Continuing with P_N boundary conditions:

Recall last time we discussed vacuum, interface, and reflective boundary conditions.

We also derived the P_1 marshak boundary condition if the interface of the boundary is a vacuum.

Let us continue our exploration of the marshak b.c. in a more general form:

$$2\pi \int_{\mu in} P_i(\mu)\, \varphi(\mu)\, d\mu = 2\pi \int_{\mu in} P_i(\mu)\, \varphi_b(\mu)\, d\mu$$

for $i = 1, 3, 5, \ldots N$

here we have set the flux in our system to some known flux at the boundary φ_b

Note: in our last lecture we said the incoming flux was 0, because we were at a vacuum. $\varphi_b = 0$. However, this boundary condition (the Marshak boundary condition) can be used for any $\varphi_b(\mu)$.

We know how to represent $\varphi(\mu)$ using legendre polynomials

$$\varphi(\mu) = \sum_{n=0}^{N} \frac{2n+1}{4\pi} \Phi_n P_n(\mu)$$

where Φ_n contains all other phase space variables from φ, like $\vec{r}$ or E.

Using this representation in our boundary condition:

$$2\pi \int_{\mu_{in}} P_i(\mu) \sum_{n=0}^{N} \frac{2n+1}{4\pi} \Phi_n P_n(\mu)\, d\mu = 2\pi \int_{\mu_{in}} P_i(\mu)\, \varphi_b(\mu)\, d\mu$$

for $i = 1, 3, 5, \ldots, N$

yields $\frac{(N+1)}{2}$ equations w boundary

Let us now consider this B.C. for P_3:

$$2\pi \int_0^1 P_1(\mu) \sum_{n=0}^{3} \frac{2n+1}{4\pi} \Phi_n P_n(\mu)\, d\mu = 2\pi \int_0^1 P_1(\mu)\, \varphi_b(\mu)\, d\mu$$

$$2\pi \int_0^1 P_3(\mu) \sum_{n=0}^{3} \frac{2n+1}{4\pi} \Phi_n P_n(\mu)\, d\mu = 2\pi \int_0^1 P_3(\mu)\, \varphi_b(\mu)\, d\mu$$

we have two equations, one for $i=1$ and one for $i=3$.

Rather than making the boundary a vacuum, let us consider an isotropic flux on the boundary

$$\psi_b(\mu) = \frac{\Phi_b}{4\pi}$$

The P_3 boundary conditions resolve to:

$$\frac{1}{2}\Phi_0 + \Phi_1 + \frac{5}{8}\Phi_2 = \frac{1}{2}\Phi_b$$

$$-\frac{1}{8}\Phi_0 + \frac{5}{8}\Phi_2 + \Phi_3 = -\frac{1}{8}\Phi_b$$

again, if instead we have a vacuum boundary condition then we set $\Phi_b = 0$.

If we have reflective boundary conditions, then we know that the net current should be 0.

Reflective B.C.

$$\psi(x_{\frac{1}{2}}, \mu) = \psi(x_{\frac{1}{2}}, -\mu)$$

The odd moments (i.e. Φ_1) are associated with the current. Thus, we set

$$\Phi_i = 0 \quad \text{for } i = 1, 3, 5 \ldots \text{odd} \ldots N$$

still yields $\frac{(N+1)}{2}$ equations @ boundary.

In the P_1 approximation this is equivalent to setting the current to zero at each boundary.

$$\phi_1 = 2\pi \int_{-1}^{1} \mu\, \psi(\mu)\, d\mu = J = 0$$

Note: Marshak boundary conditions preserve/conserve particles / crossing the boundary.

Mark boundary conditions

$$\psi(0, \mu > 0) = \psi(a, \mu < 0) = 0$$

and

$$\psi(0, \mu_i) = \psi(a, \mu_i) = 0$$

where $i = 1, 2, \ldots \frac{N+1}{2}$ up to N odd

The points μ_i are a finite set chosen from positive roots of

$$P_{N+1}(\mu_i) = 0$$

Supplement your knowledge of Mott
+ Marshak B.C.S by reading Bell
+ Glasstone 2.5 (d)