Physics 131
General Physics I
Grinnell College
Spring 2012

Summary of Units I, II, III
Introduction to Forces and Motion

In these units, we are first introduced to the measures of motion: position, velocity, and acceleration. Typically, we are interested in calculating or measuring the position as a function of time of an object. The first two time derivatives of position turn out to be essential for the solution of these problems, so we give them special names. The velocity is the first derivative

$$v(t) = \frac{dx(t)}{dt} \approx \frac{\Delta x}{\Delta t}$$

where the first, exact definition is sometimes referred to as the instantaneous velocity, and the second, approximate definition is referred to as the average velocity over the time interval Δt. Similarly, we define the acceleration as

$$a(t) = \frac{dv(t)}{dt} = \frac{d^2x(t)}{dt^2} \approx \frac{\Delta v}{\Delta t}$$

where the final approximation is called the average acceleration. We investigated this acceleration experimentally in a bit of detail by looking at freefall. We discovered that the velocity increased linearly with time, which implied a constant acceleration for falling objects. The value of this acceleration is 9.8 m/sec^2.

We then introduced the concept of force, a push or a pull. We discovered that contrary to much common misconception, a large force does not necessarily give rise to fast motion of an object, that is a large velocity, but rather it gives rise to a large acceleration. Further, we found that this acceleration was inversely proportional to the amount, or mass, of the object being pushed; heavy things accelerate more slowly than light things for the same force. This is expressed mathematically as Newton's second law:

$$F = ma.$$

We then investigated several different types of forces, and the type of motion that results from them. We already were familiar with the motion resulting from a constant force, which thereby gives a constant acceleration, as we saw in the freefall case. We observed this for the sliding friction case, which has the force law

$$F_{friction} = -\mu F_{normal}$$

where μ is the coefficient of friction, and the normal force is the force pushing the two surfaces together, and the negative sign is a reminder that the force acts against the motion. For the gravity force, we have a force law

$$F=-mg$$

where g is 9.8 m/sec^2. The motion that results from constant acceleration is described by

$$x(t) = \tfrac{1}{2}at^2 + v_o t + x_o \text{ and}$$

$$v(t) = at + v_o$$

where v_o and x_o are the initial velocity and position of the object. The acceleration a is calculated from the ratio of the force to the mass, using Newton's second law.

We also looked at the spring force law

$$F = -kx$$

which is also known as Hooke's law. We saw that this gives rise to oscillatory motion. This type of motion is so important to many areas of physics that we will return and spend an entire unit investigating it.

There remained one other major possible force law, one that gave a force proportional to velocity. This is characteristic of viscous drag forces, as we investigated moving a weight through corn syrup, and in watching a light object fall through the air. This force law is given by

$$F = -\alpha v.$$

This law gives rise to an exponential decrease in the velocity. The complete solution of position as a function of time is

$$x(t) = x_{final} - \frac{mv_o}{\alpha} e^{\left(-\frac{\alpha}{m} t\right)}$$

and velocity as a function of time as

$$v(t) = v_o e^{\left(-\frac{\alpha}{m} t\right)}.$$

We will often encounter situations in which the force is a combination of two or more of these force laws, and we can often gain analytical solutions in those cases as well.

However, the real world rarely behaves in such nice ways. Actual forces do not have these simple laws, or simple solutions. Often we can use these simple cases as reasonable approximations to reality, but sometimes we cannot. In those cases we cannot solve analytically, however, we can always resort to numerical techniques. In general, these techniques involve using the forces to calculate accelerations, and thereby update the velocity and position over short time intervals, using our average velocity and acceleration formulae

$$\Delta v = a\Delta t$$
$$\Delta x = v\Delta t.$$