Phy 232
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Hw for 11/6/17

QHR3, Q15 M10

QHR3

Imagine that you are one of the first astronauts to visit mars. Your crew stumbles on evidence of ruins created by some alien race an unknown amount of time ago. You personally discover some kind of device that proves to have spots that are faintly radioactive. When you run an analysis of the spots you find that about 0.012% of their atoms are radioactive $^{99}_{43}Tc$. The spots contain a variety of other materials which all look reasonably normal except that 22% are $^{99}_{44}Ru$ with no other isotopes of ruthenium present. This is odd because in natural samples of ruthenium, only 13% would be this particular isotope. Your measurements indicate a sample of purified technicium with a mass of 1.98 μg reads about 1260 decays/s. How long ago was the device made?

The activity (rate of decay) of a radioactive sample is $A = \frac{dN}{dt} = \lambda N$, where λ is a constant called the decay constant and N is the number of undecayed nuclei. For a radioactive substance the number of undecayed nuclei as a function of time is $N(t) = N_0 \cdot e^{-\lambda t}$, where N_0 is the initial number of nuclei, and λ is the decay constant. At a time T, if $X \cdot N_0$ nuclei are still undecayed, then we have $X \cdot N_0 = N_0 \cdot e^{-\lambda T}$. We can solve for the elapsed time by manipulating this expression to $-\ln(X)/\lambda = T = -\frac{N}{A} \cdot \ln(X)$.

We are given the activity of a 1.98 μg sample of pure ^{99}Tc is 1260 Bq $= A$. The molar mass of ^{99}Tc is $98.906 \frac{g}{mol}$, so the number of ^{99}Tc atoms in a 1.98 μg sample is $\frac{6.022 \cdot 10^{23} \frac{1}{mol}}{98.906 \frac{g}{mol}} \cdot 1.98 \cdot 10^{-6} g = 1.206 \cdot 10^{16}$ atoms $= N$

It is reasonable to assume that $^{99}_{43}Tc$ undergoes β^- decay making $^{99}_{44}Ru$, as this is the only isotope of ruthenium present which does not happen for "naturally" occurring ruthenium. A quick search shows that $M_{99Tc} = 98.9062547u$, $M_{99Ru} = 98.905939u$, so neutron decay is energetically possible.

Let's assume there are Q atoms in all the spots. Then $1.2\cdot10^{-4}\cdot Q$ atoms are ^{99}Tc, $0.022\cdot Q$ atoms are ^{99}Ru. Assuming no other ruthenium was initially present in the spots, the total number of ^{99}Tc atoms initially present is $(1.2\cdot10^{-4}+0.022)\cdot Q=0.02212\cdot Q$

So the ratio of ^{99}Tc atoms when I find the device to the initial amount of ^{99}Tc atoms is $\frac{1.2\cdot10^{-4}Q}{0.02212\,Q}=0.005$. This means at time T, if there were initially N_0 undecayed ^{99}Tc atoms, there are $0.005\cdot N_0$ atoms, so $x=0.005$

Plugging everything into $T=\frac{-N\cdot\ln(x)}{A}$, we get $T=\frac{-1206\cdot10^{15}\cdot\ln(0.005)}{1200\text{ Bq}}=5.3\cdot10^{13}$

This is about 1.6 Myr

This is reasonable as ^{99}Tc has a half life of about 0.2 Myr and for N_0 initial atoms, going through 8 half lives gives $\frac{N_0}{2^8}=0.004\cdot N_0$ which is very close to what is given in the problem and 8 half lives is $8\cdot0.2\text{ Myr}=1.6\text{ Myr}$, which agrees with our answer.

Q15M.10

Fusion in both magnetic confinement reactors and in the stars occurs at lower temps than one might expect because of quantum tunneling. In the chapter we estimated that in order for two hydrogen nuclei to get close enough (within 2 fm) to react, they would need a KE of 360 keV. The Sun's central temp is is $\sim15\cdot10^{6}$ K. The probability that a hydrogen atom in the sun would have this much energy is very roughly 10^{-114}, so fusion shouldn't happen in the sun. However, quantum tunneling can allow nuclei to tunnel through the potential energy barrier separating the nuclei even if they don't get this close. A more sophisticated quantum calculation of the probability that two colliding nuclei of mass m and proton # Z will tunnel together and fuse in a gas of temp T is $P_r\approx\exp\left(-\frac{2\pi}{\hbar c}\frac{(Ze)^2}{4\pi\varepsilon_0}\sqrt{\frac{mc^2}{3k_BT}}\right)$

a) Use this expression to calculate the probability that two hydrogen atoms will fuse at the sun's center.

$$Pr \approx \exp\left(-\frac{2\pi}{\hbar c}\cdot\left(\frac{1\cdot e^2}{4\pi\epsilon_0}\right)\cdot\sqrt{\frac{m_H c^2}{3k_B T}}\right)$$

$$Pr \approx \exp\left(\frac{-2\pi}{197.3\,eV\cdot nm}\cdot 1.44\,eV\cdot nm\cdot\sqrt{\frac{511000\,eV}{3.8617\cdot 10^{-5}\,eV/K\cdot 15\cdot 10^{6}K}}\right) = e^{-0.526431} \approx 0.591$$

b) Compare this to the classical probability of $2\cdot 10^{-120}$

$$0.591/2\cdot 10^{-120} = \frac{0.591}{2}\cdot 10^{120} \approx 2.955\cdot 10^{119} \text{ times greater probability (a lot)}$$

This seems reasonable. Classical probabilities can get ridiculously small.