General Physics II
Physics 132
Grinnell College

Units I-II: Fluid Flow
Summary of Concepts and Equations

Essential to our notions of fluid flow was the concept of pressure. Pressure is a single value at any point in the fluid that determines how hard the fluid is pushing on adjacent material. That adjacent material may be a wall of the vessel holding the fluid, or it might be a surface of an object immersed in the fluid, or it might be the neighboring part of the fluid itself. To find the actual force exerted on a surface by the fluid, we take the pressure times the area of the surface, the direction of the force being perpendicular to the surface. This is expressed in an equation as:

$$P = F/A$$

where P is the pressure, F is the magnitude of the force, and A is the area of the surface.

We argued that there must be conservation of the fluid; that is, if the fluid is flowing from one part of the vessel to another, we must account for all of the fluid somewhere. Strictly speaking, this means that the mass of fluid is conserved, but liquids in general are highly incompressible, and so we can generally consider the conservation to be of volume as well. So if we have smooth flow, where the velocity of the fluid is well-defined at every point, we argued then that the flow at one point (defined as the volume of fluid passing that point per unit time) must be equal to the flow at another point along the fluid circuit. This is most useful if the fluid is flowing through tubes and we can easily determine the cross-sectional area of the tube. In that instance, we get

$$f_1 = f_2, \text{ implying}$$
$$v_1A_1 = v_2A_2$$

where the f's are the flow at the two locations, v's are the velocities, and A's are the cross-sectional areas. We also used this to show that the two dimensional flow between the Lucite plates gave a velocity as a function of radius dependence that looked like 1/r.

Two extremely important results are contained in a famous equation called the Bernoulli equation. This equation contains the equivalent of the classical mechanics expression for conservation of energy. In particular, we know that if we exert a force on an object and move it, we do some work on it that is expressed either as a change in potential energy or a change in kinetic energy. For a fluid, this means that pressure changes in the fluid must either represent changes in the fluid velocity, or a change in the gravitational potential energy, that is, the height of the fluid. This is expressed as a conservation expression like in the fluid conservation equation above, that is it refers to qualities of the fluid in two different places. As an equation, this is

$$P_1 + \rho g h_1 + (1/2)\,\rho v_1^2 = P_2 + \rho g h_2 + (1/2)\,\rho v_2^2$$

We used the static part of this (the ρgh) as a tool to help us indicate pressure throughout a system by the height of a column of water. We also (in a problem) used this to find Archimedes principle: a submerged object experiences a force upward (called the buoyant force) in the fluid that is equal to the weight of the fluid displaced, that is, the density of the fluid times the volume times g. This is expressed as an equation as

$$F_{buoyant} = \rho g V$$

where V is the submerged volume of the object.

The Bernoulli equation assumes that energy is conserved, or in other words that there is no significant friction in the fluid system. For fluids, we call this friction viscosity, or drag. Actual calculations of viscous forces are complicated, and more than we want to do in this course. However, for many circumstances, when the fluid flow is not too fast, these frictional forces are proportional to the relative velocity of the fluid. For example, if a viscous fluid is flowing through a horizontal tube of uniform cross-sectional area (where Bernoulli would tell us that the pressure is constant), a pressure drop is nevertheless observed as we go along the fluid flow path. This is because the pressure is doing work against friction, so energy is lost, and hence the pressure is lower further along the path. This energy is not actually lost, of course, but turns into thermal energy that heats the fluid. We observed that the pressure drop was roughly proportional to the flow rate. We defined the constant of proportionality to be a quantity that we called the flow resistance. This is expressed then as the equation

$$R = \Delta P/f$$

where f is the flow rate, ΔP is the pressure difference *over the tube or flow restriction* and R is this new quantity called the flow resistance. This we found to be of practical significance, because then it allowed us a way to monitor fluid flow without actually interrupting the fluid flow; we simply observe the pressure difference across the flow restriction and multiply by the flow resistance.