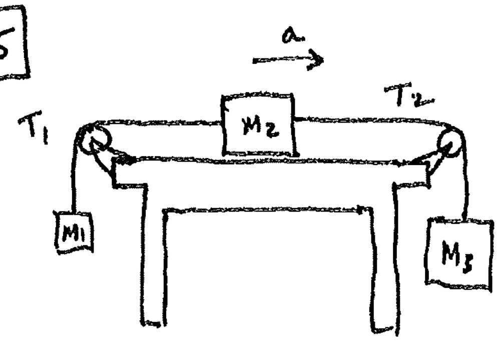
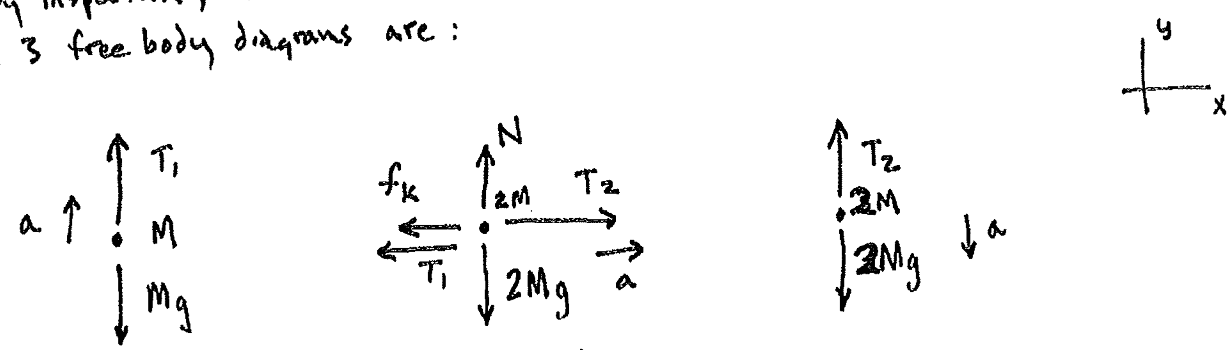
PHY 131 Problem Set #4
Halliday, Resnick & Walker, 8th Ed, Extended
Ch. 6: 25, 31, 51, 55, 65, 87,
Ch. 13: 9, 14, 17, 22

6:25



$m_1 = M$
$m_2 = 2M$
$m_3 = 2M$
$a = 0.500\ m/s^2$

By inspection, block M_2 will accelerate to the right as shown.
The 3 free body diagrams are:



Leading to this set of Newton's 2nd Law eqns:

① $T_1 - Mg = Ma$ $\quad$ $x: T_2 - T_1 - f_K = 2Ma$ ② $\quad$ $2Mg - T_2 = 2Ma$ ④

$y: N - 2Mg = 0$ ③

and we'll also need that $f_K = \mu_K N$. ⑤

③ $\Rightarrow$ $N = 2Mg$, then substitute into ⑤ and ②:

$$T_2 - T_1 - \mu_K(2Mg) = 2Ma \quad ⑥$$

Now solve ① and ④ for the tensions:

$$T_1 = M(a+g) \qquad T_2 = 2M(g-a)$$

and substitute into ⑥:

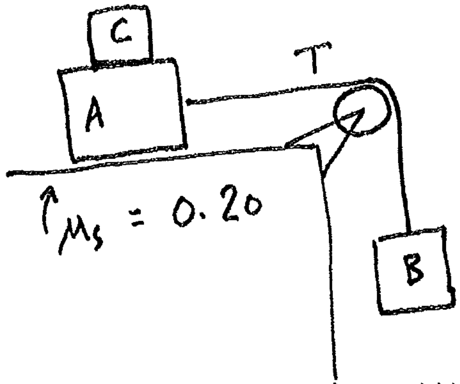
$$2M(g-a) - M(a+g) - \mu_K(2Mg) = 2Ma.$$

Rearrange to get $\mu_K(2Mg) = 2Mg - 2Ma - Ma - Mg - 2Ma$

$$\mu_K = \frac{Mg - 5Ma}{2Mg} = \frac{1}{2} - \frac{5a}{2g} = \frac{1}{2} - \frac{5 \times 0.5}{2 \cdot 9.8}$$

$$= \frac{9.8 - 2.5}{2 \cdot 9.8} = \underline{0.37}$$

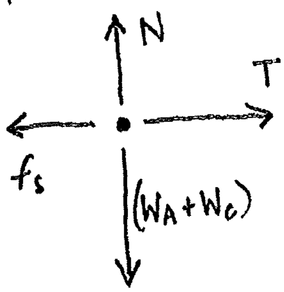
6:31

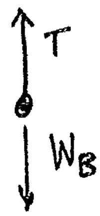


$W_A = 44N$
$W_B = 22N$

a) What minimum W_c keeps block A from slipping?

Apply Newton's 2nd, with no acceleration:





and $f_s \le \mu_s N$
look for the equality, which represents the maximum force that the friction can supply to keep block A from slipping.

x: $T - f_s = 0$ $\qquad$ $T - W_B = 0$

y: $N - (W_A + W_c) = 0$

$T - \mu_s N = 0$, $\quad N = (W_A + W_c)$, so

$T - \mu_s (W_A + W_c) = 0$

Substitute that $T = W_B$ to get

$W_B - \mu_s (W_A + W_c) = 0 \Rightarrow$

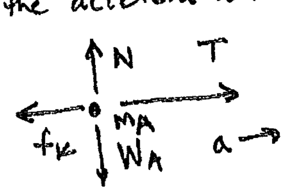
$W_B - \mu_s W_A - \mu_s W_c = 0$

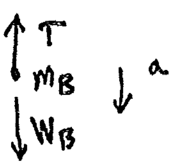
or $W_c = \dfrac{W_B - \mu_s W_A}{\mu_s} = \dfrac{22N - 0.2(44N)}{0.2}$

$= 110 - 44$

$= \underline{66\,N}$

b) Remove block C. Assume $\mu_K = 0.15$.
Find the acceleration of block A.





x: $T - f_K = m_A a$

y: $N - W_A = 0$

and $-T + W_B = m_B a$

and $f_K = \mu_K N$

6:31 cont'd

so $N = W_A$,

and $T - \mu_k N = m_A a$, so $T - \mu_k W_A = m_A a$.

Then $T = W_B - m_B a$, so eliminate T:

$$(W_B - m_B a) - \mu_k W_A = m_A a$$

$$W_B - \mu_k W_A = a(m_A + m_B), \quad \text{where } m_A = \frac{W_A}{g} \text{ and } m_B = \frac{W_B}{g}:$$

$$\frac{(W_B - \mu_k W_A)}{\frac{W_A}{g} + \frac{W_B}{g}} = a, \quad \text{so } a = g\frac{(W_B - \mu_k W_A)}{(W_A + W_B)}$$

$$= g\left(\frac{22N - 0.15 \times 44N}{(44N + 22N)}\right) = g\left(\frac{22 - 6.6}{66}\right)$$

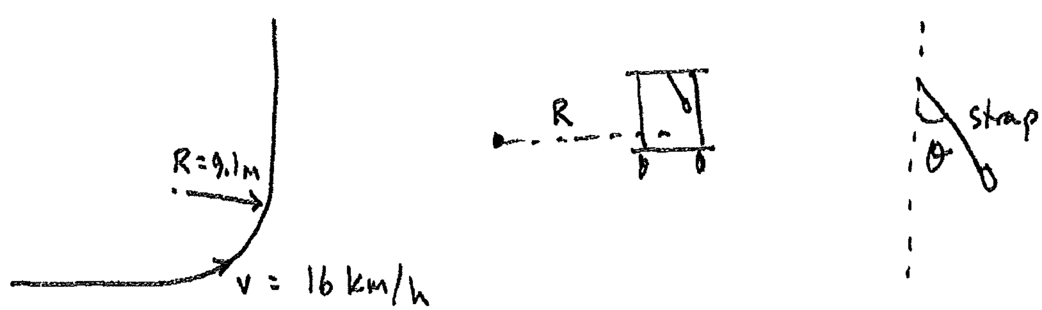
$$= g\left(\frac{1}{3} - \frac{1}{10}\right)$$

$$a = g\left(\frac{7}{30}\right)$$

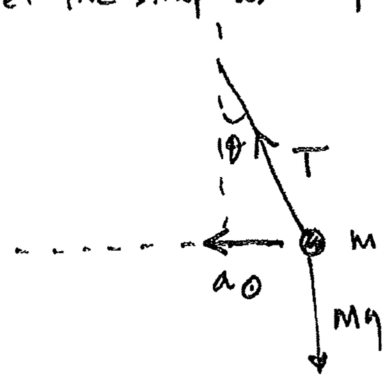
$$= g(0.23\bar{3})$$

$$\approx \underline{2.3 \text{ m/s}^2}$$

6:51



Let's model the strap as a point mass M on a string:



The tension in the string both holds up the weight of the strap and provides the force needed to allow the strap to have inward acceleration $a_\odot$.

$$x: \quad T\sin\theta = m a_\odot$$

$$y: \quad T\cos\theta - mg = 0$$

$$\text{so} \quad \left.\begin{aligned} T\sin\theta &= m a_\odot \\ T\cos\theta &= mg \end{aligned}\right\} \text{divide to give } \tan\theta = \frac{a_\odot}{g}$$

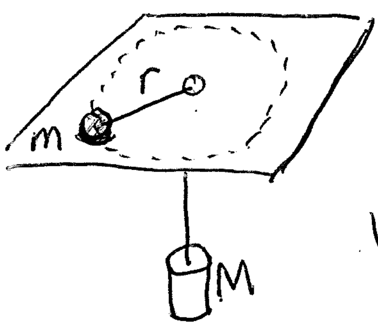
$\boxed{6:51 \text{ cont'd}}$

Now $a_\theta = \dfrac{v^2}{R} = \dfrac{\left(\dfrac{16\text{ km}}{\text{h}}\right)^2 \left(\dfrac{1\text{ h}}{60\text{ min}}\right)^2 \left(\dfrac{1\text{ min}}{60\text{ s}}\right)^2 \left(\dfrac{1000\text{ m}}{1\text{ km}}\right)^2}{(9.1\text{ m})} = 2.17\text{ m/s}^2$

so $\theta = \tan^{-1}\left(\dfrac{a_\theta}{g}\right) = \tan^{-1}\left(\dfrac{2.17}{9.8}\right) = 12.5^\circ$ $\left[12.2^\circ \text{ if } g = 10\text{ m/s}^2\right]$

$= \underline{\underline{12^\circ}}$ (2 sig. figs.)

$\boxed{6:55}$



$m = 1.5$ kg

$r = 20.0$ cm $= 0.2$ m

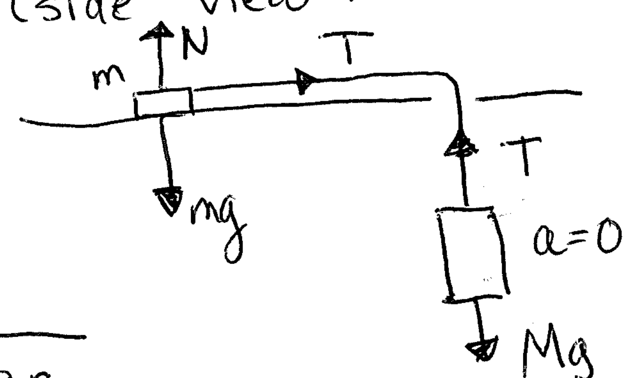
$M = 2.5$ kg

What speed does the puck need to be to keep at rest?

- Apply Newton's 2nd law to each mass separately; each mass has a different $\vec{a}_{net}$.

$T = ma_c = \dfrac{mv^2}{r}$

$T - Mg = M(0)$



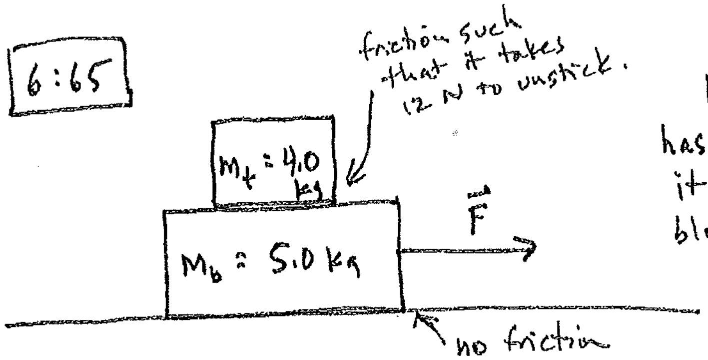
- Eliminating T gives

$\dfrac{mv^2}{r} = Mg$

$v^2 = \dfrac{M}{m} gr \Rightarrow |v| = \sqrt{\dfrac{M}{m} gr}$

$|v| = \sqrt{\dfrac{2.5}{1.5}(10\text{ m/s}^2)(0.2\text{ m})} = 1.8\text{ m/s}$

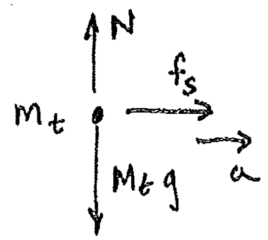
$\boxed{6:65}$



We know that when the top block has 12 N applied horizontally to it, it will slip on the bottom block.

a) Find the magnitude of the maximum horizontal force that can be applied to the lower block so that the blocks move together.

Consider M_t alone :



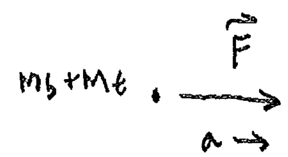
$x: \; f_s = M_t a$

$y: \; N - M_t g = 0$

and $f_s \leq \mu_s N$

The maximum frictional force is when $f_s = \mu_s N = \mu_s M_t g$

Now look at the bottom mass, which behaves as a single object of mass $M_t + M_b$ while it is stuck to the top mass :



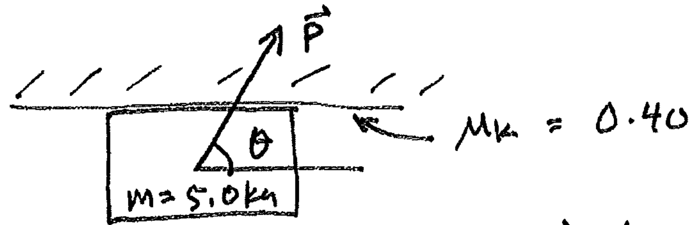
$F = (M_b + M_t)\, a$

so $a = \dfrac{F}{(M_b + M_t)}$

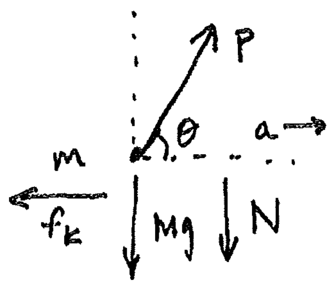
$$\therefore \; F_{max} = \left(\frac{5.0\text{ kg} + 4.0\text{ kg}}{4.0\text{ kg}} \right) 12\text{ N} = \frac{9}{4}(12\text{ N}) = 9 \cdot 3 = \underline{\underline{27\text{ N}}}$$

b) Then this maximum acceleration $a = \dfrac{f_s\,(max)}{M_t} = \dfrac{12\text{N}}{4.0\text{kg}} = \underline{\underline{3.0\text{ m/s}^2}}$

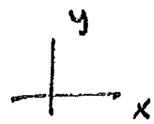
$\boxed{6:87}$



A crazed student pushes with force $\vec{P}$ (magnitude 80 N, angle $\theta = 70°$).
Find the magnitude of the block's acceleration.



Note that mg and N point in the same direction.



$x: \quad P\cos\theta - f_k = ma$ and $f_k = \mu_k N$, parallel to the ceiling

$y: \quad P\sin\theta - mg - N = 0 \quad \Rightarrow N = P\sin\theta - mg$

$$P\sin\theta = mg + N$$
$$P\cos\theta = ma + (\mu_k N)$$
$$P\cos\theta = ma + \mu_k (P\sin\theta - mg)$$
$$\frac{P\cos\theta - \mu_k P\sin\theta + \mu_k mg}{m} = a$$

$$\frac{80 \cdot (.342) - 0.4 \cdot 80 \cdot 0.940 + .4 \cdot 5 \cdot 9.8}{5} = 3.38\ m/s^2 = \underline{\underline{3.4}}\ m/s^2$$

2 sig. figs.

13-9 The gravitational force the Earth exerts on me (mass m) is $|\vec{F}_{G_E}| = \frac{GM_E m}{R_E^2} = mg$.
(where $\frac{GM_E}{R_E^2} = 9.8\,m/s^2$)

If r_{BH} is the distance between me and a tiny black hole, the gravitational force it exerts on me is $|\vec{F}_{G_{BH}}| = \frac{GM_{BH}m}{r_{BH}^2}$

Equate these two and solve for r_{BH}.

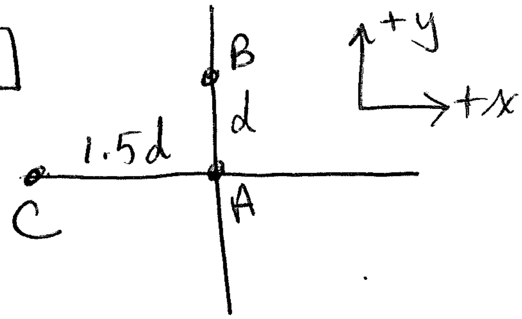
$$\frac{\cancel{G}M_E\cancel{m}}{R_E^2} = \frac{\cancel{G}M_{BH}\cancel{m}}{r_{BH}^2}$$

$$r_{BH} = \sqrt{\frac{M_{BH}R_E^2}{M_E}} = \sqrt{\frac{(1\times10^{11}\,kg)(6.37\times10^6\,m)^2}{(5.98\times10^{24}\,kg)}}$$

$r_{BH} = 0.8\,m$

Altenately: $\frac{GM_{BH}\cancel{m}}{r_{BH}^2} = \cancel{m}g \Rightarrow r_{BH} = \sqrt{\frac{GM_{BH}}{g}}$

$\boxed{13\text{-}14}$



m_A
$m_B = 2m_A$
$m_C = 3m_A$
$m_D = 4m_A$

Where to put m_D to cancel effects of m_B and m_C?

First, what are $\vec{F}_{AB}$ and $\vec{F}_{AC}$?

x-dir

$$\vec{F}_{AC} = \frac{-Gm_Am_C}{(\frac{3}{2}d)^2}\hat{i}$$

$$= \frac{-G(3m_A^2)}{\frac{9}{4}d^2}\hat{i}$$

$$\vec{F}_{AC} = -\frac{4}{3}\frac{Gm_A^2}{d^2}\hat{i}$$

y-dir

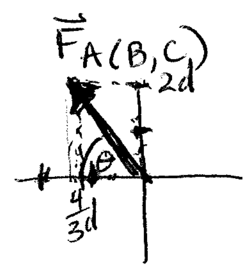
$$\vec{F}_{AB} = \frac{Gm_Am_B}{d^2}\hat{j}$$

$$= \frac{G(2m_A^2)}{d^2}\hat{j}$$

$$\vec{F}_{AB} = \frac{2Gm_A^2}{d^2}\hat{j}$$

we require $\sum \vec{F} = 0$

$$\vec{F}_{AC} + \vec{F}_{AB} + \vec{F}_{AD} = 0$$



what is $|\vec{F}_{AD}|$ and θ_{AD}?

$$|\vec{F}_{AC} + \vec{F}_{AB}| = \sqrt{\left\{\left(-\frac{4}{3}\right)^2 + (2)^2\right\}\left(\frac{Gm_A^2}{d^2}\right)^2}$$

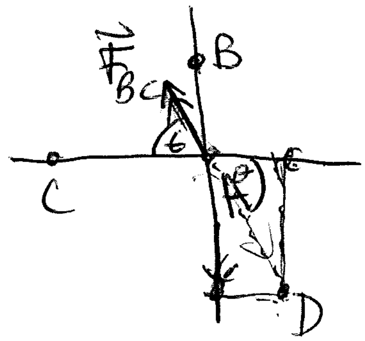
$$= 2.4\frac{Gm_A^2}{d^2}$$

$$\theta = \tan^{-1}\left(\frac{F_{AB}}{F_{AC}}\right) = \frac{-2}{4/3} = -\frac{3}{2}$$ $= -56.3°$ QII above $-x$ axis

$\boxed{13\cdot14}$ cont.

$\vec{F}_{AD}$ must have magnitude $2.4\,\dfrac{Gm_A^2}{d^2}$

and be located 56.3 Q IV
(below +x axis)



$F_B + F_C$ attract m_A

$\vec{F}_D$ must attract m_A opposite direction: $56.3°$

so $$2.4\,\frac{Gm_A^2}{d^2} \equiv \frac{Gm_A m_D}{r^2} = \frac{4Gm_A^2}{r^2}$$

$$r^2 = \frac{4\cancel{Gm_A^2}}{2.4\cancel{Gm_A^2}}\,d^2 = 1.67d^2$$

$$r = 1.29\,d$$

$$\vec{F}_{AD} = +r\cos\theta\,\hat{\imath} - r\sin\theta\,\hat{\jmath}$$
$$= (1.29d)\cos 56.3\,\hat{\imath} - (1.29\,d)\sin 56.3\,\hat{\jmath}$$
$$= 0.71d\,\hat{\imath} - 1.07d\,\hat{\jmath}$$

Here are two ways to approach this problem.

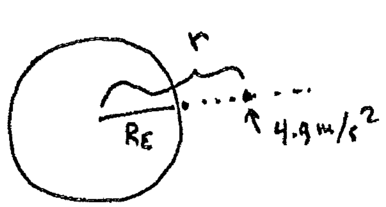
[13:17] ① The acceleration due to gravity is given by $a_g = \frac{GM}{r^2}$, where M = mass of Earth, and r is the distance from Earth's center. Substitute $r = R + h$, where R is the radius of Earth and h is the altitude. Solve for h, given $a_g = 4.9\ m/s^2$

$$a_g = \frac{GM}{(R+h)^2} \Rightarrow (R+h)^2 = \frac{GM}{a_g} \Rightarrow R+h = \sqrt{\frac{GM}{a_g}}$$

$$h = \sqrt{\frac{(6.67\times10^{-11}\ m^3/s^2\cdot kg)(5.98\times10^{24}\ kg)}{4.9\ m/s^2}} - 6.37\times10^6\ m$$

$$= 2.6\times10^6\ m$$

Compare this with the Int'l Space Station, which orbits at an altitude between 330 km to 435 km.

② 

At Earth's surface

$$F_{grav} = \frac{GM_E m}{R_E^2} = mg$$

$$\text{so} \quad g = \frac{GM_E}{R_E^2}$$

We want $g \rightarrow g/2$ (i.e. $9.8\ m/s^2 \longrightarrow 4.9\ m/s^2$), so

$$\frac{g}{2} = \frac{GM_E}{2R_E^2} = \frac{GM_E}{r^2}, \text{ where}$$

$$r^2 = 2R_E^2$$

or $r = \sqrt{2}\,R_E$ is the new radius. But the problem asks for the altitude (i.e. distance above R_E), so we get

$$r - R_E = \sqrt{2}\,R_E - R_E = (\sqrt{2}-1)R_E$$

$= 0.414\,R_E$ with $R_E = 6.37\times10^6\ m$

$= \underline{2.6\times10^6\ m} = \underline{2600\ km}$

13:22 (a) Plugging $R_h = \frac{2GM_h}{c^2}$ into the indicated expression, we find

$$a_g = \frac{GM_h}{(1.001R_h)^2} = \frac{GM_h}{(1.001)^2(2GM_h/c^2)^2} = \frac{c^4}{(2.002)^2 G}\frac{1}{M_h}$$

$$= \frac{3.02\times10^{43}\ \text{kg}\cdot\text{m/s}^2}{M_h}$$

(b) Note M_h in denominator, so a_g decreases as M_h increases.

(c) for $M_h = (1.55\times10^{12})(1.99\times10^{30}\ \text{kg}) = 3.08\times10^{42}\ \text{kg}$

$a_g = 9.82\ \text{m/s}^2$

(d) Refer to sample problem 13.3 for part of the necessary information. The difference in a_g, da_g (Eq. 13.16)

$\left[da_g = -\frac{2GM_E}{r^3}dr\right]$ becomes $da_g = \frac{-2\ GM_h\ dr}{(2.002\ GM_h/c^2)^3}$

$dr \rightarrow 1.7\ \text{m}$ (height of astronaut in sample problem 13.3)

$|da_g| = 7.3\times10^{-15}\ \text{m/s}^2$

(e) the differences of gravitational forces on the body will be negligible. Note that the difference between this problem and Sample Problem 13-3 is the mass of the black hole. A "super massive" black hole doesn't exert the same differential force over a distance 1.7 m.