Name:____________________________________

Worksheet 07

1 Vectors To Describe Objects in Space

Exercise 1. *Describe the line, L, given by*

$$y = 3x$$

using vectors, by choosing $\vec{v}$ and $\vec{w}$ so that the linear combination

$$\vec{v} + t\vec{w}$$

describes all of the points on L if we let t vary across all real numbers.

Exercise 2. *Describe the line, L, given by*

$$y = 3x + 4$$

using vectors, by choosing $\vec{v}$ and $\vec{w}$ so that the linear combination

$$\vec{v} + t\vec{w}$$

describes all of the points on L if we let t vary across all real numbers.

Exercise 3. *If we consider the line described by the components of the*

$$< 1, 3 > +t < -1, 5 >$$

as we let t vary across all real numbers. Describe this line using parametric equations, then describe it using slope-intercept form.

Exercise 4. *If we consider the line described by the components of the*

$$< 1, -2, 3 > +t < -1, 5, 1 >$$

as we let t vary across all real numbers. Describe this line using parametric equations.

Exercise 5. *Given the points*

$$P_1(1, 2, -1)$$
$$P_2(2, -1, 0)$$
$$P_3(0, 5, 0)$$

Find a vector normal to the plane containing P_1*,* P_2*, and* P_3*.*

Exercise 6. *Suppose $\vec{n}$ is a normal vector to some plane, X. If any representative of a vector, $\vec{a}$, is contained in X, then what is the value of $\vec{n} \bullet \vec{a}$? Explain your answer.*

Exercise 7. *Suppose points $P_1(2,3,4)$ and $P_2(1,-2,0)$ are both contained in the same plane, X, and let*

$$\vec{b} =< 2-1, 3-(-2), 4-0 >=< 1,5,4 > .$$

If $\vec{n}$ is normal to X, then what is the value of $\vec{n} \bullet \vec{b}$? Explain your answer.

Exercise 8. *Suppose that $P(x,y,z)$ and $P_0(x_0,y_0,z_0)$ are both contained in the same plane, X. Further, suppose that $\vec{n}$ is normal to X. Given your observations in Exercise 7, give a relationship between $\vec{r} =< x,y,z >$, $\vec{r}_0 =< x_0,y_0,z_0 >$, and $\vec{n}$.*

2 Generalizing and Application

Exercise 9. *Looking back at Exercise 4, can you eliminate the parameter and express this line using Cartesian coordinates (x, y, and z)?*
(Hint: It may take more than one equation.)

Exercise 10. *In general, can you express the line described by*

$$< x_1, y_1, z_1 > +t < x_2, y_2, z_2 >$$

as we let t vary across all real numbers, using Cartesian coordinates? When might you encounter problems? Can every line in $\mathbb{R}^3$ be expressed as a linear combination of vectors, as in (1)? Can every line in $\mathbb{R}^3$ also be expressed in Cartesian coordinates?

Exercise 11. *Given your observations in Exercise 8, can you find derive a general equation of a plane through a given point and with a given normal vector?*

Exercise 12. *Lines in* $\mathbb{R}^3$ *that are not parallel, and do not intersect, are called* skew *lines. Show that the lines, with the symmetric equations below, are skew. Also, find the distance between these lines.*

$$x = y = z \tag{1}$$

$$x + 1 = \frac{y}{2} = \frac{z}{3} \tag{2}$$

3 Challenge

Exercise 13. *Given a plane, X, and a point, P, find an equation that describes the distance from P to X.*