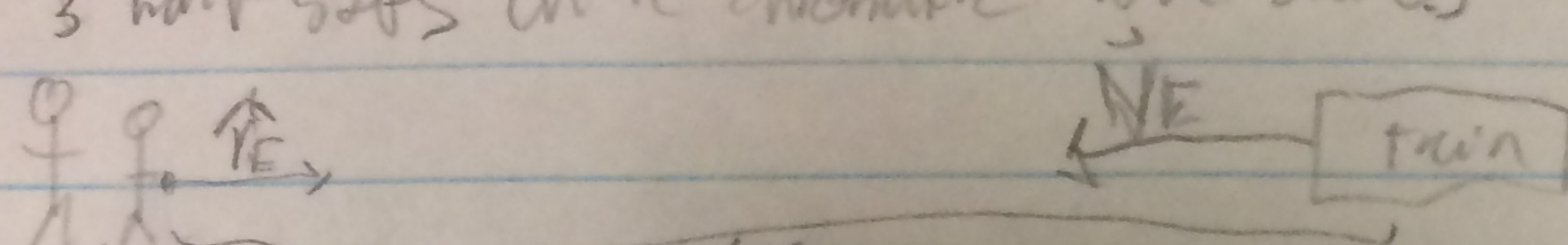
Charles Cunningham

Q1R.2

You and a friend are walking along some railroad tracks one at night. You recently heard a train blow its whistle while at rest at a station 2km away. You again hear the whistle as the train passes an intersection you know to be about 600m away. Your friend says "you know, that whistle sounds about a minor third sharp." About how much time do you have to get safely off the tracks? (minor third is 3 half steps on the chromatic music scale)



A half step on the chromatic music scale has a frequency $2^{1/12}$ times higher than the frequency below it.
According to my friend, the frequency of the train whistle f_D sounds 3 half steps above what it did when we first heard it, that is, $f_D = (2^{1/12})^3 \cdot f_0$.

At a speed $|\vec{V}_E|$, frequency of the train's whistle as heard by us is

$$f_D = \frac{f_0}{1 - \frac{|\vec{V}_E|}{|\vec{V}_W|}}$$, where $|\vec{V}_W|$ is the speed of sound in air (343 m/s)

Solving for $|\vec{V}_E|$, $f_D - \frac{f_D|\vec{V}_E|}{|\vec{V}_W|} = f_0$

$\Rightarrow \frac{f_D - f_0}{f_D} \cdot |\vec{V}_W| = |\vec{V}_E|$

but $f_D = (2^{1/12})^3 f_0$, so we get $\frac{(2^{1/12})^3 f_0 - f_0}{(2^{1/12})^3 f_0} \cdot |\vec{V}_W| = |\vec{V}_E|$

$\Rightarrow \frac{2^{1/4} - 1}{2^{1/4}} \cdot |\vec{V}_W| = |\vec{V}_E|$. So $|\vec{V}_E| = 0.1591 \cdot |\vec{V}_W| = 54.6 \text{ m/s}$

At a distance of 600m, it took the sound of the whistle $\frac{600m}{343 m/s} = 1.75s$ to reach us. During that time, the train travelled $1.75s \cdot 54.6 m/s = 95.5 m$ leaving we have $\frac{600m - 95.5m}{54.6 m/s} = \boxed{9.2 \text{ seconds to get off the tracks}}$

Q2B.1

Consider the triangle shaped waves shown in the drawing printed out and attached. Each wave moves with a speed of 5 cm/s in the direction indicated. Draw separate graphs showing what the superposition principle implies that the combined waves should look like at 2s, 3s, 4s, and 6s

We can use the superposition principle to find the function that will help us graph the waves.

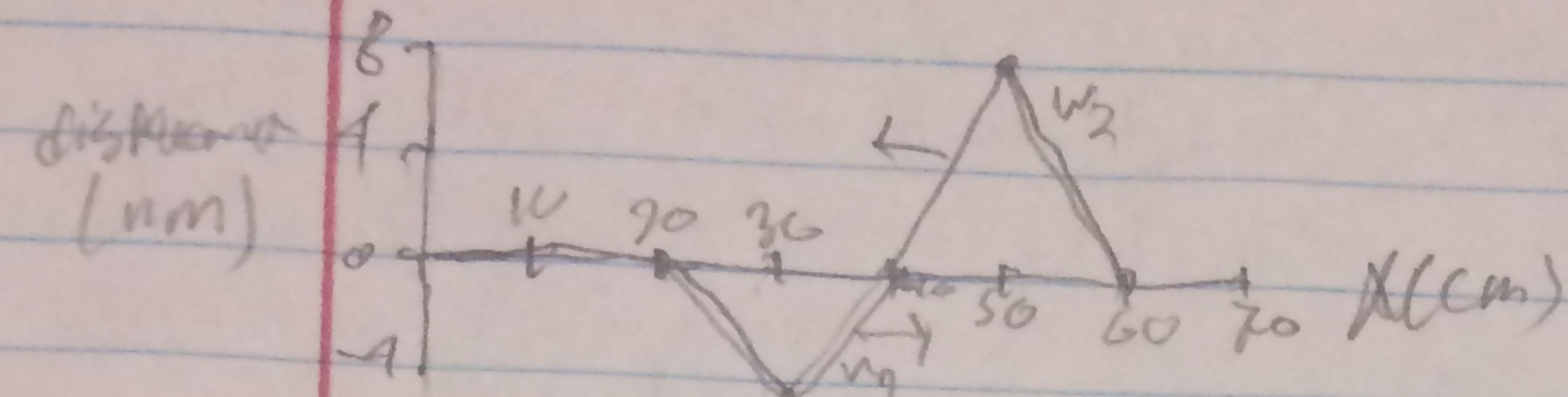
Let the wave traveling in the +x direction be $W_1(x,t)$
Let the wave travelling in the -x direction be $W_2(x,t)$.

W_1 will be of the form $W(x,t) = |m(x - x_0 - 5t)| + W_0$
For W_1, we can see that $m = \frac{4}{30-20} = \frac{4}{10} = \frac{2}{5}$.
$x_0 = 20$, and $W_0 = -4$, so $W_1(x,t) = |\frac{2}{5}(x - 20 - 5t)| - 4$
which simplifies to $W_1(x,t) = |\frac{2}{5}x - 8 - 2t| - 4$

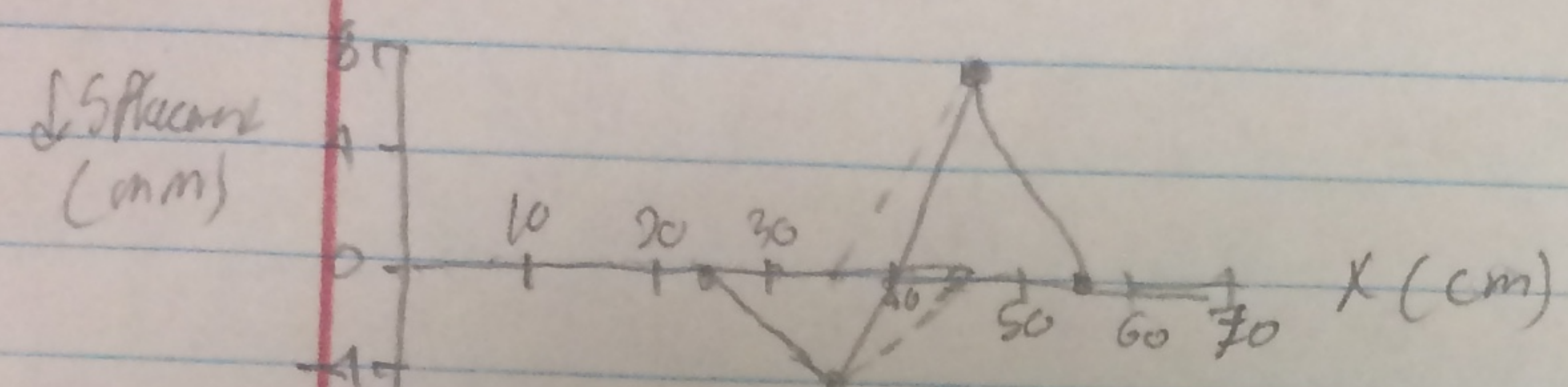
The W_2 will be of the form $W(x,t) = W_0 - |m(x - x_0 + 5t)|$
So the $m = \frac{8-0}{60-50} = \frac{8}{10} = \frac{4}{5}$. $x_0 = 60$, $W_0 = 8$.
So $W_2(x,t) = 8 - |\frac{4}{5}(x - 60 + 5t)|$ which simplifies to
$W_2(x,t) = 8 - |\frac{4}{5}x - 48 + 4t|$

When adding these two functions together, we will have to be careful to

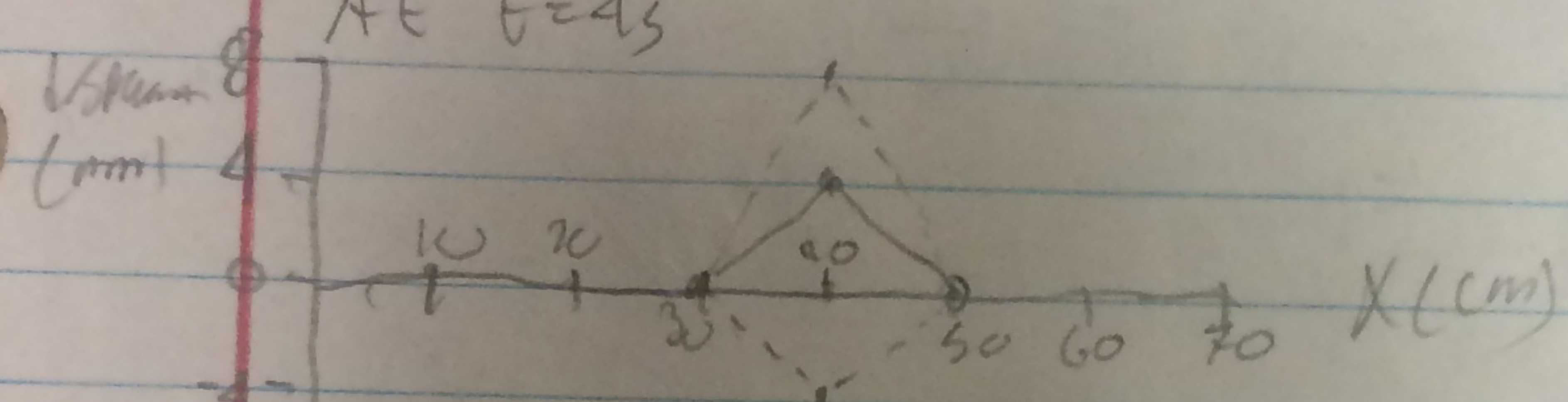
At t=2s



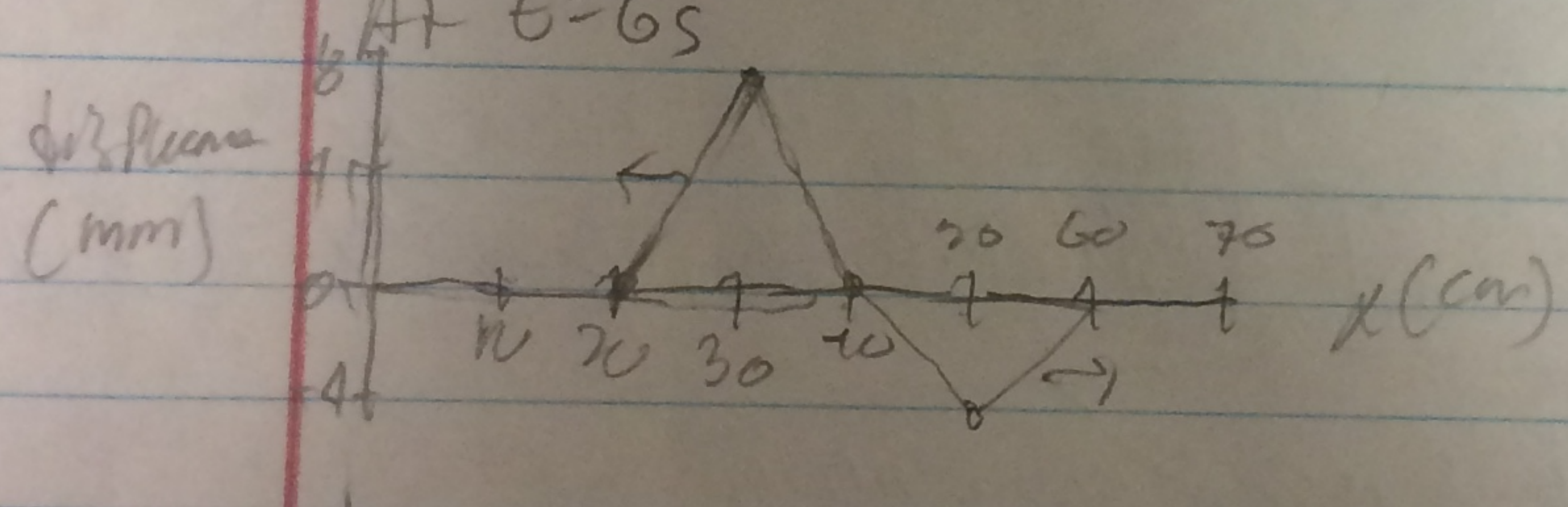
At t=3s



At t=4s



At t=6s



Q2M.2

Suppose a string on an acoustic guitar is 25 in. = 25·2.54cm = 63.5cm long between its fixed ends. The speed of waves on a stretched string is $|\vec{v}| = \left(\frac{|\vec{F}_T|}{\mu}\right)^{\frac{1}{2}}$, where $|\vec{F}_T|$ is the magnitude of the tension force on the string and μ is the string's mass per unit length. The highest E string on such a guitar has a pitch of about 329 Hz. Assume $\mu = 0.2\,g/m$

a) What tension force must be applied to this string?

We want to tune our string to have a pitch of 329 Hz. That is, we want $f = 329\,Hz$.

A string that is fixed at two ends can only produce standing waves at frequencies $f = \frac{|\vec{v}|}{2L} \cdot n$, where n is the mode.

For our string, $|\vec{v}| = \left(\frac{|\vec{F}_T|}{\mu}\right)^{\frac{1}{2}}$, so $f = \left(\frac{|\vec{F}_T|}{\mu}\right)^{\frac{1}{2}} \cdot n / 2L$

Solving for $|\vec{F}_T|$, $\frac{2fL}{n} = \left(\frac{\vec{F}_T}{\mu}\right)^{\frac{1}{2}} \Rightarrow \frac{4f^2L^2}{n^2} = \frac{|\vec{F}_T|}{\mu}$

$|\vec{F}_T| = \frac{4f^2L^2 \cdot \mu}{n}$, We want $f = 329\,Hz$, $n = 1$, and know $\mu = 0.0002\,kg/m$ and $L = 0.635\,m$, so $|\vec{F}_T| = 4 \cdot (329\,Hz)^2 \cdot (0.635\,m)^2 \cdot 0.0002\,kg/m \approx \boxed{34.9\,N}$

So there must be 34.9N of tension on the string

b) By what fraction would we have to increase the tension to tune the string to G (392 Hz)?

We would have to increase the tension by $\frac{|\vec{F}_{TG}|}{|\vec{F}_{TE}|}$:

$|\vec{F}_{TG}| = 4 \cdot (392\,Hz)^2 \cdot L^2 \cdot \mu$, $|\vec{F}_{TE}| = 4 \cdot (329\,Hz)^2 \cdot L^2 \cdot \mu$

So $\frac{F_{TG}}{F_{TE}} = \frac{(392\,Hz)^2}{(329\,Hz)^2} \approx 1.42$

So we would need to increase the tension by 142%

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Eval of answers

Q1R.1: Units look good, magnitudes seem reasonable.

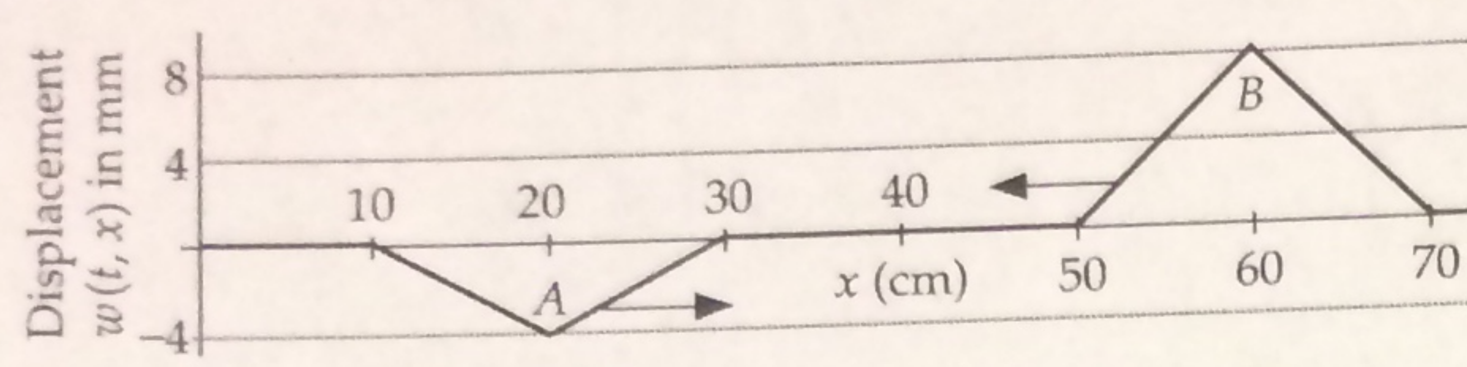
Q2B.1: I like the graphs

Q2M.2: Units good, magnitudes reasonable.

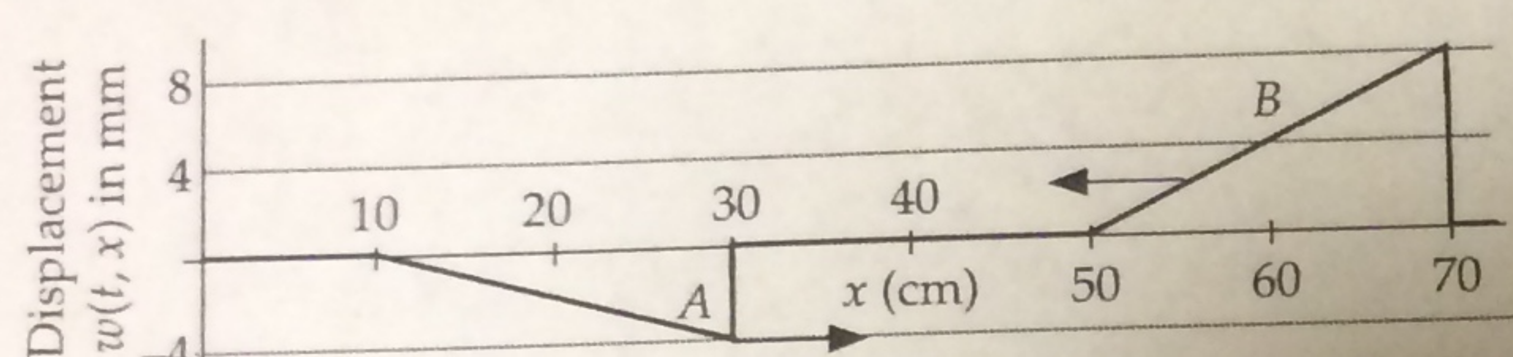
HOMEWORK PROBLEMS

Basic Skills

Q2B.1 Consider the triangle-shaped waves shown in the drawing below. Each wave moves with a speed of 5 cm/s in the direction indicated. Draw separate graphs showing what the superposition principle implies that the combined wave should look like at 2 s, 3 s, 4 s, and 6 s.



Q2B.2 Consider the sawtooth-shaped waves shown in the drawing below. Each wave moves with a speed of 5 cm/s in the direction indicated. Draw separate graphs showing what the superposition principle implies that the combined wave should look like at 2 s, 3 s, 4 s, and 6 s.



Q2B.3 Suppose we have a string 1.2 m long and fixed at both ends. We adjust the tension on the string until the speed of waves on the string is 24 m/s. What is the frequency of the string's fundamental mode of oscillation?

Q2B.4 An organ pipe open at both ends is 2.2 m long. What is the frequency of the fundamental mode of the air in the pipe?

Q2B.5 An organ pipe open at both ends has a fundamental frequency of 440 Hz (concert A). What is the length of this pipe? What are the frequencies of its first three harmonics?

Q2B.6 An organ pipe closed at one end has a fundamental frequency of 220 Hz (A below middle C). What is this pipe's length? What are the frequencies of its first three harmonics?

Q2B.7 Suppose we have a string 1.5 m long that is fixed at both ends. We adjust the string's tension so the string's fundamental frequency is 100 Hz. What is the frequency of the normal mode of the string's oscillation that has three antinodes?

Q2B.8 Suppose we have an organ pipe that is closed at one end. The pipe's length is such that the fundamental frequency of its air column is 230 Hz. What is the frequency of the normal mode that has two internal antinodes (not counting the antinode at the closed end)?

Q2B.9 To tune a woodwind instrument, one pulls apart or pushes together two sections of the instrument.
(a) Why does this change the instrument's pitch?
(b) How is this related to the purpose of the slide on a trombone?

Modeling

Q2M.1 A concert flute, as you can see from figure Q2.13 is about 2 ft long. Its lowest pitch is middle C (about 262 Hz). On the basis of this evidence, should we consider a flute to be a pipe that is open at both ends or at just one end? (The end of the flute farthest from the mouthpiece is clearly open. The other end of the flute seems to be clearly closed, so if you claim that the flute is open at both ends, you should try to explain where the other open end is.)

Q2M.2 Suppose a string on an acoustic guitar is 25 in. long between its fixed ends. As we saw in chapter Q1, the speed of waves on a stretched string is $|\vec{v}| = (|\vec{F}_T|/\mu)^{1/2}$, where $|\vec{F}_T|$ is the magnitude of the tension force on the string and μ is the string's mass per unit length. The highest E string on such a guitar has a pitch of about 329 Hz. Assume that string has a mass per unit length of 0.2 g/m.
(a) What tension force must be applied to this string?
(b) By what fraction would we have to increase the tension to tune the string up to G (392 Hz)?

Q2M.3 You may know that if you inhale helium, your voice sounds strange and high-pitched if you talk as you exhale the helium. Why is this? (*Hints:* Your sinuses are resonating chambers of air that emphasize certain pitches produced

Figure Q2.13
A woman playing a flute (Credit: © CMCD/Getty Images)