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Phy 232
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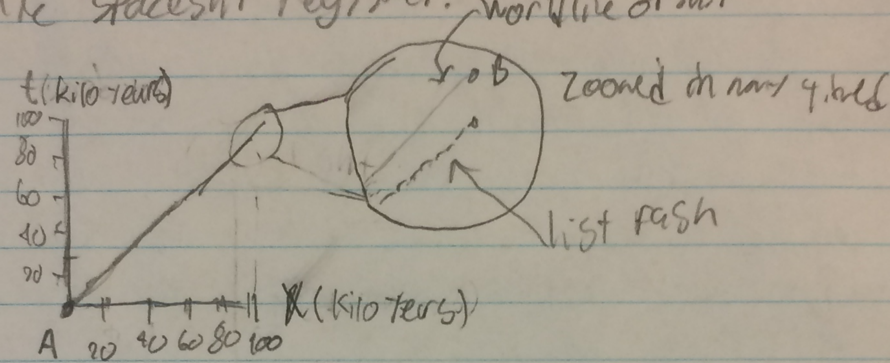
HW for 9/04/17
R3M.6, R3R.2, R3M.4

R3M.6

A spaceship travels from one end of the milky way galaxy to the other (a distance of about 100,000 y) at a constant velocity of magnitude $|\vec{v}| = 0.999$, as measured in the frame of the galaxy. How much time does a clock in the spaceship register?



Let event A be the ship leaving one side of the galaxy and event B being it's arrival at the other side of the galaxy. At a speed of 0.999, the coordinate time measured by the synchronized clocks on either end of the galaxy is $\Delta t = (100000y \div 0.999) - 0 = 100100.1001$ years (assuming that event A starts at $t=0$).

The time measured by a clock on the ship would be the spacetime interval between events A and B, as the spaceship is inertial and present at both events.

By the metric equation, $\Delta s = \sqrt{\Delta t^2 - |\Delta \vec{d}|^2} = \sqrt{(100100.1001\ years)^2 - (100,000y)^2}$

$\Rightarrow \Delta s = 4475$ years

So the clock on the spaceship would only measure a time of about 4475 years

R3R.2
Your boss is on the earth–Pluto shuttle, which travels at a constant velocity of 0.60 straight from earth to Pluto, a distance of 5.0 h in an inertial frame attached to the sun. An hour into the flight (according to your boss's watch) your boss sends a laser message to you on earth, asking you to send a wake-up call appropriately timed so that your boss can catch a 1 hour nap (as measured on your bosses watch). You immediately reply with the wake-up call and an apology that the call is late, claiming in your defence that the laws of physics prevent a timely response.

b) How long does your boss sleep (according to your boss's watch) before your message was received?

According to the watch of the boss, 1 hour had elapsed. Because the flight is inertial, we can say that the spacetime interval between when the boss's flight leaves (A) and when he sends the signal (B) is 1 hour. By the metric equation,

$1 \text{ hour} = \Delta t^2 - |\Delta \vec{d}|^2 = \left(\frac{|\Delta \vec{d}|}{0.6}\right)^2 - |\Delta \vec{d}|^2 = \left(\frac{1}{0.6^2} - 1\right)|\Delta \vec{d}|^2$

So $|\Delta \vec{d}| = \sqrt{\frac{1}{\left(\frac{1}{0.6^2} - 1\right)}} = 0.75$ hours.

A=(0,0)

B=(4.25,1.25)

This means the boss has travelled 0.75 of the 5 hours between earth and Pluto. The coordinate time between A and B is thus $\Delta t = \frac{0.75 \text{ hours}}{0.6} - 0 = 1.25$ hours (Assuming that $t = 0$ at A).

At a speed of 1, the signal sent by the boss will reach earth (event C) in $(5 - 0.75)$ hours $= 4.25$ hours, so the coordinate time between A and C is $1.25 + 4.25 = 5.5$ hrs

C=(1.7, 5.5)

The boss has travelled an additional $0.6 \cdot 4.25$ hrs $= 2.55$ hours, bringing the distance the boss has travelled once the message is recieved to be $2.55 \text{ hours} + 0.75 \text{ hours} = 3.3$ hours

Once the wakeup call is sent, it will travel at a speed of 1.
To determine when and where the ship recieves it (D), we need the equations of the world lines.
The world line of the wakeup call has the equation $(t - 5.5) = 1(x - 0)$
$\Rightarrow t = x + 5.5$

The worldline of the ship has equation $(t-5.5)=\frac{1}{-0.6}(X-1.7)$

$\Rightarrow t=\frac{x}{-0.6}+\frac{25}{3}$

Setting these equal to each other will give us the X coordinate that the ship recieves the wakeup call:

$$x+5.5=\frac{x}{-0.6}+\frac{25}{3} \Rightarrow \frac{0.6x+x}{0.6}=2.83 \Rightarrow x=\frac{4.5\cdot0.6}{(0.6+1)}=1.0625\text{ hours}$$

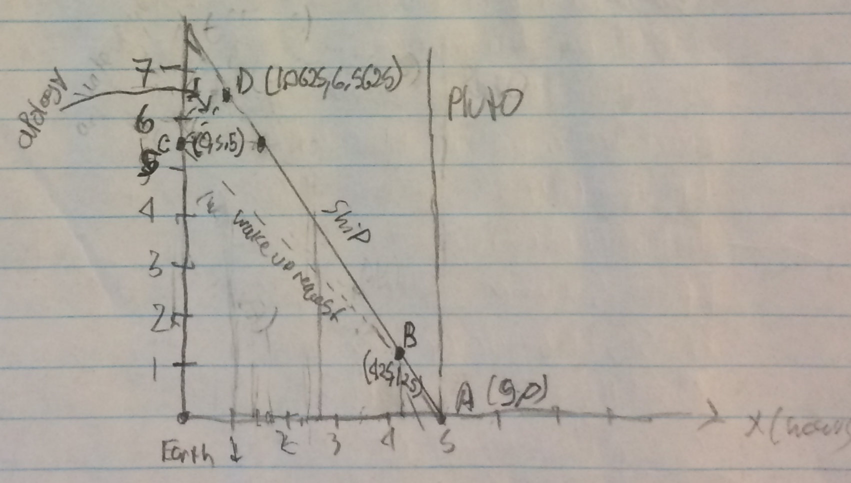
So the coordinate of the bottom A and D is

D=(1.0625, 6.5625) $1.0625+5.5=6.5625$ hours

From the bosses perspective, the elapsed time between B and D (the time which he sleeps)

$$\Delta S=\sqrt{(6.5625-1.25)^2-(1.0625-4.25)^2}=4.25\text{ hrs}$$

So the boss sleeps for 4.25 hours



a) Why was the boss's request impossible?

1 hour of boss time is equivalent to 1.25 hours of earth time.

The message takes 4.25 earth hours to be recieved, after which the boss will have already slept $\frac{4.25}{1.25}=3.4$ hours so his request is impossible.

R3MA

Suppose you and a friend are riding in trains that move relative to each other at relativistic speeds. As you pass each other, you both measure the time separation and spatial separation of two firecracker explosions that occur on the tracks between you. [You can measure the latter by measuring the distance between the scorch marks that the explosions leave on your side of ~~the~~ train]. You find the firecracker explosions to be separated by 1 μs of time and 0.40 μs of distance in your frame. By radio, your friend reports that the explosions were separated by only 0.60 μs of time in your friend's frame. Is this possible? If it is, find the spatial separation of the events in your friend's frame. If not, explain why not.

Assuming that both frames are relativistic, event A (firecracker 1) and event B (firecracker 2) must have a given spacetime interval, Δs, between them. Seeing that the distance between the firecrackers is measured by their scorch marks, then in my frame at least, the time measured is a coordinate time (measured by two synchronized clocks at two different events). Therefore, the spacetime interval is $\Delta s = \sqrt{\Delta t^2 - |\Delta \vec{d}|^2} = \sqrt{1\mu s^2 - (0.4\mu s)^2} = \sqrt{0.84}\ \mu s$.

If the situation above is possible, we should get a real value for $|\Delta \vec{d}'|$ by using the metric equation with the time measured by the friend.

$0.84\ \mu s^2 = (0.6\mu s)^2 - |\Delta \vec{d}'|^2 \Rightarrow |\Delta \vec{d}'|^2 = (0.6\mu s)^2 - (0.84\mu s) = -0.48\mu s.$

So $\Delta \vec{d}'$ is imaginary, and the situation is impossible.

In other words, there is no real $|\Delta \vec{d}'|$ that will give the same spacetime interval (with the time measured by the friend) as the spacetime interval calculated using my measurements. Because spacetime intervals are absolute (they do not depend on the inertial frame they are measured in), this situation is impossible.

Evaluation of answers

R3M.6

When I initially did this problem, I realized that I used metric equation

doesn't exist [$\Delta s = \Delta t - |\Delta \vec{d}|$] instead of $\Delta s^2 = \Delta t^2 - |\Delta \vec{d}|^2$]

After reading over it, I realized my mistake and calculated a more reasonable value of 4175 years (as opposed to my initial answer of 100 years). This seems more reasonable as an answer of 100 years at only .9999c might imply that C can be reached by massive objects.

R3R.2

I made the same mistake as in R3M.6, and correcting it was not fun. My overall answer did not change, just the numbers. A value of 4.25 hours seems reasonable as it is less than the coordinate time on earth (as it should be due to relativistic effects). It is as if the less thinks that the metric equation is $\Delta s = \Delta t - |\Delta \vec{d}|$ as opposed to $\Delta s^2 = \Delta t^2 - |\Delta \vec{d}|^2$

R3M.4

This was a tricky one. I decided that the situation was impossible because assuming it was true gave an imaginary $|\Delta \vec{d}|$