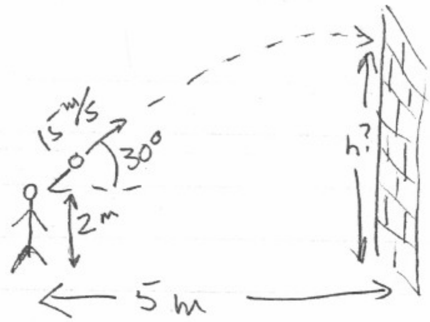
PHY131 HW Vb Solutions

1



$v_{ox} = 15\,\text{m/s} \cdot \cos 30° = 13\,\text{m/sec}$

$v_{oy} = 15\,\text{m/s} \cdot \sin 30° = 7.5\,\text{m/sec}$

x component of motion

$x(t) = v_{ox} t + \cancel{x_0}^{\,0}$

$= 13\,\text{m/s} \cdot t$

y component of motion

$y(t) = \frac{1}{2} a t^2 + v_{oy} t + y_0$

$= -5\,\text{m/s}^2\, t^2 + 7.5\,\text{m/s}\, t + 2\text{m}$

Use x equation to find <u>when</u>:

$$x(t_{hit}) = 5\text{m} = 13\,\text{m/s} \cdot t_{hit} \Rightarrow t_{hit} = \frac{5\text{m}}{13\,\text{m/s}} = 0.385\,\text{sec}$$

Use y equation to find how high:

$$h = y(t_{hit}) = -5\,\text{m/s}^2 (0.385\,\text{sec})^2 + 7.5\,\text{m/s}\,(0.385\,\text{sec}) + 2\text{m}$$

$$= -.74\text{m} + 2.88\text{m} + 2\text{m} = \boxed{4.14\text{m}}$$

2 $x(t) = 3\sin(4t)$ $y(t) = 3\cos(4t)$

$v_x(t) = \frac{dx}{dt}$ $v_y = \frac{dy}{dt}$

$\boxed{= 12\cos(4t)}$ $\boxed{= -12\sin(4t)}$

$$|v| = \sqrt{v_x^2 + v_y^2} = \sqrt{144\cos^2(4t) + 144\sin^2(4t)} = \sqrt{144\underbrace{(\cos^2(4t) + \sin^2(4t))}_{=1}} = \sqrt{144} = \boxed{12}$$

speed — magnitude of $\vec{v}$

This is the parametrization of a circle of radius 3.

3 $x(t) = 3\sin(\omega t)$ $\qquad$ $y(t) = 4\cos(\omega t)$

(Note - because x & y motions have different amplitudes, this path is <u>not</u> a circle - it is an ellipse. The speed is <u>not</u> constant here like it was in 8))

$$r^2 = x^2 + y^2 \quad \text{(Pythagoras)}$$

$$\boxed{= 9\sin^2(\omega t) + 16\cos^2(\omega t)}$$

[If you wish, you could use $\cos^2 + \sin^2 = 1$ to write as $r^2 = 9 + 7\cos^2(\omega t)$]

$$\vec{v} = \left(\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}\right) = \left(3\omega\cos(\omega t), -4\omega\sin(\omega t)\right)$$

so $v^2 = v_x^2 + v_y^2$ (Pythagoras)

$$\boxed{= 9\omega^2\cos^2(\omega t) + 16\omega^2\sin^2\omega t}$$

[If you wish, this could be written as $v^2 = 9\omega^2 + 7\omega^2\sin^2\omega t$]

4 $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(9\omega^2\cos^2\omega t + 16\omega^2\sin^2\omega t\right) = \frac{1}{2}m\omega^2\left(9\cos^2\omega t + 16\sin^2\omega t\right)$

using $\omega^2 = \frac{k}{m}$ $\quad = \frac{1}{2}k\left(9\cos^2\omega t + 16\sin^2\omega t\right)$

$$U = \frac{1}{2}kr^2 = \frac{1}{2}k\left(9\sin^2(\omega t) + 16\cos^2(\omega t)\right)$$

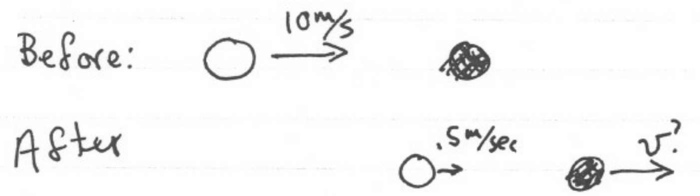
Adding $K + U$ gives:

$$E = U + K = \frac{1}{2}k\left[9\cos^2\omega t + 16\sin^2\omega t + 9\sin^2\omega t + 16\cos^2\omega t\right]$$

$$= \frac{1}{2}k\left[9\underbrace{\left(\cos^2\omega t + \sin^2\omega t\right)}_{1} + 16\underbrace{\left(\sin^2\omega t + \cos^2\omega t\right)}_{1}\right] = \frac{1}{2}k\left[9+16\right] = \boxed{\frac{25}{2}k}$$

This is just a constant – <u>total</u> energy does not change with time $\Rightarrow$ conserved

5



Same mass for both balls

We assume no external forces, so momentum is conserved—
assume all in line:

$$P_{bef} = P_{aft}$$

$$m v_{cb} + \cancel{m v_{8b}}^{\,0} = m v_{ca} + m v_{8a} \qquad \text{Divide by } m$$

$$10 \, \text{m/s} = .5 \, \text{m/s} + v_{8a} \Rightarrow \boxed{v_{8a} = 9.5 \, \text{m/s}}$$

How much KE lost?

$$KE_{bef} = \tfrac{1}{2} m v_{cb}^2 + \tfrac{1}{2} m \cancel{v_{8b}^2}^{\,0} = \tfrac{1}{2} m (10 \, \text{m/s})^2 = 50 \cdot m \, \frac{m^2}{s^2}$$

$$KE_{aft} = \tfrac{1}{2} m v_{ca}^2 + \tfrac{1}{2} m v_{8a}^2 = \tfrac{1}{2} m \left[(.5 \, \text{m/s})^2 + (9.5 \, \text{m/s})^2 \right]$$

$$= 45.25 \cdot m \, \frac{m^2}{s^2}$$

So 90.5% of KE remains $\Rightarrow$ $\boxed{9.5\% \text{ lost}}$

6 We assumed above that the 8 ball was moving in same direction as initial motion — that is, if we call the initial & final cue ball direction the x direction, there is no y direction component to $\vec{v}_{8a}$. Let's use cons of $\vec{p}$ to show this. In particular, $\vec{p}$ is conserved by components, so

$$P_{y\,bef} = P_{y\,aft}$$

$$\cancel{m v_{cby}}^{\,0} + \cancel{m v_{8by}}^{\,0} = \cancel{m v_{cay}}^{\,0} + m v_{8ay} \qquad \text{divide by } m$$

$$\boxed{0 = v_{8ay}} \Leftarrow \text{motion must be all in } x \text{ direction}$$

Hint! Lot's of subscripts on v's are essential for 2d momentum problems!

7 Now, a collision w/ both x & y components. First, momentum

$$\vec{P}_{bef} = \vec{P}_{aft}$$

x component	y component
$P_{cbx} + P_{8bx} = P_{cax} + P_{8ax}$	$P_{cby} + P_{8by} = P_{cay} + P_{8ay}$
$m\cancel{v_{cbx}}^{\,0} + m\cancel{v_{8bx}}^{\,0} = mv_{cax} + mv_{8ax}$	$m\underbrace{v_{cby}}_{1\,m/s} + m\cancel{v_{8by}}^{\,0} = mv_{cay} + mv_{8ay}$

Divide out m's

$0 = v_{cax} + v_{8ax}$	$1\,m/s = v_{cay} + v_{8ay}$

We know angle of cue after is 45°, so

$$v_{cax} = v_{ca}\cos 45° = .707\,v_{ca}$$

$$v_{cay} = \underbrace{v_{ca}}_{\text{total speed}}\sin 45° = .707\,v_{ca}$$

$$\boxed{v_{8ax} = -v_{cax} = -.707\,v_{ca}} \qquad \boxed{1\,m/s = .707\,v_{ca} + v_{8ay}}$$

So, now we have 2 equations & 3 unknowns (v_{ca}, v_{8ax}, v_{8ay}).

We can only solve this with a third equation — in this case — conservation of KE:

Recall, if we have $\vec{v}$ by components, KE is

$$\frac{1}{2}mv^2 = \frac{1}{2}m\Big(\underbrace{\sqrt{v_x^2+v_y^2}}_{\text{Pythagoras}}\Big)^2 = \frac{1}{2}m\left(v_x^2+v_y^2\right), \text{ so}$$

$$KE_{bef} = KE_{aft} \qquad \text{(by assumption of problem)}$$

$$\frac{1}{2}m\left(\cancel{v_{cbx}^2}^{\,0} + v_{cby}^2\right) + \frac{1}{2}m\left(\cancel{v_{8bx}^2}^{\,0} + \cancel{v_{8by}^2}^{\,0}\right) = \frac{1}{2}m\left(v_{cax}^2 + v_{cay}^2\right) + \frac{1}{2}m\left(v_{8ax}^2 + v_{8ay}^2\right)$$

Divide out $\frac{1}{2}m$, replace v_{cax} & v_{cay} w/ $.707\,v_{ca}$:

$$\underset{\overset{\uparrow}{(1\,m/s)^2}}{v_{cby}^2} = \underbrace{(.707 v_{ca})^2 + (.707 v_{ca})^2}_{= v_{ca}^2} + v_{gax}^2 + v_{gay}^2$$

$$\boxed{1\,m^2/s^2 = v_{ca}^2 + v_{gax}^2 + v_{gay}^2} \Leftarrow \text{our third equation.}$$

Substitute $-.707 v_{ca}$ for v_{gax} to get:

$$1\,m^2/s^2 = v_{ca}^2 + (-.707 v_{ca})^2 + v_{gay}^2 = 1.5\, v_{ca}^2 + v_{gay}^2$$

Now, use other result (p_y conservation)

$$(1\,m/s - .707 v_{ca}) = v_{gay} \quad \text{to get}$$

$$1\,m^2/s^2 = 1.5 v_{ca}^2 + (1\,m/s - .707\, v_{ca})^2$$

$$1\,m^2/s^2 = 1.5 v_{ca}^2 + 1\,m^2/s^2 - 2 \times .707 v_{ca} \times 1\,m/s + .5\, v_{ca}^2$$

$$0 = 2 v_{ca}^2 - 1.414\,m/s \cdot v_{ca}$$

Divide by v_{ca} (we know $v_{ca} \neq 0$)

$$0 = 2 v_{ca} - 1.414\,m/s \Rightarrow v_{ca} = \frac{1.414\,m/s}{2} = .707\,m/s$$

so $v_{cax} = v_{ca} \cos 45° = .707\,m/s \cdot .707 = \boxed{0.5\,m/s}$

$v_{cay} = v_{ca} \sin 45° = .707\,m/s \cdot .707 = \boxed{0.5\,m/s}$

$v_{gay} = 1\,m/s - .707 v_{ca} = 1\,m/s - .707\,m/s \cdot .707 = 1\,m/s - .5\,m/s = \boxed{0.5\,m/s}$

$v_{gax} = -v_{cax} = \boxed{-0.5\,m/s}$