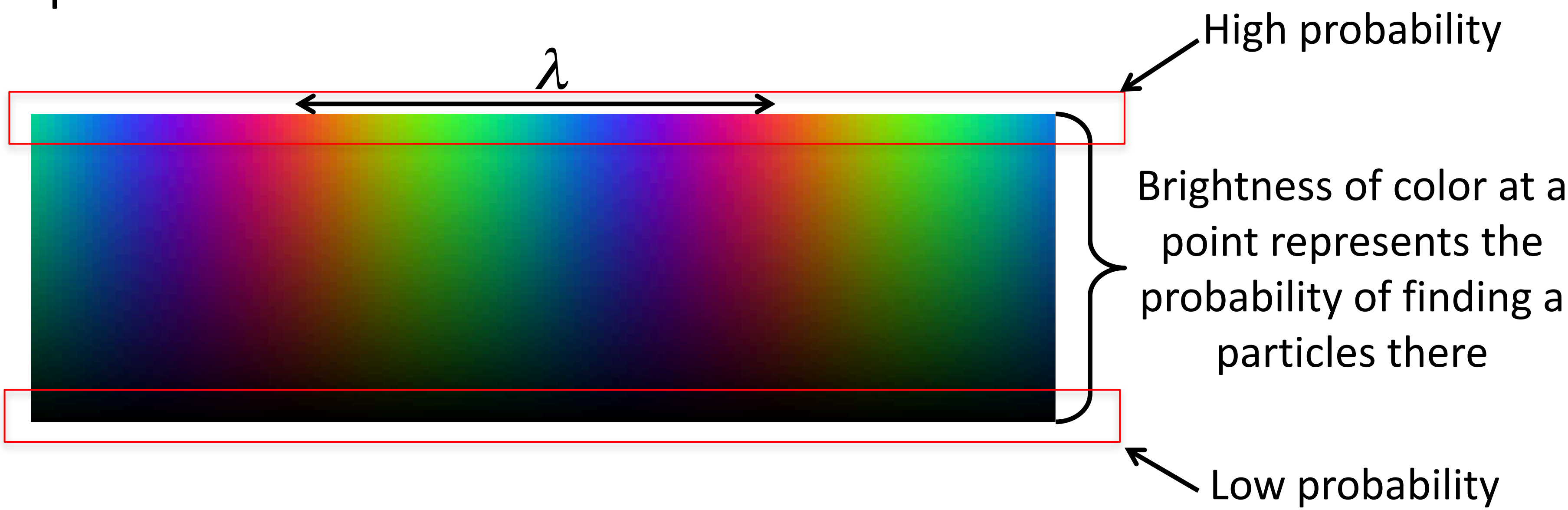
Review of Unit 11: Quantum Waves in One Dimension

- To help us visualize quantum waves, we introduced quantum palette:



- Different colors also help us understand the time evolution of quantum waves.

  Basic rule: Where the color is red becomes orange at the next time step, then yellow, green, and so on.

What is the propagation direction of the quantum wave below?

A. To the right

B. To the left

What is the propagation direction of the quantum wave below?

✓ A. To the right

B. To the left

Review of Unit 11: Quantum Waves in One Dimension

- Just like waves on a string, overlapping quantum waves undergo interference.

Constructive interference:

R overlapping with R $\Rightarrow$ brighter R

G overlapping with G $\Rightarrow$ brighter G

Destructive interference:

R overlapping with its opposite color (T) $\Rightarrow$ black

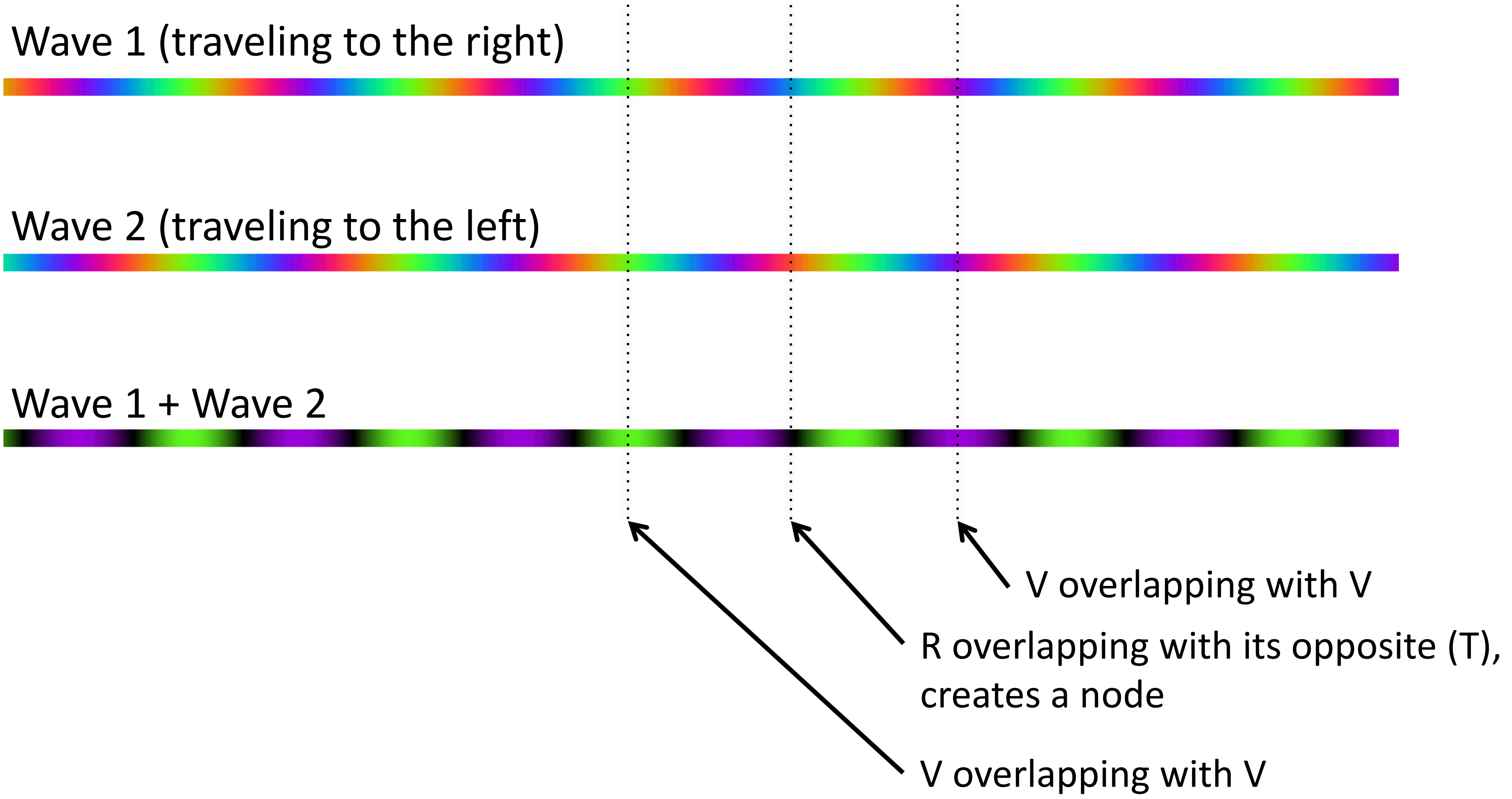
O overlapping with its opposite color (B) $\Rightarrow$ black

Y overlapping with its opposite color (I) $\Rightarrow$ black

G overlapping with its opposite color (violet) $\Rightarrow$ black

Review of Unit 11: Quantum Waves in One Dimension

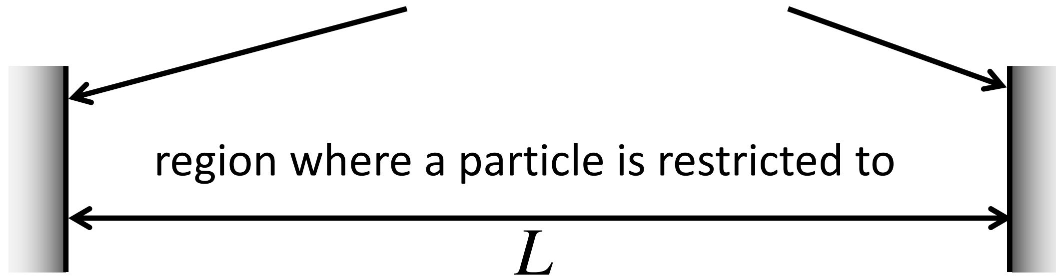
- Two counter-propagating quantum waves form a standing wave:



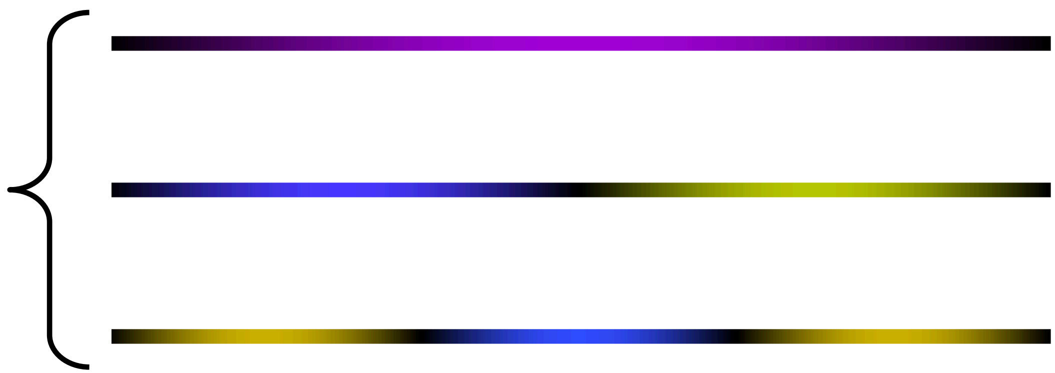
Review of Unit 11: Quantum Waves in One Dimension

- **Quantum particle in a 1-dimensional "box"**: a simple model to describe what happens if a particle (like an electron) is confined to a small region of space.

There must be nodes at the boundaries since the particle cannot be at or beyond the boundaries.



Some of the possible quantum waves



$$L = \frac{\lambda}{2}$$

$$L = 2\frac{\lambda}{2}$$

$$L = 3\frac{\lambda}{2}$$

Review of Unit 11: Quantum Waves in One Dimension

- **Quantum particle in a 1-dimensional "box"**: continued

In general,

$$L = n\frac{\lambda}{2} \Longrightarrow \lambda = \frac{2L}{n}$$

Quantum waves with certain discrete wavelengths can exist inside the "box".

n: positive integer called <u>quantum number</u>

Since $E = \dfrac{p^2}{2m} = \dfrac{h^2}{2m\lambda^2}$,

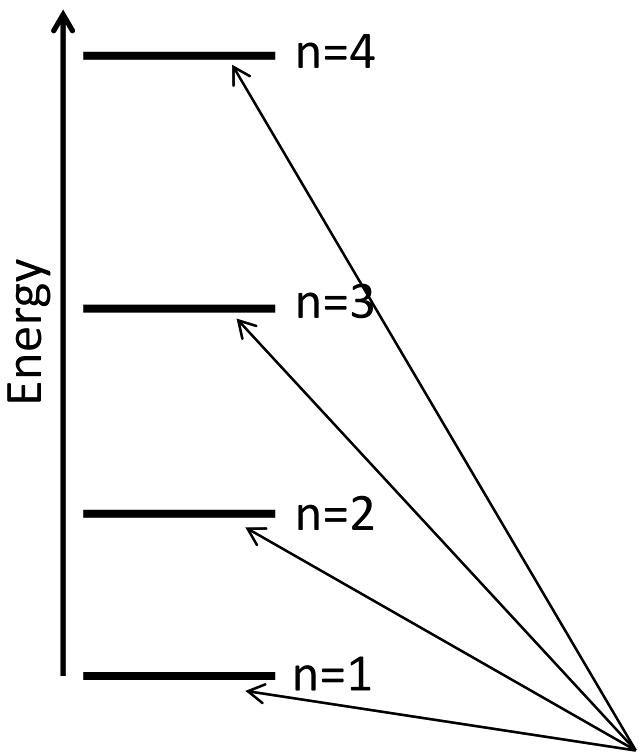
$$\boxed{E = \frac{p^2}{2m} = \frac{n^2 h^2}{8mL^2}}$$

Allowed energy levels of a particle with mass m in a box with length L

Suppose a particle with mass m in a box with length L is initially in n=2 state. It then loses energy and goes to n=1 state by emitting a photon.

What is the energy of the photon?

Hint: $E = \dfrac{n^2 h^2}{8mL^2}$

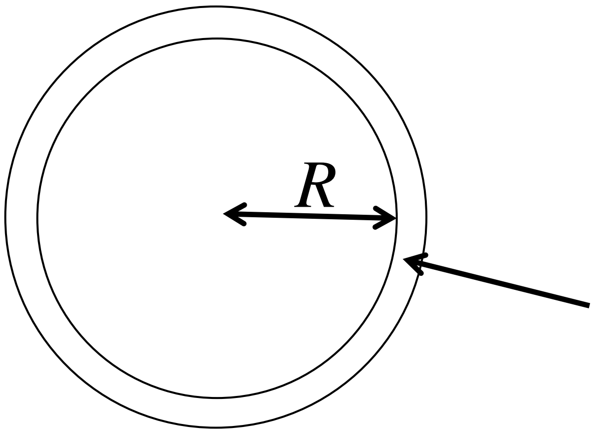


$$\Delta E = E_2 - E_1 = \frac{2^2 h^2}{8mL^2} - \frac{1^2 h^2}{8mL^2} = \frac{3h^2}{8mL^2} = E_{photon}$$

$$E_{photon} = hf \quad \Longrightarrow \quad \boxed{f = \frac{3h}{8mL^2}}$$

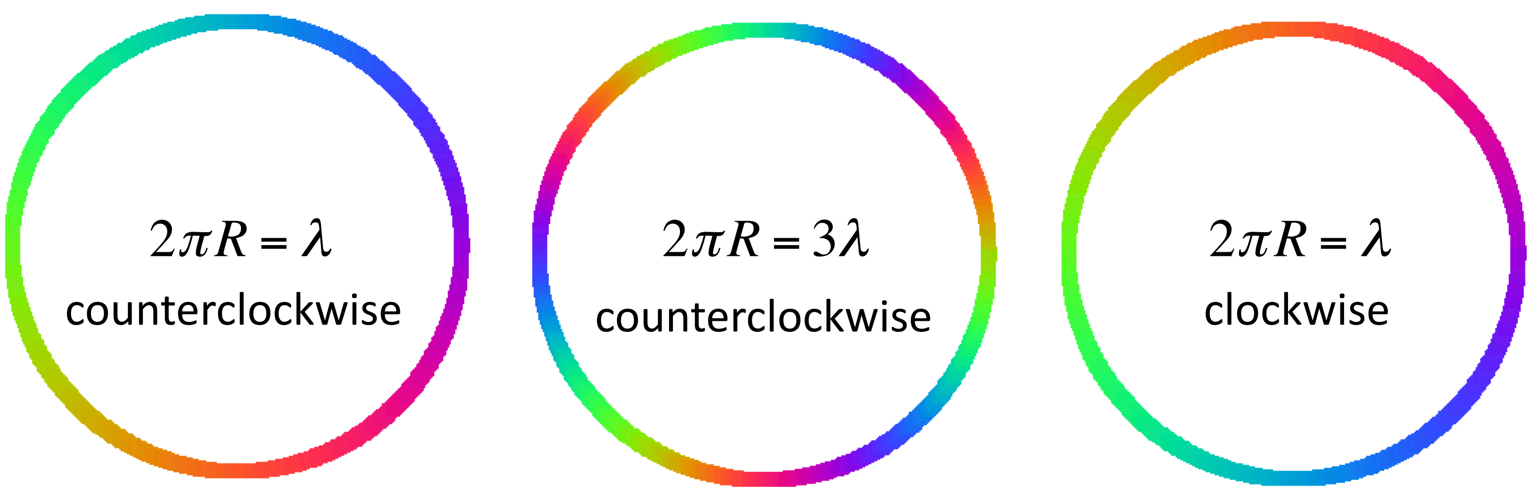
Review of Unit 11: Quantum Waves in One Dimension

- **Quantum particle on a hoop**: a simple model to describe what happens if a particle (like an electron) is confined to a circular path.



Colors must vary continuously and there can only be one color at any given point along the hoop.

Some of the possible quantum waves



Review of Unit 11: Quantum Waves in One Dimension

- **Quantum particle on a hoop**: continued

In general,

$$2\pi R = n\lambda \Longrightarrow \lambda = \frac{2\pi R}{n}$$

Integer. (By convention, we use positive n if the wave moves counterclockwise, and negative n if it moves clockwise.)

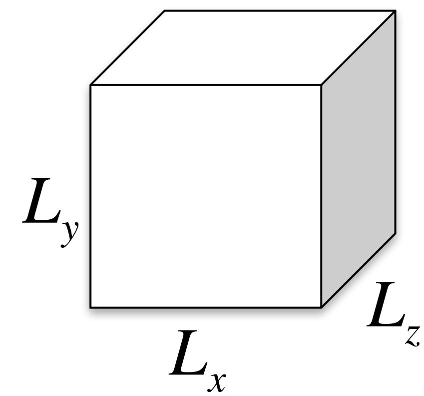
Particle on a hoop has angular momentum given by

$$L = pR = \frac{h}{\lambda}R = \frac{nh}{2\pi} = n\hbar$$

$$\hbar = \frac{h}{2\pi}$$

Review of Unit 12: Quantum Waves in Multiple Dimensions and the the Hydrogen Atom

- **Quantum particle in a 3-dimensional box**:
  In each of 3 directions (x,y,z), we need to have an integer multiple of a half wavelength.



$$L_x = n_x \frac{\lambda_x}{2}, \quad L_y = n_y \frac{\lambda_y}{2}, \quad L_z = n_z \frac{\lambda_z}{2}$$

$\Longrightarrow$ Need 3 quantum numbers (n_x, n_y, n_z) to specify a quantum state

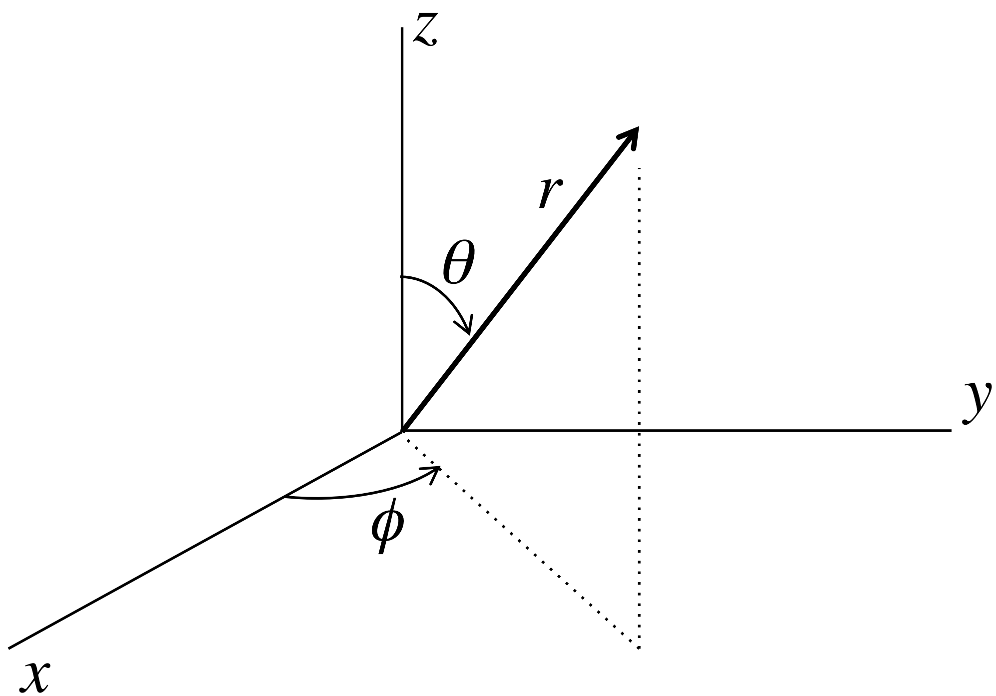
Since $E = \dfrac{p_x^2 + p_y^2 + p_z^2}{2m}$,

$$E = \frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right)$$

Allowed energy levels of a particle with mass m in a 2-dimensional box.

Review of Unit 12: Quantum Waves in Multiple Dimensions and the the Hydrogen Atom

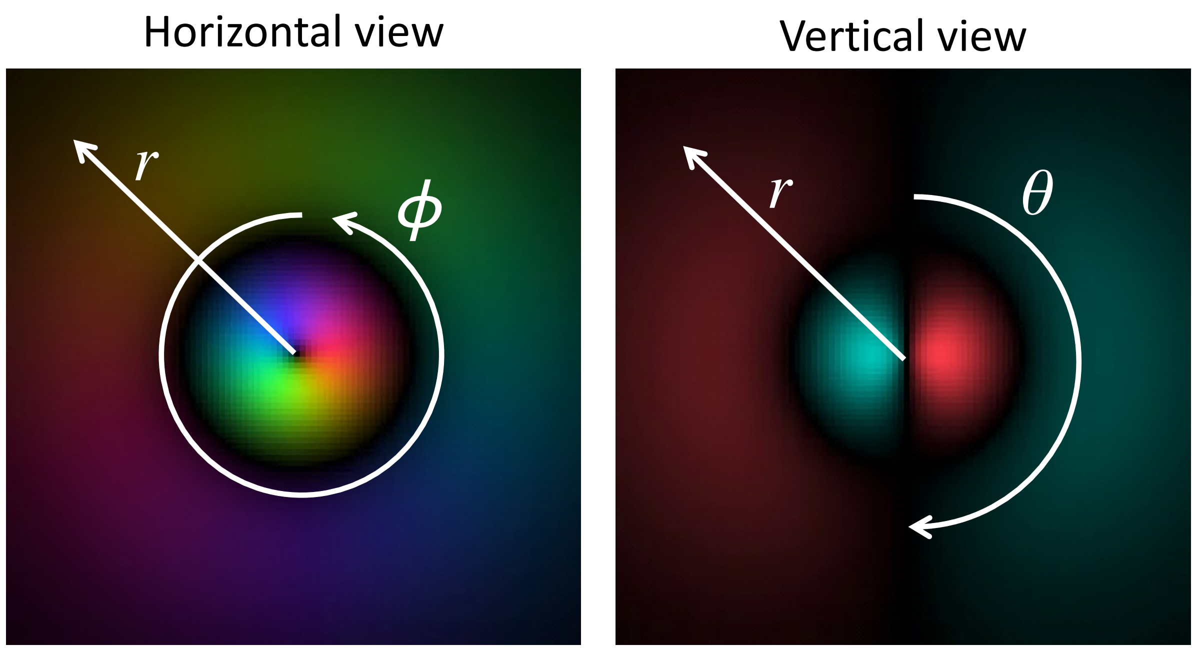
- **Hydrogen Atom**: Because of the spherical symmetry of the atom, the number of nodes (or half wavelengths) should be counted in the r, θ, and φ directions instead of x, y, and z directions.



$$0 \le r \le \infty$$
$$0^\circ \le \theta \le 180^\circ$$
$$0^\circ \le \phi \le 360^\circ$$

Review of Unit 12: Quantum Waves in Multiple Dimensions and the the Hydrogen Atom

- **Hydrogen Atom**: continued



n_r: number of nodes in the radial direction (does not include the node at the origin)
n_θ: number of nodes in the θ direction in the vertical view (does not include nodes at 0° and 180°)
n_ϕ: number of rainbows in the ϕ direction in the horizontal view (positive if counterclockwise, negative if clockwise)

Review of Unit 12: Quantum Waves in Multiple Dimensions and the the Hydrogen Atom

- **Hydrogen Atom**: continued

Instead of using of n_r, n_θ, n_ϕ to specify a quantum state, we can also use quatnum numbers n, l, m_l, where

$$n = n_r + n_\theta + \left|n_\phi\right| + 1, \qquad l = n_\theta + \left|n_\phi\right|, \qquad m_l = n_\phi$$

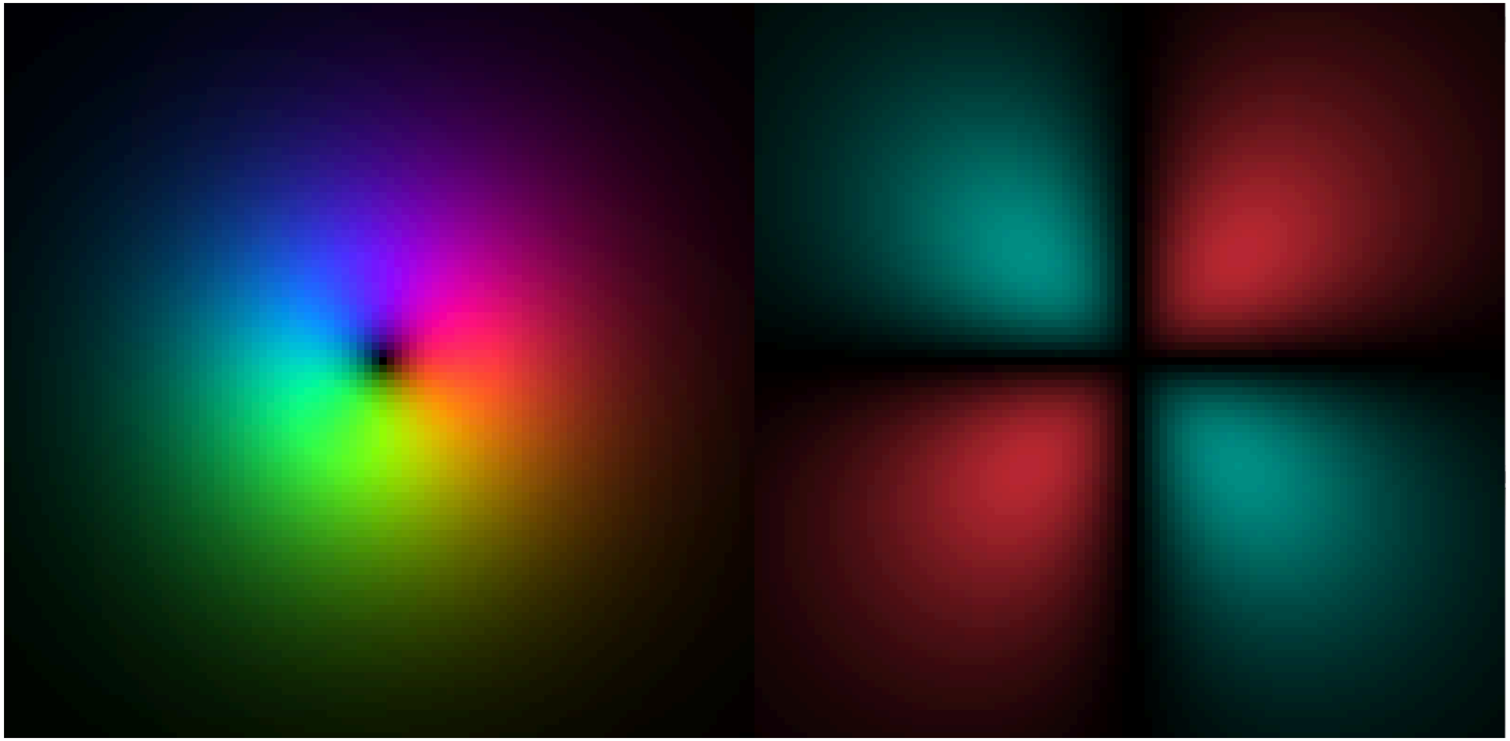
We usually prefer n, l, m_l since they are directly related to physical quantities we can measure experimentally:

$$E = -\frac{2.2\times 10^{-18}\,J}{n^2} \qquad \text{Energy}$$

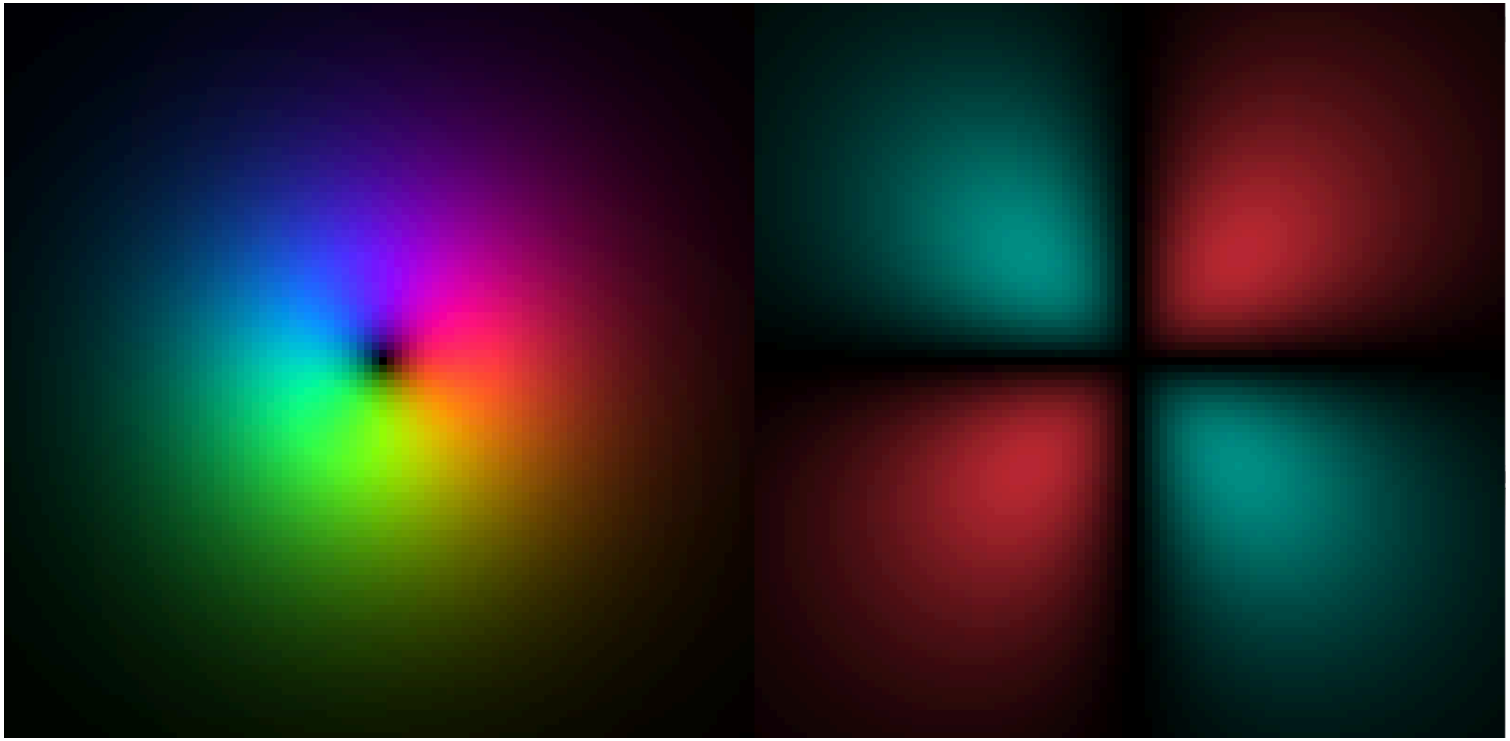
$$L^2 = l\left(l+1\right)\hbar^2 \qquad \text{Square of total angular momentum}$$

$$L_z = m_l\hbar \qquad \text{Angular momentum in z direction}$$

What are the quantum numbers n, l, m_l?



What are the quantum numbers n, l, m_l?



n=3, l=2, m_l=1

Review of Unit 13: Statistical Physics

- **Microstate**: the state of a system in which the states of all the constituents (atoms, coins, dice, etc) are specified

  Example: coin 1 = H, coin 2 = T, coin 3 = T, …

- **Macrostate**: the state of a system in which macroscopic parameters are specified.

  Example: Half of 10 coins are heads, the other half are tails.

- For a system with N particles each of which have two possible states (e.g. N coins with n of them heads):

$$\text{\# of microstates} = \frac{N!}{n!(N-n)!}$$

Review of Unit 13: Statistical Physics

- **Entropy**: “degree of disorder”

$$S = k_B \ln(\#\text{ of microstates})$$

$$k_B = 1.38\times 10^{-23}\, J/K$$:Boltzmann constant

- **Second law of thermodynamics**: entropy of an isolated system always increases or remains the same.

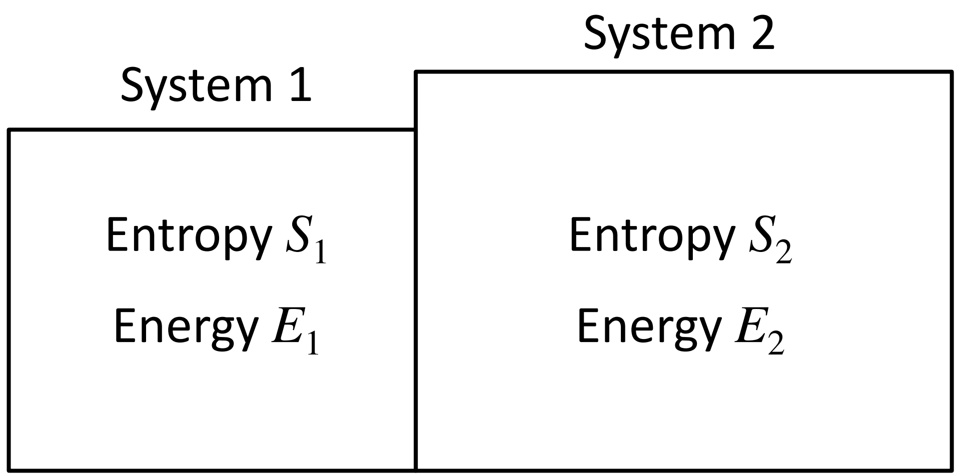
- **Temperature** is defined as

$$\frac{1}{T} = \frac{dS}{dE}$$

Review of Unit 13: Statistical Physics

- Suppose there are two systems, then

$$S_{tot} = S_1 + S_2 \qquad \text{(entropy is additive)}$$



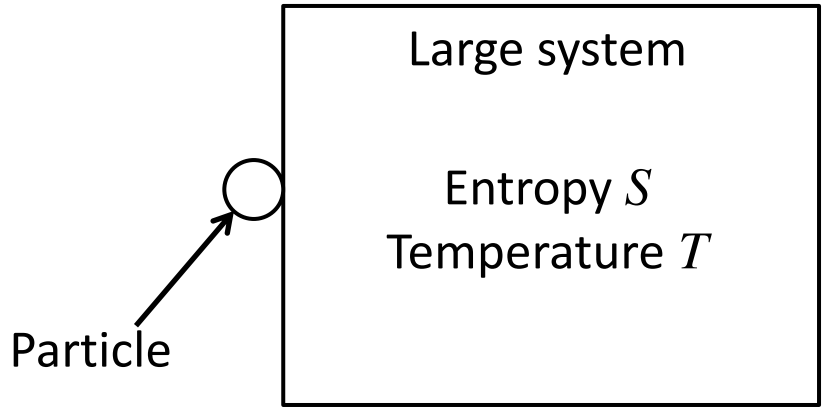
- If they are allowed to exchange energy, then they will reach equilibrium (maximum entropy) when

$$\frac{dS_1}{dE_1} = \frac{dS_2}{dE_2} \Longrightarrow \frac{1}{T_1} = \frac{1}{T_2} \Longrightarrow T_1 = T_2$$

(equal temperature)

Review of Unit 13: Statistical Physics

- Suppose a particle is in thermal equilibrium with its surrounding,



then the relative probability of finding the particle in state 1 over state 2 is

Boltzmann factor

$$\frac{\textbf{probability 1}}{\textbf{probability 2}} = \exp\left(\Delta S / k_B\right) = \boxed{\exp\left(-\Delta E / k_B T\right)}$$

Difference in the entropy of the large system when the particle is in state 1 vs state 2

Difference in the energy of the particle when the it is in state 1 vs state 2

Review of Unit 13: Statistical Physics

- The absolute probability of finding the particle in a state with energy ε:

$$\textbf{probability}(\varepsilon) = \frac{\exp(-\varepsilon / k_B T)}{Z}$$

where $$Z = \sum_s \exp(-\varepsilon_s / k_B T)$$

sum over all states of the particle

- Absolute probability can be used to calculate the average energy of a particle at temperature T:

$$E_{average} = \frac{1}{Z} \sum_s \varepsilon_s \exp(-\varepsilon_s / k_B T)$$

Review of Unit 13: Statistical Physics

- We can combine the expression for the average energy and what we learned in a particle in a 3-dimensional box to derive

$$E = \frac{3}{2} N k_B T$$ average total energy of an ideal gas

$$PV = N k_B T$$ ideal gas law

P must be in N/m^2
V must be in m^3
N is the number of particles
T must be in Kelvin

Review of Unit 14: The Doppler Effect

- **Doppler Effect**: change in observed frequency (and wavelength) of a wave due to moving source or receiver.

- **Mathematically,**

For moving source: $$f' = \frac{f}{1 + v_{source} / c}$$

For moving receiver: $$f' = f\left(1 - v_{receiver} / c\right)$$

$v_{source}, v_{receiver} > 0$ if the distance between source and receiver increasing

$v_{source}, v_{receiver} < 0$ if the distance between source and receiver decreasing