

## Homework VI

### Session VI.1

1. In Unit VI.1, we saw that circular motion could be represented as:

$$(x,y) = R(\cos \omega t, \sin \omega t)$$

What does motion look like that might be given by each the following expressions (that is, I want three separate answers here):

$$(x,y) = R(\sin \omega t, \cos \omega t)$$

*or*

$$(x,y) = R(\cos \omega t, -\sin \omega t)$$

*or*

$$(x,y) = R(\cos \omega t, \cos \omega t)$$

2. Rewrite the circular motion expression  $(x,y) = R(\cos \omega t, \sin \omega t)$  to a form appropriate for motion in a circle with a rate of 5 radians/sec, a radius of 3 m, and a *center* located at  $(x,y) = (2,3)$ .

### Session VI.2

3. Recall what you discovered in session VI.2, that

$$F_{\text{centripetal}} = -mR\omega^2 = -m\frac{v^2}{R}$$

where the minus sign indicates the force is directed inward. If a car of mass 700 kg is going around a corner which is part of a circular path of radius 10 m at 7 m/sec, what is the force the ground exerts on the wheels to do this (i.e. travel in the circle, *not* the force necessary to resist gravity and keep the car from falling to the center of the earth)? What is the direction of the force?

4. Imagine you are a passenger in the car in problem 3, and you have a mass of 65 kg. What is the force acting on you? How large is that compared to the gravity force acting on you? Will this seem like a big force or a small one?

5. We know that the earth rotates once per day, and has a circumference of 40,000,000 meters. If we are standing on the equator, what is the centripetal acceleration we experience? This acceleration lessens our apparent weight (just as accelerating at  $9.8 \text{ m/s}^2$  makes us seem weightless) by an amount equal to  $ma$ . Can we expect to notice the weight reduction associated with this centripetal acceleration relative to our normal weight (or in other words, would we weigh significantly more at the pole than at the equator)?

### Session VI.3

6. A skater in a spin can speed up her rotation by pulling her arms in. This decreases her moment of inertia, while maintaining her angular momentum. In this problem, we'll estimate how big an effect this has on her moment of inertia. First, let's estimate roughly her rotating body as having a mass of 50 kg, at an average radius of 10 cm. Find this moment of inertia. Then estimate arms have a mass of 5 kg each at an outstretched radius of 70 cm, and a pulled-in radius of 15 cm. Calculate the arm moment of inertia in each case, pulled-in and outstretched.

7. If the skater in problem 1 is initially spinning at 3 rotations per second with arms outstretched, how fast is she spinning when she pulls her arms in? (We are assuming that the contact with the ice is frictionless, so we may assume no net force on her. Is there any net torque on her?)

8. The force of gravity is given in generality (i.e. even far from the surface of the earth) as

$$F = -\frac{GMm}{r^2}$$

where M is the mass of one body (like the earth) and m is the mass of the other body attracted to it (like the moon, or you), and G is called the gravitational constant,  $6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$ . If the mass of the earth is  $6.0 \times 10^{24} \text{ kg}$  (notice that you could find this from the value of g and the radius of the earth, although you don't need to), how far away is the moon given that it orbits the earth every 28 days?