This homework was done in collaboration with Luke Seifert and Anthony Boyd

NPRE 560 Oleksandr Yardas Fall 2024

HW 3 Due 2024.10.23

101

+1 for LaTeX

- Show your work.
- This work must be submitted online as a `.pdf` through Canvas.
- Work completed with LaTeX or Jupyter earns 1 extra point. Submit source file (e.g. `.tex` or `.ipynb`) along with the `.pdf` file.
- If this work is completed with the aid of a numerical program (such as Python, Wolfram Alpha, or MATLAB) all scripts and data must be submitted in addition to the `.pdf`.
- If you work with anyone else, document what you worked on together.

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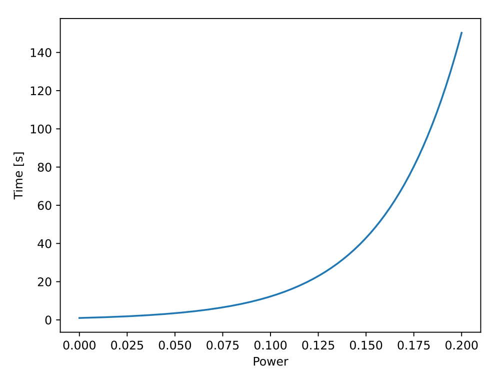
1. (Ott Review 6.20) Describe in words, with graphs, and with formulas the transient following a step change in reactivity or source:

   (a) (5 points) Without delayed neutrons.

Solution: With no delayed neutrons, we drop delayed neutrons from the kinetics equation:

$$\dot{p} = \frac{\rho(t)}{\Lambda}p(t) \tag{1}$$

Following a step reactivity insertion, the slope of the power will constantly increase at a rate of $\frac{\rho}{\Lambda}\rho$, that is, without any delayed neutrons the power blows up. We can also see this if we solve the equation analytically, assuming the reactivity stays constant, as $p(t) = p_0 e^{\frac{\rho}{\Lambda}t}$. Figure a shows a plot of the power over time using $p_0 = 1.0$, $\Lambda = 2 \cdot 10^{-5}$. The below figure shows a plot of the reactivity insertion without delayed neutrons.

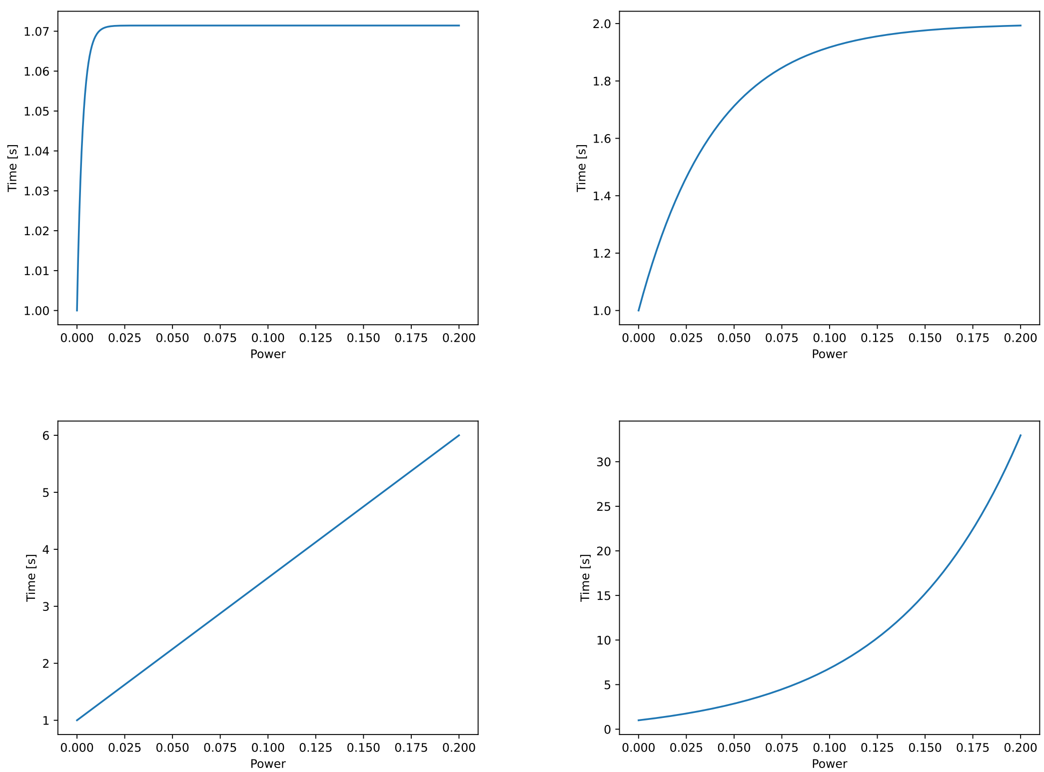


   (b) (5 points) With constant delayed neutron source.

Solution: With a constant delayed source, we approximate the delayed neutron source as constant, that is $S_d(t) = S_d = \beta p_0$. Assumuing no external source, the kinetics equation becomes

$$\dot{p} = \frac{\rho(t) - \beta}{\Lambda} p(t) + \frac{\beta p_0}{\Lambda} \tag{2}$$

The transient following a step reactivity insertion depends on the value of β. If $\beta \gg \rho$, we will have an inital jump in reactivity that quickly stabilizes. As $\beta > \rho$, the transient takes longer to stabilize. If $\beta = \rho$, the $p(t)$ term dissappaer and we have linear transient. As β becomes less than ρ, the transient blows up more quickly. The figures below, left-to-right top-to-bottom, show th transient for β 750, 100, 50, and 20 pcm



(c) (5 points) With no approximations (no formula required).

Solution: The behavior of the transient in this case will be very similar to the CDS approximation, however this time we have a time-varying delayed neutron source. This is what we studied in CP 1.

2. (Ott Review 6.34) Estimate the time it takes to establish the stable asymptotic transient for $\rho_1 < \beta$ in an initially critical reactor.

Solution: For a critical reactor with a step reactivity instertion to a stationary state ρ_1 from a stationary state ρ_0, the response in the power is also a prompt jump. So there will be a transtion to a higer flat power profile corresponding to the prompt jump in reactivity. We can find the time it takes to make this transition based on Equation 6.94 in Ott, the equation or the power when the inverse prompt period is constant (which we have when we are critical):

$$p(t) = p_0 e^{\alpha_p t} + p_0 \frac{\beta}{\beta - \rho_1}(1 - e^{\alpha_p t}) \tag{3}$$

We have a stable asymptotic transient when $\frac{dp}{dt} = 0$:

$$\frac{dp}{dt} = p_0 \alpha_p e^{\alpha_p t} - p_0 \frac{\beta}{\beta - \rho_1} \alpha_p e^{\alpha_p t}$$

Setting this equal to zero and rearranging terms yields

$$0 = p_0 \alpha_p \left(1 - \frac{\beta}{\beta - \rho_1}\right) e^{\alpha_p t} \tag{4}$$

If we try to take the log of both sides, we have log(0) which is nonphysical. So if we instead set the LHS equal to some small value δ, we can take the lograithm:

$$\log\left(\frac{\delta}{p_0 \alpha_p \left(1 - \frac{\beta}{\beta - \rho_1}\right)}\right) = \alpha t \tag{5}$$

As δ approaches zero, the LHS blows up towards $-\infty$. I am not sure how to proceed from here :/

3. (10 points) (Ott Review 6.35) Explain in terms of roots of the characteristic equation:
   (a) (5 points) the prompt jump phenomenon

   Solution: λ_7 is the smallest root so governs short time-behavior, and is reponsible for the inital dynamics of a prompt jump phenomenon.

   (b) (5 points) the delayed neutron induced transition

   Solution: The delayed neutron transition is goverened by the intermediate roots λ_i for $i = 2, 3, 4, 5, 6$.

   (c) (5 points) the stable period

   Solution: The stable period is govered by the largest root (i.e. the root yielding the longest period), λ_1

4. (30 points) (Ott Problem 10.1) Find the numerical value of p^{00}, the flux after a prompt jump for which the increase due to delayed neutrons is just compensated by Doppler feedback, for an LWR from the typical λ and γ/β values given in the text. Discuss why p^{00} may vary between reactors (e.g. the SEFOR reactor discussed in the text).

Solution: Equation 10.48 in Ott described the term p^{00}:

$$-\frac{\gamma}{\beta}p^{00} = \overline{\lambda} \tag{6}$$

Expanding $\overline{\lambda}$, we can cancel out the β term from each side:

$$-\gamma p^{00} = \sum_i \frac{\nu_{d,i}}{\nu}\lambda_i \tag{7}$$

For an LWR, a typical γ is -0.8\$/fp-s (from Equation 10.35 in Ott). We can use the values from Table 2-III in Ott to calculate $\sum_i \frac{\nu_{d,i}}{\nu}\lambda_i$. Using the $\nu_{d,i}$ values for U235 and $\nu = 2.5$, we get the RHS equals 0.0029 s^{-1}. Dividing by $-\gamma$ and plugging in our values, we get $p^{00} = 0.0036$fp/\$.

In different reactors (LWRs and fast reactors), β and $\overline{\lambda}$ will be different due to different fuels used, leading to different neutron yields, so p^{00} would also then be different.

5. (15 points) (Ott Review 10.1) Define each term, give an example of the physical phenomena involved, and an example of a transient for each:
   (a) (5 points) Energy coefficient.
   (b) (5 points) Temperature coefficient of reactvity.
   (c) (5 points) Power coefficient.

Solution:

- The energy coefficient of reactivity is given by

$$\gamma_Q = \frac{\partial \rho}{\partial Q} \tag{8}$$

  The energy coefficient of reactivity is the change in reactivity due to energy generated in the core. The physical phenomena involved are heat generation in the fuel leading to increases in temperature which cause Doppler broadening, as well as decreases in density decreasing macroscopic cross sections. An example of a transient for this is a reactivity excursion increasing the rate of energy released in the fuel, causing an increase in fuel temperature, which in turn decreases the density of the fuel. As the Doppler effect kicks in due to the higher temperature, reactivity decreases, which in turn reduces the energy deposition in the fuel

- The temperature coefficent of reactivity is given by

$$\gamma_T = \frac{\partial \rho}{\partial T} \tag{9}$$

  The temperature coefficient of reactivity is the change in reactivity due to temperature change in the fuel. The physical phenomena involved is Doppler broadening. An example of a transient for this feedback mechanism is a reactivity excursion causing an increase in fuel temperature, decreasing reactivity via the Doppler effect, and in then returning to at or near the original reactivity and temperature.

- The power coefficient of reactivity is given by

$$\gamma_P = \frac{\partial \rho}{\partial P} \tag{10}$$

The power coefficient of reactivity is the change in reactivity due to the change in power levels in the reactor. The physical phenomena involved are heat generation in the fuel leading to increases in temperature which cause Doppler broadening, decreases in density decreasing macroscopic cross sections, and build-up and decay of neutron poisons (xenon and samarium). An example of a transient would be a cyclically fluctuating power in a large, low-power reactor (e.g. Chicago Pile 1) due to buildup of xenon and samarium reducing power via parasitic absorption (which prevents further fission chains), which removes reactivity from the reactor. Then as the xenon and samarium decay away, power increases as fission chains begin to form again, inserting reactivity back into the system.