Oleh Yorbis.
Phy 202
Curtis Cunningham

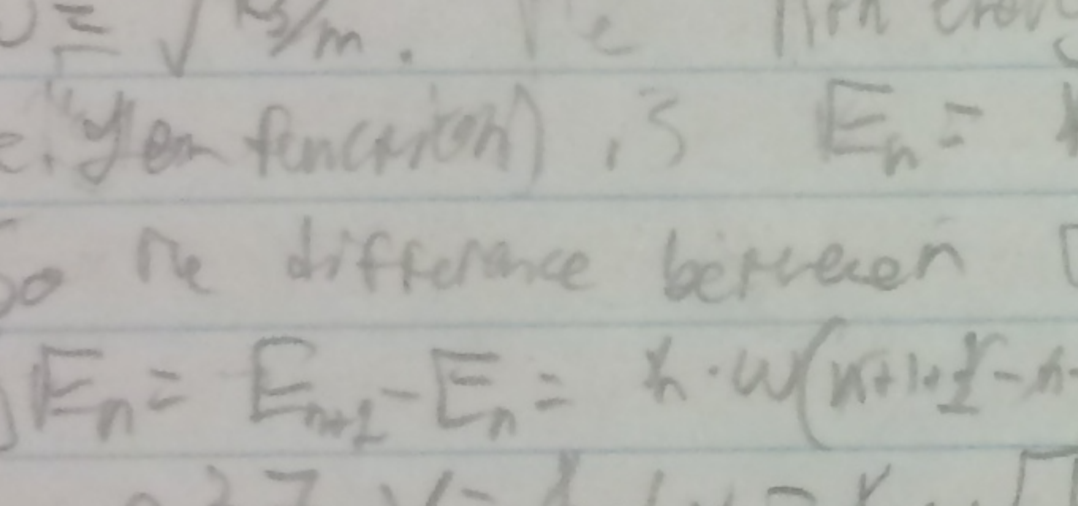
HW for 10/23/17
Q10M.9, Q AB.9, Q AM.2

Q10M.9

Consider an HCl molecule. The hydrogen atom in this molecule has a mass we can look up, and the chlorine mass is large enough that we can consider it to be fixed. The bond between these atoms has a local minimum when the atoms are separated by 0.13 nm, and for "small oscillations" around the minimum, the bond's potential energy can be modeled as a harmonic oscillator PE function. Suppose we find that the energy difference between adjacent energy eigenvalues for a vibrating HCl molecule is 0.37 eV

a) What is the effective spring constant k_s for the bond between these atoms?
The harmonic oscillator PE function is $V(x) = \frac{1}{2}\cdot m\cdot \omega^2 \cdot x^2 = \frac{1}{2}\cdot k_s \cdot x^2$
$\omega = \sqrt{k_s/m}$. The nth energy eigenvalue (corresponding to an associated energy eigenfunction) is $E_n = \hbar\cdot\omega(n+\frac{1}{2})$, $n = 0, 1, 2, \ldots$
So the difference between adjacent energy eigenvalues is
$\Delta E_n = E_{n+1} - E_n = \hbar\cdot\omega(n+1+\frac{1}{2}-n-\frac{1}{2}) = \hbar\cdot\omega\cdot 1 = \hbar\cdot\omega$
So $0.37\,eV = \hbar\cdot\omega = \hbar\cdot\sqrt{\frac{k_s}{m}} \Rightarrow$
Solving for k_s, we get $k_s = m\cdot\left(\frac{0.37\,eV}{\hbar}\right)^2 = m\cdot\left(\frac{0.37\,eV}{6.582\cdot 10^{-16}\,eV\cdot s}\right)^2$
The mass of a hydrogen atom is $m = 1.6734\cdot 10^{-27}$ kg.
So $k_s = 1.6734\cdot 10^{-27}\,kg\cdot(5.62\cdot 10^{14}\,s^{-1})^2 = 529\,\frac{kg}{s^2}$

b) The width of the energy eigenfunction is roughly equal to the distance between classical turning points for the eigenfunction's corresponding energy. Assuming the HCl molecule is in its lowest vibrational state, what is this distance?



This drawing shows the lowest energy eigenvalue E_0 on the potential energy function $V(x) = \frac{1}{2}\cdot k_s\cdot x^2$. We can find the width of the energy eigenfunction by finding the x-coordinates of the classical turning points for E_0. So we say $E_0 = \frac{1}{2}k_s\cdot x^2 \Rightarrow \frac{1}{2}\hbar\cdot\sqrt{\frac{k_s}{m}} = \frac{1}{2}\cdot k_s\cdot x^2$
$\Rightarrow \hbar\cdot\sqrt{\frac{1}{m\cdot k_s}} = x^2 \Rightarrow x = \pm\sqrt{\frac{\hbar}{\sqrt{m\cdot k_s}}} = \pm\sqrt{\frac{1.0546\cdot 10^{-34}\,J\cdot s}{\sqrt{1.6734\cdot 10^{-27}\,kg\cdot 529\,\frac{kg}{s^2}}}} = \pm 1.06\cdot 10^{-11}\,m$

So the width of the energy eigenfunction for the lowest vibrational energy state is $1.06 \cdot 10^{-11} m - -1.06 \cdot 10^{-11} m = 2 \cdot 1.06 \cdot 10^{-11} m = 2.12 \cdot 10^{-11} m = \boxed{0.0212 nm}$

c) Does this qualify as a "small oscillation"? If so, how high would n have to be for the small oscillation approximation to break down? Justify your reasoning.

A small oscillation around the minimum would be $\leq 10\%$ of ~~the~~ minimum. 10% for 0.13nm is 0.013nm, so this is not a small oscillation.

QAB.9

Suppose a quanton has the spin state $|\psi\rangle = \begin{bmatrix} \frac{5}{13} \\ i\frac{12}{13} \end{bmatrix}$

a) Show that this state is normalized.

A state vector $|\psi\rangle$ is normalized if $\langle\psi|\psi\rangle = 1$

So we ~~test~~:

$\langle\psi|\psi\rangle = \left[\left(\frac{5}{13}\right)^*, \left(i\frac{12}{13}\right)^*\right] \begin{bmatrix} \frac{5}{13} \\ i\frac{12}{13} \end{bmatrix} = \frac{5}{13}\cdot\frac{5}{13} + -i\frac{12}{13}\cdot i\frac{12}{13} = \frac{25}{13^2} + (-1)\cdot\frac{144}{13^2} = \frac{169}{13^2} = \frac{169}{169} = 1$

So the state $|\psi\rangle = \begin{bmatrix} 5/13 \\ i12/13 \end{bmatrix}$ is normalized

b) What's the probability that we will determine a quanton in such a state to have $S_y = +\frac{1}{2}\hbar$?

The probability that a quanton in state $|\psi\rangle = \begin{bmatrix} 5/13 \\ i\,12/13 \end{bmatrix}$ will be determined to have $S_y = +\frac{1}{2}\hbar$ is

$|\langle +y|\psi\rangle|^2$, where $|+y\rangle = \begin{bmatrix} \sqrt{1/2} \\ i\sqrt{1/2} \end{bmatrix}$

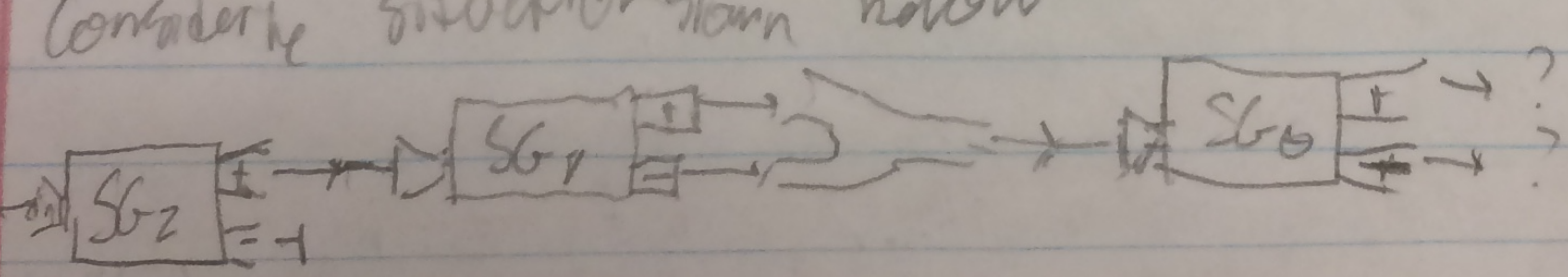
so $|\langle +y|\psi\rangle|^2 = \left|\left[\left(\sqrt{\tfrac{1}{2}}\right)^*, \left(i\sqrt{\tfrac{1}{2}}\right)^*\right] \begin{bmatrix} 5/13 \\ i12/13 \end{bmatrix}\right|^2 = \left|\sqrt{\tfrac{1}{2}}\cdot\frac{5}{13} + -i\sqrt{\tfrac{1}{2}}\cdot i\frac{12}{13}\right|^2 = \left|\sqrt{\tfrac{1}{2}}\left(\frac{5}{13} - i^2\frac{12}{13}\right)\right|^2$

$\Rightarrow \left|\sqrt{\tfrac{1}{2}}\left(\frac{5}{13} + \frac{12}{13}\right)\right|^2 = \left|\frac{17}{13\sqrt{2}}\right|^2 = \boxed{\frac{289}{338}}$

So the probability is $\frac{289}{338}$

QAM.2

Consider the situation shown below



a) Directly calculate the probability that a quanton entering the second SG device exits via the + channel of the third device

A quanton can take two possible paths from the SGy device to the SGθ device, so (assuming that we do not look at which channel of the SGy device a quanton exits) the superposition rule applies.

The sequence rule states that the quantum amplitude of a quanton prepared in an initial state $|+z\rangle$ for which the value of an observable B is determined immediately after the value of an observable A is determined is $\langle b_n|a_n\rangle\langle a_n|+z\rangle$, where b_n and a_n are the determined values for observables B and A.

So the total quantum amplitude for a quanton being determined to have state $|+\theta\rangle$ is

$$\langle +\theta|+y\rangle\langle +y|+z\rangle + \langle +\theta|-y\rangle\langle -y|+z\rangle$$

$$\Rightarrow \left(\begin{bmatrix}\cos(\tfrac{\theta}{2}), & \sin(\tfrac{\theta}{2})\end{bmatrix}\begin{bmatrix}\sqrt{\tfrac{1}{2}} \\ i\sqrt{\tfrac{1}{2}}\end{bmatrix}\right)\cdot\left(\begin{bmatrix}\sqrt{\tfrac{1}{2}}, & -i\sqrt{\tfrac{1}{2}}\end{bmatrix}\begin{bmatrix}1 \\ 0\end{bmatrix}\right) + \left(\begin{bmatrix}\cos(\tfrac{\theta}{2}), & \sin(\tfrac{\theta}{2})\end{bmatrix}\begin{bmatrix}\sqrt{\tfrac{1}{2}} \\ -i\sqrt{\tfrac{1}{2}}\end{bmatrix}\right)\cdot\left(\begin{bmatrix}\sqrt{\tfrac{1}{2}}, & i\sqrt{\tfrac{1}{2}}\end{bmatrix}\begin{bmatrix}1 \\ 0\end{bmatrix}\right)$$

$$\rightarrow \left(\cos(\tfrac{\theta}{2})\cdot\sqrt{\tfrac{1}{2}} + \sin(\tfrac{\theta}{2})\cdot i\sqrt{\tfrac{1}{2}}\right)\cdot\left(\sqrt{\tfrac{1}{2}}\right) + \left(\cos(\tfrac{\theta}{2})\cdot\sqrt{\tfrac{1}{2}} - \sin(\tfrac{\theta}{2})\cdot i\sqrt{\tfrac{1}{2}}\right)\cdot\left(\sqrt{\tfrac{1}{2}}\right)$$

$$= \tfrac{1}{2}\cdot\left(\cos\tfrac{\theta}{2} + i\sin(\tfrac{\theta}{2}) + \cos(\tfrac{\theta}{2}) - i\sin(\tfrac{\theta}{2})\right) = \tfrac{1}{2}\cdot 2\cos(\tfrac{\theta}{2}) = \cos(\tfrac{\theta}{2})$$

So the probability that the quanton will exit the +z channel is

$$|\text{Quantum amplitude}|^2 = |\cos(\tfrac{\theta}{2})|^2 = \cos^2\tfrac{\theta}{2}$$

b) Explain physically why your results make sense

Notice that when a quanton moves from the second SG device to the third, it is not known which channel of the second SG device that a given quanton came out of. No measurement of the quanton's state is performed between the first and third device. This means that between the first and third SG device, the state of all quantons is preserved, so it is as if the second SG device were not there at all. We can verify this by calculating $|\langle +\theta|+z\rangle|^2 = \left|\begin{bmatrix}\cos\tfrac{\theta}{2}, & \sin\tfrac{\theta}{2}\end{bmatrix}\begin{bmatrix}1 \\ 0\end{bmatrix}\right|^2 = |\cos\tfrac{\theta}{2}|^2 = \cos^2\tfrac{\theta}{2}$

which is what we got.

C) Now suppose we block quantons emerging from the "−" channel of the second SG device so they don't go into the funnel before the third device. What is the probability for a quanton entering the second device exits the + channel of the third SG device?

We now know with certainty that the quanton coming out of the second SG device will have state $|+y\rangle$, so the superposition rule does not apply.

The quantum amplitude of the quanton being determined to leave out of the + port is thus

$\langle +\theta|+y\rangle\langle +y|+z\rangle = \left(\begin{bmatrix}\cos\frac{\theta}{2} & \sin\frac{\theta}{2}\end{bmatrix}\begin{bmatrix}\sqrt{\frac{1}{2}} \\ i\sqrt{\frac{1}{2}}\end{bmatrix}\right)\left(\begin{bmatrix}\sqrt{\frac{1}{2}} & -i\sqrt{\frac{1}{2}}\end{bmatrix}\begin{bmatrix}1 \\ 0\end{bmatrix}\right)$

$\Rightarrow \left(\cos\left(\frac{\theta}{2}\right)\sqrt{\frac{1}{2}} + i\sin\left(\frac{\theta}{2}\right)\sqrt{\frac{1}{2}}\right)\cdot\left(\sqrt{\frac{1}{2}}\right) = \frac{1}{2}\left(\cos\left(\frac{\theta}{2}\right) + i\sin\left(\frac{\theta}{2}\right)\right)$

Euler's formula tells us that $e^{i\theta} = \cos(\theta) + i\sin(\theta)$. So

$\frac{1}{2}e^{i\frac{\theta}{2}} = \frac{1}{2}\left(\cos\left(\frac{\theta}{2}\right) + i\sin\left(\frac{\theta}{2}\right)\right)$

So our quantum amplitude is $\frac{1}{2}e^{i\frac{\theta}{2}}$

The probability that the quanton will be determined to exit the + channel of the SGθ device is $\left|\frac{1}{2}e^{i\frac{\theta}{2}}\right|^2 = \frac{1}{2}e^{i\frac{\theta}{2}}\left(\frac{1}{2}e^{i\frac{\theta}{2}}\right)^* = \frac{1}{2}e^{i\frac{\theta}{2}}\cdot\frac{1}{2}e^{-i\frac{\theta}{2}} = \frac{1}{4}e^{i\left(\frac{\theta}{2}-\frac{\theta}{2}\right)} = \boxed{\frac{1}{4}}$

D) Explain physically why the latter result makes sense?

The state of a quanton leaving the second SG device is known with certainty, which collapses the previous state so that the quantum amplitude is dependent on θ only. Notice, however, that the form of the quantum amplitude is $e^{i\theta/2}$. The absolute square of $e^{i\theta/2}$ is 1 ALWAYS, so strangely enough, the PROBABILITY that the quanton exits the + channel of the third SG device is NOT dependent on the value of θ at all. This is very difficult to give a physical connotation. It simply is how it behaves, we just use the mathematical model to describe it. It makes ZERO sense physically.

Eval O' answers

Q10 M.9
Reasonable magnitude, right units

Q AB.9
Reasonable magnitude, right units

Q AM.2
Reasonable magnitude (but strange to think about), right units