The facts on this sheet may be used without further proof or justification, unless that is the content of a given exercise.

When in doubt, you should justify all of the steps in your solutions that make use of a fact on this sheet (or on previous reference sheets). For example, if a step in your argument uses the fact that $\sin^2(x) = 1 - \cos^2(x)$, then you would write, "by 1.1" next to that step.

1 Trigonometric Identities

1. $\sin^2(x) + \cos^2(x) = 1$
2. $\sec^2(x) - \tan^2(x) = 1$
3. $2\sin^2(x) = 1 - \cos(2x)$
4. $2\cos^2(x) = 1 + \cos(2x)$
5. $2\left[\sin(A)\cos(B)\right] = \sin(A-B) + \sin(A+B)$
6. $2\left[\sin(A)\sin(B)\right] = \cos(A-B) - \cos(A+B)$
7. $2\left[\cos(A)\cos(B)\right] = \cos(A-B) + \cos(A+B)$

3 Derivatives

1. $\frac{d}{dx}(a^x) = a^x \ln(a)$
2. $\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)}$
3. $\frac{d}{dx}(\sin(x)) = \cos(x)$
4. $\frac{d}{dx}(\cos(x)) = -\sin(x)$
5. $\frac{d}{dx}(\tan(x)) = \sec^2(x)$
6. $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$
7. $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$
8. $\frac{d}{dx}(\cot(x)) = -\csc^2(x)$
9. $\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$
10. $\frac{d}{dx}\left(\cos^{-1}(x)\right) = -\frac{1}{\sqrt{1-x^2}}$
11. $\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$
12. $\frac{d}{dx}\left(\csc^{-1}(x)\right) = -\frac{1}{x\sqrt{x^2-1}}$
13. $\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{x\sqrt{x^2-1}}$
14. $\frac{d}{dx}\left(\cot^{-1}(x)\right) = -\frac{1}{1+x^2}$

5 Integrals

For functions u and v satisfying the appropriate hypotheses, we have

1. $\int (a^u) du = a^u / \ln(u) + C$
2. $\int (1/u) du = \ln|u| + C$
3. $\int \sin(u) du = -\cos(u) + C$
4. $\int \cos(u) du = \sin(u) + C$
5. $\int \sec^2(u) du = \tan(u) + C$
6. $\int \csc(u)\cot(u) du = -\csc(u) + C$
7. $\int \sec(u)\tan(u) du = \sec(u) + C$
8. $\int \csc^2(u) du = -\cot(u) + C$
9. $\int \frac{1}{\sqrt{a^2-u^2}} du = \sin^{-1}(u/a) + C$
10. $\int \frac{1}{a^2+u^2} du = (1/a)\tan^{-1}(u/a) + C$
11. $\int \frac{1}{u\sqrt{u^2-a^2}} du = (1/a)\sec^{-1}(u/a) + C$
12. $\int \tan(u) du = \ln|\sec(u)| + C$
13. $\int \cot(u) du = \ln|\sin(u)| + C$
14. $\int \sec(u) du = \ln|\sec(u) + \tan(u)| + C$
15. $\int \csc(u) du = \ln|\csc(u) - \cot(u)| + C$

6 Vectors

6.1 Properties of Vectors

If (V_n, F) is a vector space, with $\vec{a}, \vec{b}$ and $\vec{c}$ in V_n, and c and d in F, then

Theorem 6.1.1.

1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$
3. $\vec{a} + \vec{0} = \vec{a}$
4. $\vec{a} + (-\vec{a}) = \vec{0}$
5. $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$
6. $(c + d)\vec{a} = a\vec{a} + d\vec{a}$
7. $(cd)\vec{a} = c(d\vec{a})$
8. $1\vec{a} = \vec{a}$

6.2 Properties of the Dot Product

In the vector space $(\mathbb{R}^n, \mathbb{R})$, with $\vec{a}, \vec{b}$ and $\vec{c}$ in $\mathbb{R}^n$, and c is in $\mathbb{R}$, the *dot product* has the following properties:

Theorem 6.2.1.

1. $\vec{a} \bullet \vec{a} = ||\vec{a}||^2$
2. $\vec{a} \bullet \vec{b} = \vec{b} \bullet \vec{a}$
3. $\vec{a} \bullet (\vec{b} + \vec{c}) = \vec{a} \bullet \vec{b} + \vec{a} \bullet \vec{c}$
4. $(c\vec{a}) \bullet \vec{b} = c(\vec{a} \bullet \vec{b}) = \vec{a} \bullet (c\vec{b})$
5. $\vec{0} \bullet \vec{a} = 0$

Theorem 6.2.2. *If θ is the angle between vectors $\vec{a}$ and $\vec{b}$, then*

$$\vec{a} \bullet \vec{b} = ||\vec{a}|| \cdot ||\vec{b}|| \cdot \cos(\theta).$$

Theorem 6.2.3. *Vectors $\vec{a}$ and $\vec{b}$ are orthogonal if and only if*

$$\vec{a} \bullet \vec{b} = 0.$$

6.3 Properties of the Cross Product

In the vector space $(\mathbb{R}^3, \mathbb{R})$, with $\vec{a}, \vec{b}$ and $\vec{c}$ in $\mathbb{R}^3$, and c is in $\mathbb{R}$, the *cross product* has the following properties:

Theorem 6.3.1.

1. $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$
2. $(c\vec{a}) \times \vec{b} = c(\vec{a} \times \vec{b}) = \vec{a} \times (c\vec{b})$
3. $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$
4. $(\vec{a} + \vec{b}) \times \vec{c} = (\vec{a} \times \vec{c}) + (\vec{b} \times \vec{c})$
5. $\vec{a} \bullet (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \bullet \vec{c}$
6. $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \bullet \vec{c})\vec{b} - (\vec{a} \bullet \vec{c})\vec{c}$

Theorem 6.3.2. *The vector $\vec{a} \times \vec{b}$ is orthogonal to both $\vec{a}$ and $\vec{b}$.*

Theorem 6.3.3. *If θ is the angle between $\vec{a}$ and $\vec{b}$, with $0 \leq \theta \leq \pi$, then*

$$||\vec{a} \times \vec{b}|| = ||\vec{a}|| \cdot ||\vec{b}|| \cdot \sin(\theta)$$

Theorem 6.3.4. *Two non-zero vectors $\vec{a}$ and $\vec{b}$ are parallel if and only if*

$$\vec{a} \times \vec{b} = \vec{0}.$$

6.4 Calculus and Vector Valued Functions

Theorem 6.4.1. *In the vector space $(\mathbb{R}^n, \mathbb{R})$, if we are given vector-valued functions $\vec{u}(t)$and $\vec{v}(t)$, real-valued function $f(t)$, and constant c in $\mathbb{R}$, then we have that*

1. $[\vec{u}(t) + \vec{v}(t)]' = \vec{u}\,'(t) + \vec{v}\,'(t)$
2. $[c\vec{u}(t)]' = c(\vec{u'}(t))$
3. $[f(t)\vec{u}(t)]' = f'(t)\vec{u}(t) + f(t)\vec{u}\,'(t)$
4. $[\vec{u}(t) \bullet \vec{v}(t)]' = (\vec{u}\,'(t) \bullet \vec{v}(t)) + (\vec{u}(t) \bullet \vec{v}\,'(t))$
5. $[\vec{u}(t) \times \vec{v}(t)]' = (\vec{u}\,'(t) \times \vec{v}(t)) + (\vec{u}(t) \times \vec{v}\,'(t))$
6. $[\vec{u}(f(t))]' = f'(t)\vec{u}\,'(f(t))$

when differentiating with respect to t.