Name: Worked Solution

Collaborator(s): ______________________

This tutorial will walk you through solving a projectile motion physics problem. Following similar steps will enable you to solve a wide variety of multi-dimensional kinematics problems.

Hercules throws a 7.25 kilograms shot put from a height of 1.6 meters. The release angle of the shot put is 50 degrees and its initial velocity is 14.6 m/s. What is the magnitude of the shot put velocity when it strikes the ground and what angle does it hit at?

Identify the missing and relevant information

1. List all the relevant variables for which you know a value. List the variable(s) you will be solving for.

$|\vec{V_0}| = 14.6 \frac{m}{s}$

$\theta_0 = 50°$

$Y_0 = 1.6\,m$

$Y_f = 0$

$a_y = -g$

$V_{0y} = V_0 \sin\theta$

$V_{fy} = ?$

$t = ?$

$t_x = t_y$

$X_0 = 0$

$X_f = ?$

$a_x = 0$

$V_{0x} = V_0 \cos\theta$

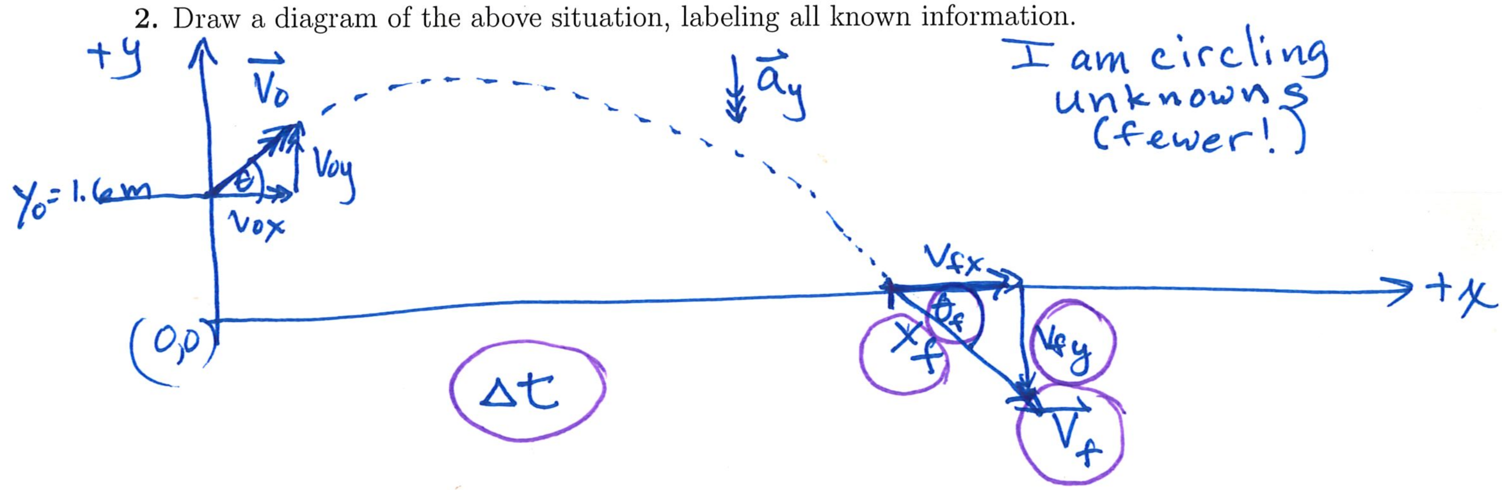
$V_{fx} = V_0 \cos\theta$

$|\vec{V_f}| = ?$

$\theta_f = ?$

Draw a diagram

2. Draw a diagram of the above situation, labeling all known information.



I am circling unknowns (fewer!)

Divide the problem into smaller steps

3. Write equations for $a_x(t)$ and $a_y(t)$.

$a_x = 0$

$a_y = -g$

4. Write equations for $v_x(t)$ and $v_y(t)$.

$V_{fx} = V_{ox} + \cancel{at}^{\,\phi}$ $\quad V_{fx} = V_{ox} = |\vec{V}_0| \cos\theta$

$V_{fy} = V_{oy} - gt$

5. Write equations for $x(t)$ and $y(t)$.

$x_f = \cancel{x_0}^{\,\phi} + V_{ox}t + \cancel{\tfrac{1}{2}at^2}^{\,\phi}$

$\cancel{y_f}^{\,\phi} = y_0 + V_{oy}t - \tfrac{1}{2}gt^2 \Rightarrow$

$x_f = V_{ox}t \Leftarrow$ two unknowns

$\tfrac{1}{2}gt^2 - V_{oy}t - y_0 = 0$

$\Uparrow$ only <u>one</u> unknown

Solve for unknowns

6. Solve for the time at which the shot put will strike the ground.

use quadratic formula

$at^2 + bt + c = 0$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = \frac{g}{2} \quad b = -V_{oy} \quad c = -y_0$

$$t = \frac{V_{oy} \pm \sqrt{(-V_{oy})^2 - 4\left(\frac{g}{2}\right)(-y_0)}}{2\left(\frac{g}{2}\right)}$$

$$= \frac{V_{oy} \pm \sqrt{V_{oy}^2 + 2gy_0}}{g} = \frac{11.18 \pm \sqrt{125.1 + 31.86}}{9.8}$$

$y_0 = 1.6\,m$

$V_{oy} = (14.6\,\frac{m}{s})(\sin 50^\circ) = 11.18\,\frac{m}{s}$

$$t = \begin{cases} \dfrac{11.18 + 12.5}{9.8} = \boxed{2.4\ s} \\ \dfrac{11.18 - 12.5}{9.8} \times \text{negative} \end{cases}$$

7. What are the x and y components of the velocity at the time the shot put strikes the ground?

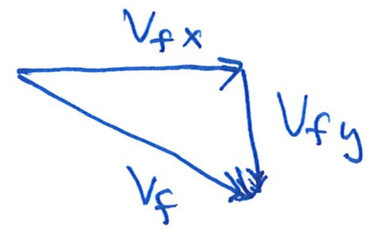
$V_{fx} = V_{ox} = |\vec{V}_0| \cos\theta = (14.6\,\frac{m}{s})(\cos 50^\circ) =$

$= 9.38\ m/s$

$V_{fy} = V_{oy} - gt = 11.18\,\frac{m}{s} - (9.8)(2.4\,s)$

$\boxed{V_{fy} = -12.5\,\frac{m}{s}}$

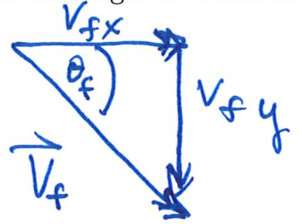
8. Determine the magnitude of the velocity at this point in time.



$$|\vec{V}_f| = \sqrt{V_{fx}^2 + V_{fy}^2}$$
$$= \sqrt{(9.38)^2 + (-12.5)^2}$$
$$|\vec{V}_f| = 15.6 \text{ m/s}$$

$V_f^2 = V_{fx}^2 + V_{fy}^2$
Pythagorean Thm.

9. Determine the angle at which the shot put is striking.



$$\theta_f = \tan^{-1}\left(\frac{opp}{adj}\right)$$
$$= \tan^{-1}\left(\frac{V_{fy}}{V_{fx}}\right) = \tan^{-1}\left(\frac{-12.5}{9.38}\right)$$
$$\theta_f = 53.1^\circ, \text{ Below the Horizon}$$

Check the reasonableness of your answer

10. How does the final velocity compare to the initial velocity? Is this what you would expect and why?

It is a little bigger in magnitude.
Yes: When $\Delta y = 0$, $\vec{V}_f = -\vec{V}_0$. Since the shot put fell farther than y_0, gravity accelerated it for a bigger final speed

11. How does the angle the shot put strikes at compare with the angle it was released at? Is this what you would expect and why?

θ_f is greater than θ_i, but pointed "down" rather than "up"

This makes sense, again because V_{fy} was greater in magnitude than V_{oy}, and reversed in direction. Since $\vec{V}_f$ had a bigger y-component, this created a bigger angle below the horizon.