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Phy 232
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HW RCA 8/30/17
R1M.7, R1M.19 R2M.3

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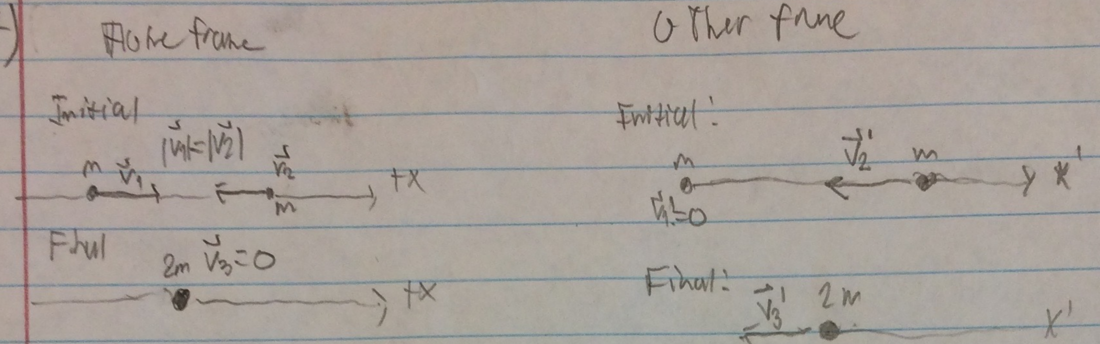
R1M.7)



Statement: The figure above shows an inelastic collision between two blobs as observed in two different inertial reference frames. Assume the galilean transformation equations are true.

(a) Which frame is the Home frame, according to the convention established in this chapter? What is the sign of β? Explain your reasoning.

According to the convention established in this chapter, which is that the Home frame and Other frame are distinguished by adding a "prime" to variables in the Other frame so as to signify the constant velocity $\vec{\beta}$ that the Other frame moves at with respect to the Home frame. The presence of primes on variables in the frame on the right identifies that frame as the Other frame, and so the frame on the left must be the Home frame. The sign of β depends on the frame being observed; in the Home frame, the Other frame moves at a velocity $+\beta\uparrow$, and in the Other frame, the Home frame moves at a velocity $-\beta\uparrow$.

(b) What is V_{2x}' in terms of $|\vec{V_1}|$?

Because the galilean transformation equations are true, we can say that $\vec{V}'=\vec{V}-\vec{\beta}$. According to the Home frame, we can deduce that $-\vec{V_1}=\vec{V_2}$. In the Other frame $\vec{V_1}'$ initially has a value of 0. $\vec{V_1}'=\vec{V_1}-\beta \Rightarrow 0=\vec{V_1}-\vec{\beta} \Rightarrow \vec{\beta}=\vec{V_1}$

Using the fact that $\vec{\beta}=\vec{V_1}$, $\vec{V_2}'=\vec{V_2}-\vec{V_1}=-\vec{V_1}-\vec{V_1}=-2\vec{V_1}$.

All motion in the frames are along the x-axis. Therefore, $V_{2x}'=-2|\vec{V_1}|$

(c) What is V'_{3x} in terms of $|\vec{V_1}|$?

Using the fact that $\vec{\beta} = \vec{V_1}$, $\vec{V_3'} = \vec{V_3} - \vec{V_1} \Rightarrow \vec{V_3'} = 0 - \vec{V_1}$

Since all motion is along the x axis, we can say that $V'_{3x} = -V_{1x} = -|\vec{V_1}|$

Therefore, $V'_{3x} = -|\vec{V_1}|$

(d) What is the system's total x-momentum in the Home frame, both initially and finally, in terms of $m|\vec{V_1}|$?

We know $-\vec{V_1} = \vec{V_2}$

Initial momentum is $m\vec{V_1} + m\vec{V_2} \Rightarrow m\vec{V_1} - m\vec{V_1} = m|\vec{V_1}| - m|\vec{V_1}| = 0$

$\vec{P}_{initial} = \vec{P}_{final}$

Therefore, $\vec{P}_{initial} = m|\vec{V_1}| - m|\vec{V_1}| = 0$

$P_{final} = m|\vec{V_1}| - m|\vec{V_1}| = 0$

(e) What is the system's total x momentum in the other frame, both initially and finally, in terms of $m|\vec{V_1}|$?

We know $\vec{V_1'} = 0$, $\vec{V_2'} = -2|\vec{V_1}|$

So $\vec{P}'_{initial} = m \cdot \vec{V_1'} + m \cdot \vec{V_2'} = -2m|\vec{V_1}|$

$\vec{P}_{final} = M_{final} \cdot \vec{V}_{final}$, $\vec{V_3'} = -|\vec{V_1}|$

So $\vec{P}_{final} = -2m|\vec{V_1}|$

Therefore, $\vec{P}'_{initial} = -2m|\vec{V_1}|$

$\vec{P}'_{final} = -2m|\vec{V_1}|$

(f) Is ~~momentum~~ conserved in the Home frame?

Yes, because $\vec{P_i} = \vec{P_f}$

(g) Is momentum conserved in the other frame?

Yes, because $\vec{P_i'} = \vec{P_f'}$

(h) Assuming the energy is conserved in the home frame, show that it is also conserved in the other frame.

$$\Delta K = K_f - K_i = \frac{1}{2} 2m \cdot 0\,m/s - \left(\frac{1}{2} \cdot m(\vec{v}_1)^2 + \frac{1}{2} m \cdot (-\vec{v}_1)^2\right)$$
$$\Rightarrow 0 - m(\vec{v}_1)^2 = -m(\vec{v}_1)^2$$

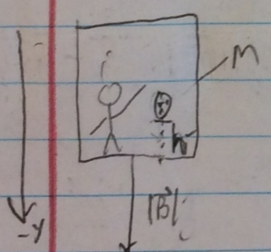
Because energy is conserved, $\Delta E = \Delta K + \Delta U = 0$. So ΔU must be equal to $m(\vec{v}_1)^2$ which is the amount of thermal energy released when the blobs collide.

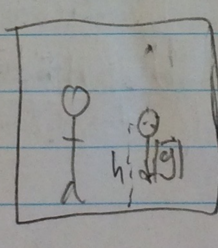
$$\Delta K' = K'_f - K'_i = \frac{1}{2} 2m \cdot (-\vec{v}_1)^2 - \left(\frac{1}{2} \cdot m \cdot 0\,m/s + \frac{1}{2} \cdot m \cdot (-2\vec{v}_1)^2\right)$$
$$\Rightarrow m\vec{v}_1^{\,2} - \left(0\,m/s + \frac{1}{2} 4m\vec{v}_1^{\,2}\right)$$
$$\Rightarrow m\vec{v}_1^{\,2} - 2m\vec{v}_1^{\,2} = -m\vec{v}_1^{\,2}$$

$\Delta K = \Delta K'$, so $\Delta U' = \Delta U$

Therefore, $\Delta E' = 0$ (energy is conserved in the other frame)

R1M.10) A person in an elevator drops a ball of mass m from rest from a height h above the elevator floor. The elevator is moving at a constant speed $|\vec{\beta}|$ downward with respect to its enclosed building.





$h = \frac{1}{2}\vec{g} \cdot t^2 \rightarrow t = \sqrt{\frac{2h}{g}}$

(a) How far will the ball fall in the building frame before it hits the floor? $h = \frac{1}{2} \cdot |\vec{g}| \cdot t^2 \Rightarrow t = \sqrt{\frac{2h}{|\vec{g}|}}$

$$h_{fall}(t) = \left(\frac{1}{2}|\vec{g}|t^2 + |\vec{\beta}|t\right)\hat{j},\ \Rightarrow h_{fall} = \left(\frac{1}{2}|\vec{g}|\frac{2h}{|\vec{g}|} + |\vec{\beta}|\sqrt{\frac{2h}{|\vec{g}|}}\right) - \hat{j}$$

$\Rightarrow \boxed{h_{fall} = h + |\vec{\beta}|\sqrt{\frac{2h}{g}}}$

(b) What is the ball's initial vertical velocity in the building frame?

$-|\vec{\beta}|\hat{j}$

(c) Use the law of conservation of energy in the building frame to compute the ball's final speed (as measured in that frame) just before it hits the elevator floor.

$\Delta U = \Delta K \Rightarrow \cancel{m}|\vec{g}|\,h_{ball} = \frac{1}{2}\cdot\cancel{m}\cdot\left(|\vec{v}_f|^2 - (-|\vec{\beta}|)^2\right)$

$\Rightarrow |\vec{g}|\cdot h_{ball} = \frac{1}{2}\left(|\vec{v}_f|^2 - |\vec{\beta}|^2\right) \Rightarrow 2|\vec{g}|\,h_{ball} + |\vec{\beta}|^2 = |\vec{v}_f|^2$

$\Rightarrow 2|\vec{g}|h + 2|\vec{g}|\,|\vec{\beta}|\sqrt{\frac{2h}{|\vec{g}|}} + |\vec{\beta}|^2 = |\vec{v}_f|^2 = \left(|\vec{\beta}| + \sqrt{2|\vec{g}|h}\right)^2$

So $\boxed{\hat{j}\,|\vec{v}_f| = -\hat{j}\left(|\vec{\beta}| + \sqrt{2|\vec{g}|h}\right)}$

(d) Use the galilean velocity transformation equations and the result of part (c) to find the ball's final speed in the elevator frame.

$\vec{v}' = \vec{v} - \vec{\beta} \Rightarrow \boxed{\vec{v}_f' = -\hat{j}|\vec{\beta}| - \hat{j}\sqrt{2|\vec{g}|h} + |\vec{\beta}|\hat{j} = -\hat{j}\sqrt{2|\vec{g}|h}}$

$\boxed{\vec{v}_f' = -\hat{j}\sqrt{2|\vec{g}|h}}$

(e) Assume that the conservation of energy applies in the building frame, use the result of part (d) and the fact that the ball's acceleration is $|\vec{g}|$ to show that energy is also conserved in the elevator frame.

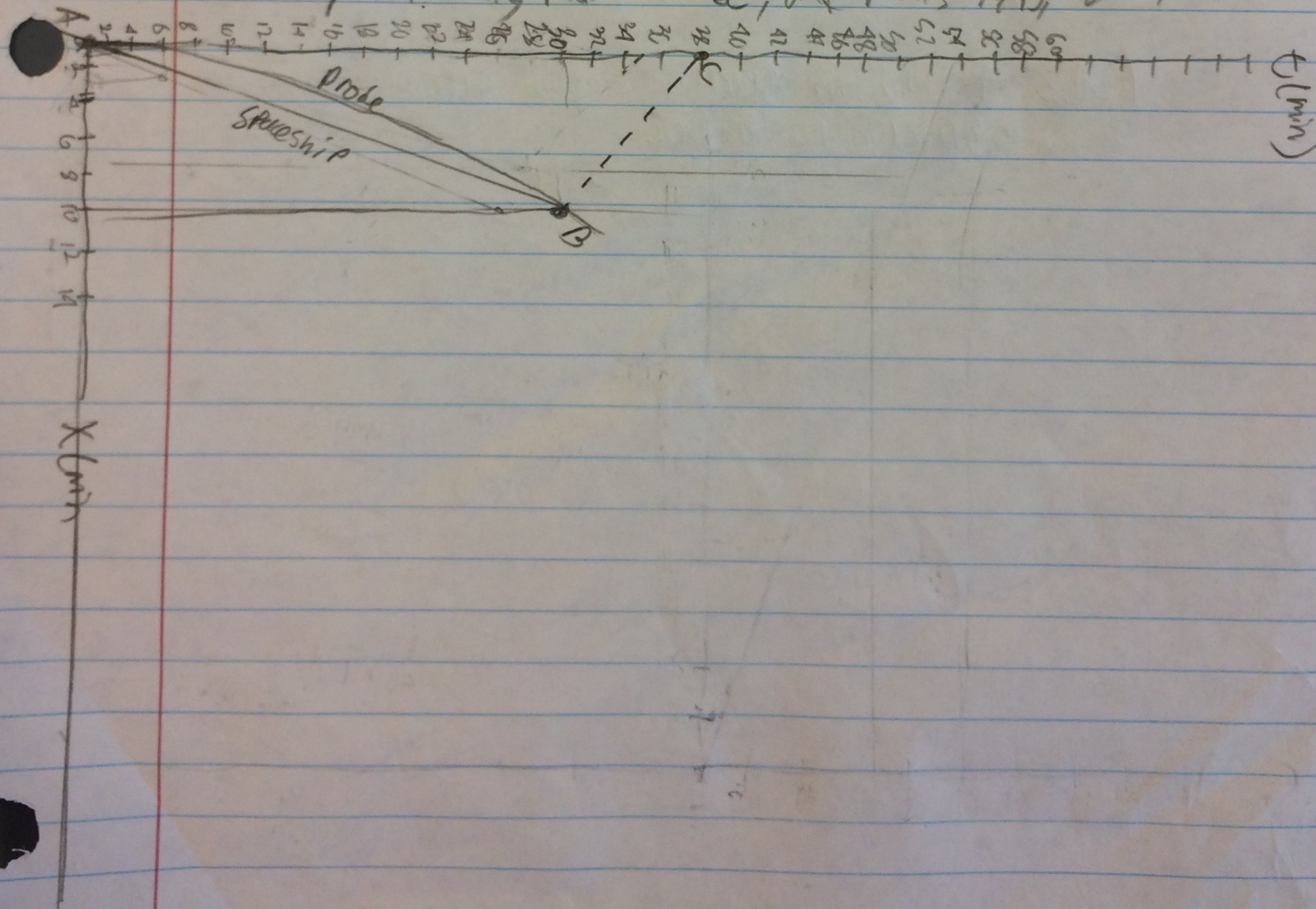
If energy is conserved in the elevator frame then $\Delta K' = \Delta U'$

$\cancel{m}\cdot g\cdot h \overset{?}{=} \frac{1}{2}\cdot\cancel{m}\cdot\left(|\vec{v}_f'|^2 - |\vec{v}_0'|^2\right) \Rightarrow 2|\vec{g}|h \overset{?}{=} |\vec{v}_f'|^2 - |\vec{v}_0'|^2 \Rightarrow 2|\vec{g}|h = \left(\sqrt{2|\vec{g}|h}\right)^2$

$\Rightarrow 2|\vec{g}|h = 2|\vec{g}|h$, so, $\boxed{\Delta K' = \Delta U' \text{ and energy is conserved}}$

$+|\vec{\beta}|^2$

R2M.3) An alien spaceship travelling at a constant velocity of $\frac{1}{3}$ in the $+x$ direction passes the earth (call this event A) at time $t=0$. Just as the spaceship passes people on the earth launch a probe, which accelerates from rest towards the space ship at such a rate that it catches up to and passes the alien spaceship (call this event B). when both are 10min of distance from earth. As it passes the alien spaceship the probe takes a photo and sends it back to earth as an encoded radio message at the speed of light. The message reaches earth at event C. Draw a quantitatively accurate spacetime diagram of the situation (as observed in a frame attached to earth, with the earth at $x=0$) that clearly shows the worldlines of earth, the spaceship, the probe, the returning radio message, and events A, B, and C.



Evaluations of answers

R1M.7) I find it plausible that momentum is conserved in both the home frame and the other frame, seeing as they are both inertial due to the principal of relativity.

R1M.10) In part (a), I calculated the time it would take for the ball to fall a height h in the elevator frame, and used that time as the value for my time variable to find the height the ball fell in the building frame; I found that energy was conserved in the elevator frame on the assumption that it was conserved in the building frame, which should be true based on my assumption. I cannot verify the validity of the assumption because neither the building frame nor the elevator frame are inertial.

R2M.3) I think my spacetime diagram accurately represents the events posed in the problem