Phy 232
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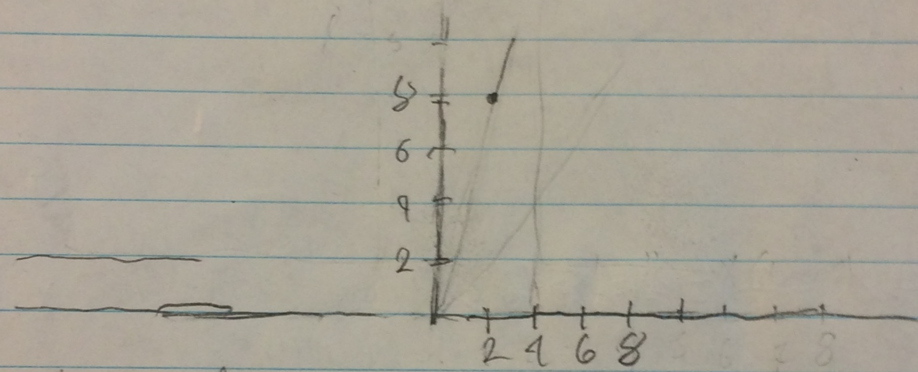
Hw for 7/7/20
R2M.6, ~~R2M.8~~, R3M.1

R2M.6

Suppose a spaceship is docked at a space station floating deep in space. Assume the space station defines the origin in its own frame. At $t=0$ (call this event A) the spaceship starts accelerating in the $+x$ direction away from the space station at a constant rate (as measured in the station frame). The spaceship reaches a cruising speed of 0.5 after 8 h as measured in the station frame.



a) Where does event B occur?

If $\vec{v}_f = 0.5$ and $\Delta t = 8$ hours, we can find Δx_B by using

$\vec{v} = \vec{a}t$ and $\Delta \vec{x}_B = \frac{1}{2}\vec{a}t^2$

$\vec{a} = \frac{\vec{v}}{t} \rightarrow \Delta \vec{x}_B = \frac{1}{2}\frac{\vec{v}}{t}t^2 = \frac{1}{2}\vec{v}t$

So $\Delta \vec{x}_B = \frac{1}{2} \cdot 0.5 \cdot 8\text{hours} = \boxed{2\text{ hours}}$

B) Find the magnitude of the ship's acceleration in SR units and as a multiple of $|\vec{g}|$

$\vec{a} = \frac{\vec{v}}{t} = \frac{0.5}{8\text{hours}} \cdot \frac{1\text{hour}}{60\text{min}} \cdot \frac{1\text{min}}{60\text{s}} \approx 1.7 \cdot 10^{-5} \approx \boxed{2 \cdot 10^{-5}\ s^{-1}}$

$|\vec{g}| = 9.8\text{m/s}^2 \cdot 1 = 9.8\frac{m}{s^2} \cdot \frac{1s}{299792458m} = 3.2 \cdot 10^{-8} \approx 3 \cdot 10^{-8}\ s^{-1}$

$\frac{\vec{a}}{|\vec{g}|} = \frac{2 \cdot 10^{-5}\ s^{-1}}{3 \cdot 10^{-8}\ s^{-1}} \Rightarrow \vec{a} = \frac{2 \cdot 10^{-5}\ s^{-1}}{3 \cdot 10^{-8}\ s^{-1}}|\vec{g}| \approx \boxed{300\,|\vec{g}|\ s^{-1}}$

So a shock proof clock will not survive.

c) At event B, the spaceship sends a radio signal, reporting it has reached cruising speed. This signal reaches the station at event C. When and where does this event occur?

a radio signal sent

Electromagnetic waves travel at speed 1. So at $(x, t) = (2\text{ hours}, 8\text{ hours})$ will reach $x = 0$ hours at $t = 10$ hours. So Event C occurs at $C(x, t) = (0, 10\text{ hours})$

d) The technician responds to this message 0.5 h later after returning from a coffee break: call this event D. When and where does the ship recieve the acknowlegement (event E)

Assuming the ship does not speed up and stays at a cruising speed of 0.5, after 0.5 hours + 2 hours it has travelled $2.5\text{ hours} \div \left(\frac{1}{0.5}\right) = \frac{2.5}{2} = 1.25$ hours further, so it is at $(x, t) = (3.25\text{ hours}, 10.5\text{ hours})$ when the technician sends the signal.

We can find the time t that the signal will reach the space ship by using the equations of the signal world line and the ship world line.

The signal world line has a slope of 1 and leaves $x = 0$ at $t = 10.5$, so the equation is $(t - 10.5) = 1(x - 0) \to t = x + 10.5$

The ship world line has a slope of 2 after $t = 8$ hours, and is $x = 2$ hours from the origin, so the equation is $(t - 8) = 2(x - 2)$
$\Rightarrow t = 2x - 4 + 8 = 2x + 4$

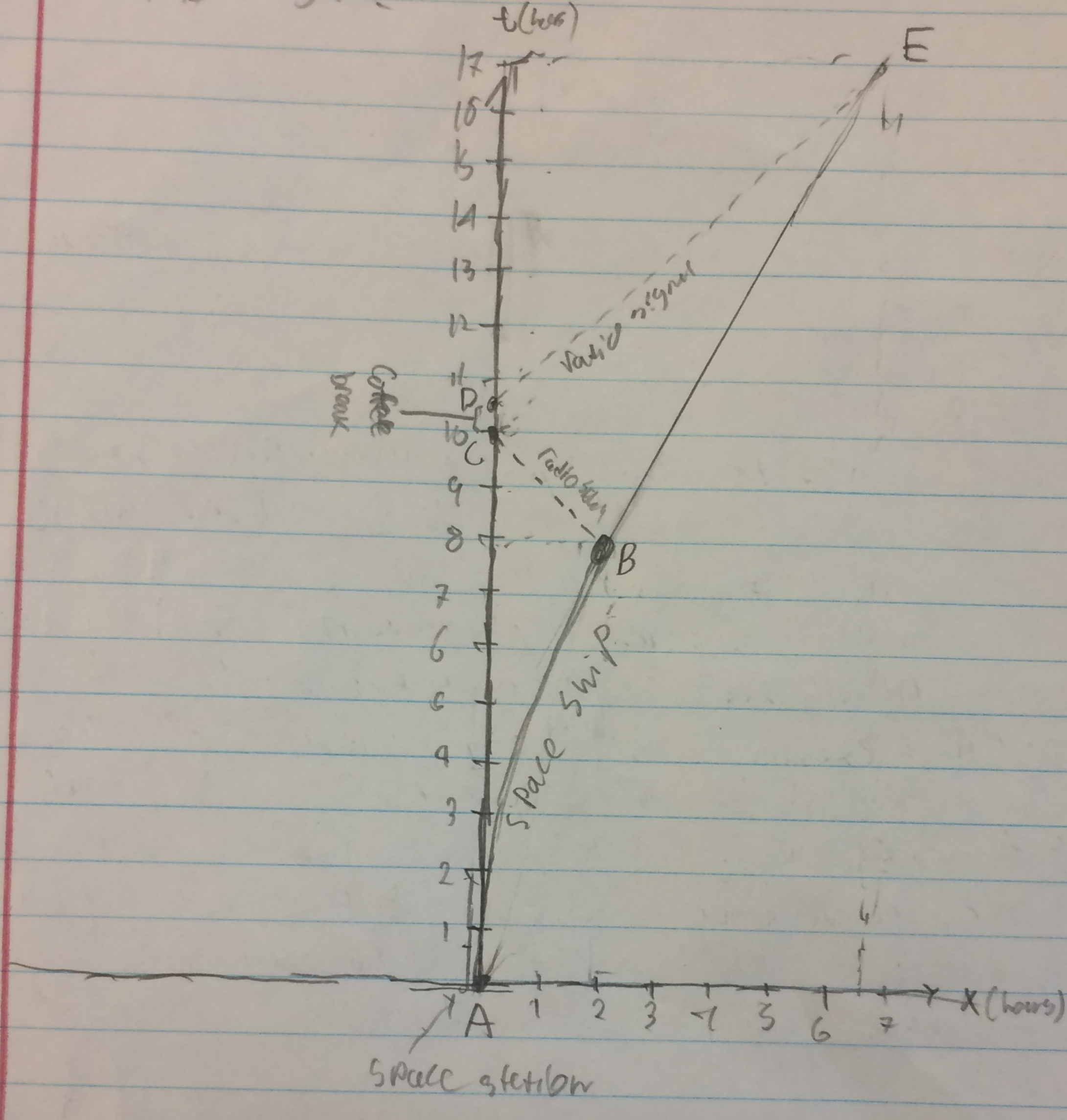
Setting these equal to each other to find their position in space,
$x + 10.5 = 2x + 4 \to \boxed{6.5\text{ hours} = x}$

Plugging in $x = 6.5$ hours to both equations to find t,

$6.5\text{ hours} + 10.5\text{ hours} = 2(6.5)\text{ hours} + 4\text{ hours} \Rightarrow 17\text{ hours} = 13\text{ hours} + 4\text{ hours} \Rightarrow 17\text{ hours} = 17\text{ hours}$

Therefore, the signal reaches the ship at $(x, t) = (6.5\text{ hours}, 17\text{ hours})$

e) Draw a careful spacetime diagram of the situation, showing the worldlines of the space station, space ship, radio signals, events A through E



Evaluation of answers:

(a) I used the velocity and displacement equations to find the time coordinate of the spaceship. I should have gotten an answer in hours, which I did, so my answer seems reasonable.

(b) In SR units, acceleration has units of s^{-1}, and my answer has units of s^{-1}. Additionally, the magnitude of my answers seems to be on a reasonable order of magnitude (10^{-5} for converting hours to seconds)
(10^{-8} for converting the acceleration of gravity to SR units by a 7 orders of magnitude reduction)

c) The question asked for the time place in hours, and my answer was in hours and of a reasonable magnitude (10^1).

d) My answer of (6.5, 17) hours seems of a reasonable magnitude when the initial conditions were (0, 10.5) hours.

e) My space time diagram accurately portrays all the events in the problem.

R3M.7)
Imagine that in the year 2065, you are watching a live broadcast from the space station at the planet Neptune, which is 4.0 light-hours from earth at the time. (assume that the TV signal from Neptune is sent to earth via a laser light communication system) At exactly 6:17 pm (as registered by the clock on your desk) you see a technician on the TV suddenly exclaim, "Hey! We've just detected an alien spacecraft passing by here!" Let this be event A. Exactly 1 hr later, the alien space ship is detected passing by earth. Let this be event B. Assume the earth and neptune stations can be considered parts of the inertial reference frame of the solar system, and assume that the spaceship travels at a constant velocity.

a) During the broadcast, you can see on your TV screen the face of a clock sitting on the technician's desk. What time should you see on this clock face at 6:17 pm your time if that clock is synchronized with yours?

Because the broadcast takes 4.0 light hours · 1 = 4 hours to travel to the earth, the time on the clock on the TV should read 2:17 pm

b) What is the coordinate time between Events A and B in the solar system frame?

The coordinate time is defined as the time measured in an inertial reference frame between two events by a pair of synchronized clocks.
Because the clocks at events A and B are synchronized (just like the same clock), the coordinate time is 1 hour.

c) What is the speed of the alien spaceship, measured in the solar system frame?

The spaceship takes 4+1 = 5 hours to go 4 hours
so its speed is $\frac{4 \text{ hours}}{5 \text{ hours}} = \boxed{\frac{4}{5} = 0.8}$

d) What kind(s) of time would the spaceship's clock measure between A and B?

The spaceship's clock cannot ~~measure~~ time between ~~events~~ A and B because it is not synchronized with the clock at A and B, nor is it present at event A

Evaluation of answers:

a) The order of magnitude of the time difference is reasonable given a distance of 4.0 hours between the broadcast and its reception

b) The coordinate time is of a reasonable size (it is not huge and it is not tiny) within the scale of this problem.

c) ~~A~~ speed of 0.8, while very fast, is not unreasonable for the given conditions. If I did the fraction wrong and speed is $\frac{5}{4}$, this would mean the ship is faster than light, which wouldn't make sense because it would never have been detected in the first place.

d) I believe question d to be a trick because we do not know anything about the clock on the alien ship, and because the ship is only present at event B, we cannot say anything about the kinds of time registered by the clock on the alien ship for A and B.