Worksheet 01 - Statements and Sets

Definition. A *mathematical statement* is an assertion that is objectively true or objectively false, based on the axioms and definitions that we accept as universally true.

Definition. A *set* is a collection of elements without regard to repetition or order.

Note: Elements can be any mathematical object that we can define (including sets!)

Definition. The set with no elements is called *the empty set*

Exercise 1. Determine which of the following are mathematical statements. For each mathematical statement, determine its truth value and explain, as precisely as you can, how you determined the truth value. If you are unable to determine the truth value, explain what you tried.

a) The sum of two odd integers is always an even integer.

b) $a^2 + b^2 = c^2$

c) There is a rational number between any distinct real numbers.

d) There exists a simultaneous solution to the three equations

$$
\begin{aligned}
2x + \quad + 8z &= 6 \\
x - 3y + 18z &= 15 \\
3x + 3y - 2z &= -1
\end{aligned}
$$

   in $\mathbb{R}^3$.

e) The remainder when dividing 35^{396} by 397 is 1.

f) There exist real numbers x and y such that $2x = y + 2x$.

g) Edgar Allan Poe wrote *The Raven*.

h) The sum of 17 and 9 is equal to 26.

i) All continuous real valued functions, of a single real variable, have an elementary anti-derivative.

j) There exists a, non-constant, real valued functions of a single variable, f such that $f(a + b) = f(a) + f(b)$ for all inputs a and b.

k) There exists positive natural numbers a, b, c with $a^4 + b^4 = c^4$.

l) The digits of e eventually form a repeating sequence.

m) Elliott Smith is a great songwriter.

n) The numbers 1 and 2 occur equally frequently as digits in the infinite decimal expansion of e.

o) There set of rational numbers is the same size as the set of integers.

STOP

Exercise 2. Determine the truth value of the statements, and explain how you would prove that your claims are correct.

a) For any real number, x, we have that $2x + 1 = 7$.

b) There is a real number, x, so that $2x + 1 = 7$.

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Example. You are familiar with several important sets:

- The Natural Numbers: $\mathbb{N} = \{0, 1, 2, 3, \dots\}$

- The Positive Natural Numbers: $\mathbb{N}^+ = \{1, 2, 3, \dots\}$

- The Integers: $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2 \dots\}$

- The Rational Numbers: $\mathbb{Q}$

- The Real Numbers: $\mathbb{R}$

- The set of infinitely differentiable functions: C^∞

Two ways we construct sets, from these examples and other mathematical objects, include

1. building a new set from elements, from a another set, each with a particular property, or

2. using the elements of one set to "parameterize" the elements of a new set.

Example. If $\mathbb{Z}$ is the set of integers, we can build the set of even integers by writing:

$$E = \{x \in \mathbb{Z} : \text{ There is a } y \in \mathbb{Z} \text{ with } x = 2y\}$$

Or "The set E is the set of all integers, x, such that there is an integer, y, with the property that $x = 2y$".

Alternatively, I could get the same set by using the integers to parameterize the even integers. That is, we could write:

$$E = \{2y : y \in \mathbb{Z}\}$$

Or, "The set E is the set of all products of the form $2y$ such that y is an integer.

Building sets, then comparing, and combining them in various ways, helps us organize and describe our mathematical objects. The more complicated our sets and objects become, the more careful we have to be when we make statements about them.

Exercise 3. Describe the set of odd integers using both methods described in this section.

Exercise 4. Describe the set of points on the line $y = 2x + 3$, in $\mathbb{R}^2$, using both methods described in this section.

Assignment

- Read Sections 1.1 and 1.2. This is only seven pages, but you should expect seven pages of math to go by much more slowly than pages from other subjects. If it takes you less than three hours, read it twice.

- Read the syllabus.

- Stop by my office to say 'hello'. If I'm not there, slide a note under my door letting me know that you tried.

- Write me an e-mail, before 9pm on 1/23, with the subject: `MAT215-Day01`

  - Include your preferred name with phonetic pronunciation, and your preferred pronouns, as well as

  - three questions from the reading, and one from the syllabus, written in complete sentences.