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R6R.7, R7M.7

R6R.7) Two spacecraft of equal rest length $L_R = 100$ns pass very close to each other as they travel in opposite directions at a relative speed of $|\beta| = \frac{3}{5}$. The captain of ship O has a laser cannon at the tail of her ship. She intends to fire the cannon at the instant her bow is lined up with the tail of ship O'. Since ship O' is Lorentz contracted to 80ns in the frame of ship O, she expects the laser burst to miss the other by 20ns. To the observer in ship O', it is ship O that is contracted to 80 ns. Therefore, the observer on O' concludes that if the captain of O carries out her intention, the laser burst will strike ship O' 20ns behind the bow!

Assume the captain of O carries out her intentions exactly as described, according to the measurements in her own frame, and analyze what really happens as follows

a) Construct a carefully calibrated two-observer spacetime diagram of the situation described. Define event A to be the coincidence of the bow of ship O and the tail of ship O' and event B to be the firing of the laser cannon. Choose A to define the origin event in both frames, and locate B according to the description of the intention of O' above. When and where does this event occur as measured in the O' frame, according to the diagram?

The scale of the t' and x' axes are calculated using
$\Delta t = \Delta t' \gamma$:
If we want 5ns intervals on the t' axis, and $\beta = \frac{3}{5}$, then the t-interval between each t' mark will be $\Delta t = 5\text{ns} \cdot \frac{1}{\sqrt{1-(\frac{3}{5})^2}} = 1.25 \cdot 5\text{ns} = 6.25$

According to the spacetime diagram I constructed [ATTACHED]
Event B occurs at $(x', t') = (125\text{ns}, -75\text{ ns})$

$t' = \gamma(t + \beta x)$
$x' = \gamma(x + \beta t)$

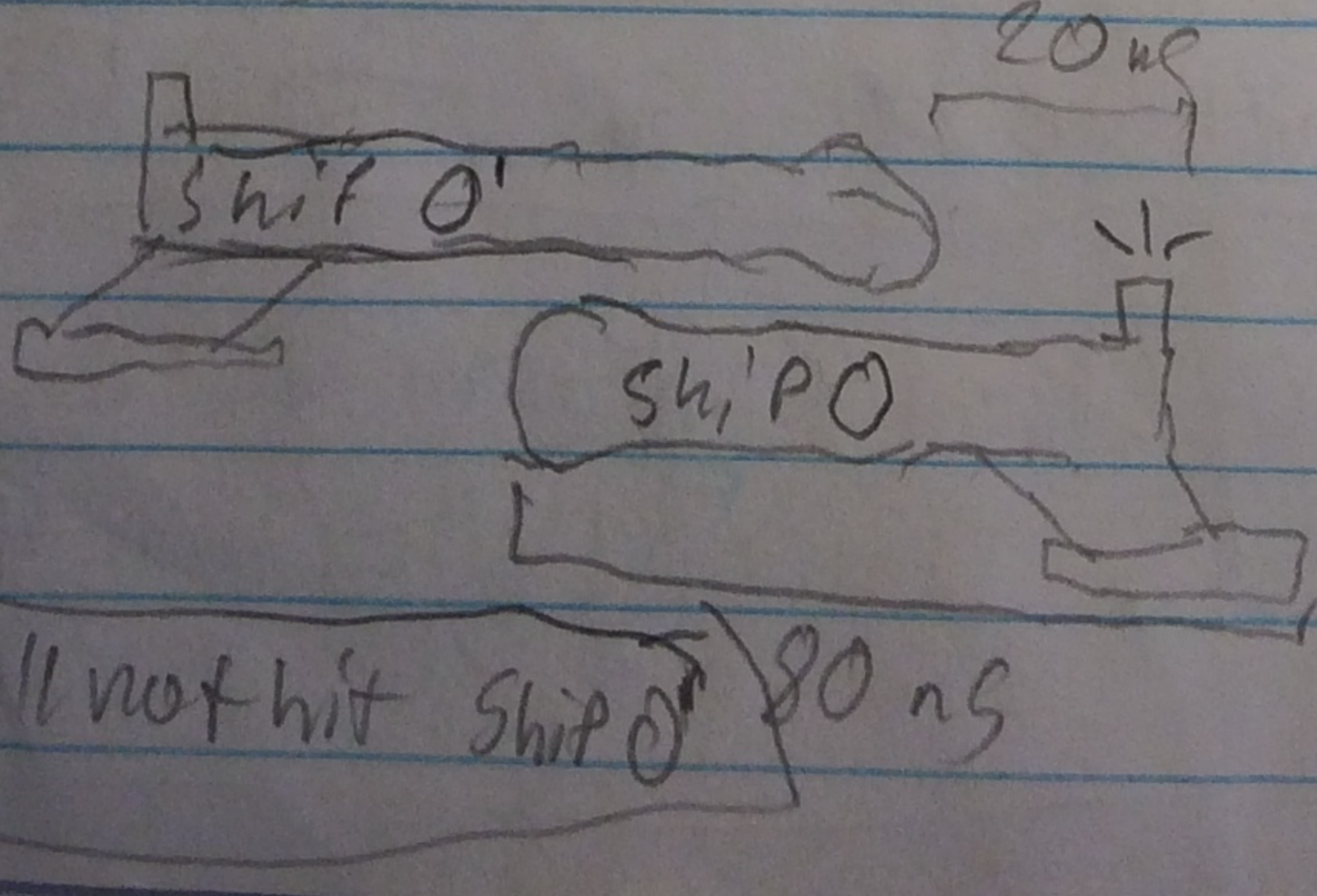
b) verify the coordinates of B
If my space time diagram is correct, then I should get the same coordinates in a Lorentz transform.
Event B has Home frame coordinates $(x, t) = (100, 0)$
So using

$$x' = \gamma(x - \beta t) \Rightarrow x' = 1.25 \cdot (100 - 0) = 125$$
$$t' = \gamma(t - \beta x) \Rightarrow t' = 1.25 \cdot (0 - \tfrac{3}{5} \cdot 100) = -75$$

So the Lorentz transform supports my answer in (a).

c) write a short paragraph describing whether the cannon must really hit or not. Discuss the hidden assumption in the statement of the apparent paradox, and point out how one of the drawings in Figure R6.10 is misleading

In the frame of ship O', Event B happens 75ns before Event A. This means that when the bow of ship O is in line with the tail of ship O', the laser has already been fired in the ship O' reference frame. This fact is swept under the rug by the misleading picture in figure R6.10b, which shows the bow of ship O inline with the tail of ship O' at the moment the laser is fired from the frame of ship O'. This is simply incorrect, as we can read from our spacetime diagram that at this moment, the bow of ship O' is 20ns behind the tail of ship O' as opposed to the 20ns in front of the tail of ship O as shown in figure R6.10b. A correct view from ship O' would look something like this:



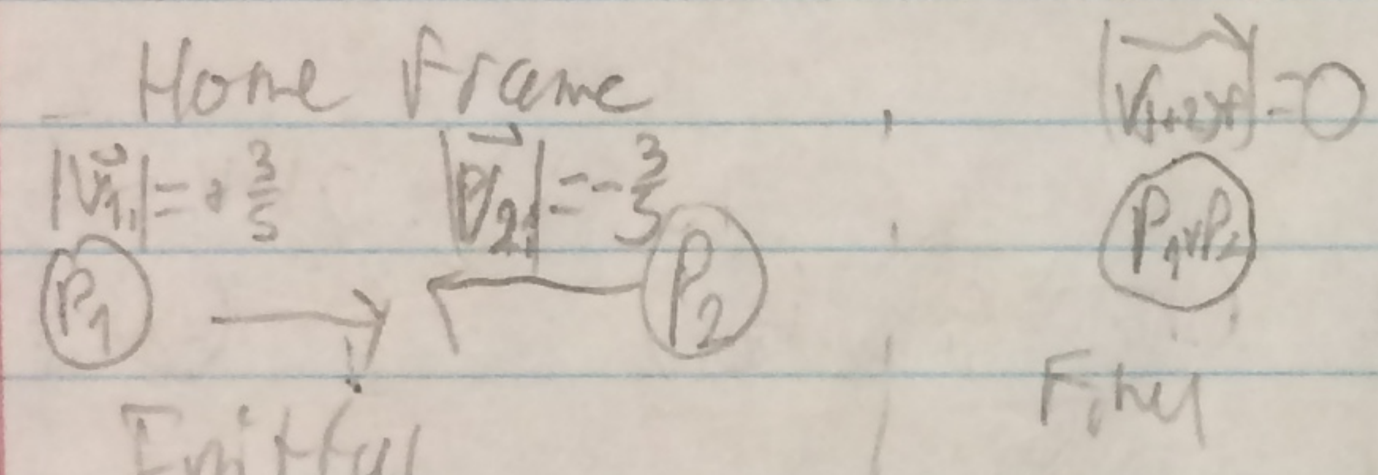
Corrected version of Figure R6.10b

So the laser will not hit ship O'

R7M.7)

Suppose that in the Home Frame, two particles of equal mass m are observed to move along the x-axis with equal and opposite speeds $|\vec{v}| = \frac{3}{5}$. The particles collide and stick together, becoming one big particle which remains at rest in the home frame. Now imagine observing the same situation from the vantage point of an other frame that moves in the +x direction with an x-velocity of $\beta = \frac{3}{5}$ with respect to the home frame.

a) Find the velocities of all particles as observed in the other frame, using the appropriate Einstein velocity transformation equations.



Using the Einstein velocity transform $v_x' = \frac{v_x - \beta}{1 - \beta v_x}$, we can find $\vec{v}_{1i}'$, $\vec{v}_{2i}'$, and $\vec{v}_{(1+2)f}'$

$$\vec{v}_{1i}' = \frac{\vec{v}_{1i} - \beta}{1 - \beta \vec{v}_{1i}} = \frac{\frac{3}{5} - \frac{3}{5}}{1 - (\frac{3}{5})^2} = 0$$

$$\vec{v}_{2i}' = \frac{\vec{v}_{2i} - \beta}{1 - \beta \cdot \vec{v}_{2i}} = \frac{-\frac{3}{5} - \frac{3}{5}}{1 + (\frac{3}{5})^2} = \frac{-6/5}{34/25} = \frac{-6 \cdot 25}{5 \cdot 34} = \frac{-6 \cdot 5}{34} = \frac{-3 \cdot 5}{17} = \frac{-15}{17} \approx -0.88$$

$$\vec{v}_{(1+2)f}' = \frac{\vec{v}_{(1+2)f} - \beta}{1 - \beta \cdot \vec{v}_{(1+2)f}} = \frac{0 - \frac{3}{5}}{1 - \frac{3}{5} \cdot 0} = \frac{-\frac{3}{5}}{1} = -\frac{3}{5}$$

So
$$\boxed{\begin{aligned} \vec{v}_{1i}' &= 0 \\ \vec{v}_{2i}' &= -\frac{15}{17} \\ \vec{v}_{(1+2)f}' &= -\frac{3}{5} \end{aligned}}$$

b) We have defined the momentum of a particle with mass m and velocity $\vec{v}$ to be $\vec{p} = m\vec{v}$. Is the system's total momentum conserved in the Home frame?

If total momentum is conserved, then $\underbrace{\vec{V_{1i}} \cdot m_1 + \vec{V_{2i}} \cdot m_2}_{P_i} = \underbrace{\vec{V_{(1+2)f}} \cdot (m_1 + m_2)}_{P_f}$
we know that $m_1 = m_2$, so we get

$\frac{3}{5} \cdot m_1 - \frac{3}{5} \cdot m_2 \stackrel{?}{=} 0 \cdot 2m_1$

$\Rightarrow 0 = 0$, so $\vec{P_i} = \vec{P_f}$, thus momentum is conserved in the Home frame.

c) Is the system's total momentum conserved in the Other frame? Is this a problem?

Assuming that $m_1 = m_1'$ and $m_2 = m_2'$, then if momentum is conserved, $\underbrace{\vec{V_{1i}}' \cdot m_1 + \vec{V_{2i}}' \cdot m_2}_{\vec{P_i}'} = \underbrace{\vec{V_{(1+2)f}}' \cdot (m_1 + m_2)}_{\vec{P_f}'}$

So we get $\vec{P_1}' = m_1 \cdot 0 - \frac{15}{17} \cdot m_1 = -\frac{15}{17} \cdot m_1$

$\vec{P_2}' = -\frac{3}{5} \cdot 2m_1 = -\frac{6}{5} \cdot m_1$

Therefore, $\vec{P_1}' \neq \vec{P_2}'$, so momentum is not conserved in the Other frame.

This result is most likely due to the use of a Newtonian definition of momentum, one that is inconsistent with Relativity.

Evaluation of Problems

R6R.1

The magnitude of time and position yielded from ~~both~~ my space time diagram and my Lorentz transform are ~~in~~ the correct units and seem to be of a reasonable magnitude.

R7M.7

~~The~~ magnitude of the velocities in the other frame seem to be of a reasonable velocity.

R6R 7 Two Observer Spacetime diagram