

## DC (Direct Current) Circuits

### Summary of Concepts and Equations

We developed our notions of electrical current flow following our understanding of fluid flow. We found that electrical current only flowed in a complete loop of conducting material (generally metals), or a network of complete loops. We developed a number of analogies:

Voltage  $\longleftrightarrow$  Pressure  
Electrical Current  $\longleftrightarrow$  Fluid Flow  
Battery  $\longleftrightarrow$  Pump  
Resistors and Bulbs  $\longleftrightarrow$  Flow Restrictions

We also learned to draw schematic diagrams of circuits using symbols for the various circuit elements: batteries, resistors, bulbs, switches, capacitors, voltmeters, and ammeters. We learned how to draw diagrams of real circuits, and how to construct real circuits from diagrams. We found that ammeters only work usefully when they are in series with the current we wish to measure, and voltmeters work meaningfully only when they are in parallel across the voltage source or voltage drop we wish to measure.

We studied the notion of resistance with some care. The entire concept is only meaningful for materials in which the current is proportional to the applied voltage difference (sometimes termed "Ohmic" materials). This allows us to define a resistance for a conducting material given implicitly by Ohm's law,

$$\Delta V = IR .$$

We also found that the resistance of a material was proportional to the length of the conducting material. The resistance is also inversely proportional to the cross-sectional area. The intrinsic resistance of the material can be expressed in terms of a constant for the material known as the resistivity  $\rho$  such that the resistance for some material of length  $l$  and cross-sectional area  $A$  is given by

$$R = \rho \frac{l}{A}.$$

We also found that combining two resistors in series or parallel gave a circuit that was externally equivalent to a single resistor, given by the expressions

$$\begin{aligned} \text{Series: } R_{total} &= R_1 + R_2 \quad \text{and} \\ \text{Parallel: } \frac{1}{R_{total}} &= \frac{1}{R_1} + \frac{1}{R_2}. \end{aligned}$$

These rules can be applied repeatedly to solve for currents through rather complex circuits.

There are some circuits, however, which are not solvable using simply equivalent resistors. For these circuits, one must apply a more systematic technique utilizing Kirchhoff's rules. These rules may be summarized as:

- 1) The sum of the currents flowing into a junction is equal to the sum of the currents flowing out of that same junction (or equivalently, the algebraic sum of all the currents at a junction is equal to zero).
- 2) The sum of the voltage changes over any complete loop in the circuit is equal to zero.

Application of these rules to solve a problem also must be systematic.

- 1) Define a separate current for each circuit leg, i.e. from each junction to the next junction by each possible path. Label your currents, and include a defined direction (which is arbitrary).
- 2) Write junction equations for each independent junction (typically all but one). Make sure to pay attention to your *defined* current direction.
- 3) Write loop equations for each independent loop (typically all but one). Here you must carefully assign signs to each voltage change as you travel an imaginary trip around the complete loop:
  - a. negative to positive on battery is a positive voltage change, positive to negative on battery is a negative voltage change,
  - b. moving with the defined current direction across a resistor is a negative voltage change, moving against the current direction is a positive change.
- 4) Solve the resulting equations simultaneously for (typically) each current. You must have an independent equation for each unknown. Make sure that you have each current, each battery voltage, and each resistance included in at least one equation.