# Problem Set 22: Due Monday, May 7

**Problem 1:** Consider the matrix
$$A = \begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix}.$$
We know from Proposition 3.3.16 that $A$ is invertible, and we also know a formula for the inverse. Now compute $A^{-1}$ using our new method by applying elementary row operations to the matrix
$$\begin{pmatrix} 3 & 2 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{pmatrix}.$$

**Problem 2:** Let $V$ be the vector space of all $2 \times 2$ matrices. Explain why there is no injective linear transformation $T\colon \mathcal{P}_4 \to V$.

**Problem 3:** Determine whether each of the following matrices is invertible, and if so, find the inverse.

a. $\begin{pmatrix} 1 & 1 & 3 \\ 0 & 2 & 4 \\ -1 & 1 & 0 \end{pmatrix}$

b. $\begin{pmatrix} 0 & 4 & 4 \\ 1 & -2 & 0 \\ 3 & -4 & 2 \end{pmatrix}$

c. $\begin{pmatrix} 0 & 1 & 5 \\ 0 & -2 & 4 \\ 2 & 3 & -2 \end{pmatrix}$

**Problem 4:** Either prove or find a counterexample: If $A$ and $B$ are invertible $n \times n$ matrices, then $A + B$ is invertible.

**Problem 5:** Consider the matrix
$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

a. Explain why $A$ has no left inverse.

b. Show that $A$ has infinitely many right inverses.