Written Assignment 2: Due Friday, February 9

Note: On this homework, be sure to use the formal definitions of injective and surjective given in Definition 1.6.8. Also, carefully examine the two bullet points before Proposition 1.6.9 that explain how to prove that a function is injective or surjective.

Problem 1: Consider the function $f\colon \mathbb{Z} \to \mathbb{Z}^2$ given by $f(n) = (3n^2 - 77, 5n + 6)$.
a. Is f injective? Justify your answer carefully.
b. Is f surjective? Justify your answer carefully.

Problem 2: Suppose that A, B, and C are sets and that both $f\colon A \to B$ and $g\colon B \to C$ are surjective functions. Show that the function $g \circ f\colon A \to C$ is surjective.
Hint: You are trying to prove that the function $g \circ f\colon A \to C$ is surjective. Following the guidelines before the proof of Proposition 1.6.9, you should start by taking an arbitrary $c \in C$. With this c in hand, your goal is to build an $a \in A$ with $(g \circ f)(a) = c$.

Problem 3: Suppose that we have a function $f\colon \mathbb{R} \to \mathbb{R}$ with the property that $f(x \cdot y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$. Suppose that $f(2) = 5$ and $f(3) = 7$. What is $f(\frac{1}{6})$? Explain.
Hint: What can you say about $f(1)$?