Written Assignment 8: Due Friday, April 20

Problem 1: Let V be a vector space, and let $\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n, \vec{w}_1, \vec{w}_2, \dots, \vec{w}_m \in V$. Assume that $(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n)$ is linearly dependent. Show that $(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n, \vec{w}_1, \vec{w}_2, \dots, \vec{w}_m)$ is linearly dependent.

Problem 2: Let V be a vector space and let $\vec{u}, \vec{v}, \vec{w} \in V$. Assume that $(\vec{u}, \vec{v}, \vec{w})$ is linearly independent. Show that $(\vec{u}+\vec{v}, \vec{u}+\vec{w}, \vec{v}+\vec{w})$ is linearly independent.
Hint: Think carefully about how to start your argument. Remember that you want to prove that $(\vec{u}+\vec{v}, \vec{u}+\vec{w}, \vec{v}+\vec{w})$ is linearly independent, which is a "for all" statement.

Problem 3: Let V be a vector space, and let $\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n, \vec{w}_1, \vec{w}_2, \dots, \vec{w}_m \in V$. Assume that both $(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n)$ and $(\vec{w}_1, \vec{w}_2, \dots, \vec{w}_m)$ are linearly independent.
a. Give an example of this situation where $(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n, \vec{w}_1, \vec{w}_2, \dots, \vec{w}_m)$ is linearly dependent.
b. Assume also that

$$\mathrm{Span}(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n) \cap \mathrm{Span}(\vec{w}_1, \vec{w}_2, \dots, \vec{w}_m) = \{\vec{0}\}.$$

Show that $(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n, \vec{w}_1, \vec{w}_2, \dots, \vec{w}_m)$ is linearly independent.