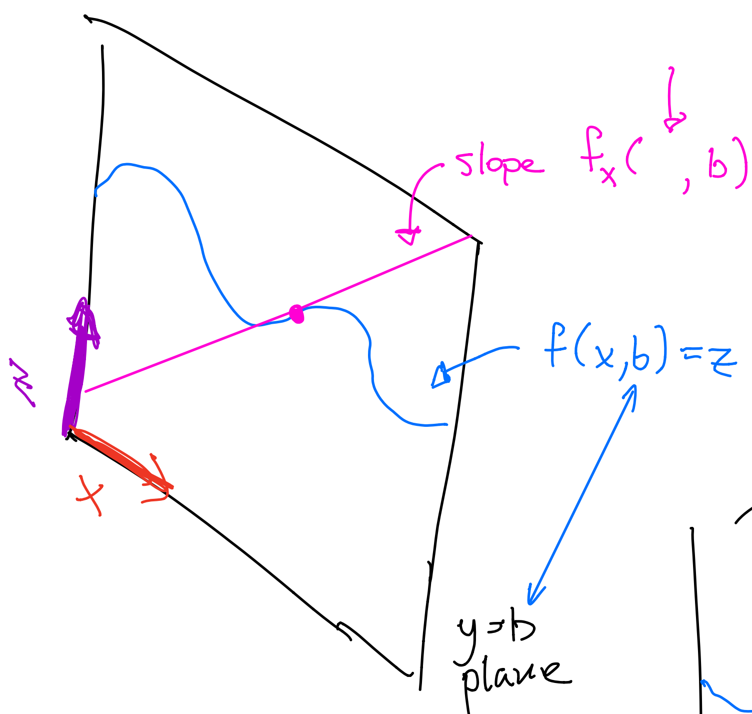
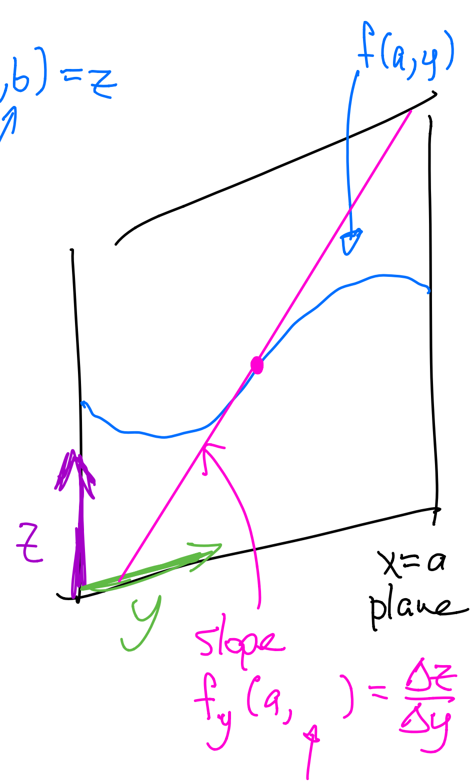
Partial Derivatives

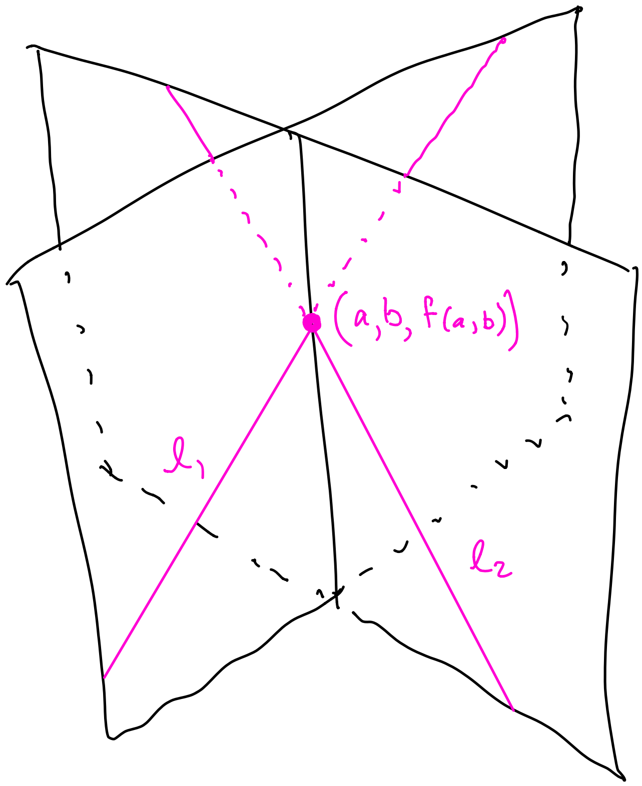
$f(x,y) = z$

$f_x(a,b) \longrightarrow$ rate of change in z w/ respect to changes in x at the point $(a,b,f(a,b))$

$f_y(a,b) \longrightarrow$ rate of change in z w/ ~~respect~~ to changes in y at the point $(a,b,f(a,b))$







l_1 and l_2 are tangent to the graph of $z = f(x,y)$ at the point $(a, b, f(a,b))$

l_1 has slope $\frac{\Delta z}{\Delta x} = f_x(a,b)$

all points on l_1 have y-value b.

l_1 has direction $\langle 1, 0, f_x(a,b) \rangle$

one unit change in x

$\frac{\Delta z}{\Delta x}$

l_2 has slope $\frac{\Delta z}{\Delta y} = f_y(a,b)$

l_2 has direction $\langle 0, 1, f_y(a,b) \rangle$

$\vec{n} = \langle 1, 0, f_x(a,b) \rangle \times \langle 0, 1, f_y(a,b) \rangle$

$P(a, b, f(a,b))$

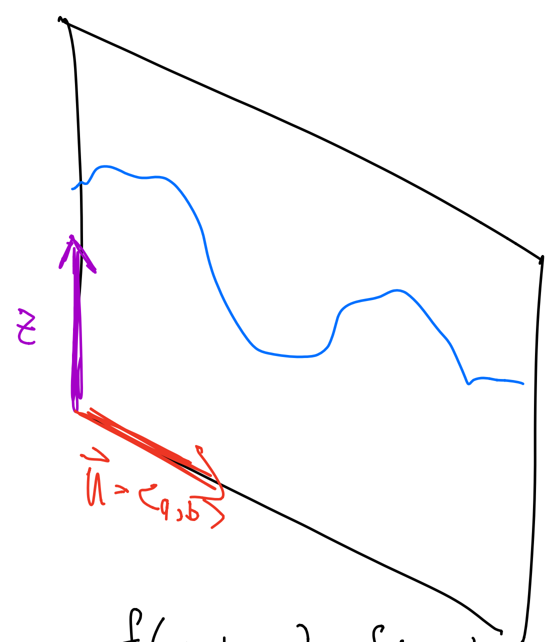
$$\vec{n} = \begin{vmatrix} 0 & f_x(a,b) \\ 1 & f_y(a,b) \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & f_x(a,b) \\ 0 & f_y(a,b) \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \hat{k}$$

$$= \langle -f_x(a,b), -f_y(a,b), 1 \rangle$$

$$\vec{n} \cdot \langle dx, dy, dz \rangle = 0$$

$$-f_x(a,b)\,dx - f_y(a,b)\,dy + dz = 0$$

$$dz = f_x(a,b)\,dx + f_y(a,b)\,dy$$



$$f_x(a,b) = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$D_{\vec{u} = \langle a,b \rangle} = \lim_{h \to 0} \frac{f(x+ah, y+bh) - f(x,y)}{h}$$

Thm If $\vec{u} = \langle a, b \rangle$ w/ $\|\vec{u}\| = 1$

then $D_{\vec{u}} f(x,y) = f_x(x,y)\, a + f_y(x,y) \cdot b$

$$\vec{u} \cdot \underbrace{\langle f_{x_1}, f_{x_2}, \dots, f_{x_n} \rangle} = D_{\vec{u}} f(x_1, \dots, x_n)$$

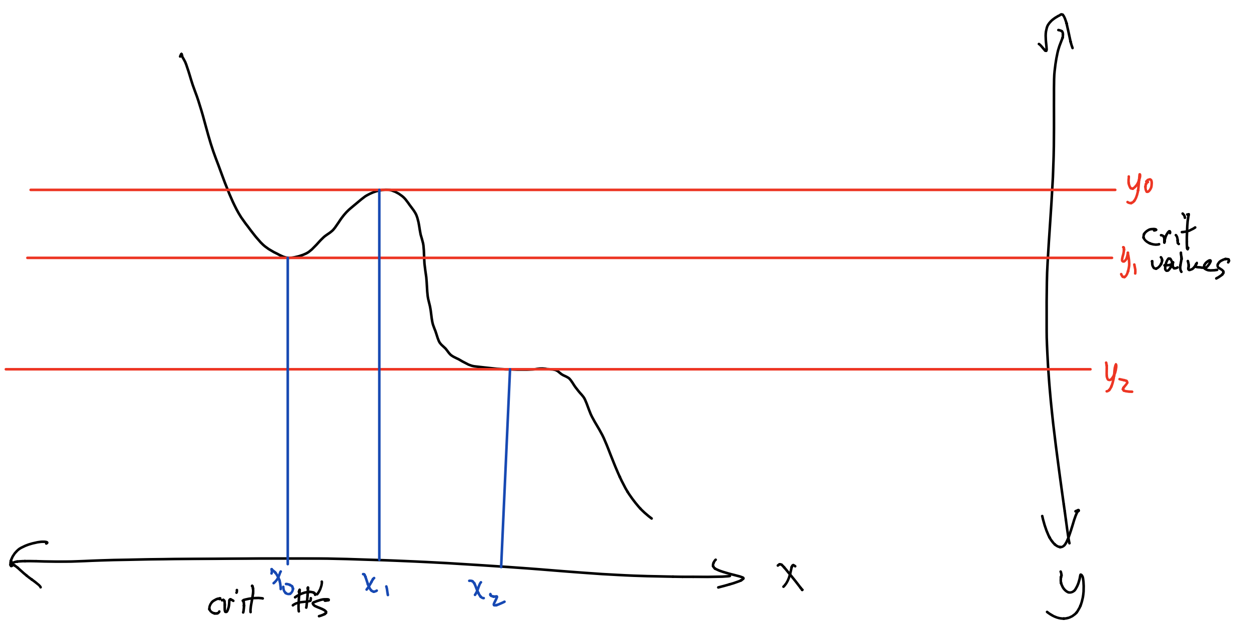
"The Gradient vector" of f or ∇f

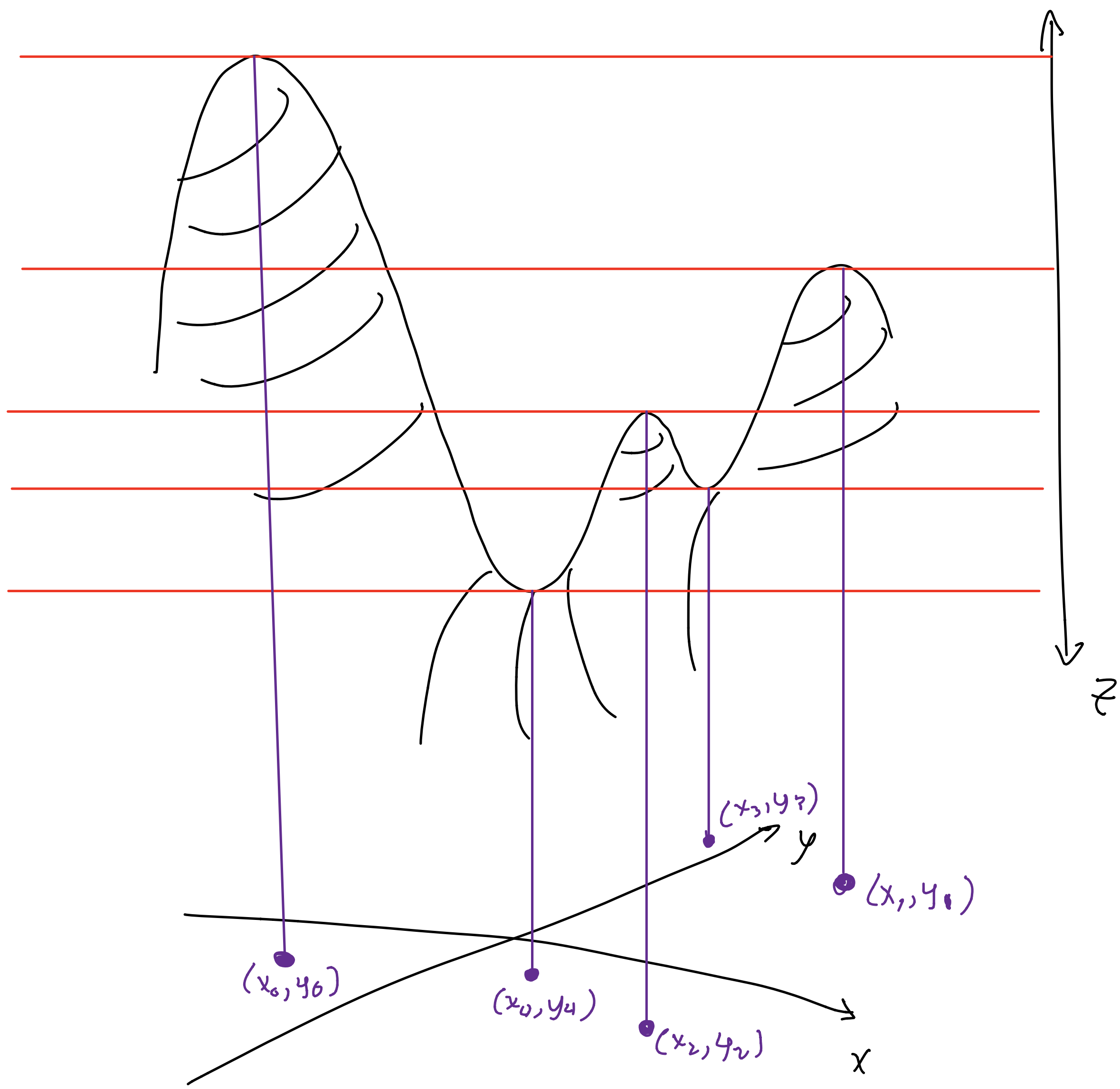
$D_{\vec{u}} = \nabla f \cdot \vec{u}$

when is $|\nabla f \cdot \vec{u}|$ the largest?

when ∇f and $\vec{u}$ are parallel

Critical values and critical numbers





z
y
x
(x_3, y_3)
(x_1, y_1)
(x_0, y_0)
(x_4, y_4)
(x_2, y_2)