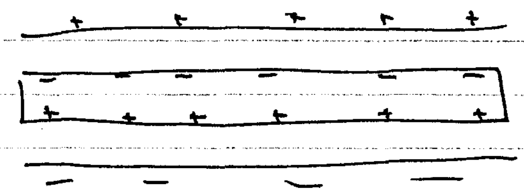
Homework VIII - IX

Session VIII.3

1a) $|E| = \left|\frac{dV}{dx}\right| = \frac{\Delta V}{\Delta x} = \boxed{\frac{V}{d}}$

b) We know $E = \sigma/\epsilon_0$ (field from 2 planes)

so $Q = \sigma \cdot A = \epsilon_0 E \cdot A = \boxed{\frac{\epsilon_0 V A}{d}}$

c) 

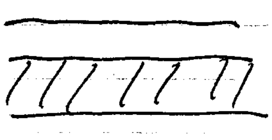
Charges appear on surfaces of conductor

d) E in (static) conductor is always zero

e) Q on conductor? Plane of charge on conductor must exactly balance plane of charge above it, so $|\sigma|$ is same as on capacitor (can also view planes of charge on conductor as another capacitor within original capacitor, or view system as two capacitors in series).

f) Since $E = \frac{\sigma}{\epsilon_0}$, & σ does not change, E is unchanged by addition of conductor

g) $\Delta V_{total} = \Sigma \Delta V_i \leftarrow$ over 3 regions, each $V = E \cdot d$



i $\rightarrow \Delta V_i = \int E \cdot dl = E \cdot d/4 = Ed/4$

ii $\rightarrow \Delta V_{ii} = \int E \cdot dl = 0 \cdot d/2 = 0$

iii $\rightarrow \Delta V_{iii} = E \cdot d/4$

$\Delta V_{total} = \frac{Ed}{2}$ (Half original voltage

h) $C = \frac{Q}{V} = \frac{\sigma \cdot A}{Ed/2} = \frac{2\sigma A}{Ed} = \frac{2\sigma A}{\sigma/\epsilon_0 \cdot d} = \frac{2\epsilon_0 A}{d} \Leftarrow$ twice original capac.

i) $U_{before} = \frac{1}{2} Q V_0 \qquad U_{after} = \frac{1}{2} Q V_{after} = \frac{1}{2} Q V_0/2 \Leftarrow$ half the stored energy

2) $q_{\pm} = \pm 10^{-10}\,C$ $\quad d = 10^{-3}\,m$ $\quad$ a) $p = qd = 10^{-13}\,Cm$

b) $\vec{\tau} = \vec{p}\times\vec{E} \Rightarrow \tau_{max} = p\cdot E = 10^{-13}\,Cm \cdot 10^{5}\,V/m = 10^{-8}\,\text{"J"} = 10^{-8}\,Nm$

units of energy, but not really energy – $J = N\cdot m$

3) We estimate

$$p = qd = \underbrace{1.6\times10^{-19}\,C}_{e}\ \underbrace{2\times10^{-10}\,m}_{\text{molec size}} = 3.2\times10^{-29}\,Cm$$

we want $p\dfrac{dE}{dx} = F = mg$

$$\text{So}\quad \frac{dE}{dx} = \frac{mg}{p} = \frac{18\times1.67\times10^{-27}\,kg\cdot10\,m/s^2}{3.2\times10^{-29}\,Cm} = 9.4\times10^{3}\,\frac{V}{m^2}$$

(In case you are concerned about units $kg\,m^2/s^2 = J = V\cdot C$

$$\text{so}\quad \frac{kg\,m/s^2}{Cm} = \frac{kg\,m^2/s^2}{Cm^2} = \frac{J}{Cm^2} = \frac{VC}{Cm^2} = \frac{V}{m^2}\ )$$

So, this is about $10^4\,\frac{V}{m^2}$. We saw E's of $1000V/1cm$ $\}$ (this was our capacitor on the balance)

or $10^5\,V/m$. This went to zero over a distance also a few cm, giving $\frac{dE}{dx} \approx 10^7\,V/m^2$ — well beyond our required $10^4\,V/m^2$. Alternatively, we saw $V \approx 10^4\,V$ on the end of the Wimshurst machine — with a radius of $\approx 1\,cm = .01\,m$. For a spherical distribution, $V = \frac{q}{4\pi\epsilon_0 r} = \frac{k}{r}$

So, at $r = .01\,m$, $V = 10^4\,V$, or

$$10^4 V = \frac{k}{.01\,m} \Rightarrow k = 10^2\,Vm$$

$$E = -\frac{\partial V}{\partial r} = \frac{-k}{r^2} = \frac{-10^2\,Vm}{r^2}$$

$$\frac{dE}{dr} = \frac{2k}{r^3} = \frac{10^2\,Vm}{r^3}$$, So, at $r = .1\,m$,

not very close

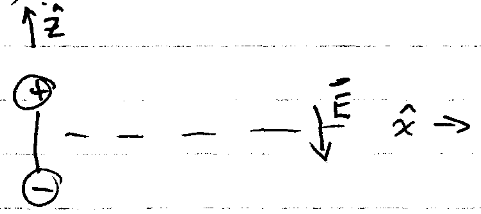
$$\frac{dE}{dr} = \frac{10^2\,Vm}{10^{-3}\,m^3} = 10^5\,V/m^2$$ $\Leftarrow$ more than enough!

If we went to surface ($r = .01\,m$), we get $\frac{dE}{dr} = 10^8\,V/m^2$!

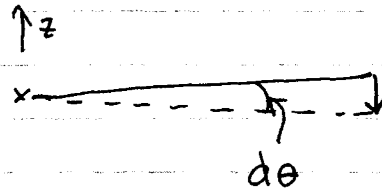
4)

object	E	V
dipole	$1/r^3$	$1/r^2$
point	$1/r^2$	$1/r$
line	$1/r$	$\ln r$
plane	constant (r^0 or z^0)	r (or z)

5)



a) Along the x axis, $\vec{E}$ is directed down – in the $-\hat{z}$ direction

b) 

$-dz = r d\theta$, so $\frac{d}{dz} = \frac{-d}{r d\theta}$

$$E_z = -\frac{\partial V}{\partial z} = +\frac{1}{r}\frac{d}{d\theta}\left(\frac{p\cos\theta}{4\pi\epsilon_0 r^2}\right)$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\frac{d}{d\theta}\cos\theta = \frac{-p\sin\theta}{4\pi\epsilon_0 r^3}$$

Along the ~~x~~ axis, $\theta = 90°$, so $\sin\theta = 1$, giving

$$E_z\Big|_{\cancel{x}\text{axis}} = \frac{-p}{4\pi\epsilon_0 r^3}$$ $\Rightarrow$ our result from p20 of Unit VII

6) A point charge produces a non-uniform E field. So, a dipole aligned w/ $\vec{E}$ (i.e. the stable configuration) will be attracted to the stronger field regions. If it is reverse, it will be repelled. Presumably such a dipole would orient itself with the field (assume rotation has some damping & doesn't oscillate forever) and then be attracted.