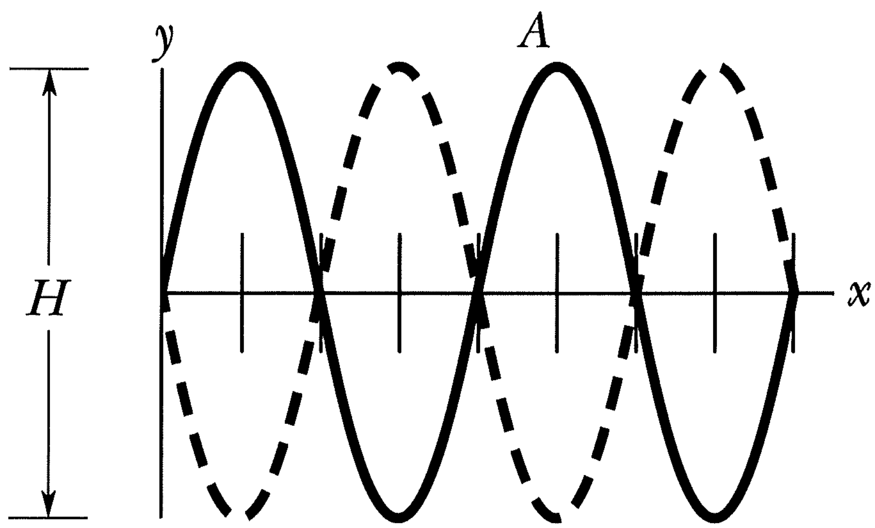
1. Two sinusoidal waves with the same amplitude and wavelength travel through each other along a string that is stretched along an x-axis. Their resultant wave is shown below. The antinode A travels from an extreme upward displacement to an extreme downward displacement in 6.0 ms. The tick marks along the axis are separated by 10 cm. the height H is 1.8 cm. Let the equations for one of the two waves be of the form

$$y(x,t) = y_m \sin(kx+\omega t).$$



In the equation for the other wave what are

(a) y_m

(b) k

(c) ω

(d) the sign in front of

Reference to point A as an anti-node suggests that this is a standing wave pattern and thus that the waves are traveling in opposite directions. Thus, we expect one of them to be of the form $y = y_m \sin(kx \pm \omega t)$ and the other to be of the form $y = y_m \sin(kx - \omega t)$. $y_1 + y_2 = [2 y_m \sin(kx)][\cos \omega t]$

(a) We conclude that $y_m = \frac{1}{2}(9.0 \text{ mm}) = 4.5$ mm, due to the fact that the amplitude of the standing wave is $\frac{1}{2}(1.80 \text{ cm}) = 0.90$ cm = 9.0 mm.

(b) Since one full cycle of the wave (one wavelength) is 40 cm, $k = 2\pi/\lambda \approx 16 \text{ m}^{-1}$.

(c) The problem tells us that the time of half a full period of motion is 6.0 ms, so T = 12 ms and $\omega = 5.2 \times 10^2$ rad/s.

(d) The two waves are therefore

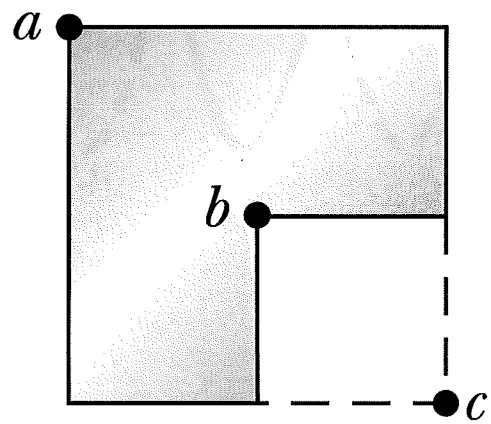
and

$$y_1(x, t) = (4.5\text{ mm})\sin[(16\text{ m}^{-1})x + (520\text{ s}^{-1})t]$$

$$y_2(x, t) = (4.5\text{ mm})\sin[(16\text{ m}^{-1})x - (520\text{ s}^{-1})t]\,.$$

If one wave has the form $y(x,t) = y_m \sin(kx + \omega t)$ as in y_1, then the other wave must be of the form $y'(x,t) = y_m \sin(kx - \omega t)$ as in y_2. Therefore, the sign in front of ω is minus.

2. The figure shows a uniform metal plate that was square before 25% of it was snipped off. Three lettered points are indicated. Rank, from largest to smallest, the rotational inertia of the plate if it pivots around a perpendicular axis through each point.



Key idea: $I = mR^2$ ← the farther the mass is from the axis of pivot, the bigger I

Largest I @ pt c: most of mass is farther from that point

I @ pt a:

Smallest I @ pt b: more of mass is centrally arranged around that point

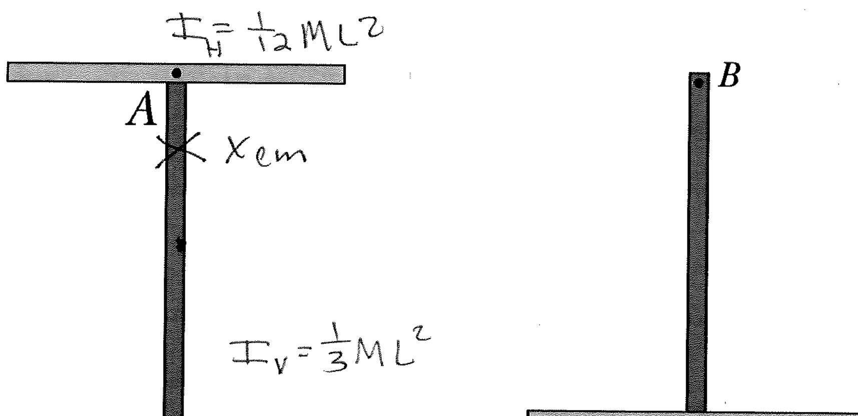
3. A horizontal vinyl record of mass 0.1 kg and radius 0.1 m rotates freely about a vertical axis through its center with an angular speed of 4.7 rad/s. The rotational inertia of the record about its axis of rotation is 0.0005 kg m2. A wad of wet putty of mass 0.02 kg drops vertically onto the record from above and sticks to the edge of the record. What is the angular speed of the record immediately after the putty sticks to it?

For simplicity, we assume the record is turning freely, without any work being done by its motor (and without any friction at the bearings or at the stylus trying to slow it down). Before the collision, the angular momentum of the system (presumed positive) is $I_i\omega_i$ where $I_i = 5.0\times10^{-4}\ \text{kg}\cdot\text{m}^2$ and $\omega_i = 4.7$ rad/s. The rotational inertia afterwards is

$$I_f = I_i + mR^2$$

where m = 0.020 kg and R = 0.10 m. The mass of the record (0.10 kg), although given in the problem, is not used in the solution. Angular momentum conservation leads to

1. $I_i\omega_i = I_f\omega_f \Rightarrow \omega_f = \dfrac{I_i\omega_i}{I_i + mR^2} = 3.4\ \text{rad/s}.$



4. A physical pendulum consists of two meter-long sticks joined together as shown. What is the pendulums period of oscillation about a pin inserted through point A at the center of the horizontal stick? What is the period of oscillation if the pin is inserted through point B at the end of the vertical stick?

We need to locate the center of mass and we need to compute the rotational inertia about A. The center of mass of the stick shown horizontal in the figure is at A, and the center of mass of the other stick is 0.50 m below A. The two sticks are of equal mass so the center

~~Let A be 0~~, Let A be at $y = 0$

@ A → O for horiz bar

$$h = \frac{y_{1cm} m_1 + y_{2cm} m_2}{m_1 + m_2} = \frac{\cancel{m}(y_{1cm} + y_{2cm})}{2\cancel{m}} = \frac{.5}{2}$$

of mass of the system is $h = \frac{1}{2}(0.50 \text{ m}) = 0.25\text{m}$ below A, as shown in the figure. Now, the rotational inertia of the system is the sum of the rotational inertia I_1 of the stick shown horizontal in the figure and the rotational inertia I_2 of the stick shown vertical. Thus, we have

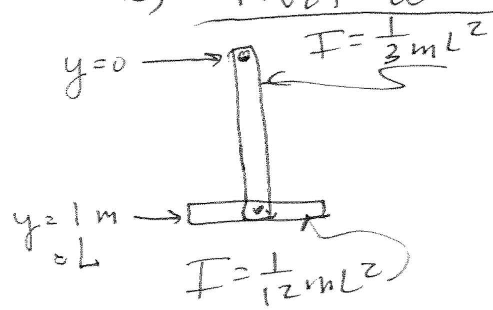
$$I = I_1 + I_2 = \frac{1}{12}ML^2 + \frac{1}{3}ML^2 = \frac{5}{12}ML^2$$

where L = 1.00 m and M is the mass of a meter stick (which cancels in the next step). Now, with $m = 2M$ (the total mass), Eq. 15–29 yields

$$h = \frac{L}{4} \qquad T = 2\pi\sqrt{\frac{\frac{5}{12}ML^2}{2Mgh}} = 2\pi\sqrt{\frac{5L}{6g}}$$

where $h = L/4$ was used. Thus, T = 1.83 s.

B) Pivot at top of inverted T:



$$h = \frac{\cancel{m}(y_{1cm} + y_{2cm})}{2\cancel{m}} = \frac{L}{2} = .5\text{m}$$

$$I = I_1 + I_2 = \frac{5}{12}ML^2$$

$$T = 2\pi\sqrt{\frac{\frac{5}{12}ML^2}{2Mg\frac{L}{2}}} = 2\pi\sqrt{\frac{5}{12}\frac{L}{g}}$$

$$T = 1.28\text{s}$$

5. The length of a bicycle pedal arm is 0.152m and a downward force of 111 N is applied to the pedal by the rider. What is the magnitude of the torque about the pedal arm's pivot when the arm is at angle

(a) 30° ?

(b) 90° ?

(c)180° ?

We compute the torques using $\tau = rF \sin \phi$. $= \vec{r} \times \vec{F}$

(a) For $\phi = 30°$, $\tau_a = (0.152 \text{ m})(111 \text{ N}) \sin 30° = 8.4 \text{ N} \cdot \text{m}$.

(b) For $\phi = 90°$, $\tau_b = (0.152 \text{ m})(111 \text{ N}) \sin 90° = 17 \text{ N} \cdot \text{m}$.

(c) For $\phi = 180°$, $\tau_c = (0.152 \text{ m})(111 \text{N}) \sin 180° = 0$.

6. What are (a) the lowest frequency, (b) the second lowest frequency and (c) the third lowest frequency for standing waves on a wire that is 10.0 m long, has a mass of 100 g and is stretched under a tension of 250 N?

Possible wavelengths are given by $\lambda = 2L/n$, where L is the length of the wire and n is an integer. The corresponding frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu} = \sqrt{\tau L/M}$, where τ is the tension in the wire, μ is the linear mass density of the wire, and M is the mass of the wire. $\mu = M/L$ was used to obtain the last form. Thus

$$f_n = \frac{n}{2L}\sqrt{\frac{\tau L}{M}} = \frac{n}{2}\sqrt{\frac{\tau}{LM}} = \frac{n}{2}\sqrt{\frac{250 \text{ N}}{(10.0 \text{ m})\ (0.100 \text{ kg})}} = n\ (7.91 \text{ Hz}).$$

(a) The lowest frequency is $f_1 = 7.91$ Hz. (the fundamental or first harmonic)

(b) The second lowest frequency is $f_2 = 2(7.91 \text{ Hz}) = 15.8$ Hz. (the second harmonic)

(c) The third lowest frequency is $f_3 = 3(7.91 \text{ Hz}) = 23.7$ Hz. (the third harmonic)