Phy 232
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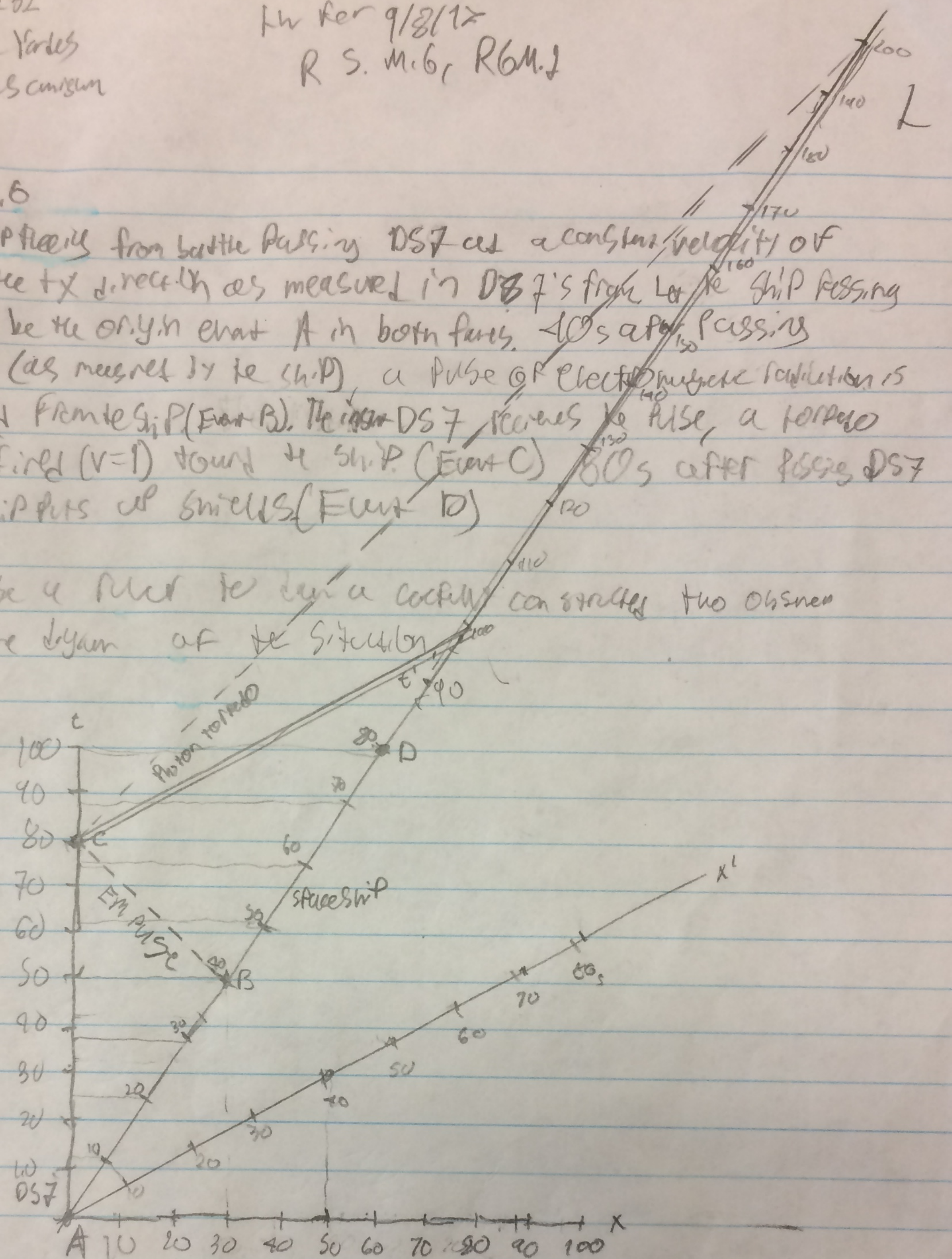
hw for 9/8/17
R S. M.6, R6M.1

RSM.6

A ship fleeing from battle passing DS7 at a constant velocity of $\frac{3}{5}$ in the +x direction as measured in DS7's frame. Let the ship passing DS7 be the origin event A in both frames. 40s after passing DS7 (as measured by the ship), a pulse of electromagnetic radiation is emitted from the ship (Event B). The ~~instant~~ DS7 receives the pulse, a torpedo is fired (v=1) toward the ship (Event C). 80s after passing DS7 the ship puts up shields (Event D)

a) Use a ruler to draw a carefully constructed two observer spacetime diagram of the situation



$\Delta t = \gamma \cdot \Delta t'$, for $\Delta t' = 10s$, $\Delta t = \gamma \cdot 10s = 12.5s$

b) When and where did event B occur in the home frame?

Event B occurs at $(x_B, t_B) = (30, 50)$ in the home frame, based on the spacetime diagram. The Lorentz transform for verifying this would be

$t_B = \gamma(t'_B + \beta x'_B) \Rightarrow t = \frac{1}{\sqrt{1-\beta^2}} \cdot (40s + \frac{3}{5} \cdot 0s) = 1.25 \cdot (40s) = 50s$

$x_B = \gamma(x'_B + \beta t'_B) \Rightarrow x = \frac{1}{\sqrt{1-\beta^2}} \cdot (0s + \frac{3}{5} \cdot 40s) = 1.25 \cdot (\frac{120}{5}s) = 30s$

So my answer from the spacetime diagram is verified by the Lorentz transform

c) When does event C occur in the home frame? Explain how you located this event on the diagram.

Event C occurs at $(x_c, t_c) = (0, 80s)$ in the home frame.
I obtained this answer by drawing a line of slope -1 from event B to the worldline of the DS7. My spacetime diagram is good enough that the math agrees: Event B is 30s away from DS7, so it will take an EM pulse 30s to reach DS7. As Event B occurred at $t_B = 50s$ in the home frame, the pulse will reach DS7 at $t_c = 50s + 30s = 80s$, which agrees with the line I drew on my diagram.

d) When does event C occur in the ship frame? Explain how you can read t'_c from the diagram, and use an appropriate Lorentz transform to verify your result

We can determine the time event C occurs in the ship frame by drawing a line parallel to the diagram x' axis from Event C. As all events on a given diagram x' axis occur at the same time, the intersection of the parallel diagram x' axis and the t' axis should tell us when Event C occurs in the frame of the ship.
I drew a line parallel to the diagram x' axis (it is a double line) and it intersects t' at about $\boxed{100\ s}$

The Lorentz transformation for this would be
$t'_c = \gamma \cdot (t_c - \beta x_c) = 1.25 \cdot (80s - 0) = 100s$
This verifies my result.

e) Which event, C or D, occurs first in the DS7 frame? In the ship frame? Explain

Event C occurs first in the DS7 frame as clearly shown by the space time diagram.
We can use the Lorentz transform to verify:
$t_D = \gamma(t'_D + \beta x'_D) = 1.25(80s + \frac{3}{5} 0) = 1.25 \cdot 80s = 100s$
Seeing that $t_C = 80s$, and that $100s > 80s$, (within the context of this inertial reference frame), Event C occurs before Event D (in the DS7 frame)

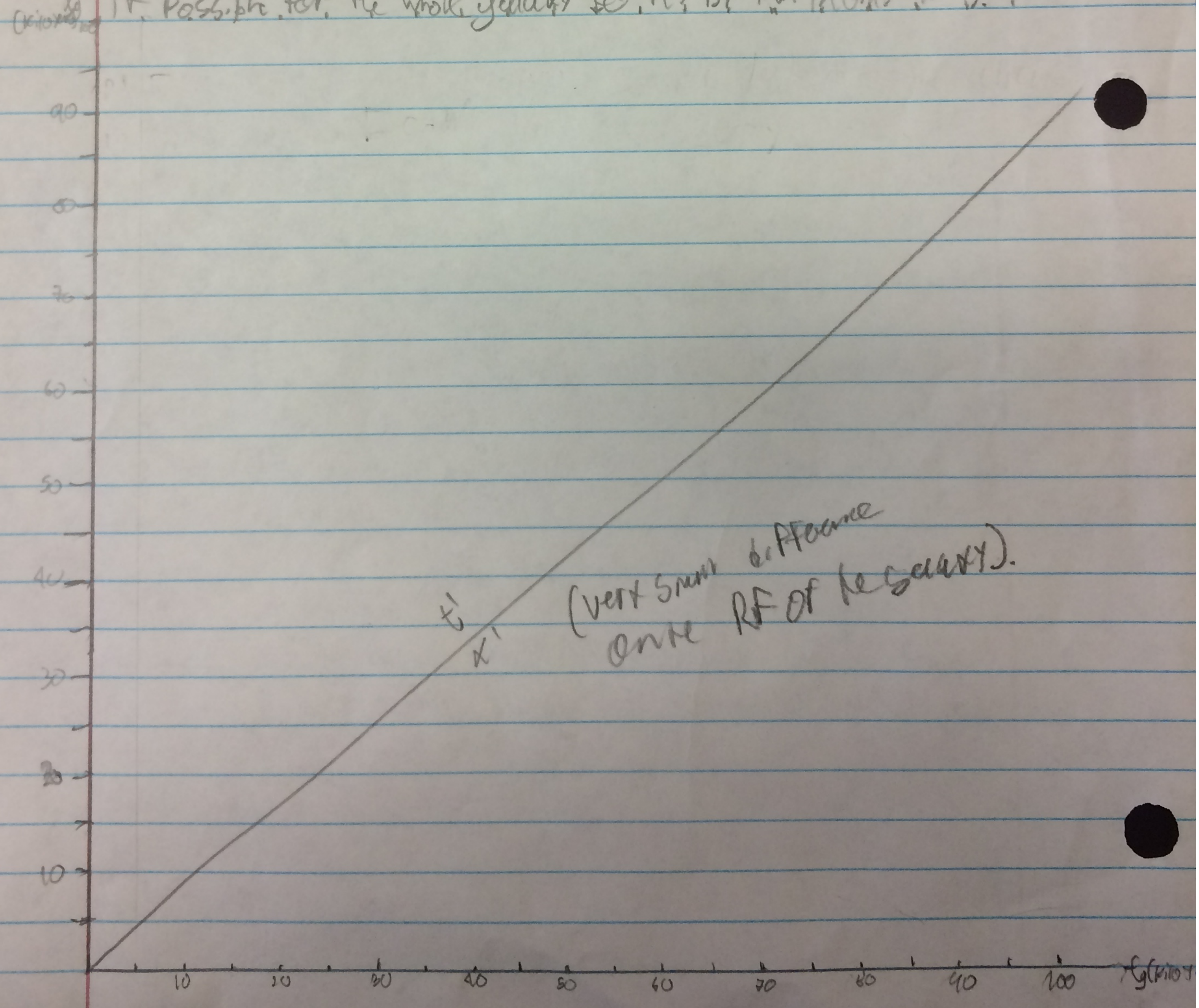
As we saw in part (d), Event C occurs at $t'_C = 100s$ in the ship frame, knowing that $t'_D = 80s$, Event D occurs before Event C in the ship frame.

f) Could the ship have made the decision to raise its shields as a consequence of observing that DS7 had fired a torpedo? Why or why not?

The ship could not have made such a decision based on observing DS7 firing a torpedo. While event C must occur at $t'_C = 100s$ in the ship frame, the ship cannot observe the firing of the torpedo until about 100s later as shown in the diagram. Such an observation (at time $t'_D = 80s$) would mean that somehow an observation had been made, and the signal which carries the information about the observed event would have to move faster than light, which is impossible.

PGM.7

Imagine an alien spaceship travelling so rapidly that it crosses our galaxy (whose rest diameter is 100,000 ly) in only 100 y of spaceship time. Observers at rest in the galaxy say that this is possible because the ship's speed $|\beta|$ is so close to 1 that the proper time it measures between its entry into and departure from the galaxy is much shorter than the galaxy frame coordinate time between those events (~100,000 y). But how does this look to the aliens? To them, the galaxy moves backward relative to them at the speed $|\beta| \approx 1$ and so is Lorentz contracted to a bit less than 100 ly across. THIS is what makes it possible for the whole galaxy to fly by them in only 100 y.



(c) Find the exact value of the speed $|\beta|$ that the aliens must achieve to cross the galaxy in 100y.

Let's define ~~Event~~ Event A as the ~~Event~~ where the spaceship starts to cross the galaxy, and set the origin of the galaxy frame and the ship frame at A.

Let's call Event B the event where the ship reaches the other end of the galaxy, 100,000y away from the galaxy frame's origin.

So $|\Delta \vec{d}_{AB}| = 100{,}000\,y$

The spacetime interval between events A and B as measured by the ship (which in the galaxy frame is moving at a speed of $|\beta|$) is $\Delta s_{AB} = 100\,y$.

Because the spacetime interval between the same events in ANY IRF is constant, we can use the metric equation to say
$(100y)^2 = \Delta t_{AB}^2 - (100{,}000y)^2$

The distance that the ship travels ($|\Delta d_{AB}| = 100{,}000y$) divided by its speed ($|\beta|$) would give the coordinate time between events A and B in the frame of the galaxy (Δt_{AB})

So $(100y)^2 = \left(\frac{100{,}000y}{|\beta|}\right)^2 - (100{,}000y)^2$
$\Rightarrow \frac{(100y)^2}{(100{,}000y)^2} = \frac{1}{|\beta|^2} - 1 \Rightarrow \frac{1.0 \cdot 10^4 y^2}{1.0 \cdot 10^{10} y^2} + 1 = \frac{1}{|\beta|^2} \Rightarrow \frac{1}{1+1.0 \cdot 10^{-6}} = |\beta|^2$

So $\boxed{|\beta| = \sqrt{\frac{1}{1+1.0\cdot10^{-6}}} = 0.9999995}$

b) Find the galaxy's diameter in the alien's frame, and verify that a galaxy with such a diameter moving at a speed $|\beta|$ will completely pass the alien's ship in 100 y

An object of length L_R, where L_R is the measured length of that object in an IRF where it is at rest, will be Lorentz contracted by a factor of $\sqrt{1-\beta^2}$ when its length along the direction of motion is measured in an IRF where it's moving at a speed $|\beta|$. Mathematically, this is

$$L = L_R\sqrt{1-|\beta|^2}$$

So our galaxy of length $L_R = 100{,}000\,y$ (in the galaxy frame) when viewed from the frame of the spaceship will have a length of $L = 100{,}000\,y\sqrt{1-(0.9999995)^2} = 100\,y$.

A galaxy of diameter $100\,y$ moving at a speed $|\beta|$ past the spaceship will completely pass it in a time $\Delta t'_{AB} = \frac{100\,y}{0.9999995} \approx 100\,y$, so this confirms that the aliens would indeed only experience 100 y of time to cross the galaxy.

Eval o' answers

ASM.6)
The magnitudes don't seem crazy, and the units all appear to be correct.

RM6.2
The magnitudes seem to be reasonable, units are good.