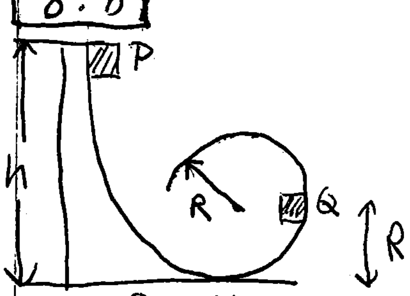
PHY 131 HW 6 Solutions

Ch. 8 8, 9, 25, 29, 34, 40, 53

Ch. 13 37, 40, 63

$\boxed{8:8}$



no friction

$m = 0.032\ kg = 3.2\times10^{-2}\ Kg$

$R = 12\ cm = 0.12\ m$

$h = 5R$

How much work does F_G do on the block as it travels from point P to:

a) point Q?

+y ↑ ↓ F_G ↓ $d = 4R$ angle between F_G & d is 0

Note: Δy 4R

$W_G = \vec{F}_G \cdot \vec{d} = |F||d| \cos 0°$

$= mg(4R)$

$= 4(10\ m/s^2)(3.2\times10^{-2}\ kg)(0.12\ m)$

$= 0.15\ J$

b) top of loop?

Note $\Delta y = 3R$ everything else the same

$W_G = \vec{F}_G \cdot \vec{d} = mg(3R)$

$= 0.11\ J$

c) For $U_G = 0$ at bottom of loop, what is U_G at point P? Note $y = 5R$

$U_G = mgy = mg5R$

$= 0.19\ J$

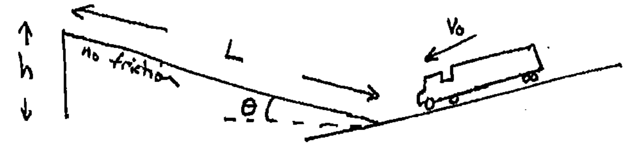
d) point Q? Note $y = R$

$U_G = mgR = 0.038\ J$

e) at top of loop, $y = 2R$ $U_G = mg2R = 0.075\ J$

f) $|\vec{V}_o|$ doesn't change any of these calculations; all results remain the same

$\boxed{8:9}$ A runaway truck with failed brakes is moving down a hill at 130 km/h just before going up a frictionless (!) emergency escape ramp with an inclination of $\theta = 15°$. The truck's mass is 1.2×10^4 kg.



[Real truck escape ramps use coarse gravel with very high friction]

a) What minimum length L must the ramp have if the truck is to stop (momentarily) along it? (Assume the truck is a particle & justify that assumption.)

The mass of the truck $M = \sum_i m_i$ where m_i are the particles (atoms) that make up the truck. The work done on it (them) is

$$W_{gravity} = Mgh = \left(\sum_i m_i\right) gh = \sum_i m_i g h$$

and the kinetic energy

$$KE_{truck} = \frac{1}{2} M v^2 = \frac{1}{2}\left(\sum_i m_i\right) v^2 = \sum_i \left(\frac{1}{2} m_i v^2\right),$$

where the m_i's are the same in each case. So we need look at any one of them, or at all of them together. Simply put, we assume that the truck stays together in one piece, all moving at velocity v at any one time whose height variation is small compared to h.

On the slippery up-ramp we want gravity to do negative work on the truck so that

$$W_{grav} = -Mgh = \underset{\downarrow \atop 0}{KE_{final}} - \underset{\downarrow \atop \frac{1}{2}Mv^2}{KE_{initial}}$$

so $-Mgh = \frac{1}{2} M v^2$

$$gh = \frac{1}{2} v^2 \implies h = \frac{v^2}{2g}$$

(independent of mass, another justification for treating it as a particle)

But $h = L \sin\theta$ so $L = \dfrac{v^2}{2g \sin\theta}$

$$v = \frac{130\,\cancel{km}}{\cancel{h}} \cdot \frac{1\,\cancel{h}}{60\,\cancel{min}} \cdot \frac{1\,\cancel{min}}{60\,s} \cdot \frac{1000\,m}{1\,\cancel{km}} = 36.\bar{1}\ m/s$$

Hence $L = \dfrac{(36.\bar{1})^2}{(9.8\,m/s^2)\, 2 \sin 15°} = \dfrac{(2519)}{9.8\,m/s^2} = 257\ m$

$\underline{260\ m}$

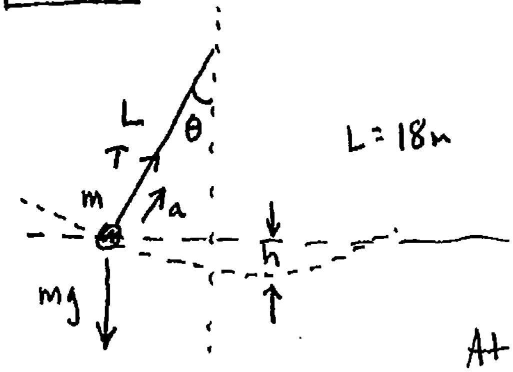
[250 m if g = 10]

b) Does L change if M changes? No, it stays the same.

c) Does L change if v decreases?

L scales as v^2, so if v decreases, L decreases.

$\boxed{8:25}$ I'm not drawing Tarzan. Nor Jane. 7/7



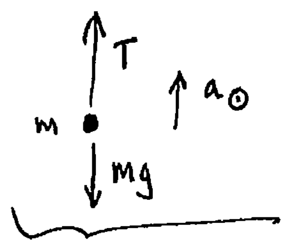
$L = 18\,m$

At the bottom of the swing, $h = 3.2\,m$

T_{max} of the vine is $950\,N$.

$W_{Tarzan} = 688\,N$

At the bottom:



$$T - mg = m a_\odot = \frac{mv^2}{L}$$

Conservation of energy:

Leaving the cliff, Tarzan has $KE = 0$, and $PE_{grav} = mgh$.

At the bottom of the swing he has $KE = \frac{1}{2}mv^2$ and $PE_{grav} = 0$

$$E_{after} = E_{before}$$

$$\underbrace{KE_{after}}_{\frac{1}{2}mv^2} + \underbrace{PE_{after}}_{0} = \underbrace{KE_{before}}_{0} + \underbrace{PE_{before}}_{mgh}$$

$$\frac{1}{2}v^2 = gh \implies v^2 = 2gh = 2 \times 9.8 \times 3.2 = 62.7\,m^2/s^2$$

Now substitute this v^2 into our equation for tension:

$$T = mg + \frac{mv^2}{L} \qquad m = \frac{W_{Tarzan}}{g}$$

$$= W_{Tarzan} + \frac{(70.2\,kg)(62.7\,m^2/s^2)}{18.0\,m} \qquad = \frac{688}{9.8} = 70.2\,kg$$

[68.8 kg]

$$= 688 + 244.5$$

$$= 933\,N$$ (whew! Tarzan is saved)

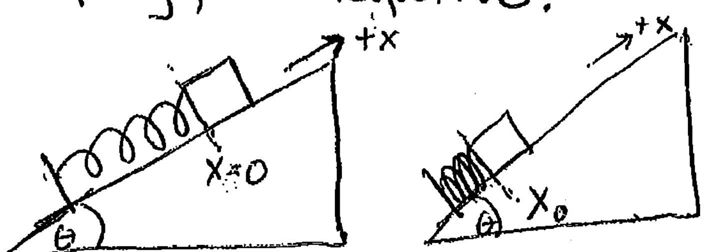
a) The vine doesn't break

b) [For $g = 10\,m/s^2$, $v^2 = 64$ so $T = 688 + \frac{68.8 \cdot 64}{18} = 933N$

This is the same because $\frac{mv^2}{L} = \frac{m(2gh)}{L} = \frac{2h}{L}(mg)$ ← Given as 688N.]

$\boxed{8:29}$ We must choose a reference point for the gravitational potential energy U_g (and h, the height). Choose the point when the spring is maximally compressed.

The highest point is reached when the block, after accelerating up the ramp, reaches a point where its speed is $v_f = 0$ (momentarily). Choose the x-axis pointing up the incline, with $x=0$ the relaxed position of the spring. So x_0, the initial position of the compressed spring, is negative.



$x_0 = -20$ cm
$= -.2$ m (SI units)

$k = 19.6$ N/cm
$= 1960$ N/m (SI units)

(a) The elastic potential energy
$U_s = \frac{1}{2} k x_0^2 = 39.2$ J

(b) Since U_g initially is 0 (by our choice of reference point) the change in gravitational potential energy is mgh.

8:29 (cont.)

(b) continued ....

this change in U_g has to be equal to the change in U_s; $\Delta U_g = U_s = 39.2\,J$

(c) The principle of mechanical energy conservation leads to

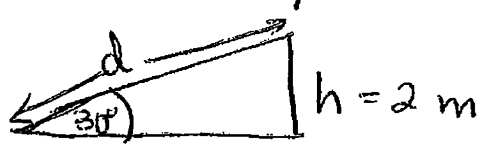
$$K_0 + U_0 = K_f + U_f$$

$$0 + \tfrac{1}{2}kx_0^2 = 0 + mgh$$

$m = 2.0\,kg$, so we solve for h

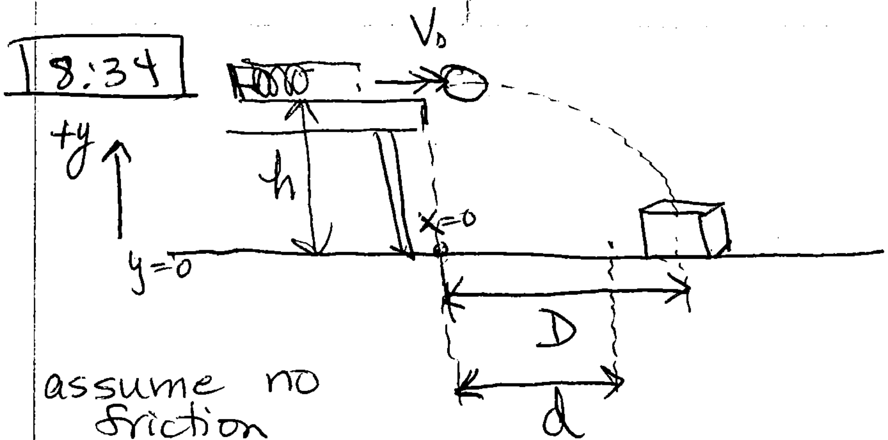
$$h = \frac{\tfrac{1}{2}kx_0^2}{mg} = \frac{39.2\,J}{(2.0\,kg)(9.8\,m/s^2)} = 2.0\,m$$

BUT, the question asks us for the distance d up the incline, so we must use geometry and trig



$$d = \frac{h}{\sin 30°} = \frac{2\,m}{1/2}$$

$d = 4.0\,m$



$D = 2.20$ m

$\Delta l_{SPG} = 1.10$ cm

$d = 2.20 - 0.27$ m
$= 1.93$ m

$h = ?$ $m = ?$

$k_s = ?$

assume no friction

We missed the box by 27 cm when we compressed the spring 1.10 cm. How much should we compress the spring in trial 2 in order to score a direct hit?

We need more V_i to get farther in the x-direction before $h = -\frac{1}{2}gt^2 = 0$ (we have no control over how long it will take to hit the floor). The same time to hit the floor must also allow the ball to travel D.

vertical

$$h_f = -\tfrac{1}{2}gt^2 = 0$$

$$|h| = \tfrac{1}{2}gt^2$$

horizontal

$$x_f = V_{ox}t$$

$$t = \frac{x_f}{V_0}$$

$$h = \tfrac{1}{2}g\left(\frac{x}{V_0}\right)^2 \Rightarrow x_f = V_0\sqrt{\frac{2h}{g}}$$

I have too many unknowns so I look for relationships that allow me to cancel or ignore them. Look: $x_f \propto V_0$

$$\text{So } \frac{x_2}{x_1} = \frac{V_{02}}{V_{01}} \Rightarrow \frac{2.20}{1.93} = \frac{V_{02}}{V_{01}} \Rightarrow V_{02} = 1.14\,V_{01}$$

We also know that V_0 is $\propto$ the spring compression by conservation of energy

$$\tfrac{1}{2}kl^2 = \tfrac{1}{2}mv^2 \Rightarrow v \propto l$$

$\boxed{8{:}34}$ cont

So I seek another ratio that allows me to cancel variables I don't know:

$$\frac{V_{02}}{V_{01}} = \frac{l_2}{l_1} \Rightarrow V_{02} = \frac{l_2}{l_1} V_{01} \qquad l_1 =$$

I can put these two relationships together.

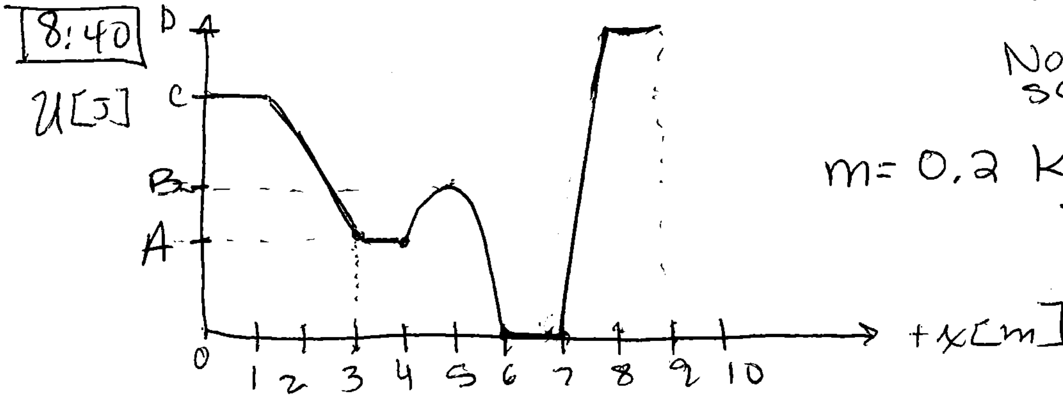
$$V_{02} = 1.14\, V_{01} \qquad \text{and} \qquad V_{02} = \frac{l_2}{l_1} V_{01}$$

$$1.14\, \cancel{V_{01}} = \frac{l_2}{l_1} \cancel{V_{01}}$$

$$l_2 = (l_1)(1.14) = (1.10\text{ cm})(1.14)$$

$$= 1.25\text{ cm}$$

$\boxed{8{:}40}$



NOT TO SCALE!

m = 0.2 kg

$U_A = 9$ J $\quad U_B = 12$ J $\quad U_C = 20$ J $\quad U_D = 24$ J

Particle is released @ point where $U_B = 12$ J. At that moment, the particle has $K = 4$ J. We want to find its velocity at other points and the "position" of the turning points.

Key:
- Initial Emech is constant
- We know y-component @ turning points (from Emech → U_{TOT} + zero K) but we need x-component to identify "position." (y-comp = U)
- If we know the slope and the y-component we can find x

8:40 cont.

a) What is the speed of the particle at $x = 3.5\,m$?

• Note that @ 3.5, $U = 9J$

$$E_{TOT} = U_i + K_i = 12 + 4 = 16\,J$$

$$16\,J = U_2 + K_2 = 9\,J + K_2$$

$$\text{so } K_2 = 16\,J - 9\,J = 7\,J$$

$$K = \tfrac{1}{2}mv^2 \longrightarrow V_2 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(7.0\,J)}{(.2\,kg)}}$$

$$V_2 = 8.37\,m/s \text{ @ } x = 3.5\,m$$

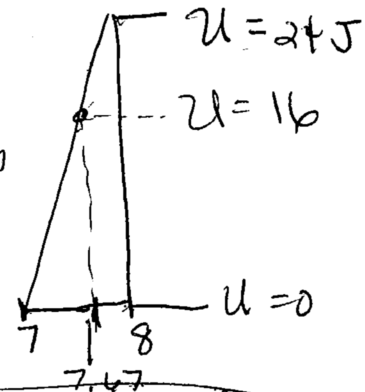
b) ~~At~~ $x = 6.5$, $U = 0$, so all the mechanical energy is kinetic energy.

$$16\,J = \tfrac{1}{2}mv^2 \longrightarrow V_3 = \sqrt{\frac{2(16)J}{.2\,kg}} = 12.6\,m/s$$

@ $x = 6.5\,m$

c) At the turning point, $K = 0$, $E_{TOT} = U = 16\,J$

16J is our "y-coordinate." Look at the line from 7m to 8m:



the slope is $\frac{\Delta U}{\Delta x} = \frac{24}{1} = 24\,J/m$

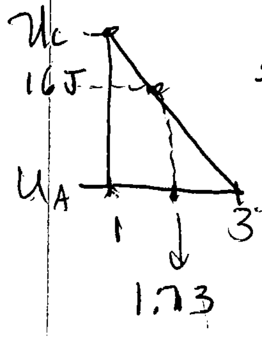
$$\text{so } 16\,J = 24\,J \cdot x$$

$$y = mx + \cancel{b}^{\,0}$$

$$\Delta x = \frac{16}{24} = \frac{4}{6} = \frac{2}{3}$$

$$\text{so } X_R @ 16\,J = 7 + 2/3 = 7.67\,m$$

d) Similary at the left turning point, we find the slope



$$\frac{\Delta U}{\Delta x} = \frac{20 - 9J}{2\,m} = \frac{-11}{2}\,J/m$$

our particle still has $16J = E_{TOT}$

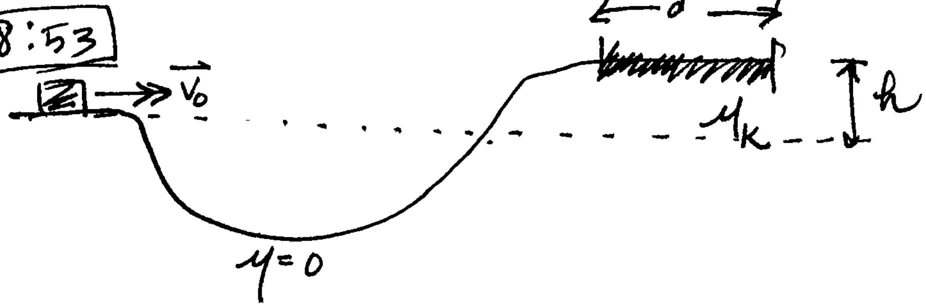
$$y = mx + b$$

$$16 = -\frac{11}{2}x + 20$$

$$(-4)\left(-\frac{2}{11}\right) = .73 = \Delta x$$

$$X_L = 1 + \Delta x = 1.73$$

$\boxed{8:53}$



Since the valley is frictionless, the only reason for the speed to diminish when it reaches the plateau is the <u>gain</u> in U_G.

The gain $= \Delta U_G = mgh$ where $h = 1.1$ m.

Sliding along the rough surface of the higher level, the block finally stops when its remaining kinetic energy K has turned into thermal energy E_{Th}, where

$$\Delta E_{Th} = W_f = f_k d = \mu m g d \qquad \mu = 0.60$$

$$U_i + K_i = U_f + K_f + E_{Th}$$ (treat initial posn as $h=0$ for U_G)

$$0 + \frac{1}{2}\cancel{m}v_0^2 = \cancel{m}gh + 0 + \mu \cancel{m} g d$$

$$\frac{1}{2}v_0^2 - gh = \mu g d$$

$$d = \frac{v_0^2}{2\mu g} - \frac{h}{\mu} = \frac{(6\,\text{m/s})^2}{2(0.6)(9.8)} - \frac{1.1}{0.6} \qquad \left[\frac{\frac{m^2}{s^2}}{\frac{m}{s^2}} - m\right] = [m]$$

$$d = 1.2 \text{ m}$$

13:37

$K_i = \frac{1}{2}mv^2$

m

asteroid

$U_i = -\frac{GM_A m}{R_A}$

$R_A = 500$ km

M_A = asteroid

m = object

Escape speed → "just enough" K to reach infinity where $K \to 0$, $U \to 0$

condn: $-\frac{GMm}{R} + \frac{1}{2}mv^2 = 0$

We are given the asteroid equivalent of $g = \frac{GM_E}{R_E^2}$ on earth

$$g_{AST} = \frac{GM_A}{R_A^2} = 3 \text{ m/s}^2$$

(a) We substitute this into our conservation of energy equation for escape speed

$$\frac{GMm}{R} = \frac{1}{2}mv^2$$

$$\left(\frac{GM}{R^2}\right)\cancel{m}R = \frac{1}{2}\cancel{m}v^2$$

$$\sqrt{2g_{AST}R} = v = \sqrt{2(3 \text{ m/s}^2)(500\times10^3 \text{ m})}$$

$$v = 1.7\times10^3 \text{ m/s}$$

(b) what if $v_i = 1000$ m/s? We see immediately that it won't escape: it will rise to some height – let's call it h – and then return. What's the condition for h? $v = 0$

By conservation of energy

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{(R+h)} + 0$$

again use g_A: $-g_A\cancel{m}R + \frac{1}{2}\cancel{m}v^2 = -\frac{g_A R^2 \cancel{m}}{R+h}$

[note: $\frac{GMm}{R+h} = \left(\frac{GM}{R^2}\right)\left(\frac{mR^2}{R+h}\right)$]

$\boxed{13{:}37}$ cont.

Solve for $h = \dfrac{2g_A R^2}{2g_A R - v^2} - R$

$$h = \frac{2(3\,\text{m/s}^2)(500\times10^3\,\text{m})}{2(3\,\text{m/s}^2)(500\times10^3\,\text{m}) - (1000\,\text{m/s})^2} - 500\times10^3\,\text{m}$$

$h = 2.5\times10^5\,\text{m}$

(c) Note "dropped" from $h = 1000$ km. This implies $v_i = 0$. It's potential energy is $U_i = -\dfrac{GMm}{R+h}$ and $K_i = 0$

By Conservation of energy: $U_i + K_i = U_f + K_f$

$$-\frac{GM\cancel{m}}{R+h} = -\frac{GM\cancel{m}}{R} + \frac{1}{2}\cancel{m}v_f^2$$

again $\dfrac{GM}{R+h} = \dfrac{g_A R^2}{R+h}$ $\qquad \dfrac{GM}{R} = g_A R$

$$\frac{-g_A R^2}{R+h} = -g_A R + \frac{1}{2}v_f^2$$

solving for v_f

$$v = \sqrt{2g_A R - \frac{2g_A R^2}{R+h}}$$

$$= \sqrt{2(3\,\text{m/s}^2)(500\times10^3\,\text{m}) - \frac{2(3\,\text{m/s}^2)(500\times10^3\,\text{m})^2}{(500\times10^3\,\text{m} + 1000\times10^3\,\text{m})}}$$

$v = 1.4\times10^3\,\text{m/s}$

13:40

(a) find $\mathcal{U}$ of system

$$\mathcal{U} = -\frac{G M_A M_B}{R} = \frac{-\left(6.67\times10^{-11}\ \frac{m^3}{s^2\,kg}\right)(20\ kg)(10\ kg)}{0.8\ m}$$

$$\mathcal{U} = -1.67\times10^{-8}\ J$$

(b) find K of sphere B when it has moved 0.2 m toward A.

Note r between spheres is now 0.6 m

By conservation of energy

$$\mathcal{U}_i = K_f + \mathcal{U}_f$$

$$-1.67\times10^{-8}\ J = \underbrace{\tfrac{1}{2} m_B v_f^2}_{K_f} - \frac{G M_A M_B}{.6\ m}$$

$$K_f = -1.67\times10^{-8}\ J + \left(\frac{\left(6.67\times10^{-11}\ \frac{m^3}{s^2\,kg}\right)(20\ kg)(10\ kg)}{.6\ m}\right.$$

$$K_f = 5.6\times10^{-9}\ J$$

13:63

The energy required to raise a satellite of mass m to an altitude h (at rest)

$$E_1 = \Delta U = \frac{GM_E m}{R_E} - \frac{GM_E m}{R_E + h} \qquad [-U_f - U_i]$$

$E_1 = E_{LIFT}$

compare this with the energy required to put the satellite in orbit at that altitude

$$E_2 = \frac{1}{2} m v_{ORB}^2 = \frac{GM_E m}{2(R_E + h)}$$

$E_2 = E_{ORBIT}$

a) We are asked to find the height h where these two energies are equal.

$$E_1 - E_2 = 0$$

$$\frac{GM_E m}{R_E} - \frac{GM_E m}{R_E + h} - \frac{GM_E m}{2(R_E + h)} = 0$$

$$GM_E m \underbrace{\left(\frac{1}{R_E} - \frac{3}{2(R_E + h)} \right)}_{\text{set this equal to zero + solve for } h} = 0$$

$$h = \quad \frac{1}{R_E} = \frac{3}{2(R_E + h)}$$

$$R_E + h = \frac{3R_E}{2}$$

$$h = \frac{3R_E}{2} - R_E = \frac{R_E}{2}$$

$$h = 3.19 \times 10^6 \text{ m}$$

b) For greater height $h_2 > h_1$, $\Delta E > 0$ implying $E_1 > E_2$. Thus the energy of lifting is greater