1 Recap

Last time, we developed the ***integration by parts formula***. We started with the Product Rule for differentiation:

If f and g are differentiable functions then

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

From this, we used integration to come up with a new formula.

$$\begin{aligned}
(f(x)g(x))' &= f'(x)g(x) + f(x)g'(x) && \text{(by The Chain Rule)} \Rightarrow && (1)\\
\int (f(x)g(x))' &= \int f'(x)g(x)dx + \int f(x)g'(x)dx && \text{(integrating both sides)} \Rightarrow && (2)\\
\int f'(x)g(x)dx &= f(x)g(x) - \int f'(x)g(x)dx. && \text{(solving for } \int f'(x)g(x)dx\text{)} && (3)
\end{aligned}$$

The ***integration by parts formula*** says that

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx \tag{4}$$

2 An Example

Compute $\int x \sec^2(2x)dx$.

- Let

  $f(x) = x$ and $g'(x) = \sec^2(2x)$.

- Now,

  $$I = \int x \sec^2(2x)dx = \int f(x)g'(x)dx$$

- To use integration by parts, we compute

  $f'(x) = 1$ and $g(x) = \frac{1}{2}\tan(2x)$

- Then plug these in to our integration by parts formula that is Equation (4).

  $$I = \frac{1}{2}\tan(2x) - \frac{1}{2}\int \tan(2x)dx \tag{5}$$

- If we are lucky, we can keep going. Here, we are lucky. We can look back at integral formulas we know and use them. In the integral table from your book, we have

  12. $\int \tan(u)du = \ln|\sec(u)| + C$

- So, we have that

$$\begin{aligned}
I &= \frac{1}{2}\tan(2x) - \frac{1}{2}\int \tan(2x)dx && \text{using the I.B.P. formula} && (6)\\
&= \frac{1}{2}\tan(2x) - \frac{1}{4}\ln|\sec(2x)| + C && \text{using substitution and number 12 from the integral table.} && (7)
\end{aligned}$$

So, to summarize the strategy, we do the following:

- We are given an integral
- We choose f and g'
- We compute f' and g
- We plug that in to integration by parts
- We see if we know $\int f' g dx$
- If we do, we're done. If we don't we can try using integration by parts on $\int f' g dx$, or we can change our choices for f and g'.