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Phy 232
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HW for 11/22/17

TSR.3, T6M.1

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TSR 3

Consider two identical imaginary boxes of air with volume V, one at sea level (altitude $z=0$) and one at an altitude z that is much larger than the size of either box. A molecule of mass m in a given quantum state in the upper box will have an energy that is $m|\vec{g}|z$ higher than the exactly corresponding quantum state in the lower box b/c of the extra grav. PE of the upper box. Now suppose that we connect these boxes with a hose so that molecules are free to flow between the boxes. Suppose also that the gases in both boxes have the same abs. temp. T.

a) Argue that the Boltzmann factor implies that, in equilibrium, the ratio of the number $N(z)$ of molecules in the upper box to the number $N(0)$ in the lower box must be

$$N(z)/N(0) = \exp(-m|\vec{g}|z/k_BT)$$

The Probability ~~that~~

b) Argue that the ratio of the gas pressures must be

$\frac{P(z)}{P(0)} = \exp(-m|g|z/k_BT)$.

The ideal gas law states the relationship between pressure, temperature, volume and number of molecules in a gas:

$PV = Nk_BT$

$\Rightarrow P = \frac{Nk_BT}{V}$

For the box at elev. z, $P(z) = \frac{N(z)\,k_BT_0}{V}$

For the box at elev. 0, $P(0) = \frac{N(0)\,k_BT_0}{V}$

$\frac{P(z)}{P(0)} = \frac{N(z)\,\cancel{k_BT_0}}{\cancel{V}} \bigg/ \frac{N(0)\,\cancel{k_BT_0}}{\cancel{V}} = \frac{N(z)}{N(0)}$

As we argued in part (a), this is $\exp(-m|g|z/k_BT)$

So $\frac{P(z)}{P(0)} = \exp(-m|g|z/k_BT)$

c) We can model the earth's atmosphere as a series of vertically stacked and connected boxes of gas, so the last equation should apply to the earth's atmosphere as long as the entire atmosphere has a temp. independent of z. Use this model to calculate the approximate air pressure at the top of Mount Everest ($z = 8848\text{m}$), if $T \approx 295\text{K}$

From problem T5B.10, the molar mass of air is $\frac{29.0\text{g}}{\text{mol}}$

so the mass of a single molecule of air is $\frac{29.0\text{g}}{1\text{mol}} \cdot \frac{1\text{kg}}{1000\text{g}} \cdot 6.022\cdot10^{23}$ molecules

$\Rightarrow m = 4.82\cdot10^{-26}\text{kg}$

So the pressure of the atmosphere at elevation $z = 8848\text{m}$ (at $T = 295\text{K}$), is

$P(8848\text{m}) = P(0)\cdot\exp\left(-4.82\cdot10^{-26}\text{kg}\cdot9.8\text{m/s}^2\cdot8848\text{m}\cdot\left(295\text{K}\cdot1.38\cdot10^{-23}\frac{\text{J}}{\text{K}}\right)^{-1}\right)$

$\Rightarrow P(0)\cdot0.36$

So the pressure at $z = 8848\text{m}$ is about 0.36 times that of the pressure at $z = 0$.

Note that $P(0) \approx 1.013\cdot10^5\text{Pa}$, so $P(8848\text{m}) \approx 1.013\cdot10^5\cdot0.36 = \boxed{3.632\cdot10}$

d) What's the approximate density of air at Coors Stadium in Denver, Colorado ($z = 1610\,m$)?

$\frac{kg}{ms^2}$

$$\rho(z) = \frac{P(0) - P(z)}{g \cdot z}$$

$$\rho(1610m) = \frac{P_0 \cdot (1 - \exp[-4.82 \cdot 10^{-26} kg \cdot 9.8 m/s^2 \cdot 1610m / (295K \cdot 1.38 \cdot 10^{-23} \frac{J}{K})])}{9.8 m/s^2 \cdot 1610m}$$

$$\Rightarrow \rho(1610m) = P_0 \cdot 1.08 \cdot 10^{-5} \frac{kg}{m^3} \approx 1.1\, kg/m^3$$

e) Argue that this significantly effects how far a well hit baseball travels in that stadium compared to a baseball stadium at coastal cities.

Note that $\rho(0) \approx 1.2\, kg/m^3$

This means at coastal cities, a well hit ball has to push against more air than at Coors Stadium due to the higher air density, so the ball will lose KE more quickly and not go as far.

[GM.1]

Suppose that in a certain physics experiment a plasma of ionized oxygen atoms emerges from an opening in a furnace with a temp of 3200K, we then use a velocity selector that passes only those ions whose speeds are within $\pm 0.01\, km/s$ of 1.40 km/s. If we need $2 \cdot 10^{12}$ ions/s with that speed for our exp, how many ions/s must the furnace emit?

$$Pr(V \pm \tfrac{1}{2}dv) = \frac{4}{\pi^{1/2}} \cdot \left(\frac{V^2 m}{2k_BT}\right) \exp\left(-\left(\frac{V^2 m}{2k_BT}\right)\right) \cdot \frac{dV\sqrt{m}}{\sqrt{2k_BT}} = \frac{4 V^2}{\sqrt{\pi}} \cdot \left(\frac{m}{2k_BT}\right)^{3/2} \cdot \exp\left(-\left(\frac{V^2 m}{2k_BT}\right)\right) \cdot dV$$

hence $2 \cdot 10^{12}$ ions/s $= Pr(1400 m/s \pm 5 m/s) \cdot X$ ions/s

$$\Rightarrow Pr(1400 m/s \pm 5 m/s) = \frac{4 (1400 m/s)^2}{\sqrt{\pi}} \cdot \left(\frac{2.657 \cdot 10^{-26} kg}{2 \cdot 1.38 \cdot 10^{-23} \frac{J}{K} \cdot 3200K}\right)^{3/2} \cdot \exp\left(-\frac{(1400 m/s)^2 \cdot 2.657 \cdot 10^{-26} kg}{2 \cdot 1.38 \cdot 10^{-23} \frac{J}{K} \cdot 3200K}\right) \cdot 10 m/s$$

$$\Rightarrow Pr(1400 m/s \pm 5 m/s) = 0.00409$$

$$X\, \text{ions/s} = \frac{2 \cdot 10^{12}\, \text{ions/s}}{0.00635} = 4.94 \cdot 10^{14}\, \text{ions/s}$$

Evals: [SD.3]: Check her first [GM.1]: Check her many / [illegible]