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(Name)

Exam I

Mathematics 220

September 27, 2017

There are five problems on this exam. Books, notes, and calculators are not permitted. If you can't complete a problem, please show all your work for partial credit.

1. Determine whether the equation is exact, and if it is, find the solution.

$$(e^x \sin y - y) + (e^x \cos y + 2y - x)y' = 0$$

2. A tank contains 100 gallons of water and 50 ounces of salt. Water containing a salt concentration of $\frac{1}{4}(1 + \frac{1}{2}\sin t)$ ounces per gallon flows into the tank at a rate of 2 gallons per minute, and the mixture in the tank flows out at the rate of 3 gallons per minute.

   - Identify quantities that change with time, and assign them variables.

   - Find a differential equation for the amount of salt in the tank at any time t. **Do not solve the differential equation!**

3. Find the general solution of the differential equation:

$$y'' + 4y' + 13y = 0$$

What happens as $t \to \infty$? Does this depend on the initial conditions? Why or why not?

4. Give the *form* of the particular solution for the following differential equations. You do not need to solve these!

   (a) $u'' + 4u = t^2 e^t \cos(2t)$

   (b) $u'' + 2u' + 5u = (t+2)e^{-t} \sin(2t)$

5. Assume $t > 0$. One solution of $2t^2y'' + 3ty' - y = 0$ is $y_1(t) = t^{-1}$. Find the general solution using reduction of order.