Physics 131 - Homework IX-XI - Solutions

1. We know $d\sin\theta = n\lambda$, and $\lambda = 633\text{ nm} = 6.33\times10^{-7}\text{ m}$

We also have 13,400 lines/inch, so

$$d = \frac{1}{13{,}400\ \text{lines}/\text{in}} = 7.46\times10^{-5}\ \frac{\text{in}}{\text{line}} * \frac{2.54\text{cm}}{1''} = 1.9\times10^{-4}\text{ cm}$$

$$= 1.9\times10^{-6}\text{ m}$$

So, if $d \sin\theta = n\lambda$,

$$\theta = \sin^{-1}\left(\frac{n\lambda}{d}\right) = \sin^{-1}\left(\frac{n \cdot 6.33\times10^{-7}\,m}{1.9\times10^{-6}\,m}\right)$$

$$= \sin^{-1}\left(n \cdot 0.3339\right)$$

So, for $n=1$, $\theta_1 = .34$ radians $= 19.5^\circ$

$\boxed{n=2,}$ $\theta_2 = .73$ radians $= 42^\circ$

But for $n=3$, $\frac{n\lambda}{d} > 1$, so no $n=3$ peak.

(You may have found $n=3$ worked if you rounded intermediate values — that's ok!)

2. Waves diffract more when openings are small & λ is large:

$$d \sin\theta = n\lambda \quad \text{or} \quad \sin\theta = \frac{n\lambda}{d}$$

So if $\frac{\lambda}{d}$ is big, θ is also large

So, $\boxed{\text{large } \lambda\text{'s diffract}}$ more — these correspond to $\boxed{\text{low } f}$ $\left(\lambda = \frac{v_{wave}}{f}\right)$

3. We get *destructive* interference when the difference in path is a *half* integer (e.g. ½, 1½, 2½, etc) number of wavelengths.

$$\text{So } \left(n+\tfrac{1}{2}\right)\lambda = \Delta \text{path} = 20\,\text{cm}$$

$$\text{So if } n=0, \quad \frac{\lambda_0}{2} = 20\,\text{cm} \Rightarrow \lambda_0 = 40\,\text{cm} = .4\,\text{m}$$

$$f_0 = \frac{v}{\lambda_0} = \frac{300\,\text{m/s}}{.4\,\text{m}} = \boxed{750\ \text{Hz}}$$

$$\text{Now, if } n=1, \quad \frac{3\lambda_1}{2} = 20\,\text{cm} \Rightarrow \lambda_1 = \frac{40\,\text{cm}}{3} = .133\,\text{m}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{300\,\text{m/s}}{.133} = \boxed{2250\ \text{Hz}}$$

So, lowest n's give lowest f.

Session X.1

4. $\lambda = 633\,\text{nm} = 6.33\times10^{-7}\,\text{m} \Rightarrow f = \frac{c}{\lambda} = \frac{3\times10^{8}\,\text{m/s}}{6.33\times10^{-7}\,\text{m}} = 4.7\times10^{14}\,\text{Hz}$

$E = hf = 6.6\times10^{-34}\,\text{J}\cdot\text{sec}\cdot 4.7\times10^{14} \boxed{= 3.1\times10^{-19}\,\text{J}}$ ⇐ like typical atomic energies

$$p = \frac{h}{\lambda} = \frac{6.6\times10^{-34}\,\text{J}\cdot\text{sec}}{6.33\times10^{-7}} = \boxed{1.04\times10^{-27}\ \text{kg m/sec}}$$

5. An electron has mass $m = 9.1\times10^{-31}\,\text{kg}$, $E = 5\times10^{-19}\,\text{J}$

$\lambda = \frac{h}{p}$, so we need to find p. Use $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ $(p = mv)$

$$\text{So } p = \sqrt{2mE} = \sqrt{2\cdot 9.1\times10^{-31}\,\text{kg}\cdot 5\times10^{-19}\,\text{J}} = 9.54\times10^{-25}\,\text{kg m/sec}$$

$$\lambda = \frac{h}{p} = \frac{6.6\times10^{-34}\,\text{J sec}}{9.54\times10^{-25}\,\text{kg m/s}} = \boxed{6.9\times10^{-10}\,\text{m}}$$

Very close to a nanometer

XI.1 6. $m = .2\,kg$ $v = 45\,m/s$ typically for baseball

a) If $m = .2\,kg$, what v gives $\lambda = 0.1\,m$?

$$\lambda = \frac{h}{p} \Rightarrow mv = \frac{h}{\lambda} \Rightarrow v = \frac{h}{m\lambda} = \frac{6.6\times10^{-34}\,J\,sec}{.2\,kg \cdot .1\,m} = 3.3\times10^{-32}\,m/s$$

This is ridiculously slow – taking 10^{17} sec to travel the width of a nucleus!

b) If $v = 45\,m/s$, what m gives $\lambda = 0.1\,m$?

$$\text{Again } mv = \frac{h}{\lambda} \Rightarrow m = \frac{h}{\lambda v} = \frac{6.6\times10^{-34}\,J\,sec}{.1\,m \cdot 45\,m/sec} = 1.5\times10^{-34}\,kg$$

This is also ridiculously small – even single elementary particles (like the electron) have masses greater than this!

The point is, because h is so small, we cannot see quantum effects with ordinary sized objects at ordinary speeds.

7. For a quantum wave: $A \sin(kx - \omega t)$

- A is the overall brightness
- k is $2\pi/\lambda$ where λ is the distance from one red to the nearest next red (or blue to blue or ...)
- ω is $2\pi/T$ where T is the time between successive occurrences of the same color (red to red, or blue to blue, or ...)