Ch 13

CC #6 How are dot products useful?

- check for orthogonality
- * tells us the angle between two vectors (6.2.2)
- finding projections of vectors

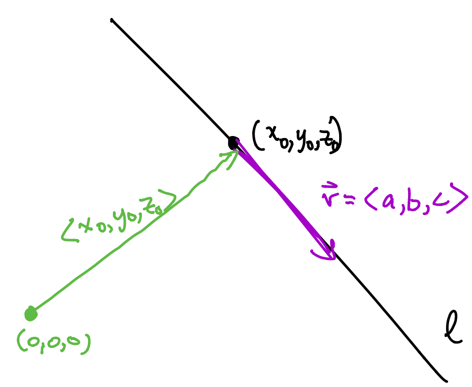
CC #9 How are cross products useful?

- detect parallel vectors
- generate orthogonal vectors

CC #12 How do we find the angle between ~~intersecting~~ planes?

find the angle between the normal vectors

CC #13 equations for lines in $\mathbb{R}^3$



vector $\quad \ell = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$

parametric
$$\left.\begin{aligned} x(t) &= x_0 + ta \\ y(t) &= y_0 + tb \\ z(t) &= z_0 + tc \end{aligned}\right\} \Longrightarrow \quad t = \frac{x - x_0}{a} \quad t = \frac{y - y_0}{b} \quad t = \frac{z - z_0}{c}$$

symetric $\quad \dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

CC#14 equations of planes

$\langle a,b,c\rangle = \vec{n}$ is normal to the plane
(x_0,y_0,z_0) is a point in the plane

vector $\vec{n}\cdot(\overset{\vec{r}}{\langle x,y,z\rangle}-\overset{\vec{r}_0}{\langle x_0,y_0,z_0\rangle})=0$

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

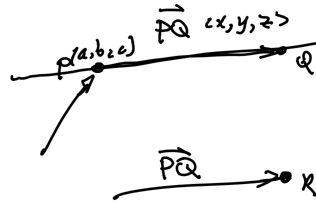
scalar equation $ax+by+cz+(\ d\ )=0$

CC #16

How to tell if 3 points are colinear
or 4 points are coplanar

Given P,Q,R,

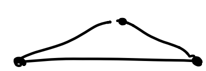
$\overrightarrow{PQ} \parallel \overrightarrow{QR}$



$\ell = \langle a,b,c\rangle + t\langle x,y,z\rangle$

$|\overrightarrow{PQ}| + |\overrightarrow{QR}| = |\overrightarrow{PR}|$

$\vec{v}\in V$ $\quad \vec{w} = c\vec{v}$ for some c
$c \in \mathbb{R}$



Given P,Q,R,S are they coplanar?

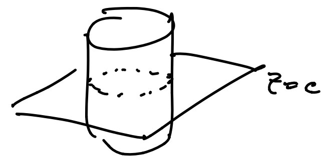
$\overrightarrow{PQ}\times\overrightarrow{QR} = c(\overrightarrow{QR}\times\overrightarrow{RS}) \leftarrow$

$\vec{n} = \overrightarrow{PQ}\times\overrightarrow{QR}$ find an equation for the plane w/ $\vec{n}$ normal containing P.

TF #3 $|\vec{u}\times\vec{v}| = |\vec{v}\times\vec{u}|$ T ✓ (6.3.1.1)
#9 $(\vec{u}\times\vec{v})\cdot\vec{u}=0$ T ✓ (6.2.3) (6.3.2)
#11 The cross product of two unit vectors is a unit vector. F ✓

$\|\vec{v}\|=1$
$\|\vec{u}\|=1$ $\quad$ 6.3.3 says $\|\vec{u}\times\vec{v}\| = \|\vec{u}\|\cdot\|\vec{v}\|\sin\theta$
$\neq 1$ unless $\sin\theta=1$

\#13 $\{(x,y,z) \mid x^2+y^2=1\}$ is a circle.

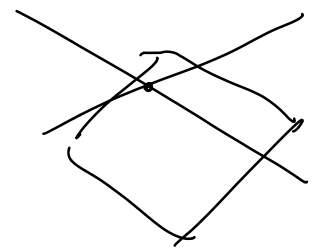


ex 17

line through $(-2,2,4)$

perpendicular to plane $2x-y+5z=12$

direction $\langle 2,-1,5\rangle$

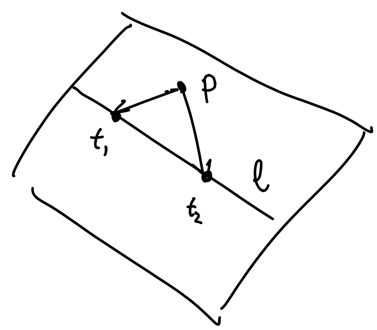


$l = \langle -2,2,4\rangle + t\langle 2,-1,5\rangle$

$x = -2 + t\cdot 2$

$y = 2 + t(-1)$

$z = 4 + t(5)$



ex 20 $P(1,2,-2)$ } pt

$\left.\begin{array}{l} x = 2t \\ y = 3-t \\ z = 1+3t \end{array}\right\}$ line

$t=0$ $Q(0,3,1)$ is on the line

$t=1$ $R(2,2,4)$ is on the line

$\vec{PQ} = \langle 1-0, 2-3, -2-1\rangle = \langle 1,-1,-3\rangle$

$\vec{QR} = \langle 0-2, 3-2, 1-4\rangle = \langle -2,1,-3\rangle$

$\langle 1,-1,-3\rangle \times \langle -2,1,-3\rangle$

$\begin{vmatrix} -1 & -3 \\ 1 & -3 \end{vmatrix}\hat{i} - \begin{vmatrix} 1 & -3 \\ -2 & -3 \end{vmatrix}\hat{j} + \begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix}\hat{k}$

$6\hat{i} + 9\hat{j} + (-1)\hat{k}$

$\vec{n} = \langle 6,9,-1\rangle$

$P(1,2,-2)$

$\langle 6,9,-1\rangle \cdot \left(\langle x,y,z\rangle - \langle 1,2,-1\rangle\right)$

14.

CC 7. (omit)

CC 9.

T/F #1 $\quad t^3\langle 1, 2, 3\rangle$ is this a line? $\quad$ yes! (despite my poor intuition)

this is just the line $x = s$, $y = 2s$, $z = 3s$

composed w/ t^3, which goes through all the same values, only more quickly

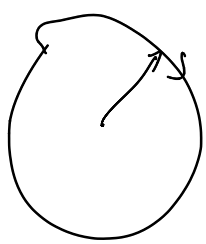
This is a good example of a curve w/ two parameterizations w/ different tangent vectors

$\vec{r}_1(t) = \langle t^3, 2t^3, 3t^3\rangle \qquad \vec{r}_2(t) = \langle t, 2t, 3t\rangle$

$\vec{r}_1{}'(t) = \langle 3t^2, 6t^2, 9t^2\rangle \qquad \vec{r}_2{}'(t) = \langle 1, 2, 3\rangle$

Think of tracing the same curve at different speeds, the velocity vectors will be different.

9) $|\vec{r}(t)| = 1$ for all t then $|r'(t)|$ is constant $\quad$ False

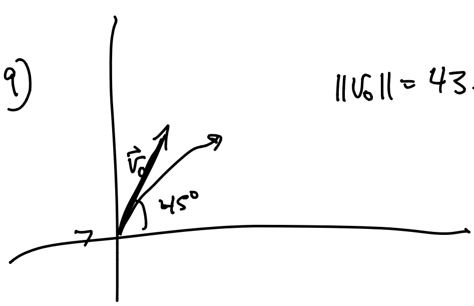


$\langle \sin(t), \cos(t)\rangle$

$\vec{r}(t)\ \langle \sin(t^3), \cos(t^3)\rangle$

$\vec{r}\,'(t) = \langle 3\cos(t^3)t^2, -3\sin(t^3)t^2\rangle$

19)



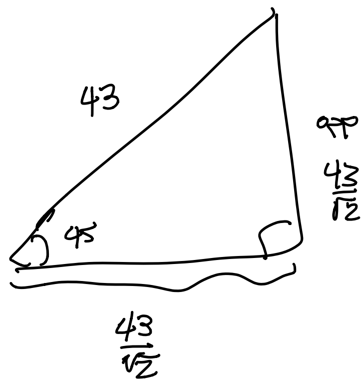
$\|v_0\| = 43$ ft/s $\qquad \vec{a} = -g\hat{j}$

$\vec{v}(t) = \int_0^t \vec{a}(u)\,du = -gt\hat{j} + C$

$\vec{v}(t) = -gt\hat{j} + \vec{v}_0$

$\vec{r}(t) = \int_0^t \vec{v}(u)\,du$

$= -g\frac{t^2}{2}\hat{j} + \vec{v}_0 t + 7\hat{j}$ $\leftarrow$ starting position



$\sin(45) = \frac{opp}{43}$

$\cos(45) = \frac{adj}{43}$

$V_0 = \frac{43}{\sqrt{2}} i + \frac{43}{\sqrt{2}} j$

$\vec{r}(t) = -g\frac{t^2}{2}\hat{j} + \frac{43}{\sqrt{2}}t\hat{i} + \frac{43}{\sqrt{2}}t\hat{j} + 7\hat{j}$

$= \left\langle \frac{43}{\sqrt{2}}, -g\frac{t^2}{2} + \frac{43}{\sqrt{2}} + 7 \right\rangle$