Capacitor: any two conductors separated by an insulator.



• In a circuit diagram, a capacitor is represented by:

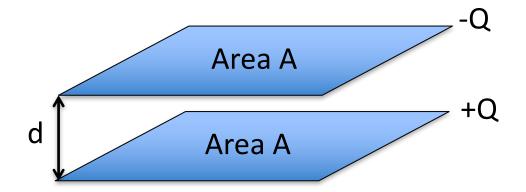


• Capacitance: $C = \frac{Q}{\Delta V}$ Measure of a capacitor's ability to store charge

Unit: Farad = F = C/V

Parallel-plate capacitor:

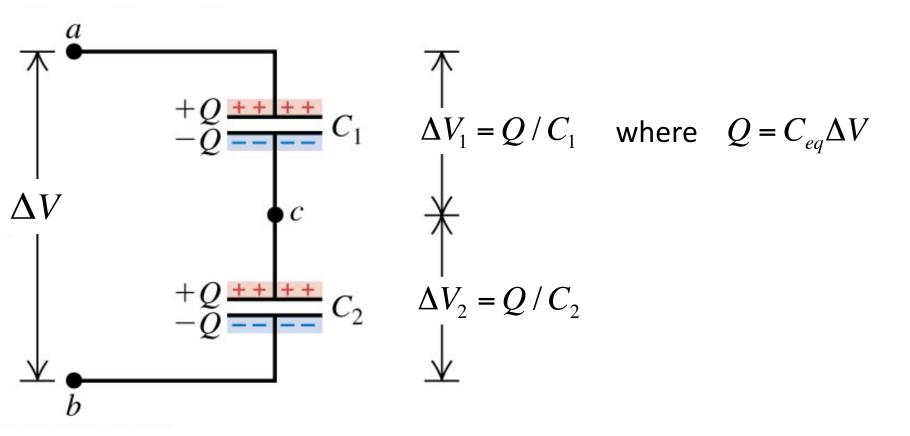
$$C = \frac{A\epsilon_0}{d}$$



Two capacitors in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

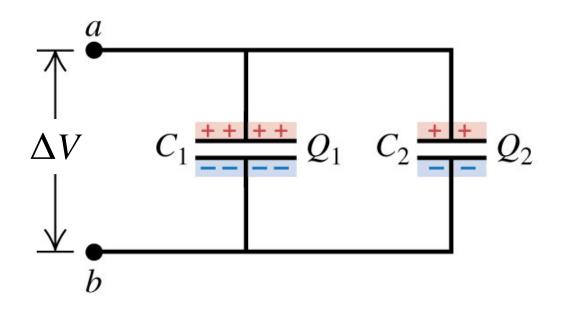
Capacitors in series store the same amount of charge:



Two capacitors in parallel:

$$C_{eq} = C_1 + C_2$$

Capacitors in parallel have the same voltage difference:



Charge stored in C_1 :

$$Q_1 = C_1 \Delta V$$

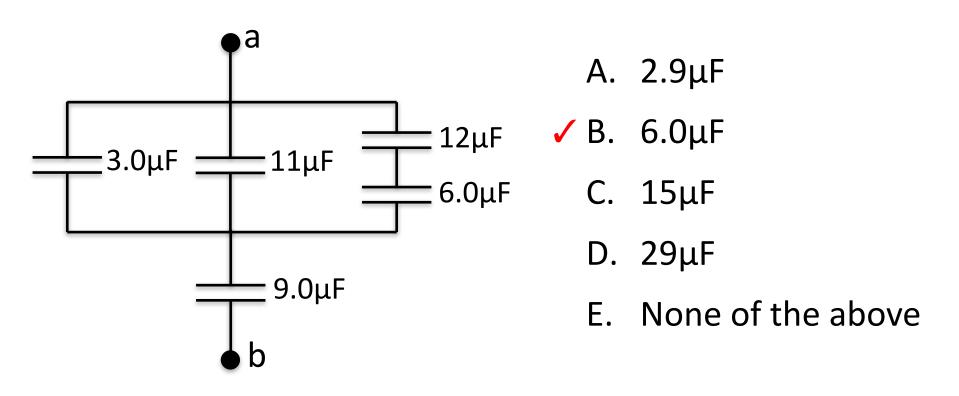
Charge stored in C_2 :

$$Q_2 = C_2 \Delta V$$

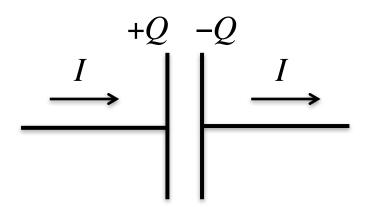
Total charge stored:

$$Q_{tot} = Q_1 + Q_2 = C_{eq} \Delta V$$

What is the equivalent capacitance of the five-capacitor network shown below?



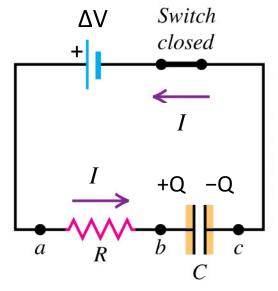
 Charge on a capacitor is related to the current into/out of the capacitor:



Mathematically:

$$Q = \int I \, dt$$
 and $I = \frac{dQ}{dt}$

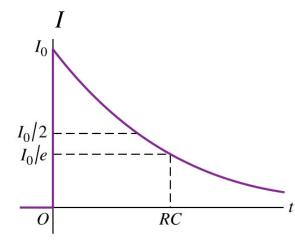
Capacitor charging with constant voltage source:



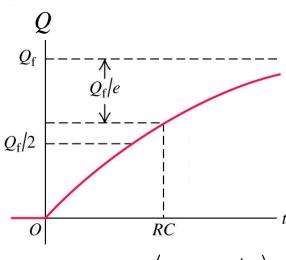
When the switch is closed, the charge on the capacitor increases over time while the current decreases.

Characteristic time: $\tau = RC$

 Time it takes for the current to decrease to 1/e of the initial value

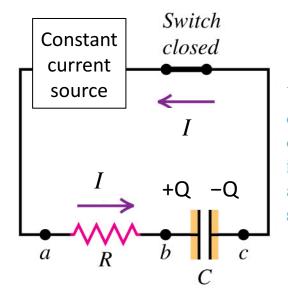


$$I(t) = I_0 e^{-t/\tau}$$



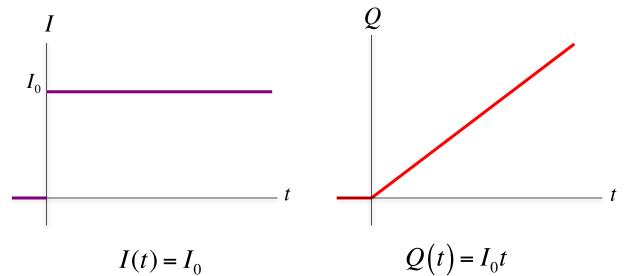
$$Q(t) = Q_{final} \left(1 - e^{-t/\tau} \right)$$

Capacitor charging with constant current source:

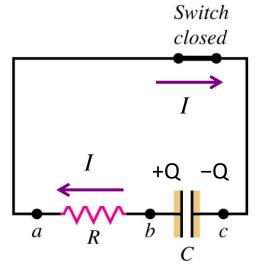


When the switch is closed, the charge on the capacitor increases linearly and the current stays constant.

No characteristic time because the current is constant and the charge on the capacitor keeps increasing.



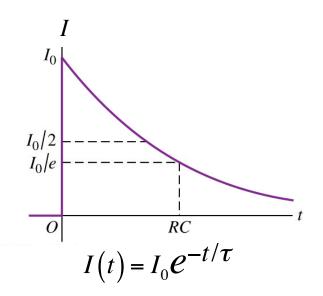
Capacitor discharging:

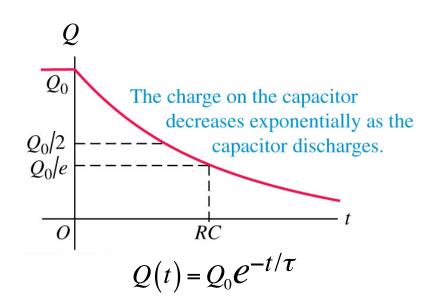


When the switch is closed, the charge on the capacitor and the current both decrease over time.

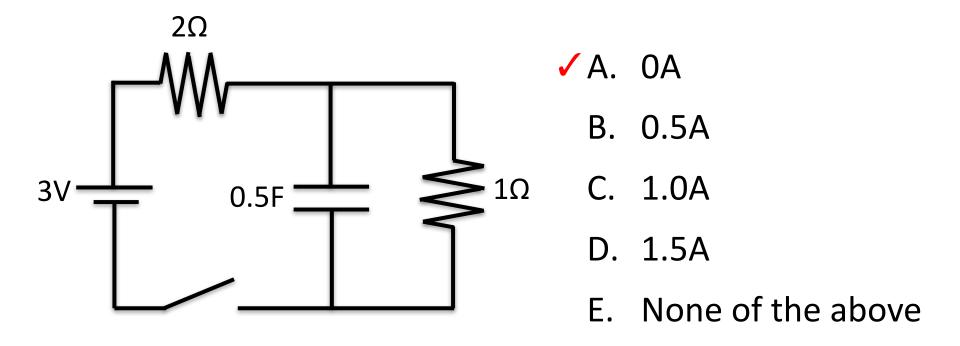
Characteristic time: $\tau = RC$

- Time it takes for the current to decrease to 1/e of the initial value
- Time it takes for the charge to decrease to 1/e of the initial value



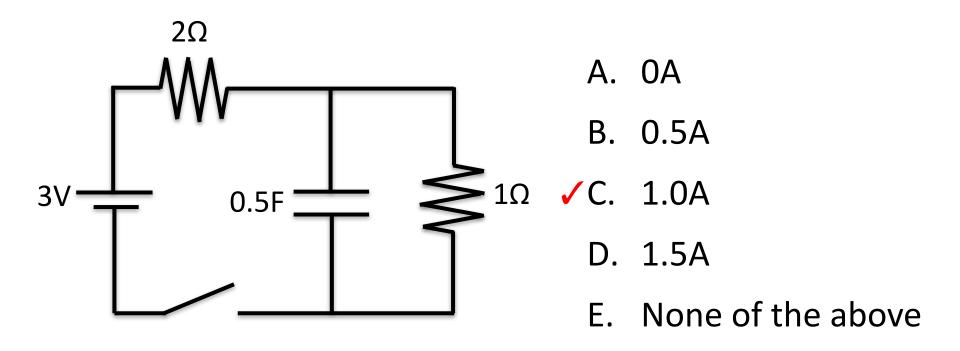


What is the current through the 1Ω resistor immediately after the switch is closed?



When a capacitor is uncharged, the potential difference is zero. So it behaves like a conducting wire at that moment.

What is the current through the 1Ω resistor a long time after the switch is closed?



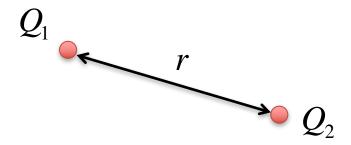
When a capacitor is fully charged, it allows no current to flow through it.

- **Electric force**: force between charges
- Three key facts about electric charge & electric force:
 - 1. Charges come in two flavors: positive & negative
 - 2. Two positive charges or two negative charges repel
 - 3. A positive and a negative attract

SI unit of charge: Coulomb (C)

• Coulomb's Law: describes force between two point charges

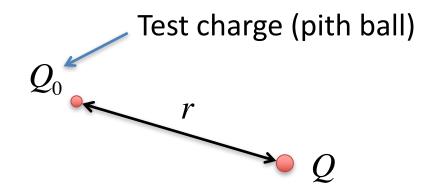
$$F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r^2}$$
 where $\varepsilon_0 = 8.85 \times 10^{-12} C^2 / N \cdot m^2$ (permittivity of free space)



Direction of the force:

Toward each other if q_1 and q_2 have the same sign. Away from each other if q_1 and q_2 have opposite signs.

 Electric field: The electric field is the electric force that a test charge (like the pith ball) would feel (at that particular location) divided by the charge on the ball.

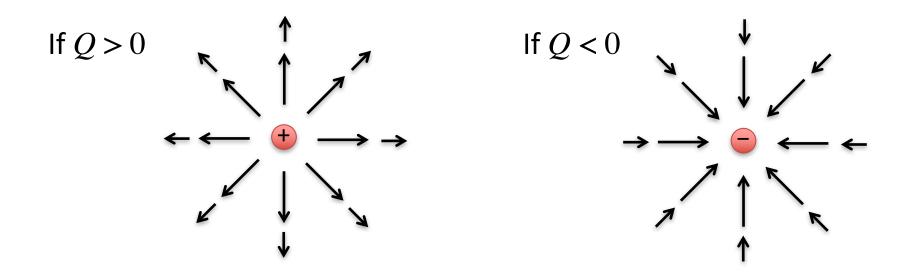


Mathematically:

$$E = \frac{F}{Q_0} = \frac{\left(\frac{1}{4\pi\varepsilon_0} \frac{Q_0 Q}{r^2}\right)}{Q_0} \qquad \Longrightarrow \qquad E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

Independent of Q_0 . So we consider the electric field as something that exists all around Q even in the absence of the test charge.

• Graphically, the electric field of a point charge Q looks like:



• Electric force on a test charge Q_0 :

$$\vec{F} = Q_0 \vec{E}$$

- Three ways to calculate the electric field:
 - 1. For a point charge, use <u>Coulomb's law</u>:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

2. For a charge distribution with a high degree of symmetry, use Gauss's law:

$$EA = \frac{q_{enclosed}}{\varepsilon_0}$$

3. When 1 & 2 fail, but you can break the charge distribution up into parts, use the principle of superposition.

- Tips for finding the electric field using Gauss's law:
 - 1. Identify the symmetry of charge distribution.
 - 2. Pick a Gaussian surface that reflects the symmetry of the charge distribution. (If the Gaussian surface is chosen properly, electric field should be either parallel or perpendicular to the surface.)
 - 3. Find the charge enclosed by the Gaussian surface (q_{enclosed}).
 - 4. Find the area of the Gaussian surface (A) where the electric field is perpendicular to the surface.
 - 5. Simplify the expression $EA = q_{enclosed} / \varepsilon_0$, and solve for E.

- Three types of charge densities:
 - 1. Linear charge density:

$$\lambda$$
 = charge per unit length $\Longrightarrow q = \lambda l$

2. Surface charge density:

$$\sigma$$
 = charge per unit area $\Rightarrow q = \sigma A$

3. Volume charge density:

$$\rho$$
 = charge per unit volume $\implies q = \rho V$

For which of the following charge distributions would Gauss's law *not* be useful for calculating the electric field?

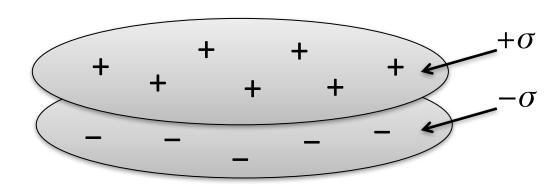
- A. a uniformly charged sphere of radius R
- B. a spherical shell of radius R with charge uniformly distributed over its surface
- ✓ C. a right circular cylinder of radius R and height h with charge uniformly distributed over its surface
 - D. an infinitely long circular cylinder of radius R with charge uniformly distributed over its surface
 - E. Gauss's law would be useful for finding the electric field in all of these cases.

 The principle of superposition: total electric field is the vector sum of the electric fields generated by charged objects in space.

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_N$$

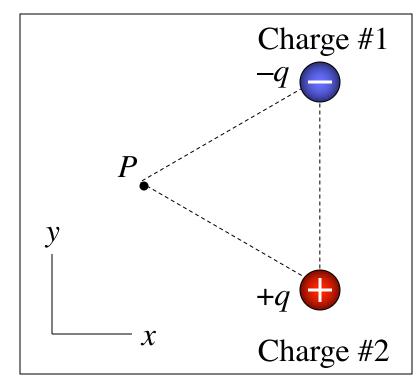
where \vec{E}_i = electric field due to ith charged object

Example: Electric field due to two infinite charged planes is the vector sum of the electric field due to the positively charged plane and the electric field due to the negatively charged plane.



Two point charges and a point P lie at the vertices of an equilateral triangle as shown. Both point charges have the same magnitude q but opposite signs. There is nothing at point P.

The net electric field that charges #1 and #2 produce at point *P* is in



A. the +x-direction.

B. the –*x*-direction.

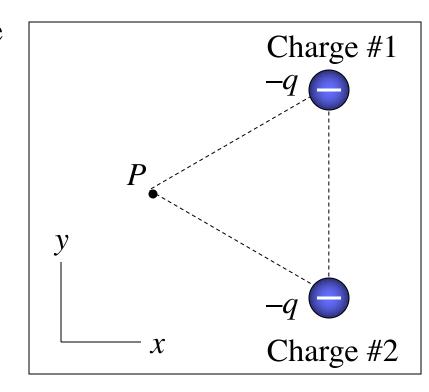
 \checkmark C. the +y-direction.

D. the -y-direction.

E. none of the above

Two point charges and a point P lie at the vertices of an equilateral triangle as shown. Both point charges have the same negative charge (-q). There is nothing at point P.

The net electric field that charges #1 and #2 produce at point *P* is in



 \checkmark A. the +x-direction.

B. the -x-direction.

C. the +y-direction.

D. the –y-direction.

E. none of the above

 Electric potential (also called voltage): electric potential energy of a test charge divided by its charge:

$$V = \frac{U}{q_{test}} \qquad \text{(Unit: volt = V)}$$

Note: Electric potential energy (U) and electric potential (V) are related but they are not the same. When we say "potential", we mean electric potential.

• Electric field and electric potential are related:

$$V = -\int E \ dx \quad \langle \longrightarrow \rangle \quad E_x = -\frac{dV}{dx}$$

The difference in the electric potential between two points can be calculated using

$$\Delta V = -\int_{0}^{b} E \ dx \qquad a \longrightarrow b$$

• Electric potential due to a point charge:

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

 If you have multiple point charges, you can calculate the total electric potential as

$$V = V_1 + V_2 + ... + V_N$$
 where $V_i =$ electric field due to ith charge

Example: Suppose two point charges q_1 =+1C and q_2 =-1C are separated by 10.0cm. Which one of the following correctly describes the relationship between the values of electric potential at point a, b, and c?

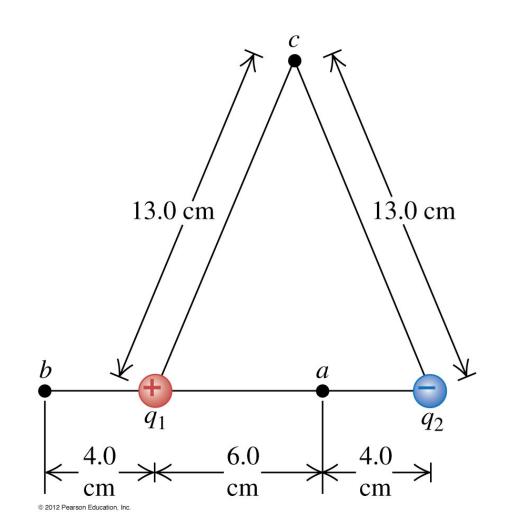
A.
$$V_a > V_b > V_c$$

B.
$$V_a > V_c > V_b$$

C.
$$V_b > V_a > V_c$$

$$\checkmark$$
 D. $V_b > V_c > V_a$

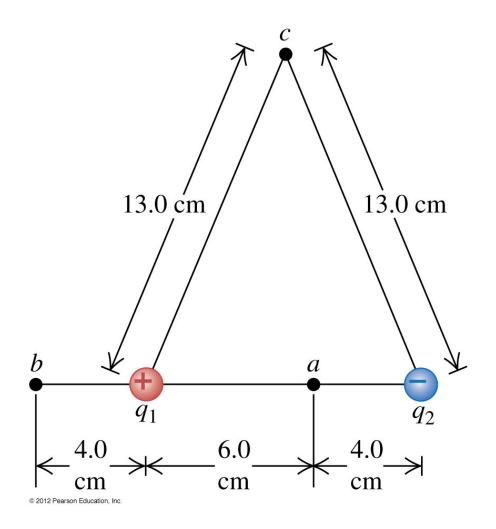
$$E. V_c > V_a > V_b$$



Example: The electric potential at point c is zero. Does that imply that the electric field is also zero at point c?

A. Yes

✓ B. No



Important: Electric field is negative slope of the electric potential.

Energy stored in a capacitor:

$$U = \frac{1}{2}Q\Delta V = \frac{1}{2}C\Delta V^2 = \frac{Q^2}{2C}$$

Power dissipated by a resistor:

$$P = I\Delta V = \frac{\left(\Delta V\right)^2}{R} = I^2 R$$

- In a conductor, charges would move until the electric field inside is zero.
- When charges stop moving in a conductor:
 - 1. The electric field is zero inside.
 - 2. The electric potential is constant inside.
 - 3. Any excess charge is on the surface. Inside is neutral.

- **Electric dipole**: two charges, +q and -q, separated by a fixed distance d.
 - \vec{d}
- Electric dipole is characterized by a quantity called <u>electric dipole</u> moment:

$$p = qd$$
 or $\vec{p} = q\vec{d}$

• Electric potential energy of a dipole in the presence of external electric field, \vec{E} : +q \uparrow \vec{E}

$$U = -\vec{p} \cdot \vec{E} = -pE\cos\theta$$

In a uniform external electric field:

$$\vec{F} = 0$$
 but $\tau = -\frac{dU}{d\theta} = pE\sin\theta$

If a dipole is free to move, it will rotate until θ =0.

In a non-uniform electric field,

$$F = p \frac{dE}{dx} \qquad \text{(assuming } \theta = 0\text{)}$$

 An electric dipole creates its own electric field and electric potential. You can calculate them using

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$
 and $V = V_+ + V_-$

 Their general expressions are complicated. But if you are far from the dipole, the electric potential is approximately equal to

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$