Physics 132
General Physics II
Grinnell College

Capacitors, Charge, and Force
Summary of Concepts and Equations

In these two units, we examined the concepts of electric charge and the forces they give rise to through the use of capacitors, which is an important concept in itself. A capacitor consists typically of two large metal plates that are close to one another. These plates store charge on them, and can do so with much greater efficiency than a single plate because the positive charge stored on one plate attracts the negative charge stored on the other plate. Because this improvement in storage ability is so great, the charge one normally encounters on one of the plates is always equal in magnitude and opposite in sign to the charge on the other plate. The efficiency of charge storage is measured through a quantity called the capacitance, defined by the relationship

$$C = Q/\Delta V$$

where Q is the charge in Coulombs, ΔV is the potential difference across the plates in volts, and C is the capacitance in Farads. We found, through measurements of our homemade capacitors, that the capacitance was proportional to the area of the plates, and inversely proportional to the separation between the plates.

Combinations of capacitors, like combinations of resistors, produce a circuit element that is functionally identical to a single capacitor. The rules for the effective capacitance of parallel and series combinations of capacitors is given by the formulae

$$C_{parallel} = C_1 + C_2, \text{ and}$$

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2}$$

where C_1 and C_2 are the capacitance values of the individual capacitors. You should note that this is inverted from the effective resistor case, where series resistors add, and parallel resistors add as inverses. The reason for the difference between capacitor and resistor rules is the fact that voltage is proportional to R for a resistor (Ohm's law, V = IR), whereas voltage is proportional to 1/C for a capacitor (from the definition C = Q/V).

We started our electrical investigations by observing electric current flow. We only late in the game addressed the issue of *what* was flowing, and called this stuff charge. In contrast to the fluid case, here we appear to have two different kinds of stuff, positive and negative. The combination of the freedom with which this charge flows through conductors, and the repulsive force associated with like charges conspires to keep electrical components with no net charge on them. We even observed that although we can store separated charges in a capacitor, the net charge on the capacitor is still zero--the amount of current that flows in one lead appears to be exactly equal to the current flowing out the other lead. We assumed that what we were measuring as current was in fact the time derivative of the charge stored on the capacitor, or equivalently the rate of charge passing any given point in a wire. In an equation, this is expressed as

$$I = dQ/dt$$

where Q is the charge in Coulombs, t is time in seconds, and I is the current in Amperes, or Coulombs/second. Although we didn't measure charge directly, we did measure it implicitly by integrating the current flowing into a capacitor. We found that even when the charging and discharging processes were considerably different in character, the amount of charge stored on the capacitor is equal to the charge we pull off later. This gave credence to the notion that current is the time derivative of a conserved quantity, charge. Charge can never be created or destroyed; we can only change the net charge of an object by moving positive or negative charges on or off it. This charge conservation proves to be extremely important in many areas of physics. We had already seen it in another guise: the Kirchhoff junction rule.

We investigated the rate at which a capacitor can charge or discharge. Both of these processes involve an exponential dependence. For discharging a capacitor through a resistor, the charge as a function of time is

$$Q(t) = Q_0 e^{-t/\tau}$$

where Q_0 is the initial charge (at t = 0) on the capacitor, and τ is a characteristic time given by $\tau = RC$. This characteristic time has the same role in the exponential as our characteristic time for exponential draining of a cylinder of fluid through a capillary. For charging the capacitor, the charge function must approach a final value of Q_{final} rather than zero, so the functional form for that looks like

$$Q(t) = Q_{final}\left(1 - e^{-t/\tau}\right),$$

where again $\tau = RC$. If one wishes to have a function of the voltage on the capacitor, this can easily be found using $C = Q/\Delta V$ to convert these expressions, or simply to replace Q with ΔV, Q_0 with ΔV_0, and Q_{final} with ΔV_{final}.

Finally we actually measured the dependence of the electric force on distance, and found it to decrease as the separation between the two charges increased, and that the dependence was plausibly $1/r^2$. This law expressing the force exerted by a point-like charge Q_1 on a second point charge Q_2 is formally represented by Coulomb's law:

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2}$$

where $\varepsilon_o = 8.85 \times 10^{-12}\ C^2/Nm^2$, a constant that characterizes the intrinsic strength of the force. The overall positive sign (in contrast to the negative sign in the rather similar gravitational force law) indicates that the force is repulsive—pushing to more positive r—when the charges have the same sign, and is attractive for charges with opposite sign. Although often we consider cases that are effectively one dimensional, combinations of many charges require that we remember force is a vector, and we therefore have to add the electric forces as vectors, where the direction is along the line of the two charges participating in that particular interaction.