PHY 131 HW 10

Ch. 16: 4, 10, 11, 13, 31, 33, 44, 47, 80, 86

16-4

a) The speed of the wave is the distance divided by the required time:

$$v = \frac{853 \text{ seats}}{39 \text{ s}} = 21.87 \frac{\text{seats}}{\text{s}} \approx 22 \text{ seats/s}$$

b) The width w is equal to the distance the wave has moved during the average time required by a spectator to stand and then sit.

$$w = vt = (21.87 \frac{\text{seats}}{\text{s}})(1.8 \text{ s}) \sim 39 \text{ seats}$$

16-10 With length in cm and time in seconds

$$u = \frac{du}{dt} = 225\pi \sin(\pi x - 15\pi t)$$

square this and add it to the square of $15\pi y$

$$u^2 + (15\pi y)^2 = (225\pi)^2 [\sin^2(\pi x - 15\pi t) + \cos^2(\pi x - 15\pi t)]$$

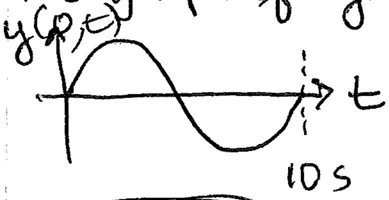
this means that

$$u = \sqrt{(225\pi)^2 - (15\pi y)^2}$$
$$= 15\pi \sqrt{15^2 - y^2}$$

Therefore when $y = 12$, $u = \pm 135\pi$. So the speed there is 424 cm/s = 4.25 m/s

16-13

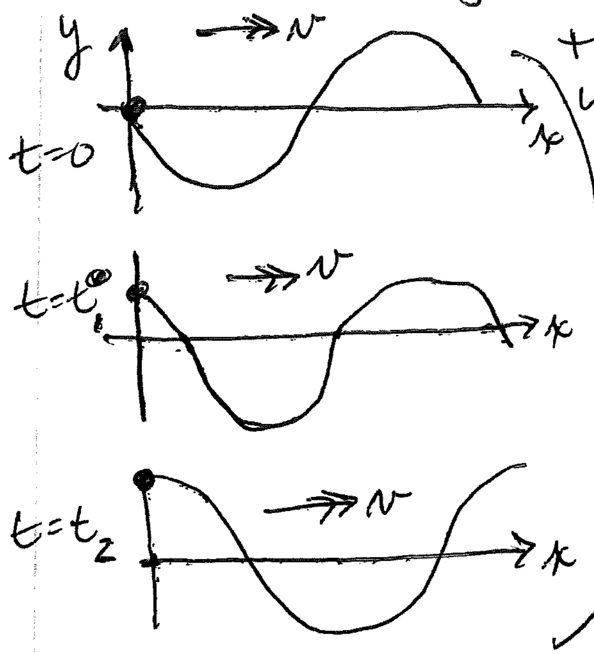
the graph of y vs. t shows $y(x=0)$ as a function of t



so @ $t=0$ $y=0$
as t increases, $y(x=0)$ increases in the positive y direction immediately, then declines: it is a sine wave.

so $T = 10$ s

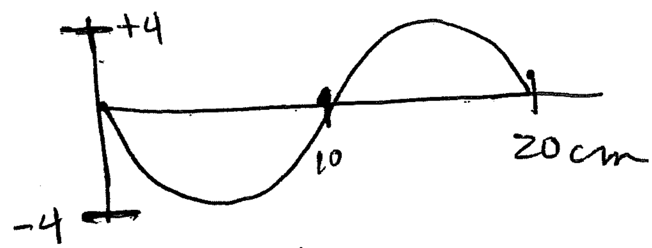
this means that the snapshot of y vs x must be a negative sine wave @ $t=0$



this is the only form of wave where $x=0$ will become more positive in the next few snapshots

therefore $y(x,t) \Rightarrow y(x,0)$

$Y_m(kx \pm \omega t + \phi)$



must have the form

a) $y(x,0) = -Y_m \sin kx$

b) $Y_m = 4$ cm

e) So $\phi \equiv \pi$

$y(x,t) =$

c) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{20\text{ cm}} = \frac{\pi}{10}$

c) $k = 0.31$ [cm^{-1}]

f) since wave is traveling to the right: $-\omega t$

d) $\omega = \frac{2\pi}{T} = \frac{2\pi}{10} = \frac{\pi}{5}$

d) $\omega = 0.63$ rad/s

g) $f = \frac{1}{T}$ $f = .10$ Hz

$v = f\lambda = \frac{\omega}{k} = 2$ cm/s

$\boxed{16\text{-}13}$ cont.

$$y(x,t) = 4.0 \sin\left(\frac{\pi}{10}x - \frac{\pi}{5}t + \pi\right) \quad [cm, s]$$

$$y(x,t) = -4.0 \text{ cm } \sin\left(\frac{\pi}{10}x - \pi/5\, t\right)$$

$$\frac{dy}{dt} = (-4)\left(-\frac{\pi}{5}\right)\cos\left(\frac{\pi}{10}x - \frac{\pi}{5}t\right)$$

$$= \frac{4}{5}\pi \cos\left(\frac{\pi}{10}x - \frac{\pi}{5}t\right)$$

$$v_T(0,5) = \frac{4}{5}\pi \cos\left(\frac{\pi}{10}(0) - \frac{\pi}{5}(5)\right)$$

$$= 4/5\,\pi \cos(-\pi)$$

$$v_T = -\frac{4}{5}\pi = -2.5 \text{ cm/s}$$

$\boxed{16\text{-}31}$ The displacement of the string is given by

$$y = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi)$$

$$= 2\, y_m \cos\left(\frac{\phi}{2}\right) \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

where $\phi = \pi/2$

$$A = 2\, y_m \cos\left(\frac{\phi}{2}\right) = 2\, y_m \cos\left(\frac{\pi}{4}\right) = 1.41\, y_m$$

16-33

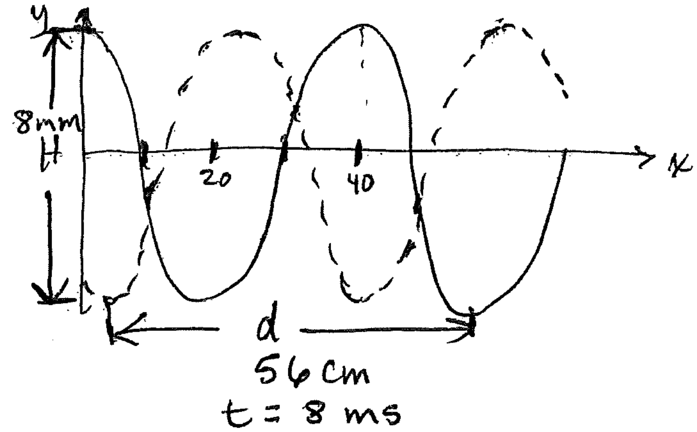
a) $A = y_m = 9.00$ mm (given)

b) tick marks @ 10 cm

so $\lambda = 40$ cm

$\Rightarrow k = \frac{2\pi}{\lambda} = \frac{2\pi}{40 \text{ cm}}$

$k = 16$ [m^{-1}]



c) we know $v_T = \frac{56}{8} = 70$ m/s, since $\omega = kv$

$\omega = 16 \cdot 70$

$\omega = 1120$ [s^{-1}]

d) given $H = 8$ mm $\longrightarrow$ $y' = 4$ mm

$y' = 2 y_m \cos\left(\frac{\phi}{2}\right)$

$\cos\frac{\phi}{2} = \frac{y'}{2y_m} = \frac{0.004}{2(.009)}$

$\phi = 2 \cos^{-1}(.222)$

$\phi = 2.69$ rad

e) wave moves to left so phase is of the form $(kx + \omega t)$ so

$y_1 = 0.009 \sin(16x + 1120t)$

$y_2 = 0.009 \sin(16x + 1120t + 2.7)$

$\boxed{16\text{-}44}$ The string is flat each time the particle passes through its equilibrium position. A particle may travel up to its positive amplitude point and back to equilibrium during this time. This describes half of one complete cycle, so we conclude $T = 2(0.50\,s) = 1.0\,s$. Thus $f = 1/T = 1.0$ Hz and the wavelength is

$$\lambda = \frac{v}{f} = \frac{10\text{ cm/s}}{1\text{ Hz}} = 10\text{ cm}$$

$\boxed{16\text{-}47}$

a) The resonant wavelengths are given by $\lambda = \frac{2L}{n}$ where L is the length of the string and n is an integer. The resonant frequencies are given by $f = \frac{v}{\lambda} = \frac{nv}{2L}$, where v is the wave speed

Let the lower frequency be associated w/ n the higher frequency is then associated w/ n+1 (there are no resonant frequencies in between). So

$$f_1 = \frac{nv}{2L} \quad \text{and} \quad f_2 = \frac{(n+1)v}{2L}$$

the ratio is $\frac{f_2}{f_1} = \frac{n+1}{n} \Rightarrow n = (n+1)\frac{f_1}{f_2}$

$$n = \frac{f_1}{f_2}n + \frac{f_1}{f_2}$$

$$n - n\frac{f_1}{f_2} = \frac{f_1}{f_2}$$

$$n\left(1 - \frac{f_1}{f_2}\right) = \frac{f_1}{f_2}$$

$$n = \frac{f_1}{f_2}\left(\frac{f_2}{f_2 - f_1}\right)$$

$$n = \frac{f_1}{f_2 - f_1}$$

$$n = \frac{315\text{ Hz}}{420\text{ Hz} - 315\text{ Hz}}$$

$n = 3$

$\boxed{16\text{-}47}$ cont.

The lowest possible frequency $f = \frac{v}{2L} = \frac{f_1}{n} = \frac{315\text{ Hz}}{3}$

$f = 105\text{ Hz}$

The longest possible wavelength $\lambda = 2L$

$v = \lambda f = 2Lf = 2(0.75\text{ m})(105\text{ Hz})$

$v = 158\text{ m/s}$

$\boxed{16\text{-}80}$ $f = 600\text{ Hz}$ tuning fork

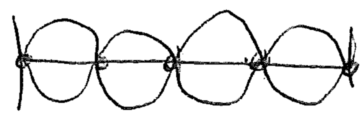
$v = 400\text{ m/s}$ wave speed

4 loops $\rightarrow$ $n = 4$ $\quad$ 4^{th} harmonic

$A = 2\text{ mm} = 0.002\text{ m}$ amplitude

a) length of string?

4 loops $\longrightarrow L = 2\lambda$



λ and f are related by the wave speed

$\lambda = \frac{v}{f} = \frac{400\text{ m/s}}{600\,[s^{-1}]} = \frac{2}{3}\text{m}$

so $L = \frac{4}{3} = 1.3\text{ m}$

b) Equation for $y(x,t) = y_m \sin(kx)\cos(\omega t)$

we need $k = \frac{2\pi}{\lambda} = \frac{2\pi 3}{2} = 3\pi\,[m^{-1}]$

we need $\omega = 2\pi f = 2\pi(600\text{ Hz}) = 1200\pi\,[s^{-1}]$

$y(x,t) = 0.002\,[m]\sin(3\pi x)\cos(1200\pi t)$

16-86 $y_1 = 0.05 \cos(\pi x - 4\pi t)$
$y_2 = 0.05 \cos(\pi x + 4\pi t)$

This is similar to the example in the book except cosine functions are used to describe the waves. This will require the use of the identity $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

let $\alpha = (\pi x - 4\pi t)$ $\quad\beta = (\pi x + 4\pi t)$

$\alpha + \beta = \pi x - \cancel{4\pi t} + \pi x \cancel{+4\pi t}$
$= 2\pi x$ $\qquad \frac{\alpha+\beta}{2} = \pi x$

$\alpha - \beta = \pi x - 4\pi t - \pi x - 4\pi t$
$= -8\pi t$ $\qquad \frac{\alpha-\beta}{2} = -4\pi t$

$y_1 + y_2 = y' = 2 y_m \cos(\pi x)\cos(-4\pi t)$
$= (.05)2 \cos\pi x \cos 4\pi t$
$y' = 0.10 \cos(\pi x)\cos(4\pi t)$

a) For non-negative x the smallest value to produce $\cos\pi x = 0$ is $x = \frac{1}{2} \rightarrow x = 0.5\ m$

b) Taking the derivative

$v_T = \frac{dy'}{dt} = (0.1 \cos\pi x)(-4\pi \sin 4\pi t)$

the sin factor $= 0$ when $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

Therefore the first time the particle @ $x=0$ has zero velocity is $t=0$.

c) The second time the velocity at $x=0$ is $=0$ is $t = 0.25\ s$.

d) The third time is $t = 0.5\ s$