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Phy 202
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HW Rec: 10/21/17
Q4R.1, QSM.3, QSM.10

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Q4R.1

Experiments have shown that the nervous system of the human eye effectively takes about 30 "frames" per second, and that when the eye is fully dark adapted it needs to receive only about 500 visible photons per frame from an object to register it. Our sun radiates a power of about $3.9\cdot10^{26}$ W at all wavelengths, peaking in the yellow region of the spectrum (but only about $\frac{1}{2}$ of that energy is in the visible range). The pupil of your dark adapted eye has a diameter of about 8 mm. Estimate how far away a star like the sun could be and still be visible to the naked eye. The nearest visible star is about 4 ly away; most stars we see in the sky are hundreds of light years away. What does this mean for most visible stars' intrinsic brightness compared to the sun's?

A fully dark adapted eye needs to receive $\frac{30\text{ frames}}{s}\cdot\frac{500\text{ photons}}{\text{frame}} = 15000$ photons

$\Rightarrow$ 15000 photons per second from an object to register it. This means that the effective intensity of the light needs to be $\frac{E\cdot 15000\text{ photons}}{s}\cdot\frac{1}{\pi(0.004m)^2}\approx E\cdot 3\cdot10^{8}\frac{W}{m^2}$, where

E is the energy of a photon with wavelength λ, given by

$E=\frac{hc}{\lambda}$, where c = speed of light, $h=6.63\cdot10^{-34}$ J·s
$\approx 3.0\cdot10^{8}$ m/s

A star like the sun radiates $\sim3.9\cdot10^{26}\frac{J}{s}$ in all wavelengths, peaking in the yellow spectrum (but only $\frac{1}{2}$ of this energy is in the visible range), giving the sun an effective visual power of $\sim1.95\cdot10^{26}\frac{J}{s}$.

The wavelength of yellow light is ~580 nm.

So the effective intensity of light emitted from a star like the sun needs to be at least $\frac{h\cdot c}{580\cdot10^{-9}m}\cdot3\cdot10^{8} = \frac{6.63\cdot10^{-34}J\cdot s\cdot3.0\cdot10^{8}\frac{m}{s}}{580\cdot10^{-9}m}\cdot3\cdot10^{8}\,s^{-1}m^{-2}$

$\Rightarrow 1.02\cdot10^{-10}\frac{W}{m^2}$. The intensity of light emitted at a power P a distance r meters away from its source is $I=\frac{P}{4\pi r^2}$, so a star like the sun whose effective visual power is $1.95\cdot10^{26}$ W, must be at a distance r away from the eye such that $1.02\cdot10^{-10}\frac{W}{m^2}=\frac{1.95\cdot10^{26}W}{4\pi\cdot r^2}$

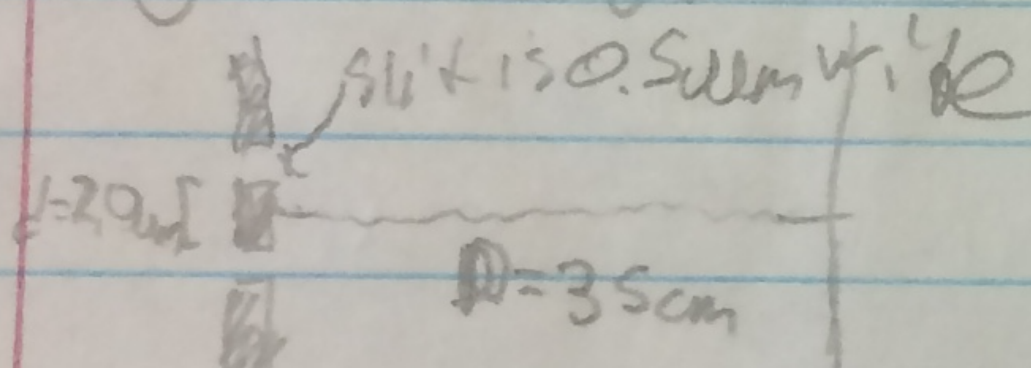
Solving for r, we get

$$r = \sqrt{\frac{1.95 \cdot 10^{26}\ W}{4 \cdot \pi \cdot 1.02 \cdot 10^{-10} \frac{W}{m^2}}} = 3.9 \cdot 10^{17}\ m \approx \boxed{41 \text{ light years}}$$

Most visible stars that we see in the sky are much more than 41 light years away, which means that most visible stars have an intrinsic brightness much greater than that of our sun.

QSM.3

Verify that in the Jönsson experiment, electrons with 50 keV of KE going through slits 2.0 μm wide will produce an interference pattern on a screen 35 cm away having adjacent bright spots ~1 μm apart.



The slits are a distance d = 2.0 μm apart, and the interference pattern is to be projected on a screen a distance D = 35 cm away from the slits. D >> d, so we can use $\frac{s}{D} = \frac{\lambda}{d}$, where s = distance between bright spots.

nonrelativistic approximation of $\lambda = \frac{h}{|\vec{p}|}$

In this case, $\lambda = \frac{hc}{\sqrt{2K \cdot mc^2}}$ where $h \cdot c = 1240\ eV \cdot nm$, $m \cdot c^2 = 511{,}000\ eV$ for an electron, and $K = 50{,}000\ eV$.

Solving for s, we get $s = \frac{\lambda \cdot D}{d} = \frac{1240\ eV \cdot nm}{\sqrt{2 \cdot 50{,}000\ eV \cdot 511000\ eV}} \cdot 35\ cm \cdot \frac{1\ nm}{10^{-7}\ cm} \div 2.0\ \mu m \cdot \frac{1\ nm}{10^{-3}\ \mu m}$

$\Rightarrow \frac{1240\ eV \cdot nm}{\sqrt{5.11 \cdot 10^{10}\ eV^2}} \cdot 3.5 \cdot 10^{8}\ nm \div (2.0 \cdot 10^{3}\ nm) = 968\ nm \approx \boxed{1.0\ \mu m}$

e)

QSM.10
The electrons used in the Jönsson experiment (with KE K = 50 keV) are not really nonrelativistic (they are moving at about 40% of c)

(a) Use the result of QSP.1 to calculate a relativistically correct wavelength for this electron beam.

From problem QSD.1a, we are given
$\lambda = \frac{hc}{\sqrt{K(K+2mc^2)}}$

So $\lambda = \frac{1240\,eV\cdot nm}{\sqrt{50000\,eV\cdot(50000\,eV + 2\cdot 511{,}000\,eV)}} \approx 0.005\,nm$

c^2

b) By about what percentage is problem QSM.3's nonrelativistic calculation of the bright-spot spacing in error

Using $\frac{s}{D} = \frac{\lambda}{d}$, $s = \frac{D\lambda}{d} = 3.5\cdot 10^8_{nm}\cdot 0.005_{nm} \div (2.0\cdot 10^3_{nm}) = 937\,nm$

The percent error of ~~problem~~ QSM 3's nonrelativistic calculation is $\frac{|968-937|}{937}\cdot 100\% = 3.3\%$ in error

Eval of answer

Q4D.7
15000 Photons per second is not a lot, so it seems reasonable that 41 lightyears is the maximum distance that a star like the sun could be (order of magnitude wise).

QSM.3
I did it! Right units

QSM.10
3.3% is lower than I expected, but not inconceivable.