Name:

Worksheet 02

1 Using Trigonometric Identities To Compute Integrals

1.1 An Example

$$\int \cos^4(x)dx = \int \left(\cos^2(x)\right)^2 dx \tag{1}$$

$$= \int \left(\frac{1 + \cos(2x)}{2}\right)^2 dx \tag{2}$$

$$= \frac{1}{4} \int (1 + 2\cos(2x) + \cos^2(2x)) dx \tag{3}$$

$$= \frac{1}{4} \int \left(1 + 2\cos(2x) + \frac{1 + \cos(4x)}{2} \right) dx \tag{4}$$

$$= \frac{1}{4} \int \left(\frac{3}{2} + 2\cos(2x) + \frac{1}{2}\cos(4x)\right) dx \tag{5}$$

For the following, use complete mathematical sentences and include any intermediate calculations that are needed.

Exercise 1. What justifies step (1) in the above computation?

Exercise 2. What justifies step (2) in the above computation?

Exercise 3. What justifies step (3) in the above computation?

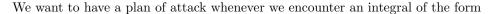
Exercise 4. What justifies step (4) in the above computation?

Exercise 5. What justifies step (5) in the above computation?

Exercise 6. Evaluate the following:

$$\frac{1}{4} \int \left(\frac{3}{2} + 2\cos(2x) + \frac{1}{2}\cos(4x)\right) dx$$

1.2 Devising a Plan of Attack



$$\int \left(\sin^m(x)\cos^n(x)\right)dx\tag{6}$$

Where m and n are integers.

Exercise 7. If we are trying to compute an integral in the form of equation (6), and m is odd, how can we use trigonometric identities to make the computation possible with methods we already have?

Exercise 8. If we are trying to compute an integral in the form of equation (6), and n is odd, how can we use trigonometric identities to make the computation possible with methods we already have?

Exercise 9. If we are trying to compute an integral in the form of equation (6), and m and n are both even, how can we use trigonometric identities to make the computation possible with methods we already have?

2 Trigonometric Substitution

3 The Building Blocks

Recal this important identity:

$$\sin^2(x) + \cos^2(x) = 1 \tag{7}$$

Exercise 10. Divide both sides of equation (7) by $\cos^2(x)$ to get a new trigonometric identity.

Exercise 11. Using equation (7) and the new identity you came up with in Exercise 10, complete the following statements:

- (a) $\cos^2(x) =$
- (b) $\sec^2(x) =$
- (c) $\tan^2(x) =$

3.1 Practice

Exercise 12. Evaluate the following:

$$\int \left(\frac{20x}{\sqrt{25 - 25x^8}}\right) dx$$

Exercise 13. Evaluate the following:

$$\int \left(\frac{10x^4}{9+4x^{10}}\right) dx$$

Exercise 14. Evaluate the following:

$$\int \left(\frac{2 \cdot \csc^2(2x)}{\cot(2x) \cdot \sqrt{\cot^2(2x) - 1}}\right) dx$$

4 Optional Practice

If your group is waiting for the class to move forward, take some time to practice the problems in this section.

1. Evaluate
$$\int_0^{\pi} \cos^6(\theta) d\theta$$

2. Evaluate
$$\int \cos^2(\theta) \tan^3(\theta) d\theta$$

3. Show that
$$\int_{-\pi}^{\pi} \sin(mx) \cos(nx) dx = 0$$
 for all positive integers m and n

4. Evaluate
$$\int_0^1 \sqrt{x^2 + 1} dx$$

5. Show that
$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2}) + C$$