Name:____________________________________

Worksheet 01

1 Introduction

Welcome to the first worksheet of the semester. The goal when completing these worksheets is to give an opportunity to develop and practice concepts in an environment where we can help one another. If you find the exercises easy, great! Make sure that your group-mates understand the concepts in hand. You should not move past even a single line that you do not fully understand, or that any member of your group does not fully understand.

If you and all of your group feel that you have mastered the material before the class has moved on, use that time to generate your own examples to test one another. Being able to write your own exercises is an excellent way to test your mastery of a concept. If you need further challenges, see the last section of the worksheet.

2 Recap

Up to this point, it is assumed that you have mastered differentiation of real valued functions, and have begun to develop your knowledge of anti-differentiation and integrals. In particular, you should be comfortable with recognizing some standard derivatives and, thus, their anti-derivatives, as well as basic applications of the Substitution Rule for integration.

As you may have noticed already, integration is not as straightforward as differentiation. In fact, while you should be able to compute the derivative of any function that you can write down, it is fairly easy to write down a function whose integral has no closed form (if interested, see the Wikipedia article: Nonelementary integral). The first part of this course will be devoted to learning some basic techniques for computing the anti-derivatives of certain, relatively nice, functions.

3 Integration by Parts

The Substitution Rule for integration corresponded to "undoing" the Chain Rule for differentiation. You might hear me refer to it as the Anti-Chain Rule. The next differentiation rule we want to "undo" is the Product Rule.

The Product Rule If f and g are both differentiable functions, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Or, in other notation,

$$[f(x)g(x)]' = f(x)g'(x) + g(x)f'(x).$$

Exercise 1. *Compute*

$$\frac{d}{dx}[x\cos(x)]$$

using the Product Rule. Include all steps.

Recall that

$$\int \left(\frac{d}{dx} \left[x \cos(x) \right] \right) dx = x \cos(x) + C. \tag{1}$$

Exercise 2. *Take the anti-derivative of both sides of the equality in your final answer for Exercise 1 and simplify until you have a single anti-derivative left to compute. Isolating this term on one side of the equation yields a new integral identity.*

Exercise 3. *Rephrase your calculations in Exercise 2 in terms of f and g from Exercise 1.*

This process yields a new integration strategy:

Integration By Parts For differentiable functions f and g we have that

$$\int f(x)g'(x)dx =$$

Exercise 4. *Use this strategy to compute:*

$$\int x \sin(x) dx$$

Exercise 5. *Use this strategy to compute:*

$$\int \ln(x) dx$$

(Hint: Your options for f and g' are fairly limited.)

Exercise 6. *Use this strategy to compute*

$$\int xe^x dx$$

Exercise 7. *Use this strategy to compute*

$$\int x^2 e^x dx$$

Exercise 8. *Use this strategy to compute*

$$\int e^x \cdot \cos(x) dx$$

(Hint: Don't stop trying, even if it seem like you are going nowhere.)

Exercise 9. *Recall that*

$$\frac{d}{dx}(\cot^{-1}(x)) = -\frac{1}{1+x^2}$$

Compute

$$\int \cot^{-1}(x) dx$$

4 Optional Practice

If your group is waiting for the class to move forward, take some time to practice the problems in this section.

1. Evaluate $\int x\cos(5x)dx$
2. Evaluate $\int \frac{\ln(y)}{\sqrt{y}}dy$
3. Evaluate $\int (\ln(x))^2 dx$
4. Evaluate $\int (\ln(x))^3 dx$
5. Evaluate $\int (\ln(x))^4 dx$
6. Complete the following general rule the:

$$\int (\ln(x))^n dx =$$

(Note: Your answer may contain an integral.)