Name:____________________________________

Test Your Skills - 06
Deadline - 4/14

Answer these exercises, in complete mathematical sentences and using mathematical notation properly. You are to work on these individually, without collaboration. You may consult your book and myself, but **not the math lab** or other resources. To earn extra credit, stop into my office hours (or make an appointment) and present your solutions. Partial credit will be given for any earnest attempt.

Exercise 1. *Supposes* $f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$. *Show that* $f_{xx} + f_{yy} + f_{zz} = 0$

Exercise 2. *Let* $F(x,y,z) = 4x^2 + 2y^2 + z^2$. *Consider the intersection of level surface* $F(x,y,z) = 16$, *and the plane* $y = 2$. *Find an equation for the line tangent to this intersection, at the point* $(1,2,2)$.

Exercise 3. *Suppose* a, b, c, *and* d *are real numbers, with* $0 < a, b, c, d \leq 50$. *Alice computes the product of these four numbers, exactly. Bob first rounds each of the numbers to the nearest whole number, then computes their product. Show how differentials can be used to estimate the maximum difference between Alice's and Bob's results.*

Exercise 4. *Consider*

$$g(s,t) = e^s \cos(t),$$
$$h(s,t) = e^s \sin(t).$$

If $F(g(s,t), h(s,t))$ *has continuous first and second partial derivatives, show that*

$$\left(\frac{\partial F}{\partial g}\right)^2 + \left(\frac{\partial F}{\partial h}\right)^2 = e^{-2s}\left[\left(\frac{\partial F}{\partial s}\right)^2 + \left(\frac{\partial F}{\partial t}\right)^2\right]$$

and

$$\frac{\partial^2 F}{\partial g^2} + \frac{\partial^2 F}{\partial h^2} = e^{-2s}\left[\frac{\partial^2 F}{\partial s^2} + \frac{\partial^2 F}{\partial t^2}\right]$$