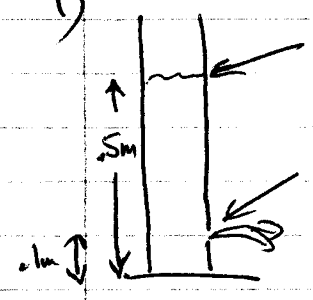
Physics 132 - HW II - Solutions

Bernoulli supplement

1)



We assume a velocity in hole that is considerably much greater than the velocity in the cylinder (I should have encouraged this by saying a small hole, or directly saying $v_{top} \approx 0$). Both top & hole sections are in contact w/ the air, so $P = 1atm$. Comparing top & hole:

$$P_{top} + \rho g h_{top} + \frac{1}{2}\rho \underbrace{v^2_{top}}_{=0} = P_{bottom} + \rho g h_{bottom} + \frac{1}{2}\rho v^2_{bottom}$$

$$\underbrace{(P_{top} - P_{bottom})}_{0} + \rho g (h_{top} - h_{bottom}) = \frac{1}{2}\rho v^2_{bottom}$$

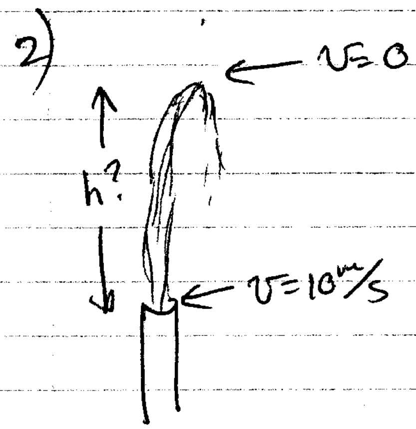
$$g(.5m - .1m) = \frac{1}{2} v^2_{bottom}$$

$$v_{bottom} = \sqrt{2 \cdot 10 \, m/s^2 \cdot .4m} = 2.8 \, m/sec$$

(You could take intermediate step of finding P at depth of .4m in the cylinder, but that is not necessary)

Phy 132 – II. Bernoulli Solutions

2)

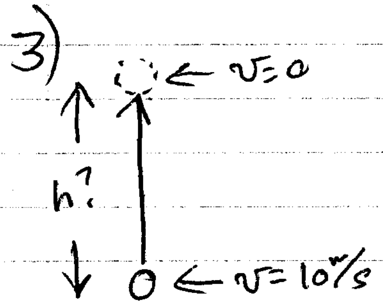


$$P_b + \tfrac{1}{2}\rho v_b^2 + \rho g h_b = P_t + \tfrac{1}{2}\rho \cancel{v_t^2}^{\,0} + \rho g h_t$$

$$\cancel{(P_b - P_t)}^{\,0} + \tfrac{1}{2}\rho(10\,m/s)^2 = \rho g(h_t - h_b)$$

$$h = h_t - h_b = \frac{\tfrac{1}{2}(10\,m/s)^2}{(10\,m/s^2)} = 5\,m$$

3)



Ball of mass m thrown up @ $10\,m/s$

$$E_{bef} = E_{after}$$

$$K_b + U_b = K_t + U_t$$

$$\tfrac{1}{2}mv_b^2 + \cancel{mgh_b}^{\,0} = \tfrac{1}{2}\cancel{mv_t^2}^{\,0} + mgh_t$$

$$\tfrac{1}{2}\cancel{m}v_b^2 = \cancel{m}gh$$

$$h = \frac{\tfrac{1}{2}v_b^2}{g} = \frac{\tfrac{1}{2}(10\,m/s)^2}{10\,m/s^2} = 5\,m$$

Session II.1

1) We saw for draining a solution $h(t) = h_0 e^{-t/\tau}$. So, we guess for radioactive decay $N(t) = N_0 e^{-t/\tau}$. Question is, does this satisfy $\frac{dN}{dt} = -\alpha N$? Take derivative of our guess solution

$$\frac{dN}{dt} = \left(\frac{-1}{\tau}\right)\underbrace{N_0 e^{-t/\tau}}_{N} = -\left(\frac{1}{\tau}\right)N \quad \text{so, this satisfies}$$

II.1 Solutions

our rule of $\frac{dN}{dt} = -\alpha N$ <u>if</u> $\tau = \frac{1}{\alpha}$.

We also know $T_{1/2} = \ln 2 \cdot \tau$, so

$$\boxed{T_{1/2} = \frac{\ln 2}{\alpha}}$$

2) $h(t) = h_0 e^{-t/\tau}$ $\Leftarrow$ assuming flow dominated by viscosity.

We know $T_{1/2} = 20$ sec (given), and $T_{1/2} = \ln 2 \cdot \tau$,

So $\tau = \frac{20\,\text{sec}}{\ln 2} = 28.9\,\text{sec}$

We want time when $\frac{h(t)}{h_0} = .1$ (90% empty)

So $e^{-t/\tau} = 0.1$

$$\ln e^{-t/\tau} = \ln(.1)$$

$$-t/\tau = \ln(.1)$$

$$t = -\ln(.1) \cdot \tau = -\ln(.1) \cdot 28.9\,\text{sec} = 66.4\,\text{sec}$$

II.2 – Solutions

1) We are given $f = 5\,cm^3/sec$ & $\Delta P = 1000\,Pa$. You can do this in all SI (meter, kg, sec) units, or in mixed units – I'll do both

What is R?

SI

$$R = \Delta P/f = \frac{10^3\,Pa}{5\,cm^3/sec} \cdot \left(\frac{10^2\,cm}{1\,m}\right)^3$$

$$= 2\times10^8\ \frac{Pa\text{-}sec}{m^3}$$

Mixed

$$R = \frac{\Delta P}{f} = \frac{10^3\,Pa}{5\,cm^3/sec}$$

$$= 200\ \frac{Pa\text{-}sec}{cm^3}$$

What is f if $\Delta P \Rightarrow 4000\,Pa$?

SI

$$f = \frac{\Delta P}{R} = \frac{4\times10^3\,Pa}{2\times10^8\ \frac{Pa\text{-}sec}{m^3}}$$

$$= 2\times10^{-5}\ m^3/sec$$

$$\left(= 20\ cm^3/sec\right)$$

Mixed

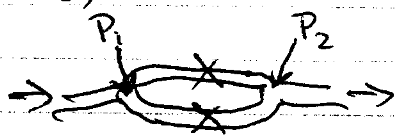
$$f = \frac{\Delta P}{R} = \frac{4\times10^3\,Pa}{2\times10^2\ \frac{Pa\text{-}sec}{cm^3}}$$

$$= 20\ cm^3/sec$$

2) R's in parallel $\Rightarrow$ $\Delta P = 200\,Pa$, $f = 10^{-6}\,m^3/sec$

a) R? $R = \frac{\Delta P}{f} = \frac{200\,Pa}{10^{-6}\,m^3/s} = \boxed{2\times10^8\ \frac{Pa\text{-}sec}{m^3}}$

b) Since ΔP is same across second restriction, flow must be same through each restriction.



$f_{total} = f_1 + f_2 = \boxed{2\times10^{-6}\ m^3/sec}$

$\Delta P = P_1 - P_2 =$ same for both (P_1 determined by height, $P_2 = 1$ atm) and as for part a)

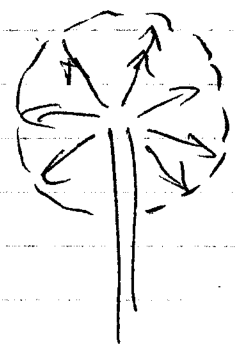
II.2 - Solutions

2c) $R_{eff} = \frac{\Delta P}{f_{total}} = \frac{200\,Pa}{2\times10^{-6}\,m^3/s} = 10^8\ \frac{Pa\text{-}sec}{m^3}$

(half the value of a single R)

Session II.3

1) Flowing out into area defined by a sphere, so



$$f = v \cdot A = v \cdot 4\pi r^2$$

so $\boxed{v(r) = \frac{f}{4\pi r^2}}$

If $f = 100\ cm^3/sec$ & $r = 20\ cm$,

$$v = \frac{100\ cm^3/sec}{4\pi\,(20\,cm)^2} = \boxed{.0199\ cm/sec}$$

[Some people saw the picture as flowing into a ~~half~~ hemisphere , so then

$v(r) = \frac{f}{2\pi r^2}$ & $v = .398\ cm/sec$ @ $r = 20\,cm$]