Physics 131
General Physics I
Grinnell College
Spring 2012

Summary of Units 8 and 9
Waves

We most commonly think of waves as phenomena that oscillate sinusoidally in both time and space. In fact, the oscillation part is not essential; pulses are governed by wave rules as well as sinusoidal waves are . Most typically we talk about a function that moves through space, such that

$$f(x,t) = f(kx-\omega t), \text{ which may be } f(x,t) = A\sin(kx-\omega t)$$

where in the latter instance we see the format for a sinusoidal wave that travels in the positive x direction. The constant A gives the amplitude of the oscillation, while the constants k and ω relate to the periodicity of the oscillation. In particular, k relates to the wavelength as

$$k = 2\pi/\lambda$$

and ω relates to the frequency and period of oscillation as

$$\omega = 2\pi f = 2\pi/T.$$

The speed of this wave is given by the ratio of k and ω

$$c = \omega/k = \lambda f = \lambda/T$$

where the latter parts of this expression give relations involving wavelength, frequency and period.

A wave on a string that encounters a fixed end is reflected in an inverted fashion. This is easiest to see for pulses, but it occurs for sinusoidal waves as well. In the case of sinusoidal waves, a fixed end automatically produces two identical waves traveling in opposite directions. The resulting wave pattern is the literal sum of the two waves, which gives rise to a standing wave

$$f(x,t) = 2A\sin(kx)\cos(\omega t)$$

in which some points in space experience no oscillation whatsoever, and are known as nodes. This wave does not travel. If we fix both ends of the string, this constrains the motion further, such that a standing wave will exist only if it has a node at each end. This requires then that the length of the string be a multiple of half the wavelength (which is the distance between adjacent nodes). This can be expressed as an equation

$$\ell = n\frac{\lambda}{2}$$

where n is an integer (1, 2, 3, etc). This equation also constrains the frequencies to be

$$f = \frac{c}{\lambda} = \frac{nc}{2\ell}$$

where c is the speed of wave motion on the string, which is determined by the tension and density of the string.

In multiple dimensions, we studied waves that moved in a straight line, which are called plane waves, since the location of a crest is no longer defined by a point but rather by a plane. The wave function is given by

$$\mathrm{f}(x,y,z,t) = \mathrm{A}\sin(k_x x + k_y y + k_z z - \omega t).$$

The period of oscillation is as for one dimensional waves, that is $T = 2\pi/\omega$. The wave travels in a direction defined by the vector

$$\mathbf{k} = (k_x, k_y, k_z).$$

The wavelength is given by $k = 2\pi/\lambda$ where

$$k = \sqrt{k_x^2 + k_y^2 + k_z^2}.$$

We also looked at spherical and circular waves which radiated from a central point. The wave function is given by an expression like

$$f(r,t) = A\sin(kr \pm \omega t)$$

where the "-" sign gives waves radiating out, and the "+" sign gives waves that move inward. We can superpose two waves like this separated by a distance d, and find that we get interference maxima at a distance at angles that are given by the expression

$$d\sin\theta_n = n\lambda,$$

where $n = 0, \pm 1, \pm 2,$ etc. We found that plane waves traveling through an opening comparable or smaller than the wavelength produced spherical waves that appeared to radiate from the center of the opening. This process is known as diffraction. As a result, plane waves incident on slits spaced by d give rise to interference maxima also given by the formula above for the interference maxima of two spherical waves, as for two slit diffraction and diffraction gratings.