Preparation for October 11

Suppose L is some positive number. If we define a function $f(x)$ on the interval $(0, L)$, we can define two extensions of $f(x)$ to the interval $(-L, L)$.

On the one hand, we could make an odd function $f_{\text{odd}}(x)$ as follows:

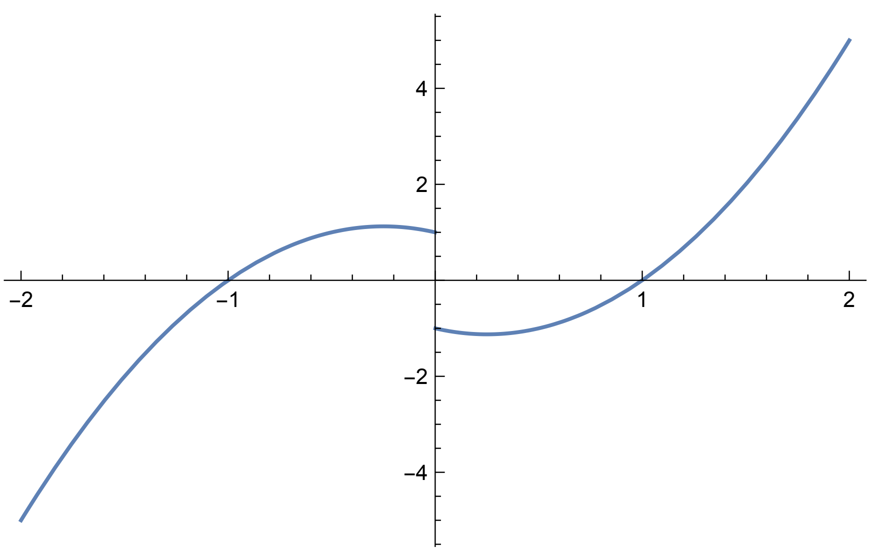
$$f_{\text{odd}(x)} = \begin{cases} -f(-x) & -L < x < 0 \\ f(x) & 0 < x < L \end{cases}$$

That is, for negative values of x, we look at the corresponding positive number $-x$, take the value of f at that point, and then take the negative of that value.

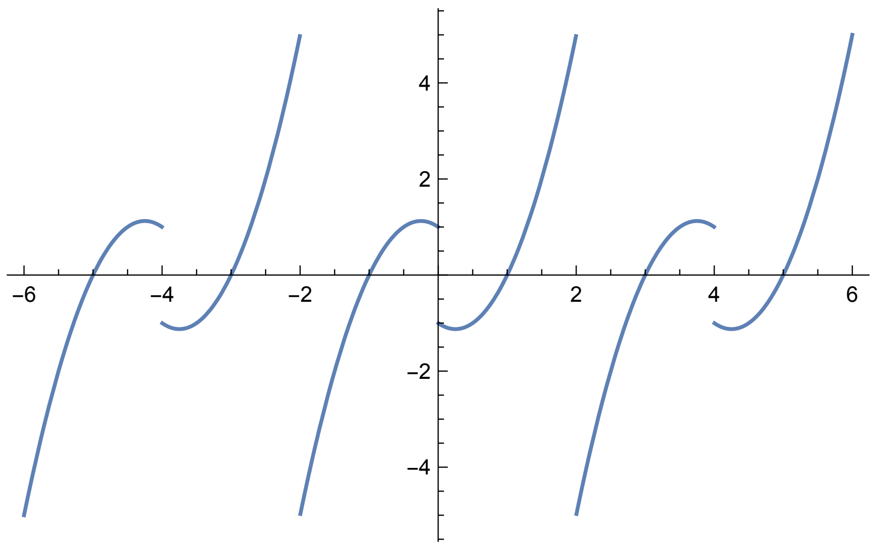
For, example, let's suppose $L = 2$. If $f(x) = 2x^2 - x - 1$ on $(0, 2)$, then for $-2 < x < 0$, we have

$$f_{\text{odd}}(x) = -(2(-x)^2 - (-x) - 1) = -2x^2 - x + 1.$$

Here is a graph:



We could then make $f_{\text{odd}}(x)$ a periodic function with period $2L = 4$. Here is what the graph of that would look like:



On the other hand, we could make an even function $f_{\text{even}}(x)$ as follows:

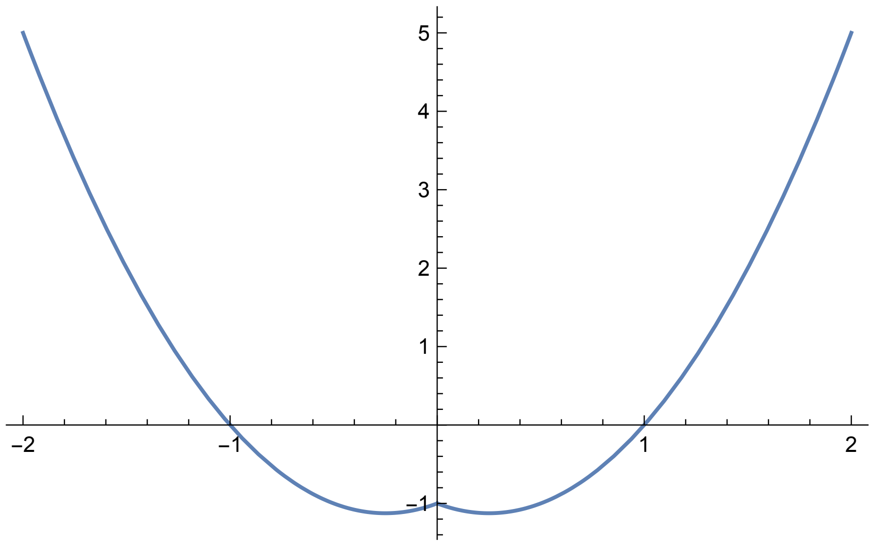
$$f_{\text{even}(x)} = \begin{cases} f(-x) & -L < x < 0 \\ f(x) & 0 < x < L \end{cases}$$

That is, for negative values of x, we look at the corresponding positive number $-x$, and just take the value of f at that point.

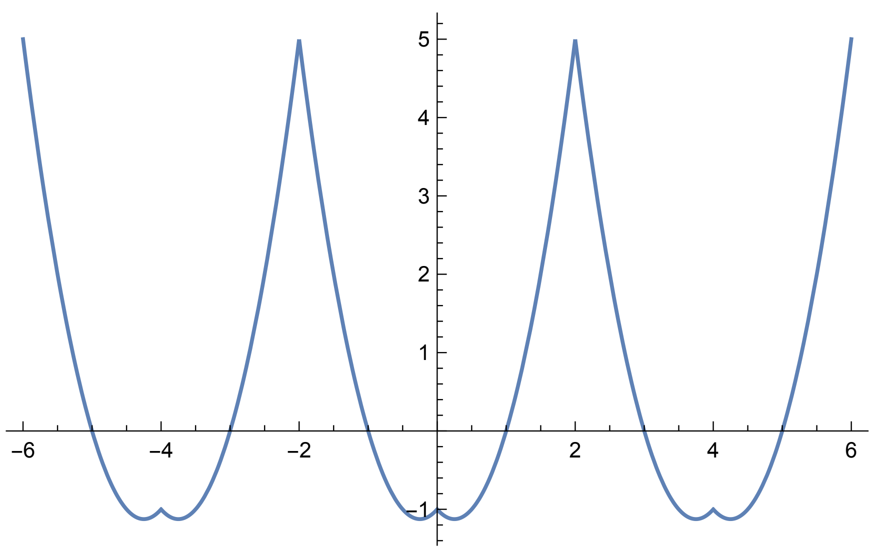
For, example, let's suppose $L = 2$. If $f(x) = 2x^2 - x - 1$ on $(0, 2)$, then for $-2 < x < 0$, we have

$$f_{\text{even}}(x) = 2(-x)^2 - (-x) - 1 = 2x^2 + x - 1.$$

Here is a graph:



We could then make $f_{\text{even}}(x)$ a periodic function with period $2L = 4$. Here is what the graph of that would look like:



Notice that all of these graphs are the same on $(0, 2)$.