Ollie Yarles
Phy 232
Charles Cunningham

HW for 10/6/17
QGR.2, Q7M.7

1

QGR.2

A bucky ball consists of 60 carbon atoms arranged in the form of a hollow ball. Say that we illuminate a free, initially non rotating buckyball with visible light, with the intensity of sunlight ($1000\frac{W}{m^2}$), whose photons all have their spins alligned with their direction of motion. About how long after the illumination begins will the buckyball (on the average) be spinning a million times persecond, assuming it absorbs all the light falling on it?

Molar mass of carbon atom: $12.01\frac{g}{mol}$
Mass of buckyball: $12.01\frac{g}{mol}\cdot\frac{1kg}{1000g}\cdot\frac{1mol}{6.022\cdot10^{23}atoms}\cdot 60\,atoms = 1.197\cdot10^{-24}kg$

A photon has a spin of $\hbar$, we can be treated as if an angular momentum

The rate at which photons will hit, and thus be absorbed by the bucky ball is $R = \frac{Intensity\cdot Area}{Energy/photon}$. So the total angular momentum transfered after t seconds is $R\cdot t\cdot\hbar$

The angular momentum of a spining hollow ball is $|\vec{L}| = \frac{2}{3}m r^2 |\vec{\omega}|$, where m is the mass, r is the radius, and $|\vec{\omega}|$ is the angular frequency.

By dividing the angular frequency by 2π, we get the rotational frequency, $f = 1\cdot10^{6}Hz$

So we can rewrite our expression for angular momentum as $\frac{|\vec{L}|}{2\pi} = \frac{2}{3}m r^2 f$

Substituting $R\cdot t\cdot\hbar$ for $|\vec{L}|$, we get $\frac{R\cdot t\cdot\hbar}{2\pi} = \frac{2}{3}m r^2 f$

$\rightarrow \frac{I\cdot A\cdot t\cdot\hbar}{E_p\cdot 2\pi} = \frac{2}{3}\cdot m r^2 f$. Solving for t, we get

$t = \frac{2\pi m r^2 f\cdot E_p\cdot 2\pi}{3 I\cdot A\cdot\hbar} = \frac{2\cdot m\cdot f\cdot E_p}{3\cdot I\cdot\hbar}$. $E_p = \frac{hc}{\lambda}$ is the energy of a photon with wavelength λ.

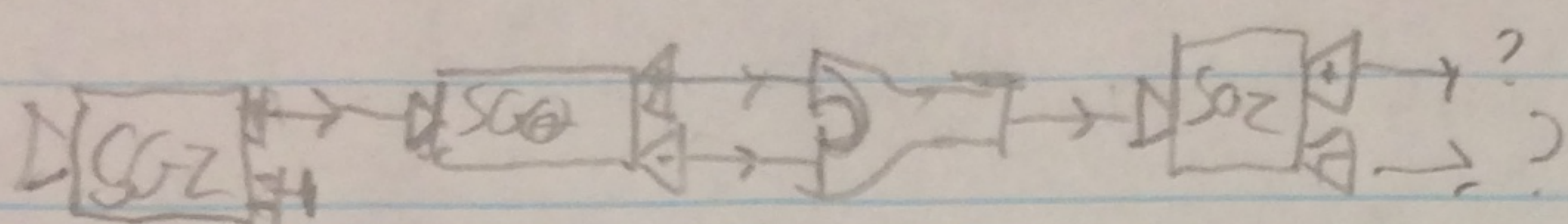
In this case, we have visible light which has an average wavelength of 550nm (see Q6M.3)

Substituting $\frac{hc}{\lambda}$ for E_p we get $t = \frac{2\cdot m\cdot f\cdot hc}{3\cdot I\cdot\frac{h}{2\pi}\cdot\lambda} = \frac{2\cdot m\cdot f\cdot 2\pi\cdot c}{3\cdot I\cdot\lambda}$

$$t = \frac{2\cdot 1.197\cdot10^{-24}kg\cdot 1\cdot10^{+6}Hz\cdot 2\pi\cdot 3\cdot10^{8}\frac{m}{s}}{3\cdot 1000\frac{W}{m^2}\cdot 550\cdot10^{-9}m} = \boxed{2.7\cdot10^{-6}s}$$

Q7M.1

Suppose that N electrons enter the second Stern-Gerlach device shown in the figure below. Let the angle of θ be such that $\cos\frac{1}{2}\theta = \frac{3}{5}$, $\sin\frac{1}{2}\theta = \frac{4}{5}$.



(a) How many electrons come out of the "$-$" channel of the third device?

An SG device forces electrons into one of two well defined spin orientation states (aligned or antialigned)

The first SG device prepares the electrons in the $|+z\rangle$ state. The second SG device prepares the electrons in either the $|+\theta\rangle$ or $|-\theta\rangle$ state. So by the outcome probability rule, the quantum amplitude that the spin orientation states of electrons will be aligned or antialigned in the second device are, respectively, $\langle +\theta|+z\rangle$ and $\langle -\theta|+z\rangle$.

Notice that by looking at the outputs of the third SG device only, we cannot know which path the electrons actually took, so the superposition rule applies. By the sequence rule, the quantum amplitude that electrons will be determined to be in antialigned spin orientation state by the third device is $\langle -z|+\theta\rangle\langle +\theta|+z\rangle + \langle -z|-\theta\rangle\langle -\theta|+z\rangle$

We are given that $|+z\rangle = \begin{bmatrix}1\\0\end{bmatrix}$, $|-z\rangle = \begin{bmatrix}0\\1\end{bmatrix}$, $|+\theta\rangle = \begin{bmatrix}\cos\frac{1}{2}\theta\\ \sin\frac{1}{2}\theta\end{bmatrix}$, $|-\theta\rangle = \begin{bmatrix}-\sin\frac{1}{2}\theta\\ \cos\frac{1}{2}\theta\end{bmatrix}$.

So the total quantum amplitude is $\left([0, 1]\begin{bmatrix}\cos\frac{1}{2}\theta\\ \sin\frac{1}{2}\theta\end{bmatrix}\right)\left([\cos\frac{1}{2}\theta, \sin\frac{1}{2}\theta]\begin{bmatrix}1\\0\end{bmatrix}\right) + \left([0,1]\begin{bmatrix}-\sin\frac{1}{2}\theta\\ \cos\frac{1}{2}\theta\end{bmatrix}\right)\left([-\sin\frac{1}{2}\theta, \cos\frac{1}{2}\theta]\begin{bmatrix}1\\0\end{bmatrix}\right)$

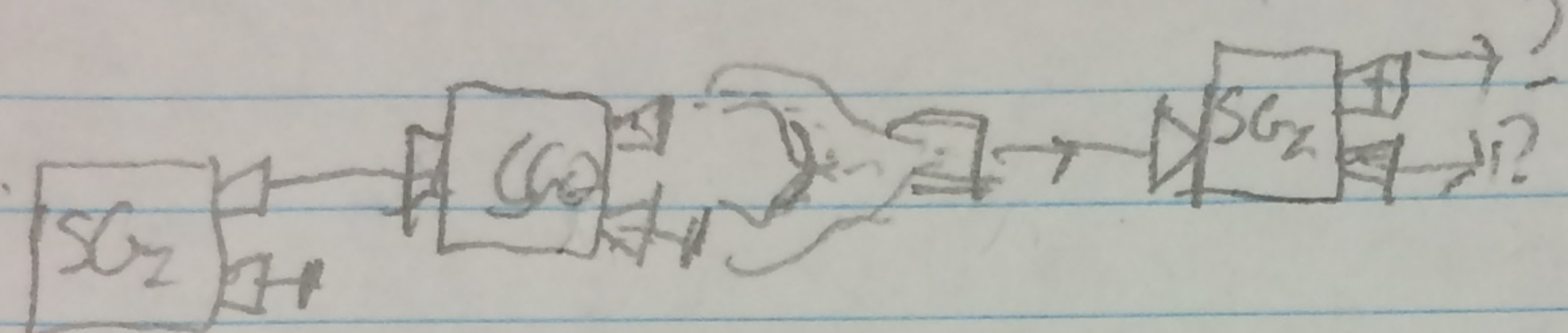
$\Rightarrow (0\cdot\cos\frac{1}{2}\theta + 1\cdot\sin\frac{1}{2}\theta)\cdot(\cos\frac{1}{2}\theta\cdot 1 + \sin\frac{1}{2}\theta\cdot 0) + (0\cdot -\sin\frac{1}{2}\theta + 1\cdot\cos\frac{1}{2}\theta)\cdot(-\sin\frac{1}{2}\theta\cdot 1 + \cos\frac{1}{2}\theta\cdot 0)$

$= \sin\frac{1}{2}\theta\cdot\cos\frac{1}{2}\theta - \sin\frac{1}{2}\theta\cdot\cos\frac{1}{2}\theta = 0$

The probability of a given result is the square of its quantum amplitude. So the probability that the electrons will exit the "$-$" port is $0^2 = 0$. So for N electrons, $0\cdot N = 0$ electrons will come out of the "$-$" port.

(b) Now suppose that we block the electrons that come out of the "−" channel of the second device before they go into the U-shaped funch. How many come out of the + channel in the third device now?

In part (a) we found that the quantum amplitudes for electrons being aligned or anti aligned were, respectively, $\langle +\theta|+z\rangle$ and $\langle -\theta|+z\rangle$



Now the electrons that go out of the "−" channel of the second device are blocked so we know that all the electrons are in the $|+\theta\rangle$ state, giving a quantum amplitude of $\langle +\theta|+z\rangle$.
By the sequence rule, the total quantum amplitude of electrons that leave the "−" port is

$$\langle -z|+\theta\rangle\langle +\theta|+z\rangle = \left([0,1]\begin{bmatrix}\cos\tfrac{1}{2}\theta\\ \sin\tfrac{1}{2}\theta\end{bmatrix}\right)\left([\cos\tfrac{1}{2}\theta, \sin\tfrac{1}{2}\theta]\begin{bmatrix}1\\0\end{bmatrix}\right)$$

$$\Rightarrow \left(0\cdot\cos\tfrac{1}{2}\theta + \sin\tfrac{1}{2}\theta\right)\cdot\left(\cos\tfrac{1}{2}\theta\cdot 1 + \sin\tfrac{1}{2}\theta\cdot 0\right) = \sin\tfrac{1}{2}\theta\,\cos\tfrac{1}{2}\theta$$

The probability the electrons will come out of the "−" channel is the square of the quantum amplitude that is

$$\left(\sin\tfrac{1}{2}\theta\,\cos\tfrac{1}{2}\theta\right)^2 = \sin^2\tfrac{1}{2}\theta\cos^2\tfrac{1}{2}\theta = \left(\tfrac{4}{5}\right)^2\cdot\left(\tfrac{3}{5}\right)^2 = \frac{16\cdot 9}{25^2} = \frac{144}{625}$$

So the number of electrons that leave the "−" port is

$$\boxed{N\cdot\frac{144}{625}}$$

Evals: QGR.2: right units, reasonable magnitude
Q7M.7: right units, reasonable magnitude