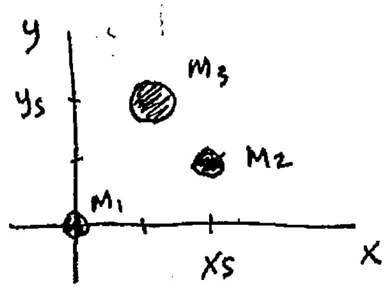
PHY 131 HW Ch.9

Problems 2, 15, 20, 39, 44, 53, 62, 66, 72, 75

9:2



$m_1 = 3.0\ kg$
$m_2 = 4.0\ kg$
$m_3 = 8.0\ kg$

$x_s = 2.0\,m$
$y_s = 2.0\,m$

What are
a) the x coordinate

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{(3.0\,kg)(0) + (4.0\,kg)(2m) + (8kg)(1}{(3+4+8)\,kg}$$

$$= \frac{0+8+8}{15} = \underline{\underline{\frac{16}{15}\,m}}$$

b) the y coordinate

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{(3.0\,kg)(0) + (4.0\,kg)(1m) + (8\,kg)(2m)}{(3+4+8)\,kg}$$

$$= \frac{0+4+16}{15} = \frac{20}{15} = \underline{\underline{\frac{4}{3}\,m}}$$

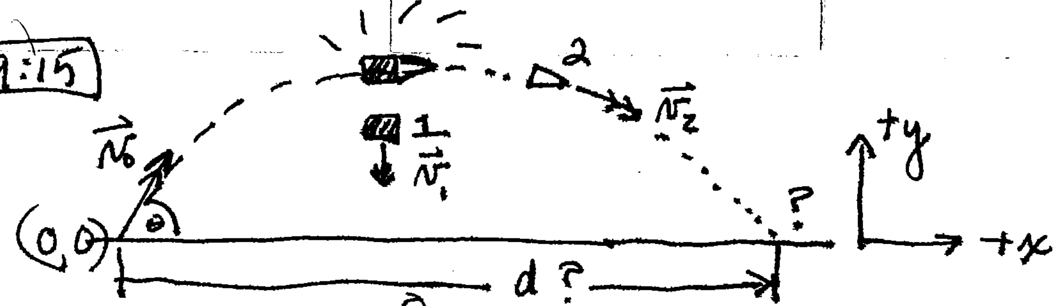
of the center of mass?

So $(x_{cm}, y_{cm}) = \left(\frac{16}{15}, \frac{4}{3}\right)$

c) If m_3 is gradually increased, does the center of mass shift toward or away from that particle, or does it remain stationary?

The center of mass shifts toward m_3 as m_3 increases.

9:15



$\theta = 60°$
$V_0 = 20\ m/s$

We need to find ① the coordinates of the point where the shell explodes and ② the velocity of the fragment that doesn't fall straight down.

Choose origin at launch point.

<u>x dir</u>

$V_x = V_{0x} = 20\ m/s$

$x = V_{0x} t$

$x = V_0 t \cos\theta$

$a_x = 0$

<u>y-dir</u>

$V_{fy} = V_{0y} - gt$

$y_f = y_i + V_{0y}t - \frac{1}{2}gt^2$

Find t for $V_y = 0$: top of trajectory

$$t = \frac{V_{0y}}{g} = \frac{V_0 \sin\theta}{g} = \frac{(20\ m/s)(\sin 60°)}{9.8\ m/s^2} = 1.8\ s = t$$

x at top of trajectory:

$$x = V_0 t \cos\theta = (20\ m/s)(1.8\ s)(\cos 60°) = 17.7\ m$$

y at top of trajectory:

$$y = V_{0y}t - \frac{1}{2}gt^2 =$$
$$= (V_0 \sin\theta)(1.8\ s) - 5(1.8\ s)^2$$
$$= (20)(\sin 60°)(1.8) - 5(1.8\ s)^2$$
$$= 15.0\ m$$

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conservation of momentum?

No forces in horizontal direction: P_x is conserved.

two fragments equal mass: $M \to \frac{M}{2} + \frac{M}{2}$

Before = After

$$M V_{xi} = \cancel{\frac{M}{2}(0)} + \frac{M}{2} V_{xf}$$

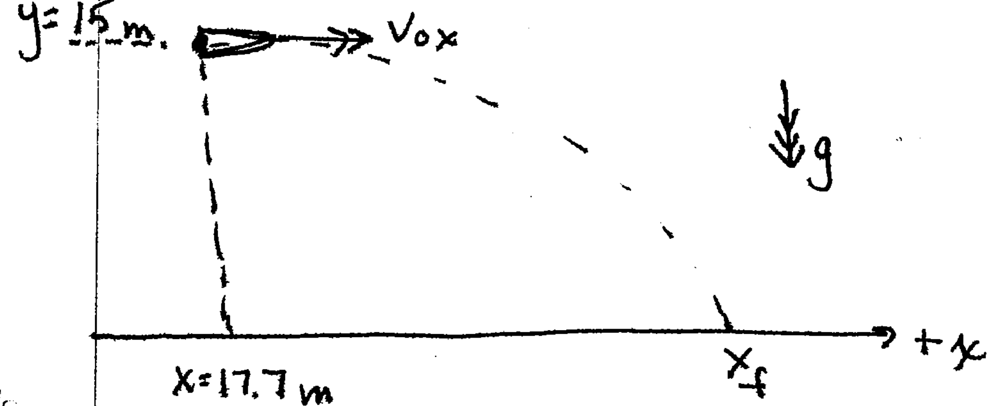
$$\cancel{M} v_0 \cos\theta = \frac{1}{2}\cancel{M}\, V_{fx}$$

V_{fx} (at top of arc) $= 2 v_0 \cos\theta = 2(20)\cos 60°$

$$V_{fx} = 20\ m/s$$

9:15 cont.

Second part of trajectory : kinematics



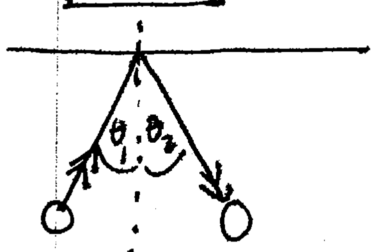
Now :

$x_0 = 17.7$ | $y_0 = 15\text{ m}$
$x_f = ?$ | $y_f = 0$
$v_{0x} = 20\text{ m/s}$ | $v_{0y} = 0$
$v_{fx} = 20\text{ m/s}$ | $v_{fy} = ?$
$a_x = 0$ | $a_y = -9.8\text{ m/s}^2$
$t = ?$ | $t = ?$

$$\Delta y = -5t^2$$
$$-15\text{ m} = -5t^2$$
$$t = \sqrt{3}\text{ s}$$

$$x_f = x_0 + v_{0x}t + \cancel{\tfrac{1}{2}a_x t^2}^{\,0}$$
$$= 17.7\text{ m} + 20\text{ m/s}(\sqrt{3}\text{ s})$$
$$x_f = 52.3\text{ m}$$

9:20



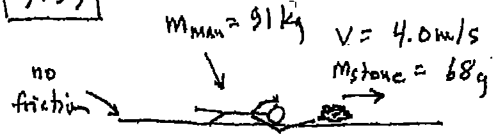
$m = 0.165\text{ kg}$
$v_0 = 2\text{ m/s}$
$\theta_1 = 30^\circ$
$V_{x_i} = V_{x_f}$
$V_{y_i} = -V_{y_f}$

Note: Impact force is in y-direction so P_x is conserved.

$$P_{xi} = P_{xf}$$
$$m v_i \sin\theta_1 = m v_i \sin\theta_2$$
requires $\theta_2 = \theta_1 = 30^\circ$

$$\Delta P_y = P_{yf} - P_{yi}$$
$$= -m v_i \cos\theta_2 - m v_i \cos\theta_1 \;\hat{j}$$
$$= -2(0.165\text{ kg})(2\text{ m/s})\cos 30^\circ \hat{j}$$
$$\Delta P_y = -0.572\text{ kg}\cdot\text{m/s}\;\hat{j}$$

$\boxed{9:39}$



man + stone are initially at rest, so total momentum is zero. After shoving the stone to the right, the man moves to the left because momentum is conserved:

$$\vec{P}_{after} = \vec{P}_{before}$$

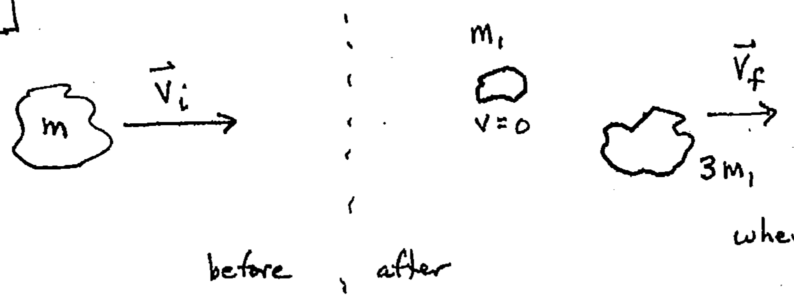
$$m_{stone}\vec{V}_{stone} + m_{man}\vec{V}_{man} = 0$$

$$\therefore \quad \vec{V}_{man} = \frac{-m_{stone}\vec{V}_{stone}}{m_{man}} = \frac{-(0.068\,kg)(4.0\,m/s)}{91\,kg}$$

$$= -0.0030\ m/s$$

i.e. his speed is $|V_{man}| = \underline{\underline{3.0\ mm/s}}$ (opposite direction from stone)

$\boxed{9:44}$



where $m = m_1 + 3m_1$
$m = 4m_1$
so $m_1 = m/4$.

Outer space $\Longrightarrow$ no external forces, so the fragments need to have the same momentum as the object:

$$P_{before} = P_{after}$$

$$m\vec{V}_i = m_1(0) + 3m_1\vec{V}_f$$

$$\text{so} \quad \vec{V}_f = \frac{m\vec{V}_i}{3m_1} = \frac{m\vec{V}_i}{3/4\,m} = \frac{4}{3}\vec{V}_i$$

Looking at the energy:

$$E_{before} = \frac{1}{2}mV_i^2$$

$$E_{after} = \frac{1}{2}m_1(0)^2 + \frac{1}{2}(3m_1)V_f^2$$

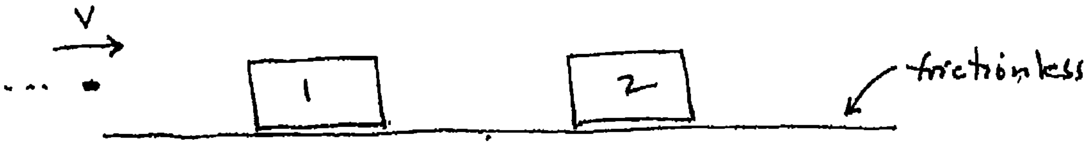
$$= 0 + \frac{1}{2}\left(\frac{3}{4}m\right)\left(\frac{4}{3}V_i\right)^2$$

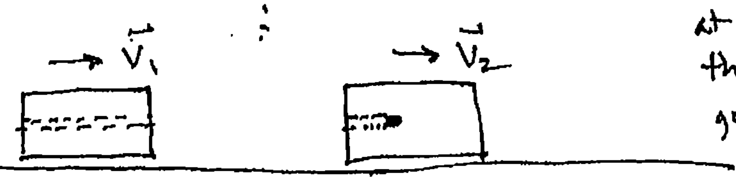
$$= \frac{1}{2}\left(\frac{3}{4}\cdot\frac{16}{9}\right)mV_i^2$$

$$= \frac{1}{2}mV_i^2\left(\frac{16}{12}\right) = \frac{4}{3}\left(\frac{1}{2}mV_i^2\right)$$

So it appears that the fragments have 1/3 more KE than the object had initially. Thus the explosion released $\frac{1}{3}\left(\frac{1}{2}mv^2\right) = \frac{1}{6}mv^2$ energy

9:53





Blocks are initially at rest. Bullet passes through block 1 and gets stuck in block 2

$m_1 = 1.20$ kg $v_1 = 0.630$ m/s $M_{bullet} = 3.50$ g
$m_2 = 1.80$ kg $v_2 = 1.40$ m/s

Find the speed of the bullet as it a) leaves and b) enters block 1.

Frictionless surface $\Rightarrow$ no external forces $\Rightarrow$ momentum is conserved,

so
$$\vec{P}_{before} = \vec{P}_{after}$$
$$m_b\vec{V} = m_1\vec{V}_1 + (m_2 + m_b)\vec{V}_2$$

Hence
$$\vec{V} = \frac{m_1\vec{V}_1 + (m_2 + m_b)\vec{V}_2}{m_b}$$

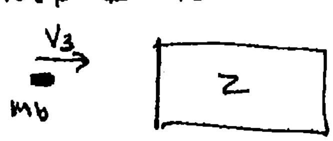
(could ignore m_b here)

$$= \frac{(1.20\text{ kg})(0.630\text{ m/s}) + (1.80\text{ kg} + 0.0035\text{ kg})(1.40\text{ m/s})}{0.00350\text{ kg}}$$

$$|\vec{V}| = \frac{0.756 + 2.52}{0.00350} = \frac{3.281}{.00350} = \underline{\underline{937\text{ m/s}}}\ \text{(b)}$$

This is part b), the ~~velocity~~ speed when it enters block 1.

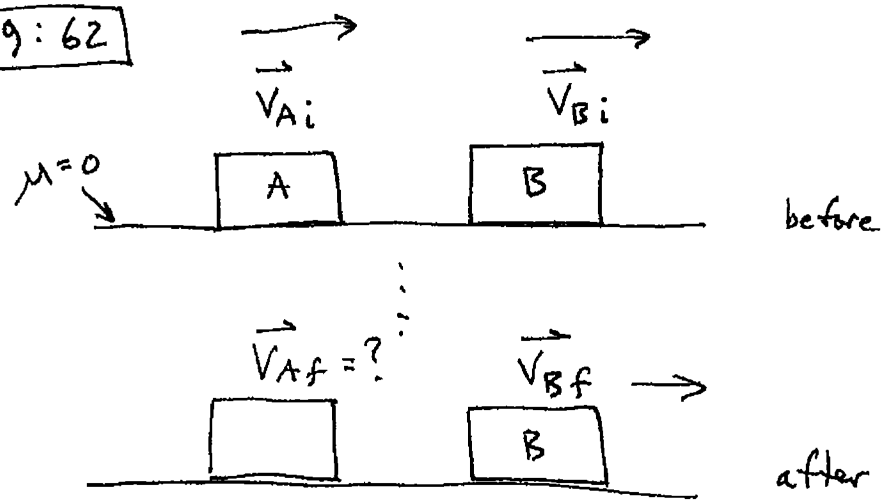
To get part a), note that the bullet carries $(m_b+m_2)\vec{V}_2$ momentum through block 1 to block 2:



Block 2 gets $(m_2+m_b)\vec{V}_2 = 2.52$ kg m/s so the bullet must carry this:

$$m_b V_3 = 2.52\text{ kg m/s}$$
$$\text{so } V_3 = \frac{2.52\text{ kg}}{.00350} = \underline{\underline{721\text{ m/s}}}\ \text{(a)}$$

$\boxed{9:62}$



$m_A = 1.60$ kg
$m_B = 2.40$ kg

$V_{Ai} = 5.5$ m/s
$V_{Bi} = 2.5$ m/s
$V_{Bf} = 4.9$ m/s

a) Find speed and b) direction of $\vec{V}_{Af}$.

No friction with surface, so $F_{ext} = 0 \Longrightarrow$ momentum is conserved.

$$\vec{P}_{before} = \vec{P}_{after}$$

$$m_A \vec{V}_{Ai} + m_B \vec{V}_{Bi} = m_A V_{Af} + m_B \vec{V}_{Bf}$$

Solve for V_{Af} : $m_A \vec{V}_{Af} = m_A \vec{V}_{Ai} + m_B \vec{V}_{Bi} - m_B \vec{V}_{Bf}$

so $\vec{V}_{Af} = \vec{V}_{Ai} + \frac{m_B}{m_A} \vec{V}_{Bi} - \frac{m_B}{m_A} \vec{V}_{Bf}$

$$= 5.5 \text{ m/s} + \frac{2.40}{1.60}(2.5 \text{ m/s}) - \frac{2.4}{1.60}(4.9 \text{ m/s})$$

$$= 5.5 + 3.75 - 7.35$$

$\vec{V}_{Af} = 1.90$ m/s , positive so it points to right

a) speed = $\underline{\underline{1.90 \text{ m/s}}}$

b) to the right.

c) Is the collision elastic?

The energy beforehand is $\frac{1}{2} m_A V_{Ai}^2 + \frac{1}{2} m_B V_{Bi}^2$

$$= \frac{1}{2}(1.60)(5.5)^2 + \frac{1}{2}(2.40)(2.5)^2$$

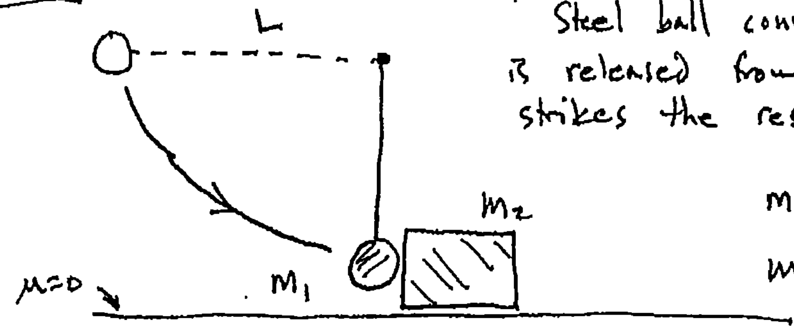
$$= 24.2 + 7.5 = \underline{31.7 \text{ J}}$$

The energy afterwards is $\frac{1}{2}(1.60)(1.90)^2 + \frac{1}{2}(2.40)(4.9)^2$

$$= 2.89 + 28.81 = \underline{31.7 \text{ J}}$$

SAME $\Longrightarrow$ yes, it is an elastic collision

9: 66



Steel ball connected to a string is released from the horizontal. It strikes the resting block elastically.

$m_1 = 0.500$ kg

$m_2 = 2.50$ kg

$L = 70.0$ cm

Find a) the speed of the ball just after the collision.

This collision conserves both momentum (no ext. forces) and energy (elastic):

$$m_1 \vec{v}_{1i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \qquad \text{momentum}$$

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \qquad \text{energy}$$

2 equations, 3 unknowns.

Before the collision we can apply conservation of energy to the ball:

$$E_{\text{at top}} = E_{\text{at bottom}}$$

$$(KE + PE)_{\text{top}} = (KE + PE)_{\text{bottom}}$$

$$0 + mgL = \tfrac{1}{2} m v_{1i}^2 + 0$$

where we've taken the zero of the gravitational potential energy to be at the bottom.

$$\text{So} \quad v_{1i}^2 = 2gL$$

$$\text{or} \quad v_{1i} = \sqrt{2gL}$$

This now allows us to solve for $\vec{v}_{1f}$ and $\vec{v}_{2f}$. I'm going to skip the algebra for now because it's a Friday afternoon before break, and just use eqns 9-67 and 9-68, derived for just this case:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} \qquad v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i}$$

9:66 (cont.)

$$V_{1f} = \left(\frac{0.500 - 2.50}{0.500 + 2.50}\right)\sqrt{2gL} \quad ; \quad V_{2f} = \frac{2\,(0.500)}{(0.500 + 2.50)}\sqrt{2gL}$$

$$= \frac{-2}{3.00}\sqrt{2gL} \quad ; \quad +\frac{1}{3.00}\sqrt{2gL}$$

where $\sqrt{2gL} = \sqrt{2 \cdot 9.8 \cdot 0.70} = 3.70$ m/s [3.74 if $g = 10$]

so $V_{1f} = -\frac{2}{3.}(3.70\text{ m/s}) = \cancel{2.80\text{ m/s}}\ \underline{\underline{-2.47}}$ m/s [-2.49 m/s]

and $V_{2f} = +\ \underline{\underline{1.23}}$ m/s [$+1.25$ m/s]

Aside:

Here's a technique to solve these 1-dimensional elastic problems quickly without using eqns 9-67 and 9-68:

① Find the center of mass velocity of the objects just before the collision:

$$V_{cm} = \frac{m_1 V_{1i} + 0}{m_1 + m_2} = \frac{0.500\text{ kg}}{3\text{ kg}}(3.70\text{ m/s})$$

$$V_{cm} = +0.617\text{ m/s}$$

② Transform into the center of mass frame by subtracting off V_{cm} from V_{1i} and V_{2i}:

$$u_{1i} = V_{1i} - V_{cm} = 3.70\text{ m/s} - 0.617\text{ m/s} = 3.08\text{ m/s}$$

$$u_{2i} = 0 - V_{cm} = 0 - 0.617\text{ m/s} = -0.617\text{ m/s}$$

③ If the two masses collide, their c.m. momenta reverse (this is because the momentum in the center of mass is zero throughout, and the only way to conserve energy is that $u_{2f} = \pm u_{2i}$ and $u_{1f} = \pm u_{1i}$).

$$\text{so} \quad u_{1f} = -u_{1i} = -3.08\text{ m/s}$$

$$\text{and} \quad u_{2f} = -u_{2i} = +0.617\text{ m/s}$$

④ Finally, transform back into the laboratory frame by adding back V_{cm}:

$$V_{1f} = u_{1f} + V_{cm} = -3.08\text{ m/s} + 0.617\text{ m/s} = \underline{-2.47\text{ m/s}}$$

$$V_{2f} = u_{2f} + V_{cm} = 0.617 + 0.617 = \underline{\underline{+1.23\text{ m/s}}}$$

in agreement with the previous results. Note that this technique is completely general, in that we could have started with both masses moving, unlike the derivation of 9-67 and 9-68, which assumes $V_{2i} = 0$. You can derive 9-75 & 76 from this.

9:72

a) Conservation of linear momentum:

$$M\vec{V} + m\vec{v} = M\vec{V}' + m\vec{v}'$$

Since $M = m = 2.0\,kg$, the masses divide out.

$$\vec{v}' = \vec{V} + \vec{v} - \vec{V}'$$

Resolve into x- and y-components: $\begin{bmatrix} v_x \\ v_y \end{bmatrix}$

$$\vec{V} = \begin{bmatrix} 15 \\ 30 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} -10 \\ 5 \end{bmatrix} \quad \vec{V}' = \begin{bmatrix} -5 \\ 20 \end{bmatrix} \, m/s$$

$$\vec{v}' = \begin{bmatrix} v'_x \\ v'_y \end{bmatrix} = \begin{bmatrix} 15 - 10 - (-5) \\ 30 + 5 - 20 \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \end{bmatrix} \, m/s$$

$$\vec{v}' = 10\hat{i} + 15\hat{j} \; m/s$$

b) Initial + final KE

$$K_f = \frac{1}{2} M V'^2 + \frac{1}{2} m v'^2 = \frac{1}{2}(2\,kg)\left\{((-5)^2 + 20^2) + (10^2 + 15^2)\right\}$$

$$= 800\,J$$

$$K_i = \frac{1}{2} M V^2 + \frac{1}{2} m v^2 = \frac{1}{2}(2\,kg)\left\{(15^2 + 30^2) + ((-10)^2 + 5^2)\right\}$$

$$= 1300\,J$$

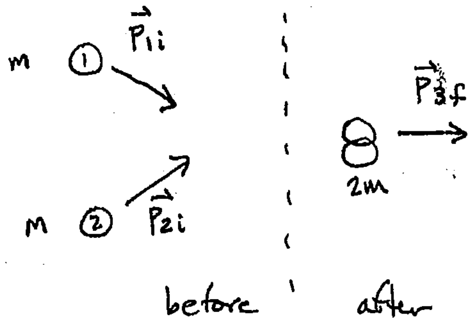
$$\Delta K = K_f - K_i = (800 - 1300)\,J$$

$$= -500\,J$$

-500 J of the initial kinetic energy is lost.

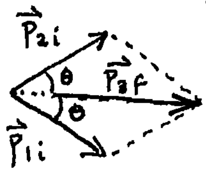
9:75 2-D collisions

After a completely inelastic collision, two objects of the same mass and same initial speed move away together at half their initial speed. Find the angle between the initial velocities of the objects.



Inelastic collision ⟹ energy is not conserved. No external forces means that momentum vector is conserved.

So $\qquad \vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{3f}$



along x : $\quad P_{2ix} + P_{1ix} = P_{3f}$

along y : $\quad P_{2iy} - P_{1iy} = 0$

where $p_1 = m_1 v_1$, $p_2 = m_2 v_2$, $p_3 = (m_1+m_2) V_3$ and $m_1 = m_2 = m$.

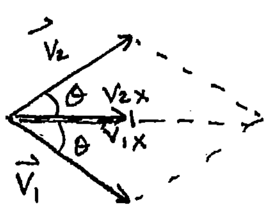
so x : $\quad m v_{2x} + m v_{1x} = (2m) V_3$

and y : $\quad m v_{2y} - m v_{1y} = 0 \implies |v_{1y}| = |v_{2y}|$.

We know that $|v_3| = |\frac{1}{2} v_1| = |\frac{1}{2} v_2|$ so the x equation gives

$$v_{2x} + v_{1x} = 2 V_3$$

v_y components add to zero:



$v_{2x} = v_2 \cos\theta$

$v_{1x} = v_1 \cos\theta$

Now with $|v_1| = |v_2| = |v|$ and $|v_3| = |\frac{v}{2}|$,

$$\underbrace{v\cos\theta + v\cos\theta} = 2\left(\frac{v}{2}\right) = v$$

$$2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$

$\therefore\ \theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$. But θ is the half angle between the vectors, so the angle is

$$\Phi = 2\theta = \underline{\underline{120°}}$$