Worksheet 21 - Summary of Concepts and Results

1 Exam 3

Exam 3 will cover all of Chapter 4. It will have the same format as previous exams.

1.1 Short Answer

You will be asked to answer at least one of the following:

- Explain why the successive applications of elementary row operations to a system of equations results in a new system of equations with the same solution set.
- Precisely describe (using proper terminology) how you can apply Proposition 4.2.12 to describe the solution sets of the systems of equations encoded in the following augmented matrices:

  1. $B_1 = \begin{pmatrix} 1 & 0 & 1 & 4 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ $\quad B_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad B_3 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

  2. $B_4 = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad B_5 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 4 & 5 \end{pmatrix}$ $\quad B_6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

  3. $B_7 = \begin{pmatrix} 1 & 2 & \boxed{1} & 1 \\ 0 & 1 & \boxed{1} & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad B_8 = \begin{pmatrix} 1 & \boxed{2} & 0 & \boxed{1} & 1 \\ 0 & 0 & 1 & \boxed{1} & 3 \end{pmatrix}$ $\quad B_9 = \begin{pmatrix} 1 & \boxed{2} & \boxed{3} & \boxed{4} & \boxed{5} & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

- Describe a general strategy for determining whether a collection of n vectors in $\mathbb{R}^n$ are linearly independent.
- Describe a general strategy for demonstrating that a collection of $n+1$ vectors in $\mathbb{R}^n$ are linearly dependent.
- How are Proposition 4.2.14 and Proposition 4.3.3. and Theorem 4.4.6 (and the corollaries corresponding to each) similar? How are they different?
- How is Proposition 3.3.4 similar to results in Chapter 4? Do the claims in Corollary 3.3.5 hold in this, more general, context?

1.2 Results

You will be asked to prove at least one of the following:

- 4.1.8
- 4.1.16
- 4.2.12
- 4.2.14
- 4.3.2
- 4.3.3
- 4.4.2
- 4.4.4
- 4.4.10
- 4.4.15

Exercise 1. In groups of at least 3, discuss the items above and identify:

- one short answer question;
- one result;
- and one question not covered here (other theorems, topics, worksheet problems, homework, etc.) that you want to review more thoroughly before the exam.

Write these questions down, with your names, and give it to me before the end of class.

Exercise 2. Let V be a vector space, and let $\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n, \vec{w}_1, \vec{w}_2, \ldots, \vec{w}_m \in V$. Assume that $(\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n)$ is linearly dependent. Show that $(\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n, \vec{w}_1, \vec{w}_2, \ldots, \vec{w}_m)$ is linearly dependent.

Exercise 3. Let V be a vector space and let $\vec{u}, \vec{v}, \vec{w} \in V$. Assume that $(\vec{u}, \vec{v}, \vec{w})$ is linearly independent. Show that $(\vec{u}+\vec{v}, \vec{u}+\vec{w}, \vec{v}+\vec{w})$ is linearly independent.
Hint: Think carefully about how to start your argument. Remember that you want to prove that $(\vec{u}+\vec{v}, \vec{u}+\vec{w}, \vec{v}+\vec{w})$ is linearly independent, which is a "for all" statement.

Exercise 4. Let V be a vector space, and let $\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n, \vec{w}_1, \vec{w}_2, \ldots, \vec{w}_m \in V$. Assume that both $(\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n)$ and $(\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_m)$ are linearly independent.
a. Give an example of this situation where $(\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n, \vec{w}_1, \vec{w}_2, \ldots, \vec{w}_m)$ is linearly dependent.
b. Assume also that

$$\mathrm{Span}(\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n) \cap \mathrm{Span}(\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_m) = \{\vec{0}\}.$$

Show that $(\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n, \vec{w}_1, \vec{w}_2, \ldots, \vec{w}_m)$ is linearly independent.

Exercise 5. Without referring to the proofs in the book, prove Proposition 5.1.2 and Proposition 5.1.3.