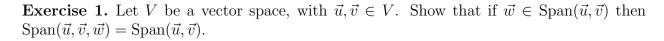
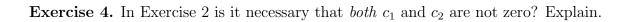
Worksheet 19 - Spans



Exercise 2. Let V be a vector space, with $\vec{u}, \vec{v} \in V$. Show that if $\vec{w} = c_1 \vec{u} + c_2 \vec{v}$ for non-zero $c_1, c_2 \in \mathbb{R}$, then $\operatorname{Span}(\vec{u}, \vec{v}) = \operatorname{Span}(\vec{u}, \vec{w}) = \operatorname{Span}(\vec{w}, \vec{v})$.

Exercise 3. Let *V* be a vector space, with $\vec{u}_1, \vec{u}_2, \vec{u}_3 \in V$. Show that if $\vec{v} = c_1 \vec{u}_1 + c_2 \vec{u}_2 + c_3 \vec{u}_3$ for non-zero $c_1, c_2, c_3 \in \mathbb{R}$, then $\text{Span}(\vec{u}_1, \vec{u}_2, \vec{u}_3) = \text{Span}(\vec{u}_1, \vec{u}_2, \vec{v})$.



Exercise 5. In Exercise 3 is it necessary that c_1, c_2 and c_3 are all not zero? Explain.

Exercise 6. Let V be a vector space, with $\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{w} \in V$. If $\operatorname{Span}(\vec{u}_1, \vec{u}_2, \vec{u}_3) \neq \operatorname{Span}(\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{w})$, then what can you say about \vec{w} ?

Exercise 7. Let V be a vector space with $\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n, \vec{w} \in V$. Under what conditions can we say that $\operatorname{Span}(\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n) = \operatorname{Span}(\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n, \vec{w})$? Prove that your answer is correct.

Exercise 8. Let V be a vector space with $\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n \in V$, and suppose

$$\vec{w} = d_1 \vec{u}_1 + d_2 \vec{u}_2 + \dots + d_n \vec{u}_n$$

for $d_i \in \mathbb{R}$. State conditions under which we can replace \vec{u}_k with \vec{w} in $\mathrm{Span}(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n)$ and conclude that

$$\mathrm{Span}(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_{k-1}, \vec{u}_k, \vec{u}_{k+1}, \dots, \vec{u}_n) = \mathrm{Span}(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_{k-1}, \vec{w}, \vec{u}_{k+1}, \dots, \vec{u}_n)$$

Prove that your answer is correct.