# PHY 131 HW 8 Solutions

$\boxed{10:8}$ a) We integrate (with respect to time)

$\alpha = 6.0t^4 - 4.0t^2$ [rad/s²]

$$\omega(t) = \int_0^t \alpha \, dt = \int_0^t 6.0t^4 - 4.0t^2 \, dt$$

$$= \frac{6}{5}t^5 - \frac{4}{3}t^3 + C$$

given $\omega_0 = 2.0$ rad/s

$$\omega = 1.2t^5 - 1.3t^3 + 2.0 \quad [\text{rad/s}]$$

b) We integrate (with respect to time)

$\omega = 1.2t^5 - 1.3t^3 + 2.0$

$$\theta(t) = \int_0^t \omega \, dt = \int_0^t 1.2t^5 - 1.3t^3 + 2.0 \, dt$$

$$= \frac{1.2}{6}t^6 - \frac{1.3}{4}t^4 + 2.0t + C$$

$$\theta = 0.20t^6 - 0.33t^4 + 2.0t + 1.0 \quad [\text{rad}]$$

Given @ time $t=0$, $\theta = 1.0$ rad

$\boxed{10{:}28}$ First I'll put everything in SI units

$$\omega_0 = 150\left(\frac{\cancel{rev}}{\cancel{min}}\right)\left(\frac{1\ \cancel{min}}{60\,s}\right)\left(\frac{2\pi\,rad}{\cancel{rev}}\right) = 15.7\ rad/s$$

$$\Delta t = 2.2\ \cancel{h}\left(\frac{60\ \cancel{min}}{\cancel{h}}\right)\left(\frac{60\,s}{1\ \cancel{min}}\right) = 7920\ s$$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_0}{\Delta t} = \frac{0 - 15.7}{7920\ s}\left[\frac{rad}{s^2}\right]$$

a) $\alpha = -1.98\times10^{-3}\ s^{-2}$

b) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$$= (15.7)(7920\,s) + \tfrac{1}{2}(-1.98\times10^{-3})(7920\,s)^2$$

$\theta = 66{,}245\ rad$

$\# Rev = \frac{\theta}{2\pi} = 9.9\times10^3\ rev$

c) for $\omega = 75\ \frac{rev}{min} = 7.85\ rad/s$

for $r = 0.5\,m$ $\quad a_{tan} = \alpha r = (-1.98\times10^{-3})(0.5\,m)$

$a_t = -9.9\times10^{-4}\ m/s$

d) $a_r = a_{centrip} = \omega^2 r$

$$= (7.85\ rad/s)^2\,(0.5\,m)$$

$a_r = 30.8\ m/s^2$

Note $|a_{net}| = \sqrt{a_r^2 + a_t^2}$

but $a_r \gg a_t$ so

$|a_{net}| \approx 31\ m/s^2$

$\boxed{10:33}$

Calculate the rotational inertia of a wheel that has a kinetic energy of 24400 J when rotating at 602 rev/min.

$$K.E. = \frac{1}{2} I \omega^2 \implies I = \frac{2\,K.E.}{\omega^2}$$

$$= \frac{2\,(24400\,J)}{3974\ rad^2/s^2}$$

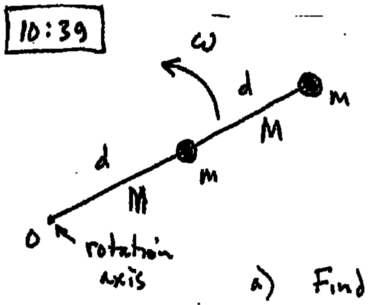
$$\omega = \frac{602\ \cancel{rev}}{\cancel{min}} \cdot \left(\frac{2\pi\ rad}{1\ \cancel{rev}}\right)\left(\frac{1\ \cancel{min}}{60\,s}\right)$$

$$= (63.0\ rad/s)$$

$$\therefore\ \omega^2 = 3974\ rad^2/s^2$$

$$I = \underline{\underline{12.3\ kg\,m^2}}$$

$\boxed{10:39}$




$d = 5.6$ cm

$M = 1.2$ kg

$m = 0.85$ kg

$\omega = 0.30$ rad/s

a) Find $I_{\text{about } O}$.

$$I = I_{\text{point masses } m} + I_{\text{rods}}$$

rods behave as a single rod of mass 2M and length 2d.

$$= md^2 + m(2d)^2 + \frac{1}{3}(2M)(2d)^2$$

$$= md^2(1+4) + \frac{1}{3}\cdot 8 \cdot M d^2$$

$$= 5md^2 + \frac{8}{3} M d^2$$

$$= d^2\left[5m + \frac{8}{3}M\right]$$

$$= \underbrace{(5.6\times10^{-2}\,m)^2}_{31.36\times10^{-4}}\left[\underbrace{\underbrace{5\cdot 0.85\ kg}_{4.25} + \underbrace{\frac{8}{3}\,1.2\,kg}_{3.2}}_{7.45}\right]$$

$$= 233.6\times10^{-4}\ kg\cdot m^2$$

$$I = \underline{0.02[illegible]3\ kg}\cdot m^2$$

b) Find its KE about the origin.

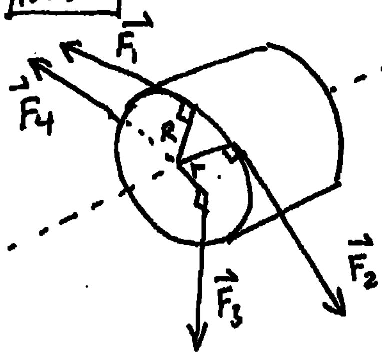
$$K.E. = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2}(0.0234\ kg\,m^2)(0.30\ rad/s)^2 = 0.0011\ J$$

$$= 1.1\times10^{-3}\ J$$

$$= \underline{\underline{1.1\ mJ}}$$

$\boxed{10{:}54}$




Solid Cylinder with mass $M = 2.0$ kg

$F_1 = 6.0\,N$ $\quad r = 5.0\,cm$

$F_2 = 4.0\,N$ $\quad R = 12\,cm$

$F_3 = 2.0\,N$

$F_4 = 5.0\,N$

Find a) magnitude, and b) direction of the angular acceleration of the cylinder

$F_1$ will rotate the cylinder CCW

$F_2$, $F_3$ will rotate the cylinder CW

$F_4$ has no effect because its ~~momentum~~ moment arm is zero.

Let's call a CW rotation positive. Then $\tau_1 = -R\,F_1 \sin 90^\circ$ ↑ indicates CCW

$\tau_2 = +R F_2 \sin 90^\circ$ CW

$\tau_3 = +r F_3 \sin 90^\circ$ CW

$\tau_4 = (0) F_4 = 0$

Net torque is thus

$$-(12\times10^{-2}\,m)(6.0\,N) + (12\times10^{-2}\,m)(4.0\,N) + (5\times10^{-2}\,m)(2.0\,N)$$

$\tau_{net} = -(14\times10^{-2})\,N\cdot m$ [neg. sign indicates actual torque is CCW or out of page]

It's a solid cylinder, thus its rotational inertia is $I = \frac{1}{2} M R^2$.

Now Newton's 2nd Law (rotational form) is

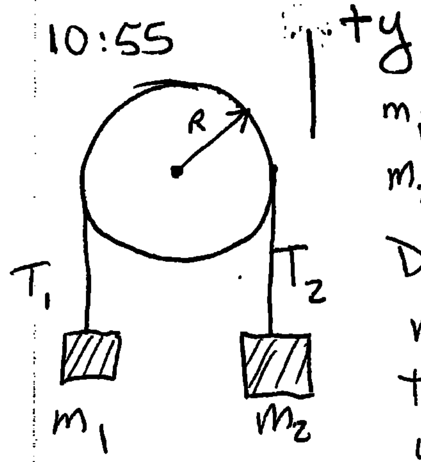
$$\sum \vec{\tau} = I\alpha$$

so $\vec{\alpha} = \dfrac{\vec{\tau}_{net}}{I} = \dfrac{-14\times10^{-2}}{\frac{1}{2}(2.0\,kg)(12\times10^{-2}\,m)^2} = \underline{\underline{-9.7\ rad/s^2}}$ where the minus sign indicates CCW (out of page)

a) $|\alpha| = 9.7$ rad/s

b) direction is CCW, or out-of-the-page by the RH rule

10:55




$m_1 = 460g = 0.460$ Kg
$m_2 = 500g = 0.500$ kg
Disk: $R = 5.00$ cm $= 0.05$ m
neglect friction of axle, BUT the disk is <u>NOT</u> massless and we must consider its rotational inertia $I$ when we consider the system as a whole.

$v_0, \omega_0 = 0$; at $t=0$, blocks released and $m_2$ drops $y_{2,f} = 75$ cm $= 0.75$ m in $t_f = 5$ s.
No slippage means that $m_1$ had to rise $y_{1,f} = 0.75$m and that 0.75m of the cord also moved around the rim of the disk, i.e. $s = R\theta = 0.75$ m (the arclength).

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a) What is $|\vec{a}|$, the linear acceleration of the blocks? From Kinematics $y = \frac{1}{2}at^2$

$$|\vec{a}| = \frac{2y}{t^2} = \frac{2(0.75m)}{(5\,s)^2} = 0.06 \text{ m/s}^2$$

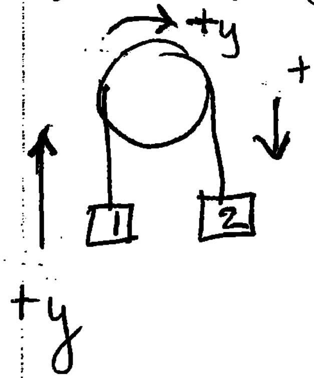
Block 2 moves down at this acceleration. Block 1 moves up with same acceleration. A point on the rim of the disk moves with that tangential acceleration. $a_T = \alpha R = 0.06$ m/s²

b) We write Newton's 2nd Law for each component of the system.

$m_1$: $\Sigma F = m_1 a$ $m_2$: $\Sigma F = m_2 a$ d: $\Sigma \tau = I\alpha$

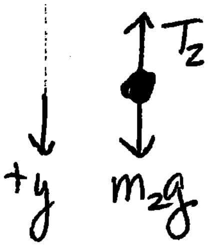
10:55 cont.

b) cont. (for block 1)




Note: since system moves @ $\vec{a}$ the $\oplus$ direction of motion must be the same for all components (regardless of RHR for rotational motion). Here I choose the given direction of motion as +y

$\Sigma F = m_2 a$

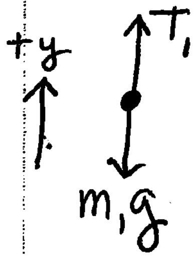



$m_2 g - T_2 = m_2 a$

$T_2 = m_2(g-a) = (0.5\ kg)(9.8\ m/s^2 - 0.06\ m/s^2)$

$T_2 = 4.87\ N$

c) ~~for~~ block 2




$\Sigma F = m_2 a$

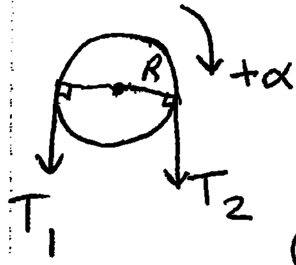
$T_1 - m_1 g = m_1 a$

$T_1 = m_1(g+a) = (0.46\ kg)(9.8 + .06)\ m/s^2$

$T_1 = 4.54\ N$

d) $\alpha_{DISK} = \frac{a_{TAN}}{R} = \frac{0.06\ m/s^2}{0.05\ m} = 1.20\ \frac{rad}{s^2} = \alpha$

e) for Disk: $\Sigma \tau = I\alpha$ ($\sin 90° = 1$)

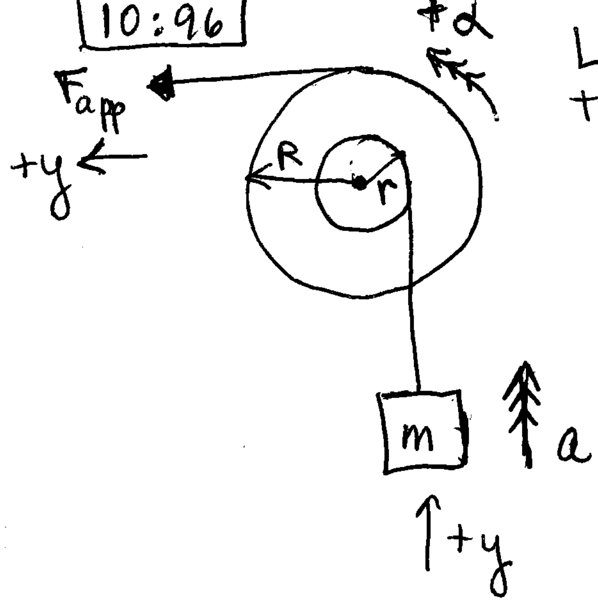



$R \times T_2 - R \times T_1 = I\alpha$

$\frac{R(T_2 - T_1)}{\alpha} = I = \frac{(0.05m)(4.87-4.54)N}{1.20\ rad/s^2}$

$I = 0.014\ kg \cdot m^2$

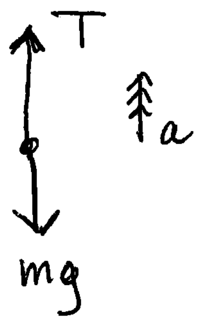
$\boxed{10:96}$




Let T be the tension in the rope

Linear motion: hanging mass has upward $\vec{a}$ : choose $+y$ dir therefore acceleration of disk $+\alpha$ is CCW and $F_{APP}$ points in $+y$

hanging mass




$\Sigma F = ma$

$T - mg = ma$

$T = mg + ma$

$T = (30\ kg)(9.8 + 0.8\ m/s^2)$

$T = 318\ N$

$\alpha = \frac{a}{r} = \frac{0.8}{0.2} = 4 s^{-2}$

(NOTE HANGING mass is attached to disk w/ r = .20m)

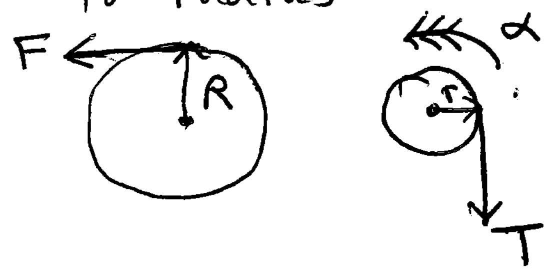
disk

$\Sigma \tau = I\alpha$

We don't know $I$

$\Sigma \tau = +\tau_{F_{APP}} - \tau_{\substack{HANG\\WT}} = I\alpha$

both forces are perpendicular to radius




$(\vec{R} \times \vec{F}_{APP}) - (\vec{r} \times \vec{T}) = I\alpha$

$RF \sin 90 - rT \sin 90 = I\alpha$

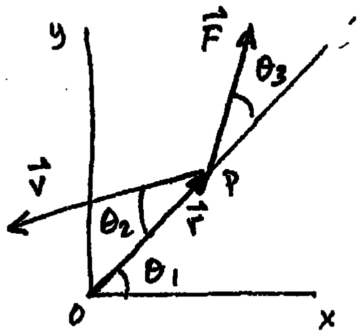
$I = \frac{RF - rT}{\alpha}$

$= \frac{(0.5m)(140N) - (0.2)(318N)}{4\ s^{-2}}$

$I = 1.6\ kg\ m^2$

# Chapter 11

$\boxed{11:28}$




$m = 2.0\ kg$ ← mass of particle P

$\vec{r}_o = 3.0 m$

$\theta_1 = 45°$

$|\vec{v}| = 4.0\ m/s\ ,\quad \theta_2 = 30°$

$|F| = 2.0 N\ ,\quad \theta_3 = 30°$ acts on P

All vectors lie in the xy plane

About the origin, what are

a) the magnitude and b) direction of the angular momentum of P.

$$\vec{l} = \vec{r} \times \vec{p} = rp \sin \phi = r\,(mv) \sin \theta_2$$

$$= (3.0\,m)(2.0\,kg)(4.0\,m/s)\left(\tfrac{1}{2}\right)$$

$$= \underline{\underline{12\ kg\ m^2/s}}$$

b) Direction is from the RH rule: it points <u>out</u> of the page

c) the magnitude and d) direction of the torque acting on P?

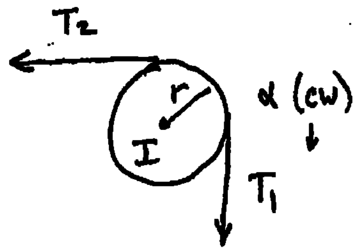
$$\vec{\tau} = \vec{r} \times \vec{F} = rF \sin \phi = r F \sin 30°$$

$$= (3.0\,m)(2.0\,N)(1/2)$$

$$= \underline{\underline{3.0\ N \cdot m}}$$

d) Direction is from RH rule: it points <u>out</u> of the page.

d) To find $T_2$, let's look at the net torque acting on the pulley:




Newton's $2^{nd}$: $\sum \vec{\tau} = I\vec{\alpha}$

$$rT_1 - rT_2 = I\alpha$$

$$\therefore\ T_2 = \frac{rT_1 - I\alpha}{r}$$

$$T_2 = T_1 - \frac{I\alpha}{r}$$

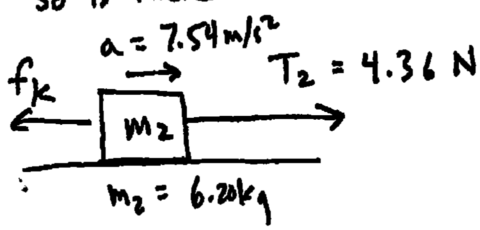
(note that this reduces to $T_2 = T_1$ if $I = 0$).

$$\text{so}\quad T_2 = 14.0\text{ N} - \underbrace{\frac{(7.40\times10^{-4})(314)}{2.40\times10^{-2}}}_{9.68\text{ N}}$$

$$\underline{\underline{T_2 = 4.36\text{ N}}}$$

<u>Optional</u> (& really embarrassing)

So is there friction between $m_2$ and the table?




$$T_2 - f_k = m_2 a$$

$$f_k = T_2 - m_2 a$$

$$= 4.36\text{ N} - (6.20\text{ kg})(7.54\text{ m/s}^2)$$

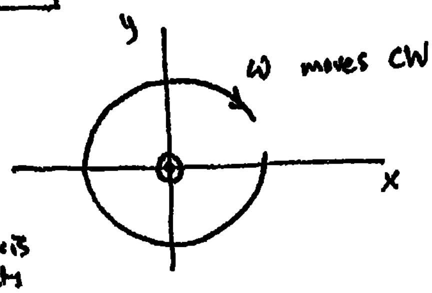
$$f_k = -42.4\text{ N}$$

Hmm. this suggests that the friction acts to the right, and so <u>increases</u> the acceleration by adding to $T_2$.

Looking back, we can see an obvious problem: How can a pair of equal masses, which we know in the limit of a massless pulley, with $m_2$ on a frictionless surface, and $m_1$ hanging down gives

$$a = \frac{m_1 g}{m_1 + m_1} = 4.9\text{ m/s}^2,$$ go <u>faster</u> if it has to put energy into the massive pulley? This must be a <u>very</u> special table top — it has the ability to exert a force to the right on the block!

$\boxed{11:32}$




z axis points up, so we are looking from the positive side of the z axis.

a) $\vec{\tau} = I\vec{\omega}$ and $\tau = \frac{d\ell}{dt}$

if $|\ell| = 4.0 \text{ kg m}^2/\text{s}$ then

$\frac{d\ell}{dt} = 0$ ($\ell$ has no time dependence)

Hence $\underline{\underline{\tau = 0}}$

b) $|\ell| = 4.0 t^2 \text{ kg m}^2/\text{s}$

$\frac{d\ell}{dt} = 2(4.0 t)$, hence $|\tau| = \underline{8.0 t} \text{ kg m}^2/\text{s}^2 (-\hat{k})$

<u>into the page</u> (or negative if you believe CCW motion is positive)

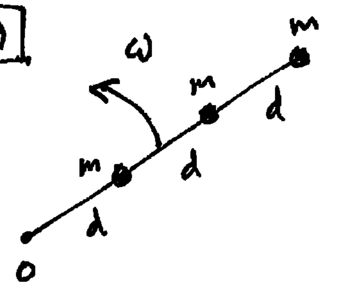
c) $\ell = 4.0\sqrt{t} \text{ kg m}^2/\text{s}$

$\frac{d\ell}{dt} = \frac{d}{dt}\left(4.0 t^{1/2}\right) = \underline{\underline{2.0 t^{-1/2}}} \text{ kg m}^2/\text{s}^2 (-\hat{k})$

d) $\ell = \frac{4.0}{t^2} \Rightarrow \frac{d\ell}{dt} = \frac{d}{dt}\left(4t^{-2}\right) = -8t^{-3} \text{ kg m}^2/\text{s}^2 (-\hat{k})$

$= \underline{\underline{+\frac{8}{t^3} \text{ kg m}^2/\text{s}^2 (+\hat{k})}}$

$\boxed{11:39}$




$m = 23 \text{ g}$
$d = 12 \text{ cm}$
massless rods
$\omega = 0.85 \text{ rad/s}$

a) Find $I$: 3 pt. masses, at $d$, $2d$, $3d$:

$I = md^2 + m(2d)^2 + m(3d)^2$

$= md^2(1+4+9) = 14md^2$

$= 14(23\times10^{-3})(12\times10^{-2})^2$

$= 4.6\times10^{-3} \text{ kg m}^2$

$\boxed{11:39 \text{ cont'd}}$

b) magnitude of angular momentum of middle particle:

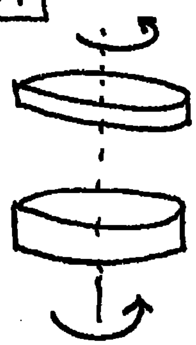
$$v = r\omega = (2d)\,\omega \qquad \left(\text{or } \ell = mr^2\omega = m(2d)^2\omega\right)$$

$$\text{so} \quad \ell = mvr = m(2d\omega)(2d)$$
$$= m\,4d^2\omega$$
$$= (23\times10^{-3}\ \text{kg})(4)(12\times10^{-2}\ \text{m})^2(0.85\ \text{rad/s})$$
$$= 1.1\times10^{-3}\ \text{kg m}^2/\text{s}$$

c) magnitude of angular momentum of the whole thing:

$$\ell = I\omega = (4.6\times10^{-3}\ \text{kg m}^2)(0.85\ \text{rad/s}) = 3.9\times10^{-3}\ \text{kg m}^2/\text{s}$$

$\boxed{11:47}$



$I_1 = 3.30\ \text{kg m}^2 \quad \omega_1 = 450\ \text{rev/min} \quad$ CCW

$I_2 = 6.60\ \text{kg m}^2 \quad \omega_2 = 900\ \text{rev/min} \quad$ CCW

a) They collide and stick together

$$L_{before} = I_1\omega_1 + I\omega_2$$
$$I_{after} = I_1 + I_2$$

Ang. Momentum is conserved (bearings have no friction and thus cannot exert an external torque)

$$L_{before} = L_{after}$$
$$I_1\omega_1 + I\omega_2 = I_{after}\,\omega_3$$
$$\therefore \quad \omega_3 = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{(3.3)(450) + (6.6)(900)}{9.9}$$
$$\omega_3 = \underline{\underline{750}}\ \text{rev/min}$$

$\boxed{11{:}47}$ cont.

b) If $L_2$ is CW, then it reverses sign:

$$L_{before} = L_{after}$$

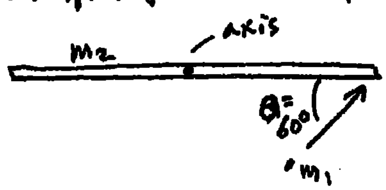
$$I_1\omega_1 - I_2\omega_2 = I_{total}\ \omega_3$$

$$\therefore\ \omega_3 = \frac{(3.3)(450) - 6.6\,(900)}{9.9} = -450 \text{ rev/min}$$

c) where the negative sign indicates that instead of being CCW, the rotation is $\underline{\text{CW}}$.

$\boxed{11{:}55}$ A uniform thin rod of length 0.500 m and mass 4.00 kg can rotate in a horizontal plane about a vertical axis through its center. The rod is at rest when a 3.00 g bullet traveling in the rotation plane is fired into one end of the rod at $\theta = 60°$.

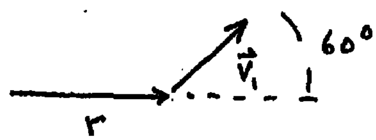
If the bullet lodges in the rod and the angular velocity of the rod is 10 rad/s immediately after the collision, what was the bullet's speed just before impact?




$$I_{rod} = \frac{1}{12} M_2 L^2$$

Conservation of angular momentum (no external torques).

$$\ell_{initial} = \vec{r} \times \vec{p} = r p \sin\phi = r m_1 v_1 \sin\phi$$




$$= \left(\frac{L}{2}\right) m_1 v_1 \frac{\sqrt{3}}{2}$$

$$\ell_{final} = I\omega_2 \quad \text{where} \quad I = I_{rod} + \underbrace{m_1\left(\frac{L}{2}\right)^2}$$

a piece due to the bullet at radius $R = L/2$.

These are equal, hence

$$I\omega_2 = \frac{L}{2} m_1 v_1 \frac{\sqrt{3}}{2}$$

$$\left(\frac{1}{12} M_2 L^2 + \frac{M_1}{4} L^2\right) \frac{\omega_2 \cdot 4}{\sqrt{3}\ L m_1} = v_1$$

$$\left[\frac{1}{12}(4.00\,\text{kg}) + \frac{3\times10^{-3}}{4}\right](0.50\,\text{m})^{\cancel{2}}\left(\frac{10\ \text{rad/s}}{\sqrt{3}}\right)\left(\frac{4}{\cancel{(0.50)}(3\times10^{-3})}\right) = v_1$$

$$\left[\frac{1}{3} + \underset{\text{(can ignore)}}{0.00075}\right]\left(\frac{20}{3\sqrt{3}\times10^{-3}}\right) = 1286 \text{ m/s}$$

$$= \underline{\underline{1.3\times10^{3}\ \text{m/s}}}$$