Olek Yardas
Phy 232
Charles Conway Lin

Prm for 11/03/17

Q13 R.3, Q14M.1

Q13 R.3

Suppose you have two samples of radioactive material, one with a half life of $t_A = 1.0 y$ and one with a half life of $t_B = 4.0 y$. Assume both samples are roughly equal at the time $t = 0$ when you mix them. How long an amount will you have to wait for the activity of the combined sample to decrease to one half of its original activity?

$$\text{Let } C(t) = 2A_0 e^{-\frac{t \cdot \ln(2)}{t_{1/2}}} = A_0 \left(e^{-\frac{t \cdot \ln(2)}{1.0y}} + e^{-\frac{t \cdot \ln(2)}{4.0y}} \right)$$

$$\Rightarrow 2e^{-\frac{t \ln(2)}{t_{1/2}}} = e^{-\frac{t \ln 2}{1.0y}} + e^{-\frac{t \ln 2}{4.0y}}$$

$$\Rightarrow 2e^{-\frac{T \ln(2)}{t_{1/2}}} = e^{-\frac{T \ln(2)}{1.0y}} + e^{-\frac{T \ln(2)}{4.0y}}, \text{ take } T = t_{1/2}, \text{ so}$$

$$e^{-\frac{T \ln(2)}{1.0y}} + e^{-\frac{T \ln(2)}{4.0y}} = 1$$

I used Mathematica to plot $y = e^{-\frac{t \ln(2)}{1.0y}} + e^{-\frac{t \ln(2)}{4.0y}}$ and $y = 0$ and solve directly and found that $\boxed{T \approx 1.85983\,y}$

Q14M.1

One way to remember whether it is a neutrino or an antineutrino (or electron or positron) that is involved in a weak-interaction process is to use the conservation laws. The weak interaction, in addition to conserving energy, momentum, and angular momentum, conserves charge, a quantity called the lepton number (e^-, ν have lepton number +1, e^+, $\bar{\nu}$ have lepton number −1, p^+, n have lepton number 0), and a quantity called the baryon number (p^+ and n have baryon number +1, e^-, e^+, ν, $\bar{\nu}$ have baryon number 0)

a) Show the processes described by equations Q14.1 through Q14.4 conserve charge, lepton #, and baryon #.

If charge is conserved, $\sum$ charge before $= \sum$ charge after
If lepton # is conserved, $\sum$ lepton $\#_i = \sum$ lepton $\#_f$
If baryon # is conserved, $\sum$ baryon $\#_i = \sum$ baryon $\#_f$

	Q14.1	Q14.2	Q14.3	Q14.4	
	$\nu + n \rightarrow e^- + p^+$	$n \rightarrow e^- + p^+ + \bar{\nu}$	$p^+ \rightarrow e^+ + n + \nu$	$p^+ + e^- \rightarrow n + \nu$	Conserved for all?
Charge	$0+0 \overset{?}{=} -1e + 1e \overset{\checkmark}{=} 0$	$0 \overset{?}{=} -1e + 1e + 0 \overset{\checkmark}{=} 0$	$+1e \overset{?}{=} +1e + 0 + 0 \overset{\checkmark}{=} +1e$	$+1e - 1e \overset{?}{=} 0 + 0 \overset{\checkmark}{=} 0$	Yes
Lepton #	$+1 + 0 \overset{?}{=} +1 + 0 \overset{\checkmark}{=} +1$	$0 \overset{?}{=} +1 + 0 - 1 \overset{\checkmark}{=} 0$	$0 \overset{?}{=} -1 + 0 + 1 \overset{\checkmark}{=} 0$	$0 + 1 \overset{?}{=} 0 + 1 \overset{\checkmark}{=} +1$	Yes
baryon #	$0 + 1 \overset{?}{=} 0 + 1 \overset{\checkmark}{=} +1$	$+1 \overset{?}{=} 0 + 1 + 0 \overset{\checkmark}{=} +1$	$+1 \overset{?}{=} 0 + 1 + 0 \overset{\checkmark}{=} +1$	$+1 + 0 \overset{?}{=} +1 + 0 \overset{\checkmark}{=} +1$	Yes

b) Imagine that if we hit a proton with particle X having sufficient energy, it turns into a neutron and emits particle Y. If X is either a neutrino or antineutrino, which is it? What is particle Y? Explain your reasoning.

This process is caused by the weak interaction, so charge, baryon #, and lepton # must be conserved:
X is either a neutrino or antineutrino.

	Neutrino	Antineutrino
	$\nu + p^+ \rightarrow n + Y$	$\bar{\nu} + p^+ \rightarrow n + Y$
Charge	$0 + 1e \overset{?}{=} 0 + Y$	$0 + 1e \overset{?}{=} 0 + Y$
Lepton #	$+1 + 0 \overset{?}{=} 0 + Y$	$-1 + 0 \overset{?}{=} 0 + Y$
baryon #	$0 + 1 \overset{?}{=} +1 + Y$	$0 + 1 \overset{?}{=} +1 + Y$

We can see that in both cases Y must be a particle with baryon $\# = 0$, charge $= +1e$. This means that Y is a positron (e^+).
A positron has a lepton # of -1, so lepton number is conserved only if X is an antineutrino ($\bar{\nu}$).

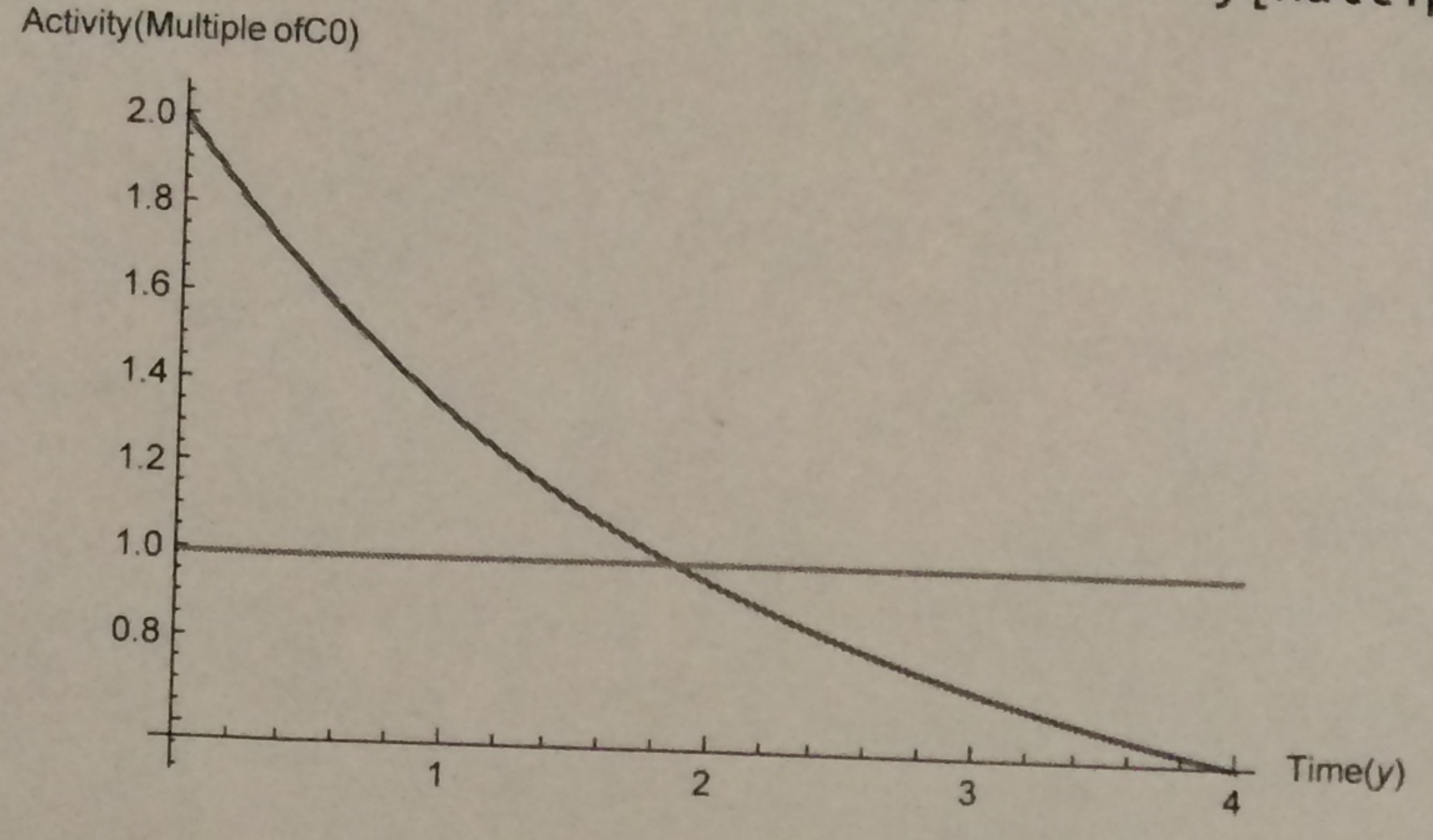
So X is an antineutrino ($\bar{\nu}$) and Y is a positron (e^+)

$\bar{\nu} + p^+ \rightarrow n + e^+$

Eval O'casions:
Q13R 3: reasonable magnitude, not units
Q14M1: good reasons

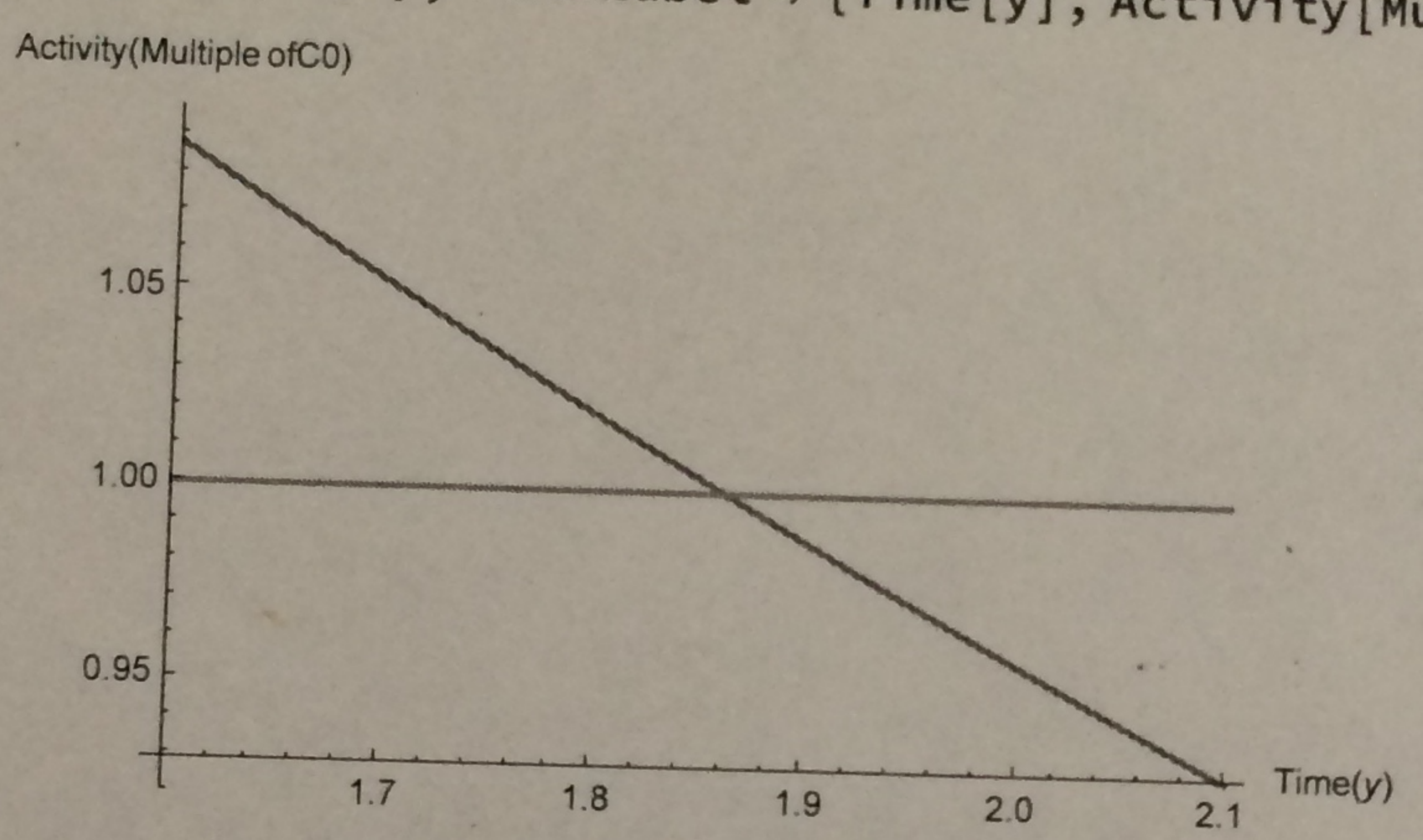
In[22]:=
```
Plot[{(Exp[-t * Log[2] / 1]) + (Exp[-t * Log[2] / 4]), 1},
  {t, 0, 4}, AxesLabel → {Time[y], Activity[Multiple ofC0]}]
```

Out[22]=



In[23]:=
```
Plot[{(Exp[-t * Log[2] / 1]) + (Exp[-t * Log[2] / 4]), 1},
  {t, 1.6, 2.1}, AxesLabel → {Time[y], Activity[Multiple ofC0]}]
```

Out[23]=




```
Solve[{(Exp[-t * Log[2] / 1]) + (Exp[-t * Log[2] / 4]) == 1, t ∈ Reals}, t]
```

$$\left\{\left\{t \to \frac{4\,\text{Log}\left[\text{Root}\left[-1 - \#1^3 + \#1^4 \,\&,\, 2\right]\right]}{\text{Log}[2]}\right\}\right\}$$

$$N\left[\left\{\left\{t \to \frac{4\,\text{Log}\left[\text{Root}\left[-1 - \#1^3 + \#1^4 \,\&,\, 2\right]\right]}{\text{Log}[2]}\right\}\right\}\right]$$

```
{{t → 1.85983}}
```