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PHY 232-01
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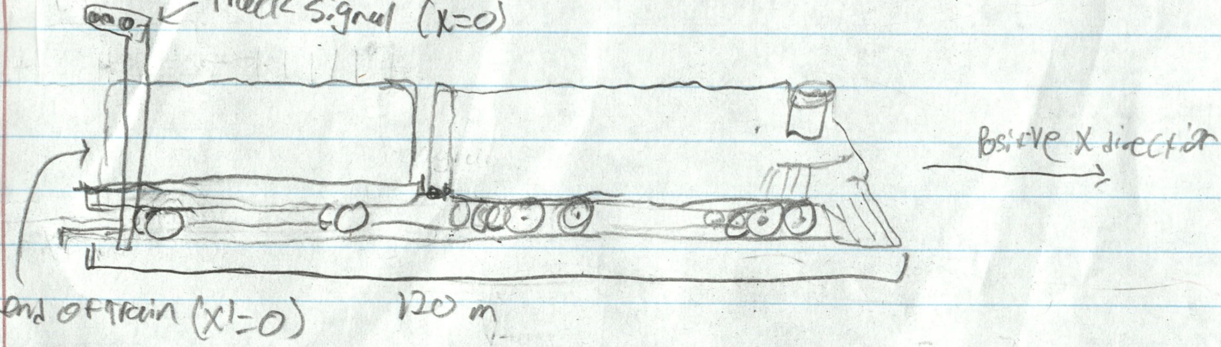
Problems for 8/28/17
R1B.2, R1B.5

R1B.2) Suppose we select the rear end of a 120 m long train to define the origin $x'=0$ in the train frame, and we define a certain track signal light to define the origin $x=0$ in the track frame. Suppose the train's rear end passes this light at $t=t'=0$ as the train moves in the $+x$ direction at a constant speed of 25 m/s. 12 s later, the engineer turns on the train's headlight. Assume the galilean transformation equations are true.



Goal: We want to find where the engineer turns on the lights in (a) the train frame, and (b) the track frame.

Conditions:
- A) t' = time in train frame
- B) $x'=0\,m$ is the origin in train frame and is located at the end of the train
- C) t = time in track frame
- E) The headlights of the train are 120 m in the $+x$-direction from $x'=0\,m$
- F) $x=0$ is the origin in the track frame and is located at the track signal
- G) At $t=t'=0$, $x=x'=0\,m$
- H) The velocity of the train relative to the track ($\vec{\beta}$) is $\hat{\imath}\,25$ m/s
- I) $\vec{r}'$ = position vector of headlights in frame, $\vec{r}$ = position vector of headlights in track frame
- J) $t'=t$ due to the assumption that the galilean transformation equations are true

Worked out Solution

We assume that the galilean transformation equations are true (Problem Statement). So the position of the headlights is expressed as

$$\vec{r}'(t') = \vec{r}(t) - (\hat{\imath}\,25\,m/s)\,t \quad \text{(by A, C, I, H)}$$

$$\Rightarrow \vec{r}'(t) = \vec{r}(t) - (\hat{\imath}\,25\,m/s)\,t \quad \text{(by J)}$$

(Problem continued on next page)

By (E), $\vec{r}'(t) = 120m$, so

$\uparrow 120m = \vec{r}(t) - (\uparrow 25 m/s)t \Rightarrow (\uparrow 120m) + (\uparrow 25 m/s)t = \vec{r}(t)$

The engineer turns on the headlights at $t = 12s$, so

$\vec{r}(12s) = (\uparrow 120m) + (\uparrow 25 m/s) 12\cancel{s} = \uparrow 120m + \uparrow 300m = \uparrow 420m$

(b) Solution

$\boxed{\text{Therefore, the position of the headlights when the the conductor turns them on in the track frame is } \uparrow 420m \text{ from } x = 0m \text{ at } t = 12s}$

In the train frame, the headlights are always $\uparrow 120m$ from $x' = 0m$ (this is not affected by the time).

(a) Solution

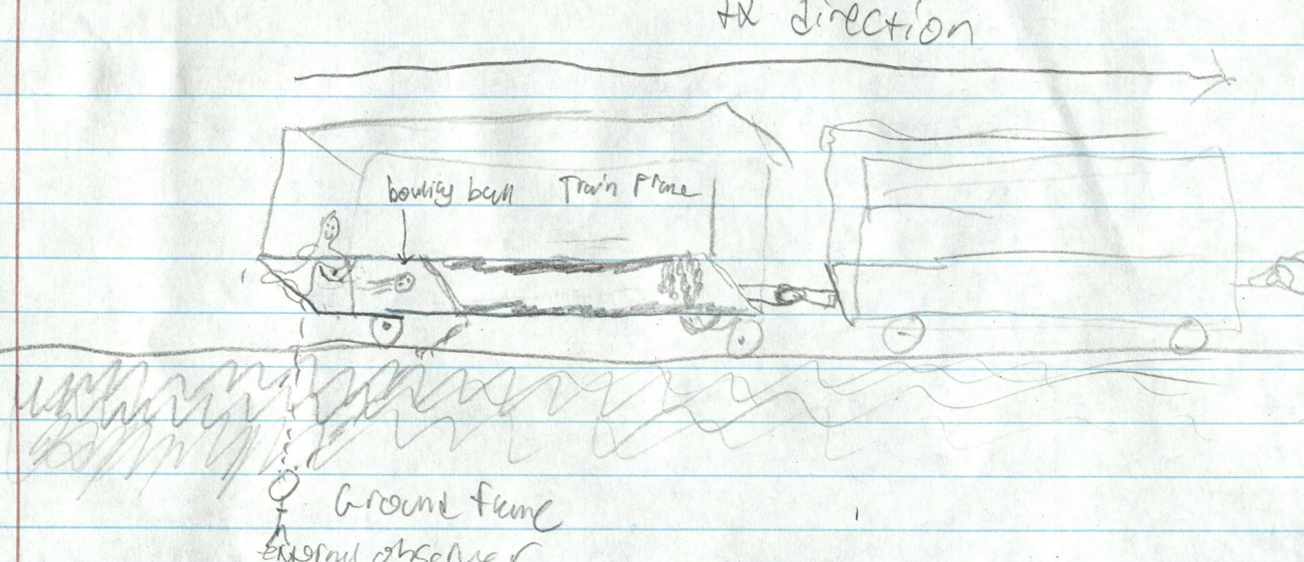
$\boxed{\text{Therefore, the position of the headlights when the conductor turns them on in the train frame is } \uparrow 120m \text{ from } x' = 0m \text{ at } t = 12s}$

Evaluation of answers:

In the solution to (b), I set the rear of the train and the signal light at $x = x' = 0$ at time $t' = t = 0$ as this was a detail that was explicitly stated in the problem statement. My final answer of $\uparrow 420m$ from the origin of the track frame assumed that the "where" (see "Goal" on previous page) refered to the location of the headlight relative to the origin of the track frame at $t = 12s$. Given the train's velocity of $\uparrow 25 m/s$, over 12 seconds, any point on the train will have travelled $\uparrow 300m$. Adding the initial displacement of $\uparrow 120m$ from the traffic signal to the headlight with the travelled displacement of $\uparrow 300m$ yields my answer of $\uparrow 420m$ at 12s.

In the solution to (a) I assumed the "where" (see "Goal" on previous page) was the headlight. I also assumed that the rear of the train would move at the same velocity of $\uparrow 25 m/s$ as the front (ignored possible decoupling, coupling lag, curvature of track etc.) so the distance between the headlights and the origin at the rear of the train would be a constant $\uparrow 120m$ regardless of the time, which yields my answer of $\uparrow 120m$ at 12s.

R1B.5) Suppose that in an effort to get more passengers, Amtrack trains now offer free bowling in a specially constructed "bowling alley" car. Imagine that such a train is travelling at a constant speed of 35m/s relative to the ground. A bowling ball is hurled by a passenger on the train in the same direction the train is travelling. Assume the Galilean transformation equations are true.



General conditions:

Galilean transformation equations are true. (A)

The speed of the train relative to the ground ($|\vec{\beta}|$) is 35 m/s (B)

The bowling ball and the train move in the same direction. (C)

(D) $\vec{V}'$ = Velocity vector of bowling ball in train frame

(E) $\vec{V}$ = Velocity vector of bowling ball in ground frame

(F) The speeds and velocities are constant

Conditions specific to (a): $|\vec{V}| = 42$ m/s (F_A)

Conditions specific to (b): $\vec{V}' = \uparrow 8$ m/s (F_B)

(a) Goal: Find the ball's speed in the train frame.

By (E, F_A) and (E, B), we can say that $\vec{V}(t) \uparrow 42$ m/s, $\vec{\beta} = \uparrow 35$ m/s

So the ball's velocity in the train frame is expressed as (see next page)

(b) Goal: Find the ball's speed in the ground frame

By (E, B) we can say that $\vec{\beta} = \uparrow 35$ m/s.

So the ball's velocity in the train frame is expressed as

$\uparrow 8\text{m/s} = \vec{V}(t) - \uparrow 35\text{m/s}$ (by A, D, F_B)

(see next page)

$\vec{v}'(t) = \hat{i}\,42\,m/s - \hat{i}\,35\,m/s$ (by A, C)

$\Rightarrow \vec{v}'(t) = \hat{i}\,7\,m/s$

but we want $|\vec{v}'(t)|$, so

$|\vec{v}'(t)| = \sqrt{(7\,m/s)^2} = 7\,m/s$

(a) solution $\boxed{\text{Therefore, the speed of the ball in the frame of the } \cancel{\text{train}} \text{ is } 7\,m/s}$

$\Rightarrow \vec{v}(t) = \hat{i}\,8\,m/s + \hat{i}\,35\,m/s = \hat{i}\,43\,m/s$

but we want $|\vec{v}(t)|$, so

$|\vec{v}(t)| = \sqrt{(43\,m/s)^2} = 43\,m/s$

$\boxed{\text{Therefore, the speed of the ball relative to the ground is } 43\,m/s}$ (b) solution

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Evaluation of answers:

In the solution to (a), I used the fact that the ball and train moved in the same direction so I could express the velocity of the ball relative to the ground and the velocity of the train relative to the ground as vectors with their respective speeds in the same component, which I chose to be the x direction. I also used the fact that the velocities were constant (see problem statement), which allowed me to ignore the time variable. I found the speed by taking the norm of the velocity vector of the bowling ball in the train frame, which gave me the answer of 7m/s.

In the solution to (b), I used the fact that the ball and train moved in the same direction, so I could express the velocity of the train as a vector with its speed in one component, which I chose to be the x direction. I also used the fact that the velocities were constant (see problem statement), which allowed me to ignore the time variable. I found the speed by taking the norm of the velocity vector of the bowling ball in the ground frame, which gave me the answer of 43 m/s.