Quantum Physics
Phy 233
Charles Cunningham

Hw for 10/13/17
Q9M.5, Q9M.7, Q10M.4

1

Q9M.5

Suppose a quanton's wavefunction at a given time is $\psi(x) = A \cdot e^{-\frac{1}{2}(x/a)^2}$, where A is an unspecified constant and $a = 1.5$nm. According to the table of integrals, $\int_{-\infty}^{\infty} [e^{-(x/a)^2}]\,dx = a\sqrt{\pi}$

If we perform an experiment to locate the quanton at this time, what would be the probability of a result within ± 0.1nm of the origin?

The probability that a quanton will be determined to be within a certain range (a, b) is
$\Pr(\Delta x) = \int_a^b |\psi(x)|^2\,dx \,/ \int_{-\infty}^{\infty} |\psi(x)|^2\,dx$

In this case, $\psi(x) = A \cdot e^{-\frac{1}{2}(x/a)^2}$ so $|\psi(x)|^2 = A^2 \cdot e^{-(x/a)^2}$

We know that $A^2 \int_{-\infty}^{\infty} e^{-(x/a)^2}\,dx = A^2 a\sqrt{\pi}$.

Let $(a,b) = (-0.1\text{nm}, +0.1\text{nm})$. So $\int_a^b |\psi(x)|^2\,dx = A^2 \cdot \int_{-0.1\text{nm}}^{0.1\text{nm}} e^{-(x/a)^2}\,dx$

Notice that between -0.1 and 0.1, $e^{-(\frac{x}{a})^2} \approx 1$. So we can say that
$A^2 \cdot \int_{-0.1\text{nm}}^{0.1\text{nm}} e^{-(x/a)^2}\,dx \approx A^2 \int_{-0.1\text{nm}}^{0.1\text{nm}} 1\,dx = A^2 \cdot x \Big|_{-0.1\text{nm}}^{0.1\text{nm}} = A^2 \cdot 0.2_{\text{nm}}$

So $\Pr(\Delta x) = \frac{A^2\, 0.2\text{nm}}{A^2\, 1.5\text{nm} \cdot \sqrt{\pi}} = \frac{0.2\text{nm}}{1.5\text{nm} \cdot \sqrt{\pi}} = 0.075$

Q9M.7

Imagine that a quanton's wavefunction at a given time is $\psi(x) = A \cdot e^{-|x/a|}$, where A is an unspecified constant and $a = 2.0_{\text{nm}}$. If we were to perform an experiment to locate the quanton at this time what would be the probability of a result within $\pm 2.0_{\text{nm}}$ of the origin

Because of the absolute value, the function's value at $-x$ is the same as at $+x$

So in this case, $\int_{-c}^{c} [e^{-|x/a|}]^2\,dx = 2\int_0^c [e^{-x/a}]^2\,dx = 2\int_0^c e^{-2x/a}\,dx$

$2\int_0^c e^{-2x/a}\,dx = 2 \cdot \frac{a}{-2} e^{-2x/a}\Big|_0^c = -a \cdot e^{-2x/a}\Big|_0^c = -a \cdot e^{-\frac{2 \cdot c}{a}} + a \cdot e^0 = a(1 - e^{-\frac{2 \cdot c}{a}})$

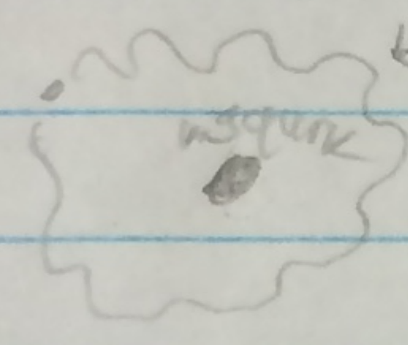
$\boxed{\text{This}}$. The table of integrals gives me $\int_0^{\infty} e^{-ax}\,dx = \frac{1}{a}$, for $\text{Re}(a) > 0$.

So $2 \cdot \int_0^{\infty} e^{-x(\frac{2}{a})}\,dx = 2 \cdot \left(\frac{1}{\frac{2}{a}}\right) = 2 \cdot \frac{a}{2} = a$.

given that $C = 2.0_{\text{nm}}$, $\Pr(\Delta c) = \frac{2.0\text{nm}\,(1 - e^{-\frac{2 \cdot 2.0\text{nm}}{2.0\text{nm}}})}{2.0\text{nm}} = 1 - e^{-2} \approx \boxed{0.86}$

Q10M.4

The potential energy of the strong interaction between two quarks has the approximate form $V(r) = br$, where b is some constant. Assume we have a light quark of mass m interacting with a very massive quark. Use the same basic approach we used for the Bohr model, find the possible energies and orbital radii of the system. If the light quark's rest energy $mc^2 = 310$ MeV for the light quark and $b = 160$ kN, estimate the minimum radius of the light quark's "orbit".



← little quark wave function

The model we are using assumes that the little quark will "orbit" around the big quark with a well defined and constant wavelength $\lambda = \frac{h}{|\vec{p}|}$ for a given orbital radius r.
Because λ must be constant for a given r, if the little quark starts at some position in its oscillation and completes one full orbit, it should end at that same position, meaning the distance travelled is equal to an integer multiple of the wavelength. So $2\pi \cdot r = n\lambda$, $n = 1, 2, 3, \ldots$

The rate of change of the little quark's momentum is $\left|\frac{d\vec{p}}{dt}\right| = \frac{m|\vec{v}|^2}{r}$.
This is equal to the magnitude of the force exerted on it, $F = \left|\frac{dV}{dr}\right| = b$.
So $\frac{m|\vec{v}|^2}{r} = b \Rightarrow r \cdot m \cdot |\vec{v}|^2 = r \cdot m \cdot b \Rightarrow |\vec{p}|^2 = r \cdot m \cdot b$
Using the de Broglie hypothesis, $|\vec{p}| = \frac{h}{\lambda}$, and using our expression for λ we get $r \cdot m \cdot b = \frac{h^2}{\frac{4\pi^2 r^2}{n^2}} \Rightarrow r \cdot m \cdot b = \frac{\hbar^2 \cdot n^2}{r^2} \Rightarrow r^3 = \frac{\hbar^2 \cdot n^2}{m \cdot b} \Rightarrow r_n = \left(\frac{\hbar^2 \cdot n^2}{m \cdot b}\right)^{\frac{1}{3}}$

The classical energy for our quark is $E = \frac{1}{2} \cdot m \cdot |\vec{v}|^2 + V(r) = \frac{1}{2} \cdot m|\vec{v}|^2 + b \cdot r$
We derived above that $m|\vec{v}|^2 = b \cdot r$. So, $E = \frac{1}{2} \cdot b \cdot r + b \cdot r = \frac{3}{2} b \cdot r$

$$E_n = \frac{3}{2} \cdot b \cdot \left(\frac{\hbar^2 n^2}{m \cdot b}\right)^{1/3} = \left(\frac{27 \cdot \hbar^2 \cdot n^2 \cdot b^3}{m \cdot 8 \cdot b}\right)^{\frac{1}{3}} = \frac{3}{2} \cdot \left(\frac{\hbar \cdot n \cdot b^2}{m}\right)^{1/3}$$

8 m

We are given that $b = 150\,kN$, and for $n=1$ $E_1 = m \cdot c^2 = 310\,MeV$.
So $m = \frac{310}{c^2} \cdot 10^6\,eV$

$$r_1 = \left(\frac{\hbar^2 \cdot c^2}{310 \cdot 10^6\,eV \cdot 1.602 \cdot 10^{-19}\,\frac{J}{eV} \cdot 150000\,N}\right)^{\frac{1}{3}} = \boxed{5.12 \cdot 10^{-16}\,m = 0.512\,fm}$$

Even # answers

Q9m.5 Right units, reasonable magnitude

Q9m.7: Right units reasonable magnitude

Q10m.4: Right units, reasonable magnitude