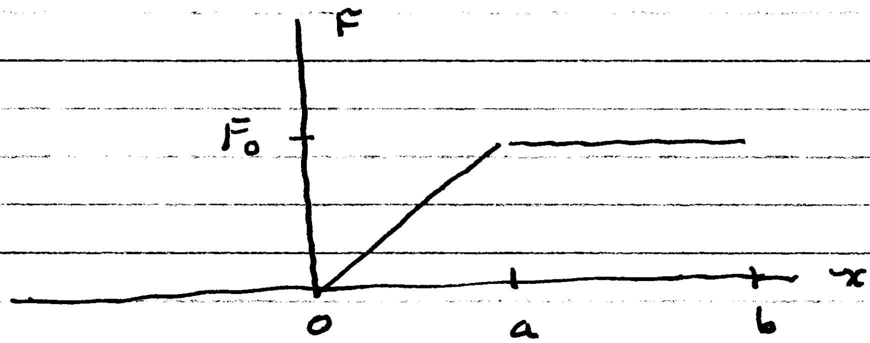
Problem Set #3

1-13



$$F = -\frac{dU}{dx}$$

so for $x < 0$ $\quad U = \text{Const.}$ call it 0

for $0 < x < a$ $\quad F = \frac{x}{a} F_0$

$$\Delta U = -\int_0^{x_f} F\,dx = -\frac{F_0}{a}\int_0^{x_f} x\,dx = -\frac{F_0 x^2}{2a}\Big|_0^{x_f}$$

$$U(x_f) - U(0) = -\frac{F_0 x_f^2}{2a}$$

so for $0 < x < a$ $\quad U = -\frac{F_0}{2a} x^2$

Note $U(a) = -\frac{a F_0}{2}$

for $x > a$

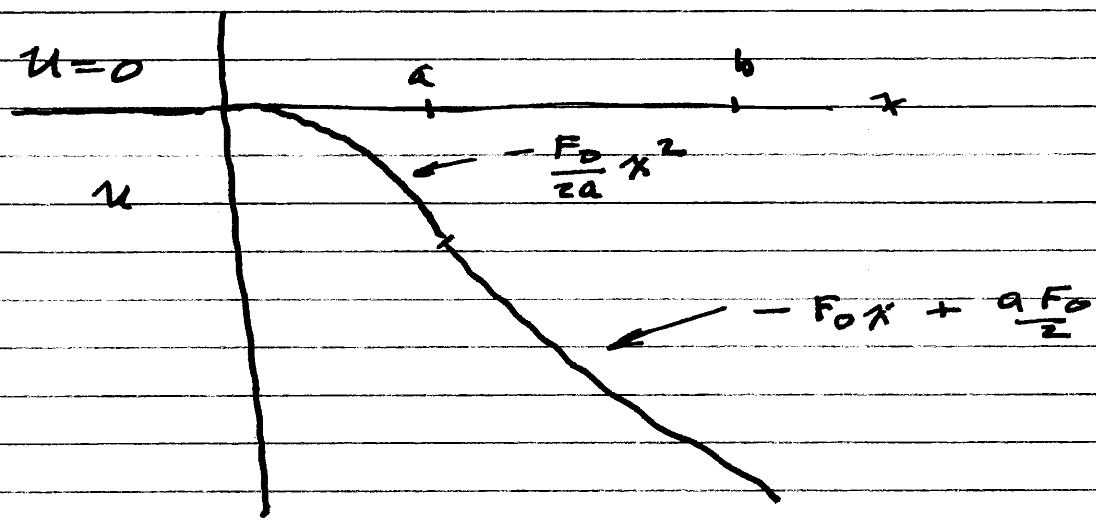
$$\Delta U = -\int_a^{x_f} F_0\,dx = -F_0 (x_f - a)$$

$$U(x_f) - U(a) = -F_0 (x_f - a)$$

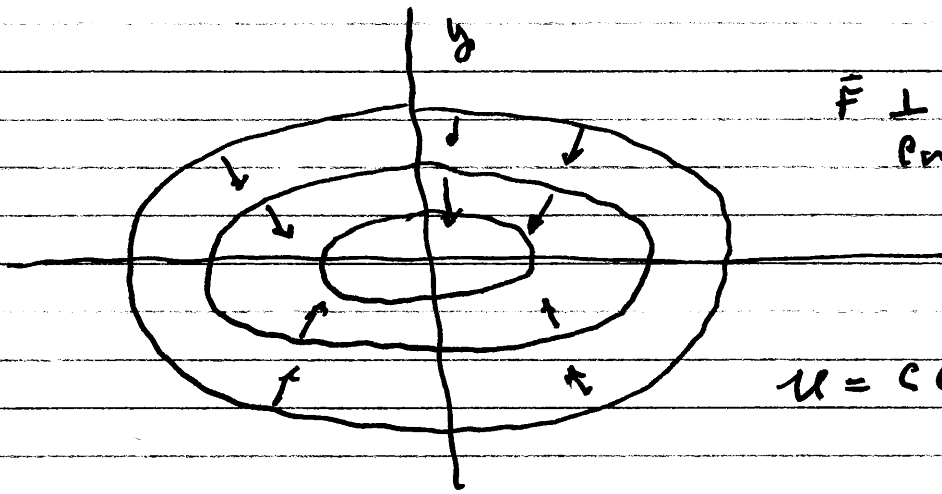
$$U(x_f) = -F_0 (x_f - a) + \left(-\frac{a F_0}{2}\right)$$

$$U(x) = -F_0 x + F_0 a - \frac{a F_0}{2}$$

$$= -F_0 x + \frac{a F_0}{2}$$



2 - 20



$\vec{F} \perp$ lines of const. U.

$U = C(x^2 + 4y^2)$

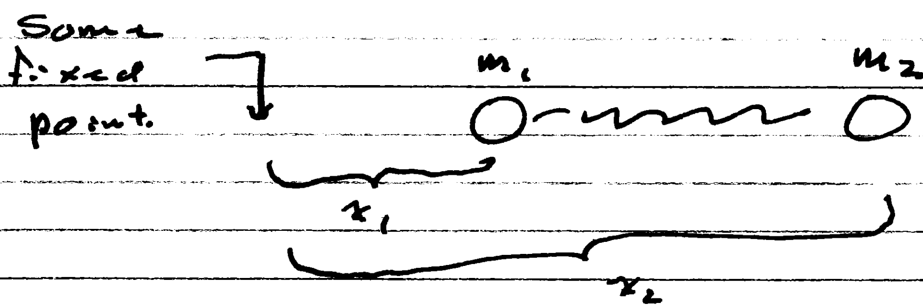
$$\vec{F} = -\vec{\nabla} U = -\frac{\partial U}{\partial x}\hat{x} - \frac{\partial U}{\partial y}\hat{y}$$

$$\vec{F} = -2Cx\,\hat{x} - 8Cy\,\hat{y}$$

equilibrium at $x = 0,\ y = 0$

It is stable —

2-1



$$m_1 \frac{d^2 x_1}{dt^2} = k\left(\underbrace{\underbrace{x_2 - x_1}_{\text{separation}} - l_0}_{\text{separation} - l_0}\right) \qquad (1)$$

Note if separation $- l_0$ is pos. force on m_1 is to right. (pos)

$$m_2 \frac{d^2 x_2}{dt^2} = -k\left(x_2 - x_1 - l_0\right) \qquad (2)$$

If spring is stretched force on m_2 is neg.

(1) $\Rightarrow$

$$\frac{d^2 x_1}{dt^2} = \frac{k}{m_1}(x_2 - x_1 - l_0)$$

(2) $\Rightarrow$ $$\frac{d^2 x_2}{dt^2} = -\frac{k}{m_2}(x_2 - x_1 - l_0)$$

Take second − first

$$\frac{d^2(x_2 - x_1)}{dt^2} = -\left(\frac{k}{m_1} + \frac{k}{m_2}\right)(x_2 - x_1 - l_0)$$

Define $x_- = x_2 - x_1$

$$\frac{d^2 x_-}{dt^2} = -k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)(x_- - l_0)$$

~~Define~~ equilibrium when $x_- = l_0$

Define

$y = x_- - l_0$ (displacement from equilib.)

$$\ddot{y} = -k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)y$$

$$\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = \frac{m_2 + m_1}{m_1 m_2}$$

Define $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ Reduced mass

$$\ddot{y} = -\frac{k}{\mu} y$$

$$y = A \cos\left(\sqrt{\tfrac{k}{\mu}}\, t\right) + B \sin\left(\sqrt{\tfrac{k}{\mu}}\, t\right)$$

~~Harmonic~~ $(x_2 - x_1) = y + l_0$

$$(x_2 - x_1) = A \cos\left(\sqrt{\tfrac{k}{\mu}}\, t\right) + B \sin\left(\sqrt{\tfrac{k}{\mu}}\, t\right) + l_0$$

Acts like HO but with mass $= \mu$.

Take Second + first

$$m_1 \frac{d^2 x_1}{dt^2} + m_2 \frac{d^2 x_2}{dt^2} = 0$$

$x_+ \equiv$ ~~$x_1 + x_2$~~ ~~m_1~~ $m_1 x_1 + m_2 x_2$

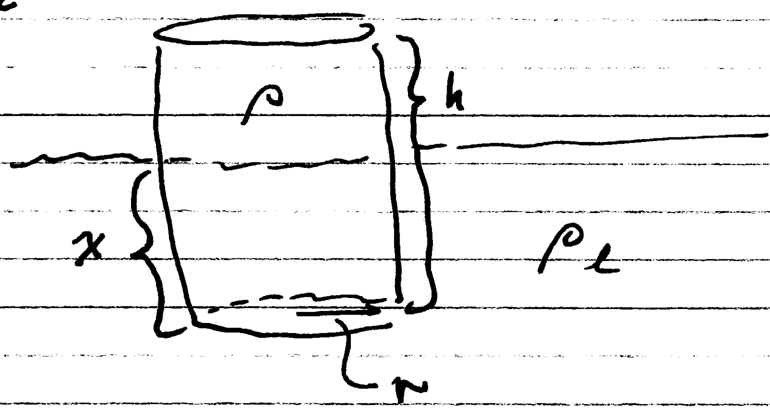
~~$\dot{x}_+ = 0 \Rightarrow \dot{x}_+ = \text{Const.}$~~

~~$\dot{x}_1 + \dot{x}_2 = \text{Const.}$ (motion.)~~

$$\ddot{x}_{+} = 0$$

$$m_1 \dot{x}_1 + m_2 \dot{x}_2 = \text{Const}$$

2-2

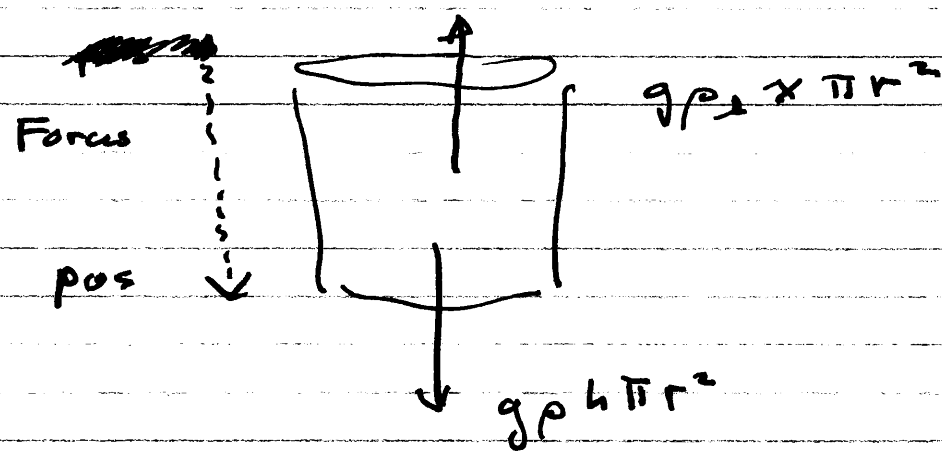


Vol. of cylinder $= h\pi r^2$

mass of cylinder $= \rho h \pi r^2 = m_c$

Vol. of liquid displaced. $= x \pi r^2$

mass of liquid displaced. $= \rho_\ell x \pi r^2$



$$m_c \ddot{x} = F = m_c g - g\rho_\ell \pi r^2 x$$

$$\ddot{x} = g - \frac{g\rho_\ell \pi r^2}{m_c} x$$

$$\ddot{x} = g - \frac{g \rho_\ell \pi r^2}{\rho h \pi r^2} x$$

$$\ddot{x} = g - g \frac{\rho_\ell}{\rho h} x$$

equilibrium $\quad \ddot{x} = 0$ when

$$g = g \frac{\rho_\ell}{\rho h} x_0 \implies x_0 = \frac{h\rho}{\rho_\ell}$$

Take $\quad y = x - x_0 \quad$ or $\quad x = y + x_0$

Sub.

$$\ddot{y} = g - g \frac{\rho_\ell}{\rho h} \left(y + \frac{h\rho}{\rho_\ell} \right)$$

$$\ddot{y} = g - g \frac{\rho_\ell}{\rho h} y - g$$

$$\ddot{y} = - g \frac{\rho_\ell}{\rho h} y$$

HO with $\quad \omega = \sqrt{\frac{g \rho_\ell}{\rho h}}$

Might be able to determine ρ_ℓ by measuring ω. May be a lot of damping.