Olle Fwdes

Phy 232

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hw for 11/27 ++

T6M.4, T7M.5

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T6M.4

In this problem, we will consider the energy budget requirement for a human body to emit thermal photons

a) Consider a relatively small person w/ a SA $1.0m^2$, body temp of 310K. Show that such a person would need to consume ~11000 kcal/day to replenish the energy emitted by their body (assume $\epsilon=1$)

We can use the Stefan Boltzmann law to show this.

SB states that the power of an object at some absolute temp. T with surface area A is $P=\epsilon\sigma A T^4$, where ϵ is the object's emissivity, $\sigma = 5.67 \cdot 10^{-8} \frac{W}{m^2K^4}$

In this case, $A = 1.0m^2$, $\epsilon=1$, $T=310K$, so

$P = 1 \cdot 5.67 \cdot 10^{-8} \frac{W}{m^2K^4} \cdot 1.0m^2 \cdot (310K)^4 = 524 \frac{J}{s}$

$\to 524 \frac{J}{s} = 524 \frac{J}{s} \cdot \frac{86400s}{day} \cdot \frac{0.239\,cal}{1J} = 10812882.35 \frac{cal}{day} \approx 11000 \frac{kcal}{day}$

This means the energy emitted by the body is ~11000 kcal/day, so to replenish this they must consume equivalent of at least this much.

b) This is for naked in vacuum. A small person actually requires something like 2000 $\frac{kcal}{day}$ to maintain body temp. Calculate the net energy per day that a naked person in a room at 295K would emit

Equation T6.28 also describes the energy a surface absorbs from surroundings at an absolute temp T. The net power is the rate of energy absorbed minus rate of energy emitted:

$P_{net} = \Delta P = \epsilon \cdot \sigma \cdot A \cdot (T_{abs}^4 - T_{em}^4) = 1 \cdot 5.67 \cdot 10^{-8} \frac{W}{m^2K^4} \cdot 1.0m^2 \cdot [(295K)^4 - (310K)^4] = -94\,W$

$\to -94W = -94 \frac{J}{s} = -94 \frac{J}{s} \cdot \frac{86400s}{1\,day} \cdot \frac{0.239\,cal}{1J} = -1945758.668 \frac{cal}{day} \approx \boxed{-1946 \frac{kcal}{day}}$

c) Clothing also helps. Suppose the person's entire SA is surrounded by a layer of clothing that completely absorbs their thermal photons, but also emits thermal photons in both directions from its surface, half to the person and half to the surroundings. Let P_P/A be the power per unit area emitted by the person, P_C/A be the power per unit area emitted by each side of the clothing layer, and P_S/A be the power per unit area the layer absorbs from the room. When the clothing is in equilibrium with the person at 310K and the room at 295K, the net energy flow to the layer must be zero, meaning $2 \cdot \frac{P_C}{A} = \frac{P_P}{A} + \frac{P_S}{A}$. Show that the layer must have an absolute temp $T_C = 302.8K$

$P_C = \epsilon_C \cdot \sigma \cdot A \cdot T_C^4$

$P_P = \epsilon_P \cdot \sigma \cdot A \cdot T_P^4$

$P_S = \epsilon_C \cdot \sigma \cdot A \cdot T_S^4$

The clothes ~~are~~ described as a perfect black body, so $\epsilon_C = 1$

So $2 \cdot \frac{P_C}{A} = \frac{P_P}{A} + \frac{P_S}{A} \Rightarrow 2 \cdot \epsilon_C \cdot \cancel{\sigma} \cdot T_C^4 = \epsilon_P \cdot \cancel{\sigma} \cdot T_P^4 + \epsilon_C \cdot \cancel{\sigma} \cdot T_S^4$

$\Rightarrow T_C^4 = \frac{T_P^4 + T_S^4}{2} \Rightarrow T_C = \left(\frac{T_P^4 + T_S^4}{2}\right)^{1/4} = \boxed{302.8K}$

d) Calculate, therefore, the net rate at which a clothed person loses energy in kCal/day

$P_{net\ clothes} = P_{in} - P_{out} = P_{clothes} - P_{person} = \epsilon \cdot \sigma \cdot A\,(T_C^4 - T_P^4)$

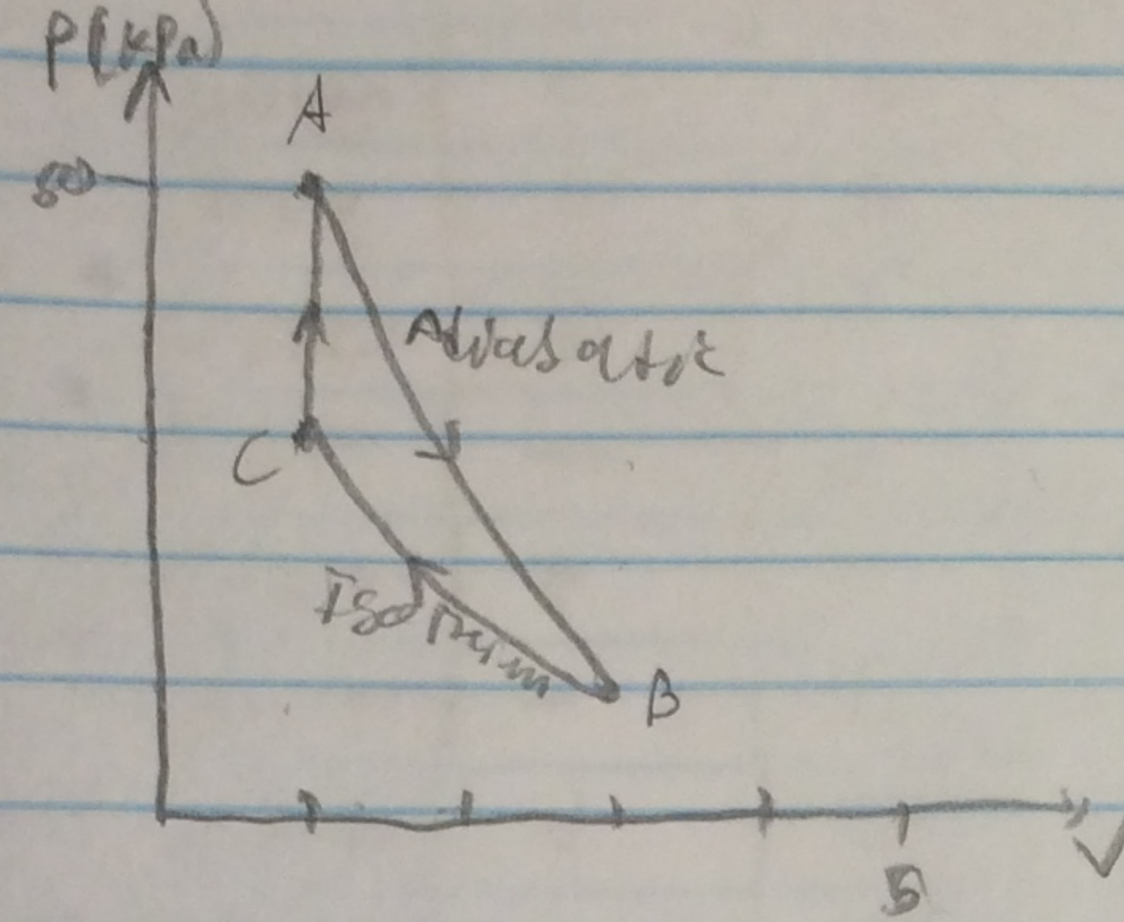
$P_{net} = 1 \cdot 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4} \cdot \left[(302.8K)^4 - (310K)^4\right] \cdot 1.0m^2 = -47.1W$

$\rightarrow -47.1W = -47.1 \frac{J}{s} \cdot \frac{86400s}{day} \cdot \frac{0.239\,cal}{J} = -972859.3332 \frac{cal}{day} = \boxed{-973 \frac{kCal}{day}}$

P.

T7M.5

Suppose we constrain a gas to follow the three step cyclic process shown in the graph below. Prepare a chart that specifies the sign of Q, W, and ΔU for each step in the process.



	Q	W	ΔU
$A \to B$	0	−	−
$B \to C$	−	+	0
$C \to A$	+	0	+

$A \to B$ is an adiabatic expansion, meaning $P\downarrow, V\uparrow$ and $Q = 0$
so by $\Delta U = W + Q + [F]$, $\Delta U = W + 0 + 0 = W$. Moreover $\Delta U = \frac{f}{2}\Delta(PV)$
This is good enough to tell us the overall sign of ΔU, W.
$P_A V_A = 50 \cdot 10^3 Pa \cdot 1 m^3$, $P_B V_B = 10 \cdot 10^3 Pa \cdot 3 m^3$
$P_A V_A > P_B V_B$, so $\Delta U_{A\to B} < 0$, $W_{A \to B} < 0$

$B \to C$ is isothermal compression, meaning $P\uparrow, V\downarrow$, and $P_n V_n = $ constant
during an infinitesimally compression $dV < 0$ so $dW = -P\,dV > 0$, so
$W_{B\to C} > 0$. Because $T =$ constant and $\Delta U \propto \Delta T$, $\Delta U = 0$
$\Delta U = W + Q$, so Q must be negative

$C \to A$ is an isochloric heating, meaning $P\uparrow$, V constant, $Q > 0$
By $dW = -P\,dV$, $dW = 0$, so $W_{C\to A} = 0$.
$\Delta U = W + Q$, so $\Delta U = Q$, $\Delta U > 0$

Evaluations:
T6M.4: right units, reasonable magnitude
T7M.5: reasonable reasoning.