Physics 234 - Mechanics **Spring '18**
Exam #3 **Prof. Case**

1. Two point masses of masses m_1 and m_2 are attracted by a central potential given by

$$U = \frac{1}{2}kr^2$$

where k is a positive constant.

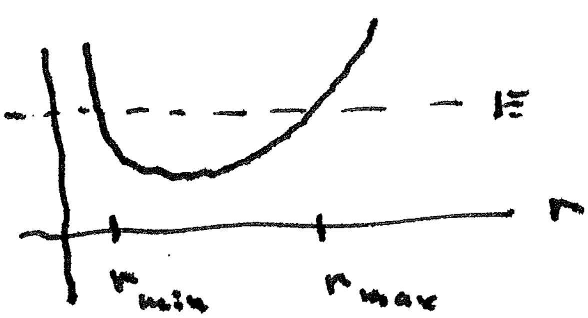
a. Make a plot of the effective potential U_{eff} vs. r. Based on this plot what can one conclude about the motion?

b. For bound orbits obtain an expression for the minimum and maximum separations between the bodies.

c. What inequality relating to energy and angular momentum is implied by your answer to part b?

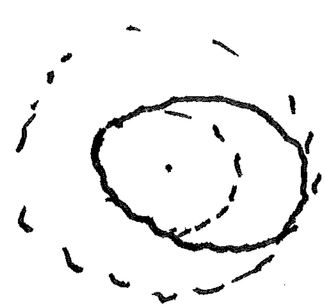
d. Suppose the bodies move so that the distance between them is constant. Obtain an expression for this distance in terms of k, the angular momentum and the masses of the bodies.

a) $U_{eff} = \frac{L^2}{2\mu r^2} + \frac{1}{2}kr^2$



$\mu = \frac{m_1 m_2}{m_1 + m_2}$

will move between r_{min} & r_{max}



All motions are bound.

b)

$$E = \frac{1}{2}\mu \dot{r}^2 + \frac{L^2}{2\mu r^2} + \frac{1}{2}kr^2$$

r_{min} & r_{max} $\Rightarrow \dot{r} = 0$

$$E = \frac{L^2}{2\mu r^2} + \frac{1}{2}kr^2$$

$$0 = \frac{1}{2}kr^4 - Er^2 + \frac{L^2}{2\mu}$$

$$0 = r^4 - \frac{2E}{k}r^2 + \frac{L^2}{k\mu}$$

r_{min} & r_{max} $\rightarrow$ $$r^2 = \frac{\frac{2E}{k} \pm \sqrt{\left(\frac{2E}{k}\right)^2 - 4\frac{L^2}{k\mu}}}{2}$$

$$r^2 = \frac{E}{k} \pm \sqrt{\left(\frac{E}{k}\right)^2 - \frac{L^2}{k\mu}}$$

$$r_{min}^2 = \frac{E}{k} - \sqrt{\left(\frac{E}{k}\right)^2 - \frac{L^2}{k\mu}}$$

$$r_{max}^2 = \frac{E}{k} + \sqrt{\left(\frac{E}{k}\right)^2 - \frac{L^2}{k\mu}}$$

c) must have $\left(\frac{E}{k}\right)^2 \geq \frac{L^2}{k\mu}$

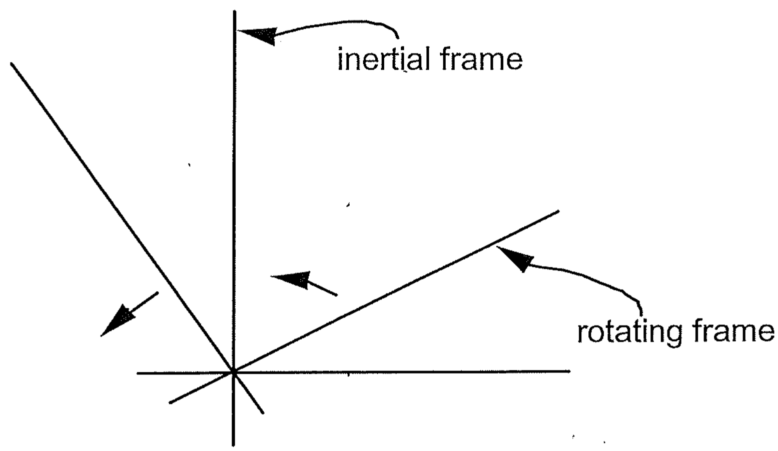
d) must have $\left(\frac{E}{k}\right)^2 = \frac{L^2}{k\mu}$ & $r^2 = \frac{E}{k}$

$$r^2 = \frac{L}{\sqrt{k\mu}}.$$

2. We showed that the acceleration observed by an inertial observer $\vec{a}_I$ is related to the acceleration seen by a non-inertial observer by:

$$\vec{a}_I = \ddot{\vec{R}} + \vec{a} + 2\vec{\omega} \times \vec{v} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right) + \dot{\vec{\omega}} \times \vec{r}$$

You should be familiar with the other quantities in this relation. The figure below shows an inertial coordinate system and a rotating coordinate system. The coordinate systems are placed so that their origins coincide. The rotating system rotates at a constant rate ω about an axis perpendicular to the page.

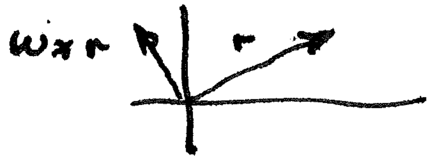


a. A point mass of mass m is seen by the rotating observer as being at rest at a distance r_o from the origin. Using the above relation, find the force acting on this object and show that it is in agreement with well-known results of Intro. Physics (PHY 131).

b. A second mass point, also of mass m, is seen by the rotating observer to move in a circle concentric with the origin of a radius r_o and an angular velocity ω in a clockwise direction. Using the above relation, find the force on this body. Show that it, too, is consistent with what is learned in Intro. Physics (PHY 131).

a) $a_I = 0 + 0 + 0 + \omega \times (\omega \times r) + 0$

$F = m a_I = \omega \times (\omega \times r)$



ω out

$\omega \times (\omega \times r) = -\omega^2 \vec{r}$

So $F = -m\omega^2 \vec{r}$

From intro for body moving in ⊙ $a = r\omega^2$ inward
$F = mr\omega^2$

b)

~~a~~

Since the mass goes around in a clockwise direction it is moving opposite to the original ω.

$$v = \frac{dr}{dt} = (-\omega)\times r = -\omega\times r$$

$$a = \frac{dv}{dt} = (-\omega)\times(-\omega\times r) = \omega\times(\omega\times r)$$

so

$$a_I = \underset{0}{\underset{\shortparallel}{\ddot{R}}} + a + 2\omega\times v + \omega\times(\omega\times r) + \underset{0}{\underset{\shortparallel}{\dot{\omega}\times r}}$$

$$= \omega\times(\omega\times r) + 2\omega\times(-\omega\times r) + \omega\times(\omega\times r)$$

$$a_I = 0$$

In fact the mass is at rest in the inertial frame & Force $= 0$