PHY 131 PROBLEM SET #1

HALLIDAY, RESNICK & WALKER, 8^{th} Ed.

CH. 2 : Q2, Q4, P1, P6, P13, P16, P18, P21, P44, P45

Q2 a) at $t=0$, x is negative
velocity is positive (the slope is pos.)

b) at $t=1s$, x is zero
v is positive

c) at $t=2s$, x is positive
v is zero

d) at $t=3s$, x is zero
v is negative

e) the particle goes through $x=0$ twice,
at $t=1s$ and at $t=3s$

Q4 The graph shows acceleration vs. time.

Since $a=\frac{dv}{dt}$, $a=0$ when $\frac{dv}{dt}=0$

$\frac{dv}{dt}=0$ when the chihuahua runs at constant speed OR stands still

$\boxed{a(t)=0 \text{ in the segment of the graph labeled } E}$

$\boxed{2:1}$ Automobile travels 40 km at 30 km/h, and then another 40 km at 60 km/h. a) Find the average velocity of the car during the complete trip.

$$\bar{v}_1 = \frac{\Delta x_1}{\Delta t_1} \Longrightarrow \Delta t_1 = \frac{\Delta x_1}{\bar{v}_1} = \frac{40\text{ km}}{30\text{ km/h}} = 4/3\text{ h}$$

is the time taken to travel the first half of the trip.

Similarly,

$$\bar{v}_2 = \frac{\Delta x_2}{\Delta t_2} \Longrightarrow \Delta t_2 = \frac{\Delta x_2}{\bar{v}_2} = \frac{40\text{ km}}{60\text{ km/h}} = 2/3\text{ h}$$

is the time taken to travel the second half of the trip.

Overall, then,

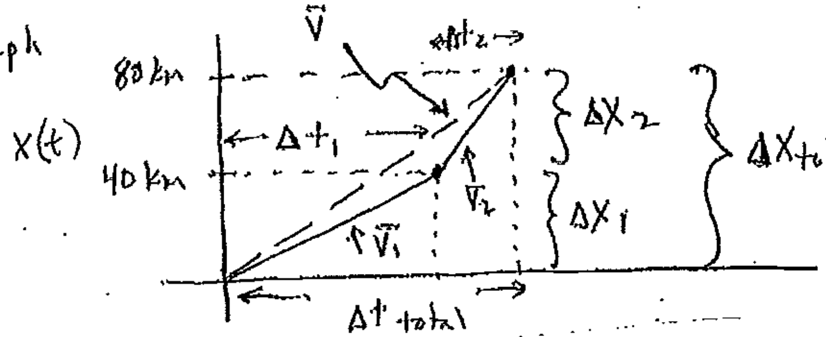
$$\bar{v}_{\substack{\text{complete}\\ \text{trip}}} = \frac{\Delta x_1 + \Delta x_2}{\Delta t_1 + \Delta t_2} = \frac{80\text{ km}}{\frac{4}{3}\text{ h} + \frac{2}{3}\text{ h}} = \frac{80\text{ km}}{2\text{ h}} = \underline{\underline{40\frac{\text{km}}{\text{h}}}}$$

All distances and times are measured in the positive direction, so the result is positive as well.

b) What is the average speed?

speed $= |\bar{v}| = 40$ km/h. No surprises here, because the displacement is always positive. (If you were to repeat the journey in the reverse direction you'd still find the same average speed, but the average velocity would now be 0 because the net displacement is zero.)

c) Graph



Note that the slopes $\bar{v}_1$, $\bar{v}_2$, and $\bar{v}$ refer to the velocities for the various segments, and that

$$\bar{v}_2 > \bar{v} > \bar{v}_1$$

The average velocity for the entire trip is closer to 30 km/h than to 60 km/h because the automobile spends more time during t_1 than during t_2.

2:6 Compute your average velocity in the following two cases:

a) Walk 73.2 m at a speed of 1.22 m/s, then run 73.2 m at a speed of 3.05 m/s along a straight track.

$$V_1 = \frac{\Delta X}{\Delta t} \Longrightarrow \Delta t = \frac{\Delta X}{V}, \text{ so}$$

$$\Delta t_1 = \frac{73.2\,m}{1.22\,m/s} = 60.0\,s$$

$$\Delta t_2 = \frac{73.2\,m}{3.05\,m/s} = 24.0\,s$$

Hence $\overline{V}_{overall} = \frac{\text{Total distance}}{\text{total time}} = \frac{73.2\,m + 73.2\,m}{(60.0 + 24.0)\,s} = \frac{146.4\,m}{84.0\,s}$

$$= 1.743\ m/s$$

Keep only 3 significant figures (to match 3 s.f. in problem)

so $\overline{V}_{overall} = 1.74\ m/s$

b) You walk for 1.00 min at a speed of 1.22 m/s and then run for 1.00 min at 3.05 m/s along a straight track:

$$\vec{V} = \frac{\Delta X}{\Delta t} \Longrightarrow \Delta X_1 = V_1 \Delta t_1$$

$$= (1.22\,m/\cancel{s})(1.00\,\cancel{min})\left(\frac{60\,s}{1\,\cancel{min}}\right) = 73.2\,m$$

$$\Delta X_2 = V_2 \Delta t_2$$

$$= (3.05\,m/\cancel{s})(1.00\,\cancel{min})\left(\frac{60\,s}{1\,\cancel{min}}\right) = 183\,m$$

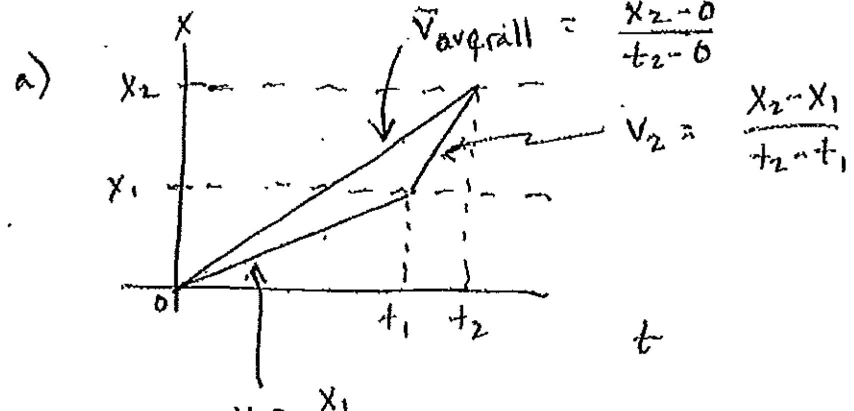
$$\overline{V}_{overall} = \frac{\text{total distance}}{\text{total time}} = \frac{(73.2\,m + 183\,m)}{2.00\,\cancel{min}}\left(\frac{1\,\cancel{min}}{60\,s}\right)$$

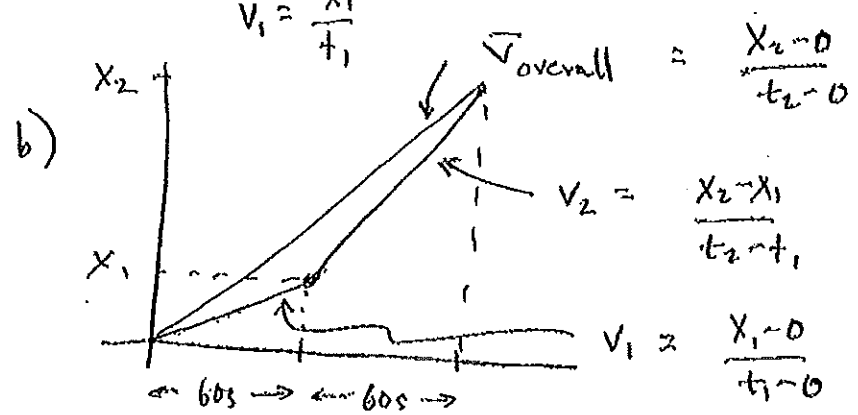
$$= 2.135\ m/s$$

Keep 3 significant figures: $2.14\ m/s$

2:6 cont'd.

c) graph both cases a) & b)





2:13 You drive on Interstate 10 from San Antonio to Houston, half the time at 55 km/h and the other half at 90 km/h. On the way back you travel half the distance at 55 km/h and the other half at 90 km/h. What is your average speed a) from San Antonio to Houston, b) from Houston back to San Antonio, and c) for the entire trip?

a) Here your average speed is the simple average of the two segments. Let L be the total distance traveled, t_1 the time for the 1st half, and t_2 the time for the second half (Here $t_1 = t_2 = \frac{1}{2}T$, the total time). In the 1st half you travel L_1, in the second L_2, and the total is $L = L_1 + L_2$

2.13 a) cont'd.

So $\bar{v} = \dfrac{L}{t_1+t_2} = \dfrac{L_1+L_2}{t_1+t_2} = \dfrac{v_1 t_1 + v_2 t_2}{t_1+t_2}$

But $t_1 = t_2$ so $|\vec{V}_{\text{on the way there}}| = \dfrac{(V_1+V_2)\,t_1}{2t_1} = \dfrac{V_1+V_2}{2}$

$= \dfrac{55\text{ km/h} + 90\text{ km/h}}{2}$

$= \underline{\underline{72.5}}\text{ km/h}$ (round to $\underline{73\text{ km/h}}$ 2 significant figures)

b) $\bar{v} = \dfrac{L}{T} = \dfrac{L_1+L_2}{t_1+t_2} = \dfrac{L_1+L_2}{\frac{L_1}{V_1}+\frac{L_2}{V_2}}$, with $L_1 = L_2$ this time,

so $\dfrac{2L_1}{\frac{L_1}{V_1}+\frac{L_1}{V_2}} = \dfrac{2L_1}{L_1\left(\frac{V_2+V_1}{V_1V_2}\right)} = \dfrac{2V_1V_2}{V_1+V_2} = \dfrac{2\cdot 55\cdot 90\,(\text{km/h})^2}{(55+90)\text{ km/h}}$

$= 68.3\text{ km/h}$

$= \underline{\underline{68\text{ km/h}}}$ (2 sig. figures)

c) Entire trip (average $\underline{\text{speed}}$)

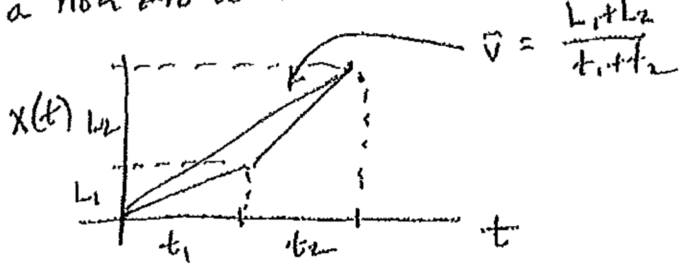
$|\vec{V}| = \dfrac{L_{total}}{T_{total}} = \dfrac{2L}{T_a+T_b} = \dfrac{2L}{\frac{L}{\bar{V}_a}+\frac{L}{\bar{V}_b}} = \dfrac{2\bar{V}_a\bar{V}_b}{\bar{V}_a+\bar{V}_b} = \dfrac{2(72.5)(68.3)}{(72.5+68.3)}$

$= 70.3\text{ km/h}$

$= \underline{70\text{ km/h}}$ (2 sig. fig.)

d) Average velocity for entire trip is $\underline{\text{zero}}$ because the signed displacement is zero and it takes a non zero amount of time.

e) Sketch for case a)



$\boxed{2:16}$ An electron travels along x at

$$x(t) = 16\,t\,e^{-t}$$, where x is in meters, t is in seconds.

How far is the electron from the origin when it momentarily stops?

The instantaneous velocity is the derivative of $x(t)$ with respect to time:

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}\left[16\,t\,e^{-t}\right].$$

$\left(\text{i.e. } \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\right)$

Use the product rule (#6, Appendix E, A-11)

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= 16\left[t\cdot(-1e^{-t}) + 1\cdot e^{-t}\right]$$

Here $u = t$, $v = e^{-t}$ and $dx = dt$

(because $e^{-x} = -e^{-x}$)

$$v(t) = 16\left[(1-t)\,e^{-t}\right]$$

This function goes to zero when $t = 1$. This occurs when at

$$x(1) = 16 \cdot 1 \cdot e^{-1} = 16\,(0.37) = \underline{5.9\text{ m}}$$

$\boxed{2:18}$ If $x = 20t - 5t^3$ (x in meters t in seconds)

$$v(t) = \frac{dx}{dt} = 20 - 15t^2$$ (#4, Appendix E, A-11) applied twice

This is zero when
$$0 = 20 - 15t^2$$
$$15t^2 = 20$$
$$t^2 = \frac{4}{3} \implies t = \pm\sqrt{\frac{4}{3}} = \underline{\pm 1.15\text{ m}}$$

b) When is $a = 0$?

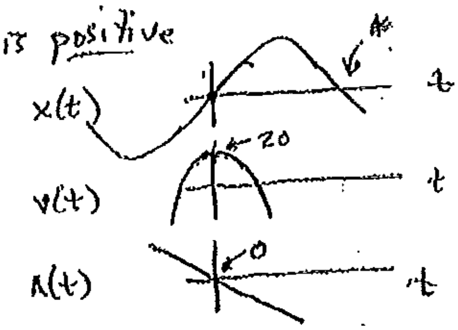
$$a(t) = \frac{d}{dt}v(t) = \frac{d}{dt}\left[20 - 15t^2\right] = 0 - 30t = -30t.$$

$0 = -30t$ occurs at $\underline{t = 0}$

c) Note that a is negative when $t > 0$, so t is positive

d) a is positive when $t < 0$, or t is negative

e) Graphs



$\boxed{2.21}$ $x(t) = ct^2 - bt^3$ is an equation giving position (in units of length, or meters)

[x] has units of [length], therefore

[ct²] must have units of [length]

[bt³] must have units of [length]

[ ] denotes units

a) since $[ct^2] = [\text{length}] = [c][\text{seconds}^2]$

c must have inverse units of seconds² and length units of meters

$[c] = [m/s^2]$ in SI units

b) since $[bt^3] = [\text{length}] = [b][\text{seconds}^3]$

$[b] = [m/s^3]$ in SI units

c) when a particle reaches its maximum (or minimum) position, its velocity $v = 0$.

$$v(t) = \frac{dx}{dt} = 2ct - 3bt^2$$

$v = 0$ when $\boxed{t = 0}$ AND

when $2ct - 3bt^2 = 0$

$$2c\cancel{t} = 3bt^{\cancel{2}}$$

$$\boxed{\frac{2c}{3b} = t}$$

for $c = 3\ m/s^2$ and $b = 2\ m/s^3$

$$\boxed{t = \frac{2(3\ m/s^2)}{3(2\ m/s^3)} = 1\ s}$$

2.21 cont'd $\quad -x \longleftrightarrow +x$

d) In the first 4s, the particle moves from

$x(t=0) = 0$ to $x(t=4s) = -80$ m

But notice the total distance:

$x(t=1) = 1$ m (in the positive direction)

$x(t=2) = -4$ m (in the negative direction)

$x(t=3) = -17$ m

The particle first moves 1 m to the right then reverses direction and continues to the left, passing through zero (again) and continuing in a negative direction

The total distance traveled is 82 m

e) The displacement, $\boxed{x(t=4s) - x(t=0) = -80\text{m}}$

f) The velocity is given by $v(t) = 2ct - 3bt^2$

at $t = 1s$, for $c = 3\ \text{m/s}^2$ and $b = 2\ \text{m/s}^3$

$$v(t=1s) = 2(3\ \text{m/s}^2)(1s) - 3(2\ \text{m/s}^3)(1s)^2$$

$$\boxed{v(1s) = \emptyset}$$

g) at $t = 2s$,

$$v(t=2s) = 2(3\ \text{m/s}^2)(2s) - 3(2\ \text{m/s}^3)(2s)^2$$

$$\boxed{= -12\ \text{m/s}}$$

h) $\boxed{v(t=3s) = -36\ \text{m/s}}$

i) $\boxed{v(t=4s) = -72\ \text{m/s}}$

2.21 cont'd

j) Acceleration is given by $a = \frac{dv}{dt}$

$a(t) = 2c - 6bt$

for $c = 3\ m/s^2$ and $b = 2\ m/s^3$

$a(t) = 6\ m/s^2 - (12\ m/s^3)t$

at $t = 1s$,

$\boxed{a(t=1s) = 6 - 12 = -6\ m/s^2}$

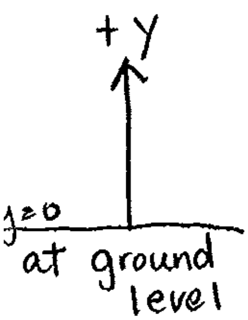
k) at $t = 2s$, $\boxed{\begin{aligned} a(t=2s) &= 6\ m/s^2 - (12\ m/s^3)(2s) \\ &= -18\ m/s^2 \end{aligned}}$

l) at $t = 3s$, $\boxed{a(t=3s) = -30\ m/s^2}$

m) at $t = 4s$ $\boxed{a(t=4s) = -42\ m/s^2}$

2.44 Ignoring air resistance, we can set $a = g = -9.8 \frac{m}{s^2}$

Down is our $-y$ direction



We know:

$y_0 = 1700\ m$

$y_f = 0\ m$ (ground level)

$a = -9.8\ m/s^2$

$v_0 = 0$ (assume raindrops fall from rest)

We want to find: v_f

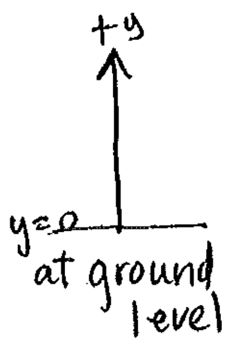
$v_f^2 = v_0^2 + 2a(y_f - y_0)$

$v_f = \pm\sqrt{-2(9.8\ m/s^2)(-1700\ m)} = \pm 183\ m/s$ (negative root is correct for our direction)

$\boxed{|v_f| = 183\ m/s}$

2.44 cont'd b) probably not safe, but we'd need more data to make our case. There is a LOT of air resistance, so rain drops fall much more slowly, actually

2.45



We know:

$V_0 = \varnothing$ (assume the wrench was dropped from rest)

$V_f = -24$ m/s (given our choice of direction; DOWN is negative y)

$a = -9.8$ m/s² ($a = g$ neglecting air resistance)

$Y_f = 0$ (ground level)

We want to know: Y_0

$$V_f^2 = V_0^2 + 2a(Y_f - Y_0)$$

$$V_f^2 = V_0^2 - 2g\,\Delta y$$

$$V_f^2 - \cancel{V_0^2}^{\,0} = -2g\,\Delta y$$

$$\Delta y = -\frac{V_f^2}{2g} = -\frac{(-24\ m/s^2)^2}{2(9.8\ m/s^2)} = -29.4\ m$$

$$\cancel{Y_f}^{\,\varnothing} - Y_0 = -29.4\ m$$

$$\boxed{Y_0 = 29.4\ m}$$

We want to know: Δt

$$V_f = V_0 - gt$$

$$t = \frac{V_0 - V_f}{g} = \frac{0 - (-24\ m/s)}{9.8\ m/s^2}$$

$$\boxed{t = 2.45\ s}$$

2.45 cont'd

c) I don't show the acceleration graph, which is a horizontal line at $-9.8\ m/s^2$