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Phy 202
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HW per 10/1/17
Q12D.4 Q12D.3

Q12D.4

In this problem we will consider the covalent bond, which illustrates how "sharing" a quantum can lead to an attractive force that has no easy classical explanation. Consider an H_2^+ molecule, where a pair of protons share a single electron. In order to make the problem tractable, we will ignore the repulsion between protons and model their interaction with a shared electron as if the protons were two square wells separated by a small distance, each about 0.1 nm wide.

Run SchroSolver and choose "Symmetric Well" PE function and a fixed well width of 0.5 nm.

a) Start with a center-to-center well separation of 0.75 nm. Show that the symmetric EEF has a slightly lower energy than the antisymmetric one.

As we can see from the SchroSolver results, the first energy for the symmetric case is about 0.9839 eV, whereas the first energy for the asymmetric case is about 0.9968 eV

b) Explain qualitatively (physically) why this must be so

The curvature of a wave function $\psi(x)$ is $K(x) = \psi''(x)/\psi(x)$. The local wavelength of the wave function $\psi(x)$ is

$[\lambda(x)]^2 = \frac{-4\pi}{K(x)}$

The energy of the wavefunction at a given x is

$[E(x)]^2 = \frac{(hc)^2}{[\lambda(x)]^2} = \frac{hc \cdot c(x)}{-4\pi} = -\frac{hc\psi''(x)}{4\pi\,\psi(x)}$

So the magnitude of the energy is just a constant multiple of the curvature of the wavefunction, that is, the greater the curvature of the wave function, the greater the energy of the wavefunction.

We

to well

We can see from the graphs that the curvature of both wave functions are about the same (in magnitude) except on the interval corresponding to the "middle" in between the two wells (this interval is approximately [-7.5nm, 7.5nm]). On this interval, the magnitude of the wave function is greater at every point for the symmetric case than the asymmetric case. The second derivatives of both wave functions will have about the same magnitude at all points, so we can see that $\frac{\psi''_{sym}(x)}{\psi_{sym}(x)} < \frac{\psi''_{asym}(x)}{\psi_{asym}(x)}$ on [-7.5nm, 7.5nm]. This is what makes the energy of the asymmetrical wave function slightly larger than the symmetrical one (because larger curvature means larger energy).

c) Since the symmetric E.F.F is the true ground state for this system, select this type of function and vary the wells' center to center separation from 0.55nm in increments of 0.05nm up to 1.10nm, and make a chart of ground state energies; you should see that the system's energy increases as the ten wells overlap.

(Chart) Ground Energy	Center to center distance (nm)
0.716308	0.55
0.862742	0.69
0.941987	0.65
0.977931	0.72
0.983946	0.76
0.987161	0.8
0.988889	0.84
0.989821	0.89
0.990461	0.96
0.990744	1.01
0.990823	1.05
0.990889	1.10

d) Explain why this works.

e) Explain why this means that sharing the electron creates an attractive force between the wells.

The system's energy increases when the wells are farther apart. This means that it takes positive work to move the wells further apart (PE is higher) so the wells will be "pushed" back together by the strong force.

f) Show that if the electron is in the antisymmetric state, sharing an electron leads to a repulsive force between the wells, and explain why this is the case.

Using Schro Solver, we can see that the energy decreases as the wells move apart. This means it takes negative work to move the wells apart (PE is decreasing) so the force is repulsive between the wells.

Q13 D.3

One can calculate the gravitational binding energy of a spherical object as follows. Assume constant density ($\rho = \frac{M}{\frac{4}{3}\pi R^3}$). Imagine disassembling the earth by removing a thin shell of thickness dr and so on.

a) Show that the energy required to move a shell of thickness dr away from the earth when its radius has already been pulled down to r is then $E = \frac{3GM^2 r^4 dr}{R^6}$

The gravitational potential energy at the system is $E_a = \frac{G \cdot M \cdot m}{r_0}$, where M is the earth's mass, m is the shell's mass

The potential energy at the bound system is $E_b = \frac{G \cdot M^3 m}{}$

The potential energy of the unbound system (separated by infinite distance) is

$E_{parts} = \frac{G \cdot M \cdot m}{\infty} = 0$

So the work required to move the shell away from the earth is

$\Delta U = E_b - E_{parts} = \frac{G \cdot M \cdot m}{r}$ where r is the pulled down radius of the earth

$M = \rho \cdot \frac{4}{3} \cdot \pi r^3 = \frac{3M}{4\pi R^3} \cdot \frac{4}{3}\pi r^3 = \frac{Mr^3}{R^3}$

$m = \rho \cdot \frac{4}{3}\pi (r^3 - (r-dr)^3)$. Using the binomial approximation this becomes

$m = \rho \cdot \frac{4}{3}\pi (r^3 - r^3(1-\frac{dr}{r})^3) \approx \frac{M}{R^3} \cdot (r^3 - r^3(1-3\frac{dr}{r})) = \frac{M}{R^3} \cdot (r^3 - r^3 + 3 \cdot r^2 dr)$

So $m = \frac{3Mr^2 dr}{R^3}$

So $E_b = \frac{G \cdot M \cdot m}{r} = \frac{G \cdot M \cdot r^3 \cdot 3 \cdot M \cdot r^2 \cdot dr}{R^3 \cdot R^3 \cdot r} = \boxed{\frac{G \cdot 3 \cdot M^2 \cdot r^4 dr}{R^6}}$

b) Do an appropriate integral to find the gravitational binding energy of the sphere.

The binding energy in this case is equal to the sum of the energies it takes to move all shells. We construct a Riemann sum:

$\sum_{n=1}^{\infty} \frac{3G \cdot M^2 \cdot r_n^4}{R^6} \Delta r$. In this case we want Δr to depend on n.

The interval of our sum is $r=0$ to $r=R$, so $\Delta r = \frac{R-0}{n} = \frac{R}{n}$

So we get an integral $\int_0^R \frac{3G M^2 r^4}{R^6} dr = \frac{3 \cdot G \cdot M^2 \cdot r^5}{R^6} \cdot \frac{1}{5} \Big|_0^R = \frac{3 \cdot G \cdot M^2 R^5}{5 R^6}$

c) Use this to find the earth's gravitational mass deficit (in kg) assuming the system's constituent parts are its atoms.

$$\Delta M = \frac{\text{Binding energy}}{c^2} \Rightarrow \Delta M = \frac{3GM^2}{5R} \Big/ c^2 = \frac{3 \cdot 6.67\cdot 10^{-11} \frac{N\cdot m^2}{kg^2} \cdot (5.98\cdot 10^{24} kg)^2}{5 \cdot 6.38\cdot 10^{6} m \cdot (3.0\cdot 10^{8} m/s)^2}$$

$$\Rightarrow \frac{3\cdot 6.67\cdot (5.98)^2}{5\cdot 6.38\cdot 9} \cdot \frac{kg\cdot\frac{m}{s^2}\cdot m^2/kg^2 \cdot kg^2}{m \cdot \frac{m^2}{s^2}} \cdot \frac{10^{-11}\cdot 10^{48}}{10^{6}\cdot 10^{16}}$$

$$\Rightarrow 2.49\cdot 10^{15} \frac{\cancel{kg^3\cdot m^3 \cdot s^2}}{\cancel{s^2\cdot kg^2\cdot m^3}} = \boxed{2.49\cdot 10^{15} kg}$$

Exam 0 answers:

Q12D4

Couldn't figure out part d, otherwise good

Q13D3

Right units, reasonable magnitude.