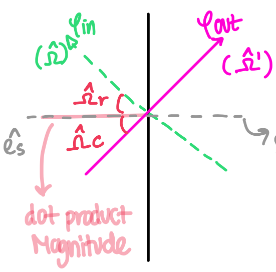
2) Albedo BC:

$$\varphi_{in}(\vec{r},E,\hat{\Omega},t)=\alpha(E)\,\varphi_{out}(\vec{r},E,\underbrace{\hat{\Omega}'}_{\text{different angle}},t)$$

, $\alpha(E)=1$ → Reflective

$\alpha(E)=0$ → vacuum

$0<\alpha(E)<1$ → some n leave, some reflect back



$(\hat{\Omega}')$ $\varphi_{in}(\vec{r},\hat{\Omega},E,t)=\alpha(E)\,\varphi_{out}(\vec{r},\hat{\Omega}',E,t)$

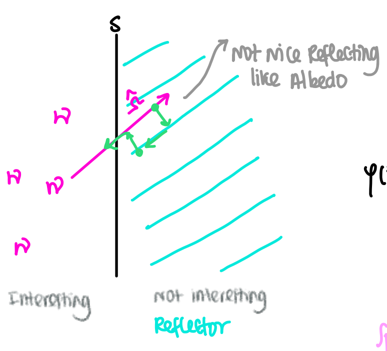
Spectral Reflection = Mirror

↳ $\underbrace{\hat{e}_s\cdot\hat{\Omega}}_{\text{out Region}}=-\underbrace{\hat{e}_s\cdot\hat{\Omega}'}_{\text{into Region}}$

↳ $(\hat{\Omega}\times\hat{\Omega}')\cdot\hat{e}_s=0$

3) White BC:

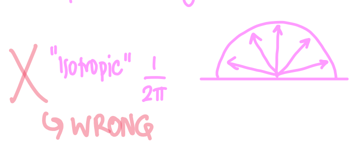
↳ Assume all neutrons passing out of volume V, over the surface S will return with an angular distribution that is isotropic



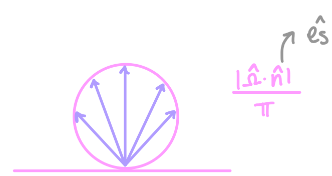
↳ This is appropiate for a thick scattering medium assumed at the boundary

$$\varphi(\vec{r},E,\hat{\Omega},t)=\int_{\underbrace{\hat{e}_s\cdot\hat{\Omega}'>0}_{\text{from outside}}}\frac{|\hat{e}_s\cdot\hat{\Omega}'|}{\pi}\,\varphi(\vec{r},E,\hat{\Omega}',t)\,d\hat{\Omega}'$$ for all $\hat{e}_s\cdot\hat{\Omega}<0$ & all $\vec{r}$ on s

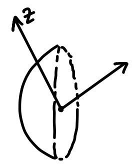
Sphere Solid Angle = 2π



→ "Isotropic"

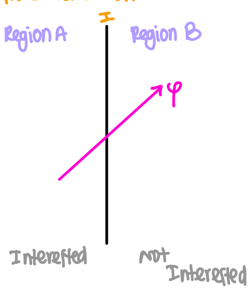


Half sphere =



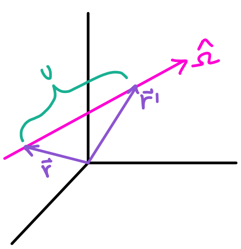
↳ for small θ, there are fewer "solid solid angles" than large θ. (Theory Manuals for Monte Carlo code)

Interface Condition:



$$\varphi_B(\vec{r}_I, E, \hat{\Omega}, t) = \varphi_A(\vec{r}_I, E, \hat{\Omega}, t)$$

Transport Equation in 1-D



↳ Neutron stream some distance u from $\vec{r}$ to $\vec{r}\,'$ along a characteristic path in the direction $\hat{\Omega}$.

$$\frac{\partial \varphi}{\partial u} = \frac{\partial \varphi}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial \varphi}{\partial y} \cdot \frac{\partial y}{\partial u} + \frac{\partial \varphi}{\partial z} \cdot \frac{\partial z}{\partial u}$$

↳ so this becomes:

$$\frac{d\varphi}{du} = \mu \frac{d\varphi}{dx} + \eta \frac{d\varphi}{dy} + \xi \frac{d\varphi}{dz} = \hat{\Omega} \cdot \nabla \varphi$$

easier to work with

$$\mu = \cos\theta, \quad \eta = \sin\theta \sin\gamma, \quad \xi = \sin\theta \cos\gamma$$

↳ for one-dimension in cartesian geometry:

from 3D to 1D ⇐ Integrated by γ from 0 to 2π

$$\left[\frac{1}{v} \frac{\partial \varphi}{\partial t} + \mu \frac{\partial \varphi}{\partial x} + \Sigma_t \varphi = \underbrace{\int_{-1}^{1} \int_{0}^{\infty} \Sigma_s(x, E' \to E, \mu' \to \mu) \varphi(x, E', \mu', t)\, dE'\, d\mu' + Q_{ex}(x, E, \mu, t)}_{\text{Depends on angle of variable } \mu \text{, can still be insotropic.}} \right.$$

↳ Depends on angle of variable μ, can still be insotropic.

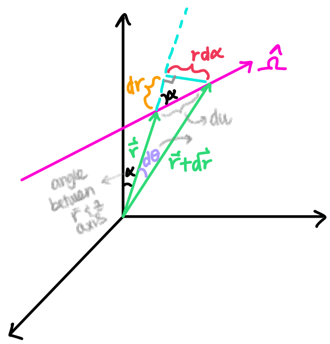
↳ for isotropic scattering: $\Sigma_s(x, E' \to E)$ ⇒ Assuming elastic neutron scattering (normalized, one in = one out)

Normalization factor

$$\Sigma_s(x, E' \to E, \mu' \to \mu)\, d\mu = \int_{-1}^{1} c\, \Sigma_s(x, E' \to E)\, d\mu = 2c\, \Sigma_s(x, E' \to E) \Rightarrow c = \frac{1}{2}$$

$$\boxed{\Sigma_s(\text{isotropic}) = \frac{1}{2}\int_{-1}^{1}\int_0^{\infty} \Sigma_s(x, E' \to E)\, \varphi(x, E', \mu', t)\, dE'\, d\mu'}$$

In spherical geometry: (1-D)



① $\dfrac{\partial r}{\partial u} = \cos\alpha = \mu$

$-\dfrac{r\,d\alpha}{du} = \sin\alpha = \sqrt{1-\mu^2}$

Chain Rule

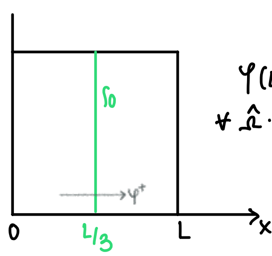
② $\dfrac{\partial \mu}{\partial u} = \dfrac{\partial(\cos\alpha)}{\partial u} = \dfrac{\partial(\cos\alpha)}{\partial \alpha}\dfrac{\partial \alpha}{\partial u} = \dfrac{\partial(\cos\alpha)}{\partial \alpha}\dfrac{\sin\alpha}{-r} = \dfrac{\sin^2\alpha}{r} = \dfrac{1-\mu^2}{r}$

$$\Rightarrow \frac{\partial\varphi}{\partial u} = \frac{\partial\varphi}{\partial r}\frac{\partial r}{\partial u} + \frac{\partial\varphi}{\partial\mu}\frac{\partial\mu}{\partial u} = \frac{\partial\varphi}{\partial r}\mu + \frac{\partial\varphi}{\partial\mu}\frac{(1-\mu^2)}{r}$$ Streaming term in spherical geometry (full form in B&G 1.7)

Example problem #1:

↳ Solve for the angular flux in a purely absorbing slab with a plane isotropic source at $x = L/3$. Assume vacuum BCs on both sides of slab. (single energy, 1-speed)

$\varphi(0) = 0$
for $(\forall)$ $\hat{\Omega}\cdot\hat{x} > 0 = \mu > 0$



$\varphi(L) = 0$
$\forall$ $\hat{\Omega}\cdot\hat{x} < 0 = \mu < 0$

↳ No scattering
↳ No fission
↳ fixed source

0 ⇒ purely Abs slab

GE: $\hat{\Omega}\cdot\nabla\varphi(\vec{r},\hat{\Omega}) + \Sigma_t\varphi(\vec{r},\hat{\Omega}) = \int_{4\pi} d\hat{\Omega}'\, \Sigma_s(\vec{r}, \hat{\Omega}' \to \hat{\Omega})\varphi(\vec{r},\hat{\Omega}') + S(\vec{r},\hat{\Omega})$

3-D (single E)

$$\mu\frac{\partial\varphi}{\partial x} + \Sigma_a\varphi(x,\mu) = S(x,\mu)$$

↳ Integrating factor: $e^{\Sigma_a x/\mu}$ ⇒ Multiply Both sides by Integrating factor

$$\frac{\partial}{\partial x}\left[\varphi(x,\mu)\, e^{\Sigma_a x/\mu}\right] = \frac{S(x,\mu)}{\mu}\, e^{\Sigma_a x/\mu}$$

The isotropic plane source can be expressed as $\int_{-1}^{1} S(x,\mu)\,d\mu = S_0$

isotropic ⇒ μ dependence goes away

check w/ MMK

$S(x)\int_{-1}^{1} d\mu = S_0$

$S(x) = \frac{S_0}{2}$

$\mu<0$ $\int_{-1}^{0} S_0\,\delta(x-L/3)\,d\mu$

$$S(x,\mu) = \frac{\int_0^{\overbrace{1}^{\mu>0}} S_0\,\delta(x-L/3)\,d\mu}{\int_{-1}^{1} d\mu}$$

normalizing

isotropic (no longer a function of μ)

$$= \frac{S_0}{2}\,\delta(x-L/3) = \begin{cases} \frac{S_0}{2} & x = L/3 \\ 0 & x \neq L/3 \end{cases}$$

for $\mu>0$, we integrate between 0 & x

$\mu>0$

from BC's

$$\varphi^{+}(x,\mu)\,e^{\Sigma a x/\mu} - \underbrace{\varphi^{+}(0,\mu)}_{0}\,e^{0} = \int_0^x \frac{S(x,\mu)}{\mu}\,e^{\Sigma a x'/\mu}\,dx'$$

$$\int_0^x \frac{S}{2}\,\delta(x'-L/3)\,e^{\Sigma a x'/\mu}\,dx'$$

$$\Rightarrow \varphi^{+}(x,\mu) = \begin{cases} 0 & x < \frac{L}{3} \\ \frac{S_0}{2\mu}\,e^{-\Sigma a (x-L/3)/\mu} & x > \frac{L}{3} \end{cases}$$

for $\mu<0$, we integrate from $\underbrace{L \text{ to } x}_{\text{you want source downstream}}$

vacuum BC

$$\varphi^{-}(x,\mu)\,e^{\Sigma a x/\mu} - \underbrace{\varphi^{-}(L,\mu)}_{0}\,e^{\Sigma a L/\mu} = \int_L^x \frac{S(x',\mu)}{\mu}\,e^{\Sigma a x'/\mu}\,dx'$$

$$\Rightarrow \varphi^{-}(x,\mu) = \begin{cases} 0 & x > \frac{L}{3} \\ \frac{S_0}{2\underbrace{|\mu|}_{\mu<0 \Rightarrow \text{deal with this}}}\,e^{\Sigma a[(x-L/3)/|\mu|]} & x < \frac{L}{3} \end{cases}$$