Name:_____

Test Your Skills - 07
Deadline - 4/21

Answer these exercises, in complete mathematical sentences and using mathematical notation properly. You are to work on these individually, without collaboration. You may consult your book and myself, but **not the math lab** or other resources. To earn extra credit, stop into my office hours (or make an appointment) and present your solutions. Partial credit will be given for any earnest attempt.

Exercise 1. *Suppose $f(x, y)$ and $g(x, y)$ are differentiable functions, and a and b are any constants, and n is a natural number. Prove the following:*

a) $\nabla(af + bg) = a\nabla f + b\nabla g$

b) $\nabla(fg) = f\nabla g + g\nabla f$

c) $\nabla\frac{f}{g} = \frac{g\nabla f - f\nabla g}{g^2}$

d) $\nabla(f^n) = n(f^{n-1})\nabla f$

Exercise 2. *Find an equation for the plane that passes through the point $P(2, 3, 4)$ and cuts off the smallest volume in the first octant.*

Exercise 3. *Consider a rectangular box with a diagonal (the distance from one bottom corner to the corner corresponding to the opposite corner on top) of length L, what is the maximum volume of the box? (Justify your answer.)*

Exercise 4. *Let*

$$f(x_1, ..., x_n) = \sqrt[n]{x_1 \cdot x_2 \cdots x_n},$$

where $x_i > 0$ for all $i = 1, 2, ..., n$, and let

$$g(x_1, ..., x_n) = x_1 + x_2 + \cdots + x_n,$$

where $x_i > 0$ for all $i = 1, 2, ..., n$.

a) If $g(a_1, ..., a_n) = c$, what is the maximum value of $f(a_1, ..., a_n)$?

b) Under what conditions do each of the following hold?:

i) $f(a_1, \ldots, a_n) < \dfrac{g(a_1, \ldots, a_n)}{n}$

ii) $f(a_1, \ldots, a_n) = \dfrac{g(a_1, \ldots, a_n)}{n}$

iii) $f(a_1, \ldots, a_n) > \dfrac{g(a_1, \ldots, a_n)}{n}$