Problem Set 5: Due Monday, February 12

Problem 1: Let $f\colon \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^3 - 8x$. Show that f is not injective.

Problem 2: Determine if the three lines $2x + y = 5$, $7x - 2y = 1$, and $-5x + 3y = 4$ intersect. Explain your reasoning using a few sentences.

Problem 3: For each part, explain your reasoning using a sentence or two.
a. Find an example of a choice for $\vec{v}, \vec{u} \in \mathbb{R}^2$ such that the solution set to $-x + 9y = -6$ is $\{\vec{v} + t\vec{u} : t \in \mathbb{R}\}$.
b. Find an example of $\vec{u} \in \mathbb{R}^2$ such that the solution set of $5x + 3y = 0$ is $\mathrm{Span}(\vec{u})$.

c. Find an example of a choice for $a, b, c \in \mathbb{R}$ such that the solution set of $ax + by = c$ is $\mathrm{Span}\left(\begin{pmatrix} 2 \\ -7 \end{pmatrix}\right)$.

Problem 4: Let

$$A = \left\{ \begin{pmatrix} 3 \\ -1 \end{pmatrix} + c \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix} : c \in \mathbb{R} \right\} \qquad \text{and} \qquad B = \left\{ \begin{pmatrix} 5 \\ 7 \end{pmatrix} + c \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix} : c \in \mathbb{R} \right\}.$$

In this problem, we will prove that $A = B$ by giving a double containment proof.
a. Fill in the blanks below with appropriate phrases so that the result is a correct proof of the statement that $A \subseteq B$.

Let $\vec{u} \in A$ be arbitrary. By definition of A, we can _____. Now notice that _____ $= \vec{u}$.
Since _____ $\in \mathbb{R}$, we conclude that $\vec{u} \in B$. Since $\vec{u} \in A$ was arbitrary, the result follows.

b. Fill in the blanks below with appropriate phrases so that the result is a correct proof of the statement that $B \subseteq A$.

Let $\vec{w} \in B$ be arbitrary. By definition of B, we can _____. Now notice that _____ $= \vec{w}$.
Since _____ $\in \mathbb{R}$, we conclude that $\vec{w} \in A$. Since $\vec{w} \in B$ was arbitrary, the result follows.

Problem 5: Given $a, b \in \mathbb{R}$, define two functions $f_a\colon \mathbb{R} \to \mathbb{R}$ and $g_b\colon \mathbb{R} \to \mathbb{R}$ by letting $f_a(x) = ax$ and letting $g_b(x) = x + b$. Determine, with explanation, all possible values of $a, b \in \mathbb{R}$ so that $f_a \circ g_b = g_b \circ f_a$.
Hint: Recall that to prove that two functions are equal, you need to argue that they give the same output for every input. To prove that two functions are not equal, you just need to give one example of an input that produces different outputs.