Homework Assignment 11

PHYSICS 314 - THERMODYNAMICS & STATISTICAL PHYSICS (Spring 2018)

Due Friday, May 11th, by noon, Noyce 1135

I cannot award full credit for work that I am unable to read or follow. For my benefit and for yours, please:

- *Write neatly*
- *Show and EXPLAIN all steps*
- *Make diagrams large and clearly-labeled*

You are welcome to collaborate with others on this assignment. However, the work you turn in should be your own. Please cite collaborators and outside sources. See the syllabus for details.

Regardless of the number of parts, all homework problems are weighted equally. Regardless of the number of questions, all homework assignments are weighted equally.

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1) Gaussian integrals and v_{rms}
   a) Use the result of the first integral to evaluate the second integral.
   Hint: Use an integration trick similar to one used on HW10.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty x^4 e^{-ax^2}\,dx = ?$$

   b) Beginning with the Maxwell speed distribution, find the root-mean-square speed, v_{rms}.
   Hint: There is a reason that this question appears directly after part a).

2) More Maxwell speed distributions
   a) Beginning with the Maxwell speed distribution, find the most probable speed, v_{max}.
   b) Beginning with the Maxwell speed distribution, find the average speed, $\bar{v}$.
   Hint: Use an integral result from HW10.
   c) Calculate v_{rms}, v_{max} and $\bar{v}$ for oxygen molecules at room temperature.

3) Maxwell speed distribution plots
   a) Use a computer program to plot the Maxwell speed distributions for nitrogen molecules at $T = 150\ K$, $T = 300\ K$, and $T = 600\ K$. Plot all three distributions on the same set of axes. Include axis labels with units.
   Briefly comment on how temperature affects the distribution.
   b) Use a computer program to plot the Maxwell speed distribution at $T = 300\ K$ for hydrogen molecules, nitrogen molecules, and oxygen molecules. Plot all three on the same set of axes. Include axis labels with units.
   Briefly comment on how particle mass affects the distribution.

4) The *escape speed* of a planet (or other celestial body) is defined as the minimum speed at which an object must be moving in order to escape the gravitational attraction of the planet. The escape speed is thus the speed for which the kinetic energy of the object is just enough to overcome the gravitational potential energy.

   a) Derive an expression for the escape speed in terms of the mass and radius of the planet. Use this expression to calculate the escape speed on Earth and the escape speed on Jupiter. You will probably need to look up properties of the planets.
   b) Gases in the upper atmosphere with high enough speeds can escape into space. The region of the atmosphere from which gases escape is known as the *exobase*, and it can be very hot. Make the reasonable assumption that both for Earth and for Jupiter, the temperature of the exobase is $1000\ K$. Use a computer program to calculate the probability that hydrogen molecules in the exobase have speeds higher than the escape velocity of Earth. Repeat the calculation for the escape velocity of Jupiter. *Hint: The values will be small, but do not just round them to zero.*
   c) Based on your result in part b), what can you infer about the hydrogen content in the atmosphere of the Earth compared to the atmosphere of Jupiter?

5) Suppose that there is a system that consists of ten particle states. To make the calculations easier, assume that all the states have the same energy, $E \equiv 0$.
   a) What is the partition function of the system, Z, if the system contains only one particle?
   b) What is the partition function of the system, Z, if the system contains only two distinguishable particles?
   c) What is the partition function of the system, Z, if the system contains only two identical bosons? *Hint: The answer is not 50.*
   d) What is the partition function of the system, Z, if the system contains only two identical fermions?
   e) For each of the parts a)-d), what is the probability of finding the system in a double-occupancy state (that is, with two particles in the same state)?

6) Fermions and Bosons
   a) For a system of fermions at $T = 300\ K$, calculate the probabilities that single-particle states with the following energies, ϵ, are occupied.
      i) $\epsilon = \mu + 1\ eV$
      ii) $\epsilon = \mu + 0.01\ eV$
      iii) $\epsilon = \mu$
      iv) $\epsilon = \mu - 0.01\ eV$
      v) $\epsilon = \mu - 1\ eV$
   b) Suppose there is a system of bosons at $T = 300\ K$. For states with each of the following energies, ϵ, find the probabilities that the state is unoccupied. Then find the probability that the state contains one particle. Then find the probability that the state contains two particles. Then find the probability that the state contains three particles. Finally, find the average occupancy of the state.
      i) $\epsilon = \mu + 1\ eV$
      ii) $\epsilon = \mu + 0.1\ eV$
      iii) $\epsilon = \mu + 0.01\ eV$

NOTICE!!! THIS PROBLEM IS TIME SENSITIVE!

7) You have two choices for the reflection problem this week. You may choose to do either Option I or Option II. *I strongly encourage you to do Option I if you are able,* but both options are worth the same number of points.

Option I

Attend the final Physics seminar of the year on Tuesday, May 8th, at noon.
The speaker is Professor Steve Kawaler from Iowa State University. The presentation is entitled, "Sounding stars while hunting for planets with the Kepler, K2, and TESS spacecraft". This talk may not be as directly related to thermodynamics as others, but it is still important to broaden your physics horizons.

After attending the lecture, write a half-page reflection on the presentation.

- Summarize the main points of the presentation.
- Discuss the connections between the science discussed in the presentation and the material covered in this course.
- Fulfilling the above requirements will earn you a $^{3.25}/_{4}$. The rest of the points will be awarded based on the depth and quality of your reflection.

Option II

Read the following material related to recent topics from class.

Satyendra Nath Bose obituary
https://physicstoday.scitation.org/doi/10.1063/1.3128568
New state of matter revealed: Bose-Einstein Condensate
https://www.nobelprize.org/nobel_prizes/physics/laureates/2001/popular.html

Write a short response to the two articles. One paragraph per article is sufficient.

- Summarize the main points of each article in a few sentences.
- For each article, discuss in a few sentences the connections between the science discussed in the article and the material covered in this course.
- Which article did you feel better conveyed the relevant science? Why? (Keep in mind the intended audience of each piece.)
- Fulfilling the above requirements will earn you a $^{3.25}/_{4}$. The rest of the points will be awarded based on the depth and quality of your explanations of the connections. (For example, a response that discusses the energy flow of a negative temperature system will earn more points than a response that simply states that negative temperature is something we saw in class.)

8) Peer Problem
Complete the problem created by your classmate. I will distribute problems in class on Tuesday.