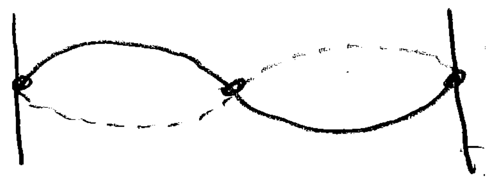
PHY 131 Worksheet on Traveling and Standing Waves

1. A rope, under a tension of 200 N and fixed at both ends, oscillates in a second-harmonic standing wave pattern. The displacement of the rope is given by

$$y(x,t) = 0.10\sin(\tfrac{\pi}{2}x)\sin(12\pi t)\ [\text{m}],$$

where $x = 0$ at one end of the rope, x is in meters and t is in seconds.

(a) Sketch the shape of the second-harmonic standing wave.



(b) What is the length of the rope?

- To know the length, we recall the relationship between the wavelength λ and the length L of the rope.
  - For n= 1, L = $\frac{1}{2}$ * λ [m]
  - For n = 2, L = 1 * λ [m]
- To learn spatial information about the wave we first identify the wave number k.

For this wave, The wave number k = $\pi/2$ [m^{-1}]
  - What is the relationship between k and λ ?

$$k = \frac{2\pi}{\lambda}$$

- Solve this equation for λ.

$$k = \frac{2\pi}{\lambda} = \frac{\pi}{2} \Rightarrow \lambda = 4.0\ m$$

- Use this value of λ to solve for the length L of the rope.

$$L = 1\lambda = 4.0\ m$$

(b) What is the frequency *f*, in Hz, of the oscillations of the rope?

- To learn temporal information about the wave we first identify the angular frequency ω.
  - For this wave, ω = 12π [rad/s]
  - What is the relationship between ω and *f*?

    $\omega = 2\pi f$

- Solve this equation for *f*.

  $f = \frac{\omega}{2\pi} = 6.0 \text{ Hz}$

- What is the period *T* of the oscillations?

  $T = \frac{1}{f} = 0.17 \text{ s}$

(c) What is the linear speed *v*, in m/s, of the waves on the rope?

- Linear speed is related to a number of wave characteristics.
  - How is speed *v* related to frequency *f*?

    $v = f\lambda$
  - The period?

    $v = \frac{\lambda}{T}$
  - The angular speed ω?

    $v = \frac{\omega}{k}$
  - The wavelength λ?

    $v = f\lambda$ or λ/T
  - The wave number *k*?

    $v = \frac{\omega}{k}$

- The speed of the wave *v* = 24 [m/s]

  $v = \frac{12\pi}{\pi/2} = 24 \text{ m/s}$

(d) What is the mass *m* of the rope?

- To find the mass, we first identify the mass density μ, in kg/m, of the rope.
  - How is μ related to the speed *v*?

$$v = \sqrt{\frac{T}{\mu}}$$

  - Solve this equation for μ.

$$\mu = \frac{T}{v^2}$$

- Find the mass m from the definition of μ.

$$\mu = \frac{m}{L} \qquad m = L\mu$$

$$m = L \cdot \frac{T}{v^2} = 1.4 \text{ kg}$$

(e) If the rope oscillates in a 3rd harmonic standing wave pattern, what will be the period *T* of oscillations?

- The period *T* and the frequency *f* are related in what way?

$$T = \frac{1}{f}$$

- How does frequency change with respect to the *n*th harmonic?
  - f_n = $\frac{nv}{2L}$ [Hz]
- Solve this equation for f_3.

$$f_3 = \frac{3v}{2L} = \frac{3}{2}\frac{24 \text{ m/s}}{4.0\text{ m}} = 9 \text{ Hz}$$

- Does the speed *v* change? Why or why not?

no: the tension didn't change, μ didn't change, the length didn't change

2. A nylon guitar string has linear density $\mu = 7.2$ g/m and is under tension T = 150 N. The distance between the fixed supports is D = 90 cm. The string is oscillating in the 3rd harmonic.

- Sketch the pattern of the third harmonic standing wave.

- For the traveling waves whose superposition produces ~~this standing~~ wave, calculate
  - the speed v [m/s],

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{150\,N}{7.2 \times 10^{-3}\ kg/m}}$$

$V = 144$ m/s

  - the wavelength λ [μ],

$$\lambda = \frac{2L}{n} = \frac{2(90\ cm)}{3} = 60\ cm$$

  - and the frequency f [Hz].

$$f = \frac{V}{\lambda} = \frac{144\ m/s}{0.6\ m} = 240\ Hz$$