

Homework II

Extra Bernoulli Problems

1. Consider a cylinder with water filled to a height of 50 cm. At a height of 10 cm is a hole in the side of the cylinder. Assuming that there is no viscous drag of the fluid flowing through the hole, what is the velocity of the fluid squirting out of it? (Hint: you will need to use Bernoulli, and make the assumption that the pressure of the water just as it leaves the hole, since it is in contact with the air, is just atmospheric pressure.)
2. Water is flowing out of a hose at a fluid velocity of 10 m/s pointed directly up in the air. Since the water leaving the hose is in contact with the air, it is always at $P=1$ atm. Use Bernoulli to determine how high this fountain of water will go. (Hint: assume that the water at the top of the fountain is not moving, i.e. $v = 0$.)
3. A ball is thrown directly upward at 10 m/sec. Use conservation of energy to determine how high the ball goes. How does this answer and this method compare to problem 2?

Homework II.1

1. You found in Monday's lab that if the rate of change in the height of the water column (i.e. the negative of the flow rate divided by the area) was proportional to height (since the height is proportional to the pressure difference across the capillary), the height versus time function was a decreasing exponential. Use the same reasoning to show, by finding the explicit function, that if the rate of radioactive decay (i.e. change of radioactive atoms into stable ones) is proportional to the number of radioactive atoms left:

$$\frac{dN}{dt} = -\alpha N$$

where α is a constant called the activity of the isotope, and N is the number of radioactive atoms left at any time, then N as a function of t is exponential. Find the half-life in terms of α .

2. We are draining a cylinder through a single small hole, like our capillary. The capillary is at the height of the bottom of the tube, and the flow is small enough that we can ignore the velocity part of Bernoulli, and assume that flow is determined just by pressure and the viscous flow through the capillary. (In simpler words, assume the cylinder draining is exponential.) If it takes 20 seconds for the tube to drop to half its original height, how long does it take to empty 90% of the water from the tube?

Homework II.2

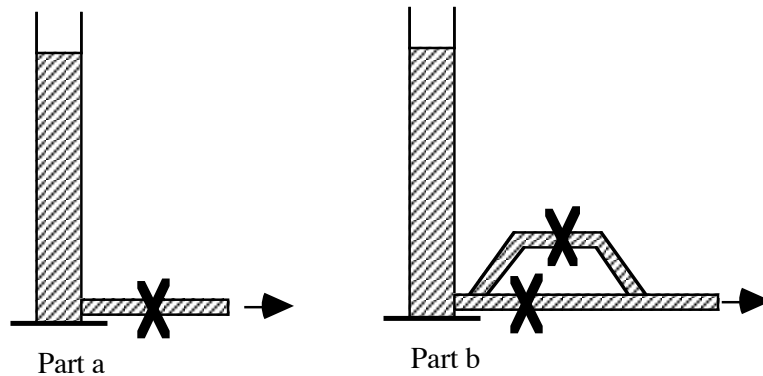
1. Recall we defined a quantity called the flow resistance at the end of Unit II Session 2 activity guide, given by

$$R = \Delta P / f.$$

If we have a modest flow rate of $5 \text{ cm}^3/\text{sec}$ through a fluid circuit element by maintaining a pressure difference of 1000 Pa across it, what is the flow resistance? Be sure to specify

the units of this resistance. What would the flow become if we increased the pressure difference to 4000 Pa?

2. A water column is attached at the bottom to a tube with a flow restriction (symbolized by the x) in it. The pressure of the fluid before that flow restriction is 200 Pa (above atmospheric pressure), and just atmosphere at the outlet. The flow through the restriction is $10^{-6} \text{ m}^3/\text{sec}$.



- What is the fluid resistance R of the restriction? You may wish to recall that $\Delta P = f R$.
- A second tube and restriction identical to the first (and at the same height) is added as shown. (This configuration is called parallel.) Now what is the *total* fluid flow?
- What is the effective fluid resistance of the two restrictions together in parallel? (This should follow from part a and b and the definition of flow resistance, *not* from some rule you may have learned earlier from a different class.)

Homework II.3

1. In the last Guidebook Entry (II.10) you found the radial dependence of fluid velocity where it was free to spread out in two dimensions. Now imagine that we have a source of fluid that then flows out like a fountain, or a mushroom, into three dimensions? How does the flow depend on radial distance? In particular, if the flow is $100 \text{ cm}^3/\text{sec}$, and is free to flow out in all upward directions (see figure), what is the fluid velocity at a distance of 20 cm from the source?

