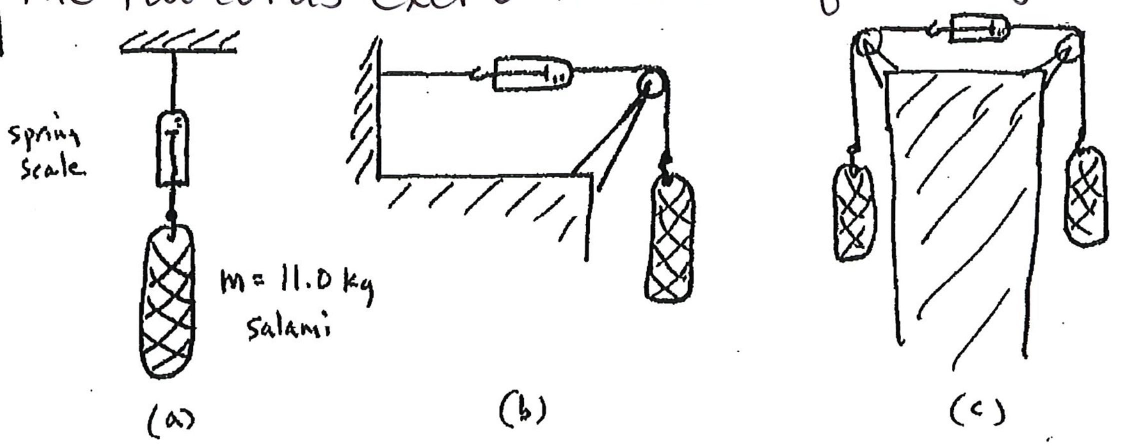
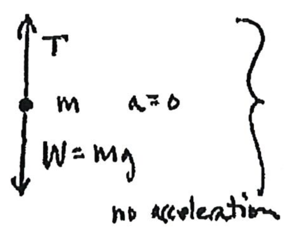
PHY 131 Problem Set 3
HRW 8th Ed.
Ch. 5: 13, 19, 21, 32, 36, 50, 54, 55, 57, 59

Note: in all three cases the scale isn't accelerating, so the two cords exert forces of equal magnitude

5:13



a) The scale provides the force on the string, giving the tension needed to support the weight of the salami:



Newton's 2nd law: $\Sigma F = ma$

$\therefore \quad T - mg = ma = 0$

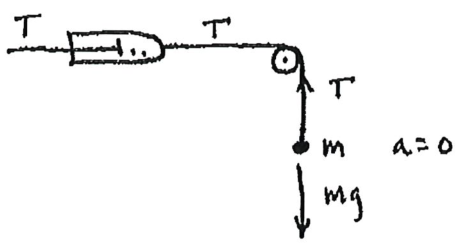
or $T = mg$

$= (11.0\text{ kg})(9.8\text{ m/s}^2)$

$= \underline{108\text{ N}}$ $\quad [110\text{ N if } g = 10]$

(For a massless scale, the upper string has the same tension)

b) Again, the scale provides (and thereby measures) the tension in the string needed to support the weight of the salami:



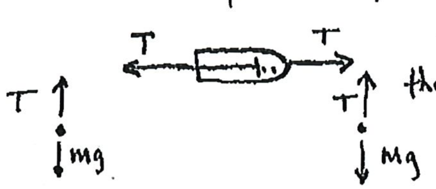
As before, $T - mg = 0$

$T = mg$

$= \underline{108\text{ N}}$

(For a massless scale, the string attached to the wall has the same tension)

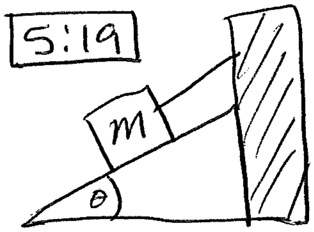
c) Same reading as parts a) and b): the scale measures the tension in the strings needing to hold either salami:



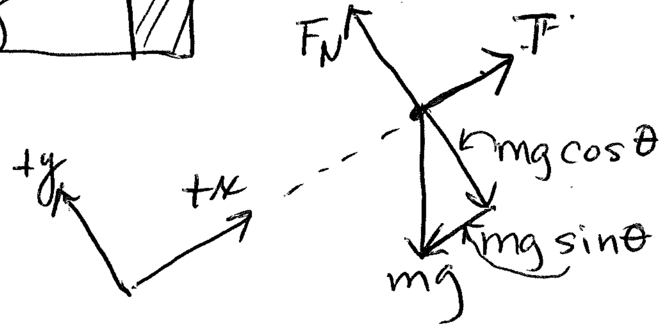
~~Hence~~ This time the left-hand salami provides the tension in the left side piece of string.

$T = \underline{108\text{ N}}$

5:19



(a) Since the acceleration of the block is zero, $\Sigma F_x = 0$ and $\Sigma F_y = 0$



x: $T - mg\sin\theta = 0$

y: $F_N - mg\cos\theta = 0$

· Solve 1st eqn for $T = (8.5\ kg)(9.8\ m/s^2)\sin 30°$

$T = 42\ N$

(b) Solve 2nd eqn for $F_N = mg\cos\theta$

$= (8.5\ kg)(9.8\ m/s^2)\cos 30°$

$F_N = 72\ N$

(c) When the string is cut, it no longer exerts a force on the block & the block accelerates down the incline under the influence of the x-component of gravity

$\Sigma F_x = ma$

$-\cancel{m}g\sin\theta = \cancel{m}a$

$a = -(9.8\ m/s^2)\sin 30°$

$\vec{a} = -4.9\ m/s^2$, down the incline

$|\vec{a}| = 4.9\ m/s^2$

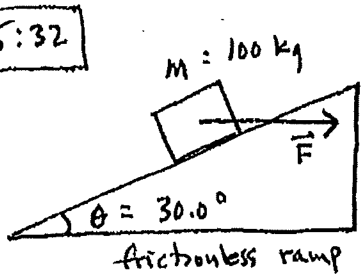
$\boxed{5:21}$ The slope of each graph gives the corresponding component of acceleration. We find $a_x = 3\ m/s^2$ and $a_y = -5\ m/s^2$. The magnitude of the acceleration vector is $|\vec{a}| = \sqrt{(3\ m/s^2)+(-5\ m/s^2)} = 5.8\ m/s^2$.

The force is obtained by $|\vec{F}_{net}| = m|\vec{a}|$

$$|\vec{F}| = (2\ kg)(5.8\ m/s^2)$$

$$|\vec{F}| = 11.7\ N$$

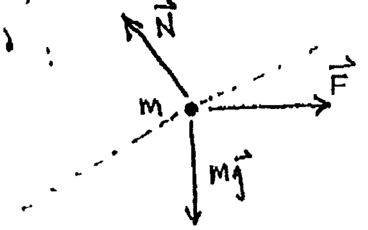
$\boxed{5:32}$



Push the crate with force $\vec{F}$ at constant speed.

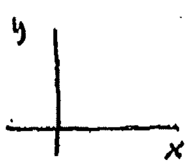
a) What is the magnitude of $\vec{F}$?

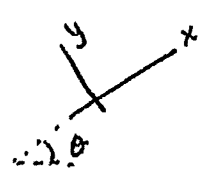
Newton's 2nd :



$\underline{a = 0}$.

We need to define an x-y coordinate system and then look at the components of the forces. Good choices are xy like so:

① 

or like so : 

②

$\boxed{\text{5:32 cont'd}}$

$$F_x = mg_x \implies F\cos 30^\circ = mg\sin 30^\circ$$

$$\therefore\ F = mg\frac{\sin 30^\circ}{\cos 30^\circ} = mg\tan 30^\circ$$

$$= 566\text{ N},$$

as before.

Then $N - F_y - mg_y = 0$

$$\implies N = F_y + mg_y$$

$$= F\sin 30^\circ + mg\cos 30^\circ$$

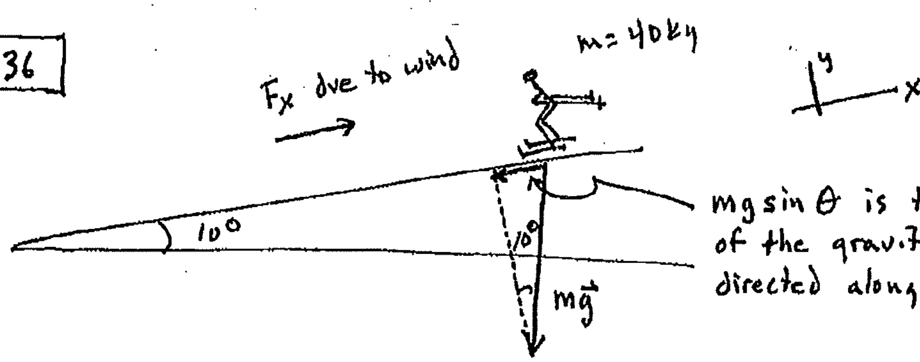
$$= \underbrace{(566\text{ N})\left(\tfrac{1}{2}\right)}_{} + \underbrace{(100\text{ kg})(9.8\text{ m/s}^2)\left(\tfrac{\sqrt{3}}{2}\right)}_{}$$

$$= 283 + 849\text{ N}$$

$$= 1132\text{ N},\ \text{as before.}$$

Note that coordinate system ① is slightly quicker to use, because you need to resolve only one of the vectors ($\vec{N}$) into its components, whereas coordinate system ② needs both $\vec{F}$ and $m\vec{g}$ to be resolved into their components.

$\boxed{\text{5:36}}$



$mg\sin\theta$ is the component of the gravitational force directed along the $-\hat{\imath}$ axis.

a) What is F_x if the magnitude of the Skier's velocity is constant?

$$\sum \vec{F} = m\vec{a}$$

Here $a_x = 0$ so

$$F_x - mg\sin\theta = m(0).$$

$$F_x = mg\sin 10^\circ = (40\text{ kg})(9.8\text{ m/s}^2)(0.174)$$

$$= \underline{\underline{68\text{ N}}}\quad \left[69\text{ N if } g = 10\text{ m/s}^2\right]$$

$\boxed{\text{5:36 cont'd}}$

b) What is F_x if the magnitude of the skier's velocity is increasing at a rate of $1.0\ m/s^2$?

$$F_x - mg\sin\theta = ma$$

$$F_x = ma + mg\sin\theta$$

$$= m(a + g\sin\theta)$$

now put $-1.0\ m/s^2$ here. Why? The skier's velocity is negative (going from right to left). In order to get the magnitude of this velocity to increase (i.e. in order to go faster) the acceleration must also be negative.

so $F_x = (40.0\ kg)(\underbrace{-1.0\ m/s^2 + 9.8\ m/s^2 \cdot 0.174}_{0.702})$

$F_x = \underline{28}\ N$ [29 N if $g = 10\ m/s^2$]

(smaller than before, allowing the skier to go faster).

c) What is F_x if the magnitude of the skier's velocity increases at a rate of $2.0\ m/s^2$?

$$F_x - mg\sin\theta = ma$$

$$F_x = m(a + g\sin\theta)$$

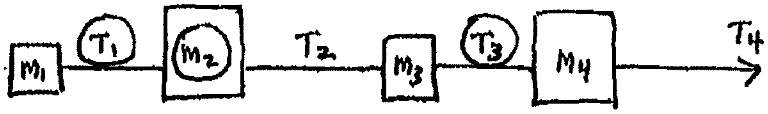
$-2.0\ m/s^2$ (same reasons as before)

$$F_x = (40.0\ kg)(-2.0\ m/s^2 + 9.8 \cdot 0.174)$$

$F_x = -11.9\ N = \underline{\underline{-12\ N}}$ [-10.5 N if $g = 10\ m/s^2$]

The wind needs to switch direction, blowing down the hill in the negative x direction in order for the skier to accelerate at this rate.

$\boxed{5:50}$ Yes, these are ~~[illegible]~~ some penguins:

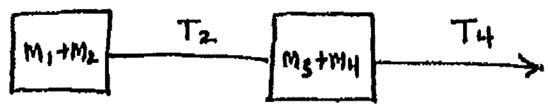


We know $m_1 = 12$ kg
$m_3 = 15$ kg
$m_4 = 20$ kg
$T_2 = 111$ N
$T_4 = 222$ N

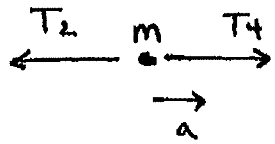
Find m_2

Circled items are unknown

We could draw free body diagrams for all the masses, but because we don't need T_1 or T_3 we can simplify things by combining $(m_1 + m_2)$ and $(m_3 + m_4)$:



Now use the right-hand combined mass to find the acceleration of the whole :



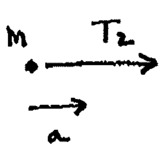
$$T_4 - T_2 = \overset{m_3+m_4}{m}a$$

$$a = \frac{T_4 - T_2}{(m_3 + m_4)}$$

$$= \frac{222 - 111}{15 + 20} \text{ N}$$

$$a = 3.17 \text{ m/s}^2$$

And now using this acceleration on the left-hand combination gives



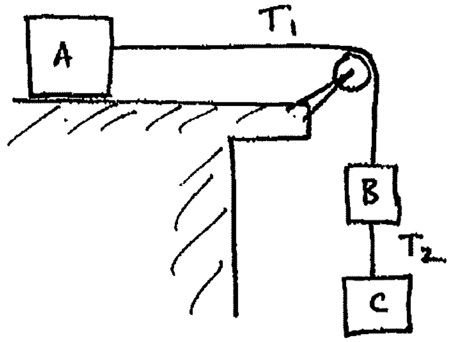
$$T_2 = \overset{m_1+m_2}{m}a = (m_1 + m_2)\,a$$

$$T_2 = m_1 a + m_2 a$$

$$\text{so} \quad \frac{T_2 - m_1 a}{a} = m_2$$

Hence $m_2 = \dfrac{111 - (12)(3.17)}{3.17} = \dfrac{111 - 38}{3.17} = \dfrac{73}{3.17} = \underline{\underline{23 \text{ kg}}}$

$\boxed{5:54}$



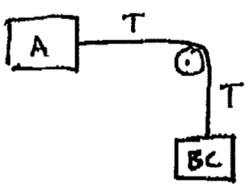
$m_A = 30.0$ kg
$m_B = 40.0$ kg
$m_C = 10.0$ kg

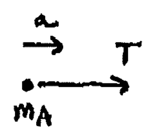
a) Find T_2 just after A is released.

Big picture: need to find the acceleration of the whole, then use it to deduce what tension T_2 makes box C have the same acceleration. So we need not find T_1.

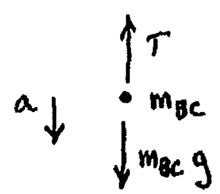
Combine B and C into mass $m_{BC} = m_B + m_C = 50.0$ kg

Then





$T = m_A a$



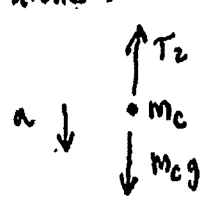
$m_{BC} g - T = m_{BC} a$

Add these to eliminate T:

$$m_{BC}\, g = (m_A + m_{BC})\, a$$

$$\therefore \quad a = \frac{m_{BC}\, g}{m_A + m_{BC}} = \frac{50 \cdot 9.8}{80} = 6.125 \text{ m/s}^2 \quad [6.25]$$

Now that we have a we can return to the original diagram, looking at T_2 alone:



$$m_c g - T_2 = m_c a$$

$$\therefore \; T_2 = m_c g - m_c a = m_c (g - a)$$

$$= 10\text{ kg}\,(9.8 - 6.125)$$

$$= \underline{\underline{36.8}} \text{ N} \quad [37.5\text{ N}]$$

$\boxed{5:54 \text{ cont'd}}$

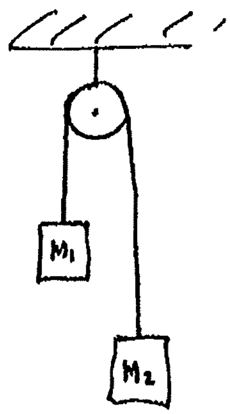
How far does A move in the first 0.250 s ?

$$(x - x_0) = v_0 t + \frac{1}{2} a t^2 \qquad \text{with } v_0 = 0 \text{ and } a = 6.125 \text{ m/s}^2$$

$$= 0 + \frac{1}{2}(6.125 \text{ m/s}^2)(0.250)^2$$

$$= \underline{0.19 \text{ m}} \qquad [0.20 \text{ m, if } g = 10 \text{ m/s}^2]$$

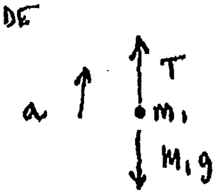
$\boxed{5:55}$ Atwood's machine



$m_1 = 1.30$ kg
$m_2 = 2.80$ kg

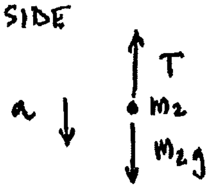
a) Find magnitude of acceleration. Define an assumed direction of a as positive. In this case the pulley will rotate clockwise so a is down on the right (and up on the left): [If you choose a to be in the other direction, you'll get a negative value. No problem – just note that a must go the other way]

LEFT SIDE



$$T - m_1 g = m_1 a$$

RIGHT SIDE



$$m_2 g - T = m_2 a$$

add these: +

$$m_2 g - m_1 g = m_1 a + m_2 a$$

$$\therefore \quad a = \frac{(m_2 - m_1) g}{m_1 + m_2} = \frac{2.80 - 1.30}{2.80 + 1.30} g = \frac{1.5}{4.1} g$$

$\boxed{5{:}55 \text{ cont'd}}$

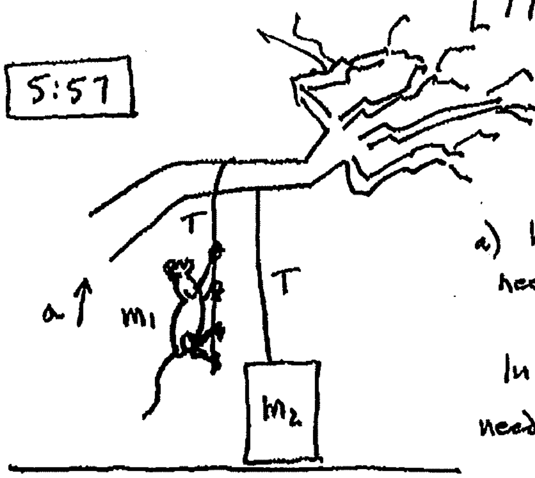
$$a = \frac{(1.5)}{4.1}(9.8) = \underline{\underline{3.59 \ m/s^2}} \quad \text{downward on the right}$$

$$\left[3.66 \ m/s^2 \text{ for } g = 10 \ m/s^2\right]$$

b) The tension in the cord can be found from either equation:

$$T = m_1(a+g) = (1.30 \text{ kg})(3.59 + 9.8)$$
$$= \underline{\underline{17.4 \text{ N}}}$$
$$\left[17.8 \text{ N for } g = 10 \ m/s^2\right]$$

$\boxed{5{:}57}$



$m_1 = 10$ kg
$M_2 = 15$ kg

a) What is the magnitude of the least acceleration needed to lift M_2 off the ground?

In order to lift the box M_2, the minimum tension needs to be:

↑ T_{min}
● M_2
↓ $M_2 g$

$$T_{min} - M_2 g = 0$$

no acceleration, T is just enough to lift the box

so $\boxed{T_{min} = M_2 g}$

On the monkey side:

a ↑

↑ T_{min}
● M_1
↓ $M_1 g$

$$T_{min} - M_1 g = M_1 a_{min}$$

$$\text{so} \quad a_{min} = \frac{M_2 g - M_1 g}{M_1} = \frac{(M_2 - M_1)g}{M_1}$$

$$= \frac{(15-10)g}{10} = \frac{1}{2}g$$

$$a_{min} = \underline{\underline{4.9 \ m/s^2}} \quad \left[5.0 \ m/s^2 \text{ if } g = 10 \ m/s^2\right] \ \text{upward}$$

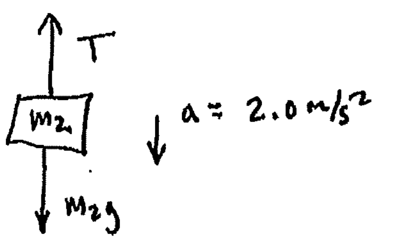
b) If the monkey then grips the rope, what is the magnitude of its acceleration?

$\boxed{\text{5:57 cont'd}}$

Same argument, but now
$$a = \frac{(m_2 - m_1)}{m_1 + m_2}\, g$$
$$= \left(\frac{15-10}{15+10}\right) g$$
$$= \frac{5}{25}\, g = \frac{1}{5}\, g = \underline{\underline{2.0\ \text{m/s}^2}}$$

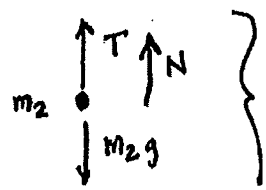
c) The direction is upward, lifting the monkey.

d) Find the tension ~~for~~ in the rope in this case:



$$m_2 g - T = m_2 \overbrace{(2.0\ \text{m/s}^2)}^{a}$$
$$T = m_2 (g - a)$$
$$= (15\ \text{kg})(9.8\ \text{m/s}^2 - 2.0\ \text{m/s}^2)$$
$$= 117\ \text{N} = \underline{120\ \text{N}}$$
(just a bit less than $W = 150$ N)

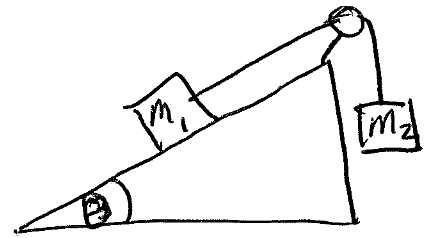
Aside: If you don't like the argument for solving for $T_{minimum}$, you could look at all the forces acting on the box. The box exerts a contact force on the ground, and the ground reacts by pushing up on the box with a normal force N:



Here's what happens as the tension in the rope increases: the normal force starts out such that $N = m_2 g$. Then as the monkey climbs faster and faster the tension in the rope gets bigger and bigger, partially supporting the weight of the box. The contact force then gets smaller, and N also shrinks. At lift-off, $N \to 0$ and $T = m_2 g$. This is the minimum tension (larger T would cause the box to accelerate).

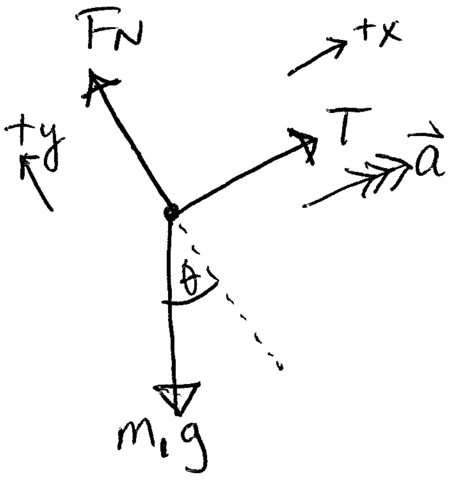
We will see further instances of the normal force vanishing just as an object lifts off of a surface, such as at the top of a loop-the-loop, if the car isn't going fast enough.

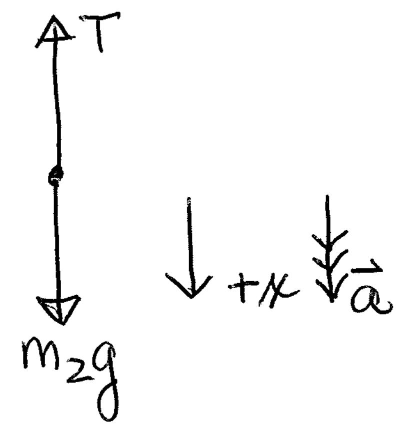
$\boxed{5:59}$



$m_1 = 3.7$ kg
$m_2 = 2.3$ kg
$\theta = 30°$

The FBD of each body is shown below





Note: same direction of motion is positive for both bodies, they have the same acceleration & the same tension in the cord.

mass 1

$\Sigma F_x = m_1 a$

$T - m_1 g \sin\theta = m_1 a$

$\Sigma F_y = 0$

$F_N - m_1 g \cos\theta = 0$

mass 2

$\Sigma F = m_2 a$

$m_2 g - T = m_2 a$

Note 2 eqns and 2 unknowns

$T = m_1 a + m_1 g \sin\theta$ $\qquad$ $T = m_2 g - m_2 a$

$m_1 a + m_1 g \sin\theta = m_2 g - m_2 a$

$m_1 a + m_2 a = m_2 g - m_1 g \sin\theta$

$$a = \frac{m_2 g - m_1 g \sin\theta}{m_1 + m_2} = \frac{[(2.3)(10) - (3.7)(10)\sin 30]}{(3.7 + 2.3)}$$

(a) $\vec{a} = 0.735$ m/s² (positive)

(b) m_1 goes up ramp, m_2 falls vertically down

$\boxed{5:59}$ (cont.)

(c) $T = m_1 a + m_1 g \sin\theta$ or $\left(\begin{aligned} T &= m_2 g - m_2 a \\ &= m_2 (g - a) \end{aligned} \right)$

$= (3.7)(.735) + (3.7)(9.8)(\sin 30^\circ)$

$T = 20.8\ N$