

## Preparation for October 30

We turn now to Chapter 7. In the remainder of the course, we will be considering situations where there are two or more dependent variables. Here is an example:

$$\begin{aligned}x' &= 3x - y \\ y' &= x + 2y\end{aligned}$$

One approach that one could take would be to transform the pair of equations into a single second order equation. To do this

- Solve for  $y$  in terms of  $x$  and  $x'$  using the first equation.
- Compute  $y'$  using what you just got.
- Plug the expressions for  $y$  and  $y'$  into the second equation.
- Simplify

In the example above,

- Using the equation  $x' = 3x - y$ , we get

$$y = 3x - x'.$$

- Thus,  $y' = 3x' - x''$ .
- Plugging these into  $y' = x + 2y$ , we get

$$(3x' - x'') = x + 2(3x - x')$$

- This simplifies to

$$x'' - 5x' + 7x = 0$$

The roots of the characteristic equation  $r^2 - 5r + 7 = 0$  are  $r = \frac{5 \pm \sqrt{-3}}{2}$ , so we get

$$x = Ce^{5t/2} \cos\left(\frac{\sqrt{3}t}{2}\right) + De^{5t/2} \sin\left(\frac{\sqrt{3}t}{2}\right)$$

We could then find  $y$  by computing  $3x - x'$ . After simplifying, we get

$$y = \left(\frac{C - \sqrt{3}D}{2}\right) e^{5t/2} \cos\left(\frac{\sqrt{3}t}{2}\right) + \left(\frac{\sqrt{3}C + D}{2}\right) e^{5t/2} \sin\left(\frac{\sqrt{3}t}{2}\right)$$