Preparation for November 17

Last time, we looked at a competing species model for population growth:

$$x' = x(1.5 - x - .5y)$$
$$y' = y(2 - .5y - 1.5x)$$

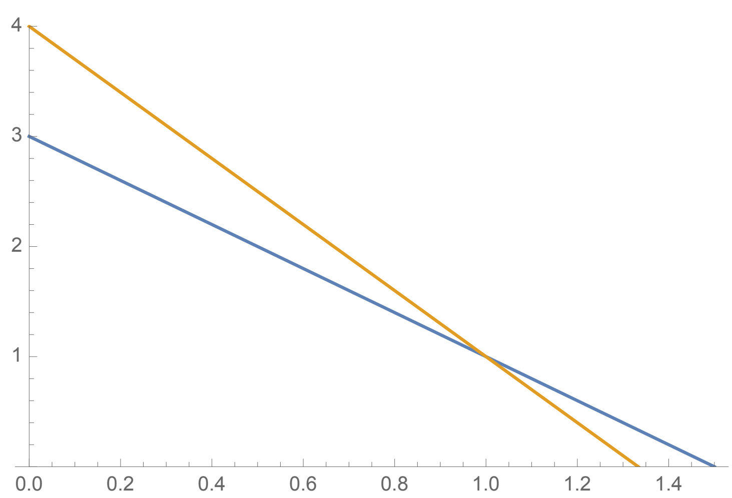
We found that an equilibrium with positive populations of both species at $(1, 1)$. To classify the type of equilibrium, we looked at the Jacobian at $(1, 1)$ and used the eigenvalues to predict that $(1, 1)$ is an unstable saddle.

There is another method to this kind of problem – we can look at *nullclines*. A nullcline is a line along which one of the populations would not be changing.

For example, if we set $x' = 0$, we get $x(1.5 - x - .5y) = 0$, so either $x = 0$ or $1.5 - x - .5y = 0$. Thus, we get two nullclines: the y-axis, and the line $x + .5y = 1.5$.

If we set $y' = 0$, we get $y(2 - .5y - 1.5x) = 0$, so either $y = 0$ or $2 - .5y - 1.5x = 0$. Thus, we get two nullclines: the x-axis, and the line $1.5x + .5y = 2$.

These nullclines are plotted below. The blue line, which passes through $(0, 3)$ and $(1.5, 0)$, is the line where $x' = 0$. The orange line, which passes through $(0, 4)$ and $(4/3, 0)$, is the line where $y' = 0$.



A solution curve must pass through the blue line vertically, since along the blue line, $x' = 0$, so there is no horizontal movement. Similarly, a solution curve must pass through the orange line horizontally, since along the orange line, $y' = 0$, so there is no vertical movement.

To the right of the blue line, the x-value is larger, so the quantity $1.5 - x - .5y$ becomes negative. That is, to the right of the blue line, $x' < 0$. Similarly, to the right of the orange line, $y' < 0$. Thus, in the upper right, all solution curves are moving down (since $y' < 0$) and to the left (since $x' < 0$). By a similar argument, in the bottom left, all the solution curves are moving up and to the right.

In the top region between the two nullclines, we are to the right of the blue line, and the left of the orange line, so $x' < 0$ and $y' > 0$. Thus, in that region, the solutions are moving to the left and up. In the tiny piece at the bottom between the two nullclines, solutions are moving down and to the right.

We can use the above three paragraphs to create a rough sketch of a phase portrait, from which we can deduce that the equilibrium point $(1, 1)$ is an unstable saddle.