Review of Unit 5: Motion in Multiple Dimensions

- **Vectors vs scalars:**

  Vectors: quantities that have both magnitude and direction.

  Scalars: quantities that have magnitude but no direction.

- **Some examples:**

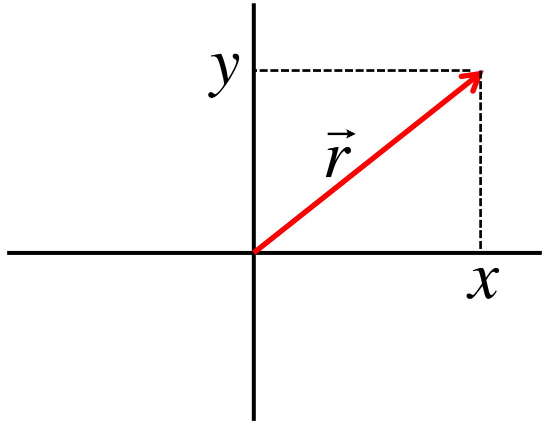
Vectors	Scalars
position	time
velocity	mass
acceleration	speed
force	energy
momentum	work

- We use an arrow to indicate a vector quantity: $\vec{r}$, $\vec{v}$, $\vec{a}$, $\vec{F}$, etc.

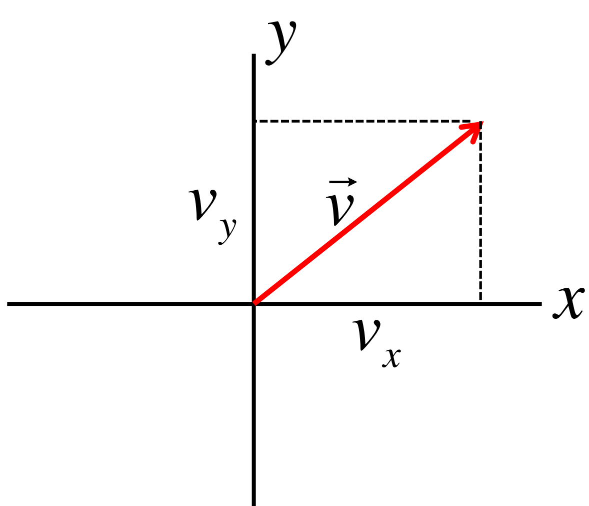
Review of Unit 5: Motion in Multiple Dimensions

- **In two dimensions:**

  We denote position by two coordinates: $\vec{r} = (x, y)$



  Similarly, we can denote other vector quantities (like velocity) with two components: $\vec{v} = (v_x, v_y)$



Review of Unit 5: Motion in Multiple Dimensions

- **In two dimensions** (continued)**:**

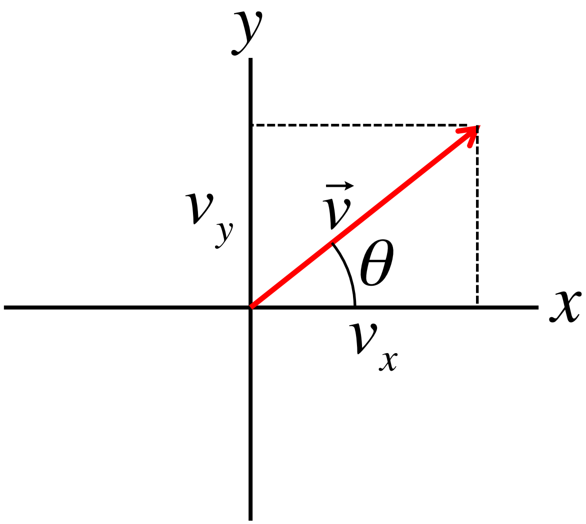
  We can calculate the magnitude and the angle relative to the x-axis from the components using:

  Magnitude (length):

  $$v = \left|\vec{v}\right| = \sqrt{v_x^2 + v_y^2}$$

  Angle relative to the x-axis:

  $$\theta = \arctan\left(\frac{v_y}{v_x}\right)$$



  We can also calculate the components from the magnitude and the angle:

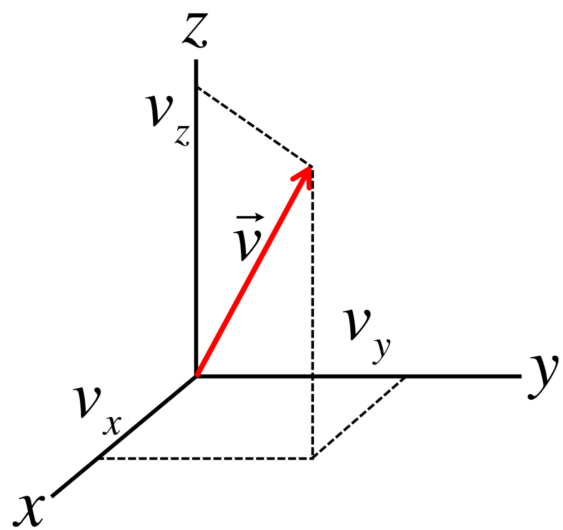
  $$v_x = v\cos\theta$$

  $$v_y = v\sin\theta$$

Review of Unit 5: Motion in Multiple Dimensions

- **In three dimensions:**

  We can denote a vector with three components: $\vec{v} = \left(v_x, v_y, v_z\right)$



  Magnitude of a vector in 3 dimensions: $v = \left|\vec{v}\right| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

  (Angles are more difficult in 3 dimensions. We need 2 angles to define a direction.)

Review of Unit 5: Motion in Multiple Dimensions

- **Newton's 2nd Law in 2 (or 3) dimension:**

$$F_x = ma_x$$

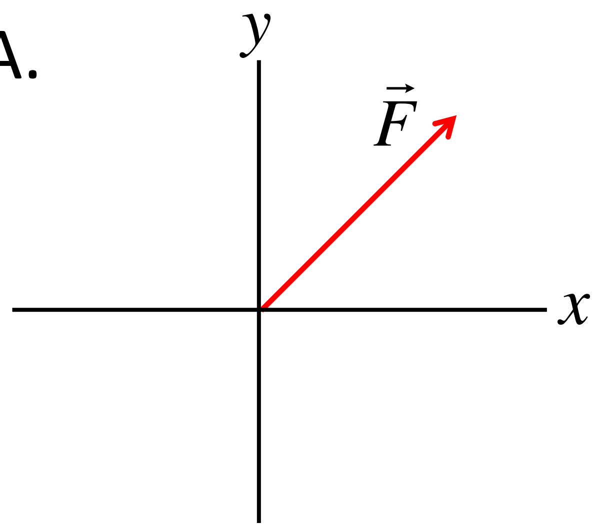
$$F_y = ma_y \qquad \text{or more simply} \qquad \vec{F} = m\vec{a}$$

$$\left(F_z = ma_z\right)$$

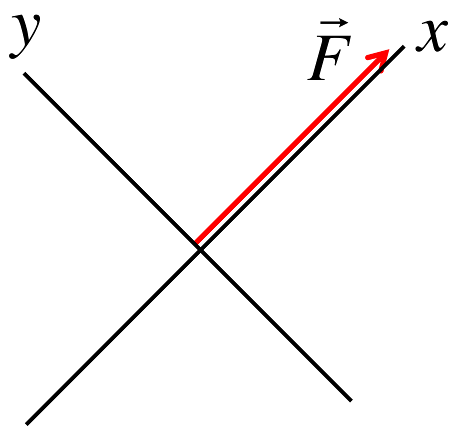
Suppose an object experiences a net force pointing in the direction shown below. We are free to choose the orientation of our xy-coordinates however we want.

Which of the following would be a good choice for our coordinates?

A.



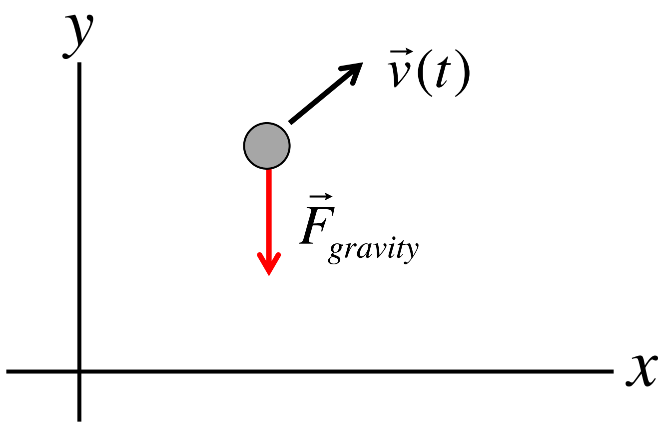
B.



In most cases, we should choose our coordinates so that one of the coordinates points in the direction of the net force.

Example: Projectile motion

An object moves under the action of gravitational force alone. Complete the following equations.



$F_x =$

$a_x =$

$v_x =$

$x = x_0 +$

$F_y =$

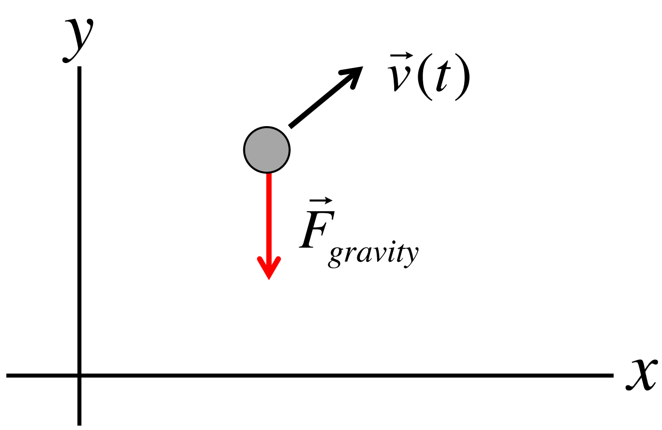
$a_y =$

$v_y = v_{y0} +$

$y =$

Example: Projectile motion

An object moves under the action of gravitational force alone.
Complete the following equations.



$$F_x = 0$$

$$a_x = 0$$

$$v_x = v_{x0}$$

$$x = x_0 + v_{x0} t$$

Motion with constant velocity

share the same t

$$F_y = -mg$$

$$a_y = -g$$

$$v_y = v_{y0} + (-g)t$$

$$y = y_0 + v_{y0} t - \frac{1}{2} g t^2$$

Motion with constant acceleration

Review of Unit 5: Motion in Multiple Dimensions

- **Conservation of momentum in 2 (or 3) dimension:** If there is no external force, then each component of the total momentum is conserved

$$\sum_i p_{i,x,before} = \sum_i p_{i,x,after}$$

x-component of the momentum of i^{th} object before

sum over all objects in the system

$$\sum_i p_{i,y,before} = \sum_i p_{i,y,after}$$

$$\left(\sum_i p_{i,z,before} = \sum_i p_{i,z,after} \right)$$

or more simply $\sum_i \vec{p}_{i,before} = \sum_i \vec{p}_{i,after}$

Review of Unit 5: Motion in Multiple Dimensions

- **Kinetic energy of an object in 2 (or 3) dimension:**

$$K = \frac{1}{2} m \left[v_x^2 + v_y^2 \right]$$

$$\left(K = \frac{1}{2} m \left[v_x^2 + v_y^2 + v_z^2 \right] \right)$$

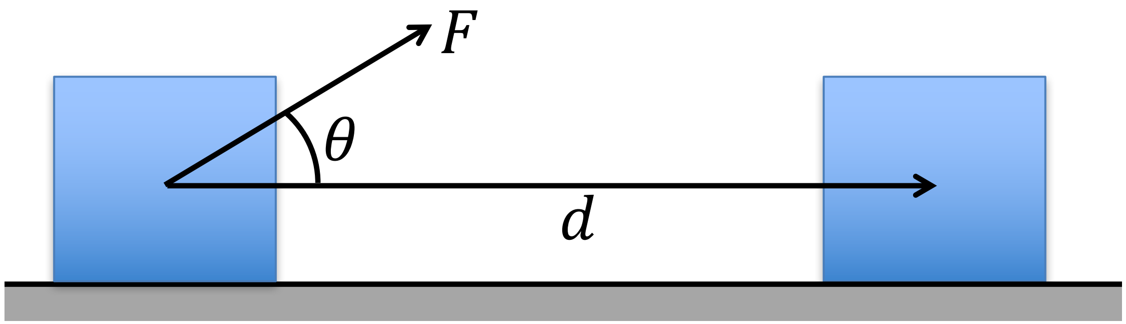
- **Conservation of energy in 2 (or 3) dimension:** The total energy of a system is conserved if all the forces acting on the system depend only on the position or constant.

$$\boxed{E_{before} = E_{after}}$$ if there is only one object

$$\boxed{\sum_i E_{i,before} = \sum_i E_{i,after}}$$ if there are multiple objects

Review of Unit 5: Motion in Multiple Dimensions

- **Work in 2 (or 3) dimension:**



If an object moves distance *d* while experiencing a constant force *F*, then work done by the force is

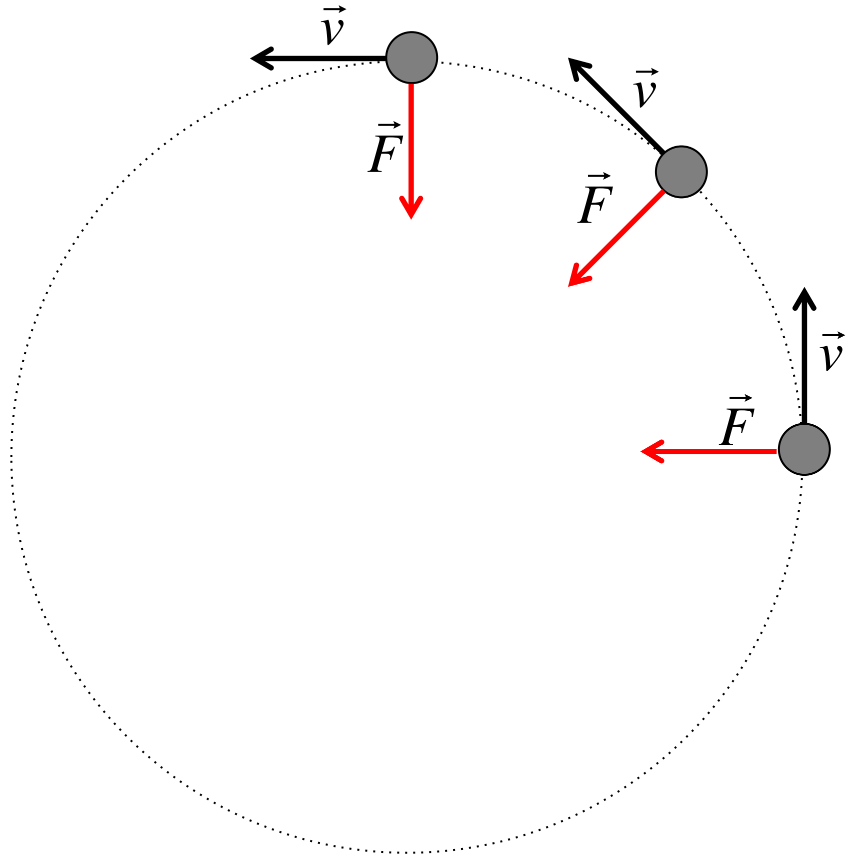
$$W = Fd\cos\theta = \vec{F}\cdot\vec{d}$$

- **Work-energy theorem in 2 (or 3) dimension:** Change in the kinetic energy of an object equals the total work done on the object

$$K_{\text{after}} - K_{\text{before}} = W \qquad \text{or more simply,} \quad \Delta K = W$$

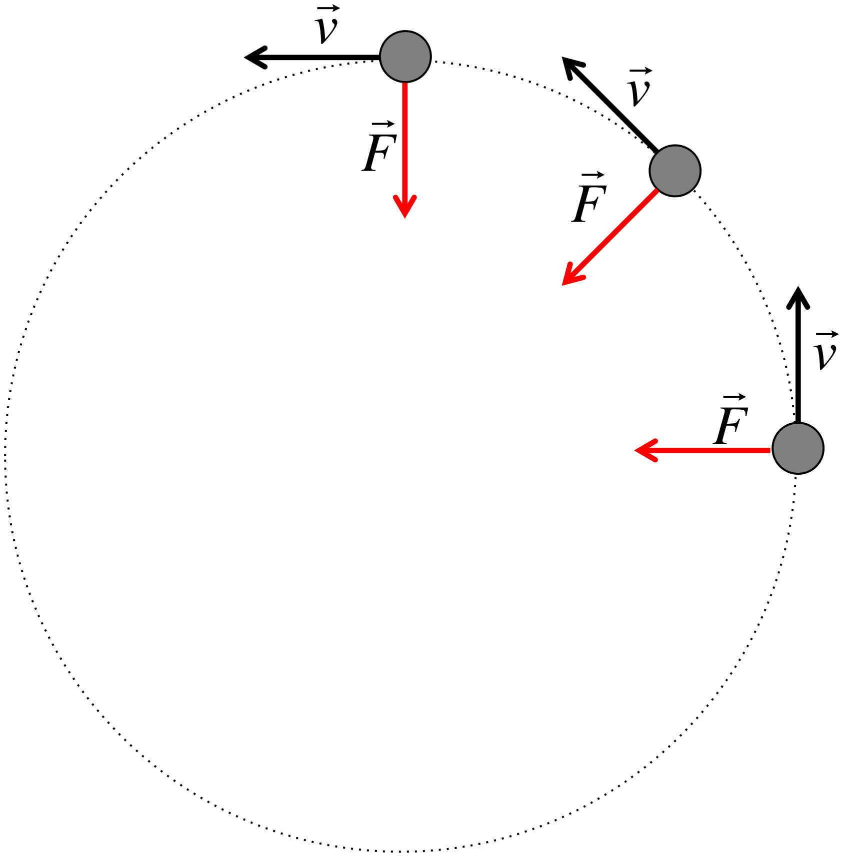
Review of Unit 6: Circular Motion & Angular Momentum

- **Uniform circular motion:** An object will undergo a circular motion with a constant speed if the net force acting on it is always constant in magnitude but perpendicular to the direction of motion.



Review of Unit 6: Circular Motion & Angular Momentum

- Mathematically:



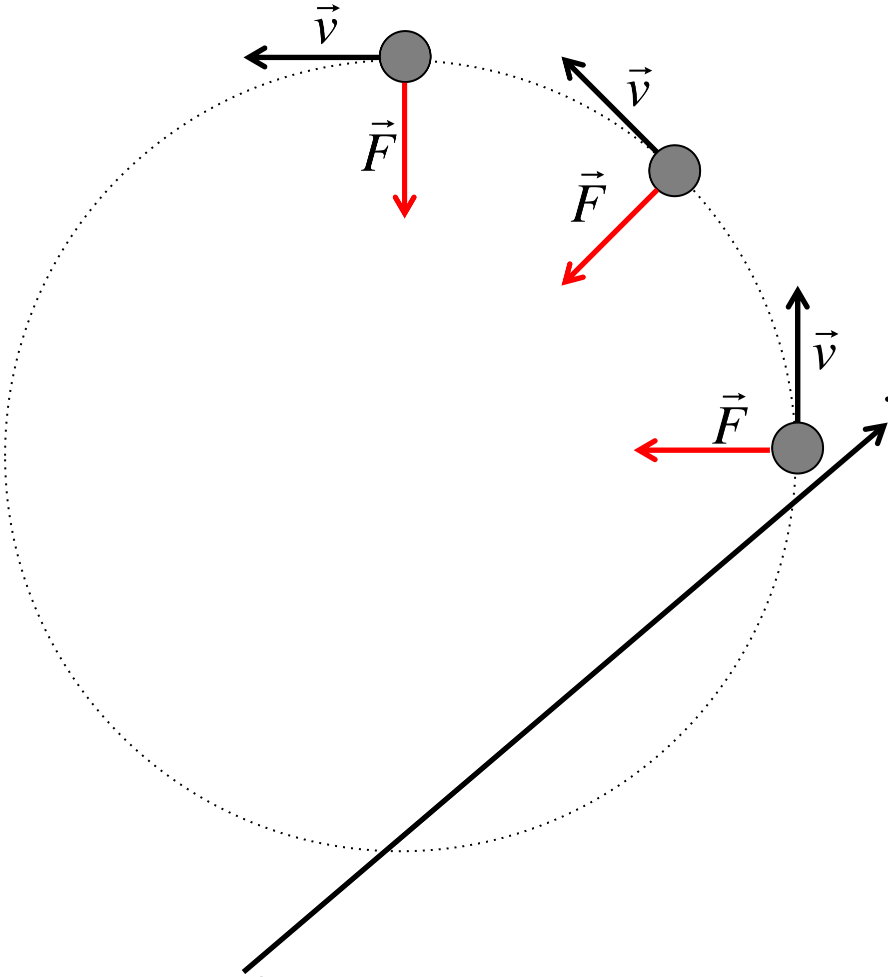
$$\begin{cases} (x, y) = R(\cos\omega t, \sin\omega t) \\ (v_x, v_y) = \left(\dfrac{dx}{dt}, \dfrac{dy}{dt}\right) \\ \qquad = R\omega(-\sin\omega t, \cos\omega t) \\ (a_x, a_y) = \left(\dfrac{d^2x}{dt^2}, \dfrac{d^2y}{dt^2}\right) \\ \qquad = R\omega^2(-\cos\omega t, -\sin\omega t) \end{cases}$$

The magnitudes of velocity and acceleration are related to the angular velocity ω (radians/sec):

$$v = \sqrt{v_x^2 + v_y^2} = R\omega \qquad a = \sqrt{a_x^2 + a_y^2} = R\omega^2$$

Review of Unit 6: Circular Motion & Angular Momentum

- Mathematically:



$$(x,y) = R(\cos\omega t, \sin\omega t)$$

$$(v_x, v_y) = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)$$

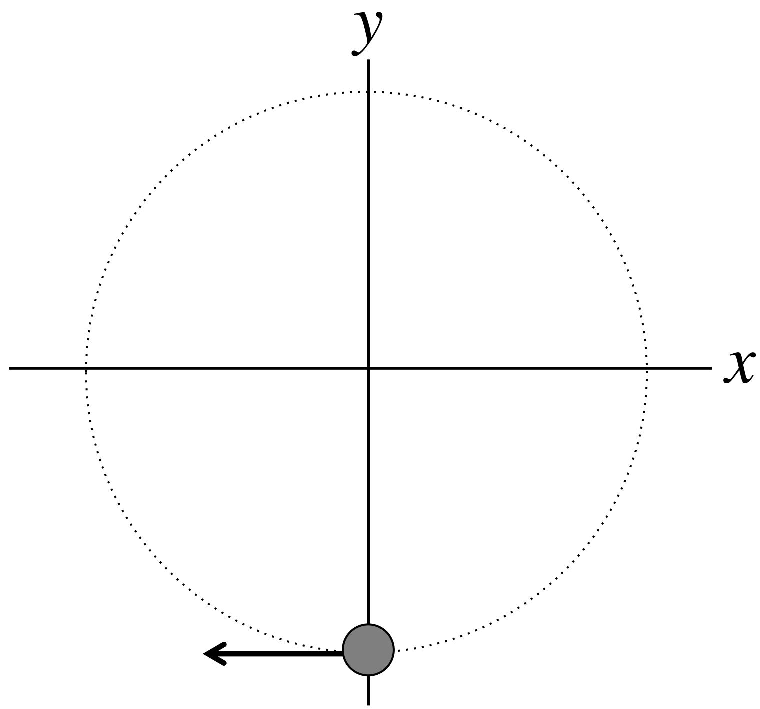
$$= R\omega(-\sin\omega t, \cos\omega t)$$

$$(a_x, a_y) = \left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}\right)$$

$$= R\omega^2(-\cos\omega t, -\sin\omega t)$$

Note: These equations assume that the object is at (R,0) at t=0 and moves counterclockwise.

Suppose an object is at (0,-R) at t=0 and moves clockwise at constant speed. Which one of the following equations describe its motion?



A. $(x,y)=R(\cos\omega t,\sin\omega t)$

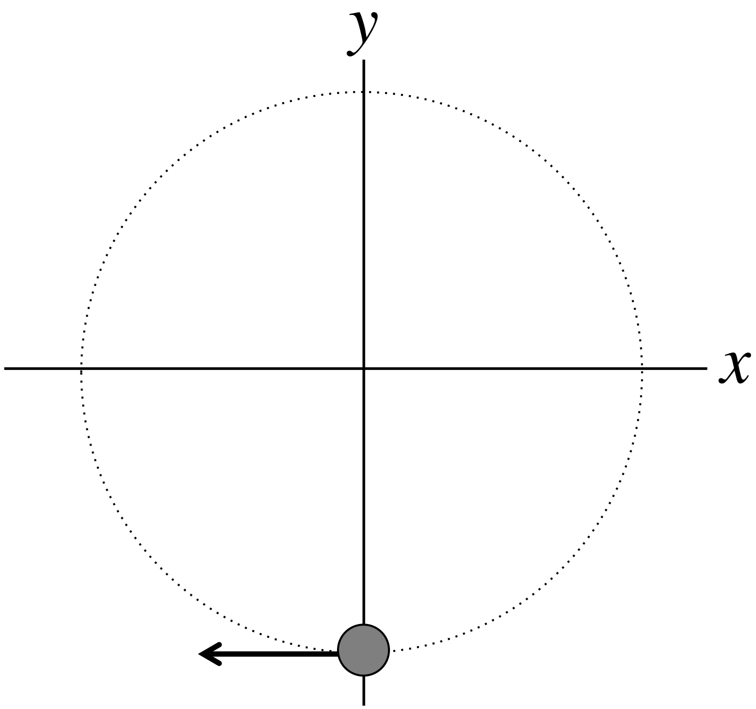
B. $(x,y)=R(-\sin\omega t,\cos\omega t)$

C. $(x,y)=R(-\sin\omega t,-\cos\omega t)$

D. $(x,y)=R(-\cos\omega t,-\sin\omega t)$

E. None of the above

Suppose an object is at (0,-R) at t=0 and moves clockwise at constant speed. Which one of the following equations describe its motion?



A. $(x,y) = R(\cos\omega t, \sin\omega t)$

B. $(x,y) = R(-\sin\omega t, \cos\omega t)$

✓ C. $(x,y) = R(-\sin\omega t, -\cos\omega t)$

D. $(x,y) = R(-\cos\omega t, -\sin\omega t)$

E. None of the above

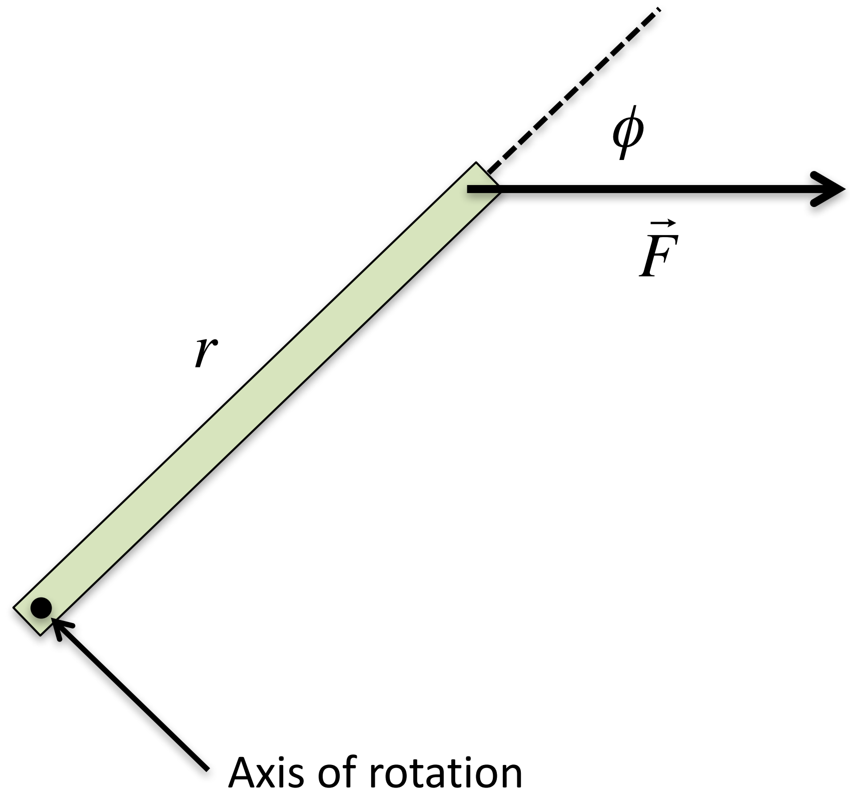
Review of Unit 6: Circular Motion & Angular Momentum

- **Torque**: tendency of a force to rotate an object about an axis of rotation.

Review of Unit 6: Circular Motion & Angular Momentum

- **Torque**: tendency of a force to rotate an object about an axis of rotation.

$$\tau = rF\sin\phi$$



- If the force is perpendicular to the lever arm,

$$\tau = rF$$

Review of Unit 6: Circular Motion & Angular Momentum

- The rate of rotation (angular velocity) of an object can change if it experiences a torque.

- **Angular acceleration**: the rate of change in angular velocity

$$\alpha = \frac{d\omega}{dt}$$

- **Newton's Second Law for rotational motion**:

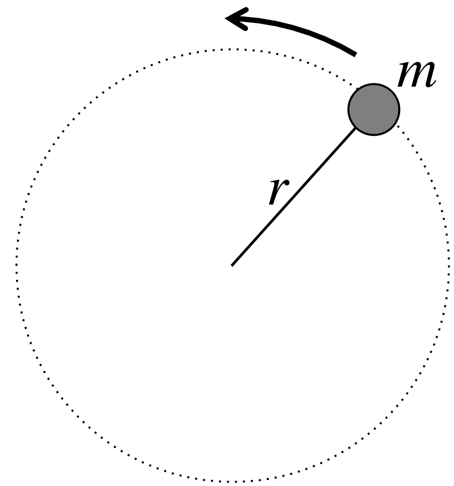
$$\tau = I\alpha$$

moment of inertia

Review of Unit 6: Circular Motion & Angular Momentum

- For a particle with mass m that is distance r away from the center of rotation, the moment of inertia is

$$I = mr^2$$



- If an object is made of multiple particles, then its moment of inertia is

$$I = \sum_i m_i r_i^2$$

Sum over all particles that make up the rotating object

Complete the following analogy table

	Linear	**Rotational**
Position	x	θ
Velocity	v	ω
Acceleration	a	α
Force/Torque	F	$\tau = rF\sin\phi$
Mass/ Moment of inertia	m	$I = \sum_i m_i r_i^2$
Momentum	$p = mv$	$L =$
Newton's 2nd Law	$F = ma$	$\tau = I\alpha$
Conservation of Momentum	$\sum_i p_{i,before} = \sum_i p_{i,after}$	
Constant acceleration	$v = at + v_0$ $x = \frac{1}{2}at^2 + v_0 t + x_0$	$\omega =$ $\theta =$

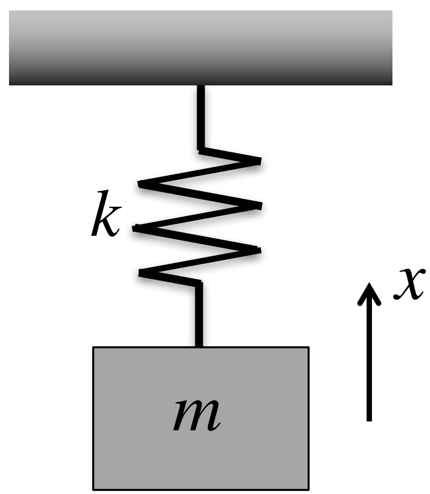
Complete the following analogy table

	Linear	**Rotational**
Position	x	θ
Velocity	v	ω
Acceleration	a	α
Force/Torque	F	$\tau = rF\sin\phi$
Mass/ Moment of inertia	m	$I = \sum_i m_i r_i^2$
Momentum	$p = mv$	$L = I\omega$
Newton's 2nd Law	$F = ma$	$\tau = I\alpha$
Conservation of Momentum	$\sum_i p_{i,before} = \sum_i p_{i,after}$	$\sum_i L_{i,before} = \sum_i L_{i,after}$
Constant acceleration	$v = at + v_0$ $x = \frac{1}{2}at^2 + v_0 t + x_0$	$\omega = \alpha t + \omega_0$ $\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$

Review of Unit 7: Oscillations

- A mass on a spring executes oscillatory motion known as simple harmonic motion:

$$x(t) = A\cos(\omega t + \phi)$$



$A =$ Amplitude

$\omega = \sqrt{\dfrac{k}{m}}$ Angular frequency

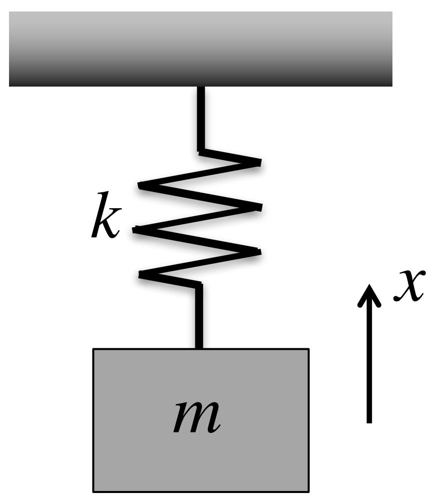
$\phi =$ determined by the value of x at $t=0$

Note: x is measured relative to the equilibrium position of the mass.

- The frequency f and the period T are related to the angular frequency:

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

If I double the amplitude of simple harmonic motion, the period of oscillation will



A. Decrease by a factor of 1/2

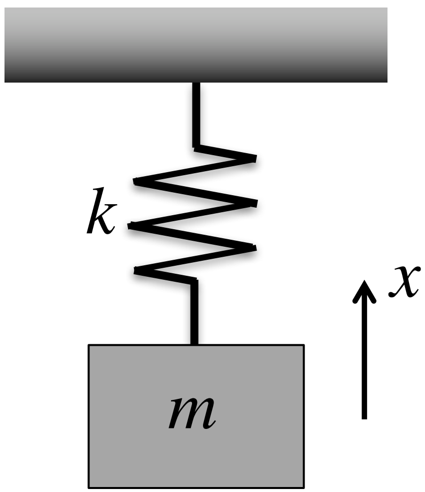
B. Not change

C. Increase by a factor of 2

D. Increase by a factor of 4

E. None of the above

If I double the amplitude of simple harmonic motion, the period of oscillation will



A. Decrease by a factor of 1/2

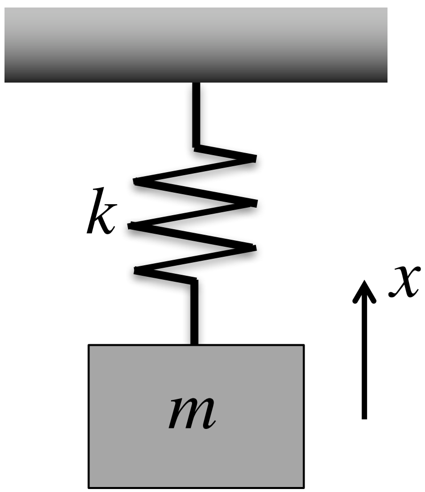
✓ B. Not change

C. Increase by a factor of 2

D. Increase by a factor of 4

E. None of the above

If I double the amplitude of simple harmonic motion, the maximum velocity of the mass will



A. Decrease by a factor of 1/2
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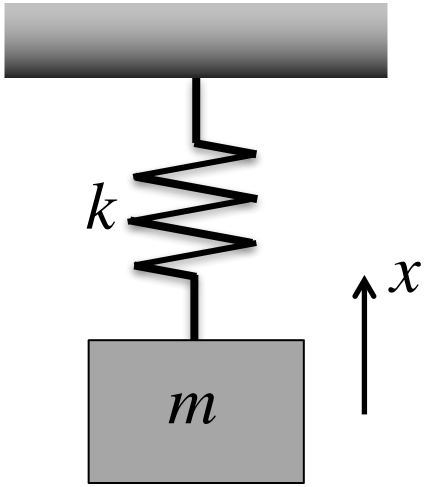
If I double the amplitude of simple harmonic motion, the maximum velocity of the mass will



A. Decrease by a factor of 1/2

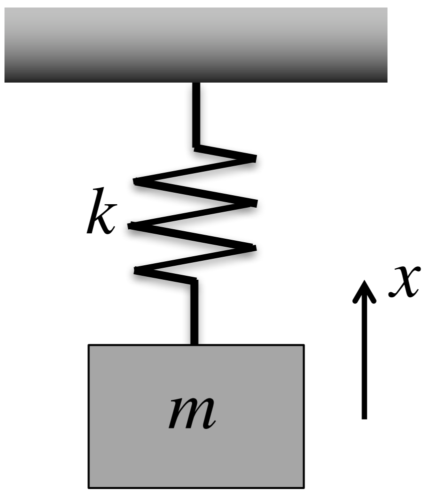
B. Not change

✓ C. Increase by a factor of 2

D. Increase by a factor of 4

E. None of the above

If I double the amplitude of simple harmonic motion, the total energy of the system will



A. Decrease by a factor of 1/2

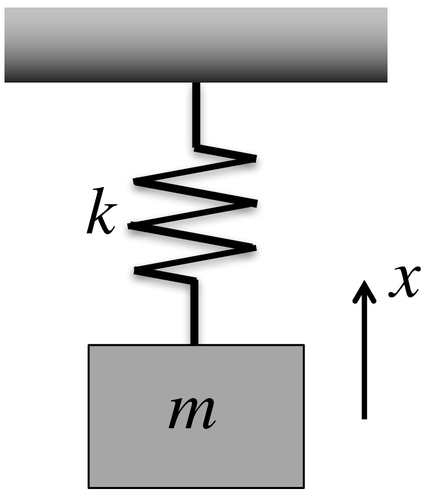
B. Not change

C. Increase by a factor of 2

D. Increase by a factor of 4

E. None of the above

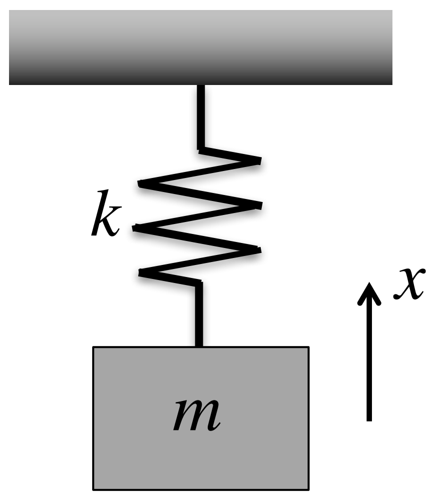
If I double the amplitude of simple harmonic motion, the total energy of the system will



A. Decrease by a factor of 1/2
B. Not change
C. Increase by a factor of 2
✓ D. Increase by a factor of 4
E. None of the above

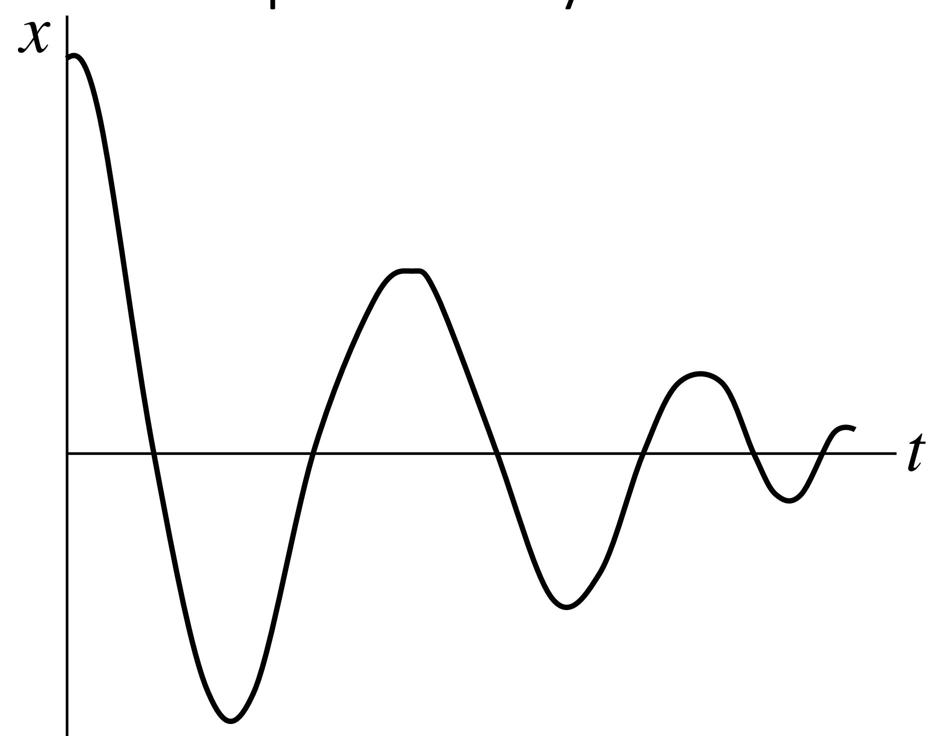
Review of Unit 7: Oscillations

- If the system experiences a drag force, then the amplitude of the oscillatory motion decreases exponentially with time



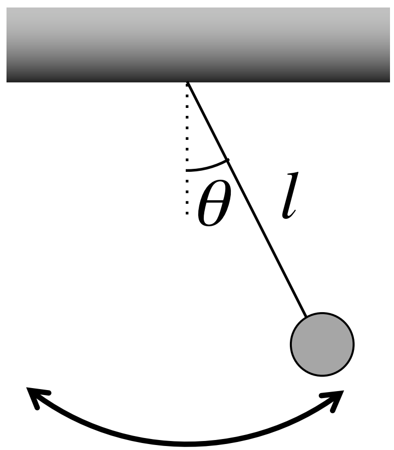
$$x(t) = Ae^{-\alpha t}\cos(\omega t + \phi)$$

$\alpha =$ determined by how quickly the amplitude decays over time



Review of Unit 7: Oscillations

- A pendulum also executes simple harmonic motion as long as the angle is not too big.



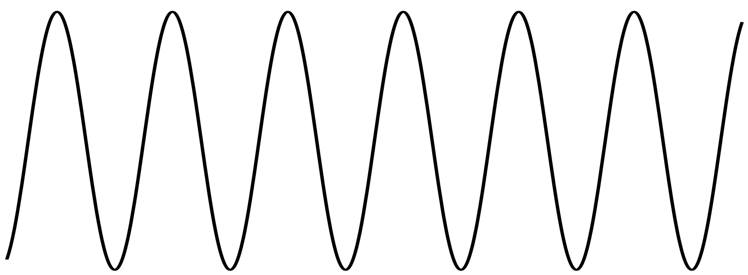
$$\theta(t) = A\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{g}{l}}$$

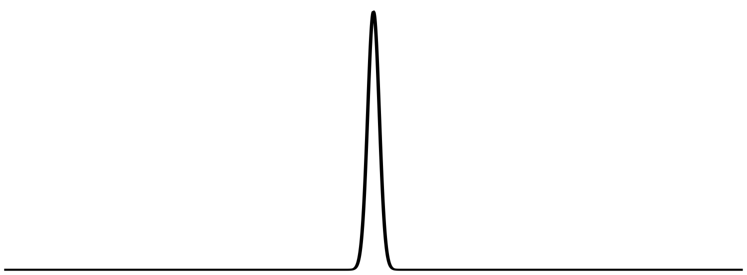
- In order for an object to undergo oscillatory motion, there must be a restoring force (or torque) that tries to bring it back to its equilibrium position.
- If the restoring force is linear (proportional to displacement from the equilibrium position), then the motion is simple harmonic.

Review of Unit 8: Waves

- **Wave:** disturbance that travels through a medium (examples: waves on a string, sound waves, electromagnetic waves).
- Waves can be periodic or non-periodic.



periodic wave



non-periodic wave

- Mathematically, any function f that depends on x and t as $f(kx\text{-}\omega t)$ or as $f(kx+\omega t)$ can represent a wave.
- The exact form of f is determined by the shape of the wave at fixed t, or the time variation of the disturbance at fixed x.

Review of Unit 8: Waves

- A sinusoidal wave whose disturbance is traveling in ±x direction can be written as

$$f(x,t) = A\sin\left(kx \mp \omega t\right)$$

k is called the **wave number** and is related to the wavelength by

$$k = \frac{2\pi}{\lambda}$$

ω is called the **angular frequency** and is related to the frequency and period by

$$\omega = 2\pi f = \frac{2\pi}{T}$$

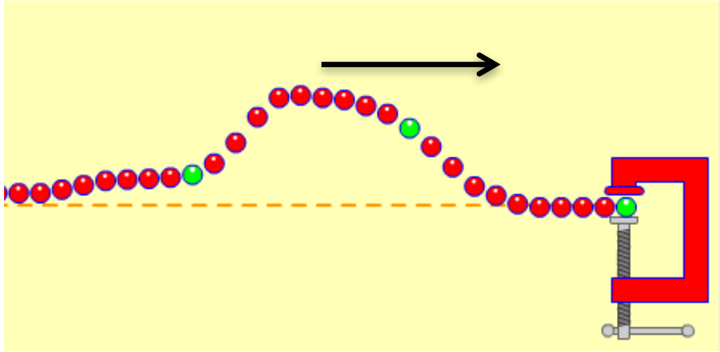
The wave speed (speed of propagation) can be expressed as

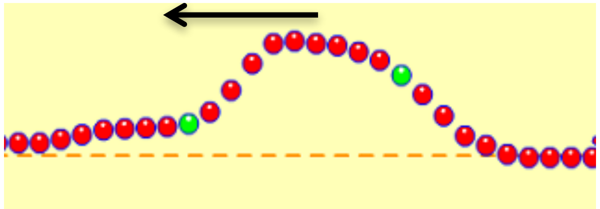
$$c = \lambda f \quad \text{or} \quad c = \frac{\omega}{k}$$

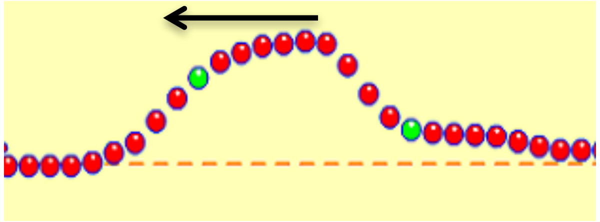
Review of Unit 8: Waves

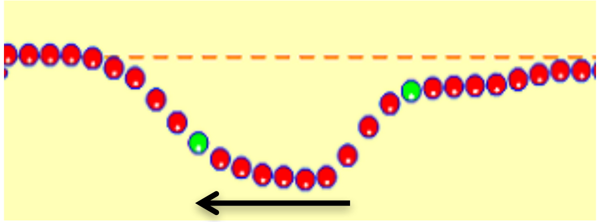
- If a medium continues indefinitely and there is no boundary, a wave could travel indefinitely (provided that the energy of the wave does not get dissipated through friction).
- If there is a boundary, then a wave gets reflected at the boundary.

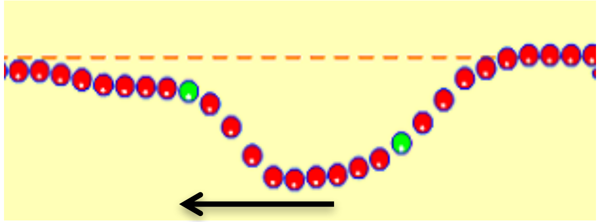
What will this wave look like after it reflects?



A. 

B. 

C. 

D. 

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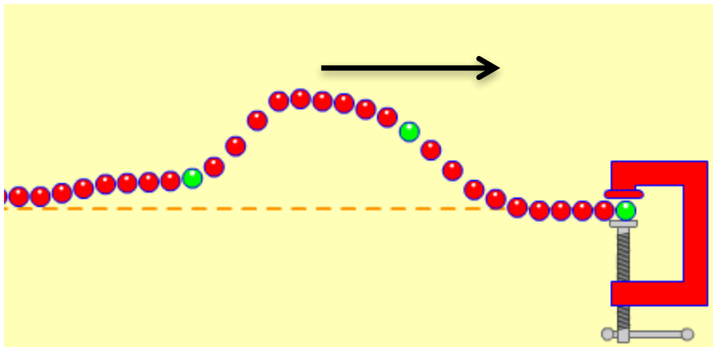
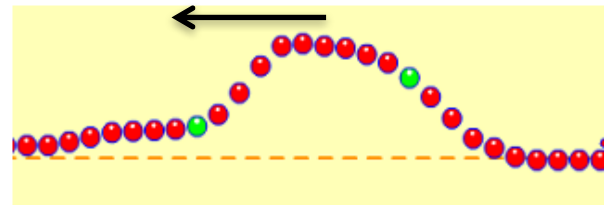
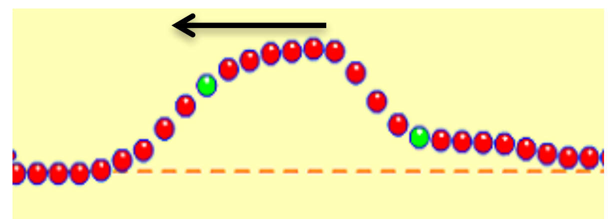


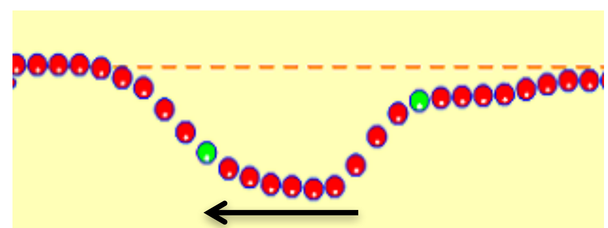
A.



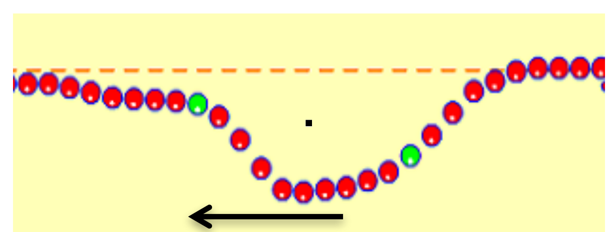
B.



✓ C.



D.

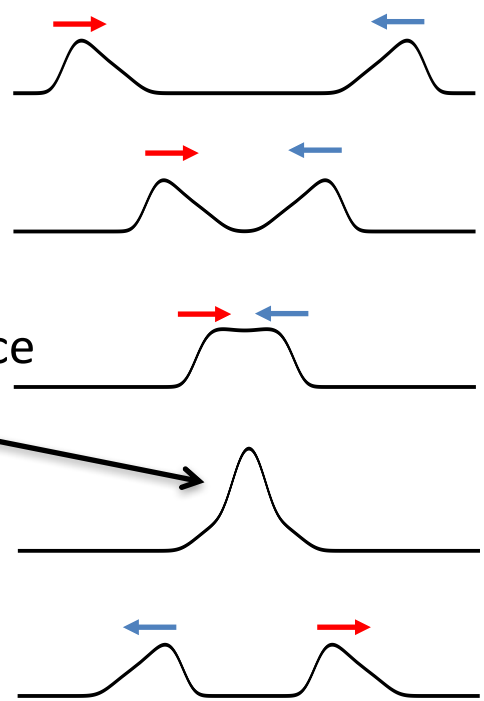


Review of Unit 8: Waves

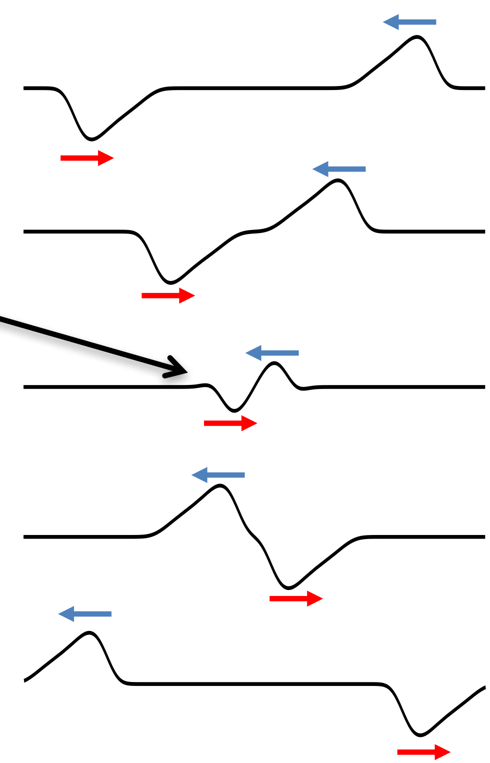
- Interference occurs when two (or more) waves overlap.
- Principle of superposition: when two waves overlap, the actual displacement at any point on the string at any time is the sum of the displacement of each waves:

$$f(x,t) = f_1(x,t) + f_2(x,t)$$

Constructive interference: two waves reinforce each other

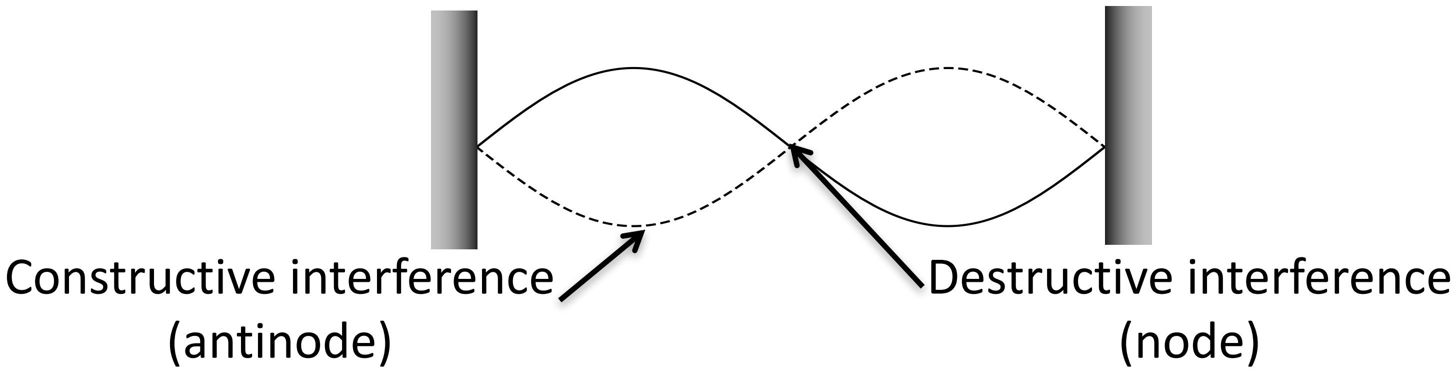


Destructive interference: two waves cancel or partially cancel each other

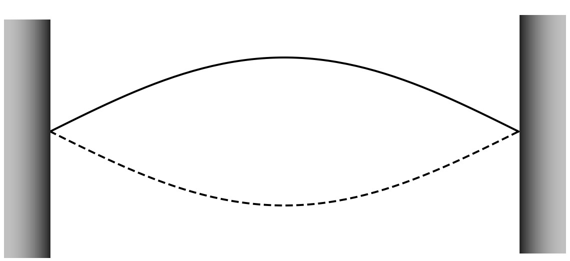


Review of Unit 8: Waves

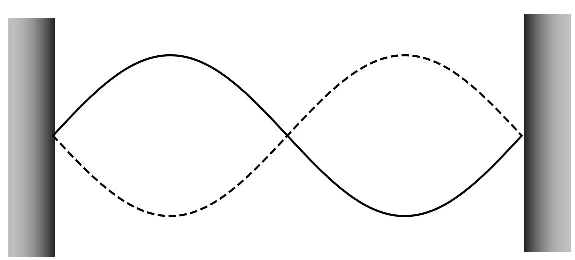
- A standing wave occurs when we have two waves travelling in opposite directions.



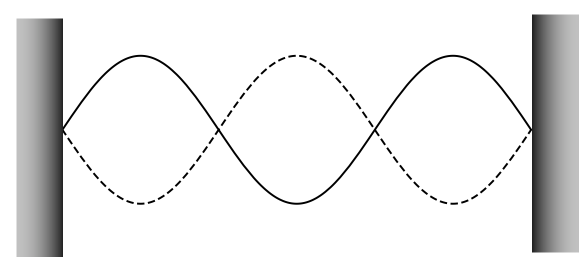
Fundamental mode
(1st harmonic, n=1)



Second overtone
(2nd harmonic, n=2)

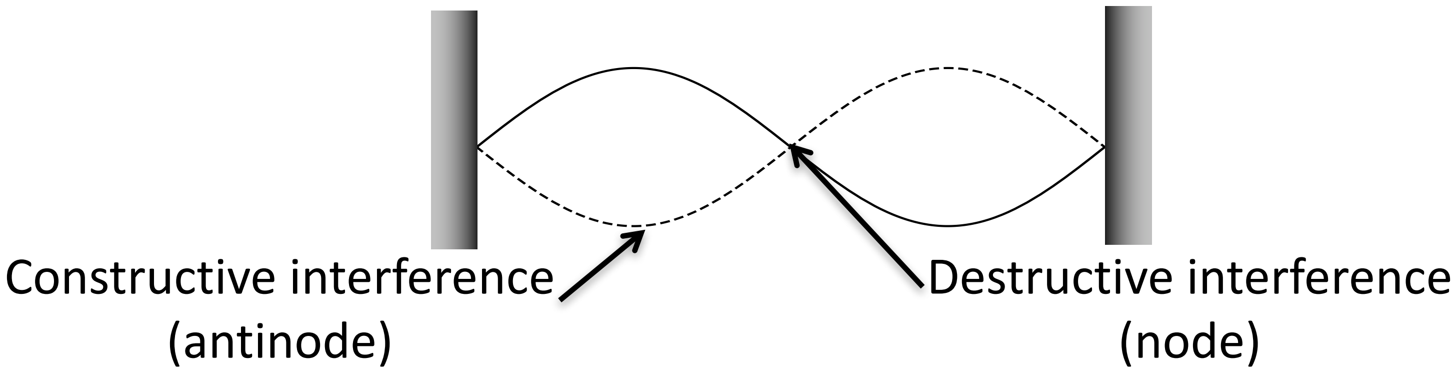


Third overtone
(3rd harmonic, n=3)



Review of Unit 8: Waves

- A **standing wave** occurs when we have two waves travelling in opposite directions.



- Because only an integer multiple of ½ wavelength can fit between the two walls,

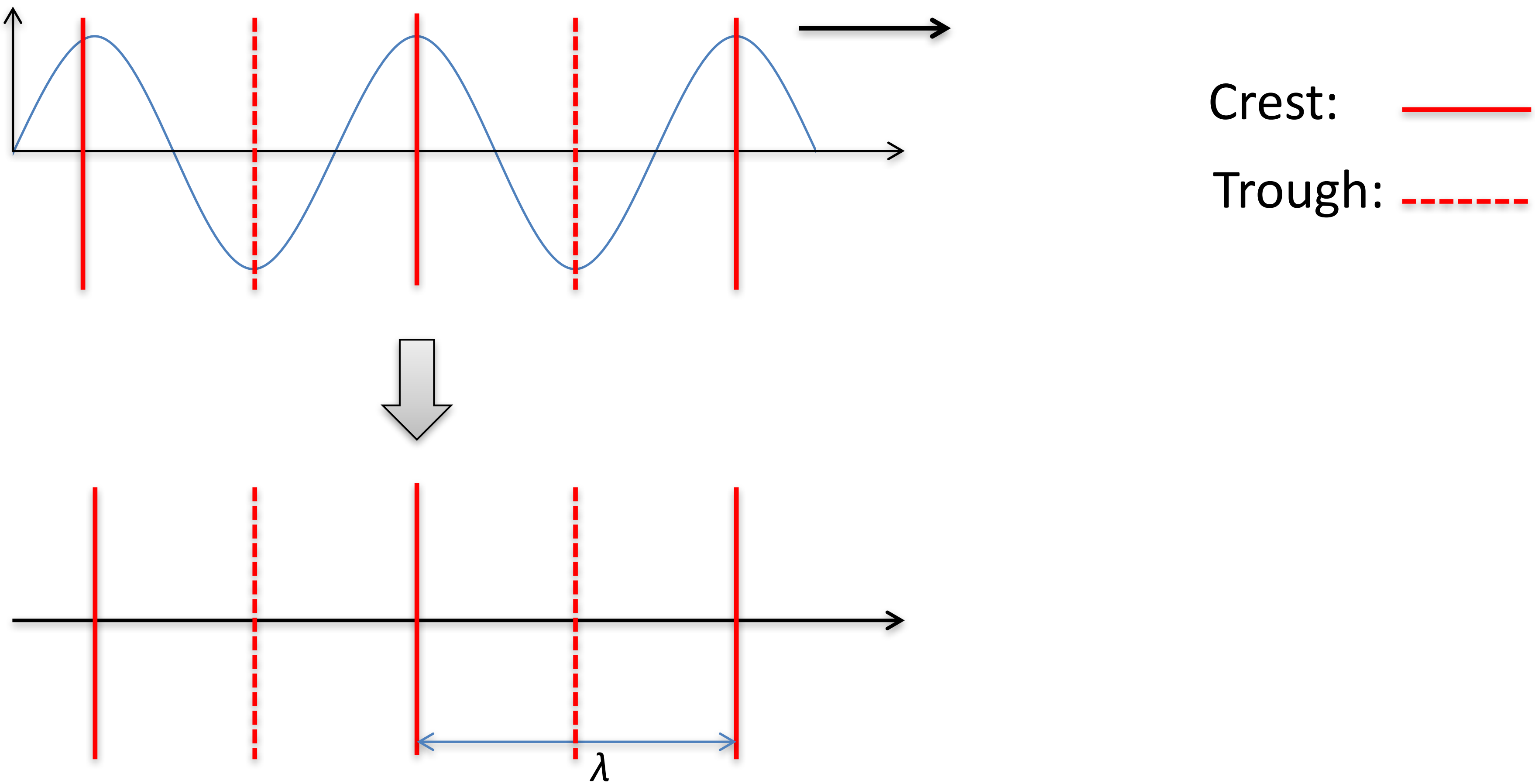
$$l = n\frac{\lambda}{2} \quad \text{where} \quad l = \text{ length of the string}$$

$$n = 1, 2, 3, \ldots$$

This leads to $\lambda = \dfrac{2l}{n}$ and $f = \dfrac{c}{\lambda} = n\dfrac{c}{2l}$

Review of Unit 9: Waves in Two and Three Dimensions

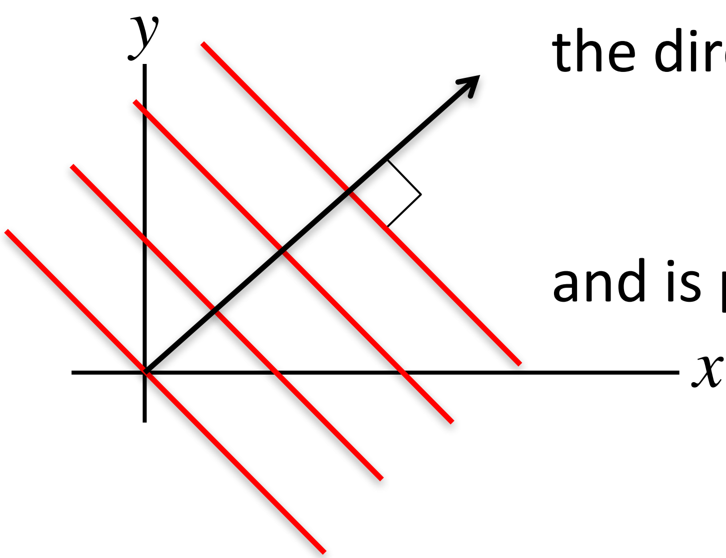
- To help us visualize waves in 2- and 3-dimensional space, we use lines to represent the locations of crests.



- We can also use dashed lines to represent the locations of troughs.

Review of Unit 9: Waves in Two and Three Dimensions

- Waves that move in a straight line are called line waves in 2D and plane waves in 3D.



the direction of travel is given by

$$\vec{k} = \left(k_x, k_y, k_z\right)$$

and is perpendicular to the crest lines

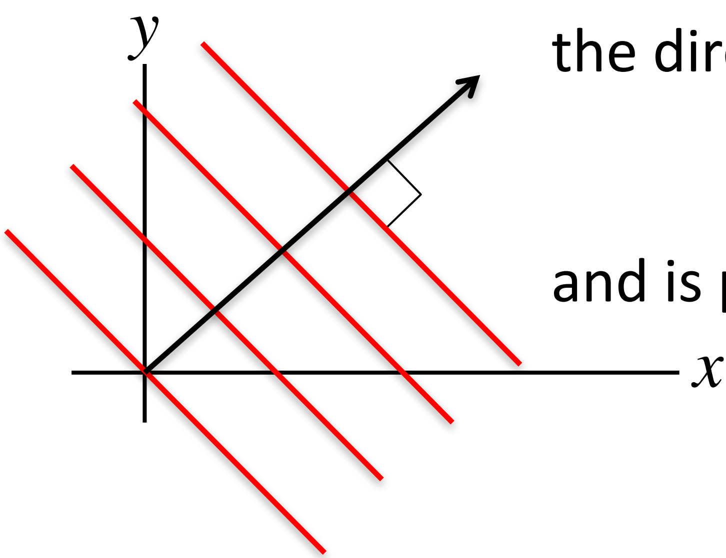
- Mathematically,

$$f\left(x, y, z, t\right) = A\sin\left(k_x x + k_y y + k_z z \mp \omega t\right)$$

Note: the wave travels in the negative $\vec{k}$ direction if the sign here is positive

Review of Unit 9: Waves in Two and Three Dimensions

- Waves that move in a straight line are called line waves in 2D and plane waves in 3D.



the direction of travel is given by

$$\vec{k} = \left(k_x, k_y, k_z\right)$$

and is perpendicular to the crest lines

- Wavelength is given by

$$\lambda = \frac{2\pi}{k} \quad \text{where} \quad k = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

Review of Unit 9: Waves in Two and Three Dimensions

- For a circular or spherical wave,

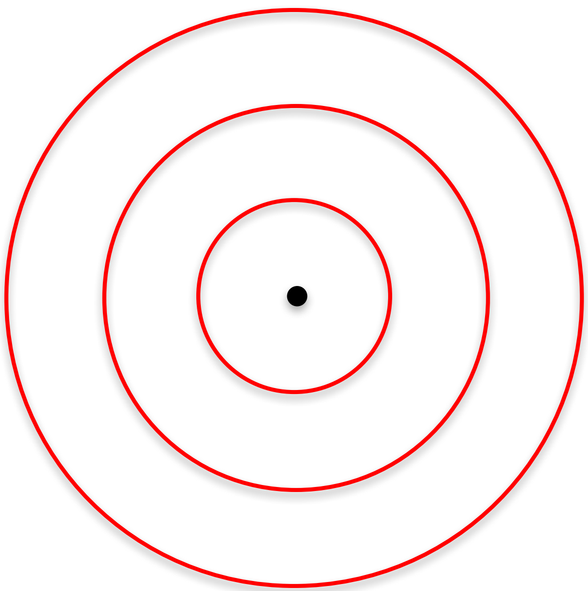
$$f(r,t) = A\sin(kr \mp \omega t)$$

+ if away from the center
− if toward the center

where $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$

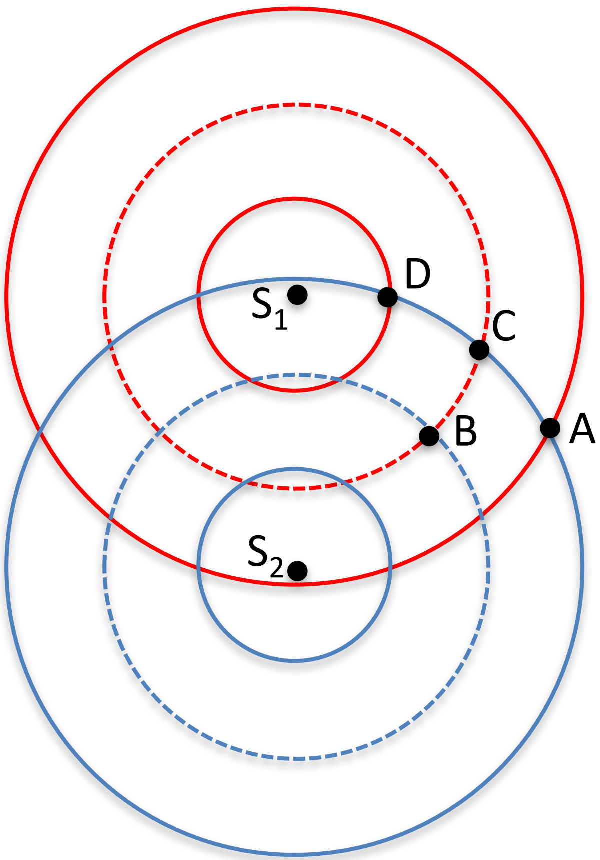
and $r = \sqrt{x^2 + y^2 + z^2}$

distance from the center



Review of Unit 9: Waves in Two and Three Dimensions

- Suppose we have two monochromatic sources of circular waves.



Point A: two crests overlap

Point B: two crests overlap

Point D: two crests overlap

Waves reinforce each other

Constructive interference

Point C: one crest and one trough overlap

Waves cancel each other

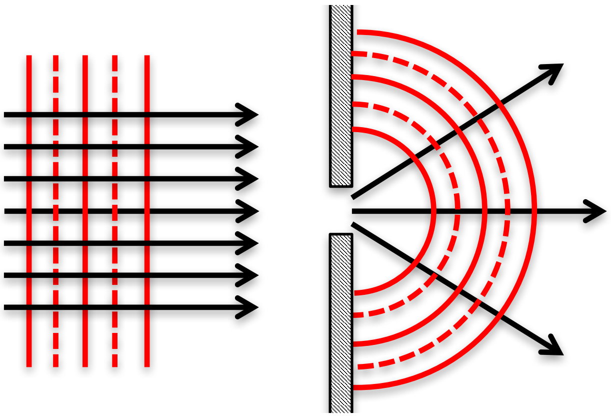
Destructive interference

- Constructive interference occurs where

$$r_2 - r_1 = n\lambda \quad \text{where} \quad n = 0, \pm 1, \pm 2, \cdots$$

Review of Unit 9: Waves in Two and Three Dimensions

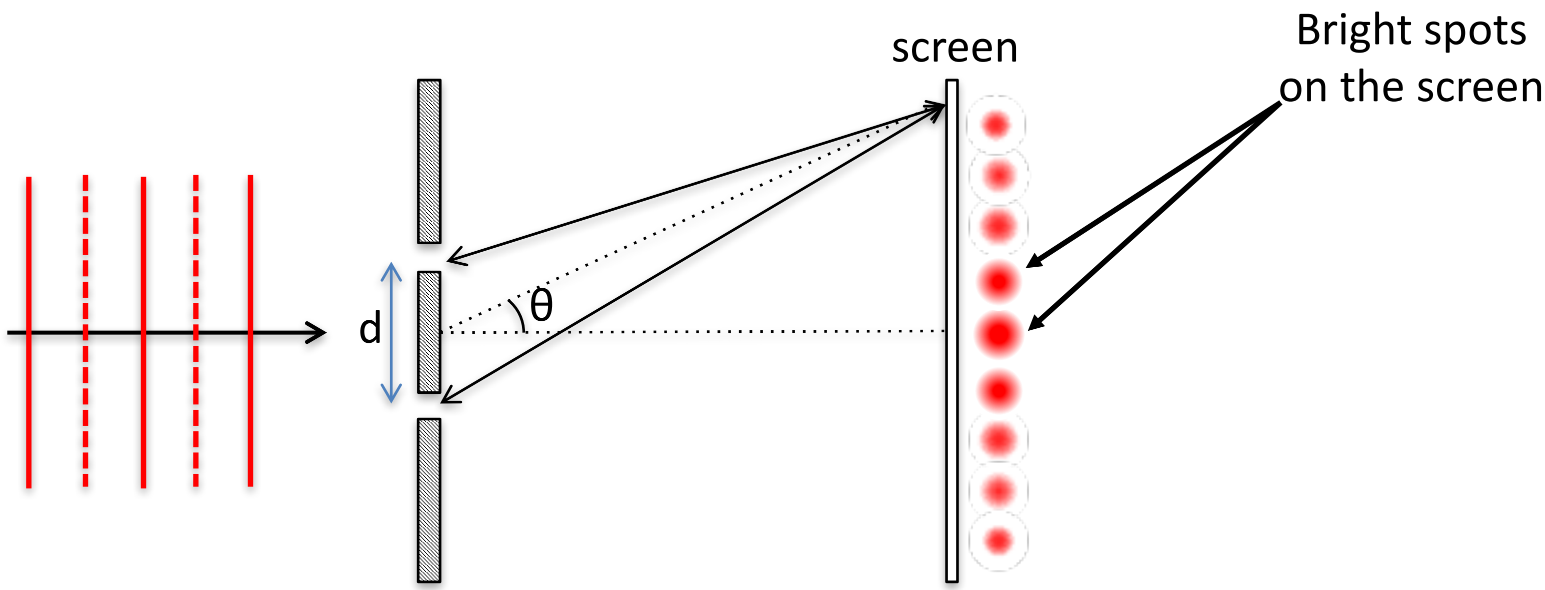
- **Diffraction**: if a wave travels through an opening that is smaller or comparable to the wavelength, then a circular wave is created on the other side of the opening.



- Sometimes, diffraction is described as the bending of waves around an obstacle.

Review of Unit 9: Waves in Two and Three Dimensions

- **Young's Double-Slit Experiment:**



- If the screen is very far, bright spots (constructive interference) are observed where

$$d \sin\theta = n\lambda \quad \text{where} \quad n = 0, \pm 1, \pm 2, \cdots$$

Review of Unit 10: Introduction to Quantized Waves

- Light is quantized, and the smallest possible packet of light is called **photon**.
- Momentum of a photon: $p = \frac{h}{\lambda}$

  where $h = 6.626 \times 10^{-34} Js$ (Planck's constant)

- Energy of a photon: $E = hf$
- Just like photons, particles (like electrons, protons and even larger objects) have wave nature and satisfy the same momentum-wavelength relationship:

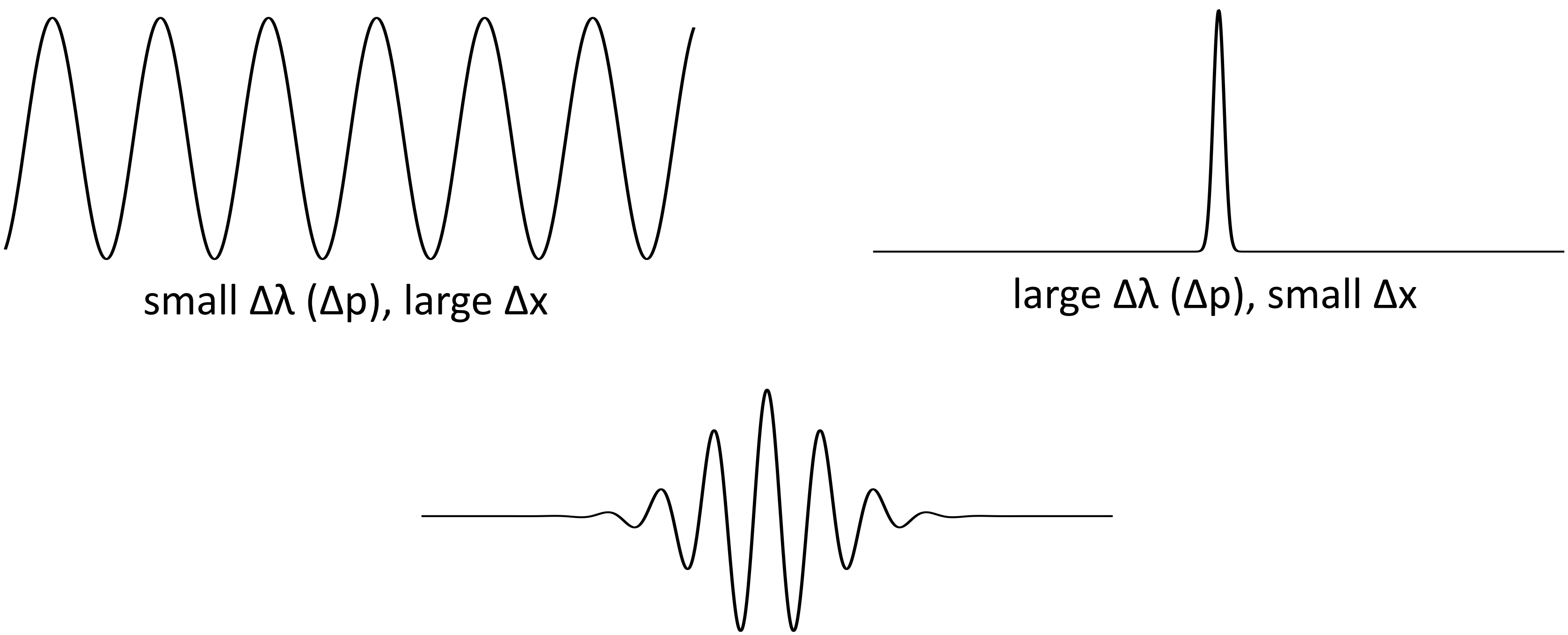
$$p = \frac{h}{\lambda}$$

- Energy of a particle is given by:

$$E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

Review of Unit 10: Introduction to Quantized Waves

- **Heisenberg's Uncertainty Principle**: it is not possible to know the position and the momentum of a particle simultaneously with unlimited precision.
- **Mathematically**, $\Delta p \Delta x > \dfrac{h}{4\pi}$
- The uncertainty principle is due to the wave nature of particles:



small Δλ (Δp), large Δx

large Δλ (Δp), small Δx

moderate Δλ (Δp) and Δx