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R7B.1, R8M.2

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R7B.1)

You are the captain of a spaceship that is moving through an asteroid belt on impulse power at a speed of $\frac{4}{5}$ relative to the asteroids. Suddenly, you see an asteroid dead ahead a distance of only 24s away, according to the sensor measurements in your ship's reference frame. You immediately shoot off a missile which travels forward at a speed of $\frac{4}{5}$ relative to the ship. The missile hits the asteroid and detonates, pulverizing the asteroid into gravel. However, you learned in Starfleet academy that it is not safe to pass through such a debris field (even with the shields on) sooner than 8 seconds (measured in the asteroid frame) after the detonation. Are you safe?

Let event A be the firing of the missile in the frame of the Asteroid. Let event B be the detonation of the missile ~~...~~ origin at the frame of the asteroid

The distance between A and B in the asteroid frame can be found by using the fact that

$\vec{d}_{AB} = d_{AB_{Rest}}\sqrt{1-\left(\frac{4}{5}\right)^2} = 24s$ so $d_{AB_{Rest}} = \frac{24s}{\sqrt{1-\left(\frac{4}{5}\right)^2}} = 40s$

So $\boxed{\vec{d}_{AB_{Rest}} = 40s}$

In the frame of the ship ($\beta = \frac{4}{5}$, relative to the asteroid frame), the speed of the missile is $\frac{4}{5}$.

Using the Einstein velocity transform to find $\vec{V}_{missile}$ in the Asteroid frame,

$$\vec{V}_{missile} = \frac{\frac{4}{5}+\frac{4}{5}}{1+\frac{4}{5}\cdot\frac{4}{5}} = \frac{8/5}{\frac{25+16}{25}} = \frac{8/5}{41/25} = \frac{200}{205} \approx 0.98$$

So $\boxed{\vec{V}_{missile} = \frac{200}{205}}$

The time it will take the missile to travel from Event A to Event B as measured in the asteroid frame is

$$\text{time} = \frac{\text{distance}}{\text{velocity}} = \frac{40s}{(200/205)} \quad 41s$$

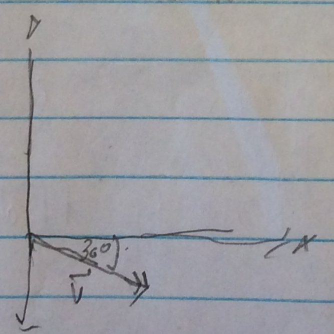
The time it takes for the ship to travel from event A to Event B as measured in the asteroid frame is

$$\text{time} = \frac{\text{distance}}{\text{velocity}} = \frac{40s}{(4/5)} = 50s,$$

So the ship will arrive at the blown up asteroid 9s after it was blown up, which is just 1 second above the minimum 8 seconds after blowing up an asteroid scores, it will be safe

RBM.2

Suppose a dust particle of mass 2.0 kg is travelling at a speed of $\frac{4}{5}$ in the XY plane at an angle of 30° clockwise from the X axis in a certain IRF. Evaluate the following in that frame:



a) The particle's Relativistic energy E

The relativistic energy is $E = \frac{m}{\sqrt{1-|\vec{v}|^2}}$ so $E = \frac{2.0\,kg}{\sqrt{1-\left(\frac{4}{5}\right)^2}} = \frac{2.0\,kg}{\sqrt{1-\frac{16}{25}}} = \frac{2.0\,kg}{\sqrt{\frac{9}{25}}} = \frac{2.0\,kg}{\frac{3}{5}}$

$= \frac{5 \cdot 2.0\,kg}{3} = \boxed{\frac{10}{3}\,kg \approx 3.3\,kg}$

b) It's relativistic momentum magnitude $|\vec{p}|$

The magnitude of the relativistic momentum $|\vec{p}|$ is $|\vec{p}| = \frac{m|\vec{v}|}{\sqrt{1-|\vec{v}|^2}}$, so

$$|\vec{p}| = \frac{2.0\mu g \cdot \frac{4}{5}}{3/5} = \frac{2.0\mu g \cdot 4 \cdot 5}{3 \cdot 5} = \frac{2.0\mu g \cdot 4}{3} = \boxed{\frac{8}{3}\mu g \approx 2.7\mu g}$$

c) Its three spatial four momentum components

The three spatial four-momentum components are

$p_x = m v_x / \sqrt{1-|\vec{v}|^2} =$

$p_y = m \cdot v_y / \sqrt{1-|\vec{v}|^2}$

$p_z = m \cdot v_z / \sqrt{1-|\vec{v}|^2}$

Because the particle is only traveling in the x and y spatial dimensions (in this frame), $p_z = 0$

The x and y components of the spatial four momentum are

$p_x = m \cdot |\vec{v}| \cdot \cos\theta / \sqrt{1-|\vec{v}|^2} = |\vec{p}| \cdot \cos\theta$

$p_y = m \cdot |\vec{v}| \cdot \sin\theta / \sqrt{1-|\vec{v}|^2} = |\vec{p}| \cdot \sin\theta$,

so

$p_x = \left(\frac{8}{3}\mu g\right) \cdot \cos(-30°) = \frac{8}{3}\mu g \cdot \frac{\sqrt{3}}{2} = \frac{4\sqrt{3}}{3}\mu g = \frac{4}{\sqrt{3}}\mu g \approx 2.3\mu g$

$p_y = \left(\frac{8}{3}\mu g\right) \cdot \sin(-30°) = \frac{8}{3}\mu g \cdot -\frac{1}{2} = -\frac{4}{3}\mu g \approx -1.3\mu g$

Therefore, the three spatial components of the four momentum are

$p_x = \frac{4}{\sqrt{3}}\mu g$

$p_y = -\frac{4}{3}\mu g$

$p_z = 0\mu g$

d) Its kinetic energy K (in joules)

The kinetic energy of an object is $K = E - m$, where E is the relativistic energy, and m is the mass or four-magnitude.

So, in SR units, $K = \frac{10}{3}\mu g - 2.0\mu g = \frac{4}{3}\mu g$

To convert to Joules, we need to convert to kg

$K = \frac{4}{3}\mu g \cdot \frac{1\,kg}{10^9\,\mu g} = \frac{4}{3}\cdot 10^{-9}\,kg \approx 1.3 \cdot 10^{-9}\,kg$

Joules are in units of $kg \cdot \frac{m^2}{s^2}$ so we multiply by $c^2\ \frac{m^2}{s^2}$, that is,

$K = \frac{4}{3}\cdot 10^{-9}\,kg \cdot \frac{(3\cdot 10^8\,m)^2}{s^2} = \frac{4}{3}\cdot 10^{-9}\,kg \cdot 9\cdot 10^{16}\,\frac{m^2}{s^2} = 4\cdot 3 \cdot 10^{7}\,kg\cdot m^2/s^2 = 1.2\cdot 10^8\,J$

So $\boxed{K = 1.2\cdot 10^8\,J}$

Eval of answers:

R7R.1

This was a tricky one. I think all our answers are of a reasonable magnitude, and have the right units.

R8M.2

Most units are all consistent, magnitudes seem correct.