Oee parles
Phy 232-01
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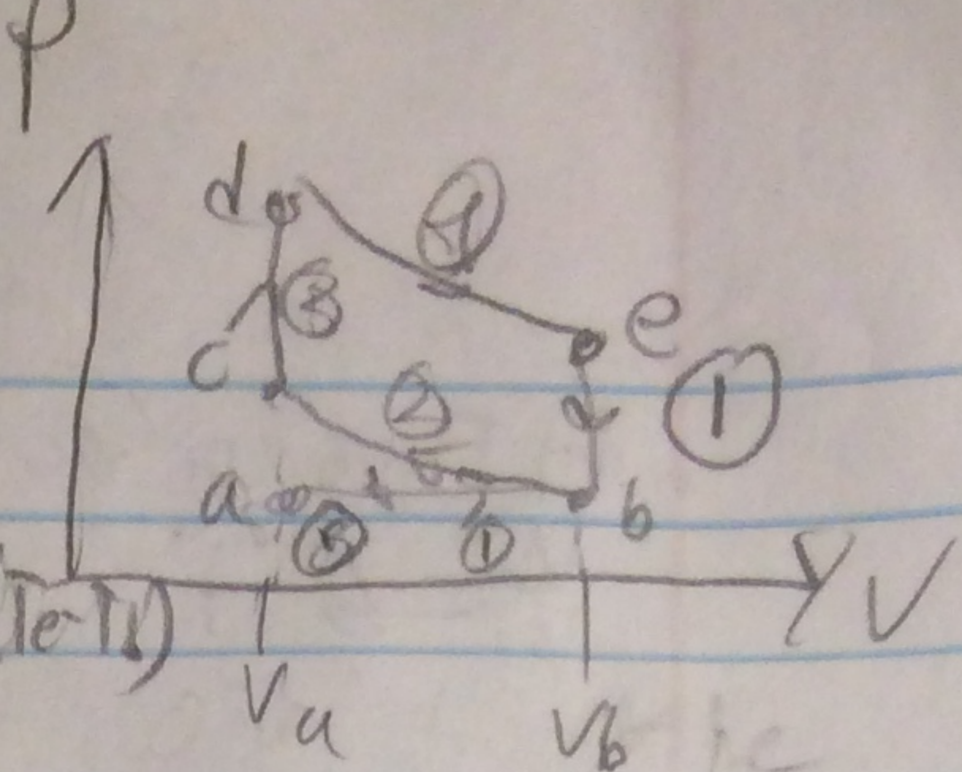
HW for 12/08/17

T9D.5
T10R.1



T9D.5

a) Prove $|Q_H| = \frac{5}{2}Nk_B(T_d - T_c)$, $|Q_c| = \frac{5}{2}Nk_B(T_e - T_b)$

We want to find Q_H and $|Q_c|$

Where $|Q_H|$ represents the heat into the engine, $|Q_c|$ represents the heat out of the engine.

We can ignore the effect on the work for step 1 and 5 because the net effect is the same as the gas hub escape but instead cooled the gas from temp T_e to T_b.

Let's re-define step 1 as this process

~~Let's make a~~ Q, W, ΔU table:

$\gamma = 1 + \frac{2}{f}$ $W_2 = \frac{P_b \cdot V_b \cdot 5}{7} \cdot \left[\left(\frac{V_b}{V_a}\right)^{2/5} - 1\right]$

$\Rightarrow \gamma = 1 + \frac{2}{5} = \frac{7}{5}$ $W_4 = \frac{P_d V_a}{7} \cdot \left[\left(\frac{V_a}{V_b}\right)^{2/5} - 1\right]$

	Q	W	ΔU
1	−	0	−
2	0	+	+
3	+	0	+
4	0	−	−

so $\Delta U = Q_1$
so $\Delta U = W_2$
so $\Delta U = Q_3$
so $\Delta U = W_4$

We can see that during step 1, Q is negative, so $|Q_1| = |Q_c|$

By the same logic, $|Q_3| = |Q_H|$

The change in internal thermal energy of an ideal gas is $\Delta U = \frac{f}{2} \cdot Nk_B \Delta T$

For the air/gas mixture, $f = 5$, so $\Delta U = \frac{5}{2} \cdot Nk_B \Delta T$

$\Delta U_1 = |Q_1| = |Q_c|$, so $|Q_c| = \frac{5}{2} \cdot Nk_B \Delta T_1$. $\Delta T_1 = T_e - T_b \Rightarrow |Q_c| = \frac{5}{2}Nk_B(T_e - T_b)$

$\Delta U_3 = |Q_3| = |Q_H|$, so $|Q_H| = \frac{5}{2}Nk_B \Delta T_3$. $\Delta T_3 = T_d - T_c \Rightarrow |Q_H| = \frac{5}{2}Nk_B(T_d - T_c)$

b) Use this to show that the engine's efficiency is $e = 1 - \frac{T_e - T_b}{T_d - T_c}$

$e = 1 - \frac{|Q_c|}{|Q_H|} \Rightarrow e = 1 - \frac{\frac{5}{2}Nk_B(T_e - T_b)}{\frac{5}{2}Nk_B(T_d - T_c)} \Rightarrow e = 1 - \frac{T_e - T_b}{T_d - T_c}$

c) Use the fact that $TV^{\gamma-1}$ = constant during an adiabatic process, to show that $\frac{T_e - T_b}{T_d - T_c} = \left(\frac{V_a}{V_b}\right)^{2/5}$.

$\gamma = 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5} \Rightarrow \gamma - 1 = \frac{2}{5}$

$T_n \cdot V_n^{2/5} = C_n \Rightarrow T_n = C_n \cdot V_n^{-2/5}$

$T_e = C_1 \cdot V_b^{-2/5}$, $T_b = C_2 \cdot V_b^{-2/5} \Rightarrow \frac{T_e - T_b}{T_d - T_c} = \frac{V_b^{-2/5}(C_1 - C_2)}{V_a^{-2/5}(C_1 - C_2)}$

$T_d = C_1 \cdot V_a^{-2/5}$, $T_c = C_2 \cdot V_a^{-2/5} \Rightarrow \frac{T_e - T_b}{T_d - T_c} = \frac{\left(\frac{1}{V_b^{2/5}}\right)}{\left(\frac{1}{V_a^{2/5}}\right)} = \left(\frac{V_a}{V_b}\right)^{2/5}$

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d) Find an automobile's efficiency if its compression ratio is $\frac{V_b}{V_a}=8$

From parts b, c we can say that
$e=1-\left(\frac{V_a}{V_b}\right)^{2/5} \Rightarrow e=1-\left(\frac{1}{8}\right)^{2/5}=$ 0.56

Exam Questions:
T4 D.8: Reasonable magnitude, right units
T10 R.4: Reasonable values, right units

T10 R.9
Ring reflects 30% of sun's light

a) Assume that the ring is thin enough that the temp on its back side is essentially the same as that on its front side. What should its diameter be as a multiple of the diameter of earth's orbit if the average temp on the ring's front surface is 14°C?

Let's assume the power output of the light is the same as the sun.
We know that at the orbit of the earth, 1 AU, $I=1367\frac{W}{m^2}$
$I_r=I_e\cdot\left(\frac{1}{r}\right)^2$

The ring absorbs 70% of all the light that hits its surface,
so $P_{in}=\frac{7}{10}\cdot I_r\cdot 2\pi r\, dw=\frac{7}{10}\cdot I_e\left(\frac{1}{r}\right)^2\cdot 2\pi r\cdot dw=\frac{7}{10}\cdot I_e\cdot\frac{2\pi}{r}\,dw$
The power radiated by the ring is such that $P_{out}=P_{in}$. However, by the Stefan-Boltzmann law saying that $\frac{P_{out}}{A_{out}}=\varepsilon\sigma T^4 \Rightarrow T=\left(\frac{P}{A}\frac{1}{\varepsilon\sigma}\right)^{1/4}$
Let's assume $\varepsilon=1$. So $T=\left(\frac{P_{out}}{A_{out}}\frac{1}{\sigma}\right)^{\frac{1}{4}}=\left(\frac{\frac{7}{10}\cdot I_e\cdot\frac{2\pi}{r}\cdot dw}{4\cdot\pi\cdot r\, dw}\cdot\frac{1}{\sigma}\right)^{1/4}=\left(\frac{7\cdot I_e}{20\, r^2\cdot\sigma}\right)^{1/4}$
Solving for $r \Rightarrow T^4=\frac{7\cdot I_e}{20\cdot r^2\sigma} \rightarrow r=\sqrt{\frac{7\cdot I_e}{20\cdot T^4\cdot\sigma}}$

$\frac{W}{m^2}$

$\frac{m^2\cdot W}{m^2 K^4}$

$\sigma=5.67\cdot 10^{-8}\frac{W}{m^2 K^4}$ $T=\frac{1K}{1°C}(273.15°C+[14°C])=287.15$
$d=2r=2\cdot\sqrt{\frac{7\cdot 1367\frac{W}{m^2}}{20\cdot(287.15)^4\cdot 5.67\cdot 10^{-8}\frac{W}{m^2K^4}}}$ $=0.07$ AU

b) Assume that the ring's back side is 100 K cooler than its front surface. What should the ring's diameter be?
Now the average surface temp is $\frac{173.15\,K+287.15\,K}{2}=230.15\,K=T$
So $d=2\cdot r=2\cdot\sqrt{\frac{7\cdot 1367\frac{W}{m^2}}{20\cdot(230.15K)^4\cdot 5.67\cdot 10^{-8}\frac{W}{m^2K^4}}}=$ 0.11 AU