Physics 132
General Physics II
Grinnell College

Electric Fields and Potentials Summary

We introduced the concept of the electric field as the electric force that a test charge would feel at an arbitrary point in space, divided by the magnitude of the test charge. This would be written as an equation as

$$E = F/q_{test}.$$

As a result of this definition, an electric field is a vector that points in the direction of the force a positive test charge would feel, and opposite to the direction of the force on a negative test charge. Of course this field arises from an arrangement of charges. We think of these charges as the *real* charges, whereas the test charges are imaginary conveniences. The electric field lines run from the positive real charges to the negative real charges. For a point charge, we get an expression that we call Coulomb's law [we used that name for the force law as well], given by

$$E = \frac{q}{4\pi\varepsilon_o r^2} \quad .$$

Actual calculations of fields from Coulomb's law involves the superposition of electric fields for several charges, or if there is a continuous charge distribution, an integral over the charge distribution. This is rarely easy. Gauss's law provides a clever trick to help us calculate electric fields in situations of high symmetry. We developed this through an analogy with fluid flow, where the continuity equation told us that the conserved quantity of the flow rate related to the flow out through a surface given by

$$f = vA$$

where f is the flow rate, v is the fluid velocity, and A is the surface area through which we are monitoring the flow. For a point source of fluid flow (like the end of a tube, or a spherical balloon inflating) the velocity of the fluid flow at a distance r from the source is given by

$$v = \frac{f}{4\pi r^2}.$$

This looks surprisingly like Coulomb's law, leading us to postulate a similar expression for electric fields

$$EA = \frac{q_{enclosed}}{\varepsilon_o}$$

where E is the electric field, A is the surface area of a closed volume called a Gaussian pillbox, and $q_{enclosed}$ is the total charge residing in that volume. In fact, the left hand side, called the electric flux, really should be an integral of the vector field against the vector surface area. However, we use it in cases of high symmetry, when E is both constant and perpendicular to the relevant area (and zero or parallel to any other portion of the surface, allowing us to neglect those areas). This allowed us to calculate the electric field as a function of distance r for a line of charge with linear charge density λ = q/length of

$$E = \frac{\lambda}{2\pi\varepsilon_o r},$$

and to find that the field due to a plane with surface charge density σ = q/Area of

$$E = \frac{\sigma}{2\varepsilon_o}.$$

In contrast, we calculated by numerical approximation the field at a distance from a dipole to drop off like $1/r^3$.

The electric potential, which we already knew as the voltage, was defined as being the potential energy per unit charge that a test charge would have at an arbitrary point in space, or $V = U/q_{test}$. Since this is analogous to the definition of the electric field, they are related to one another just as potential energy and force are:

$$E_x = -\frac{dV}{dx}, \ or \ \ V = -\int E_x dx \ .$$

We found the electric potential therefore for a constant electric field E_o in the positive x direction was linearly dependent on the distance, such as $V(x) = -E_o x$. (Note—we look at potentials due to points and dipoles in Unit IX). Most of the time, we deal with a difference of electric potential ΔV, such as with resistors or capacitors. In terms of the integral relationship between electric field and potential, using a definite integral ("integral from a to b") gives us ΔV, whereas an indefinite integral gives us the potential function $V(x)$.

We used the concepts of electric potential to help us calculate energy in capacitors and resistors, in particular

$$U = \tfrac{1}{2}Q\Delta V = \tfrac{1}{2}C\Delta V^2 = \tfrac{1}{2}\frac{Q^2}{C} \text{ , and}$$

$$P = I\Delta V = \frac{(\Delta V)^2}{R} = I^2 R \text{ ,}$$

where P is the power lost in the resistor as heat.

In addition, we found that for static cases (no circuits with batteries or the like allowing continuous current flow), the electric field inside a conductor is zero, and therefore the potential is constant everywhere within the conductor.