

Homework III

Session III.2

1. Remember that an object in freefall (e.g. a ball thrown through the air) is subject to constant acceleration of $-g$. If I throw a ball up in the air with an initial velocity straight up of 20 m/sec, how long does it take for it to reach the top of its flight, where the velocity is momentarily zero? What is the acceleration at this point?
2. Give an example where the velocity of an object is zero, but its acceleration is not. Give an example where the acceleration of an object is zero, but its velocity is not.
3. A car is rolling freely down a long, smooth hill, with an acceleration of 0.7 m/sec^2 . If the car is initially at rest ($v = 0$), how long does it take for the car to roll the 500 meters down the hill?

Session III.3

4. In this activity guide, it was suggested that a velocity function of the form

$$v = v_0 e^{\beta t}$$

would result from a position function

$$x(t) = x_{\text{final}} - \frac{mv_0}{\alpha} e^{\left(\frac{-\alpha}{m}t\right)}.$$

Take the time derivative of this position function and show that this assertion is in fact true.

For the following two questions, use your position-function-finding spreadsheet that you used to calculate the spring force example for the force function

$$F = -x^3.$$

Make sure to use a time step of 0.1 sec, as you should have done in class.

5. Show that this new force law gives rise to oscillatory motion, qualitatively like the spring case, by using initial conditions of $x = 1$ and $v = 0$.
6. Show that by changing the size of the oscillation one changes the amount of time for one complete cycle (return to the maximum positive position) by comparing the result for 2) with the result for $x = 0.5$ and $v = 0$ initial conditions. This is an important feature of force laws that have stable equilibria, but are not linear functions like Hooke's Law: the period of oscillation depends on the amplitude of the oscillation.

Session IV.1

7. A bowling ball (4 kg) is rolling down a bowling alley at 4 m/sec. It hits the single remaining pin (1 kg) head on for a spare! If the pin flies away at 5 m/sec, how much does the bowling ball slow down (that is, what is its velocity) after the collision?

8. Conservation of momentum is used heavily in forensic analysis of auto accidents. If a 800 kg car traveling at 20 m/sec runs into the rear end of a second car with mass of 600 kg which is moving only at 5 m/sec, and the two cars stick together after the accident, how fast is the combined wreckage moving? (One can apply conservation of momentum here because the forces between the cars in the collision is enormously higher than any other forces at that time, i.e. other forces are negligible.)