PHY 131 HW Set #9

Ch. 15: 4, 6, 13, 14, 16, 24, 30, 35, 43, 49

$\boxed{15:4}$ A 0.12 kg body undergoes simple harmonic motion of amplitude X_{max} = 8.5 cm and period T = 0.20 s. a) What is the magnitude of the maximum force acting on it?

$F_{max} = m\,a_{max}$, with $a_{max} = -\omega^2 X_{max}$ for simple harmonic motion

Because $T = 2\pi/\omega$, $\omega = 2\pi/T = 2\pi/0.2 = 10\pi = 31.4$ rad/s.

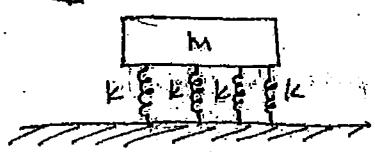
Hence $|F_{max}| = (0.12\,kg)(31.4)^2(0.085\,m) = 10.1\,N = \underline{\underline{10\,N}}$

b) If the oscillations are produced by a spring, what is the spring constant k?

Approach 1 $\quad \omega = \sqrt{\frac{k}{m}} \;\therefore\; k = m\omega^2 = (0.12\,kg)(31.4)^2 = 118\,N/m = \underline{\underline{120\,N/m}}$

Approach 2 $\quad F_{max} = -k X_{max}, \;\therefore\; k = \left|\frac{F_{max}}{X_{max}}\right| = \frac{10\,N}{0.085\,m} = 118\,N/m = \underline{\underline{120\,N/m}}$

$\boxed{15:6}$



Car supported on 4 identical springs

m = 1450 kg $\quad$ f = 3.00 Hz

a) What is k of each spring?

These four springs are arranged in parallel, so a given displacement x will produce a restoring force $F_{each} = -kx$ from each. Hence the total force is four times that of a single spring, and together they behave like a single spring of $k_{\parallel} = 4k$.

Now $\omega = \sqrt{\frac{k}{m}}$, so $k_{\parallel} = m\omega^2$, where $\omega = 2\pi f$, hence

$$k_{parallel} = m(2\pi)^2 f^2 = (1450\,kg)(2\pi)^2(3.00\,Hz)^2 = 5.15\times10^5\,N/m$$

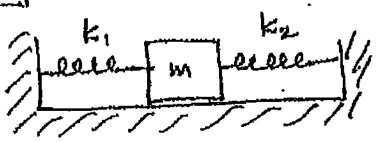
Hence $k = \frac{k_{parallel}}{4} = \underline{\underline{1.29\times10^5\,N/m}}$

b) What will be the oscillation frequency if five passengers, averaging 73.0 kg each, ride in the car with an even distribution of mass?

Now m = 1450 kg + 5 x 73.0 kg = 1815 kg

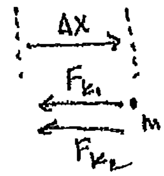
so $\omega = \sqrt{\frac{k_{total}}{1815}} = \sqrt{\frac{4(1.29\times10^5\,N/m)}{1815\,kg}} = 16.85$ rad/s and so $f = \frac{\omega}{2\pi} = \underline{\underline{2.68\,Hz}}$

$\boxed{15:13}$



These two springs are effectively in parallel, because if you displace the mass by a distance x the two forces add in the same direction

$m = 0.245\ kg$
$k_1 = k_2 = 7580\ N/m$

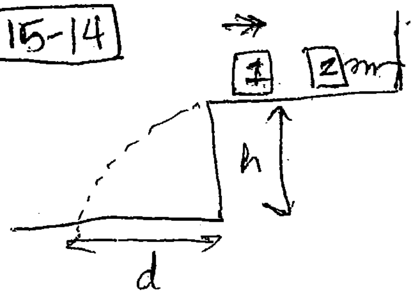


so $F_{effective} = F_1 + F_2$
$= -k_1 x - k_2 x$
$F_{effective} = -(k_1 + k_2)\,x$

Here $k_1 = k_2 = k$, so the combination behaves like a spring of constant $k_{effective} = 2k$.

Hence $\omega = \sqrt{\dfrac{k_{eff}}{m}} = \sqrt{\dfrac{2(7580\ N/m)}{(0.245\ kg)}} = 249\ rad/s$ $\therefore f = \dfrac{\omega}{2\pi} = 39.59\ Hz$
$= \underline{39.6\ Hz}$

$\boxed{15\text{-}14}$



m_1: $v_{0_1} = 8\ m/s$
$m_1 = 0.2\ kg$

m_2: $v_{0_2} = 0$
$m_2 = ?$

SPRING:
$K = 1208.5\ N/m$
$T = 0.14\ s$

$h = 4.9\ m$
$d = ?$

- We are given the period of oscillation and k, so we can find m_2
- "the spring doesn't affect collision" gives us permission to treat this as an elastic collision

- $T = 2\pi\sqrt{\dfrac{m_2}{K}} \longrightarrow m_2 = \dfrac{T^2 k}{4\pi^2}$

$m_2 = 0.6\ kg$

15:14 (cont.)

- Elastic collision : conserve momentum & KE

$\vec{p}$: $m_1 v_1 + m_2 \cancel{v_2}^{0} = m_1 v_1' + m_2 v_2'$ ①

K: $\cancel{v_2}^{0} - v_1 = -(v_2' - v_1')$ ②

① $m_1(v_1 - v_1') = m_2 v_2'$ ② $v_2' = v_1 + v_1'$

$m_1(v_1) - m_1 v_1' = m_2(v_1 + v_1')$

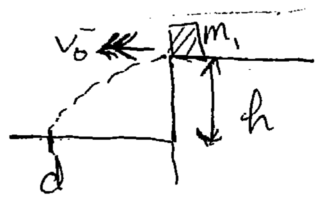
$m_1 v_1 - m_1 v_1' = m_2 v_1 + m_2 v_1'$

$(m_1 - m_2) v_1 = (m_1 + m_2) v_1'$

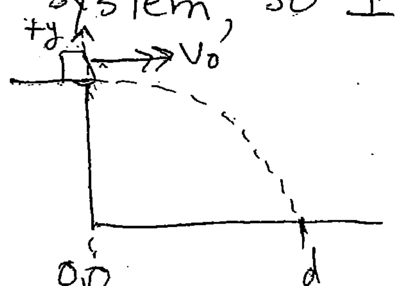
$v_1' = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right) v_1 = \dfrac{(.2 - .6)}{(.2 + .6)} \text{kg} \, (8 \text{ m/s}) = -4 \text{ m/s}$

m_1 moves to the left at 4 m/s

This is initial speed for projectile motion.



free to choose a coordinate system, so I choose



$\longrightarrow +x$

$y_0 = h = 4.9$ m
$y_f = 0$
$a_y = -9.8$ m/s
$v_{y_0} = 0$
$v_{y_f} = ?$
$\Delta t = ?$

- I can find Δt from y - eqns
- I can then find Δx

$x_0 = 0$
$x_f = d = ?$
$v_{0x} = +4$ m/s
$v_{fx} = +4$ m/s
$a_x = 0$
$\Delta t = ?$

- $\cancel{y_f}^{0} = y_0 + \cancel{v_{0y}}^{0} t + \frac{1}{2} a_y t^2 = 4.9 \text{ m} - \frac{1}{2}(9.8 \text{ m/s}^2) t^2$

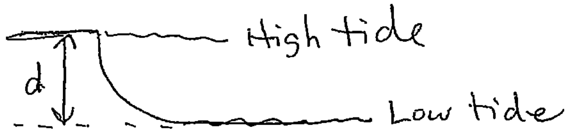
$t = \sqrt{\dfrac{4.9 \text{ m}}{4.9 \text{ m/s}^2}} = 1 \text{ s}$

- $x_f = \cancel{x_0}^{0} + v_{0x} t + \frac{1}{2} \cancel{a_x}^{0} t^2$

$x_f = (4 \text{ m/s})(1 \text{ s})$

$x_f = 4$ m

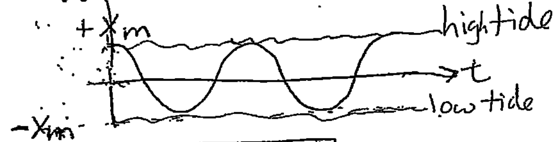
15:16



rise + fall
$T = 12.5\ h$

Note: This is the time to rise, fall, and rise again

We describe a wave



picture the oscillating wave going from $+X_m$ to $-X_m$ periodically.

$\boxed{A\cos(\omega t + \phi)}$

$A = X_m$ → $d = 2X_m$, the distance from low tide to high tide

$T = 12.5\ h = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{T} = 0.5\ \text{rad}/h$

$\phi = 0$ if we say at time $t=0$ $x = x_m$

so $x(t) = \left(\frac{1}{2}d\right)\cos(0.5t)$

where is $x(t) = \frac{1}{4}d$? when $\cos(0.5t) = \frac{1}{2}$

let $0.5t \equiv \phi$ → $\cos\phi = \frac{1}{2}$

MUST BE RADIANS

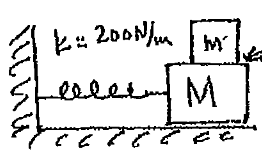
$\phi = \cos^{-1}\left(\frac{1}{2}\right)$

$\phi = 1.05\ \text{rad} = 0.5t$

$t = \frac{1.05}{.5}$

$t = 2.09$ hours

15:24



$\mu_s = 0.40$
$m = 1.8\ kg$
$M = 10\ kg$

What maximum amplitude of simple harmonic motion puts the smaller block on the verge of slipping over the larger block?

Maximum $f_{static} = \mu_s N = \mu_s mg = (0.40)(1.80)(9.8)$

$= 7.06\ N$ [7.2 N]

Now $|a_{max}| = \omega^2 x_{max}$ for simple harmonic motion.

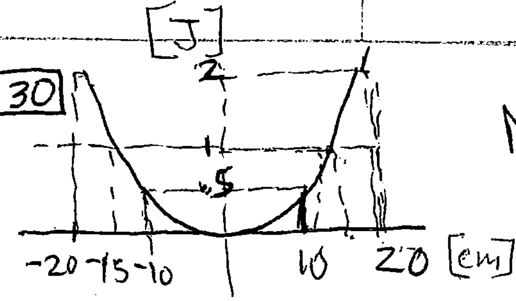
Here a_{max} at verge of slipping $= \frac{F}{m} = \frac{f_s}{m} = \frac{7.06\ N}{1.80\ kg} = 3.92\ m/s^2$ [$4.0\ m/s^2$]

We also need $\omega = \sqrt{\frac{k}{(M+m)}} = \sqrt{\frac{200\ N/m}{10\ kg + 1.8\ kg}} = 4.12\ \text{rad}/s$

$\therefore \omega^2 = 16.95\ \text{rad}^2/s^2$

Solve for $x_{max} = \frac{(a_{max})}{\omega^2} = \frac{3.92}{16.95} = \underline{\underline{0.23\ m}}$ [0.24 m]

15:30



Note that $\mathcal{U}_{SCALE} = 2\,J$

reading the graph:

$\mathcal{U} = 2 \rightarrow x = 20$ cm

$\mathcal{U} = 1 \rightarrow x = 15$ cm

$\mathcal{U} = .5 \rightarrow x = 10$ cm

$m = 2$ kg

$v = 85$ cm/s @ $x = 0$

The total mechanical energy, $K + \mathcal{U}$ is a constant. When the particle passes through $x = 0 \rightarrow$ equilibrium $\rightarrow$ the energy is all Kinetic, so

$$E = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(2\,kg)(.85\,m/s)^2$$

$$E = 0.72\,J$$

This energy will be all potential at the turning points. Elsewhere, the sum of $K + \mathcal{U}$ will always be $E_{MECH} = 0.72\,J$

Where will the potential energy $= 0.72\,J$? This will be the turning point for this particle. We know

$\mathcal{U}(x) = bx^2$ and we see $\mathcal{U}(10) = 0.5\,J$

Solve for b: $\quad b = \dfrac{\mathcal{U}(x)}{x^2} = \dfrac{0.5\,J}{100\,cm^2} = .005\,J/cm^2$

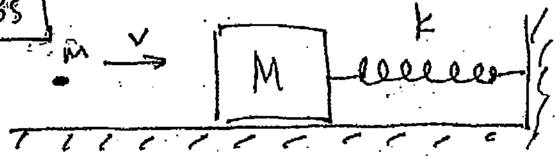
$\mathcal{U}(x) = (0.005\,J/cm^2)\,x^2 = 0.72\,J$ @ turning points

$$x = \sqrt{\frac{0.72}{0.005}} = 12\text{ cm}$$

So, YES, the particle turns back before reaching $x = 15$ cm.

$x = 12$ cm is the turning pt

15:35



$m = 9.5 \times 10^{-3}$ kg $\qquad$ $k = 6000$ N/m

$v = 630$ m/s $\qquad$ frictionless surface

$M = 5.4$ kg

Totally inelastic collision, assume spring doesn't compress until bullet is embedded.

a) Find the speed of the block just after the collision.

Momentum is conserved (no outside forces act)

$$P_{before} = P_{after}$$

$$mv = (M+m)\,V_{block}$$

$$\therefore \quad V_{block} = \frac{mv}{M+m} = \frac{(9.5\times10^{-3})(630)}{5.4 + 9.5\times10^{-3}} = \underline{\underline{1.11 \text{ m/s}}}.$$

b) Find the amplitude of the resulting simple harmonic motion.

The block, initially at rest, must start from its equilibrium position. Thus just after the collision, its total mechanical energy is all kinetic:

$$E_{\text{just after collision}} = \underbrace{KE}_{\frac{1}{2}(M+m)(V_{block})^2} + \underbrace{PE}_{0}$$

$$= \frac{1}{2}(5.4 + 9.5\times10^{-3})(1.11)^2$$

$$= 3.31 \text{ J}$$

Later, as the spring compresses, this KE goes down as energy is put into the potential energy of the spring; eventually KE = 0 at X_{max}:

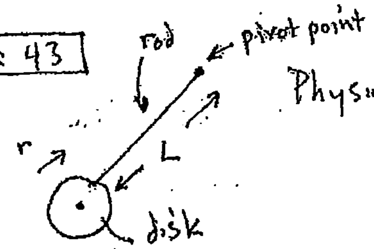
$$E_{after} = \underbrace{KE}_{0} + \underbrace{PE}_{\frac{1}{2}kX_{max}^2}$$

So $\frac{1}{2}kX_{max}^2 = 3.31$ J

$$X_{max}^2 = \frac{2(3.31)}{6000 \text{ N/m}} = 1.10\times10^{-3}$$

$$\therefore \underbrace{X_{max}}_{\text{amplitude}} = 0.033 \text{ m} = \underline{\underline{3.3 \text{ cm}}}$$

15:43



Physical pendulum

$r = 10.0\text{ cm} = 1.00\times10^{-2}\text{ m}$

$L = 500\text{ mm} = 5.00\times10^{-1}\text{ m}$

$M_{disk} = 500\text{ g} = 0.500\text{ kg}$

$M_{rod} = 270\text{ g} = 0.270\text{ kg}$

a) Calculate the rotational inertia of the pendulum about the pivot point.

$$I_{total} = \underset{\text{about end}}{I_{rod}} + \underset{\text{about c.m.}}{I_{disk}} + M_{disk} h^2 \quad \text{with } h = L + r$$

$$= \frac{1}{3} M_{rod} L^2 + \frac{1}{2} M_{disk} r^2 + M_{disk} (L+r)^2$$

$$= \frac{1}{3}(0.270)(0.5)^2 + \frac{1}{2}(0.500)(0.10)^2 + (0.500)(0.600)^2$$

$$= 0.0225 + 2.5\times10^{-3} + 0.180$$

$$= \underline{0.205\ \text{kg m}^2}$$

b) What is the distance between the pivot point and the center of mass of the pendulum? (We need this to find the restoring torque).

Call $x = 0$ the pivot point. Then the c.o.m. of the rod lies at $L/2$, and the c.o.m. of the disk lies at $r + L$.

Hence $$M_{cm} X_{cm} = m_1 x_1 + m_2 x_2$$

$$= m_{rod}\left(\frac{L}{2}\right) + m_{disk}(r + L)$$

$$= \underbrace{(0.270)\left(\frac{0.500}{2}\right)}_{0.0675} + \underbrace{(0.500)(0.500 + 0.100)}_{0.300}$$

Thus $$X_{cm} = \frac{0.3675}{(0.500 + 0.270)} = \frac{0.3675}{0.770} = \underline{0.477\text{ m}} \text{ from pivot}$$

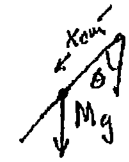
c) Calculate the period of oscillation.

Just as $\omega = \sqrt{\frac{k}{m}}$, here we have $\omega = \sqrt{\frac{K}{I}}$ where K is the restoring torque constant, as in $\tau = -K\theta$. The torque is provided by the weight of the center of mass at the distance X_{cm}:

$$\vec{\tau} = \vec{r} \times \vec{F} = -r_{cm} M g \sin\theta$$

$$\approx -\underbrace{X_{cm} M g}_{K}\, \theta \qquad \text{(small angle approximation) } \sin\theta \approx \theta$$

15:43 (cont.)



Then $\omega = \sqrt{\dfrac{x_{cm} M g}{I}}$

$= \sqrt{\dfrac{(0.477)(0.770)(9.8)}{0.205}} = 4.19$ rad/s

and so $T = 2\pi/\omega = \dfrac{2\pi}{4.19} = 1.4995\text{ s} = \underline{\underline{1.50\text{ s}}}$

[1.48 s if g = 10]

If you don't like this analogy, we could look at the restoring torque $\vec{\tau} = \vec{r} \times \vec{F} = -x_{cm} M g \sin\theta$

Take the small angle approximation ($\sin\theta \approx \theta$) so $\tau = -x_{cm} M g\, \theta$

Apply Newton's 2nd for torques:

$$\tau = I\alpha = I \frac{d^2\theta}{dt^2}$$

and write the equation like so:

$$-x_{cm} M g\, \theta = I \frac{d^2\theta}{dt^2}$$

$$I \frac{d^2\theta}{dt^2} + x_{cm} M g\, \theta = 0$$

which is identical to either the mass-spring or simple pendulum equation.

We assume
$$\theta(t) = A\cos\omega t$$
$$\frac{d\theta}{dt} = -\omega A \sin\omega t$$
$$\frac{d^2\theta}{dt^2} = -\omega^2 A\cos\omega t$$

and substitute:

$$I(-\omega^2 A\cos\omega t) + x_{cm} M g (A\cos\omega t) = 0$$

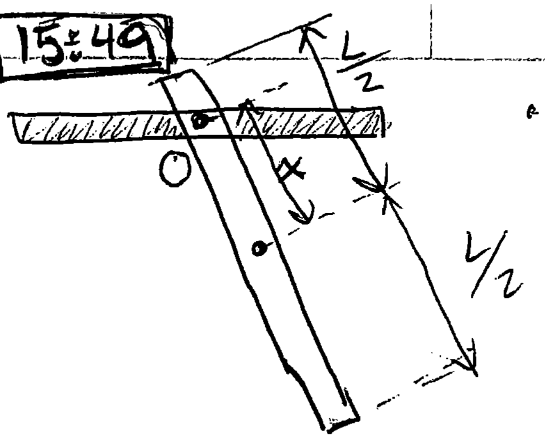
which implies that

$$-\omega^2 I + x_{cm} M g = 0$$

or $\omega^2 = \dfrac{x_{cm} M g}{I}$ so $\omega = \sqrt{\dfrac{x_{cm} M g}{I}}$

as before.

15.49



- Stick of length $L = 1.85$ m oscillates about point O.
- COM is located at $L/2$
- We can vary the distance x between the COM and point O

a) What distance x (between pivot and COM) will provide the LEAST period?

- We will have to minimize T, so we must write an expression for it in terms of x.

For a physical pendulum $T = 2\pi\sqrt{\frac{I}{mgh}}$

- h is x in this eqn.
- I, the moment of inertia, is ALSO a function of x

By parallel axis thm: $I = I_{com} + mh^2$

I_{com} of thin rod about its center is $\frac{1}{12}ML^2$

$$I = \frac{1}{12}ML^2 + Mx^2$$
$$= M\left(\frac{L^2}{12} + x^2\right)$$

so $$T = 2\pi\sqrt{\frac{\cancel{M}\left(\frac{L^2}{12}+x^2\right)}{\cancel{M}gx}} = 2\pi\sqrt{\frac{\left(L^2/12+x^2\right)}{gx}} = 2\pi\sqrt{\frac{L^2+12x^2}{12gx}}$$

$$T^2 = 4\pi^2\left(\frac{L^2+12x^2}{12gx}\right)$$

$$\frac{12gT^2}{4\pi^2} = \frac{L^2}{x} + 12x$$

$\frac{d}{dx}(\quad) = 0$ for minimum

15:49 (cont)

T is minimum for T^2 minimum

$$\frac{d}{dx}\left[\frac{12gT^2}{4\pi^2}\right] = 0 = \frac{d}{dx}\left[\frac{L^2}{x} + 12x\right]$$

$$0 = \frac{-L^2}{x^2} + 12$$

$$x = \sqrt{\frac{L^2}{12}}$$

$$x = \frac{1.85}{\sqrt{12}} = 0.53\,m$$

b) For $L = 1.85$ and $x = 0.53\,m$

$$T = 2\pi\sqrt{\frac{\cancel{M}\,.566}{\cancel{M}(9.8)(.53)}}$$

$$T = 2.07\,s$$

$$I = M\left(\frac{(1.85)^2}{12} + .53^2\right)$$

$$= M(.566$$