Name:____________________________________

Worksheet 02

1 Using Trigonometric Identities To Compute Integrals

1.1 An Example

$$\begin{aligned}
\int \cos^4(x)dx &= \int \left(cos^2(x)\right)^2 dx && (1)\\
&= \int \left(\frac{1+\cos(2x)}{2}\right)^2 dx && (2)\\
&= \frac{1}{4}\int \left(1+2\cos(2x)+\cos^2(2x)\right) dx && (3)\\
&= \frac{1}{4}\int \left(1+2\cos(2x)+\frac{1+\cos(4x)}{2}\right) dx && (4)\\
&= \frac{1}{4}\int \left(\frac{3}{2}+2\cos(2x)+\frac{1}{2}\cos(4x)\right) dx && (5)
\end{aligned}$$

For the following, use complete mathematical sentences and include any intermediate calculations that are needed.

Exercise 1. *What justifies step (1) in the above computation?*

Exercise 2. *What justifies step (2) in the above computation?*

Exercise 3. *What justifies step (3) in the above computation?*

Exercise 4. *What justifies step (4) in the above computation?*

Exercise 5. *What justifies step (5) in the above computation?*

Exercise 6. *Evaluate the following:*

$$\frac{1}{4}\int\left(\frac{3}{2}+2\cos(2x)+\frac{1}{2}\cos(4x)\right)dx$$

1.2 Devising a Plan of Attack

We want to have a plan of attack whenever we encounter an integral of the form

$$\int (\sin^m(x)\cos^n(x))\, dx \tag{6}$$

Where m and n are integers.

Exercise 7. *If we are trying to compute an integral in the form of equation (6), and m is odd, how can we use trigonometric identities to make the computation possible with methods we already have?*

Exercise 8. *If we are trying to compute an integral in the form of equation (6), and n is odd, how can we use trigonometric identities to make the computation possible with methods we already have?*

Exercise 9. *If we are trying to compute an integral in the form of equation (6), and m and n are both even, how can we use trigonometric identities to make the computation possible with methods we already have?*

2 Trigonometric Substitution

3 The Building Blocks

Recal this important identity:

$$\sin^2(x) + \cos^2(x) = 1 \tag{7}$$

Exercise 10. *Divide both sides of equation (7) by* $\cos^2(x)$ *to get a new trigonometric identity.*

Exercise 11. *Using equation (7) and the new identity you came up with in Exercise 10, complete the following statements:*

(a) $\cos^2(x) =$

(b) $\sec^2(x) =$

(c) $\tan^2(x) =$

3.1 Practice

Exercise 12. *Evaluate the following:*

$$\int \left(\frac{20x}{\sqrt{25 - 25x^8}} \right) dx$$

Exercise 13. *Evaluate the following:*

$$\int \left(\frac{10x^4}{9 + 4x^{10}} \right) dx$$

Exercise 14. *Evaluate the following:*

$$\int \left(\frac{2 \cdot \csc^2(2x)}{\cot(2x) \cdot \sqrt{\cot^2(2x) - 1}} \right) dx$$

4 Optional Practice

If your group is waiting for the class to move forward, take some time to practice the problems in this section.

1. Evaluate $\int_0^{\pi} \cos^6(\theta) d\theta$
2. Evaluate $\int \cos^2(\theta) \tan^3(\theta) d\theta$
3. Show that $\int_{-\pi}^{\pi} \sin(mx)\cos(nx)dx = 0$ for all positive integers m and n
4. Evaluate $\int_0^1 \sqrt{x^2+1}dx$
5. Show that $\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln(x + \sqrt{x^2+a^2}) + C$