
(Name)

Exam III

Mathematics 220

November 29, 2017

Please complete five out of six questions. If you do all six, cross out the one you do not wish to be graded. There are a few blank pages at the end if you need scrap paper.

1. Consider the system below:

$$\begin{aligned}x' &= 3x - 2y, & x(0) &= 3 \\y' &= 2x - 2y, & y(0) &= 1/2\end{aligned}$$

Transform the system into a single second order differential equation of the form

$$ax'' + bx' + cx = 0, \quad x(0) = d, \quad x'(0) = e.$$

2. Find the general solution of the system

$$\mathbf{x}' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}$$

3. Two competing species have population densities measured by x and y , and their populations are governed by the differential equations

$$x' = x(4 - x - 2y)$$

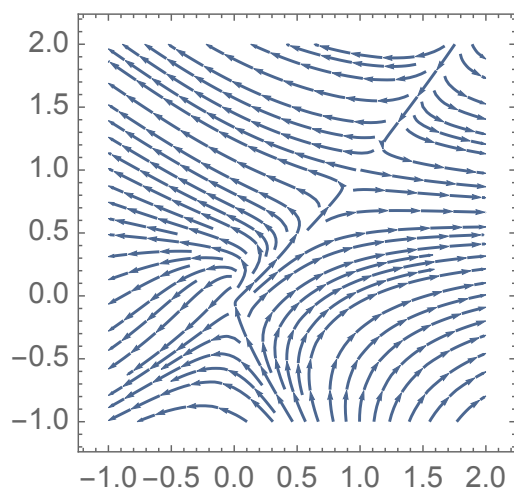
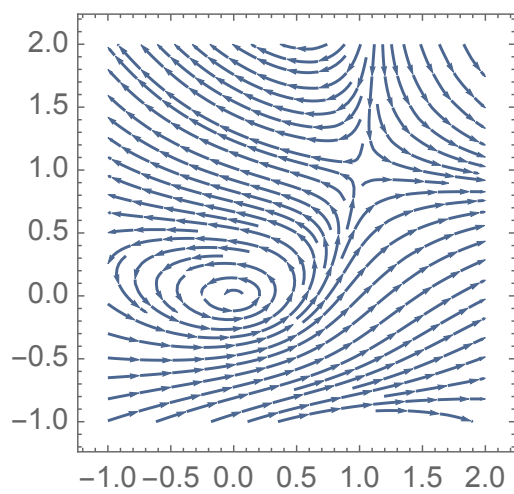
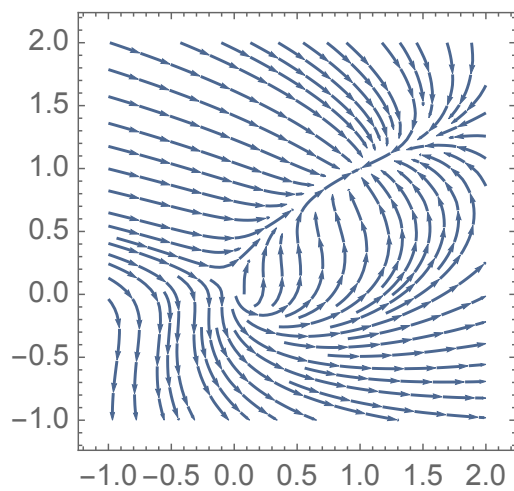
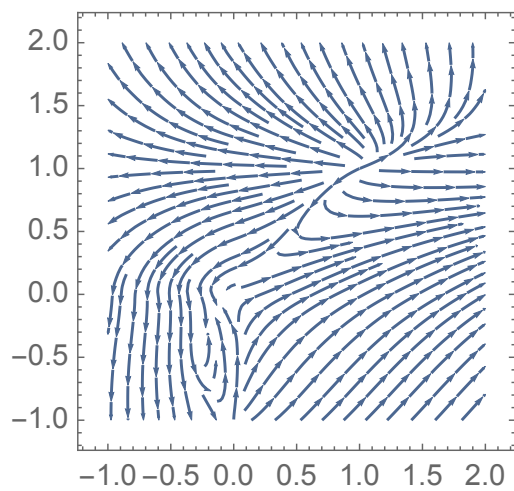
$$y' = y(6 - 3x - 2y)$$

- Find the equilibrium point at which neither species is extinct.
- Draw nullclines for the system.
- Sketch a phase portrait using the nullclines, and state whether the equilibrium point is asymptotically stable or unstable.

4. Consider the system

$$\begin{aligned}x' &= x^2 + 2xy - 3y \\y' &= 1 + x + y^2 - 2xy^2\end{aligned}$$

- Find the Jacobian of the system.
- Evaluate the Jacobian at $(1, 1)$ and at $(0, 0)$.
- Determine the eigenvalues of the Jacobian at these two points.
- Circle the phase portraits on the next page corresponding to this system.



5. Consider the system

$$\mathbf{x}' = \begin{pmatrix} 3 & -4 \\ 1 & \alpha \end{pmatrix} \mathbf{x}$$

- Determine the eigenvalues in terms of α
- Find the value or values of α where the qualitative nature of the phase portrait of the system changes.
- Describe how the phase portrait changes as α increases from -10 to 10 .

6. Find the solution of the differential equation

$$(2xy^2 + 2y) + (2x^2y + 2x + ye^y)y' = 0$$

