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Phys 2103
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HW for 9/6/17
R4M.2, R4A.2, R5M.2

R4M.2

A, B, C, and D are enjoying a trip to the amusement park. Allison rides the Ferris wheel (47.75m in diameter), Brad stays on the ground. Chris and Dylan ride a monorail that passes directly beneath the Ferris wheel. The monorail moves at 9m/s

Event A: The first time Allison passes Brad, they both look at their watches and Chris who passes under the Ferris wheel at that moment, looks at the clock on Allison's seat.

Event B: The second time Allison passes Brad, the same thing happens except it's Dylan who now passes under the Ferris wheel and looks at the clock (which we assume is synchronized with Chris's).

Everyone determines the time interval between these events. Brad measures 50s between event A and event B

a) Who measures the shortest time? The longest?

We know that $\Delta t \geq \Delta s \geq \Delta \tau$

Both Brad and Alice measure a proper time, but Alice is not inertial whereas Brad is, so Brad measures a spacetime interval, Δs. $\Delta s \geq \Delta t$, ~~but~~ Alice is not inertial, so Alice measures the shortest amount of time.

Chris and Dylan measure a coordinate time Δt, and $\Delta t \geq \Delta s$, so Chris and Dylan measure the longest time

b) How much larger or smaller is the time Allison measures than the time Brad measures?

The time Brad measures is $\Delta s = 50s$.

Allison travels a distance of $47.75m \cdot \pi \cdot \frac{1s}{3.0 \cdot 10^8 m} = 1.59 \cdot 10^{-7} \pi s$ per rotation.

So the speed per the Ferris wheel rotates must be $\frac{1.59 \cdot 10^{-7} \pi s}{50s} = 3.18 \cdot 10^{-9} \cdot \pi$

Because Allison's speed is constant, we can use $\Delta \tau_{AB} = \sqrt{1 - (3.18 \cdot 10^{-9}\pi)^2}\, \Delta t_{AB}$

Because the distance Brad travels is 0s, by the metric equation, $\Delta s^2 = \Delta t^2 - 0$

so $\Delta s = \Delta t$ in Brad's frame. Using the binomial approximation

$\Delta\tau_{AB} \approx \left(1+\frac{1}{2}\left(-(3.18\cdot10^{-9}\pi)^2\right)\right)\cdot 50s = 50s - 2.53\cdot10^{-16}\cdot\pi^2$

So Allisons $\Delta\tau$ is $2.53\cdot10^{-16}\cdot\pi^2 s$ smaller than Brad's ΔS

c) How much larger or smaller is the time Chris and Dylan measure than Brad's time?

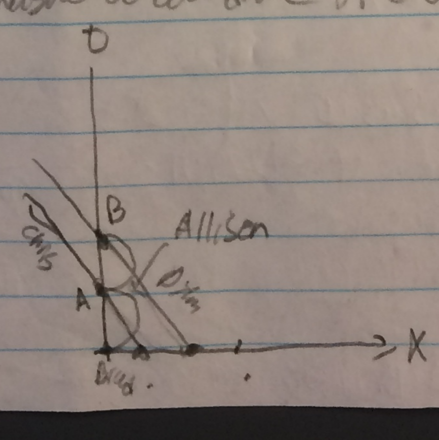
Chris and dylan are moving at a constant velocity of $\vec{v} = 9 m/s \cdot \frac{1s}{3.0\cdot10^8 m}$ $\Rightarrow 3\cdot10^{-8}$. Because Chris and Dylan measure a coordinate time Δt, we can use their speed to find it using the metric equation

$(50s)^2 = \Delta t^2 - (3\cdot10^{-8}\cdot\Delta t)^2 = \Delta t^2(1-(3\cdot10^{-8})^2)$

$\Rightarrow \Delta t^2 = \frac{50s^2}{1-(3\cdot10^{-8})^2} = \frac{2500\cdot10^{19}s^2}{9{,}999{,}999{,}999{,}999{,}991} \approx 2500$ (with a little bit more)

Chris and Dylan measure a time that is a little bit more than $50s$, but this difference is so small I cannot calculate it, but I can say that is is on an order of magnitude of $10^8 s$

d) Chris and Dylan are moving in the ground frame. Should not they ~~therefore~~ measure less time between the events than Brad?
The caveat here is that Chris and Dylan do not use clocks on the train then measure the time, but the clock on the digital display on Allison's ~~Seat~~ So the time they measure is not the time they experience. But the time as shown in the ground frame at the instant of passing. Additionally, neither Chris nor Dylan are present at both events, so even though they are experiencing the at a different rate than the ground frame, they can only measure a coordinate time between A and B which must be greater or equal to Brad's spacetime interval.



R4A.2

Consider an inertial frame at rest with respect to the earth. We observe an alien spaceship to move along the x axis of the frame in such a way that $x(t) = \frac{1}{\omega}[\sin(\omega t + \frac{1}{4}\pi) - b]$ where both x and t are measured in the inertial frame, $\omega = \pi/2$ rad/h and $b = \sin(\pi/4)$. Assume that the earth is located at $x=0$ in this frame.

a) Argue that the ship passes the earth at $t=0$ and again at $t=1.0$h.

At $t=0$, $x(0) = \frac{2}{\pi}$ h/rad $[\sin(0+\frac{1}{4}\pi) - \sin\pi/4] = 0$

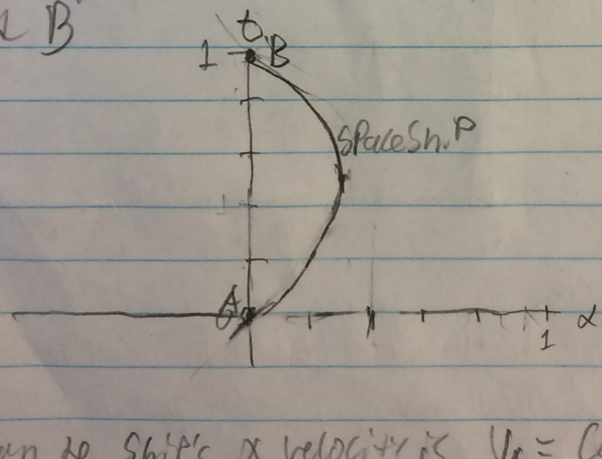
At time $t=1.0$hr, $x(t) = \frac{2}{\pi}$ h/rad $[\sin(\frac{\pi}{2} + \frac{1}{4}\pi) - \sin\pi/4]$

$= \frac{2}{\pi}(\sin(\frac{3\pi}{4}) - \sin(\pi/4))$

$= \frac{2}{\pi} \cdot (\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}) = 0$

So at $t=0$, and $t=1.0$hr, $x=0$, which means the ship passes the earth at both of these times.

b) Draw a quantitatively accurate spacetime diagram of the spaceship's worldline, including the events A where the ship passes the earth and B



c) Show the ship's x velocity is $V_x = \cos(\omega t + \pi/4)$ as measured in the inertial frame of earth

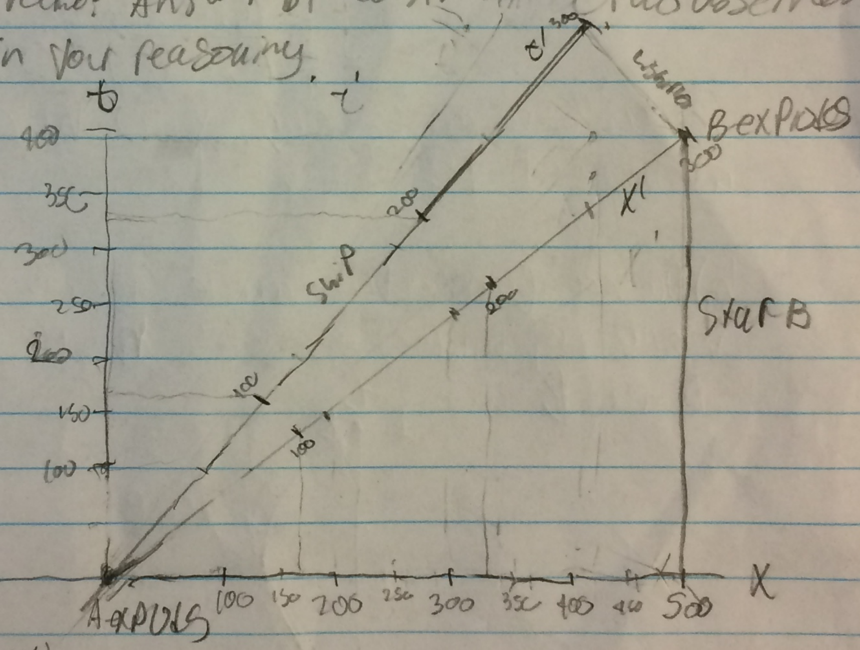
$\frac{dx}{dt} = V_x$

so $\frac{d}{dt}\left(\frac{1}{\omega}[\sin(\omega t + \frac{\pi}{4}) - b]\right) = \frac{1}{\omega} \cdot \omega \cdot \cos(\omega t + \frac{\pi}{4}) = \cos(\omega t + \frac{\pi}{4}) = V_x(t)$

RSM.1

Star A and B float essentially at rest 500 light years apart in an inertial frame that we will take to be the Home Frame. Suppose star A (whose position is $x_A = 0$ in the home frame) explodes at a time we define to be $t_A = 0$. Star B explodes at a time $t_B = 400y$ both as measured in the Home frame. A spaceship is moving from star A directly towards star B at a constant speed $|\beta|$, passing star A just as it explodes. Define the +x direction the ship is going.

a) what is the value of β if the explosions are simultaneous in the spaceship's frame? Answer by constructing a two observer diagram and explain your reasoning.



$\Delta t = \gamma \Delta t' = \frac{1}{\sqrt{1-|\beta|^2}} \Delta t'$

If the explosions occur at the same time in the spaceship's frame, then the diagram x' axis that intersects both star A exploding and star B exploding will have a slope of β.

$(t-400) = \beta(x-500) \Rightarrow \beta = \frac{400}{500} = \frac{4}{5}$ So the slope of spaceship is β

b) Where does the explosion of Star B occur in the space ship's reference frame? Answer using the two observer diagram and explain your reasoning.

I draw my diagram with t and x having increments of 100 years, & want to appropriately scale t' and x' so that we fairly accurately.

The space time interval from $t=0$ to $t=100_{yr}$ is $100yr$

The space time interval from $t'=0$ to $t'=100yr$ is also $100yr$

The set of all events a given space time interval away from the origin is described by the metric equation: $\Delta s^2 = \Delta t^2 - |\Delta \vec{d}|^2$.

So to find the t coordinate for $t'=100$ in the frame of A, we need to find the place where t' intersects with the hyperbola $(100_{yr})^2 = \Delta t^2 - |\Delta \vec{d}|^2$

$\Rightarrow (100)^2 = \Delta t^2 - (t \cdot \beta)^2 \Rightarrow (100_{yr})^2 = \Delta t^2(1-\beta^2)$

$\frac{(100)^2}{(1-\beta^2)} = \Delta t^2 \Rightarrow \sqrt{\frac{(100)^2}{(1-\beta^2)}} = \Delta t$

100

So the coordinate time between $t'=0$ and $t'=100yr$ in the frame of A is represented by the above equation. Plugging in our value for β,

$\Delta t = \frac{100\,yr}{\sqrt{1-(\frac{3}{5})^2}} = \frac{100\,yr}{\sqrt{\frac{25-16}{25}}} \Rightarrow \frac{100\,yr}{\sqrt{\frac{9}{25}}} = \frac{5 \cdot 100yr}{3} = \frac{500yr}{3} \approx 166\,yr$

So 500 yr in the A frame is 300 yr in the ship frame

so the explosion of Star B occurs at $\boxed{x' = 300yr}$

c) When does light from Star B's explosion reach the ship?

If we draw a slope of -1, coming from $x'=300yr$, this is the world line of the light flash from the explosion. So the light flash must reach the ship $(x'=0)$ at $\boxed{t'=300yr}$

d) Does the answer to part C make sense considering your answer for part B? Explain.

My answer makes sense because the speed of light is frame independent, so if Star B explodes at $t'=0, x'=300yr$, it will take 300 yr for the light to reach the ship because light travels at ± 1.

Evaluation of answers:

R4M.2

My answers for parts b and c make sense because the speeds that we're traveling at are so small that the differences in time would be miniscule.

R4A.2)

My space time diagram's pretty good I think.

R5M.2)

None of my magnitudes seem off the charts, all within reasonable orders of magnitude.