Homework Assignment 4

PHYSICS 314 - THERMODYNAMICS & STATISTICAL PHYSICS (Spring 2018)

Due Friday, March 2nd, by noon, Noyce 1135

I cannot award full credit for work that I am unable to read or follow. For my benefit and for yours, please:

- *Write neatly*
- *Show and EXPLAIN all steps*
- *Make diagrams large and clearly-labeled*

You are welcome to collaborate with others on this assignment. However, the work you turn in should be your own. Please cite collaborators and outside sources. See the syllabus for details.

Regardless of the number of parts, all homework problems are weighted equally. Regardless of the number of questions, all homework assignments are weighted equally.

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1) Derive the Sackur -Tetrode equation (Schroeder (2.49)) from the equation for the multiplicity of an ideal gas (Schroeder (2.40)). *Hint: Use techniques from HW3. Ignore the merely large* $\sqrt{2\pi N}$ *term in Stirling's approximation.*

2)

a) Use the Sackur-Tetrode equation to calculate the entropy of a mole of argon gas at room temperature and atmospheric pressure.

b) Is the entropy greater for a mole of argon or a mole of helium under the same conditions? (You do not have to calculate the entropy of the helium case.) If you use an equation in your explanation, be sure to also explain conceptually.

3) According to the Sackur-Tetrode equation, the entropy of a monatomic ideal gas can become negative when its temperature (and hence its energy) is sufficiently low. The Sackur-Tetrode equation must be invalid at very low temperatures.

a) Use the definition of entropy to explain why entropy must be positive for any accessible state.

b) Suppose you start with a sample of helium at room temperature and atmospheric pressure, then lower the temperature holding the density fixed. For the purposes of this problem, assume that the helium remains a gas and does not liquefy. (*Note: This is not an accurate assumption.*) Find the temperature below which the Sackur-Tetrode equation predicts that *S* is negative.

4) For each of the following irreversible processes, explain in a sentence or two how you can tell that the total entropy of the Universe has increased.

a) Stirring salt into a pot of soup

b) Scrambling an egg

c) Humpty Dumpty (the nursery rhyme character) having a great fall

d) A wave hitting a sand castle

e) Cutting down a tree

f) Burning gasoline in an automobile

5)

a) A particle of mass *m* is free to move in one dimension. Denote its position coordinates by *x* and its momentum by *p*. Suppose that this particle is confined within a box such that $0 \le x \le L$. Suppose that its energy is known to be between *E* and *E + dE*. Draw the classical position-momentum phase space of this particle. Be sure to indicate the regions of this phase space which are accessible to the particle. *Hint: This is similar to what we did in class with the hypersphere, except it is in 1-D.*

b) Now consider a system consisting of two weakly interacting particles, each of mass m, that are free to move in <u>one</u> dimension. Denote the respective position coordinates of the two particles by x_1 and x_2, and their respective momenta by p_1 and p_2. Both particles are confined within a box such that $0 \leq x_i \leq L$ for i=1,2. The *total* energy of the system is known to lie between E and E + dE. Instead of trying to draw a four-dimensional phase space, draw separately the part of phase space involving x_1 and x_2 (position-space), and that involving p_1 and p_2 (momentum-space). Indicate on these diagrams the regions of phase space accessible to the system.

6)

a) Start with the full expression for the multiplicity for the <u>*large*</u> Einstein solid (Schroeder (2.17)), and show that in the low-temperature limit, where $q \ll N$, the multiplicity is approximately equal to $\left(\frac{Ne}{q}\right)^q$. *Hint: Ignore the merely large $\sqrt{2\pi N}$ term in Stirling's approximation.*

b) Starting with the multiplicity for an Einstein solid in the "low-temperature" limit from part a), find a formula for the temperature of an Einstein solid in the limit $q \ll N$. Solve for the energy as a function of temperature to obtain $U = N\epsilon e^{-\epsilon/kT}$ (where ϵ is the spacing between energy levels).

7)

a) Starting with the result from part b) of the previous problem, calculate the heat capacity of an Einstein solid in the low-temperature limit.

b) Now find the heat capacity in the <u>high</u>-temperature limit. *Hint: The equipartition theorem holds in the high-temperature limit. Leave your answer in terms of f.*

c) On the same plot, sketch both the low-temperature limit (part a) and high-temperature limit (part b) expressions for heat capacity as a function of temperature. For the low-temperature limit, indicate the maximum value of C_V and the T value at which the maximum occurs.
(Note: Measurements of heat capacities of actual solids at low temperatures do not confirm the prediction that you make in this problem. We will develop a more accurate model of solids at low temperatures later this semester.)

8) On HW 3 you found the following expression for the multiplicity of an Einstein solid containing N oscillators and q energy units.

$$\Omega \approx \frac{\left(\frac{q+N}{q}\right)^q \left(\frac{q+N}{N}\right)^N}{\sqrt{2\pi q \frac{q+N}{N}}}$$

a) Find an expression for the entropy of an Einstein solid as a function of N and q. *Hint: Use only the numerator of the formula for multiplicity. <u>Explain why this is justified if both</u>* <u>N</u> *<u>and</u>* <u>q</u> *<u>are large</u>.*

b) Use the result of part a) to calculate the temperature of an Einstein solid as a function of its energy. (The energy is $U = q\epsilon$, where ϵ is the constant spacing between energy levels). Be sure to simplify your result as much as possible.

c) Invert the relation you found in part b) to find the energy as a function of temperature, then differentiate to find a formula for the heat capacity.

d) Show that the heat capacity is $C = Nk$ in the limit $T \to \infty$. Is this the result you would expect for a 1-D oscillator? Explain. *Hint: When x is very small, $e^x \approx 1 + x$.*

9) List *three* main ideas from this homework assignment. For example, you could write a few-sentence explanation of a concept, or list an equation and explain the variables and in what circumstances the equation applies.
The goal is for you to review and to reflect on the big picture. Think about what you might want to remember when you look back at this homework before the test. I hope that this will be useful for your studying. I am not looking for anything specific here; you will be graded on effort and completion.