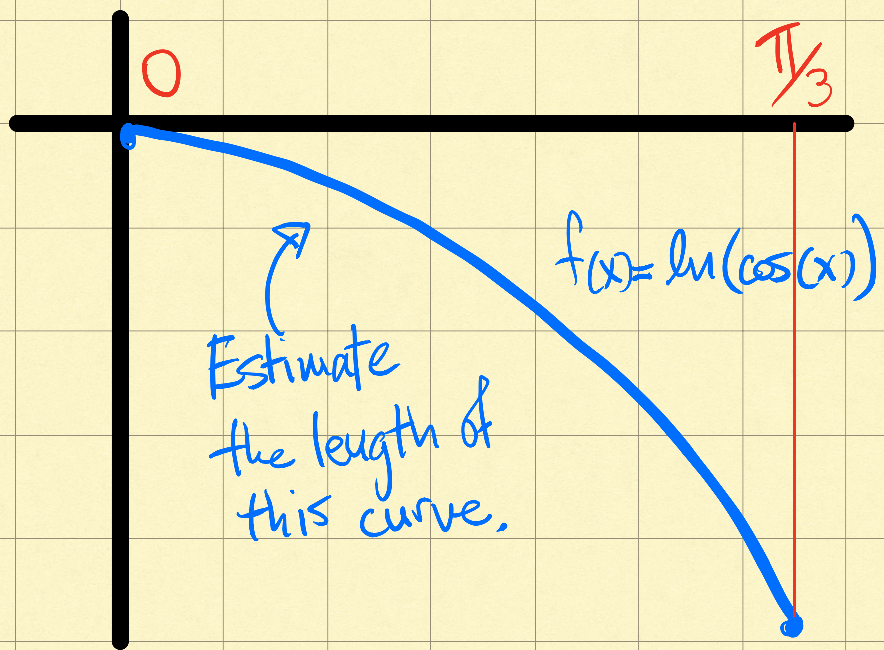
Warm-up exercise:

without using calculus, or your book:



① Review

② arc-length

③ parametric equations

Review

Integration Strategies:

1. u-sub (anti-chain rule)

trig sub

2. by parts (anti-product rule)
3. manipulate the integrand and apply 1 or 2
   a) trig ids
   b) partial fractions.

Choosing a strategy:

0. check your ref. sheet.
1. Is it a function of a function?
   - try u-sub

(2) Is it a rational function

A) Is the degree of the top $\geq$ deg. of bottom?

- long division

B) Check for u-sub

C) Do a partial fraction decomposition.

(3) Is it of the form

$$\int \sin^n(x)\cos^m(x)\,dx?$$

or

$$\int \tan^n(x)\sec^m(x)\,dx?$$

- use trig ids.

(4) Try using $\sin^2(\theta)+\cos^2(\theta)=1$ to do a trig sub.

⑤ Is it a product of functions?

- try by-parts

  tip: to choose du

  use DETAIL

  D v | E xp | T rig | A lg | I nvs trig | L og

⑥ None of these work.

- try identities
- comlete the square
- algebra
- more complicated u-sub
- hope.

Improper integrals:

Type I

one of the bounds is infinite

ex $\int_0^{\infty} f(x)\,dx$ or $\int_{-\infty}^{0} f(x)\,dx$.

Type II

one of the bounds is a vert. asymptote.

ex. $\int_0^1 \frac{1}{\sqrt{x}}\,dx$ or $\int_1^{\sqrt{2}} \frac{1}{x^2-2}\,dx$

In either case, we use limits to replace the "problem" bound.

ex. $\int_1^{\infty} \frac{1}{x^2}\,dx$

$$\overset{\text{def}}{=} \lim_{b\to\infty} \int_1^b \frac{1}{x^2}\,dx$$

$$\overset{\text{F.T.C.}}{=} \lim_{b\to\infty} \left(-\frac{1}{x}\right)\Big|_1^a$$

$$= \lim_{b\to\infty} \left(\underbrace{-\frac{1}{b}}_{\downarrow\, 0} + \underbrace{1}_{\downarrow\, 1}\right) = 1.$$

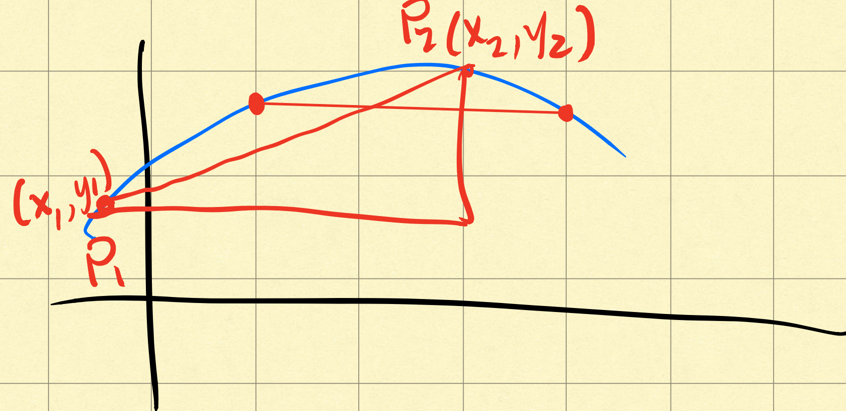
Approximating integrals:

- midpoints
- trapezoids
- Simpson's

$$\int_1^4 \frac{1}{\sqrt{x^3+1}}\,dx$$

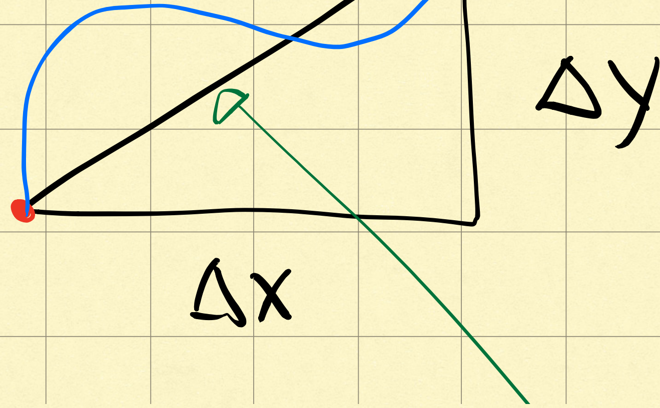
Arc length

Goal: measure the length of curves.



$$d(P_1,P_2)=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

$$=\sqrt{\Delta x^2+\Delta y^2}$$



Δx

$$\Delta y = f(x_1) - f(x_2).$$

slope $= \dfrac{\Delta y}{\Delta x}$

using MVT there is an x^* w/ $f'(x^*) = \dfrac{\Delta y}{\Delta x}$

$\Rightarrow \Delta x \cdot f'(x^*) = \Delta y$.

$$\sqrt{\Delta x^2 + \Delta y^2}$$

$$= \sqrt{\Delta x^2 + (\Delta x \cdot f'(x^*))^2}$$

$$= \sqrt{\Delta x^2 + \Delta x^2 (f'(x^*))^2}$$

$$= \left(\sqrt{1 + (f'(x^*))^2}\right) \cdot \Delta x$$

Arc length of $f(x)$ from $x=a$ to $x=b$ $= \lim_{n\to\infty} \sum_{k=1}^{n} \left(1 + (f'(x_k^*))^2\right)^{\frac{1}{2}} \Delta x_k$

$$= \int_a^b \left(1 + (f'(x))^2\right)^{\frac{1}{2}} dx$$

Find the arc length of

$\ln(\cos(x))$ from $x=0$ to $x=\frac{\pi}{3}$

$$\int_0^{\pi/3} \left(\sqrt{1+\left(\left(\ln(\cos(x))\right)'\right)^2}\; dx\right.$$

$$= \int_0^{\pi/3} \left(1+\left(\frac{-\sin(x)}{\cos(x)}\right)^2\right)^{\frac{1}{2}} dx$$ by d-ref #2 chain rule

$$= \int_0^{\pi/3} \left(1+\tan^2(x)\right)^{\frac{1}{2}} dx$$

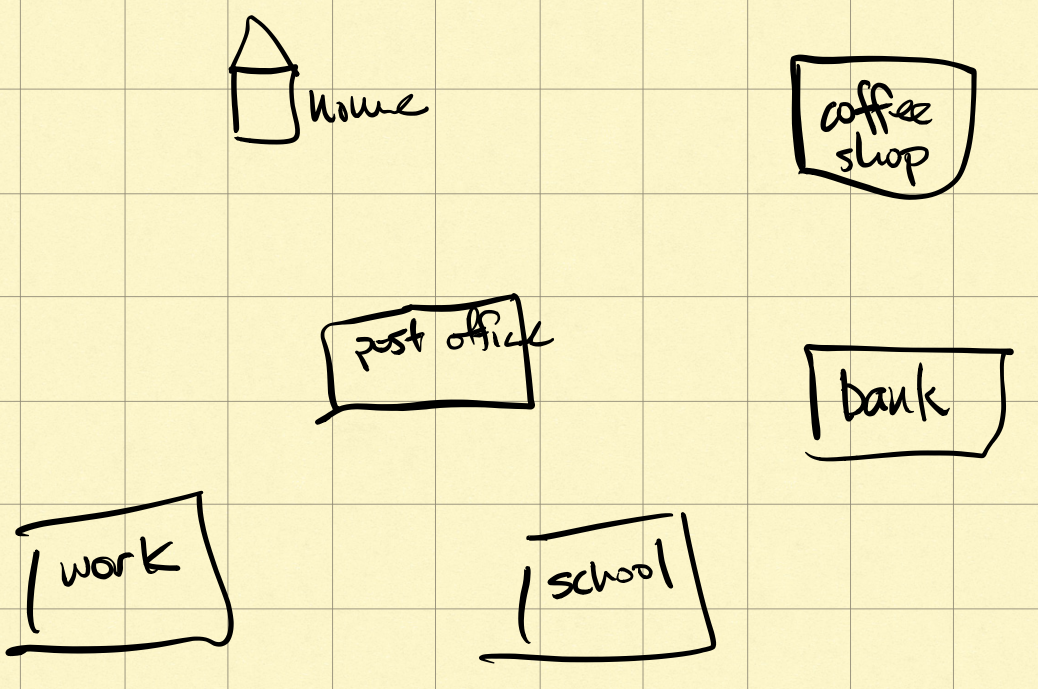
$$= \int_0^{\pi/3} \left(\sec^2(x)\right)^{\frac{1}{2}} dx$$ trig id. 2

$$= \int_0^{\pi/3} \sec(x)\,dx$$

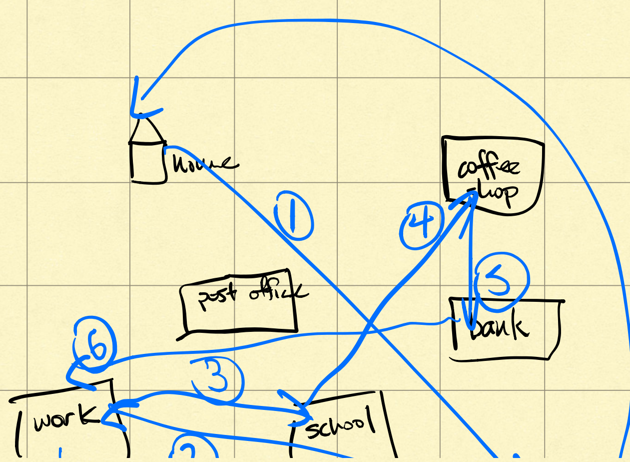
$$= \ln\left|\sec(x)+\tan(x)\right| \Big|_0^{\pi/3}$$ by d-ref 14 and FTC

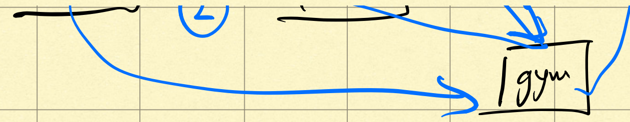
$$= \ln\left|2+\sqrt{3}\right| - \ln|1| = \ln\left|2+\sqrt{3}\right| \approx 1.317$$

Parametric Equation:



time	location
0200	home
0300	gym
0500	work
0700	school
0900	coffee
1100	bank





1300	work
1500	gym
1700	home
⋮	⋮

time	location	x	y
0200	home		
0300	gym		
0500	work		
0700	school		
0900	coffee		
1100	bank		
1300	work		
1500	gym		
1700	home		
⋮	⋮		

ex. $\left(\underbrace{t^2+t+1}_{x(t)},\ \underbrace{t\cdot\sin(t)}_{y(t)}\right)$

t	x(t)	y(t)
⋮	⋮	

any function can be used to make a parametric function

$y = f(x) \longrightarrow (x, f(x)) = (x(t), y(t))$

or let $x(t) = t$ $y(t) = f(x(t))$

everyone's favorite parametric function $(\cos(\theta), \sin(\theta))$

(the unit circle, parameterized by θ)