

Electromagnetism and Electromagnetic Waves Summary

One result of the magnetic (Lorentz) force was that moving a conductor through a magnetic field caused the free charges in that conductor to move. It turned out that this did not depend on the nature or quantity of these charges (within reason) in the sense that the voltage developed on a moving length ℓ of wire moving perpendicular to a magnetic field B at a velocity v developed a voltage ΔV along its length of

$$\Delta V = \ell v B .$$

For a complete loop of wire, this voltage was counteracted by an opposite voltage at the other end of the loop, unless the magnetic field was not uniform. Then the voltage developed was given by Faraday's law of induction

$$\Delta V = - \frac{d\Phi_B}{dt}$$

where Φ_B is the magnetic flux defined by

$$\Phi_B = \int \vec{B} \cdot d\vec{a} .$$

We found that the currents and forces that resulted from these flux changes were of a character that resisted the flux changes, which is known as Lenz's law. (The minus sign in Faraday's law is a reminder for us of this opposition to changes in flux.) Forces resisted motion, currents were developed that added to the flux for decreasing flux cases, and *vice versa* for increasing cases. We investigated this first in the form of inductance caused by permanent magnets, and then mutual inductance, where one coil produces a changing field and another exhibits the induced voltage. If the geometry is unchanging, one can define the mutual inductance M such that

$$\Delta V_2 = M \frac{dI_1}{dt} .$$

This relationship is symmetric; interchanging the roles of the two loops gives the same constant M . We also found that a single coil could induce a voltage in itself. In this case, we defined an inductance L such that

$$\Delta V = -L \frac{dI}{dt} .$$

The minus sign reminds us of Lenz's law.

Now we have determined that changing magnetic fields produce electric fields. We made a symmetry argument that suggested we should repair Ampere's law slightly to add the possibility of changing magnetic fields producing electric fields. This took the form of

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \left(I + \epsilon_o \frac{d\Phi_E}{dt} \right) = \mu_o I + \mu_o \epsilon_o \frac{d\Phi_E}{dt} .$$

These symmetric properties give rise to the ability of electric and magnetic fields to sustain themselves in the form of a wave. We looked at one example:

$$E_x = A_E \sin(kz - \omega t) \text{ and } B_y = A_B \sin(kz - \omega t) ,$$

where the velocity of the wave is given by

$$v = c = \frac{\omega}{k} = 3 \times 10^8 \text{ m/sec} .$$

We finally took a brief look at light and lenses, where we discovered the lens equation

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f},$$

and the image magnification relationship

$$M = \frac{d_2}{d_1}.$$

We also had a brief encounter with Snell's law, which tells us how light bends when going from one medium to another:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where n is the index of refraction of the medium, and θ is the angle of the light beam relative to a vector perpendicular to the surface. The index of refraction describes the extent to which an electromagnetic wave is slowed in a medium ($v = c/n$), resulting in a shorter wavelength ($\lambda = \lambda_0/n$).

We also used diffraction to measure the wavelength of light, using a diffraction formula from first semester;

$$d \sin \theta_m = m \lambda$$

which gives the angles for constructive interference, where d is the spacing between adjacent slits, and m is the order of the maximum (zero being the center, ± 1 adjacent to the center, ± 2 next, etc.).