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Phy 232
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hw for 10/9/17
Q7R.2, Q8D.7

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Q7R.2

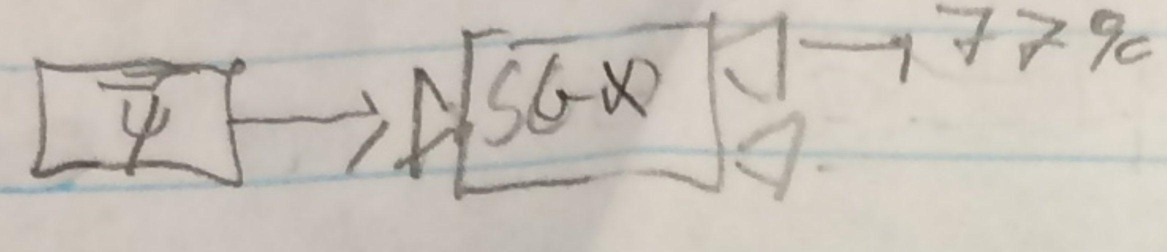
Suppose we have a box that emits electrons in a definite but unknown spin state $|\psi\rangle$. If we send electrons from this box through an SGθ device with θ such that $\cos\frac{1}{2}\theta = \frac{3}{5}$ and $\sin\frac{1}{2}\theta = \frac{4}{5}$, we find that $\frac{16}{25}$ are determined to have $S_\theta = +\frac{1}{2}\hbar$. Assuming that the components of $|\psi\rangle$ are real, argue that there are two distinct q-vectors for $|\psi\rangle$ consistent with this result. If we send electrons from this box through an SGx device and find that 77% are determined to have $S_x = +\frac{1}{2}\hbar$, what is $|\psi\rangle$?

Let the spin state vector $|\psi\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$. We know a state vector must be normalized, that is, $\langle\psi|\psi\rangle = 1$.
So we get $\langle\psi|\psi\rangle = [a^*, b^*] \begin{bmatrix} a \\ b \end{bmatrix} = a^*a + b^*b$, where a^*, b^* are the complex conjugates of a, b. However, we are told to assume that the components of $|\psi\rangle$ are real, so we can say that $a^* = a$. So we get
$a^2 + b^2 = 1 \Rightarrow b = \sqrt{1-a^2}$

We are given that the probability of electrons leaving the SGθ device have $S_\theta = +\frac{1}{2}\hbar$. By the outcome probability rule, we have
$|\langle +\theta|\psi\rangle|^2 = \frac{16}{25} \Rightarrow \langle +\theta|\psi\rangle = \frac{4}{5} \Rightarrow [\cos\frac{1}{2}\theta, \sin\frac{1}{2}\theta] \begin{bmatrix} a \\ b \end{bmatrix} = \frac{4}{5}$
$\Rightarrow \cos\frac{1}{2}\theta = \frac{3}{5}$, $\sin\frac{1}{2}\theta = \frac{4}{5}$, so we get $\frac{3}{5}\cdot a + \frac{4}{5}\cdot b = \frac{4}{5}$
One possibility is that $a = 0$, $b = 1$. $0^2 + (1)^2 = 1$, so these values work.
However, we can also solve for a:
$\frac{4}{3}(1-b) = a$; Plugging into $b = \sqrt{1-a^2} \Rightarrow b = \sqrt{1-(\frac{16}{9}(1-b)^2)}$
$b^2 = 1 - \frac{16}{9}(1-2b+b^2) \Rightarrow b^2 = 1 - \frac{16}{9} + 2\cdot\frac{16}{9}b - \frac{16}{9}b^2 \Rightarrow \frac{9+16}{9}b^2 - \frac{32}{9}b + \frac{16-9}{9} = 0$
$\Rightarrow \frac{25}{9}b^2 - \frac{32}{9}b + \frac{7}{9} = 0$
$b = \dfrac{\frac{32}{9} \pm \sqrt{(\frac{32}{9})^2 - 4\cdot\frac{25}{9}\cdot\frac{7}{9}}}{2\cdot\frac{25}{9}} = \dfrac{\frac{32}{9} \pm \sqrt{\frac{1024-700}{81}}}{\frac{50}{9}} = \dfrac{\frac{32}{9} \pm \sqrt{\frac{324}{81}}}{50/9} = \dfrac{\frac{32}{9} \pm \frac{18}{9}}{50/9}$
$b = \frac{32 \pm 18}{50}$. So we have $b = \left(\frac{32+18}{50}\right) = \left(\frac{50}{50}\right) = 1$ $b = \left(\frac{32-18}{50}\right) = \frac{14}{50} = \frac{7}{25}$
We already found a for $b = 1$, so let's find a for $b = \frac{7}{25}$:
$\left(\frac{7}{25}\right)^2 = 1 - a^2 \Rightarrow a^2 = 1 - \left(\frac{7}{25}\right)^2 = \frac{625-49}{625} = \frac{576}{625}$
So $a = \sqrt{\frac{576}{625}} = \frac{24}{25}$. So $|\psi\rangle = \begin{cases} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 24/25 \\ 7/25 \end{bmatrix} \end{cases}$

we sent directly into an SGx device

Now we are given that 77% of them have $S_x = +\frac{1}{2}\hbar$. I will assume that the situation is like that pictured below:



By the outcome probability rule, we have

$0.77 = |\langle +x|\psi\rangle|^2 \Rightarrow \sqrt{0.77} = |\langle +x|\psi\rangle|$

$\Rightarrow \sqrt{0.77} = \left|\begin{bmatrix}\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}\end{bmatrix}\begin{bmatrix}a \\ b\end{bmatrix}\right| \Rightarrow \sqrt{0.77} = \left|\sqrt{\frac{1}{2}}\cdot a + \sqrt{\frac{1}{2}}\cdot b\right|$

If we let $|\psi\rangle = \begin{bmatrix}1 \\ 0\end{bmatrix}$, we get $\sqrt{0.77} = \sqrt{\frac{1}{2}}$ which is not true.

If we let $|\psi\rangle = \begin{bmatrix}24/25 \\ 7/25\end{bmatrix}$, we get $\sqrt{0.77} = \left|\sqrt{\frac{1}{2}}\cdot\frac{31}{25}\right|$

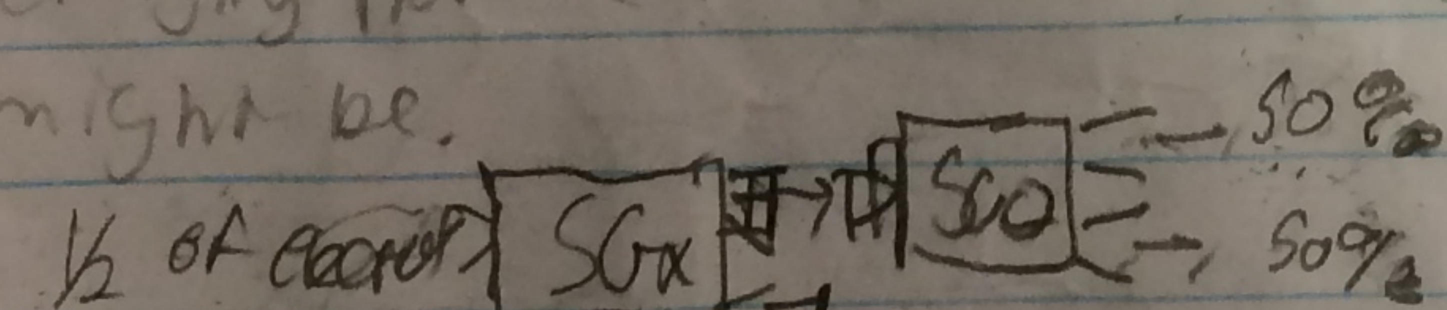
$\sqrt{0.77} = 0.877$

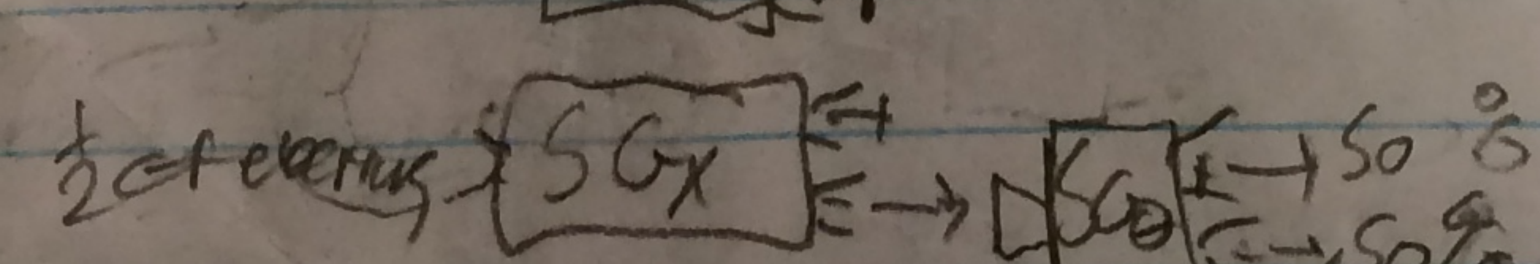
$\sqrt{\frac{1}{2}}\cdot\frac{31}{25} = 0.877$

So $|\psi\rangle = \begin{bmatrix}24/25 \\ 7/25\end{bmatrix}$

Q8D.1

Imagine that a black box prepares electrons in such a way that half have state $|+x\rangle$ and half have state $|-x\rangle$. If we then send this classical mixture of electrons through an SGθ device, show that the probabilities of emerging from the + and − channels are $\frac{1}{2}$ and $\frac{1}{2}$ no matter what the angle θ might be.





Because we have a mixture of half $|+x\rangle$ and half $|-x\rangle$ electrons, their states are already "collapsed" (they are not in superposition). This would be analogous to the situation shown above, where the state of the electron coming out of each box is already known, so the total probability that an electron comes out of the + port is $\frac{1}{2}$ and the total probability it comes out of the − port is $\frac{1}{2}$.

Evil Q answers:

Q 7R.2
Right units, Reasonable magnitudes

Q 8M.1
Trying to do a mathematical approach using the superposition rule didn't yield the right answer, so I realized that the state of the electrons were already determined, and this made it different from a system where electrons are coming out of a single source device going into parts