Phy 232

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[illegible]

HW Rem 11/8/17

Q15M.9, Q5R.1

Q15 M.9

a) Imagine in 2020 you do a measurement of the scroll fragment which may be genuine but may be a medieval fake. Suppose during the measurement process you find 77.3% of the ^{14}C that you would expect to find in a living sample. How old was the scroll?

The number of undecayed nuclei in a radioactive substance is $N(t) = N_0 e^{-\frac{\ln(2) t}{t_{1/2}}}$, where N_0 is the initial number of undecayed nuclei, and $t_{1/2}$ is the half life.

When the material that the scrolls were made on was alive, it had a constant ratio of ^{12}C to ^{14}C. After it was harvested/killed, the amount of ^{14}C began to decrease.

So if $t=0$ is around the time the substance was made not alive, then t_1 years later the amount of ^{14}C left is $\frac{N(t_1)}{N_0} = e^{-\frac{\ln(2) t_1}{t_{1/2}}}$

$\Rightarrow -\frac{\ln(2) t_1}{t_{1/2}} = \ln\left(\frac{N(t_1)}{N_0}\right) \Rightarrow t_1 = -t_{1/2} \cdot \ln\left(\frac{N(t_1)}{N_0}\right) \div \ln 2$

The half life of ^{14}C is 5730 y, and in 2020, $\frac{N(t_1)}{N_0} = 0.773$

So $t_1 = -5730\,yr \cdot \ln(0.773) \div \ln 2 = 2128$ yr old

This means the script was made around 109 BCE

b) Suppose your measurement is uncertain by ±0.2%. Estimate the uncertainty in the manuscript's age.

If we have an uncertainty of ±0.2% in our measurement, the maximum and minimum possible values are 77.5% and 77.1

$t_{max} = -5730 \cdot \ln(0.775) \div \ln 2 = 2107$ yr

$t_{min} = -5730 \cdot \ln(0.771) \div \ln 2 = 2149$ yr

So the manuscript is between 2107 and 2149 years old

This gives a uncertainty of ±1%

c) What is the range of dates during which the manuscript might have been written?

$|2020-2107-1| = 88\ BCE$
$|2020-2143-1| = 124\ BCE$

between ~~124~~ BCE and 88 BCE

d) Is the manuscript a real or a fake?
According to the problem statement, "the scrolls would not be much older than about 1120 years. These scrolls are dated to be over a thousand years older, and they are in a reasonably close time period, so they are probably legit.

Q15P.7
You need to design a nuclear "battery" for a deep-space probe. The battery must still produce at least 35 W of power after 12 yr. The available thermo electric generator converts thermal to electrical energy at an efficiency of 12%. You want to use an alpha emitter for the radioactive source because it is easy to shield electronics from alpha particles. You can obtain sufficient amounts of either $^{208}_{84}Po$ or $^{209}_{84}Po$, which are both alpha emitters with half lives of 2.9 yr and 105 yr respectively. The former decays to $^{204}_{82}Pb$, which is stable while the latter decays to $^{205}_{82}Pb$ which in turn decays by EC to $^{205}_{81}Ti$, which is stable. Which of these two isotopes would be better, and how much of that do you need?

A

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	E_b (MeV)
^{208}Po	7.839357
^{204}Pb	7.880231
^{209}Po	7.833?47
^{205}Pb	7.874?33
^{205}Tl	7.878393
^{4}He	7.073915

$^{208}_{84}Po$ $t_{1/2} = 2.9\,y$

207.981245 u

$\downarrow$ α-decay

$^{204}_{82}Pb$ (stable)

203.973043 u

$\Delta E = E_bf - E_bi$

$\Rightarrow 4 \cdot 7.073915\,MeV - 7.839357\,MeV \cdot 208$

$\Rightarrow \Delta E = 5.215328\,MeV$

$^{209}_{84}Po$ $t_{1/2} = 105\,y$

208.982430 u

$\downarrow$ α-decay

$^{205}_{82}Pb$

204.974482 u

$\downarrow$ EC

$^{205}_{81}Tl$

204.974428 u

$\Delta E = E_bf - E_bi$

$\Delta E = 4.979 \ldots\,MeV$

$\Delta E = 7.878393\,MeV \cdot 205 - 7.874\ldots\,MeV \cdot 205$

$\Delta E = 0.0\ldots\,MeV$

not the same because

$t_{1/2} = 1.5 \cdot 10^7\,y$

The power put out by the nuclear battery as a function of time is $P(t) = \Delta E \cdot 0.13 \cdot A(t)$, where ΔE is the energy released per decay, 0.13 is the 13% conversion factor, and $A(t)$ is the activity of the core in decays per second. Notice we can write $A(t)$ as $\frac{N(t)}{\tau} = \frac{\ln(2) \cdot N(t)}{t_{1/2}}$ where $t_{1/2}$ is the half life, $N(t)$ is the number of undecayed atoms, that is $N(t) = N_0 \cdot e^{-\frac{\ln(2) t}{t_{1/2}}}$

So our final expression for power is $P(t) = \Delta E \cdot 0.13 \cdot \frac{\ln(2) \cdot N_0 \cdot e^{-\frac{\ln(2) t}{t_{1/2}}}}{t_{1/2}}$

Ideally, we would like the smallest mass of core in our battery. We can express N_0 in terms of mass with $N_0 = \frac{\text{mass}}{\text{molar mass}} \cdot 6.022 \cdot 10^{23} \frac{1}{mol}$

We want the smallest value [illegible] 12 yrs to still function at 35 W

So we can rearrange to solve for the mass:

$$\frac{35\,W \cdot t_{1/2} \cdot \text{molar mass}}{\Delta E \cdot 0.13 \cdot \ln(2) \cdot 6.022 \cdot 10^{23} \frac{1}{mol}} e^{\frac{\ln(2) \cdot 12}{t_{1/2}}} = M_0$$

Let's look at $^{208}_{84}Po \rightarrow {}^{204}_{82}Pb$

$$M_0 = \frac{35\,W \cdot 2.9 \cdot 207.981245 \frac{g}{mol} \cdot 60 \cdot 60 \cdot 24 \cdot 365}{0.13 \cdot 5.215328 \cdot 10^6\,eV \cdot \ln 2 \cdot 6.022 \cdot 10^{23} \frac{1}{mol} \cdot 1.602 \cdot 10^{-19} \cdot 1000 \frac{g}{kg}} e^{\frac{\ln 2 \cdot 12}{2.9}} = 0.2601\,kg = 260\,g$$

$^{209}_{84}Po$ has a higher mass, higher half life, and lower ΔE, so it will require more mass to get the same result

Evil 0 answers:
QBM.9 (rescuing, pessimist 8).
Q4SR.7 (Right cuts, rescued)