PHY 131
HW 5 Solutions

Ch. 3: Problems 39, 41

Ch. 7: Problems 5, 11, 13, 27, 29, 34, 36, 40, 57

$\boxed{3:39}$ $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\phi = a_x b_x + a_y b_y + a_z c_z$

$$\cos\phi = \frac{a_x b_x + a_y b_y + a_z c_z}{|\vec{a}|\,|\vec{b}|}$$

$$|\vec{a}| = \sqrt{(3.00)^2 + (3.00)^2 + (3.00)^2} = 5.20$$

$$|\vec{b}| = \sqrt{(2.00)^2 + (1.00)^2 + (3.00)^2} = 3.74$$

The angle between them is found from

$$\cos\phi = \frac{(3.00)(2.00) + (3.00)(1.00) + (3.00)(3.00)}{(5.20)(3.74)}$$

$$\phi = 22°$$

$\boxed{3:41}$ $\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\phi \rightarrow \cos\phi = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|}$

Here, we're given $|\vec{A}| = 6.00$, $|\vec{B}| = 7.00$ and $\vec{A}\cdot\vec{B} = 14.0$, so

$$\cos\phi = \frac{14.0}{(6.00)(7.00)} = 0.333$$

$$\phi = 70.5°$$

$\boxed{7{:}5}$ Father and son.

Given $KE_{father} = \frac{1}{2} KE_{son}$ ①

and $M_{son} = \frac{1}{2} M_{father}$ ②

If the father speeds up by 1.0 m/s he matches his son's KE.
Find a) V_0 father and b) V_0 son.

a) Writing $KE = \frac{1}{2}MV^2$, ① becomes

$$\frac{1}{2} M_f V_{of}^2 = \frac{1}{2}\left(\frac{1}{2} M_s V_{os}^2\right). \quad \text{Using ② gives}$$

$$\frac{1}{2} M_f V_{of}^2 = \frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}M_f\right) V_{os}^2\right)$$

$$\text{so} \quad V_{of}^2 = \frac{1}{4} V_{os}^2 \quad \text{or} \quad \boxed{V_{of} = \frac{1}{2} V_{os}}$$

Now if $V_f = V_{of} + 1.0\ m/s$ we see that his KE doubles:

$$\frac{1}{2} M_f V_f^2 = 2\left(\frac{1}{2} M_f V_{of}^2\right)$$

$$\cancel{M_f}\left(V_{of} + 1\right)^2 = 2\cancel{M_f} V_{of}^2$$

$$V_{of}^2 + 2V_{of} + 1 = 2V_{of}^2$$

Putting this in standard form we get $V_{of}^2 - 2V_{of} - 1 = 0$
which has solutions

$$V_{of} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2}$$

$$V_{of} = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

Now a little interpretation: the father and son are presumably racing in the same direction, so we choose the positive square root so that when we add 1.0 m/s to the father's speed he doesn't reverse direction!

So $V_{of} = 1 + \sqrt{2} = \underline{\underline{2.4}}$ m/s

[Check: $V_{of}^2 = 5.83\ m^2/s^2$ and $(V_{of}+1)^2 = 11.66\ m^2/s^2$ which is double]

b) and then $V_{os} = 2V_{of} = \underline{\underline{4.8}}$ m/s

$\boxed{17{:}11}$

Choosing $+x \rightarrow$, $\vec{a}$ points left: negative because the luge slows down

$\vec{F} = -ma\,\hat{\imath} = -(85\text{ kg})(2\text{ m/s}^2)\,\hat{\imath} = -1.7\times10^2\text{ N}$

a) $|\vec{F}| = 170\text{ N}$

b) $W = \Delta K \rightarrow \vec{F}\cdot\vec{d} = K_f - K_i$

Note ϕ between force and distance is 180°, $K_f = 0$

$$-mad = 0 - \tfrac{1}{2}mv_i^2$$

$$d = \frac{-\frac{1}{2}mv_i^2}{-ma} = \frac{v_i^2}{2a} = \frac{(37\text{ m/s})^2}{2(2\text{ m/s}^2)}$$

$$d = 340\text{ m}$$

c) $W_{\substack{\text{DONE}\\\text{ON}\\\text{LUGE}}} = \vec{F}\cdot\vec{d} = (170\text{ N})(340\text{ m})(\cos 180^\circ)$

$$= -5.8\times10^4\text{ J}$$

d) for $\vec{a} = -4.0\text{ m/s}^2\,\hat{\imath}$

$$|\vec{F}| = |(85\text{ kg})(-4.0\text{ m/s}^2)| = 340\text{ N}$$

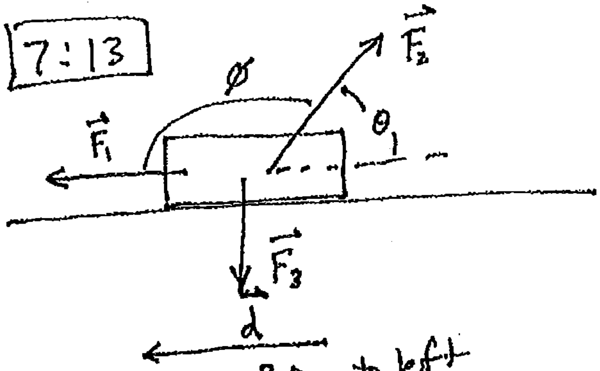
e) $d = \dfrac{(37\text{ m/s})^2}{2(4\text{ m/s}^2)} = 170\text{ m}$

f) here is the result to contemplate:

$W = \vec{F}\cdot\vec{d} = (340\text{ N})(170\text{ m})(\cos 180^\circ)$

$W = -5.8\times10^4\text{ J}$ $\boxed{\text{compare to (c)}}$

$\boxed{7:13}$



$F_1 = 5.00\,N$
$F_2 = 9.00\,N$
$F_3 = 3.00\,N$
$\theta = 60°$

a) What is the net work done on the trunk by the three forces?

F_1 does positive work $\vec{F_1} \cdot \vec{d} = F_1 d = (5.00\,N)(3m) = 15.0\,J$

F_2 does negative work $\vec{F_2} \cdot \vec{d} = F_2 d \cos\phi$
$= F_2 d \cos(180 - 60°)$
$= (9.00)(3.00)\times(-0.5) = -13.5\,J$

F_3 does no work because it is $\perp$ to the displacement: (can ignore $\vec{N}$, $m\vec{g}$ for same reason)
$W_3 = \vec{F_3} \cdot \vec{d} = F_3 d \cos(90°) = 0.$

So the net work is $+15.0\,J - 13.5\,J = \underline{+1.50\,J}$

b) The positive work goes to increase the kinetic energy of the trunk.

$\boxed{7:27}$ The work done by the spring force is

$$W_s = \frac{1}{2} k \left(x_i^2 - x_f^2\right)$$

The fact that 360N of force must be applied to pull the box $\Delta x = 4$ cm implies

$$k = \frac{F}{\Delta x} = \frac{360\,N}{0.04\,m} = 9\times10^3\,N/m$$

(a) When the block moves from $x_i = +5$ cm to $x_f = +3$ cm

$$W_s = \frac{1}{2}\left(9\times10^3\,\frac{N}{m}\right)\left\{(0.05\,m)^2 - (0.03\,m)^2\right\}$$

$W_s = 7.2\,J$

(b) Moving from $x_i = +5$ cm to $x_f = -3$ cm

$$W_s = \frac{1}{2}\left(9\times10^3\,\frac{N}{m}\right)\left\{(0.05\,m)^2 - (-0.03\,m)^2\right\}$$

$W_s = 7.2\,J$

$\boxed{7:27}$ cont.

(c) Moving from $x_i = +5$ cm to $x_f = -5$ cm

$$W_s = \frac{1}{2}\left(9\times10^3 \frac{N}{m}\right)\left\{(0.05m)^2 - (-0.05m)^2\right\}$$

$$W_s = 0$$

(d) Moving from $x_i = +5$ cm to $x_f = -9$ cm

$$W_s = \frac{1}{2}\left(9\times10^3 \frac{N}{m}\right)\left\{(0.05m)^2 - (-0.09m)^2\right\}$$

$$W_s = -25 J$$

$\boxed{7:29}$ As the body moves along the x-axis from $x_i = 3.0$ m (where $V_i = 8.0$ m/s) to $x_f = 4.0$ m (we don't know V_f ....)
We can find the work done by integral (since F is proportional to x)

$$W = \int_{x_i}^{x_f} F_x\, dx = \int_{x_i}^{x_f} -6x\, dx = -3\left(x_f^2 - x_i^2\right)$$

$$= -3(4.0^2 - 3.0^2) = -21 J = W$$

$\boxed{V_f < V_i \text{ if } W \text{ is neg}}$

$\boxed{7:29}$ cont.

By the Work-Kinetic Energy Theorem

$$-21\text{ J} = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$\tfrac{1}{2}mv_i^2 - 21\text{ J} = \tfrac{1}{2}mv_f^2$$

$$\sqrt{v_i^2 - \frac{2}{m}(21\text{ J})} = v_f$$

$$\sqrt{\left(8.0\tfrac{m}{s}\right)^2 - \frac{2(21\text{ J})}{(2.0\text{ kg})}} = 6.6\text{ m/s} = v_f$$

This makes sense: $v_f < v_i$ if W is negative

b) where will the body have $v = 5.0$ m/s?

• Reverse the sequence of our calculations.

i What is ΔK?

$$K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$
$$= \tfrac{1}{2}(2\text{kg})\left((5\tfrac{m}{s})^2 - (8\tfrac{m}{s})^2\right)$$

$$\Delta K = -39\text{ J}$$

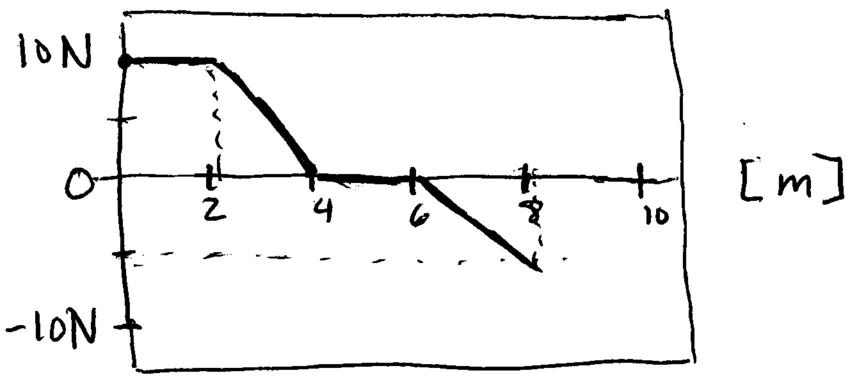
so $W = \vec{F}\cdot\vec{d} = \Delta K = -39\text{ J}$

$$\int_{x_i}^{x_f} -6x\,dx = -3\left(x_f^2 - x_i^2\right) = -39\text{ J}$$

$$x_f = \sqrt{\frac{-39\text{ J}}{-3} + (3.0\text{ m})^2}$$

$$x_f = 4.7\text{ m}$$

$\boxed{7:34}$ When given a graph of Force vs position, the area under the curve (positive or negative) is the W_{net}



we have info about the applied force from 0 to 8 m

$m = 5.0$ kg

What is the work done by the applied force over 8 m?

from 0 – 2 m : $W = (10\,N)(2\,m) = 20\,J$

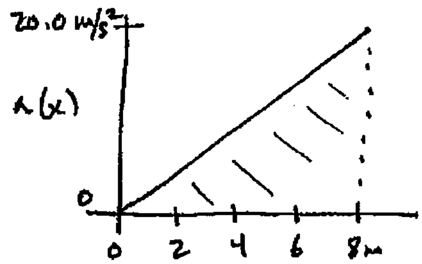
from 2 – 4 m : $W = \frac{1}{2}(10\,N)(2\,m) = 10\,J$

from 4 – 6 m : $W = 0$

from 6 – 8 m : $W = -\frac{1}{2}(5\,N)(2\,m) = -5\,N$

$W_{net} = 20 + 10 - 5 = 25\,J$

$\boxed{7{:}36}$ A 10 kg brick moves along x. a(x) is shown:



What is the net work performed on the brick by the force causing the acceleration as the brick moves from x = 0 to x = 8.0 m?

$$W = \int_{x_1}^{x_2} F(x)\,dx = \int_{x_1}^{x_2} m\,a(x)\,dx = m \int_{x_1=0}^{x_2=8.0\,\text{m}} a(x)\,dx$$

Geometrically this integral is the area under the a(x) vs. x curve, which we see is the triangle of height 20.0 m/s² and base 8.0 m, so the area is $\frac{1}{2}(20.0\,\text{m/s}^2)(8.0\,\text{m}) = 80\,\text{m}^2/\text{s}^2$. Hence

$$W = (10\,\text{kg})(80\,\text{m}^2/\text{s}^2) = 800\,\text{kg}\,\text{m}^2/\text{s}^2 = \underline{\underline{800\,\text{J}}}$$

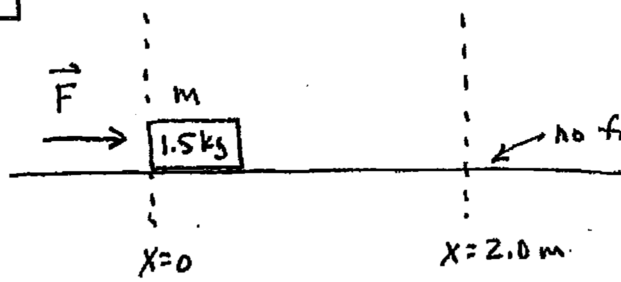
Alternatively we could write an equation for $a(x) = \left[\frac{20\,\text{m/s}^2}{8\,\text{m}}\right] x$, $\left(\text{check: } a(0) = 0\,;\ a(8\,\text{m}) = \left(\frac{20}{8}\right)8 = 20\,\text{m/s}^2\right)$, and then substitute it into the integral to get

$$W = m \int_{x_1=0}^{x_2=8.0\,\text{m}} \frac{20\,\text{m/s}^2}{8\,\text{m}}\, x\,dx = 10\,\text{kg} \cdot \frac{20\,\text{m/s}^2}{8\,\text{m}} \underbrace{\int_{x_1}^{x_2} x'\,dx}_{\frac{1}{2}x^2 \Big|_{x_1=0}^{x_2=8}}$$

$$= 10 \cdot \frac{20}{8} \cdot \frac{1}{2}\left(8^2 - 0^2\right)$$

$$= \frac{10 \cdot 20 \cdot 64}{16} = \underline{\underline{800\,\text{J}}}$$

$\boxed{7{:}40}$



$\vec{F} = \hat{i}\,(2.5 - x^2)$

Displacement is along $+\hat{i}$

so $\vec{F}\cdot\vec{d} = Fd$.

a) What is the kinetic energy of the block as it passes through $x = 2.0$ m?

The work done on the block is $W = Fd \Rightarrow \int_0^{2.0\,m} F(x)\,dx$

for a variable force.

$$\text{So } W = \int_0^{2.0\,m} (2.5 - x^2)\,dx = \underbrace{\int_0^{2.0} 2.5\,dx}_{\underbrace{2.5\int_0^{2.0} dx}_{\underbrace{2.5x\Big|_{x=0}^{x=2.0}}_{5.0}}} - \underbrace{\int_0^{2.0} x^2\,dx}_{\underbrace{-\frac{x^3}{3}\Big|_{0.0}^{2.0}}_{\underbrace{-\frac{(2.0)^3}{3}}_{-\frac{8}{3}}}}$$

$$W = 5.0 - \frac{8}{3} = \frac{15-8}{3} = \frac{7}{3}\ \text{J}.$$

This positive work goes to increase the kinetic energy of the block:

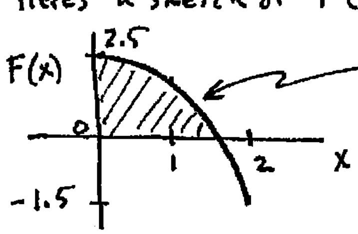
$$\underbrace{\Delta KE}_{\frac{1}{2}mv^2 - 0} = W$$

$$\frac{1}{2}mv^2 - 0 = \frac{7}{3}\ \text{J}$$

so KE of the block is $\frac{7}{3}$ J $= \underline{\underline{2.3\bar{3}\ \text{J}}}$.

b) What is the maximum KE of the block between $x = 0$ and $x = 2.0$ m?

Here's a sketch of $F(x)$:



If the force is positive, the force does positive work on the block, increasing its kinetic energy. If the force is negative, it does negative work and decreases the block's kinetic energy. The maximum KE is as the force goes to zero:

$$2.5 - x^2 = 0 \Rightarrow x^2 = 2.5 \text{ or } x = +\sqrt{2.5} = 1.58\ \text{m}$$

$\boxed{\text{7:40 cont'd}}$

So now we just need to find the work done between $x=0$ and $x=1.581$:

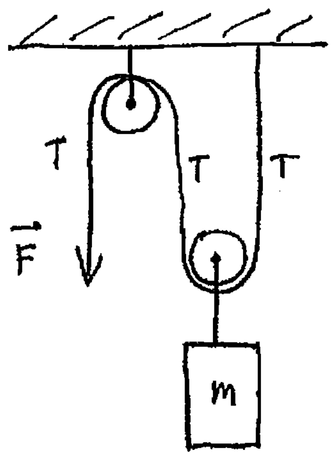
$$W = 2.5x\Big|_0^{1.581} - \frac{x^3}{3}\Big|_0^{1.581}$$

$$= 2.5\,(1.581) - \frac{(1.581)^3}{3}$$

$$= 3.953 - 1.318$$

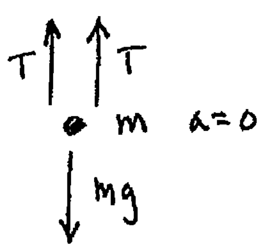
$$= \underline{\underline{2.64\ \text{J}}}$$ This is the maximum KE. It happens at $x = 1.58$ m.

$\boxed{7:57}$



$m = 20$ kg

a) What must the magnitude $|\vec{F}|$ be if you are to lift the canister at constant speed?



F provides the tension in the string, so $|F| = |T|$. The string is all at this tension because the pulleys are frictionless.

Using the free body diagram to write Newton's 2^{nd} law:

$$T + T - mg = \overset{m\cancel{a}}{0}$$

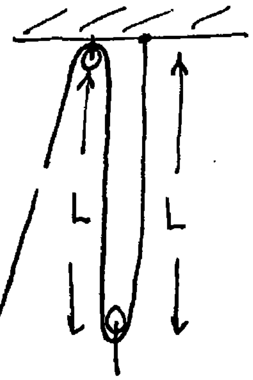
so $T = mg/2$, i.e.

$$\underline{\underline{F = mg/2}} = \frac{20\text{kg}}{2}\left(9.8\,\text{m/s}^2\right)$$

$$= 98\ \text{N} \quad \left[100\ \text{N if } g = 10\,\text{m/s}^2\right]$$

b) To lift the canister by 2.0 cm, how far must you pull the free end of the cord?

The moving pulley is a distance L from the ceiling. Note that in order to lift it to the ceiling you must pull a total length 2L of cord:



So you need to pull $2 \cdot (2.0\,\text{cm}) = \underline{\underline{4.0\,\text{cm}}}$

c) In lifting the canister by 2.0 cm, what is the work done by your force?

$$F_{\text{provided by you}} = mg/2 = 98\,\text{N}$$

Distance you have to pull : 4.0 cm.

So $W_{\text{you}} = F_{\text{from you}} \times d = (98\,\text{N})(4 \times 10^{-2}\,\text{m})$

$= \underline{\underline{3.92\,\text{J}}}$ $[4.0\,\text{J if } g = 10\,\text{m/s}^2]$

d) What is the work done by F_{gravity}?

$d = 2.0\,\text{cm} \uparrow \quad \downarrow F_{\text{grav}} = mg = (20\,\text{kg})(9.8\,\text{m/s}^2)$

Here $\vec{F} \cdot \vec{d}$ is negative because $\theta = 180°$ so $\vec{F} \cdot \vec{d} = Fd\cos(180°) = -Fd$

$W_{\text{gravity}} = -20 \times 9.8 \times 2 \times 10^{-2} = \underline{\underline{-3.92\,\text{J}}}$ $[-4.0\,\text{J}]$

(Note that the net work done on the canister is $W_{\text{you}} + W_{\text{gravity}} = 3.92\,\text{J} - 3.92\,\text{J} = 0$, consistent with the fact that you are pulling at constant speed, thus $\Delta KE = 0$.)