Physics 131 – HW-IV – Solutions

1. Before we learned cons. of energy, we found $\frac{v_i^2}{2} + gx_i = \frac{v_f^2}{2} + gx_f$

or $$\frac{v_i^2 - v_f^2}{2} = g(x_f - x_i) = g\Delta x$$

So, if $v_i = 50\,m/s$, & $v_f = 0$, & $g \approx 10\,m/s^2$, then

$$\Delta x = \frac{v_i^2}{2} \cdot \frac{1}{g} = \frac{(50\,m/s)^2}{2 \cdot 10\,m/s^2} = \boxed{125\,m!}$$ ⇐ Plenty high (about 40 stories!)

I doubt anyone can actually throw up this fast, and at this speed air resistance will be considerable & reduce the real value.

2. $F = -200N$ $\quad \Delta x = 20\,m$

↑ force opposes motion, so neg. sign.

$$\text{Work} = \int F\,dx \underset{\uparrow\text{ if F is constant}}{=} F\,\Delta x = -4000 \text{ Joules}$$

Neg sign means I lose kinetic energy.

3. Work energy theorem – $W = \Delta K$ ← kinetic energy

$$K_{bef} = \tfrac{1}{2} m v_b^2 = \tfrac{1}{2} \cdot 70kg \cdot (15\,m/s)^2 = 7875\,J$$

So $K_{aft} = K_{bef} + W = (7875 - 4000)\,J = 3875\,J$

$$K_{aft} = \tfrac{1}{2} m v_a^2 \Rightarrow v_a = \sqrt{\frac{2K_a}{m}} = \sqrt{\frac{2 \cdot 3875}{70}}\ m/s = 10.5\ m/sec$$

4. $F = mg\sin(10°)$, so $a = \frac{F}{m} = g\sin(10°)$

In session 2, we used the constant acceleration formula

$$\frac{v_f^2 - v_i^2}{2} = a\,\Delta x$$

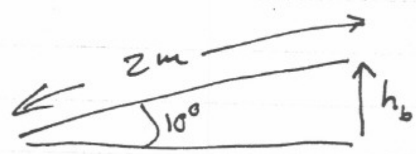
so if $v_i = 0$,

$$\frac{v_f^2}{2} = g\sin(10°) \cdot 2m$$

$$v_f = \sqrt{4 \cdot 10 \frac{m^2}{sec^2} \cdot \sin 10°} = \sqrt{40 \sin 10°}\ \frac{m}{s} = 2.6\ m/sec$$

5. Using conservation of energy,

$$E_{bef} = E_{aft}$$



$$\cancel{\tfrac{1}{2} m v_b^2}^{\,0} + mgh_b = \tfrac{1}{2} m v_a^2 + \cancel{mgh_a}^{\,0}$$

so $\cancel{m}gh_b = \frac{1}{2}\cancel{m}v_a^2$

$$v_a = \sqrt{2gh_b}$$

$h_b = 2m \cdot \sin 10°$, so $v_a = \sqrt{2 \cdot 10 m/s^2 \cdot 2m \cdot \sin 10°}$

$$= \sqrt{40 \sin 10°}\ m/s = 2.6\ m/sec$$

6. $E_{bef} = E_{after}$

$$\frac{1}{2} m v_b^2 + \cancel{\frac{1}{2} k x_b^2}^{\,0} = \frac{1}{2} \cancel{m v_a^2}^{\,0} + \frac{1}{2} k x_a^2$$

zero velocity at max position

$$\frac{1}{2}(.3\text{kg})(2\,\text{m/s})^2 = \frac{1}{2}(50\,\text{N/m})(x_a)^2$$

$$x_a^2 = \frac{.3\text{kg} \cdot 4\,\text{m}^2/\text{s}^2}{50\,\text{N/m}} = .024\ \text{m}^2$$

$$\boxed{x_a = .155\,\text{m}}$$

7. (2kg) → 5 m/s   (1kg)

before

← (2kg) 2.5 m/s   (1kg) → 15 m/s

after

$$P_{bef} = m_1 v_1 + m_2 v_2$$
$$= 2\text{kg} \cdot 5\,\text{m/s} + 0$$
$$= 10\,\text{kg m/s}$$

$$P_{after} = 2\text{kg}(-2.5\,\text{m/s}) + 1\text{kg} \cdot 15\,\text{m/s}$$
$$= -5\,\text{kg m/s} + 15\,\text{kg m/s}$$
$$= 10\,\text{kg m/s} \implies \text{yes! } p \text{ conserved}$$

$$E_{bef} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$
$$= \frac{1}{2} \cdot 2\text{kg} \cdot (5\,\text{m/s})^2 + 0$$
$$= 25\,\text{J}$$

$$E_{after} = \frac{1}{2}\, 2\text{kg}(-2.5\,\text{m/s})^2 + \frac{1}{2}\, 1\text{kg} \cdot (15\,\text{m/s})^2$$
$$= 6.25\,\text{J} + 112.5\,\text{J} = 118.75\,\text{J}$$

Energy is not conserved – in fact it increased! This is bizarre – like a bomb went off in one of the balls to release some energy.

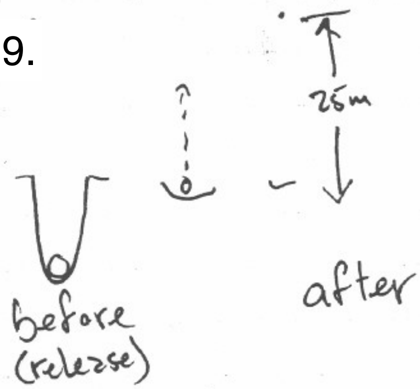
8. For energy conservation, we require that for a given system, F is always the same each time we return to any given point. So, if F depends on v, this is not true in general.

So b) & c) do <u>not</u> conserve energy, since F depends on v in addition to x. For a) Energy is conserved, and so

$$U = -\int F\,dx = -\int -x^3\,dx = \frac{x^4}{4}.$$

(One could also include an arbitrary constant.)

9.



a) $E_{before} = E_{after}$

$$\cancel{K_b}^{0} + U_b = \cancel{K_a}^{0} + U_a$$ (ball at rest at release & at top)

$$mgh_b + \tfrac{1}{2}kx_b^2 = mgh_a + \tfrac{1}{2}\cancel{kx_a^2}^{0}$$ spring is relaxed

$$\tfrac{1}{2}kx_b^2 = mg(h_a - h_b)$$

$$k = \frac{2mg(h_a-h_b)}{x_b^2} = \frac{2(.05\,kg)(10\,m/s^2)(25\,m)}{(-.3\,m)^2}$$

$$k = 278\ N/m$$

How do we know E conserved? Only forces both spring & gravity involved – both conserve energy.

b)

$$F = -kx = -(278\ N/m)(-.3\,m) = 83\,N$$

10. 

This is a collision — with no external forces, so we expect conservation of momentum.

$$P_{bef} = P_{aft}$$

$$m_1 v_{1b} + m_2 v_{2b} = m_1 v_{1a} + m_2 v_{2a}$$

$v_{1a} = v_{2a} \equiv v_a$
(I catch the book)

$$= (m_1 + m_2) v_a$$

$$(70\,kg)(5\,m/s) + 1\,kg \cdot (-1\,m/s) = (71\,kg) v_a$$

$$(350 - 1)\,kg\,m/s = 71\,kg\; v_a$$

$$v_a = \frac{349}{71}\,m/s = 4.915$$

(so, I slow by only $0.085\,m/s$)

$$E_b = \frac{1}{2} m_1 v_{1b}^2 + \frac{1}{2} m_2 v_{2b}^2 = \left(\frac{70 \cdot 25}{2} + \frac{1}{2}\right) J = 875.5\,J$$

$$E_a = \frac{1}{2}(m_1 + m_2) v_a^2 = \frac{71 \cdot 4.915^2}{2} = 857.6\,J$$

So about 18 J are lost. Notice this is much more than the kinetic energy originally of the book — only ½ J there!