Phy 232
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Parker 9/15/17

R8M.9, R9M.3

1

R8M.9

Suppose that in a certain particle physics experiment a particle of mass m_0 moving with speed $|\vec{v}_0| = \frac{3}{5}$ in the $+x$ direction in the lab frame is observed to decay into two particles, one with mass m_1 moving with speed $|\vec{v}_1| = \frac{4}{5}$ in the $+x$ direction and another with mass m_2 that is essentially at rest.

a) Show that if total particle mass and Newtonian momentum are conserved in the lab frame, that we must have $m_1 = \frac{3}{4} m_0$, $m_2 = \frac{1}{4} m_0$

For Newtonian momentum, $\vec{P}_i = \vec{P}_f$, thus,

$$m_0 \cdot |\vec{v}_0| = m_1 |\vec{v}_1| + m_2 |\vec{v}_2|$$

so for $|\vec{v}_0| = \frac{3}{5}$, $|\vec{v}_1| = \frac{4}{5}$, $|\vec{v}_2| = 0$, we get

$$m_0 \cdot \frac{3}{5} = m_1 \cdot \frac{4}{5} + 0 \Rightarrow m_1 = m_0 \frac{3}{4}$$

Because mass is also conserved, it must be that $m_0 = m_1 + m_2$

so $m_2 = m_0 - m_1 = m_0 - \frac{3m_0}{4} = \frac{m_0}{4}$

so $m_1 = \frac{3}{4} m_0$, $m_2 = \frac{1}{4} m_0$

b) Show that if four-momentum is to be conserved in the lab frame, we must have $m_1 = \frac{9}{16} m_0$, $m_2 = \frac{5}{16} m_0$.

For four-momentum to be conserved

$$\begin{bmatrix} p_{t0} \\ p_{x0} \\ p_{y0} \\ p_{z0} \end{bmatrix} = \begin{bmatrix} p_{t1} \\ p_{x1} \\ p_{y1} \\ p_{z1} \end{bmatrix} + \begin{bmatrix} p_{t2} \\ p_{x2} \\ p_{y2} \\ p_{z2} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{m_0}{\sqrt{1-|\vec{v}_0|^2}} \\ \frac{m_0 v_{x0}}{\sqrt{1-|\vec{v}_0|^2}} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{m_1}{\sqrt{1-|\vec{v}_1|^2}} \\ \frac{m_1 v_{1x}}{\sqrt{1-|\vec{v}_1|^2}} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{m_2}{\sqrt{1-|\vec{v}_2|^2}} \\ \frac{m_2 v_{2x}}{\sqrt{1-|\vec{v}_2|^2}} \\ 0 \\ 0 \end{bmatrix}$$

we can plug this into the equations

$$\frac{m_0}{\sqrt{1-(\frac{3}{5})^2}} = \frac{m_1}{\sqrt{1-(\frac{4}{5})^2}} + \frac{m_2}{\sqrt{1-0^2}} \quad \text{and} \quad \frac{m_0\frac{3}{5}}{\sqrt{1-(\frac{3}{5})^2}} = \frac{m_1\cdot\frac{4}{5}}{\sqrt{1-(\frac{4}{5})^2}} + \frac{m_2\cdot 0}{\sqrt{1-0^2}}$$

$$\to \frac{m_0}{\sqrt{\frac{16}{25}}} = \frac{m_1}{\sqrt{\frac{9}{25}}} + m_2 \quad \text{and} \quad \frac{3m_0}{5\sqrt{\frac{16}{25}}} = \frac{4m_1}{5\sqrt{\frac{9}{25}}} + 0$$

$$\to \frac{5}{4}m_0 = \frac{5}{3}m_1 + m_2 \quad \text{and} \quad \frac{3}{4}m_0 = \frac{4}{3}m_1 \to \frac{9}{16}m_0 = m_1$$

$$\to \frac{5m_0}{4} - \frac{5}{3}\left(\frac{9}{16}m_0\right) = m_2$$

$$\to \frac{60m_0 - 45m_0}{48} = m_2 = \frac{5}{16}m_0$$

so $m_2 = \frac{5}{16}m_0$, $m_1 = \frac{9}{16}m_0$

c) Consider now an IRF (the Other frame) that moves with the same speed in the +x direction as the initial particle. Use the Einstein velocity transform to show the particle's x-velocities in this frame must be $V'_{0x} = 0$, $V'_{1x} = \frac{5}{13}$, and $V'_{2x} = -\frac{3}{5}$

we will use $V'_x = \frac{V_x - \beta}{1 - \beta V_x}$

we know $|\vec{V}_0| = \frac{3}{5}$, $|\vec{V}_1| = \frac{4}{5}$, $|\vec{V}_2| = 0$

$$V'_{0x} = \frac{\frac{3}{5} - \frac{3}{5}}{1 - \frac{3}{5}\cdot\frac{3}{5}} = \frac{0}{16/25} = 0$$

$$V'_{1x} = \frac{\frac{4}{5} - \frac{3}{5}}{1 - \frac{4}{5}\cdot\frac{3}{5}} = \frac{1/5}{\frac{25-12}{25}} = \frac{25/5}{13} = \frac{5}{13}$$

$$V'_{2x} = \frac{0 - \frac{3}{5}}{1 - 0} = -\frac{3}{5}$$

so $V'_{0x} = 0$, $V'_{1x} = \frac{5}{13}$, $V'_{2x} = -\frac{3}{5}$

d) Show that if we use the outgoing particle masses that ensure that Newtonian momentum and particle mass are conserved in the lab frame the Newtonian momentum is NOT conserved

$$\vec{P}_0' \stackrel{?}{=} \vec{P}_1' + \vec{P}_2'$$

$$m_0 \cdot 0 \stackrel{?}{=} m_0 \cdot \frac{9}{16} \cdot \frac{5}{12} + m_0 \cdot \frac{9}{16} \cdot -\frac{3}{5}$$

$$0 \stackrel{?}{=} m_0 \left(\frac{45}{208} - \frac{45}{80}\right)$$

$0 \neq m_0 \left(\frac{45}{208} - \frac{15}{80}\right)$, so Newtonian momentum is not conserved

e) Show that if we use the masses that ensure that four momentum is conserved in the lab frame, the four-momentum is conserved in the Other frame.

$$\begin{bmatrix} P_{t_0}' \\ P_{x_0}' \\ P_{y_0}' \\ P_{z_0}' \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} P_{t_1}' \\ P_{x_1}' \\ P_{y_1}' \\ P_{z_1}' \end{bmatrix} + \begin{bmatrix} P_{t_2}' \\ P_{x_2}' \\ P_{y_2}' \\ P_{z_2}' \end{bmatrix} \begin{matrix} \to \gamma(P_{t_0} - \beta P_{x_0}) \stackrel{?}{=} \gamma(P_{t_1} - \beta P_{x_1}) + \gamma(P_{t_2} - \beta P_{x_2}) \\ \to \gamma(P_{x_0} - \beta P_{t_0}) \stackrel{?}{=} \gamma(P_{x_1} - \beta P_{t_1}) + \gamma(P_{x_2} - \beta P_{t_2}) \\ \to P_{y_0} \stackrel{?}{=} P_{y_1} + P_{y_2} = 0 \\ \to P_{z_0} \stackrel{?}{=} P_{z_1} + P_{z_2} = 0 \end{matrix}$$

$$P_t' = \gamma(P_t - \beta P_x)$$
$$P_x' = \gamma(P_x - P_t \beta)$$
$$P_y' = P_y = 0$$
$$P_z' = P_z = 0$$

Same yields

$$P_{t_0} - \beta P_{x_0} \stackrel{?}{=} P_{t_1} + P_{t_2} - \beta(P_{x_1} + P_{x_2}) \quad \text{A)}$$

$$P_{x_0} - \beta P_{t_0} \stackrel{?}{=} P_{x_1} + P_{x_2} - \beta(P_{t_1} + P_{t_2}) \quad \text{(B)}$$

Using these values

$$P_{t_0} = \frac{5}{4} m_0 \qquad P_{t_1} = \frac{5}{3} m_1 = \frac{5}{3} \cdot \frac{9}{16} m_0 = \frac{15}{16} m_0 \qquad P_{t_2} = m_2 = \frac{5}{16} m_0$$

$$P_{x_0} = \frac{3}{4} m_0 \qquad P_{x_1} = \frac{4}{3} m_1 = \frac{4}{3} \cdot \frac{9}{16} m_0 = \frac{3}{4} m_0 \qquad P_{x_2} = 0$$

WE GET

$$\frac{5}{4} m_0 - \frac{5}{4} \cdot \frac{3}{4} m_0 \stackrel{?}{=} \frac{15}{16} m_0 + \frac{5}{16} \cdot 0 - \frac{5}{9}\left(\frac{3}{4} m_0 + 0\right) \quad \text{(A)}$$

and

$$\frac{3}{4} m_0 - \frac{5}{4} \cdot \frac{5}{4} m_0 = \frac{3}{4} m_0 + 0 - \frac{5}{9}\left(\frac{15}{16} m_0 + \frac{5}{16} m_0\right) \quad \text{(B)}$$

(A) $\frac{20-15}{16} m_0 \stackrel{?}{=} \frac{15+5}{16} m_0 - \frac{15}{16} m_0$

$\Rightarrow \frac{5}{16} m_0 \stackrel{?}{=} \frac{5}{16} m_0$ ✓ ☺ so $P_{t0}' = P_{t1}' + P_{t2}'$

(B) $\frac{12-25}{16} m_0 \stackrel{?}{=} \frac{48}{64} m_0 - \frac{100}{64} m_0$ so $P_{x0}' = P_{x1}' + P_{x2}'$

$\Rightarrow -\frac{13}{16} m_0 \stackrel{?}{=} \frac{-52}{64} m_0 = -\frac{13}{16} m_0$ ✓ ☺

$\boxed{\text{so four momentum is conserved}}$

R9M.13

Suppose a photon with energy E_0 is travelling in the + X-direction and hits an electron of mass m at rest. The photon scatters from the electron and travels in the −X direction after the collision. Find a formula for the final energy of the photon E in terms of E_0 and m

We can use conservation of four momentum to find the value of E.

$P_{e1} + P_{\gamma 1} = P_{e2} + P_{\gamma 2}$

$$\begin{bmatrix} m \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} E_0 \\ E_0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{m}{\sqrt{1-|\vec{v}|^2}} \\ \frac{mv_x}{\sqrt{1-|\vec{v}|^2}} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} E \\ E_0 \\ 0 \\ 0 \end{bmatrix}$$

This is a system of two equations

$$m + E_0 = \frac{m}{\sqrt{1-|\vec{v}|^2}} + E \quad (A)$$

$$E_0 = \frac{mv_x}{\sqrt{1-|\vec{v}|^2}} + E \quad (B)$$

Since our motion is in the x direction, $v_x = |\vec{v}|$

(b) $\frac{m v_x}{\sqrt{1-v_x^2}} = E_0 - E$

$m^2 v_x^2 = (E_0 - E)^2 - (E_0 - E)^2 v_x^2$

$v_x^2 (m^2 + (E_0 - E)^2) = (E_0 - E)^2$

$\Rightarrow v_x^2 = \frac{(E_0 - E)^2}{m^2 + (E_0 - E)^2}$

Plugging this into (A), we get

$m + E_0 = \frac{m}{\sqrt{1 - \frac{(E_0 - E)^2}{m^2 + (E_0 - E)^2}}} + E$

$\Rightarrow m + E_0 = \cancel{\sqrt{\frac{m^2}{\frac{m^2 + (E_0 - E)^2 - (E_0 - E)^2}{m^2 + (E_0 - E)^2}}}} + E$

$\Rightarrow m + E_0 = \sqrt{\frac{m^4 + m^2 (E_0 - E)^2}{m^2}} + E$

$\Rightarrow m + E_0 = \sqrt{m^2 + (E_0 - E)^2} + E$

$(m + E_0 - E)(m + E_0 - E) = m^2 + (E_0 - E)^2$

$m^2 + mE_0 - mE + E_0 m + E_0^2 - E_0 E - Em - EE_0 + E^2 = m^2 + E_0^2 - 2E_0 E + E^2$

$\Rightarrow \cancel{m^2} + 2m(E_0 - E) - 2E_0 E + \cancel{E_0^2} + \cancel{E^2} = \cancel{m^2} + \cancel{E_0^2} - \cancel{2E_0 E} + \cancel{E^2}$

$2mE_0 = 2mE$

This doesn't seem right...
I am getting the result that $E_0 = E$, but from the conservation of 4-momentum, we should see that some of the energy that the photon initially has is transferred to the electron, but my result above implies that $V_{e_2} = V_{e_1} = 0$. I'll need to think about this some more, but I am out of time for trouble shooting so will have to leave this as it is.

Eval of answers

R8M.9: I think I did every thing correctly. but I definitely think there is a better way to compute the arithmetic that is not so imposing.

R9M.13: I got $E = E_0$, which does not make sense unless the electron does not move at all. I need to think more about this...