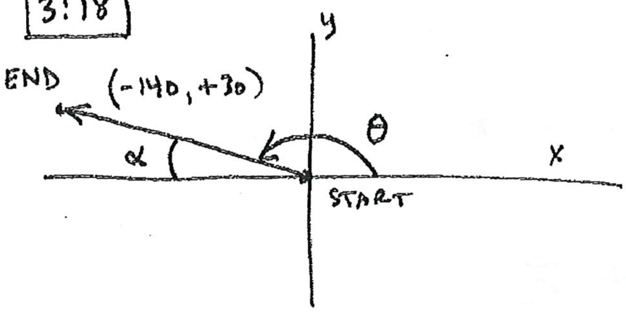
PHY 131 PROBLEM SET #2

HALLIDAY, RESNICK & WALKER, 8th Ed., Extended

CH. 3: 18, 22, 28

CH. 4: 6, 21, 24, 38, 50, 61, 62

3:18



Moves:

	x	y
	20	60
	b_x	-70
	-20	c_y
	-60	-70
NET	-140	+30

From the displacement table,

a) $20 + b_x - 20 - 60 = -140$

$b_x = -140 - 20 + 20 + 60 = \underline{-80 \text{ m}}$

b) $60 - 70 + c_y - 70 = 30$

$c_y = 30 - 60 + 70 + 70 = \underline{110 \text{ m}}$

c) Magnitude of overall displacement

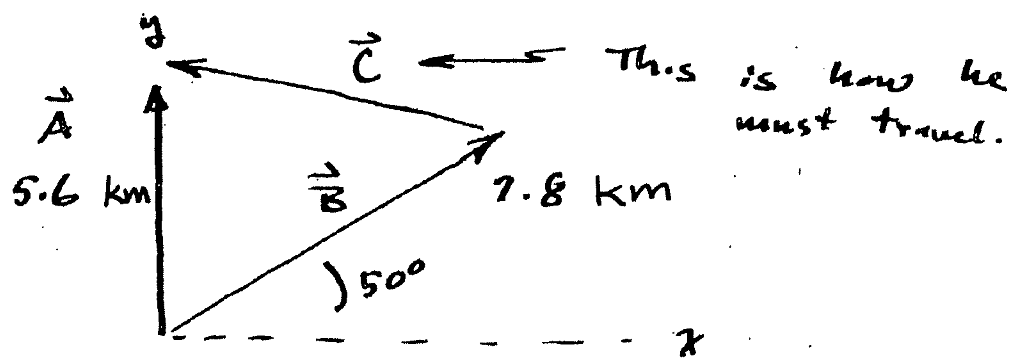
$$\sqrt{(-140)^2 + (30)^2} = \sqrt{20500} = \underline{143 \text{ m}}$$

d) Angle of overall displacement, as measured from +x axis:

$\tan\alpha = \frac{30}{140} = 0.214 \Rightarrow \alpha \approx 12.1^\circ$

so $\theta = 180 - 12.1^\circ = \underline{+168^\circ}$

Ch 3 #22



Want: $\vec{A} = \vec{B} + \vec{C}$

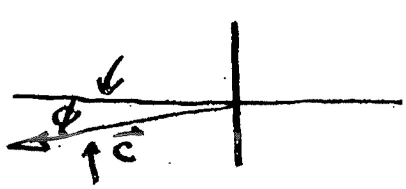
$$\vec{A} \begin{cases} 0 \\ 5.6 \end{cases} \qquad \vec{B} \begin{cases} 7.8 \cos 50 = 5.0 \\ 7.8 \sin 50 = 6 \end{cases}$$

So:

$$0 = 5 + C_x \Rightarrow C_x = -5$$

$$5.6 = 6 + C_y \Rightarrow C_y = -.4$$

Distance $\quad C = \sqrt{C_x^2 + C_y^2} = 5.02 \text{ km}$



$$\tan\phi = \frac{.4}{5} \Rightarrow \phi = 4.6^\circ$$

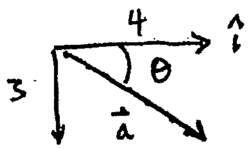
$\boxed{3:28}$ Given two vectors

$$\vec{a} = 4_m\hat{i} - 3_m\hat{j} \quad \text{and} \quad \vec{b} = 6_m\hat{i} + 8_m\hat{j}, \text{ find}$$

a) magnitude of $\vec{a}$

$$a = \sqrt{4^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = \underline{\underline{5\text{ m}}}$$

b) the angle of $\vec{a}$, relative to $\hat{i}$



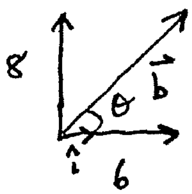
$$\tan\theta = \frac{3}{4} \Rightarrow \theta = \tan^{-1}\left(\tfrac{3}{4}\right) = \underline{\underline{37^\circ}}$$

below the $\hat{i}$ axis

c) magnitude of $\vec{b}$

$$b = \sqrt{6^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = \underline{\underline{10\text{ m}}}$$

d) angle of $\vec{b}$, relative to $\hat{i}$



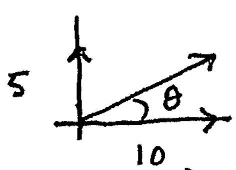
$$\tan\theta = \frac{8}{6} \Rightarrow \theta = \tan^{-1}\left(\frac{8}{6}\right) = \underline{\underline{53^\circ}}$$

above the $\hat{i}$ unit vector

e) magnitude of $\vec{a} + \vec{b} = \vec{c}$

$$\left.\begin{aligned} c_x &= a_x + b_x = 4 + 6 = 10 \\ c_y &= a_y + b_y = -3 + 8 = 5 \end{aligned}\right\} \Rightarrow c = \sqrt{10^2 + 5^2} = \underline{\underline{11.2}}$$

f) direction of $\vec{a} + \vec{b} = \vec{c}$

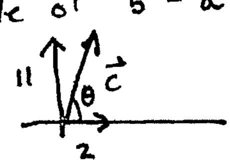


$$\tan\theta = \frac{5}{10} \Rightarrow \theta = 26.6^\circ = \underline{\underline{27^\circ}} \text{ above } \hat{i}$$

g) magnitude of $\vec{b} - \vec{a} = \vec{c}$

$$\left.\begin{aligned} c_x &= b_x - a_x = 6 - 4 = 2 \\ c_y &= b_y - a_y = 8 - -3 = 11 \end{aligned}\right\} c = \sqrt{2^2 + 11^2} = \underline{\underline{11.2}}$$

h) angle of $\vec{b} - \vec{a} = \vec{c}$



$$\tan\theta = \frac{11}{2} \Rightarrow \theta = \tan^{-1}\left(\tfrac{11}{2}\right) = 79.7 = \underline{\underline{80^\circ}}$$

above $\hat{i}$

3: 28, cont'd

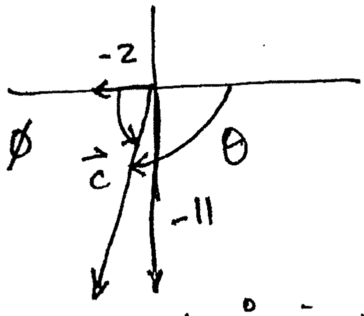
i) magnitude of $\vec{a} - \vec{b} = \vec{c}$

$c_x = a_x - b_x = 4 - 6 = -2$

$c_y = a_y - b_y = -3 - 8 = -11$

$\Rightarrow c = \sqrt{(-2)^2 + (-11)^2} = \sqrt{125} = \underline{\underline{11.2}}$

j) angle of $\vec{a} - \vec{b} = \vec{c}$



$\tan\phi = \frac{11}{2} \Rightarrow \phi = \tan^{-1}\left(\frac{11}{2}\right) = 79.7 = \underline{\underline{80^\circ}}$ below $-\hat{i}$

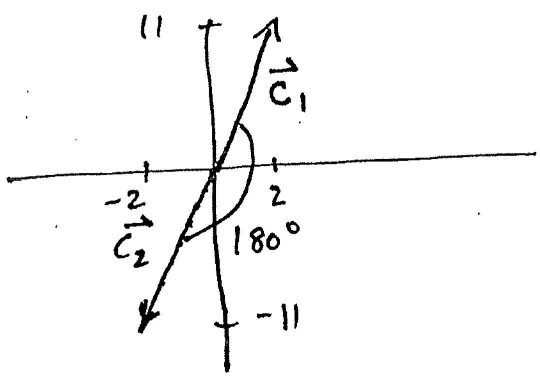
With respect to $+\hat{i}$, we need to find $\theta = 180 - 80 = \underline{100^\circ}$. We interpret this as 100° in the negative (counterclockwise) direction, so it lies at $\underline{\underline{-100^\circ}}$ with respect to $\hat{i}$.

Alternatively, we can view this as $180 + 80 = \underline{+260^\circ}$ from $\hat{i}$.

k) What is the angle between

$\vec{c}_1 = \vec{b} - \vec{a}$

and $\vec{c}_2 = \vec{a} - \vec{b}$?



From the figure, $\vec{c}_1$ and $\vec{c}_2$ point in opposite directions, so they are 180° apart.

$\boxed{4:6}$

An electron's position is given by $\vec{r} = 3.00\,t\,\hat{i} - 4.00\,t^2\,\hat{j} + 2.00\,\hat{k}$, with t in seconds, and $\vec{r}$ in meters.

a) In unit-vector notation, what is the electron's velocity $\vec{v}(t)$?

$$\vec{v} \equiv \frac{d}{dt}\vec{r} = \frac{d}{dt}\left(3.00\,t\,\hat{i} - 4.00\,t^2\,\hat{j} + 2.00\,\hat{k}\right)$$

$$= 3.00\,\hat{i} - 8.00\,t\,\hat{j} + 0\,\hat{k}$$

$$\vec{v} = \underline{\underline{3.00\,\hat{i} - 8.00\,t\,\hat{j}}}$$

At $t = 2.00\,s$, what is $\vec{v}$

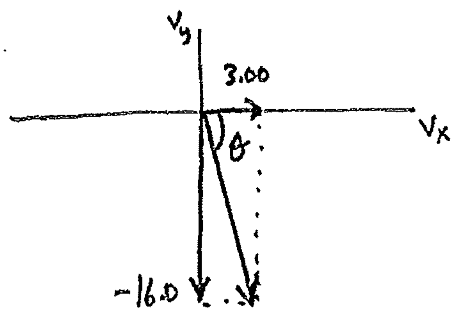
b) in unit-vector notation?

$$\vec{v}(t = 2.00s) = \underline{3.00\,\hat{i} - 16.0\,\hat{j}} \quad m/s$$

c) in magnitude

$$|v| = \sqrt{(3.00)^2 + (-16.0)^2} = \sqrt{9 + 256} = \sqrt{265} = \underline{\underline{16.3}} \quad m/s$$

d) and angle, relative to $\hat{i}$?



$$\tan\theta = \frac{-16}{3} = \underline{\underline{-79.4^\circ}}$$

$\boxed{4:21}$ A projectile is fired horizontally from a gun that is 45.0 m above flat ground, emerging from the gun at $v_0 = 250\ m/s$.

a) How long does the projectile stay in the air?

$$y - y_0 = v_{0y}\,t - \frac{1}{2}g\,t^2$$

horizontal $\Rightarrow v_{0y} = 0$.

y: 0 m, y_0: 45.0 m

so $$-45 = -\frac{1}{2}g\,t^2 \Rightarrow t^2 = \frac{2 \cdot 45\,m}{g} = \frac{90}{9.8} = 9.18\,s^2$$

or $t = \pm\sqrt{9.18} = +3.03\,s$ (choose pos. time).

$[t = \sqrt{9} = 3.00\,s$ if $g = 10]$

4:21 cont'd.

b) At what horizontal distance does the bullet strike the ground?

Along x, $\quad x - x_0 = v_{0x} t \quad$, with $t = 3.03$ (or 3.00)

and $v_{0x} = v_0 = 250$ m/s :

Distance : $x - x_0 = (250 \text{ m/s})(3.03) = \underline{758 \text{ m}}$ [750 m if assume $g = 10 \text{ m/s}^2$]

c) What is the magnitude of the vertical component of $\vec{v}$ as it strikes the ground?

Use $\quad v_y = \underbrace{v_{y0}}_{0} - g t$

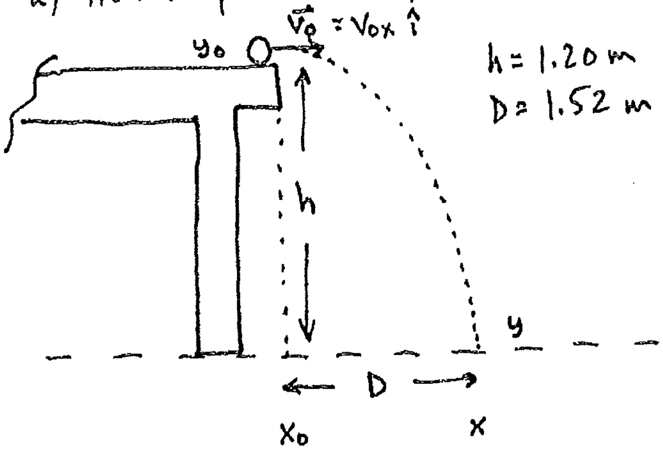
$v_y = -(9.80 \text{ m/s}^2)(3.03 \text{ s}) = -29.7$ m/s $\Rightarrow$ $\underline{29.7 \text{ m/s}}$

$-[10.0 \text{ m/s}^2][3.00 \text{ s}] = -30$ m/s $\Rightarrow$ $\underline{30 \text{ m/s}}$

4:24

A small ball rolls horizontally off the edge of a tabletop that is 1.20 m high. It strikes the floor at a point 1.52 m horizontally from the table edge.

a) How long is the ball in the air?



$h = 1.20$ m
$D = 1.52$ m

1) $(x - x_0) = v_{0x} t$

2) $(y - y_0) = v_{0y} t - \frac{1}{2} g t^2$

Use (2), with $v_{0y} = 0$ to get t:

$(y - y_0) = -\frac{1}{2} g t^2$

$y \to 0$, $y_0 \to h$

$-h = -\frac{1}{2} g t^2$

$\therefore t^2 = \frac{2h}{g} \Rightarrow t = +\sqrt{\frac{2 \cdot 1.20 \text{ m}}{9.80 \text{ m/s}^2}}$

choose later time (earlier time is to the left of origin)

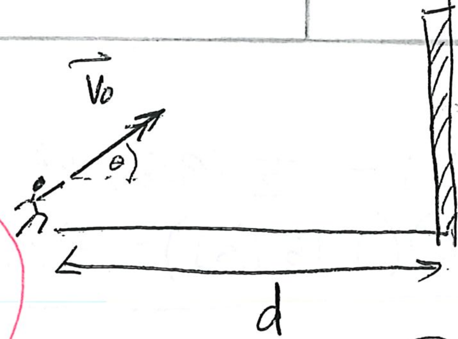
$\therefore t = \sqrt{0.245 \text{ s}^2} = \underline{0.495 \text{ s}}$

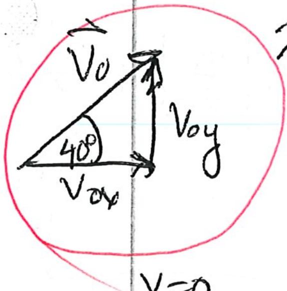
[= 0.490 s if $g = 10 \text{ m/s}^2$]

b) What is its speed at the instant it leaves the table?

Use 1) : $D = x - x_0 = v_{0x} t \Rightarrow v_{0x} = \frac{D}{t} = \frac{1.52 \text{ m}}{0.495 \text{ s}} = \underline{3.07 \text{ m/s}}$ (3.10 m/s if $g = 10 \text{ m/s}^2$)

4.38



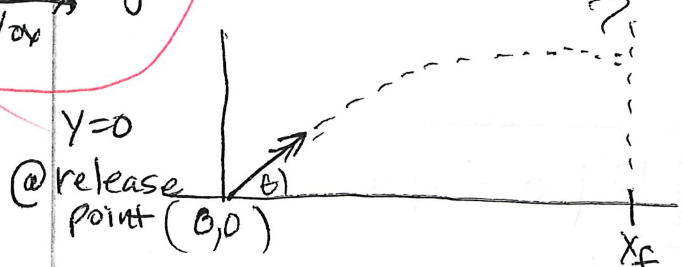


$|\vec{V_0}| = 25\ m/s \quad \theta_0 = 40°$

$X_0 = 0 \qquad Y_0 = 0$

$X_f = 22m \qquad Y_f = ?$

$V_{ox} = |V_0| \cos\theta_0 \qquad V_{oy} = |V_0| \sin\theta_0$



on the way up?
on the way down?
at maximum y
when it hits?
don't know yet.

a) what is Y_f?
b) what is V_{fx}?
c) what is V_{fy}?
d) where in trajectory is it?

$V_{fx} = V_{ox} \quad \Delta t = ? \quad a_y = -g$

$a_x = 0 \quad t_x = t_y \quad V_{fy} = ?$

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x-dir

$V_{ox} = V_{fx}$

$X_f = X_i + V_{ox} t$

ONLY ONE UNKNOWN

y-dir

$V_{fy} = V_{oy} - g t$

$Y_f = y_0 + V_{oy} t - \frac{1}{2} g t^2$

two equations
two unknowns

NOT the same two unknowns

$\Delta X = V_{ox} t$ ← start here

$t = \frac{\Delta X}{V_{ox}}$

$V_{fy} = V_{oy} - g\left(\frac{\Delta X}{V_{ox}}\right)$

$Y_f = V_{oy}\left(\frac{\Delta X}{V_{ox}}\right) - \frac{1}{2} g \left(\frac{\Delta X}{V_{ox}}\right)^2$

$$\boxed{t = \frac{22[m]}{\left(25\left[\frac{m}{s}\right]\right)(\cos 40°)} = 1.15[s]}$$

c)

$$\boxed{V_{fy} = \left(25\left[\frac{m}{s}\right]\right)(\sin 40°) - g\left[\frac{m}{s^2}\right](1.15[s]) = 4.56\left[\frac{m}{s}\right]}$$

4.38 (continued)

a) $Y_f = |V_0| \sin 40° (t) - \frac{1}{2}(9.8)t^2$

$= 25\left[\frac{m}{s}\right](\sin 40)(1.15[s]) - 4.9\left[\frac{m}{s^2}\right](1.15)^2[s^2]$

$\boxed{Y_f = 12.0 \text{ m}}$

b) $V_{fx} = V_{0x} = 25\left[\frac{m}{s}\right] \cos 40°$

$\boxed{V_{fx} = 19.15\left[\frac{m}{s}\right]}$

c) (see previous page)

$V_{fy} = 4.56\left[\frac{m}{s}\right]$ * Note that this is a positive number

d) if V_{fy} is positive, the ball is going up, so it hasn't passed through its maximum, or V_{fy} would be negative; the ball would be on its way down.

find t_{max} when $V_y = 0$ (maximum y)

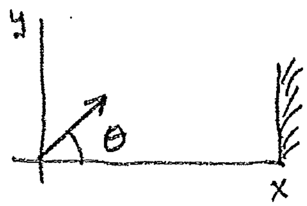
$V_y = 0 = V_{0y} - gt \Rightarrow V_{0y} = gt$

$t = \frac{V_{0y}}{g} = \frac{|V_0| \sin 40°}{9.8}$

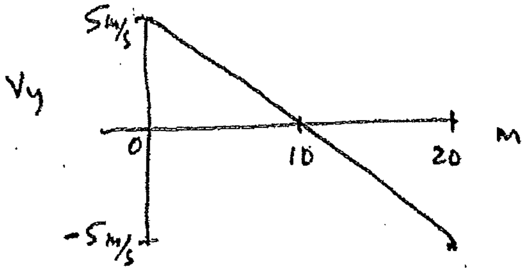
$t_{max} = 1.64$ s

we know the ball hit the wall at $t = 1.15$ s, so it had not yet reached its maximum, where $V_y = 0$ at $t = 1.64$ s

4:50 A ball is to be shot from level ground toward a wall at distance x:



The y component of the velocity is shown on this graph as a function of x, the distance to the wall:



Find the launch angle.

Clearly $V_{oy} = +5.0\ m/s$ because that is the vertical velocity if the wall were at $x = 0$ (see graph). Solve for $V_y = 0$:

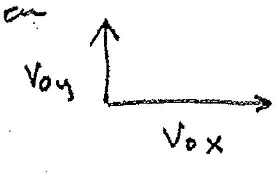
$$V_y = V_{oy} - gt$$

$$0 = V_{oy} - gt \implies t = \frac{V_{oy}}{g} = \frac{5.0\ m/s}{9.8\ m/s^2} = 0.510\,s.$$

This is the time taken to reach the top of its trajectory. Looking again at the graph, this would occur if the wall were at $x = 10\ m$. So the ball travels $10\ m$ in $0.510\,s$, i.e.

$$V_{ox} = \frac{\Delta x}{\Delta t} = \frac{10\ m}{0.510\,s} = 19.6\ m/s \qquad [20\ m/s \text{ if } g = 10\ m/s^2]$$

Then



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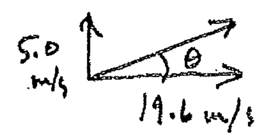


so $\tan\theta = \frac{5.0}{19.6} = 0.255$

or $\theta = \tan^{-1}(0.255) = \underline{14.3^\circ}$

$[14.0^\circ \text{ if } g = 10\ m/s^2]$

Alternative approach

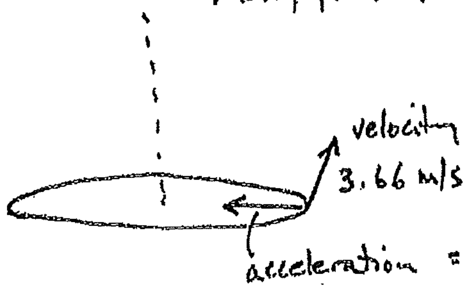
$$\frac{dV_y}{dt} = \underbrace{\left(\frac{dV_y}{dx}\right)}\left(\frac{dx}{dt}\right) \text{ must be } -g$$

$$\left(\frac{-5\ m/s - 5\ m/s}{20\ m}\right)\left(\frac{dx}{dt}\right) = -9.8\ m/s^2, \text{ thus } \left(\frac{dx}{dt}\right) = V_x = V_{ox} = 19.6\ m/s, \text{ as before.}$$

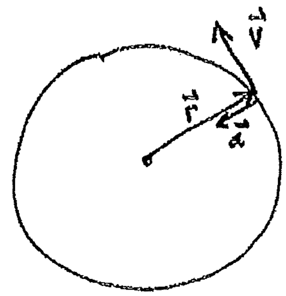
← from computing the slope of the curve from the graph

$\boxed{4:61}$

Merry-go-round



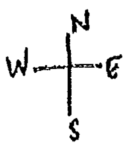
from above:

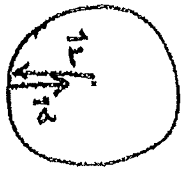


a) What is the magnitude of $\vec{r}$?

$$a_{\odot} = \frac{v^2}{r}, \text{ so } r = \frac{v^2}{a_{\odot}} = \frac{(3.66 \text{ m/s})^2}{1.83 \text{ m/s}^2} = \underline{7.32 \text{ m}}$$

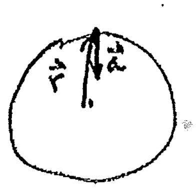
b) What is the direction of $\vec{r}$ when $\vec{a}$ is directed due east?





If $\vec{a}$ points east, then $\vec{r}$ points <u>west</u>. ($\vec{a}$ is opposite $\vec{r}$)

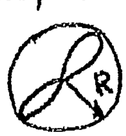
c) What is the direction of $\vec{r}$ when $\vec{a}$ is directed due south?



$\vec{r}$ must be directed <u>north</u>.

$\boxed{4:62}$ A rotating fan completes 1200 revolutions every minute. Consider the tip of a blade, at a radius of 0.15 m.

a) Through what distance does the tip move in one revolution?



Circumference $= 2\pi R = 2\pi(0.15) = \underline{0.94 \text{ m}}$

b) What is the tip's speed? $v = \frac{\Delta x}{\Delta t}$, where $\Delta x = 1200 \frac{\text{rev}}{} \times \frac{0.94 \text{ m}}{\text{rev}} = 1131 \text{ m}$

and $\Delta t = 1 \text{ min} \cdot \left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 60 \text{ s}$, so $v = \frac{1131 \text{ m}}{60 \text{ s}} = \underline{18.8 \text{ m/s}}$

c) Its acceleration $a = \frac{v^2}{r} = \frac{(18.8)^2}{0.15} = 2369 \text{ m/s}^2 = \underline{2400 \text{ m/s}^2}$ (huge!)

d) Its period $T = \frac{1 \text{ min}}{1200 \text{ rev}} \left(\frac{60 \text{ s}}{\text{min}}\right) = \underline{0.05 \text{ s}}$. Now recheck $v = \frac{0.94 \text{ m}}{0.05 \text{ s}} = 18.8 \text{ m/s}$.