Reference Sheet

The following rules and identities can be used without further proof or justification. This sheet may be updated throughout the semester, and you will be able given an applicable version for each exam.

When in doubt, you should justify all of the steps in your solutions that make use of a fact on this sheet. To make this easier, each rule or identity comes with a suggestion for abbreviation. For example, if your argument uses the fact that $\sin^2(x) = 1 - \cos^2(x)$, then you could write, "by trig 1," with the step that uses that fact.

If you spot an error on this sheet, please, notify me immediately.

1 Trigonometric Identities (trig)

(1) $\sin^2(x) + \cos^2(x) = 1$

(2) $\sec^2(x) - \tan^2(x) = 1$

(3) $2\sin^2(x) = 1 - \cos(2x)$

(4) $2\cos^2(x) = 1 + \cos(2x)$

(5) $2[\sin(A)\cos(B)] = \sin(A-B) + \sin(A+B)$

(6) $2[\sin(A)\sin(B)] = \cos(A-B) - \cos(A+B)$

(7) $2[\cos(A)\cos(B)] = \cos(A-B) + \cos(A+B)$

2 Differentiation Rules

For functions f and g satisfying the appropriate hypotheses, we have

- The Product Rule (prod. rule):
$$(f \cdot g)' = f \cdot g' + f' \cdot g$$
- The Quotient Rule (quot. rule):
$$(f/g)' = (f' \cdot g - f \cdot g')/g^2$$
- The Chain Rule (chain rule):
$$(f \circ g)' = (f' \circ g) \cdot g'$$

3 Derivatives (d-ref)

1. $\frac{d}{dx}(a^x) = a^x \ln(a)$
2. $\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)}$
3. $\frac{d}{dx}(\sin(x)) = \cos(x)$
4. $\frac{d}{dx}(\cos(x)) = -\sin(x)$
5. $\frac{d}{dx}(\tan(x)) = \sec^2(x)$
6. $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$
7. $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$
8. $\frac{d}{dx}(\cot(x)) = -\csc^2(x)$
9. $\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$
10. $\frac{d}{dx}\left(\cos^{-1}(x)\right) = -\frac{1}{\sqrt{1-x^2}}$
11. $\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$
12. $\frac{d}{dx}\left(\csc^{-1}(x)\right) = -\frac{1}{x\sqrt{x^2-1}}$
13. $\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{x\sqrt{x^2-1}}$
14. $\frac{d}{dx}\left(\cot^{-1}(x)\right) = -\frac{1}{1+x^2}$

4 Integration Strategies

For functions f and g satisfying the appropriate hypotheses, we have

- The Anti Chain Rule (u-sub):

$$\int f'(g(x)) \cdot g'(x)dx = f(g(x)) + C$$

- The Anti Product Rule (by parts):

$$\int f(x) \cdot g'(x)dx = f(x) \cdot g(x) - \int f'(x) \cdot g(x)dx$$

5 Integrals (i-ref)

For functions u and v satisfying the appropriate hypotheses, we have

(1) $\displaystyle\int (a^u)du = a^u / \ln(u) + C$

(2) $\displaystyle\int (1/u)du = \ln|u| + C$

(3) $\displaystyle\int \sin(u)du = -\cos(u) + C$

(4) $\displaystyle\int \cos(u)du = \sin(u) + C$

(5) $\displaystyle\int \sec^2(u)du = \tan(u) + C$

(6) $\displaystyle\int \csc(u)\cot(u)du = -\csc(u) + C$

(7) $\displaystyle\int \sec(u)\tan(u)du = \sec(u) + C$

(8) $\displaystyle\int \csc^2(u)du = -\cot(u) + C$

(9) $\displaystyle\int \frac{1}{\sqrt{a^2 - u^2}}du = \sin^{-1}(u/a) + C$

(10) $\displaystyle\int \frac{1}{a^2 + u^2}du = (1/a)\tan^{-1}(u/a) + C$

(11) $\displaystyle\int \frac{1}{u\sqrt{u^2 - a^2}}du = (1/a)\sec^{-1}(u/a) + C$

(12) $\displaystyle\int \tan(u)du = \ln|\sec(u)| + C$

(13) $\displaystyle\int \cot(u)du = \ln|\sin(u)| + C$

(14) $\displaystyle\int \sec(u)du = \ln|\sec(u) + \tan(u)| + C$

(15) $\displaystyle\int \csc(u)du = \ln|\csc(u) - \cot(u)| + C$