Worksheet 09 - An Approach to Organizing a Proof Attempt

The Basic Strategy

1. Write the statement to be proved.
2. Write the negation or contrapositive of the statement to be proved.
3. List the conclusions at which you hope to arrive.
4. List the assumptions you are given as your hypotheses.
5. Write any claim, calculation, or declaration that follows from previous steps.
6. Repeat Step 5 until you reach all of your conclusions.

More Detail

1. Write the statement to be proved (or disproved). To prove (or disprove) this statement, as is: **skip Step 2**.
2. Write the negation or contrapositive of the statement to be proved. **Once you do this, you have to chose <u>which of the</u> statements to be proved or disproved**. The conclusions and hypotheses in the Steps 3-6 refer to thonly one of these: the original, its negation, or the contrapositive.
   (a) Write the negation if you want to:
      i. prove the original statement: show that assuming the negation is true leads to a contradiction;
      ii. disprove the original statement: prove that the negation is true.
   (b) Write the contrapositive if you want to prove the original statement using the logically equivalent contrapositive, instead.
3. List the conclusions at which you hope to arrive. Check them off as you prove them. You may even want to include what you hope will be the final line of your proof, or a "plain English" description of your goal or strategy.
4. List the assumptions you are given as your hypotheses. Label each of these with a number inside a circle.
5. Write any claim, calculation, or declaration that follows from previous steps. Label each of these with a capital letter inside a circle.
   (a) Each claim, calculation, or declaration must be paired with a formal justification. Examples of formal justifications include:
      - A definition, by name.
      - A previously proved result or stated fact: include the name and number, e.g. "Theorem 1.1.1", or "Corollary 2.2.2", as well as the label(s) of the fact(s) you've established that show the hypotheses of the referenced theorem are satisfied.
      - Some combination of definitions, previous results, and new declarations or new claims established, so far.
   (b) Your first claims will likely be paired exactly with your given hypotheses, by listing the properties given in the definitions of the objects in those statement.
   (c) Repeat Step 5 until you reach all of your conclusions.

Writing the Proof

The rough structure of the proof will probably resemble the following:

> *Proof.* Let [*given assumptions* (1), (2),...] be arbitrary. By [*reasoning behind* (A)] we have (A). We have (C) because [*reasoning behind* (C)]. Etc. We have established [*conclusions we set out to make*], so the result is proved.

Note that your final argument only needs to make use of the claims that further progress. If you never referred to (B) to justify any of the claims that led to your final conclusions, and omitting it leaves your proof complete, then you should do so.