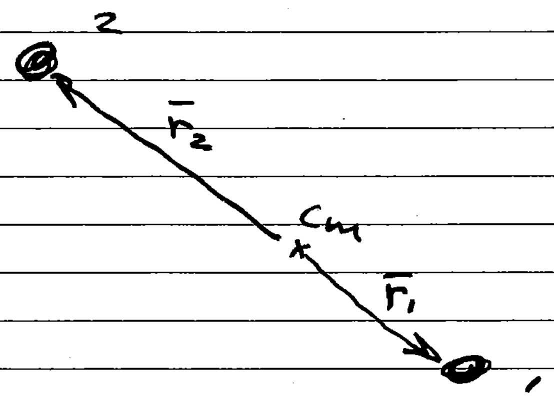
Problem Set # 9

5-1



$$\bar{r} = \bar{r}_2 - \bar{r}_1$$

location of cm seen from cm

$$0 = \frac{m_1 \bar{r}_1 + m_2 \bar{r}_2}{m_1 + m_2}$$

Thus $\quad m_1 \vec{r}_1 = - m_2 \vec{r}_2$

$$m_1 \dot{\vec{r}}_1 = - m_2 \dot{\vec{r}}_2$$

So $\quad \vec{r} = \vec{r}_2 - \vec{r}_1 = \vec{r}_2 + \frac{m_2}{m_1} \vec{r}_2$

$$= \left(1 + \frac{m_2}{m_1}\right) \vec{r}_2$$

$$\vec{r} = \frac{m_1 + m_2}{m_1} \vec{r}_2$$

Rel. to cm

$$L_{cm} = m_1 r_1 \times \dot{r}_1 + m_2 r_2 \times \dot{r}_2$$

$$= m_1 \left(-\frac{m_2}{m_1} r_2\right) \times \left(-\frac{m_2}{m_1} \dot{r}_2\right) + m_2 \vec{r}_2 \times \dot{\vec{r}}_2$$

$$= \frac{m_2^2}{m_1}(r_2 \times \dot{r}_2) + m_2(r_2 \times \dot{r}_2)$$

$$L_{cm} = \left[\frac{m_2^2}{m_1} + m_2\right] \bar{r}_2 \times \bar{\dot{r}}_2$$

$$L_{cm} = \frac{m_2^2 + m_1 m_2}{m_1} \bar{r}_2 \times \bar{\dot{r}}_2$$

$$= \frac{m_2^2 + m_1 m_2}{m_1} \cancel{\left(\frac{m_1+m_2}{m_1}\right)^2} \left(\frac{m_1}{m_1+m_2}\right)^2 \bar{r} \times \bar{\dot{r}}$$

$$= \frac{(m_2^2 + m_1 m_2)\, m_1}{(m_1+m_2)^2} \bar{r} \times \bar{\dot{r}}$$

$$= \frac{m_2(m_1+m_2)\, m_1}{(m_1+m_2)^2} \bar{r} \times \bar{\dot{r}}$$

$$L_{cm} = \frac{m_1 m_2}{m_1+m_2} r \times \dot{r}$$

This is the same as the L we used in talking of the general problem.

Kinetic energy

$$KE = \frac{1}{2} m_1 \dot{r}_1^2 + \frac{1}{2} m_2 \dot{r}_2^2$$

now $$r = \frac{m_1 + m_2}{m_1} r_2$$

$$r_2 = \frac{m_1}{m_1 + m_2} r$$

$$r = r_2 - r_1$$

so $$r_1 = r_2 - r = \frac{m_1}{m_1 + m_2} r - r$$

$$= \frac{m_1 - (m_1 + m_2)}{m_1 + m_2} r$$

$$r_1 = - \frac{m_2}{m_1 + m_2} r$$

$$KE = \frac{1}{2} m_1 \left(\frac{m_2}{m_1 + m_2} \right)^2 \dot{r}^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_1 + m_2} \right)^2 \dot{r}^2$$

$$= \frac{1}{2} \left[m_1 m_2^2 + m_2 m_1^2 \right] \frac{\dot{r}^2}{(m_1 + m_2)^2}$$

$$= \frac{1}{2} m_1 m_2 (m_2 + m_1) \frac{\dot{r}^2}{(m_1 + m_2)^2}$$

$$KE = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} \dot{r}^2 = \frac{1}{2} \mu \dot{r}^2$$

5 - 6 From Table. 5.4 - 2

Halley's Com.

perihelion .587

eccentricity .967

So

orbit $r = \dfrac{\alpha}{1 + \varepsilon \cos\theta}$

$\Rightarrow \quad \varepsilon = .967$

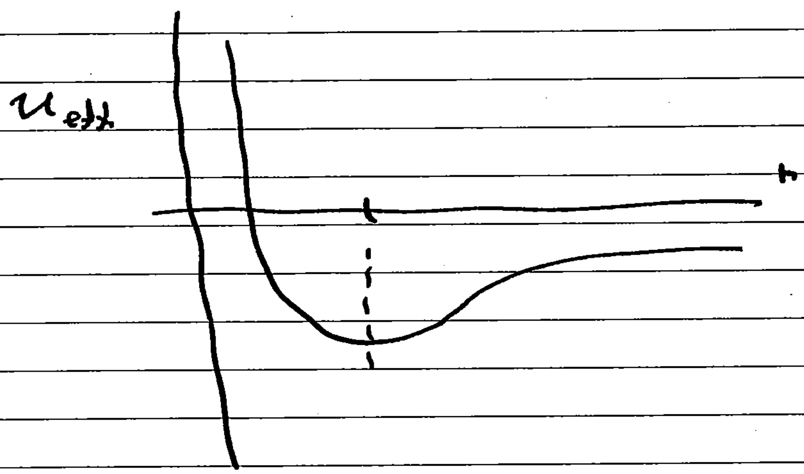
$$.587 = \frac{\alpha}{1+\varepsilon} = \frac{\alpha}{1.967}$$

$$\alpha = .587\,(1.967) =$$

max distance $= \dfrac{\alpha}{1-\varepsilon} = \dfrac{.587(1.967)}{1-.967} = 35\text{ AU}$

5 - 12

$$E = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} - \frac{GMm}{r}$$



circular orbit.

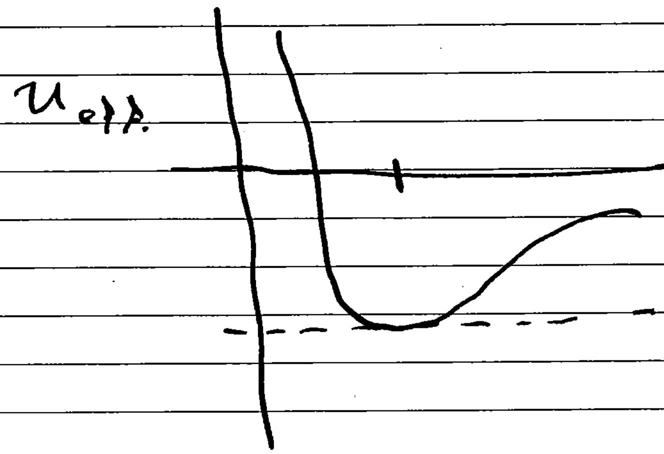
$$\frac{d}{dr}\left(\frac{L^2}{2mr^2} - \frac{GMm}{r}\right) =$$

$$= -\frac{2L^2}{2mr^3} + \frac{GMm}{r^2} = 0$$

$$\frac{L^2}{mr^3} = \frac{GMm}{r^2} \quad \text{or} \quad \frac{L^2}{mr^2} = \frac{GMm}{r}$$

so

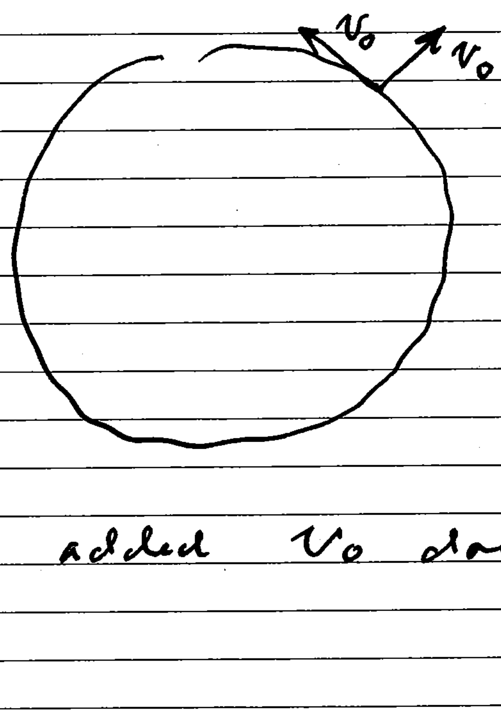
$$U_{eff} = \frac{L^2}{2mr^2} - \frac{L^2}{mr^2} = -\frac{L^2}{2mr^2}$$



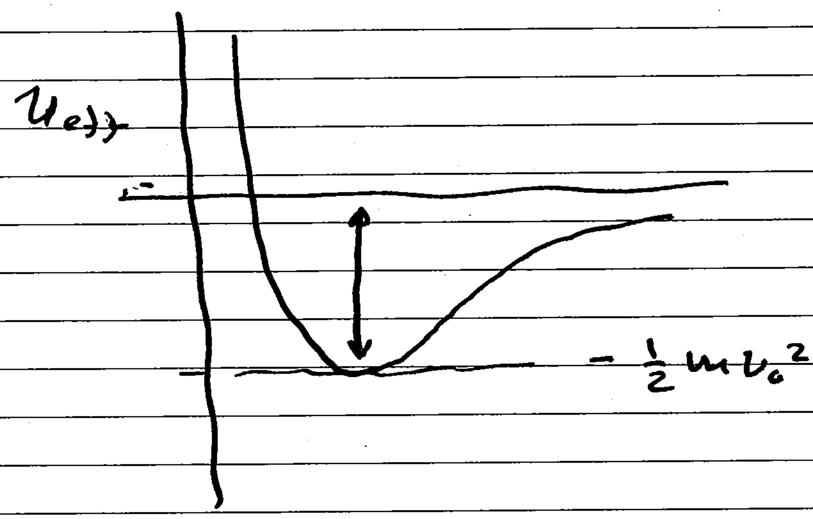
$$- \frac{L^2}{2mr^2} = E$$

But $\qquad L = m r v_0$

$$U_{eff} = - \frac{m^2 r^2 v_0^2}{2 m r^2} = - \frac{1}{2} m v_0^2$$



Note: added v_0 does not change L



So the added kinetic Energy gives

$E_{Pot} = 0$. So the body is

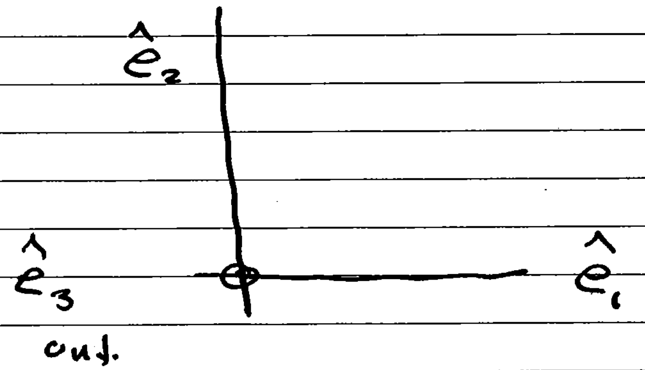
now free.

V_0 inward in or out gives

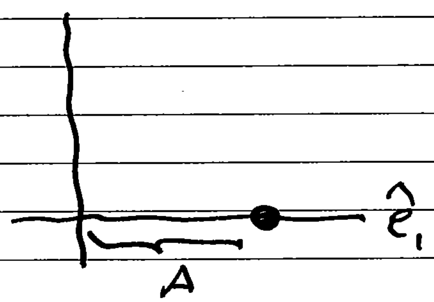
the same result.

6-3

$$\theta = \frac{\dot{\Omega} t^2}{2}$$



a)



ω cnt.

$$a_I = \underset{\substack{\| \\ 0}}{a} + \underset{\substack{\| \\ 0}}{\ddot{R}} + \underset{\substack{\| \\ 0}}{2\omega \times v} + \omega \times (\omega \times r) + \dot{\omega} \times r$$

$$\omega = \dot{\theta} = \dot{\Omega} t \hat{e}_3 \qquad r = A\hat{e}_1$$

$$\omega \times (\omega \times r) = (\dot{\Omega} t)^2 A \underbrace{\hat{e}_3 \times (\underbrace{\hat{e}_3 \times \hat{e}_1}_{\hat{e}_2})}_{-\hat{e}_1}$$

$$= -A(\dot{\Omega} t)^2 \hat{e}_1$$

$$\dot{\omega} = \dot{\Omega}\,\hat{e}_3$$

$$\dot{\omega} \times r = \dot{\Omega} A\, \hat{e}_3 \times \hat{e}_1 = \dot{\Omega} A\, \hat{e}_2$$

$$a_I = -A(\dot{\Omega} t)^2 \hat{e}_1 + \dot{\Omega} A \hat{e}_2$$

at $t = 0$

$$a_I = \dot{\Omega} A \hat{e}_2$$

$t = 3$

$$a_I = -9 A \dot{\Omega}^2 \hat{e}_1 + \dot{\Omega} A \hat{e}_2$$

$a = 0$ at all times

b)

$$r = v_0 t\, \hat{e}_1$$

$$v = v_0 \hat{e}_1$$

$$a = 0$$

$$a_I = \underset{0}{\underset{\parallel}{a}} + \underset{0}{\underset{\parallel}{\ddot{R}}} + 2\omega \times v + \omega \times (\omega \times r) + \dot{\omega} \times r$$

$$2\omega \times v = 2\dot{\Omega} t\, v_0\, \hat{e}_3 \times \hat{e}_1 = 2\dot{\Omega} t\, v_0\, \hat{e}_2$$

$$\omega \times (\omega \times r) = (\dot{\Omega} t)^2 v_0 t\, \underbrace{\hat{e}_3 \times (\hat{e}_3 \times \hat{e}_1)}_{-\hat{e}_1} = -\dot{\Omega}^2 t^3 v_0 \hat{e}_1$$

$$\dot{\omega} \times r = \dot{\Omega} v_0 t \hat{e}_3 \times \hat{e}_1 = \dot{\Omega} v_0 t \hat{e}_2$$

so

$$a_I = 2\dot{\Omega} t v_0 \hat{e}_2 - \dot{\Omega}^2 t^3 v_0 \hat{e}_1 + \dot{\Omega} v_0 t \hat{e}_2$$

$$a_I = (2\dot{\Omega} t v_0 + \dot{\Omega} v_0 t)\hat{e}_2 - \dot{\Omega}^2 t^3 v_0 \hat{e}_1$$

$$= 3\dot{\Omega} t v_0 \hat{e}_2 - \dot{\Omega}^2 t^3 v_0 \hat{e}_1$$

$$F = m 3\dot{\Omega} t v_0 \hat{e}_2 - m \dot{\Omega}^2 t^3 v_0 \hat{e}_1$$