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HW for 9/27/17

Q2R.1, Q3M.1

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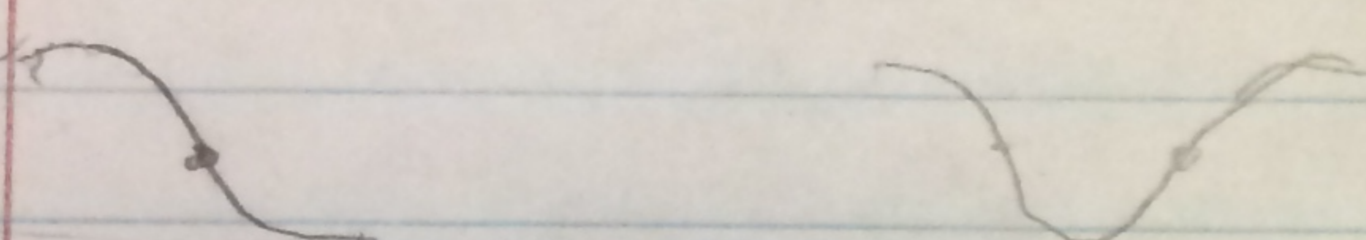
Q2R.1

A sinusoidal surface wave on a body of water that is significantly deeper than half the wave's wavelength has a phase speed of $|\vec{v}| = \sqrt{\frac{|\vec{g}|\lambda}{2\pi}}$, where $|\vec{g}| = 9.8\,m/s^2$.

Consider standing waves in a narrow rectangular pool that has length L and is sufficiently deep. Derive an expression in terms of $|\vec{g}|$, L, and n for the normal mode frequencies for waves in this pool. Show that these frequencies are NOT integer multiples of a fundamental frequency.



Since our pool is very narrow relative to its length, we can treat the surface waves as 1-dimensional. Standing waves in a body of water in a pool are analogous to open ended standing waves. The frequencies of such standing waves are

$f = n\left(\frac{|\vec{v}|}{2L}\right)$, where $n = 1, 2, 3, \ldots$, and $|\vec{v}|$ is the phase speed of the wave. Because our pool is sufficiently deep, we can use the phase speed of $|\vec{v}| = \sqrt{\frac{|\vec{g}|\lambda}{2\pi}}$. So the frequencies for standing waves in the pool are $f = n\sqrt{\frac{|\vec{g}|\lambda}{2\pi}}/2L$.

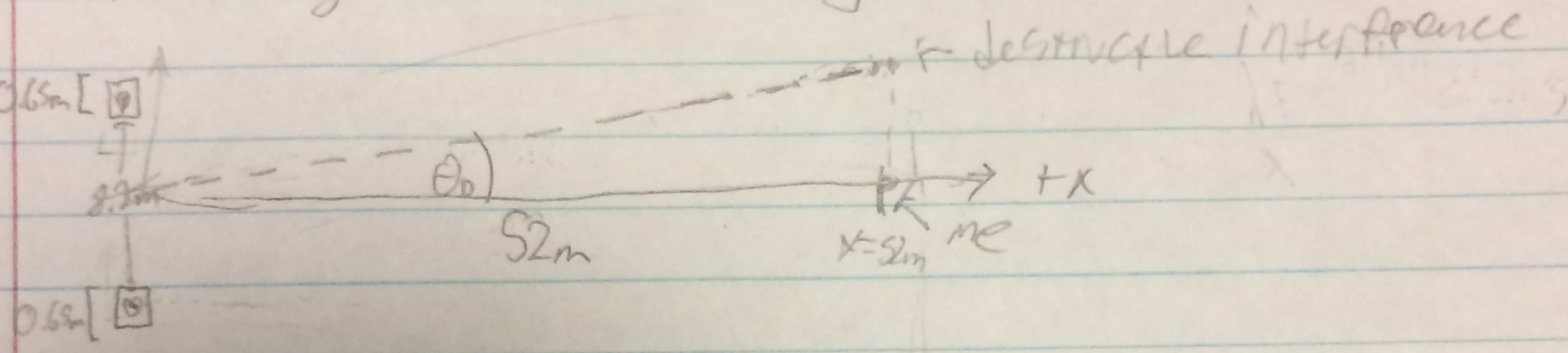
n is the number of half wavelengths in a given mode, so the first mode has 1 half wavelength, the 2nd has 2 and so on. Notice that, for constant length of the pool, the nth mode fits n half wavelengths into L, so $L = \frac{n\lambda}{2}$. Solving for λ gives $\lambda = \frac{2L}{n}$. Plugging this into our expression for f_n, we get

$$\boxed{f_n = \frac{n\sqrt{\frac{|\vec{g}|\cdot\frac{2L}{n}}{2\pi}}}{2L} = \frac{n\cdot\sqrt{\frac{|\vec{g}|\cdot L}{n\pi}}}{2L} = \sqrt{\frac{n^2\cdot|\vec{g}|\cdot L}{n\cdot\pi\cdot 4\cdot L^2}} = \sqrt{\frac{n|\vec{g}|}{4\pi L}}}$$

This means that the nth mode of vibration has a frequency proportional to $\sqrt{n}$ instead of just n, meaning that the frequencies are not integer multiples of a fundamental frequency.

Q3M.1

You are setting up a pair of PA speakers on a field in preparation for an outdoor event. Each speaker is 0.65 m wide, and the speakers are separated by 8.2 m. To test the speakers, your coworker plays a single tone through the speakers whose frequency is 440 Hz. You are standing 52 m directly in front of the speakers and facing them. Roughly how far would you have to walk to your left or right to hear the sound amplitude drop almost to zero? How much farther would you have to go to hear the amplitude go back to its original strength?



$\lambda = \frac{|\vec{v}|}{440\,Hz} = \frac{343\,m/s}{440\,Hz} \approx 0.78m.$

The sound is produced by cones in each speaker which are a bit less wide than the speakers themselves. If we think of the cones as slits, we can say that $\lambda > a$. Because we are looking for interference very far from the "slits" and is large compared to the separation of the "slits", equation Q3.3 applies.

$d \cdot \sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n \cdot \lambda}{d} = \frac{n \cdot 0.78m}{d}$

Note that $d \neq 8.2m$, but $8.2m + 0.65m$, as each "slit" produces a spherical wave with its center aligned with the center of the slit. So the distance between the center of each wave is $8.2m + \frac{0.65m}{2} + \frac{0.65m}{2}$

$\Rightarrow d = 8.85m$. Plugging this into our equation, $\sin\theta = \frac{n \cdot 0.78m}{8.85m} = 0.088 \cdot n$

We can use the small angle approximation, $\sin\theta \approx \theta$ for $\sin\theta < 0.2$ rad

The waves interfere destructively when their path lengths are out of phase, that is, $d \cdot \sin\theta = \frac{1}{2} \cdot n \cdot \lambda$. So we get

$\theta = \frac{1}{2} \cdot 0.088 \cdot n = 0.044 \cdot n$

The distance between ~~my~~ current position ($y=0$) and the first instance of destructive interference is

$|52m \cdot \sin\theta - 0m| = |52m \cdot \sin(0.044)| \approx 52m \cdot 0.044 \approx 2.3m$

So I would have to walk at least 2.3 meters parallel to the speakers in order for the amplitude to almost be zero

For amplitude to reach its initial level, I would have to walk an additional $52m \cdot 0.088 - 2.3m = 2.3m$, either in the same direction ($n=1$) or back to my original position ($n=0$)

Eval of answers:

Q2P.1
Seems like a reasonable equation

Q3m.1:
Right units, magnitudes seem reasonable