Phy 2??2
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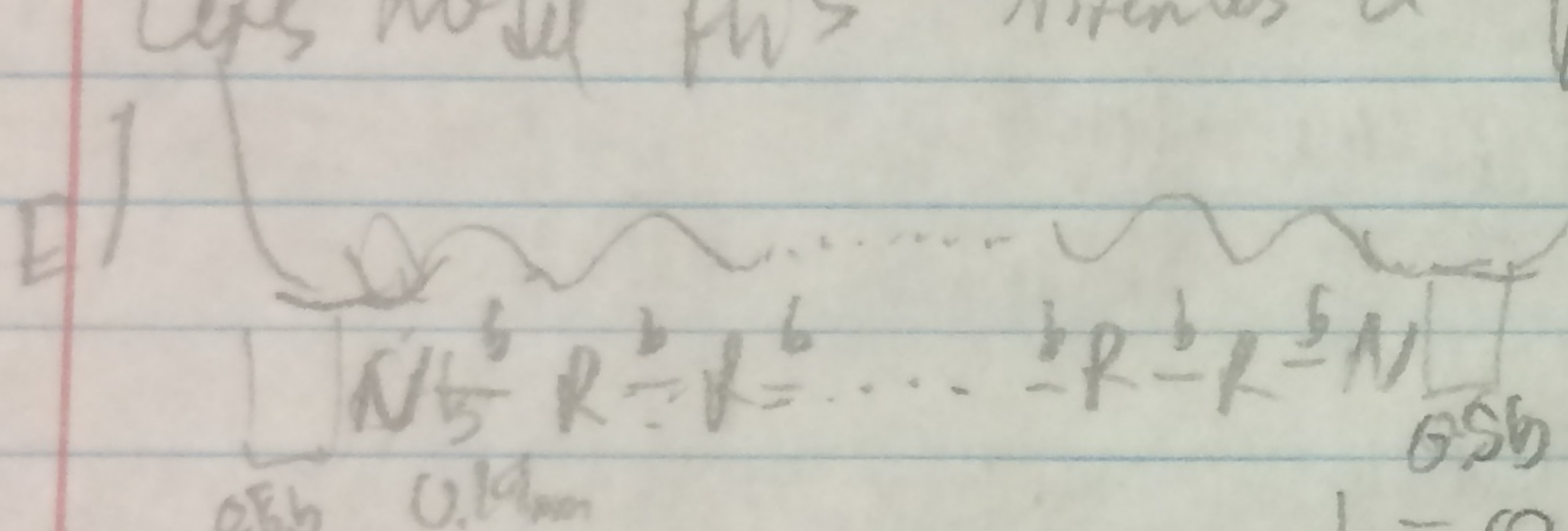
Q11 M.11
Q12 B.3
Q12 B.6

2

Q11M.11
Consider the case of an organic cyanine molecule with N bonds (where N is even) between the two nitrogen atoms. The bond length between atoms in the central chain is still $b=0.14$ nm, but the different end groups in this molecule allow electrons to penetrate roughly $0.5b$ beyond the nitrogen atom at each end.

a) Show that the lowest energy photon that the electrons along the molecule's central chain can absorb will be a wavelength of roughly $\lambda \approx \frac{8mcb^2}{hc}(N+1)$

Let's model this system as a 1d box, so the energy levels are $E_n = \frac{h^2 n^2}{8mL^2} = \frac{(hc)^2 n^2}{8mc^2L^2}$



$L = 0.5b + 0.5b + Nb = (N+1)b$

The box contains 1 electron for every bond.
So when all electrons are in their lowest level, the $n=1$ through $n=\frac{N}{2}$ levels will be filled. So the lowest energy photon that can be absorbed will kick a photon from the $n=\frac{N}{2}$ to the $n=\frac{N}{2}+1$ level.
This corresponds to a transition energy $\Delta E = E_1\left(\left(\frac{N}{2}+1\right)^2 - \left(\frac{N}{2}\right)^2\right)$

$$\Delta E = \frac{(hc)^2}{8mc^2(N+1)^2b^2}\left(\left(\frac{N+2}{2}\right)^2 - \left(\frac{N}{2}\right)^2\right) = \frac{(hc)^2(N^2+4N+4-N^2)}{8 \cdot mc^2(N+1)^2b^2 \cdot 4} = \frac{(hc)^2(N+1)}{8mc^2(N+1)^2b^2}$$

$\Delta E = \frac{(hc)^2}{8b^2mc^2(N+1)}$. This is the energy of our photon.
The wavelength of a photon with energy ΔE is $\lambda = \frac{hc}{\Delta E} = \frac{hc}{\left(\frac{(hc)^2}{8mc^2b^2(N+1)}\right)}$

$$\boxed{\lambda = \frac{8 \cdot m \cdot c^2 b^2 (N+1)}{h \cdot c}}$$

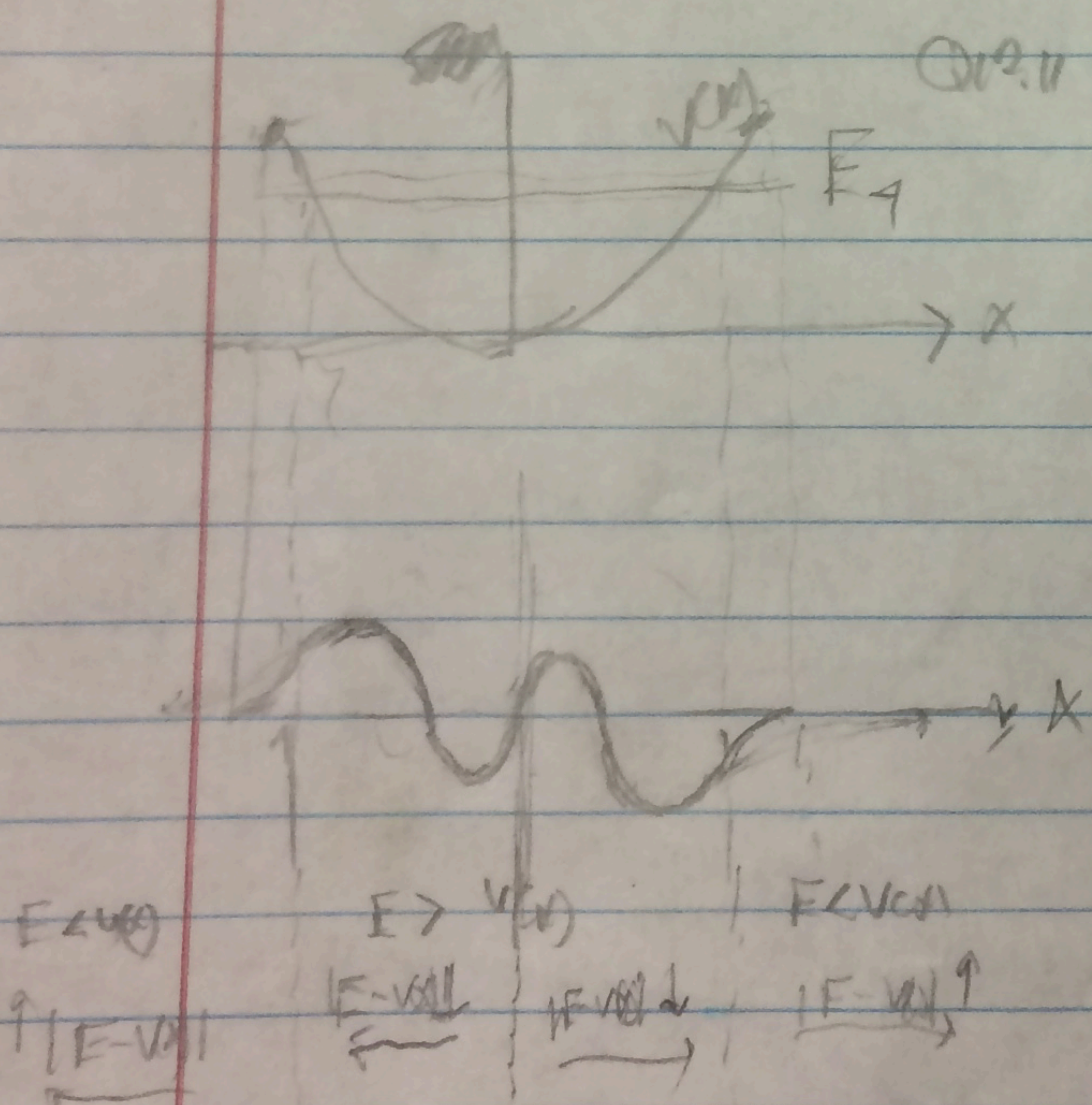
b) Predict such wavelengths for molecules with $N=6$, $N=8$ and $N=10$. Compare with the observed wavelengths, which are roughly 485 nm, 582 nm, 687 nm

$$\frac{8 m_e c^2 \cdot b^2}{hc} = \frac{8 \cdot 511000\,\text{eV}\,(0.14\,\text{nm})^2}{1240\,\text{eV}\cdot\text{nm}} = 64.617\,\text{nm}$$

So $\lambda = 64.617 \cdot (N+1)$ nm

$N=6 \rightarrow \lambda_6 = 64.617 \cdot (7)$ nm $=$ 452 nm (This is 33 nm less than actual)
$N=8 \rightarrow \lambda_8 = 64.617 \cdot (9)$ nm $=$ 582 nm (this is exactly right)
$N=10 \rightarrow \lambda_{10} = 64.617 \cdot (11)$ nm $=$ 711 nm (this is 24 nm more than the actual)

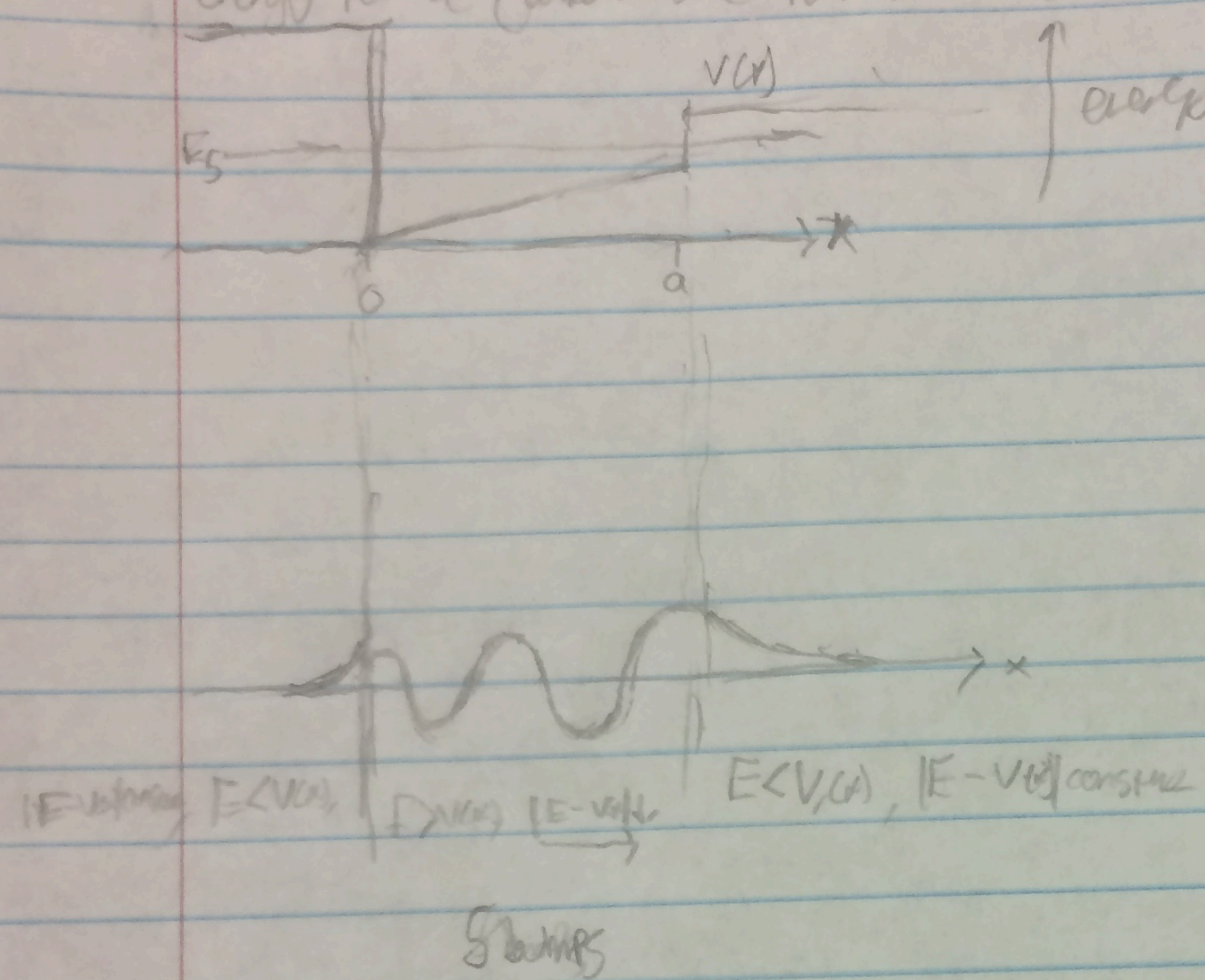
Q12B.3
Sketch the EEF corresponding to the 4th lowest bound state energy for a quanton whose PE is shown in Q12.11



Rules: 1) Physically reasonable wavefunctions approach zero as $x \rightarrow \pm\infty$
2) EEF's curve toward the horizontal axis (be wavelike) where $E > V(x)$, curve away from the axis (are exponential like) when $E < V(x)$
3) EEF's curvature ↑ with $|E-V(x)|$
a) greater curvature → shorter λ in classically allowed regions
b) greater curvature → shorter "tails" in classically forbidden regions
4) Amplitude of wavelike part grows with wavelength
5) Smooth and continuous if $V(x)$ finite
6) EEF corresponding to nth energy level will have n "bumps" in the classically allowed region

Q12B.6

Sketch the energy eigen function corresponding to the 5th lowest bound state energy for a quantum whose potential energy function is shown below



Exam 0 answers

Q11M.11: rigorous, reasonable ~~magnitude~~

Q12B.3, Q12B.6: good graphs