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Phy 232

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HW for 12/4/17

T8M.4

T9R.2

T8M.4

Suppose 22g of Helium gas in a cylinder expands quasi-statically while in contact with a resovoir at a temp of 25°C and does 85J of work on its surroundings in the process.

a) What is the gas's entropy change?

The gas expands quasistatically while in contact with a resovoir, meaning that its temp. does not change, so $\Delta U = 0$

$\Delta U = 0 = W + Q \Rightarrow W = -Q$

Gas does NEGATIVE work on surroundings, meaning $W = -85J$, so $Q = +85J$

$\rightarrow \Delta S = \frac{Q}{T}$ for $\Delta T = 0$, so $\Delta S = \frac{85J}{(273.15°C + 25°C)\,1\frac{K}{°C}} = \boxed{0.29\ J/K}$

b) What is the resovoir's entropy change?

The change of the resovoir's internal energy $\Delta U = 0$ b/c $\Delta T = 0$.

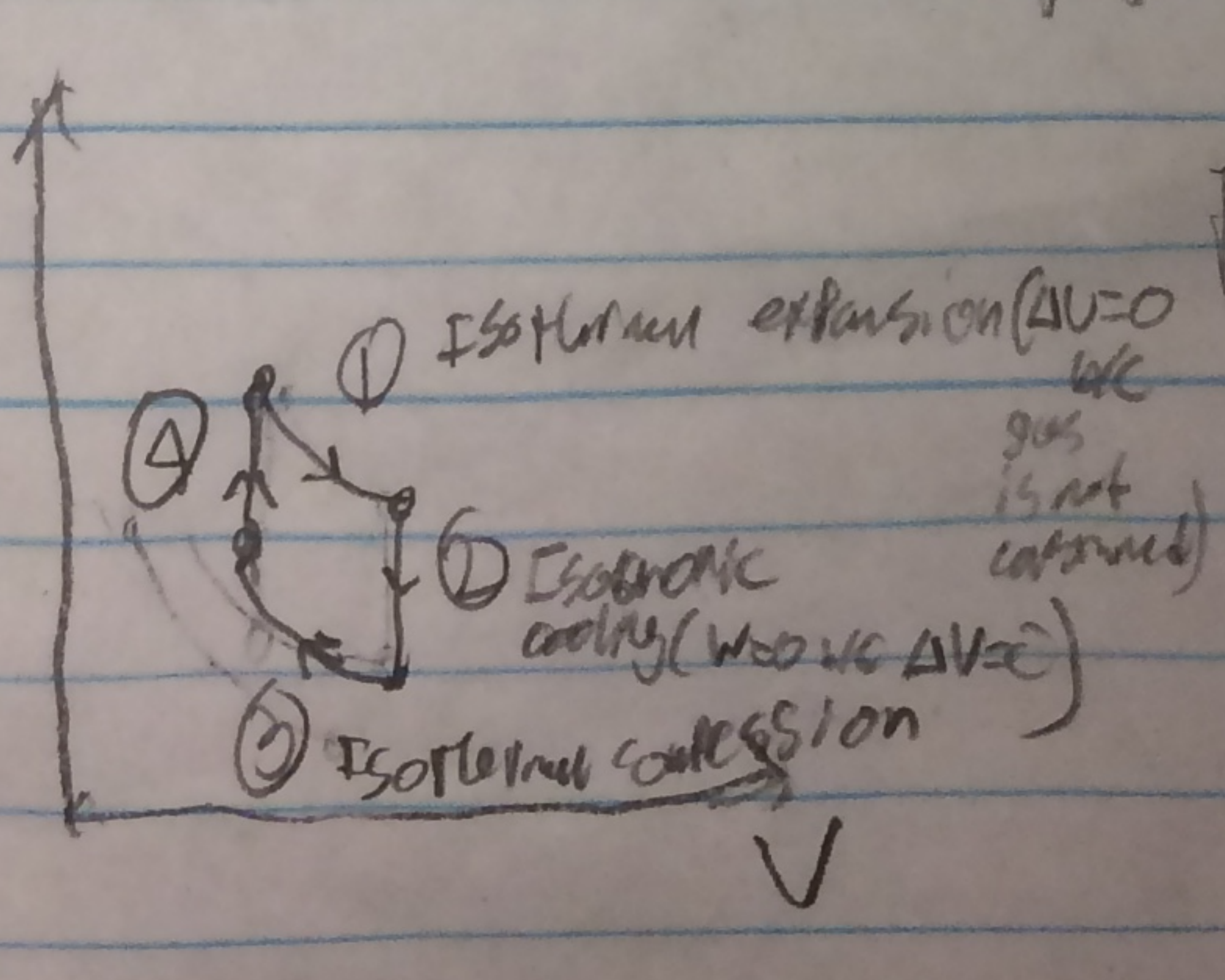
So $\Delta S = \frac{Q}{T} = \frac{-85J}{(273.15°C + 25°C)\,1\frac{K}{°C}} = \boxed{-0.29\ J/K}$

$\Delta U = Q + W$

T9R.2

$PV = Nk_BT$

a)



Isochoric

	1	2	3	4
Q	+	−	−	+
W	−	0	+	0
ΔU	0	−	0	+

Isothermal

b) use eq. + 7.10 to argue the the net work done during this cycle is $W = Nk_B(T_H - T_C)\ln\left(\frac{V_{max}}{V_{min}}\right)$

$|W|$ during the isochoric processes are 0 J

$|W|$ during process 1 is $|W_1| = -Nk_B \cdot T_H \cdot \ln\left(\frac{V_{max}}{V_{min}}\right)$
$|W|$ during process 3 is $|W_3| = -Nk_B T_C \cdot \ln\left(\frac{V_{min}}{V_{max}}\right)$

$W_{total} = \sum_{i=1}^{4} W_i = |W_1| + |W_2| + |W_3| + |W_4| = -Nk_B T_H \cdot \ln\left(\frac{V_{max}}{V_{min}}\right) + 0 - Nk_B T_C \cdot \ln\left(\frac{V_{min}}{V_{max}}\right) + 0$
$\Rightarrow -Nk_B\left(T_C \cdot \ln\left(\frac{V_{min}}{V_{max}}\right) + T_H \ln\left(\frac{V_{max}}{V_{min}}\right)\right) \Rightarrow |W| = -Nk_B(T_H - T_C)\ln\left(\frac{V_{max}}{V_{min}}\right)$

c) Heat flows into the gas in both steps 1 and 4. What is the total heat $|Q_H|$ flowing into the engine during a cycle expressed in terms of the same quantities?

During step 1, $\Delta U = 0$ so $Q = -W \Rightarrow Q_1 = -(-Nk_B \cdot T_H \cdot \ln\frac{V_{max}}{V_{min}}) = Nk_B T_H \ln\left(\frac{V_{max}}{V_{min}}\right)$
During step 2, $W = 0$, $\Delta U = Q \Rightarrow \frac{3}{2} \cdot Nk_B(T_C - T_H) = Q_2 < 0$
During step 3, $\Delta U = 0$, so $Q = -W \Rightarrow Q_3 = -(-Nk_B \cdot T_C \ln\left(\frac{V_{min}}{V_{max}}\right)) = -Nk_B T_C \frac{V_{max}}{V_{min}}$
During step 4, $W = 0$, so $\Delta U = Q \Rightarrow \frac{3}{2} \cdot Nk_B(T_H - T_C) = Q_4 > 0$

$Q_{in} = Q_i$ where $Q_i > 0$, so $Q_{in} = Q_1 + Q_4 = Nk_B\left(T_H \cdot \ln\left(\frac{V_{max}}{V_{min}}\right) + \frac{3}{2}(T_H - T_C)\right)$

d) $\frac{1}{e} = \frac{1}{e_c} + f\left(\frac{V_{max}}{V_{min}}\right)$

The efficiency of a heat engine is $e = \frac{|W|}{|Q_H|}$

From part b), we know that $|W| = NK_B(T_H - T_C)\ln\left(\frac{V_{max}}{V_{min}}\right)$

From part c), $|Q_H| = NK_B\left(T_H \cdot \ln\left(\frac{V_{max}}{V_{min}}\right) + \frac{5}{2}(T_H - T_C)\right)$

$\frac{1}{e} = \frac{|Q_H|}{|W|} = \frac{\cancel{NK_B}\left(T_H \cdot \ln\left(\frac{V_{max}}{V_{min}}\right) + \frac{5}{2}(T_H - T_C)\right)}{\cancel{NK_B} \cdot (T_H - T_C) \cdot \ln\left(\frac{V_{max}}{V_{min}}\right)} \Rightarrow \frac{T_H \cdot \ln\left(\frac{V_{max}}{V_{min}}\right) + \frac{5}{2}(T_H - T_C)}{(T_H - T_C) \cdot \ln\left(\frac{V_{max}}{V_{min}}\right)}$

$\frac{1}{e} = \frac{T_H}{T_H - T_C} + \frac{5}{2\ln\left(\frac{V_{max}}{V_{min}}\right)}$ let $e_c = \frac{T_H - T_C}{T_H}$, $f\left(\frac{V_{max}}{V_{min}}\right) = \frac{5}{2 \cdot \ln\left(\frac{V_{max}}{V_{min}}\right)}$

so $\frac{1}{e} = \frac{1}{e_c} + f\left(\frac{V_{max}}{V_{min}}\right)$

e) Argue that the efficiency of our idealized Sterling engine is less than that of a carnot engine

For a carnot engine, $e_c = \frac{T_H - T_C}{T_H}$

For a Sterling engine, $e_S = \frac{(T_H - T_C) \cdot a}{T_H \cdot a + \frac{5}{2}(T_H - T_C)}$, where $a = \ln\left(\frac{V_{max}}{V_{min}}\right)$ and is a constant.

$\frac{1}{e_c} = \frac{T_H}{T_H - T_C}$

$\frac{1}{e_S} = \frac{T_H}{T_H - T_C} + \frac{5}{2\ln\left(\frac{V_{max}}{V_{min}}\right)} = \frac{1}{e_c} + \frac{5}{2 \cdot \ln\left(\frac{V_{max}}{V_{min}}\right)}$. $\frac{1}{e_S} > \frac{1}{e_c}$, so $e_S < e_c$.

f) What is the efficiency of our idealized Sterling engine when operating between temps of 600K and 300K, if the compression ratio $\frac{V_{max}}{V_{min}} \approx 3$? compare this to the efficiency of a carnot engine.

In part d, we found $\frac{1}{e_S} = \frac{1}{e_c} + \frac{5}{2\ln\left(\frac{V_{max}}{V_{min}}\right)}$, where $\frac{1}{e_c} = \frac{T_H}{T_H - T_C}$

In this case, $\frac{1}{e_c} = \frac{600K}{600K - 300K} = \frac{600K}{300K} = 2$ $\quad \ln\left(\frac{V_{max}}{V_{min}}\right) \approx \ln 3 \approx 1.1$

So $\frac{1}{e_S} = 2 + \frac{5}{2 \cdot 1.1} = 4.3 \Rightarrow e_S = \frac{1}{4.3} = 0.23$

The efficiency of a carnot engine at the same temp range is $\frac{1}{2}$.

Eval 0 errors: TBMA: right units, reasonable magnitude
TRL2: ~~good reasoning~~, right units, reasonable magnitude