MAT133 Exam 1

Olek Yardas

TOTAL POINTS

68 / 90

QUESTION 1

1 CC1 11 / 12

- **+ 3.5 Stated the integration by parts formula. (formal)**
- + 2.5 Stated the integration by parts formula. (informal)
- + 2.5 Stated the product rule (formal)
- **+ 2 Stated the product rule (informal)**
- **+ 4 Described the relationship between the product rule and integration by parts. (formal)**
- + 2 Described the relationship between the product rule and integration by parts. (informal)
- + 3 Described when or why this strategy might useful. (some depth)
- + 3 Illustrated with an example. (formal, correct, detailed)
- + 2 Illustrated with an example. (formal, correct, less detailed)
- + 1 Illustrated with an example. (mostly correct, missing some detail)
- **+ 1.5 Well organized and clear exposition.**
- - 0.5 Missing "dx" in multiple instances.
- + 1 Illustrated with an example. (formal, incorrect)
- + 0 Illustrated with an example. (informal, incorrect)

QUESTION 2

2 CC2 5 / 12

- + 3.5 Stated definition of Type I. (formal or mostly formal)
- **+ 2 Stated definition of Type I. (informal)**
- + 3.5 Stated definition of Type II. (formal or mostly formal)
- + 2 Stated definition of Type II. (informal)
- + 2 Illustration of Type I. (detailed)
- + 1 Illustration of Type I. (minimal or informal)
- + 2 Illustration of Type II. (detailed)
- + 1 Illustration of Type II. (minimal or informal)
- **+ 1.5 Well organized and clear exposition.**
- + 3 Illustration of a combination of both types. (detailed)
- + 1.5 Illustration of a combination of both types. (minimal)
- + 3.5 Statement of Comparison Theorem. (formal)
- + 2 Statement of Comparison Theorem. (informal)
- + 2 Illustration of Comparison Theorem. (detailed)
- + 1 Illustration of Comparison Theorem. (minimal)
- - 0.5 Illustration or example contains an error.
- + 4 General discussion of the motivation behind defining improper integrals. (detailed, formal)
- + 3 General discussion of the motivation behind defining improper integrals. (detailed, informal)
- **+ 1.5 Some mention of the motivation behind defining improper integrals. (minimal, informal)**

QUESTION 3

3 CC3 10 / 12

- + 5 Statement of Arc Length Formula. (formal, equivalent to book definition)
- **+ 3.5 Statement of Arc Length Formula. (informal, formula only)**
- + 7 Derivation of Arc Length Formula. (full detail, formal)
- **+ 6 Derivation of Arc Length Formula. (some minor steps omitted or unjustified, mostly formal)**
- + 5 Discussion of derivation of the arc length formula. (less formal, includes pictorial illustration, mentions connection to distance formula and application of the Mean Value Theorem)
- + 2 Some discussion of the derivation or motivation behind the arc length formula. (informal, mentions distance formula and some other details)
- + 4 Illustration with an example. (formal, detailed)

- **+ 3** Illustration with an example. (formal, minimal)
- **+ 2** Illustration with an example. (informal or incomplete)
- **+ 1.5 Well organized and clear exposition.**
- **+ 0** Omitted, or not enough meaningful discussion to score.
- **- 1 Point adjustment**

  The (b-a) term in your derivation is your change in x, and should have disappeared at some point.

QUESTION 4

4 ES1 7 / 8

- **- 0** Correct, detailed, solution.
- **- 1.5** Final solution not reached.
- **- 2** Error followed through to a solution, despite having fundamentally altered the nature of the problem.
- **- 0.5** Failed to mention integration by parts when applied.
- **- 0.5 Minor calculation or notation error.**
- **- 0.5** Failed to properly apply the fundamental theorem of calculus to evaluate definite integrals. (Or, omitted evaluation bounds from intermediate work.)
- **- 0.5** Failed to include the constant of integration when calculating indefinite integrals.
- **- 0.5** Failed to include "dx" when expressing an integral.
- **- 2** Erroneous application of improper integrals or other reasoning to claim that the integral diverges.
- **- 0.5** Failed to include limit notation when required.
- **- 0.5 Unnecessary application of improper integral techniques.**

QUESTION 5

5 ES2 6.5 / 8

- **- 0** Correct, detailed, solution
- **- 1.5** Final solution not reached.
- **- 2** Error followed through to a solution, despite having fundamentally altered the nature of the problem.
- **- 1.5** Failed to conclude. explicitly enough, that the integral diverges because the natural logarithm diverges to negative infinity as the inputs approach zero. (Or, failed to recognize that the logarithm function is undefined at zero.)
- **- 0.5** Incorrect application of partial fraction decomposition.
- **- 0.5** Minor calculation or notation error.
- **- 0.5** Failed to properly apply the fundamental theorem of calculus to evaluate definite integrals. (Or, omitted evaluation bounds from intermediate work.)
- **- 0.5** Failed to include the constant of integration when calculating indefinite integrals.
- **- 0.5** Failed to include "dx" when expressing an integral.
- **- 0.5 Failed to include limit notation when required.**
- **- 0.5** Improper use of mathematical notation or operators.
- **- 1 Point adjustment**

  Right after correctly stating that logarithms approach negative infinity as their inputs approach zero, you state that logarithms approach negative infinity as their inputs approach negative five. Only one of these is true.

  When you justify a step by referencing the fundamental theorem of calculus, it must apply.

QUESTION 6

6 ES3 7.5 / 8

- **- 0** Correct, detailed, solution.
- **- 1.5** Failed to reach a final solution.
- **- 2** Error followed through to a solution, despite having fundamentally altered the nature of the problem.
- **- 2** Incorrect application of Trigonometric Substitution. (failed to complete the square before applying)
- **- 1** Incorrect application of completing the square.
- **- 0.5** Incorrect "unsubstitution."
- **- 0.5 Minor calculation or notation error.**

- **0.5** Failed to include the constant of integration when calculating indefinite integrals.
- **0.5** Failed to include "dx" when expressing an integral.
- **0.5** Improper use of mathematical notation or operators.
- **3** Improper use of mathematical notation or operators. (Major, fundamental, errors)
- **8** Omitted, or not enough correct work to score.

QUESTION 7

7 ES4 8 / 8

- **0 Correct, detailed, solution.**
- **1.5** Failed to reach a final solution.
- **2** Error followed through to a solution, despite having fundamentally altered the nature of the problem.
- **0.5** Minor calculation or notation error.
- **0.5** Failed to include "dx" when expressing an integral.
- **0.5** Improper use of mathematical notation or operators.
- **3** Improper use of mathematical notation or operators. (Major, fundamental, errors)
- **8** Omitted, or not enough correct work to score.

QUESTION 8

8 ES5 6.5 / 8

+ **4** Correct Cartesian equation found, with correct restrictions on x and y given.
+ **3** Correct Cartesian equation found, without correct restrictions on x and y given.
+ **2.5 Cartesian equation found, but it only describes part of the curve. (Check the domain and range of composed functions.)**
+ **2.5 Sketch is the correct shape, and obeys the restrictions on theta.**
+ **2** Sketch is the correct shape, but does not obey the restrictions on theta.
+ **1** Sketch included, and related to work, but is not well labeled enough to determine if it is correct.
+ **1.5 Direction indicated is correct, or follows from shown work.**
+ **2** Detailed, correct, table of points included.
+ **0.5** Correct table of points included, not enough to determine a complete sketch.
+ **0** Omitted, or not enough correct work to score.

QUESTION 9

9 IS1 0.5 / 5

- **0** Correct, detailed, solution.
- **1** Must explicitly summarize the entire result of Example 4, or explicitly compute the specific integral with which you are dealing, whose convergence or divergence follows from this result.
- **0.5** Did not reference the Comparison Theorem when applied.
- **1** Major calculation or notation error.
- **0.5** Improper use of mathematical notation or operators.
- **0.5** Failed to include "dx" when expressing an integral.
- **1.5** Failed to reach a final solution.
- **5 Omitted, or not enough correct work to score.**
- **2** Error followed through to a solution, despite having fundamentally altered the nature of the problem.
- **0** Click here to replace this description.

+ **0.5 Point adjustment**

  Applied the definition of Type I Improper Integral. (+0.5)

QUESTION 10

10 IS2 0 / 5

- **0** Correct, detailed, solution.
- **2** Error followed through to a solution, despite having fundamentally altered the nature of the problem.
- **3** Improper use of mathematical notation or operators. (Major, fundamental, errors)
- **0.5** Minor calculation or notation error.
- **5 Omitted, or not enough correct work to score.**

+ **2** Meaningful effort towards a solution and clear explanation of why the attempted strategy failed. (+2)

QUESTION 11

11 AS1 0 / 4

- **- 0** Correct, detailed, solution.
- **- 4 Omitted, or not enough correct work to score.**

QUESTION 12

12 TF1 0 / 2

- **- 2 Incorrect - This statement is FALSE because the degree of the numerator is larger than the denominator.**
- **- 0** Correct - This statement is FALSE because the degree of the numerator is larger than the denominator.

QUESTION 13

13 TF2 2 / 2

- **- 2** Incorrect - This statement is FALSE.
- **- 0 Correct - This statement is FALSE.**

QUESTION 14

14 TF3 2 / 2

- **- 0 Correct - This statement is TRUE. See Section 8.8.**
- **- 2** Incorrect - This statement is TRUE. See Section 8.8.

QUESTION 15

15 TF4 2 / 2

- **- 0 Correct - This statement is TRUE. See Section 9.1.**
- **- 2** Incorrect - This statement is TRUE. See Section 9.1.

QUESTION 16

16 TF5 0 / 2

- **- 2 Incorrect - This statement is FALSE. While it is usually true, we can construct situations in which it is not, especially for small values of n. A function that is constant on an interval except for very sharp spikes, very near the end, for example.**
- **- 0** Correct - This statement is FALSE. While it is usually true, we can construct situations in which it is not, especially for small values of n. A function that is constant on an interval except for very sharp spikes, very near the end, for example.

QUESTION 17

17 Bonus 0 / -10

- **+ 0 Not attempted, or not enough progress to score.**
- **+ 2** Recognized a connection with improper integrals.

Midterm Exam 1

MAT133 - Calculus II

2/17/2017

Form A

Instructions

Do not start until you are told to do so.

Please, turn off your phone and secure it in your bag. Leave your bags, pencil cases, calculators, notes, books, jackets, hats, food, and other belongings at the front of the classroom. You are permitted to have a transparent drink bottle, pens, pencils, and erasers at your desk. Please, plan to stay in the classroom for the entire duration of the exam.

You will have 80 minutes to complete this exam. Read all instructions carefully. Your responses to all items on this exam must be your own. No outside references, notes, calculators, or other aides are permitted. As it is crowded, please, refrain from glancing at the papers of those around you, and take care that your work is protected. A reference sheet, and pages for scratch work can be found attached to the end of the exam, you may detach these pages if you like. Do not detach any other pages from the exam.

The exam sections are weighted as follows:

- 36 points - Concept Check
- 40 points - Essential Skills
- 10 points - Intermediate Skills
- 4 points - Advanced Skills
- 10 points - True/False Statements
- 5 points - Bonus

Within each section, all problems are weighted the same.

If you find yourself unable to finish a question, do your best to describe your attempts and reasoning. Partial credit may be awarded for demonstrating meaningful effort towards a solution.

Raise your hand if you have any questions, or require clarification of any instructions, during the exam. Good luck!

Clearly print your name in the box below. Do not write your name in any other location unless you are submitting page(s), not attached to the rest of your exam, containing work that you want scored.

Name: Oleksandr Yardas

Leave this page blank.

1 Concept Check (36 points)

Your responses in this section should showcase what you understand about the emphasized concepts. You are encouraged to include any relevant definitions, illustrations, examples, motivation, or other insights that demonstrate your grasp on the topics in question. Your response to each item should be neat, legible, and not exceed the space provided.

1.1

Discuss *integration by parts.*

Integration by parts is a method of integrating a function using the Fundamental Theorem of Calculus and the Product rule for differentiation.

FTOC states that for a function f, $\int_a^b f\,dx = F(b) - F(a)$ where F is an anti derivative of f

The Product rule states that for functions f and g, where $S = f \cdot g$, $\frac{d}{dx} S = \frac{d}{dx}(f \cdot g) = \left(\frac{d}{dx} f\right) \cdot g + f \cdot \left(\frac{d}{dx} g\right)$

If we integrate the product rule with respect to x, we ... we

$$\int \frac{d}{dx}(f \cdot g)\,dx = \int \left(\frac{d}{dx} f\right) \cdot g\,dx + \int f \cdot \left(\frac{d}{dx} g\right) dx$$

$$\Rightarrow f \cdot g = \int f' g\,dx + \int f \cdot g'\,dx$$

By FTOC,

$$\Rightarrow \int f \cdot g'\,dx = f \cdot g - \int f' \cdot g\,dx$$

which is the definiton for integration by parts.

Put an "X" in this box if work, for this problem, continues on the back of this page.

1.2

Discuss *improper integrals.*

Improper integrals are definite integrals where the bounds are infinite or make the value of the function being integrated approach infinity.

We can use limits to help evaluate improper integrals. If we have an integral $\int_0^\infty f\,dx$, we can set

$$\int_0^\infty f\,dx = \lim_{b\to\infty}\int_0^b f\,dx$$

If $\lim_{b\to\infty}$ converges, so does the integral and likewise for $\lim_{b\to\infty}$ diverges.

If $\int_a^b f\,dx$ is improper, then a, b, or both a and b are asymptotes of f.

We can use the limit in the same way, and the rules are the same as well.

☐ Put an "X" in this box if work, for this problem, continues on the back of **this page.**

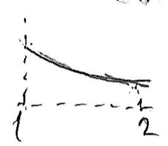
1.3

Discuss *arc length.* (Recall: The Mean Value Theorem states that if a function, f, is continuous on the interval $[a, b]$ and differentiable on the interval (a, b), then there is a number c in the interval (a, b) with $f'(c)(b-a) = f(b) - f(a)$.)

The arc length of a function is a quantity which describes the length of a curve between two distinct points.

If we try to find the length of a function, say, $\frac{1}{x}$, between 1 and 2, the "slice" would look something like this:

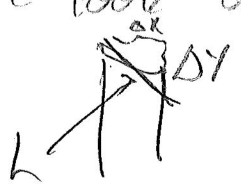
the curve



To find the length of this curve, we use the Mean Value Theorem, which states that for a function f continuous on (a,b) there is some point c on (a,b) such that

$$f'(c)\cdot(b-a) = f(b) - f(a).$$

If we look at a small slice of width n we see



we can apply the mean value theorem to find L:

$$L = \sqrt{\Delta x^2 + \Delta y^2}$$

Let $\Delta x = b-a$, $\Delta y = f'(c)\cdot(b-a)$

$$\Rightarrow L = \sqrt{(b-a)^2 + f'(c)^2\cdot(b-a)^2} \Rightarrow (b-a)\cdot\sqrt{1+f'(c)^2} = L$$

If we take a lot of these L's and use calculus to add them up between a and b, we get

[X] Put an "X" in this box if work, for this problem, continues on the back of this page.

See scratch paper (sorry was very stressed)

2 Essential Skills (40 points)

Your responses in this section should be precise and detailed, with justification given for all steps that rely on results from calculus. Clearly indicated your final answer to each exercise. Correct answers with insufficient work or justification may not receive full credit.

2.1

Evaluate, or establish the divergence of

$$\int_0^{\pi} (e^x \sin(x))\, dx.$$

we can use limits to help my attempt to evaluate this integral

$$\int_0^{\pi/2} (e^x \sin(x))\,dx + \int_{\pi/2}^{\pi} e^x \cdot \sin x\, dx = \int_0^{\pi} e^x \cdot \sin x\, dx \quad \text{by properties of integrals}$$

$$\lim_{n\to 0} \int_n^{\pi/2} e^x \cdot \sin x\, dx + \lim_{m\to\pi} \int_{\pi/2}^{m} e^x \cdot \sin x\, dx = \int_0^{\pi} e^x \cdot \sin x\, dx \quad \text{by properties of limits.}$$

To evaluate $\int e^x \cdot \sin x\, dx = I$

Let $f = \sin x$, $g' = e^x$ $\to$ so $f' = \cos x$, $g = e^x$

$I = e^x \sin x - \int e^x \cos x\, dx$ by IBP

Let $f = \cos x$, $g' = e^x$ $\to$ so $f' = -\sin x$, $g = e^x$

$\Rightarrow$ so $\int e^x \cdot \cos x\, dx = \cos x \cdot e^x + \int \sin x \cdot e^x\, dx$ by IBP

$\Rightarrow e^x \cdot \sin x - \cos x \cdot e^x - \int \sin x \cdot e^x\, dx = \int \sin x \cdot e^x\, dx$

$\Rightarrow$ therefore, $\int \sin x \cdot e^x\, dx = \frac{e^x}{2}(\sin x - \cos x) + C$

we can use this result with FTOC:

$$\lim_{n\to 0}\left(\frac{e^x}{2}\cdot(\sin x - \cos x)\right)\Bigg|_n^{\pi/2} + \lim_{m\to\pi}\left(\frac{e^x}{2}\cdot(\sin x - \cos x)\Bigg|_{\pi/2}^{m}\right)$$

$$\Rightarrow \lim_{n\to 0} \cancel{\frac{e^{\pi/2}}{2}\cdot 1} - \frac{e^n}{2}\cdot(\sin(n) - \cos(n)) + \lim_{m\to\pi} \frac{e^m}{2}\cdot(\sin(m) - \cos(m)) \cancel{- \frac{e^{\pi/2}}{2}\cdot 1}$$

$$\Rightarrow \lim_{n\to 0} -\frac{e^n}{2}\cdot(\sin(n)\cdot\cos(n)) + \lim_{m\to\pi} \frac{e^m}{2}\cdot(\sin(m)\cdot\cos(m)) = L$$

☒ Put an "X" in this box if work to be scored, for this problem, is located on a **scratch page**.

2.2

Evaluate, or establish the divergence of

$$\int_0^5 \left(\frac{x-4}{(x+7)(x-5)}\right) dx.$$

To evaluate $\frac{x-4}{(x+7)\cdot(x-5)}$ we can use partial fractions:

$$\frac{x-4}{(x+7)\cdot(x-5)} = \frac{A}{(x+7)} + \frac{B}{(x-5)}$$

$$\Rightarrow x-4 = A\cdot(x-5) + B\cdot(x+7) = (A+B)\cdot x - 5A + 7B$$

so $A+B=1 \rightarrow B = 1-A \longrightarrow B = 1-\frac{11}{12} \Rightarrow B=\frac{1}{12}$

$7B-5A=-4 \rightarrow 7\cdot(1-A)-5A=-4$

$\Rightarrow 7-7A-5A=-4$

$-12A=-11 \rightarrow$ so $A=\frac{11}{12}$

So our integral becomes

$$\int_0^5 \frac{\left(\frac{11}{12}\right)}{x+7}\,dx + \int_0^5 \frac{\left(\frac{1}{12}\right)}{x-5}\,dx$$ by FTOC and Properties of integrals

$$\Rightarrow \frac{11}{12}\cdot\left(\ln(x+7)\Big|_0^5\right) + \frac{1}{12}\cdot \ln(x-5)\Big|_0^5$$ by FTOC

$$\Rightarrow \frac{11}{12}\cdot(\ln 12 - \ln 7) + \frac{1}{12}\cdot(\ln 0 - \ln(-5))$$

$-\infty$

$\lim_{x\to 0} \ln x \to -\infty$

$\boxed{\lim_{x\to 5} \ln x \to -\infty}$

So the ~~int~~ integral diverges

Put an "X" in this box if work to be scored, for this problem, is located on a **scratch page**.

2.3

Evaluate

$$\int \left(\frac{1}{\sqrt{t^2-6t+13}}\right) dt.$$

$t^2-6t+13$

$t^2-6t+13 = (t^2-6t+9)+4.$

$\Rightarrow (t-3)^2+4$

So $\int \frac{1}{\sqrt{t^2-6t+13}}\,dt = \int \frac{1}{\sqrt{(t-3)^2+4}}\,dt = I$

(A)

We can use trig substitution to evaluate:

Let $(t-3)^2+4 = 4\sec^2\theta = 4\tan^2\theta+4$ by trig (2)

So $\Rightarrow (t-3)^2 = 4\tan^2\theta$

$\Rightarrow t-3 = 2\tan\theta \rightarrow \theta = \arctan\left(\frac{t-3}{2}\right)$

$\downarrow$

taking the derivative, we get

(B) $dt = 2\cdot\sec\theta\cdot\tan\theta\,d\theta$

By (A), (B), and the substitution rule

$I = \int \frac{2\sec\theta\cdot\tan\theta}{2\sec\theta}\,d\theta \Rightarrow \int \tan\theta\,d\theta = \ln|\sec\theta|+C$

by Iref 12

Plugging in for θ, we get $\ln\left|\sec\left(\arctan\left(\frac{t-3}{2}\right)\right)\right|+C$

Therefore,

$\int \frac{1}{\sqrt{t^2-6t+13}}\,dt = \ln\left|\sec\left(\arctan\left(\frac{t-3}{2}\right)\right)\right|+C$

☐ Put an "X" in this box if work to be scored, for this problem, is located on a **scratch page**.

2.4

Evaluate

$$\int \left(\frac{1 - \tan^2(s)}{\sec^2(s)} \right) ds.$$

(Hint: At some point, it might be helpful to consider trigonometric identities (1) and (4) on your reference pages.)

$\tan^2 s = \frac{\sin^2 s}{\cos^2 s}$

$\sec^2 s = \frac{1}{\cos^2 s}$

So $\frac{1-\tan^2 s}{\sec^2 s} = \frac{1 - \frac{\sin^2 s}{\cos^2 s}}{1/\cos^2 s} = (\cos^2 s - \sin^2 s)$

So $\int \frac{1-\tan^2 s}{\sec^2 s} ds = \underbrace{\int \cos^2(s)\, ds - \int \sin^2(s)\, ds}_{= I}$ by properties of integrals

By trig 3, 4

$\sin^2 s = \frac{1}{2} \cdot (1 - \cos(2s))$

$\cos^2 s = \frac{1}{2} \cdot (1 + \cos(2s))$

So $I = \frac{1}{2}\left[\int 1 + \cos(2s)\, ds - \int 1 - \cos(2s)\, ds\right]$

$\Rightarrow \frac{1}{2} \cdot \left[s + \frac{1}{2} \cdot \sin(2s) - s + \frac{1}{2} \cdot \sin(2s) + C\right)$

$\Rightarrow \frac{1}{2} \cdot (\sin(2s) + C)$

$\Rightarrow \frac{1}{2} \cdot \sin(2s) + C$

Therefore,

$\int \frac{1-\tan^2 s}{\sec^2 s} ds = \frac{1}{2} \cdot \sin(2s) + C$

☐ Put an "X" in this box if work to be scored, for this problem, is located on a **scratch page**.

2.5

Consider the parametric equations

$$x = \sin(\theta), \text{ and}$$
$$y = \cos(\theta),$$

$x=0$
$y=1$

when $\theta = 0$, $\sin x = \frac{\pi}{2}$

defined for $0 \leq \theta \leq \pi$. Eliminate the parameter to find a Cartesian equation of the curve. Then, sketch the curve and the direction in which the curve is traced as θ increases.

when $\theta = \pi$,
$x = 0$, $y = -1$

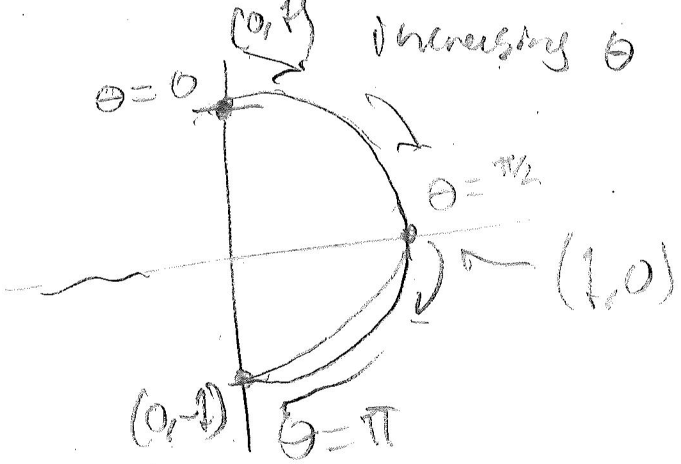
we can set $\theta = \arcsin x$

now $\theta = \frac{\pi}{2}$
$x = 1$
$y = 0$

Plugging in to $y = \cos(\theta)$, we get

$y = \cos(\arcsin(x))$

This gives us the function



Put an "X" in this box if work to be scored, for this problem, is located on a **scratch page**.

3 Intermediate Skills (10 points)

Your responses in this section should be precise and detailed, with justification given for all steps that rely on results from calculus. Clearly indicated your final answer to each exercise. Correct answers with insufficient work or justification may not receive full credit.

3.1

Determine whether

$$\int_1^\infty \left(\frac{x^4}{x^6+2}\right) dx$$

converges or diverges.

$$\lim_{t\to\infty} \int_1^t \frac{x^4}{x^6+2}\, dx$$

~~Since this is a rational function~~

~~we~~ can ~~say that~~ as

~~$x^4 <$~~ ~~$Ax+b$~~

Since the sign of $x^4 <$ sign of x^6+2

we can say that as $x \to \infty$, $\frac{x^4}{x^6+2} \to 0$

So the integral diverges.

Put an "X" in this box if work to be scored, for this problem, is located on a **scratch page**.

3.2

Evaluate

$$\int \left(\dot{e}^{\sqrt{\theta}}\right) d\theta.$$

$$\int e^{\sqrt{\theta}} d\theta$$

Let $U = \sqrt{\theta}$, so $\int e^{u} du$

$du = \frac{1}{2} \cdot \frac{1}{\sqrt{\theta}}$

☐ Put an "X" in this box if work to be scored, for this problem, is located on a **scratch page**.

4 Advanced Skills (4 points)

Your response in this section should be precise and detailed, with justification given for all steps that rely on results from calculus. Clearly indicated your final answer. Correct answers with insufficient work or justification may not receive full credit.

4.1

Evaluate

$$\int \left((2x^2+1)e^{x^2}\right) dx.$$

(Hint: It is possible to find an elementary antiderivative of $y = 2xe^{x^2}$, but not of $y = e^{x^2}$.)

☐ Put an "X" in this box if work to be scored, for this problem, is located on a **scratch page**.

5 True/False (10 points)

Determine the truth value of the following statements.
Clearly mark them as **TRUE** or **FALSE** in the space provided. Be aware that a statement is only true if it is *always* true. That is to say, if there is *even one example* that makes a statement false, then statement is false. You do not need to provide a proof, or counterexample, to justify your answers. No partial credit will be awarded.

$\frac{A}{x+3}$

1. The fraction $\dfrac{x(x^2+9)}{x^2-9}$ can be written in the form $\dfrac{A}{x+3}+\dfrac{B}{x-3}$ for some value of A and B.

TRUE FALSE

2. It is the case that

$\frac{1}{\sqrt{x}} - 1$

$\sqrt{x} - 1$

$$\int_0^1 \left(\frac{x-\sqrt{x}}{\sqrt{x}}\right) dx = \int_1^\infty \left(\frac{1}{x^2}\right).$$

TRUE FALSE

3. The improper integral $\displaystyle\int_1^\infty \frac{1}{x^p}dx$ converges whenever $p > 1$.

TRUE FALSE

4. The arc length of the curve traced by $y = \ln(\cos(x))$, for $0 \leq x \leq \pi/3$ is equal to

$\frac{1}{\cos}$

$$\int_0^{\pi/3} \left(\sqrt{1+\tan^2(x)}\right) dx.$$

TRUE FALSE

5. Simpson's Rule is always more accurate than the Midpoint Rule for approximating a definite integral.

TRUE FALSE

Bonus (5 points)

The following is an optional exercise. It is strongly recommended that you do not attempt this problem until you are confident that you have earned as many points as you are able to on the required portion of the exam.

Exercise:

Consider the following sum

$$S = 1 + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{4^2} + \cdots = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k^2}.$$

Show that $S \leq 2$. If you appeal to results that we have not covered in this class, you must prove those results.

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1.3 continued

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$$\lim_{n\to\infty} \sum_{i=1}^{n} (b-a)\cdot\sqrt{1+f'(x_i^*)^2}\cdot\Delta x$$

$\Rightarrow$ $(b-a)$ is a constant, so

$$(b-a)\cdot\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{1+f'(x_i^*)^2}\cdot\Delta x = (b-a)\cdot\int_a^b\sqrt{1+f'(x)^2}\,dx$$

which is the arc length

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2.1 Continued

as $n \to 0$, $\sin(n) \to 0$ and $\cos(n) \to 1$, $\frac{e^n}{2} \to \frac{1}{2}$

as $m \to \pi$, $\sin(m) \to 0$ and $\cos(m) \to -1$, $\frac{e^m}{2} \to \frac{e^\pi}{2}$

So by limit laws, we can say

$$L = \left(-\frac{1}{2} \cdot -1\right) + \left(\frac{e^\pi}{2} \cdot \frac{1}{2}\right) = \frac{e^\pi}{4} - \frac{1}{2}$$

So the limit converges

and $\int_0^\pi e^x \cdot \sin x \, dx = \frac{e^\pi}{4} - \frac{1}{2}$

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