Physics 131 - Homework XI-XII - Solutions

XI.2 1. $E_n = \frac{n^2 h^2}{8mL^2}$ If we look only at transitions w/ $\Delta n = 2$

(photon energy)

$$E_\gamma = \Delta E = E_n - E_{n-2} = \frac{n^2h^2}{8mL^2} - \frac{(n-2)^2h^2}{8mL^2}$$

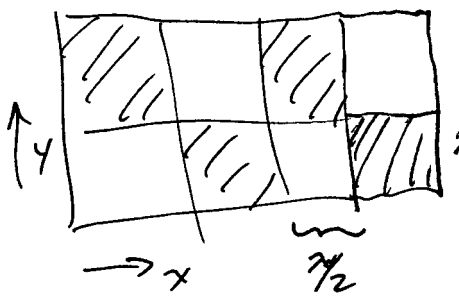
$$= \left[n^2 - (n-2)^2\right]\frac{h^2}{8mL^2} = \left[n^2 - (n^2 - 4n + 4)\right]\frac{h^2}{8mL^2}$$

$$= (4n-4)\frac{h^2}{8mL^2} = \frac{(n-1)h^2}{2mL^2}$$

2. We assume $L = \frac{h}{2\pi}$ for electron in benzene ring, where $r \approx 5\times10^{-10}$ m

$L = \frac{h}{2\pi} = mrv$, so

$$v = \frac{h}{2\pi mr} = \frac{6.6\times10^{-34}\ \text{J sec}}{2\pi(9\times10^{-31}\text{kg})(5\times10^{-10}\text{m})} = 2.3\times10^5\ \text{m/s}$$

3. 

$n_x = 4$ (4 half wavelengths)

$n_y = 2$ (2 half wavelengths)

4. $E = \frac{h^2}{8mL^2}(n_x^2 + n_y^2)$ So, to search for degeneracies in a square 2d box, we only need to show $n_x^2 + n_y^2$ is same for each state

a) $(2,3) \Rightarrow n_x^2 + n_y^2 = 4 + 9 = 13$

$(3,2) \Rightarrow n_x^2 + n_y^2 = 9 + 4 = 13$

} degenerate-by commutativity

b) $(5,5) \Rightarrow n_x^2 + n_y^2 = 25 + 25 = 50$

$(7,1) \Rightarrow n_x^2 + n_y^2 = 49 + 1 = 50$

} degenerate-by numerical accident

5. 3 dim box $L = 2 \times 10^{-10}$ m. What is ground state energy?

$$E = \frac{h^2}{8mL^2}(n_x^2 + n_y^2 + n_z^2) = \frac{(6.6 \times 10^{-34})^2}{8 \cdot 9.1 \times 10^{-31} \cdot (2 \times 10^{-10})^2}(1+1+1)$$

$= 4.5 \times 10^{-18}$ J $\Leftarrow$ not too far off from real value of around 10^{-18} J. Clearly L value is critical – and hard to estimate.