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Phx 202
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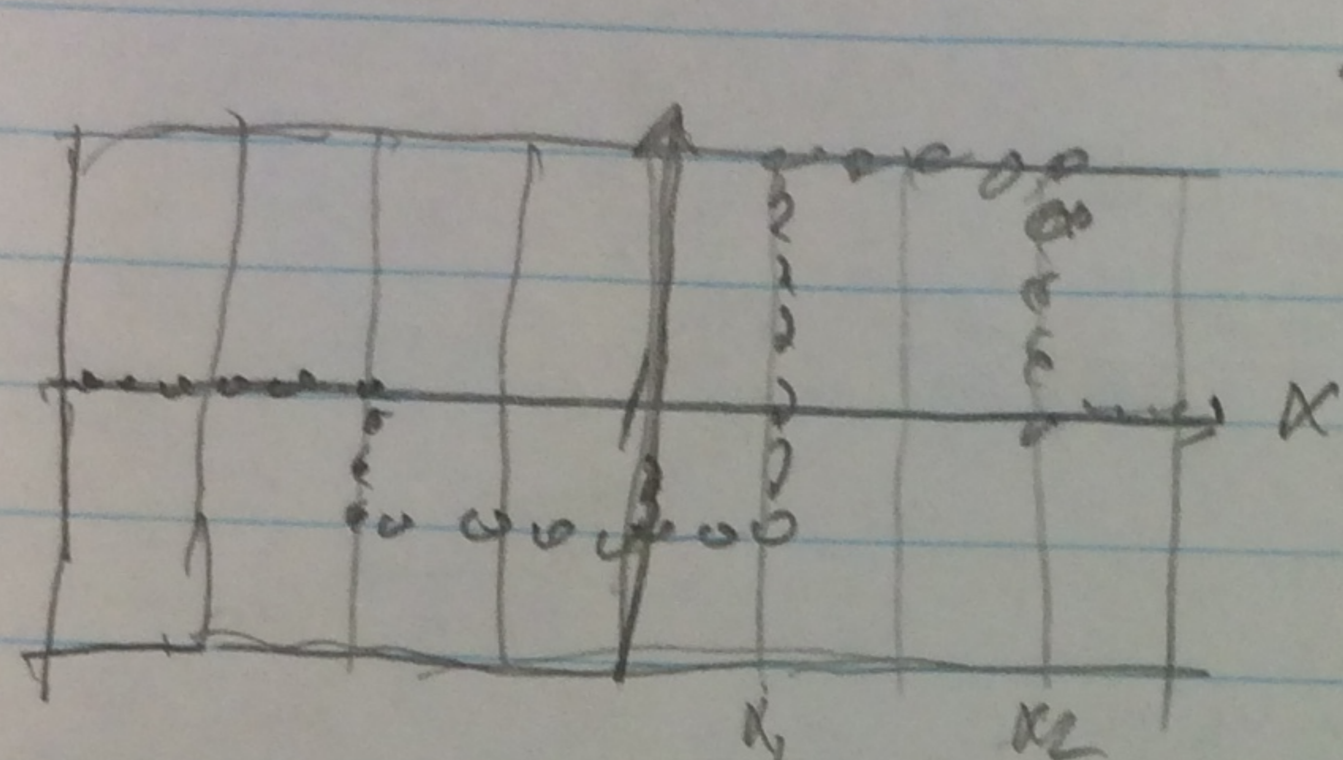
HW for W11/7c
Q9B.1, Q9B.5, Q9M.1

Q9B.1

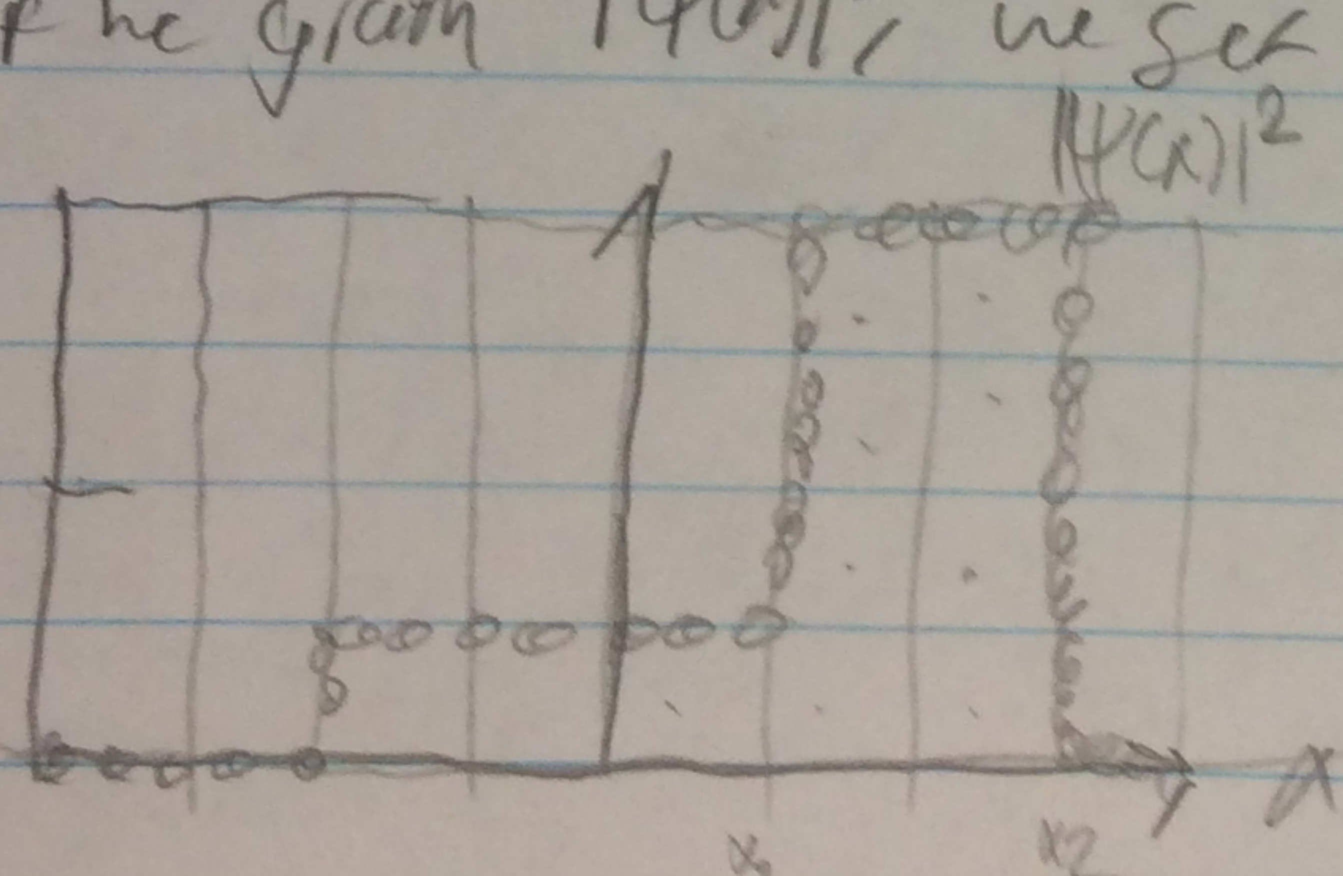
Suppose that at a given time a quanton has the wavefunction shown. If we perform an experiment to locate the quanton at that time, what is the probability that the result will be greater than zero? Explain your reasoning (Assume $\psi(x) = 0$ everywhere outside the region shown.)

$\cdots\cdots = \psi(x)$



Note that there is a section of $\psi(x)$ which is more than zero by two units for two units of x. These values of x correspond to the range of positions (x_1, x_2), where $\psi(x) > 0$. The probability that a quanton will be determined to be within the range (x_1, x_2) is
$Pr(\Delta x) = \int_{x_1}^{x_2} |\psi(x)|^2 dx \Big/ \int_{-\infty}^{\infty} |\psi(x)|^2 dx$, where $\psi(x)$ is the state as a function of its position (wavefunction).

The integral of a function between two domain values is equal to the area under the curve of that function between those values. If we graph $|\psi(x)|^2$, we get



We are told to assume that $\psi(x) = 0$ outside the shown region. So the area under the entire domain of $|\psi(x)|^2$ is 11, and the area under the range (x_1, x_2) is 8.

So $Pr(\Delta x) = \frac{8}{11}$

Q9B.5

Could the function $\psi(x) = Ae^{-|x|/a}$ (where A and a are real constants) possibly be a valid quantum wavefunction under the right circumstances? Why or why not?

A function $\psi(x)$ can only be a meaningful quantum wavefunction if $\int_{-\infty}^{\infty} |\psi(x)|^2 dx$ is a well defined nonzero number.

So we test: $\int_{-\infty}^{\infty} \left| A \cdot e^{-\frac{|x|}{a}} \right|^2 dx = A^2 \int_{-\infty}^{\infty} e^{-\frac{|x| \cdot 2}{a}} dx$

$\Rightarrow A^2 \cdot -\frac{a}{2} \cdot e^{-\frac{|x| \cdot 2}{a}} \Big|_{-\infty}^{\infty} = \frac{-A \cdot a}{2 \cdot e^{\frac{|x| \cdot 2}{a}}} \Big|_{-\infty}^{\infty} = \lim_{n \to \infty} \frac{-A \cdot a}{2 \cdot e^{\frac{|x| 2}{a}}} \Big|_{-n}^{n} = \lim_{n \to \infty} \left(\frac{-A a}{2 \cdot e^{\frac{n 2}{a}}} - \frac{-A \cdot a}{2 \cdot e^{\frac{|-n| 2}{a}}} \right)$

$= \lim_{n \to \infty} (0) = 0$

Well defined but not non-zero. So $Ae^{-|x|/a}$ is not a valid quantum wavefunction

Q9M.7

Consider an oxygen (O_2) molecule in a bottle of air. Imagine that at a certain time, we locate the molecule along the x axis to within an uncertainty of 0.1 nm.

a) What is the minimum uncertainty in the molecule's x-velocity required by the Heisenberg uncertainty principle?

The Heisenberg uncertainty principle states that we will find that $\Delta p_x \Delta x \geq \frac{1}{2}\hbar$ where Δp_x and Δx are the variances in the results of momentum and position from many experiments on quantons in same state.

In this case, $\Delta x = 0.1\text{ nm} = 0.1 \cdot 10^{-9}\text{ m} = 1 \cdot 10^{-10}\text{ m}$. The mass of an O_2 molecule is $2 \cdot \left(16.00 \frac{g}{mol} \cdot \frac{1\text{ mol}}{6.022 \cdot 10^{23}} \cdot \frac{1\text{ kg}}{1000\text{ g}}\right) = 5.31 \cdot 10^{-26}\text{ kg}$

So we have $m \cdot \Delta V_x \cdot \Delta x \geq \frac{1}{2}\hbar \Rightarrow \frac{\hbar}{2 \cdot m \cdot \Delta x} \leq \Delta V_x$, so at a minimum,

$\Delta V_x = \frac{\hbar}{2 \cdot 5.31 \cdot 10^{-26}\text{ kg} \cdot 1 \cdot 10^{-10}\text{ m}} = 9.9\text{ m/s}$

b) How does this compare (roughly) to the magnitude of the molecule's average x-velocity due to its thermal motion at room temp?

At an absolute temperature T, a molecule has an average kinetic energy of $\frac{3}{2} \cdot K_B \cdot T$ where K_B is Boltzmann's constant ($1.38 \cdot 10^{-23}$ J/K)
Room temperature is about 295 K.

The kinetic energy of a mole of O_2 molecules moving at an average speed V_{avg} is $\frac{1}{2} \cdot M \cdot V_{avg}^2$, where $M \approx 32g = 0.032kg$
dividing this energy by a mole will give us the average kinetic energy per molecule, so,

$\frac{1}{2} \cdot M \cdot V_{avg}^2 / 6.022 \cdot 10^{23} \approx \frac{3}{2} \cdot K_B \cdot T \Rightarrow V_{avg}^2 = \frac{3 \cdot K_B \cdot T \cdot 6.022 \cdot 10^{23}}{M}$

So $V_{avg} = \sqrt{\frac{3 \cdot 1.38 \cdot 10^{-23} J/K \cdot 295 K \cdot 6.022 \cdot 10^{23}}{0.032 kg}} = 479 m/s$

This answer does not seem reasonable at all, but I have been unable to get a different value.

Evals-

Q9 B.1
Reasonable magnitudes

Q9 B 5
N/A

Q9 M 2
Right units, Magnitude does not seem reasonable but I have not been able to get a different value