Physics 132
General Physics II
Grinnell College

Dipoles and Magnetic Fields Summary

An electric dipole consists of two spatially separated, equal but opposite charges. The strength of the dipole is described by the dipole moment p, given by

$$\vec{p} = q\vec{d},$$

where q is the value of the positive charge, and d is a vector giving the distance and direction from the negative charge to the positive charge. We found that if we placed a dipole in a uniform electric field, the potential energy of the system depended on the angle between the dipole moment and the electric field

$$U = -\vec{p} \cdot \vec{E} = -pE\cos(\theta)\ ,$$

which gives rise to a torque on the dipole

$$\tau = -\frac{dU}{d\theta} = pE\sin(\theta)\ .$$

If the dipole is in a non-uniform electric field, there can also be a net force exerted on the dipole. The calculation of this force is messy to do in general, but one case we examined was when the dipole moment was aligned with the electric field, in which case we found

$$F = p\frac{dE}{dx}\ .$$

This is of particular importance for induced dipoles, because the induced dipole is then always aligned with the electric field. The direction of the force on an induced dipole is such as to draw the dipole into more intense field regions.

We also did some investigation of the electric potential of an electric dipole. To do this calculation, we used a trick known as the binomial approximation, which to first order is given as

$$(1+\delta)^n = 1 + n\delta$$

where δ is a number whose absolute value is small compared to 1 (actually must be small compared to 1/n). Using this, we found that the dipole potential looks like

$$V = \frac{p\cos(\theta)}{4\pi\varepsilon_o r^2}$$

which produces electric fields that fall off with distance as $1/r^3$.

The study of electric dipoles leads nicely into the study of magnetism, for permanent magnets always consist of matched north and south poles; one never finds an isolated north pole without a matching south nearby. In fact, if one breaks a permanent magnet in half between the north and south poles, new poles appear at the break so as to make each half another dipole. The force between the [conceptually] isolated poles behaves very like the electric force, falling off like $1/r^2$. The magnetic dipole moment is symbolized by μ in contrast with the electric dipole moment p, and the relations for torque and force on a dipole are exactly the analogs of those for electric dipoles. Several important metals (iron, steel, nickel, and cobalt are the most common) are strongly magnetic; they develop very large induced magnetic dipole moments in the presence of magnetic fields. As a result, permanent magnets attract these materials, and these materials can be used either to shield or to enhance magnetic fields, depending on the arrangement of the materials.

Aside from permanent magnets, magnetic fields are also produced by currents through wires. The magnetic field lines wrap around the wire in a circular fashion, with the field direction given by the right hand rule: point your right thumb in the direction of current flow, and your fingers curl around in the direction of the magnetic field. The magnetic field from a long straight wire has a magnitude as a function of distance from the wire of

$$B = \frac{\mu_o I}{2\pi r}, \text{ where } \mu_o = 4\pi \times 10^{-7}\, N / A^2.$$

As with electrostatic problems, we have several ways of calculating magnetic fields produced by currents. The equivalent to Coulomb's law is known as the law of Biot and Savart:

$$dB = \frac{\mu_o I d\ell}{4\pi r^2}.$$

The only case that this can be trivially applied to is to find the field on axis for a current loop; we found the magnetic field at the very center of a current loop as

$$B = \frac{\mu_o I}{2R}$$

where R is the radius of the loop.

We also learned that the law of Biot and Savart can be expressed in a form like Gauss's law, which is known as Ampere's law:

$$\oint \vec{B} \cdot d\vec{\ell} = B \cdot pathlength \cdot \cos\theta = \mu_o I_{enclosed}$$

where θ is the angle between B and the path of integration. We used Ampere to solve for fields from long wires and other cylindrically symmetric currents by drawing circular Amperian loops concentric with the cylindrical axis. We also used Ampere for the solenoid by drawing a rectangular loop that sampled the field inside the solenoid, giving us

$$B = \mu_o n I$$

where n is the number of turns of wire per length on the solenoid.

Not only do magnetic fields exert forces on permanent and induced magnetic dipoles, but they also exert forces on moving charges. In fact, even the permanent and induced dipoles really are coordinated quantum mechanical motions of electrons in atoms. The magnetic force on a single moving charge is given by

$$\vec{F} = q\vec{v} \times \vec{B},$$

and the force on a current carrying wire turns out to depend only on the magnitude of the current, not on the particulars of the individual charge carriers, and is expressed as

$$d\vec{F} = Id\vec{l} \times \vec{B}, \text{ or } dF = IBdl \sin\theta .$$

In this form, we are considering the force only on a little segment dl of the wire; to find the full force we need to integrate this up over the full path of the wire. For a straight wire of length ℓ perpendicular to a uniform magnetic field, this becomes

$$F = IB\ell .$$