Worksheet 14 - More Eigenvalues and Eigenvectors

1 Recap

1.1 Interesting vectors

Eigenvectors are those for which a given transformation acts like scalar multiplication. That is, for a linear transformation encoded in a matrix A, the eigenvectors are those for which there is a non-zero scalar, λ, with the property that

$$A\vec{x} = \lambda\vec{x} \tag{1}$$

1.2 An Example

Example 1. Find the characteristic polynomial and real eigenvalues of

$$A = \begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}$$

Example 2. Find the characteristic polynomial and real eigenvalues of

$$A = \begin{pmatrix} 7 & 2 \\ -2 & 3 \end{pmatrix}$$

2 Exploration of some Theory

Let X be the set of all 2×2 matrices and define $f : X \to \mathbb{R}$ by

$$f\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = ad - bc \tag{2}$$

Exercise 1. Given matrices

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad B = \begin{pmatrix} w & x \\ y & z \end{pmatrix}$$

Show that $f(AB) = f(A)f(B)$.

Exercise 2. If A is invertible, show that

$$f(A^{-1}) = \frac{1}{f(A)}$$

Exercise 3. Explain how the function, f, is related to the characteristic polynomial of a matrix.

Exercise 4. Suppose P is an invertible matrix and A and B are matrices with the property

$$A = PBP^{-1}$$

Show that A and B have the same characteristic polynomial and hence the same eigenvalues.

For Next Time

- Finish this worksheet
- Finish Reading Section 3.5
- Go through the review sheet for the exam.