Physics 131
General Physics I
Grinnell College
Spring 2012

Summary of Units V & VI
Vectors & Motion in 2 and 3 dimensions

We developed a formalism for dealing with motion in 2 and 3 dimensions. In this system, we denote positions by x, y, [and z] coordinates, so a position $\vec{r}$ is given by

$$\vec{r} = (x, y) .$$

In a similar fashion, velocity, acceleration, and force are vector quantities given by components indicating that they have a directionality as will as a magnitude, such as

$$\vec{v} = (v_x, v_y) .$$

We can deduce the magnitude of a vector quantity from the components using the Pythagorean theorem, as in

$$|v| = \sqrt{\left(v_x^2 + v_y^2 + v_z^2\right)}$$

where we have assumed a velocity vector in this case in three-dimensional space. We can find the components (in two dimensions) if we know the magnitude of the vector and the angle relative to the x axis:

$$v_x = v\cos(\theta)$$

$$v_y = v\sin(\theta)$$

and we can invert this relationship to find that angle

$$\theta = \arctan\left(\frac{v_y}{v_x}\right).$$

Both force and acceleration are vector quantities, so Newton's second law becomes a vector equation, true in components:

$$F_x = ma_x$$

$$F_y = ma_y .$$

Since each equation is true separately, for constant force problems it is conventional to choose one of the directions to be in the direction of the constant force. This gives rise to uniformly accelerated motion in one direction, and constant velocity motion in the other. In any event, there are two separate problems that are now connected by the time. Typically, we solve one of these first for a time, insert that into the other solution to find some parameter of the motion in the other dimension.

In multiple dimensions, we discovered that momentum, being proportional to velocity, became a vector quantity giving a form to the conservation law of

$$p_{xbefore} = p_{xafter}$$

$$p_{ybefore} = p_{yafter}$$

where the direction is critically important. Energy, since velocity is squared is insensitive to direction, where we conserve the quantity

$$E_{total} = U + \frac{1}{2}m\left(v_x^2 + v_y^2 + v_z^2\right)$$

where U is the appropriate potential energy for the particular problem, and again we note that energy before is equal to energy after.

We discovered in the Harris Concert Hall that if we apply a force that is constant in magnitude but always perpendicular to the motion of an object, circular motion results. This can be expressed in x,y coordinates as

$$(x,y) = R(\cos\omega t, \sin\omega t).$$

Differentiation of this gives the velocity

$$(v_x, v_y) = R\omega(-\sin\omega t, \cos\omega t),$$

the magnitude of which is $|v| = R\omega$. Differentiating once more gives the acceleration, which is directed inward toward the center of the circle and therefore called the centripetal acceleration:

$$(a_x, a_y) = R\omega^2(-\cos\omega t, -\sin\omega t)$$

with magnitude $|a| = R\omega^2. = v^2/R$. This we can combine with Newton's second law to find an expression relating motion to the centripetal force

$$|F_{centripetal}| = mR\omega^2. = mv^2/R.$$

We extended the notions of circular motion to include motion where the angular velocity was increasing. Here we defined an angular acceleration

$$\alpha = d\omega/dt.$$

Considerations of conservation of energy led us to believe that equivalent efforts to rotate and object had equivalent torques, as defined by

$$\tau = r\,F$$

where r is the perpendicular distance from the line of force to the center of rotation. This we developed in analogy to linear motion:

	linear	**rotational**
	position, x	angle, θ
	velocity, v	angular velocity, ω
	acceleration, a	angular acceleration, α
	force, F	torque, τ
	momentum, p	angular momentum, $L=I\omega$
	mass, m	rotational inertia, I $I = \sum_i m_i r_i^2$
Newton's 2nd law	$F = ma$	$\tau = I\alpha$
Conservation	$\frac{dp}{dt} = 0$	$\frac{dL}{dt} = 0$
Constant acceleration	$x = \frac{1}{2}at^2 + v_0 t + x_0$ $v = at + v_0$	$\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$ $\omega = \alpha t + \omega_0$