Preparation for November 20

We will also look at predator-prey models. Suppose we have two populations – x represents the population of the prey and y represents the population of the predator. We could model such a situation by the system

$$x' = x(a - \alpha y)$$
$$y' = y(-c + \gamma x)$$

Can you think what the variables a, c, α, γ represent in ecological terms?

The equilibrium in which both populations are positive is found by setting x' and y' equal to 0, but insisting x and y be positive. We get one point $(c/\gamma, a/\alpha)$.

Since $P(x, y) = ax - \alpha xy$ and $Q(x, y) = -cy + \gamma xy$, the Jacobian is given by

$$\begin{pmatrix} a & -\alpha x \\ \gamma y & -c \end{pmatrix}$$

At the equilibrium point, we have

$$\begin{pmatrix} a & -c\alpha/\gamma \\ a\gamma/\alpha & -c \end{pmatrix}$$

This matrix has characteristic polynomial

$$\lambda^2 - 0\lambda + \frac{-c\alpha}{\gamma}\frac{a\gamma}{\alpha} = \lambda^2 + ac.$$

So, the eigenvalues of the linear system are

$$\lambda = \pm\sqrt{ac}\, i.$$

What can we conclude about the nonlinear system from these eigenvalues?