Physics 131
General Physics I
Grinnell College
Spring 2012

Summary of Unit IV
Conservation Principles

In this unit, we developed the principles of conservation of energy and conservation of momentum. Conservation principles allow one a method to solve some problems without having to solve explicitly for the full position-versus-time function, and sometimes without even having to know the complete details of the forces involved. They are generally expressed as

Conserved Quantity Before = Conserved Quantity After.

We derived them by integrating Newton's second law over the intervening interaction, either in time (giving rise to momentum) or space (giving rise to energy). The successful evaluation of these integrals depended on certain restrictions on the force laws, which we will review separately.

The conservation of momentum principle depended on Newton's third law of equal and opposite forces. In other words, if body one exerts a force on body two, then body two must exert an equal and opposite force back on one. Since force is equal to the time rate of change of momentum, the change in the momentum of body one is exactly balanced by the change in momentum of body two. This momentum is defined as

$$p = mv$$

where p, like v, can be positive or negative. Once we start considering motion in multiple dimensions, we will see that momentum, like velocity, is a vector, possessing both magnitude and direction. Momentum will be conserved as long as there are no forces exerted by bodies outside our system, bodies whose motion we do not or cannot keep track of. We usually express this as momentum is conserved if there are no "external forces." If momentum is conserved, we then write

$$p_{before} = p_{after}\ ,$$
$$\sum_i m_i v_{i\ \ before} = \sum_i m_i v_{i\ \ after}\ .$$

The conservation of energy principle has a very similar formulation to momentum. It depends on our ability to integrate the force law over distance, which requires that the force law depends only on position. More simply, we often say that conservation of energy depends on their being no "dissipative" forces of the frictional character (e.g. sliding friction, drag force). This is a bit restrictive, however, since forces depending explicitly on time will in general not conserve energy either (like shaking the end of a spring holding a weight at a constant frequency will pump energy into the system). The general formulation of conservation of energy, like momentum, is

$$E_{\text{before}} = E_{\text{after}}\ ,\ \text{which is expanded to}$$
$$K_{before} + U_{before} = K_{after} + U_{after}\ ,\ \text{where}$$
$$K = \tfrac{1}{2}mv^2$$

and U is separately defined for each force law through the defining equation

$$U = -\int F dx\ .$$

In particular, we have $U = mgy$ for the gravity force $F = -mg$, and $U = \frac{1}{2}kx^2$ for the spring force $F = -kx$.