Problem Set 4: Due Wednesday, February 7

Problem 1: Describe the set $\{x \in \mathbb{R} : |x| < 5\} \cup \{x \in \mathbb{R} : x \geq 3\}$ more fundamentally without using set operations, and explain why your set is the same.

Problem 2: Let $A = \{6n : n \in \mathbb{N}\} \cap \{10n : n \in \mathbb{N}\}$.
a. Write down the smallest 3 elements of A, and briefly explain how you determined them.
b. Make a conjecture about how to describe A parametrically (no need to prove this conjecture).

Problem 3: Given two sets A and B, we define

$$A \triangle B = \{x : x \text{ is an element of exactly one of } A \text{ or } B\},$$

and we call this set the *symmetric difference* of A and B. For example, we have

$$\{4, 5, 6, 8\} \triangle \{5, 6, 7, 8\} = \{4, 7\}.$$

a. Determine $\{1, 3, 8, 9\} \triangle \{2, 3, 4, 7, 8\}$.
b. What are the smallest 9 elements of the set $\{2n : n \in \mathbb{N}\} \triangle \{3n : n \in \mathbb{N}\}$?
c. Make a conjecture about how to write $\{2n : n \in \mathbb{N}\} \triangle \{3n : n \in \mathbb{N}\}$ as the union of 3 pairwise disjoint sets (no need to prove this conjecture, but do write the 3 sets parametrically).

Problem 4: Define a function $f : \{1, 2, 3, \dots, 12\} \to \mathbb{N}$ by letting $f(n)$ be the number of positive divisors of n. For example, the set of positive divisors of 6 is $\{1, 2, 3, 6\}$, so $f(6) = 4$.
a. Write out f formally as a set by listing all its elements.
b. Write down the set $\text{range}(f)$ explicitly.

Problem 5: Define a function $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ by letting $f((a, b)) = a^2 + b^2$. Does $\text{range}(f) = \mathbb{N}$? Explain your answer carefully.

Problem 6: Consider the function $f : \mathbb{Q} \to \mathbb{Q}$ given by $f(a) = 5a - 3$. We clearly have $\text{range}(f) \subseteq \mathbb{Q}$ by definition. Thus, to show $\mathbb{Q} = \text{range}(f)$, it suffices to show $\mathbb{Q} \subseteq \text{range}(f)$. To do this, we need to show how to take an arbitrary $b \in \mathbb{Q}$, and fill in the blank in $f(\underline{\quad\quad}) = b$ with an element of $\mathbb{Q}$. In this problem, we first do a few examples, and then handle a general b.
a. Fill in the blank in $f(\underline{\quad\quad}) = 7$ with an element of $\mathbb{Q}$.
b. Fill in the blank in $f(\underline{\quad\quad}) = -53$ with an element of $\mathbb{Q}$.
c. Fill in the blank in $f(\underline{\quad\quad}) = 1$ with an element of $\mathbb{Q}$.
d. Let $b \in \mathbb{Q}$ be arbitrary. Fill in the blank in $f(\underline{\quad\quad}) = b$ with an element of $\mathbb{Q}$ (your answer will depend on b), and justify that your choice works.