Midterm Exam 1

MAT133 - Calculus II

2/17/2017

Form A

Instructions

Do not start until you are told to do so.

Please, turn off your phone and secure it in your bag. Leave your bags, pencil cases, calculators, notes, books, jackets, hats, food, and other belongings at the front of the classroom. You are permitted to have a transparent drink bottle, pens, pencils, and erasers at your desk. Please, plan to stay in the classroom for the entire duration of the exam.

You will have 80 minutes to complete this exam. Read all instructions carefully. Your responses to all item on this exam must be your own. No outside references, notes, calculators, or other aides are permitted. As it is crowded, please, refrain from glancing at the papers of those around you, and take care that your work is protected. A reference sheet, and pages for scratch work can be found attached to the end of the exam, you may detach these pages if you like. Do not detach any other pages from the exam.

The exam sections are weighted as follows:

- 36 points - Concept Check
- 40 points - Essential Skills
- 10 points - Intermediate Skills
- 4 points - Advanced Skills
- 10 points - True/False Statements
- 5 points - Bonus

Within each section, all problems are weighted the same.

If you find yourself unable to finish a question, do your best to describe your attempts and reasoning. Partial credit may be awarded for demonstrating meaningful effort towards a solution.

Raise your hand if you have any questions, or require clarification of any instructions, during the exam. Good luck!

Clearly print your name in the box below. Do not write your name in any other location unless you are submitting page(s), not attached to the rest of your exam, containing work that you want scored.

Name:

$$\int \sin^m(x)\cos^n(x)\,dx$$

If m and n are even

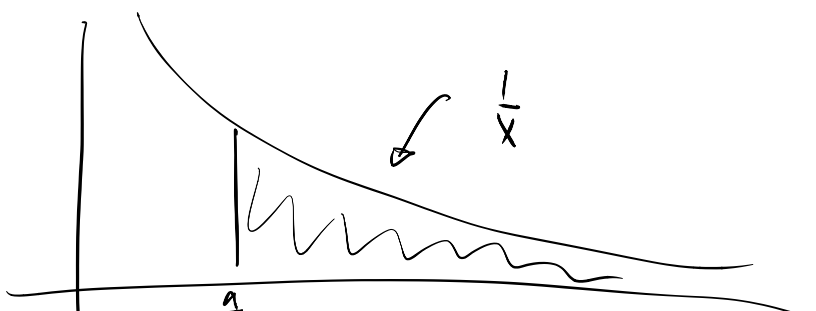
use half-angle identity to reduce the powers

$$\int \sin^4(x)\,dx$$

If one of m and n is odd

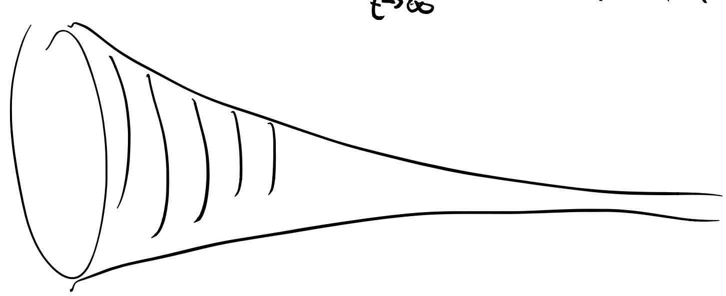
$$\int \sin^5(x)\cos^6(x)\,dx$$

$$= \int \sin(x)\sin^4(x)\cos^6(x)\,dx$$



$$\int_1^\infty \frac{1}{x}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{x}\,dx$$

$$= \lim_{t\to\infty} \ln|t| - \ln|1| = \infty$$



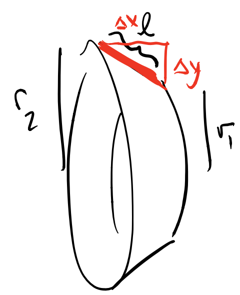
$$V = \int_1^\infty \pi \left(\frac{1}{x}\right)^2 dx$$ 1)

$$= \pi \left(\lim_{t\to\infty} \int_1^t \frac{1}{x^2}\, dx \right)$$ 2) def of imp. integral.

$$= \pi \left(\lim_{t\to\infty} -\frac{1}{x} \Big|_1^t \right)$$ 3) FTC

$$= \pi \left(\lim_{t\to\infty} -\frac{1}{t} - -\frac{1}{1} \right) = \pi$$

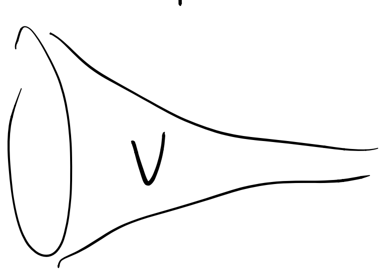
Surface area



$$S = 2\pi\left(\frac{r_1+r_2}{2}\right) l$$

$$\sum 2\pi \underbrace{\left(\frac{r_1+r_2}{2}\right)}_{f(x^*)} \underbrace{l}_{\sqrt{1+(f'(x^*))^2}\,\cdot\Delta x}$$

$$\text{Surface area} = \int_a^b 2\pi f(x)\cdot\sqrt{1+(f'(x))^2}\, dx$$



$$= \int_1^\infty 2\pi \frac{1}{x} \sqrt{1+\left(\frac{-1}{x^2}\right)^2}\, dx$$

$$V = 2\pi \int_1^\infty \frac{1}{x}\sqrt{1+\frac{1}{x^4}}\,dx. \qquad 2\pi \int \sqrt{\frac{1}{x^2}+\frac{1}{x^6}}\,dx$$

If $f(x) \le g(x)$ for all x in $[1,\infty)$

then if $\int_1^\infty f(x)\,dx$ diverges then

$\int_1^\infty g(x)$ diverges

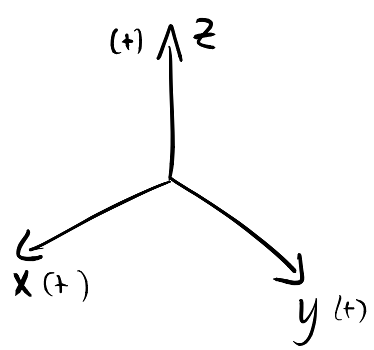
$$g(x) = \frac{1}{x}\sqrt{1+\frac{1}{x^4}} \ge \frac{1}{x}$$

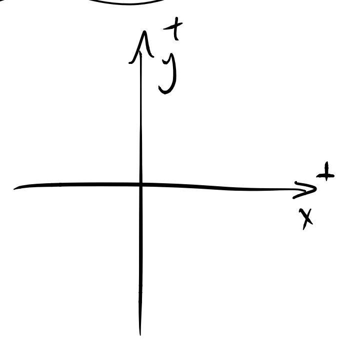
let $f(x) = \frac{1}{x}$

$g(x) = \ldots$

by comparison theorem

Surface area $= \int_1^\infty 2\pi \frac{1}{x}\sqrt{1+\frac{1}{x^4}}\,dx$ is divergent to ∞

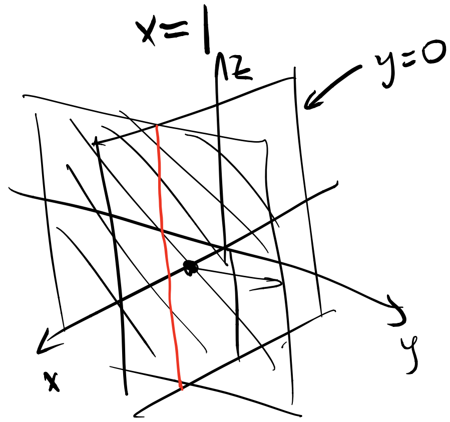


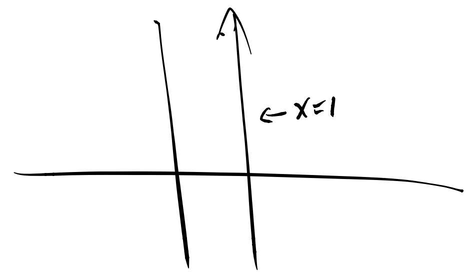


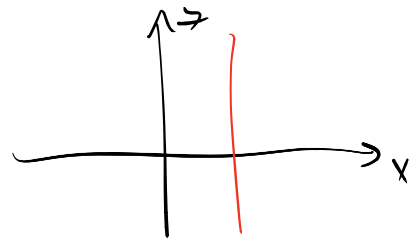
right hand rule

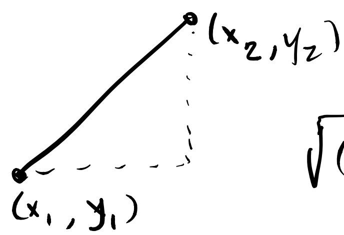
thumb → positive z-axis
pointer → positive x-axis
middle → positive y-axis

swap for left

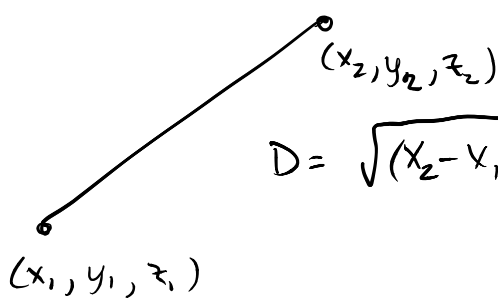




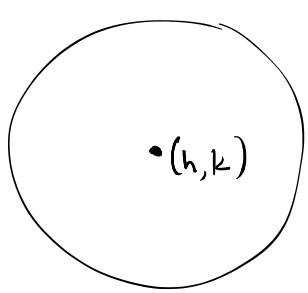




$$\sqrt{(x_2-x_1)^2+(y_1-y_2)^2}$$

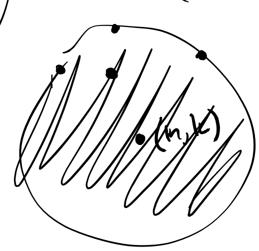


$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

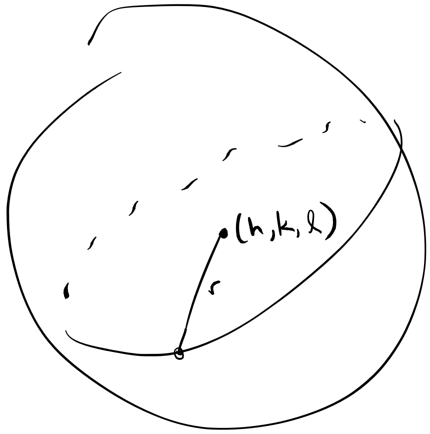


$$x^2+y^2=1$$

$$(x-h)^2+(y-k)^2=1$$



$$\leq$$



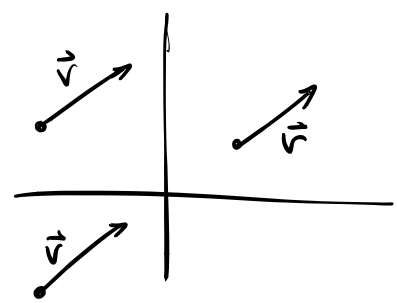
$x^2+y^2+z^2=r^2$

$(x-h)^2+(y-k)^2+(z-\ell)^2=r^2$

Sphere

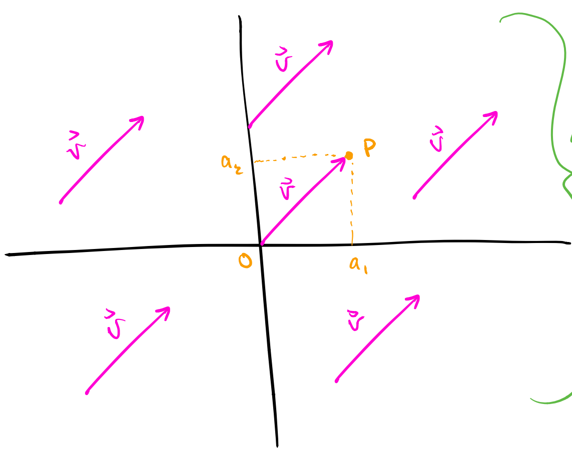
Vectors

vectors are arrows
(new objects)



$\mathbb{R}$ interact w/ vectors
by scaling

$c\cdot\vec{v}$ scales $\vec{v}$ to c times its orignal length.



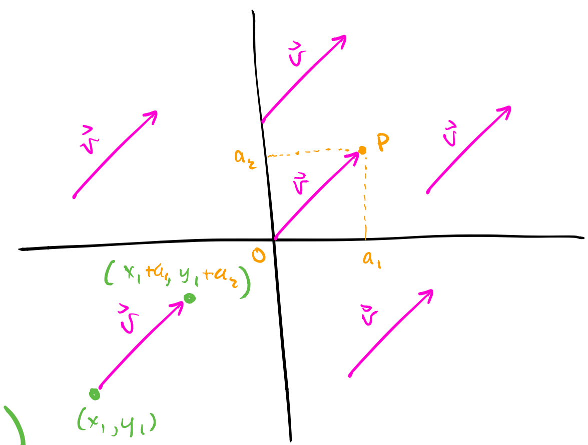
all of these are representations of the same vector.

To make vectors easier to describe, we use the position vector of the point $P(a_1,a_2)$, written $\overrightarrow{OP}$, to define the components of $\vec{v}$.

So, we write

$$\vec{v} = \langle a_1, a_2 \rangle$$

Note: if we start a a point (x_1, y_1) then $\vec{v}$ points to the point $(x_1 + a_1, y_1 + a_2)$.

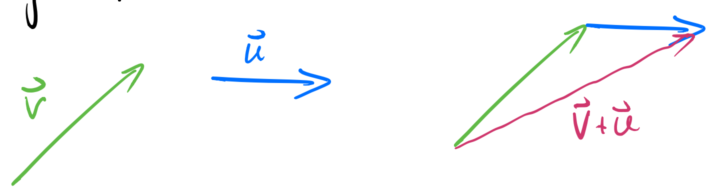


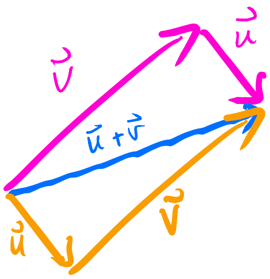
Now, we have a new object (vectors) and it interacts w/ $\mathbb{R}$ numbers ($c \cdot \vec{v}$ is a <u>scalar</u> multiple of $\vec{v}$) but we might want them to interact with each other.

First, addition ...

we can add vectors by concatenating. That is, where one vector ends, the next begins.

ex :





We get that vector addition is commutative by the "parallelogram law"

$\vec{v}+\vec{u} = \vec{u}+\vec{v}$

Given points

$A(x_1, y_1)$
$B(x_2, y_2)$

$\vec{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle$ this is the vector that, if we start it at A, points to B.

we define the magnitude of $\vec{v}$ by: $|\vec{v}| = \sqrt{a_1^2 + a_2^2}$ or $\sqrt{a_1^2 + a_2^2 + a_3^2}$

components of $\vec{v}$

Notice how this is the same as the distance from the origin to the point the vector, starting at the origin, points to.

Standard basis vectors

In two dimensions $\vec{i}$ and $\vec{j}$ w/ $\vec{i} = \langle 1, 0 \rangle$, $\vec{j} = \langle 0, 1 \rangle$

In 3-D $\vec{i}, \vec{j}, \vec{k}$ w/ $\vec{i} = \langle 1, 0, 0 \rangle$, $\vec{j} = \langle 0, 1, 0 \rangle$, $\vec{k} = \langle 0, 0, 1 \rangle$

for any $\vec{v}$ we can write $\vec{v} = a\vec{i} + b\vec{j}$ for a, b $\mathbb{R}$.

ex. If $\vec{v} = \langle -3, 2 \rangle$

then $\vec{v} = -3\vec{i} + 2\vec{j}$