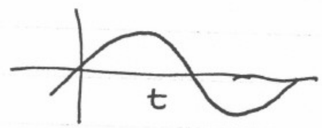
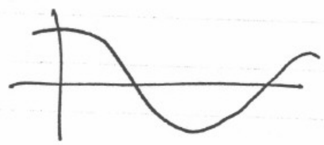
PHYSICS 131 – HOMEWORK VI - Solutions

1. I find it useful to recall

sin(t):



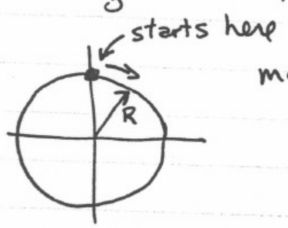
cos(t):



$(x,y) = R(\sin\omega t, \cos\omega t)$

At $t=0$, $(x,y) = (0,R)$

a smidge later, x is positive, ~~R~~ y is less than R.

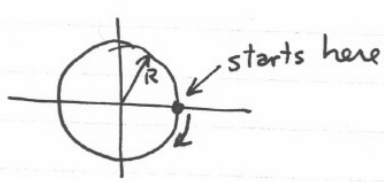


moves clockwise as shown.

$(x,y) = R(\cos\omega t, -\sin\omega t)$

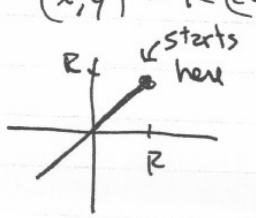
At $t=0$ $(x,y) = (R,0)$

a smidge later, x is a bit less than R, y is a bit negative



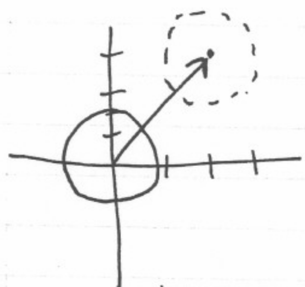
again, moves clockwise as shown

$(x,y) = R(\cos\omega t, \cos\omega t)$



At $t=0$, starts at $(x,y)=(R,R)$, which is <u>not</u> on a circle of radius R!
Since $x=y$, this oscillates along the diagonal as shown.

2. We want to shift our parametrized circle from the origin to surround the point (2,3).

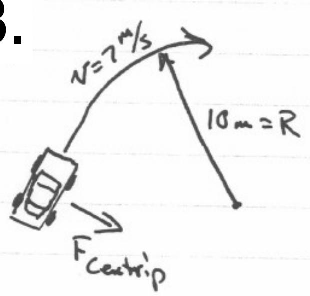


We can transform the origin (0,0) to that point by simply adding the vector (2,3) to it, so we can do that for the circle as well. We also want a radius $R = 3\text{m}$, & an angular velocity of $\omega = 5\ \text{rad/sec}$,

So $(x(t), y(t)) = (2,3) + 3\text{m}(\cos 5t, \sin 5t)$

$$\boxed{= (2 + 3\cos 5t,\ 3 + 3\sin 5t)}$$

3.



The force must be directed toward the center of the circular path, since this is the centripetal force acting on the car. Its magnitude is

$$F = m\frac{v^2}{R} = 700\text{kg}\cdot\frac{(7\text{m/s})^2}{10\text{m}}$$

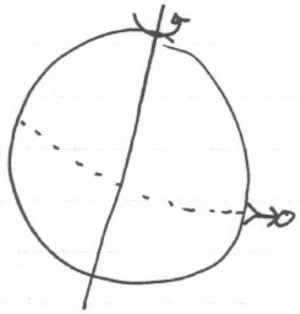
$$= 3430\ \text{N}$$

4. The person inside the car must also have a centripetal force acting on her

$$F = m\frac{v^2}{R} = 65\text{kg}\cdot\frac{(7\text{m/s})^2}{10\text{m}} = 318\ \text{N}$$

This compares to the gravity force of $mg \approx 650\text{N}$ — roughly half of this! This would certainly seem like a pretty noticeable force!

5.



The apparent force of gravity is actually the contact force of the ground — which is why we feel heavier when the elevator starts moving upward, and why we feel weightless in freefall — the contact force adjusts to give us the right acceleration. If we are accelerating inward at the equator, we appear to weigh less $\Rightarrow$ $\underset{\text{apparent weight.}}{Wt} = mg$ becomes $m(g - a_{centrip})$

[You didn't have to explain that — the problem gave you that!] So how big is this effect? The relevant comparison is $\frac{a_{centrip}}{g}$ $\Rightarrow$ that gives our % "weight loss."

$$a_{centrip} = \frac{v^2}{R}$$

$$v = \frac{4\times10^7\,m}{1\,day}\cdot\left(\frac{1d}{24hr}\right)\left(\frac{1hr}{60min}\right)\left(\frac{1min}{60sec}\right)$$

$$= 463\,m/sec$$

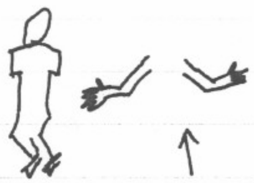
$$R = \frac{circumf}{2\pi} = \frac{4\times10^7\,m}{2\pi} = 6.37\times10^6\,m$$

$$a_{cent} = 0.034\,m/s^2$$

$$\text{so } \frac{a_{cent}}{g} = \frac{.034\,m/s^2}{10\,m/s^2} = .0034 \Rightarrow 0.34\%\text{ change —}$$

well less than a pound for most of us!

6.



$m = 50kg$

$r = .10m$

$m = 2\times 5kg$

$r = \begin{cases} .70m \\ .15m \end{cases}$

$$I_{body} = 50kg\cdot(.1m)^2 = .5\,kgm^2$$

$$I_{arms_{in}} = 10kg\cdot(.15m)^2 = .225\,kgm^2$$

$$I_{armsout} = 10kg\cdot(.7m)^2 = 4.9\,kgm^2$$

7. Initially, skater is @ 3 rot/sec, $\omega_{bef} = 2\pi\cdot 3\,rot/sec$

$$L_{initial} = I_{out}\,\omega_{bef} = (I_{body} + I_{armsout})\,\omega = (.5 + 4.9)\,kgm^2\,(2\pi\cdot 3\,rot/s)$$

$L_{after} = I_{in}\,\omega_{after} = L_{before}$ (ang momentum conserved)

So $\omega_{after} = \frac{L_{bef}}{I_{in}} = \frac{(.5+4.9)\,kgm^2\,(2\pi\cdot 3\,rot/s)}{(.5+.225)\,kgm^2} = 2\pi\, f_{rot\,after}$

so, rotation frequency is $f_{rot\,after} = \frac{5.4}{.725}\cdot 3\,rot/s = \boxed{22.3\,rot/sec}$

(about 7.4x faster!)

8.

(E) $F = -\frac{GmM}{r^2} = -m\underbrace{r\omega^2}$

Gravity force law

centrip accel of circ motion

$$\omega = 2\pi f = \frac{2\pi}{28day}\left(\frac{1day}{24hr}\right)\frac{1hr}{3600sec} = 2.6\times10^{-6}\,rad/sec$$

So $\frac{G\cancel{m}M}{r^2} = \cancel{m}r\omega^2$

or $r^3 = \frac{GM}{\omega^2}$

$$\text{so } r = \left(\frac{GM}{\omega^2}\right)^{1/3} = \left[\frac{6.67\times10^{-11}\,Nm^2/kg^2\cdot 6\times10^{24}\,kg}{(2.6\times10^{-6})^2\,sec^{-2}}\right]^{1/3} = \boxed{3.9\times10^{8}\,m}$$