Calculus Review

A number of you probably are wondering "how much calculus do I need to know?" The answer is, not very much. We will review much of it as we go along, but here we summarize a few facts and concepts.

Derivatives:

The derivative of a function is a function itself, one that gives the slope of the line tangent to the original function at any point. That is, if I have a function *f*, the derivative $f'(x) = \frac{df}{dx}$ (two different ways of writing the derivative) at some point *x* is just the slope of the original function *f*(*x*) at point *x*. Here are the derivatives for some common functions:

$$f(x) = const \rightarrow f'(x) = 0$$

$$f(x) = x^n \rightarrow f'(x) = nx^{n-1}$$

$$f(x) = e^x \rightarrow f'(x) = e^x$$

$$f(x) = \cos(x) \rightarrow f'(x) = -\sin(x)$$

$$f(x) = \sin(x) \rightarrow f'(x) = \cos(x)$$

$$f(x) = \ln(x) \rightarrow f'(x) = \frac{1}{x}$$

We more commonly use t instead of x as the independent variable--don't let that confuse you. You should get comfortable with these rules with *either* x or t as the argument (and the corresponding dx or dt in derivative and integral expressions). Often we'll encounter composite functions, two simpler functions multiplied together, for instance. Here are the rules for dealing with these:

$$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$$

$$\frac{d}{dx}(f(x) \cdot g(x)) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\frac{d}{dx} f(kx) = kf'(kx) \quad such \quad as \quad \frac{d}{dx}\sin(kx) = k\cos(kx)$$

where the last rule is a limited application of the rule known as the chain rule. Another example would be

$$\frac{d}{dx}(e^{-ax}) = (-a)(e^{-ax}).$$

Integrals:

The integral undoes the derivative, so you can write all the rules above backwards for integrals, such as

$$\int \frac{1}{x} dx = \ln(x) + const .$$

If we write an indefinite integral such as this, we are just going between one function and the anti-derivative, and because the derivative of a constant is zero, we strictly speaking only know the answer to within an arbitrary constant. We also saw the integral can be represented by the cumulative area under the curve--if we express this in terms of a specific starting point, this is called a definite integral, and this then specifies that constant. We can find the value analytically by taking the difference between the anti-derivative values at each endpoint, and the constant disappears in the subtraction. Because of the area property, the integral is often thought of as a sum of little rectangular areas of size f times Δx.