## Problem Set 9: Due Wednesday, February 28

**Problem 1:** Define $T \colon \mathbb{R}^2 \to \mathbb{R}^2$ by letting $T(\vec{v})$ be the result of first projecting $\vec{v}$ onto the line $y = 3x$, and then projecting the result onto the line $y = 4x$. Explain why $T$ is a linear transformation, and then calculate $[T]$.

**Problem 2:** Let

$$A = \begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{pmatrix}$$

a. Show that $A \cdot A = A$ by simply computing it.
b. Find an example of $\vec{w} \in \mathbb{R}^2$ such that $A = [P_{\vec{w}}]$.
c. By interpreting the action of $A$ geometrically, explain why you should expect that $A \cdot A = A$.

**Problem 3:** Define $T \colon \mathbb{R}^2 \to \mathbb{R}^2$ by letting $T(\vec{v})$ be the point on the line $y = x + 1$ that is closest to $\vec{v}$. Is $T$ is a linear transformation? Explain.

**Problem 4:** Let $\vec{w} \in \mathbb{R}^2$ be nonzero, and let $W = \text{Span}(\vec{w})$. Define $F_{\vec{w}} \colon \mathbb{R}^2 \to \mathbb{R}^2$ by letting $F_{\vec{w}}(\vec{v})$ be the result of reflecting $\vec{v}$ across the line $W$. Show that $F_{\vec{w}}$ is a linear transformation and determine the standard matrix $[F_{\vec{w}}]$.
*Hint:* Make use of projections. How can you determine $F_{\vec{w}}(\vec{v})$ using $\vec{v}$ and $P_{\vec{w}}(\vec{v})$?

**Problem 5:** Define $T \colon \mathbb{R}^2 \to \mathbb{R}^2$ by letting $T(\vec{v})$ be the result of first reflecting $\vec{v}$ across the $x$-axis, and then reflecting the result across the $y$-axis.
a. Compute $[T]$.
b. The action of $T$ is the same as a certain rotation. Explain which rotation it is.

**Problem 6:** Let $T \colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, and let $r \in \mathbb{R}$. We know from Proposition 2.4.8 that $r \cdot T$ is a linear transformation. Show that if

$$[T] = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

then

$$[r \cdot T] = \begin{pmatrix} ra & rb \\ rc & rd \end{pmatrix}.$$

In other words, if we define the multiplication of a matrix by a scalar as in Definition 3.1.14, then the standard matrix of $r \cdot T$ is obtained by multiplying every element of $[T]$ by $r$.