Homework Assignment 6

PHYSICS 314 - THERMODYNAMICS & STATISTICAL PHYSICS (Spring 2018)

Due Friday, March 16th, by noon, Noyce 1135

I cannot award full credit for work that I am unable to read or follow. For my benefit and for yours, please:

- *Write neatly*
- *Show and EXPLAIN all steps*
- *Make diagrams large and clearly-labeled*

You are welcome to collaborate with others on this assignment. However, the work you turn in should be your own. Please cite collaborators and outside sources. See the syllabus for details.

Regardless of the number of parts, all homework problems are weighted equally. Regardless of the number of questions, all homework assignments are weighted equally.

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1) **Power Plant Heat Mitigation:** A power plant produces 1 GW of electricity at an efficiency of 40%. (This is typical of a modern coal-fired plant).
   a) At what rate does this plant expel waste heat into its environment?
   b) Assume that the cold reservoir for this plant is a river whose flow rate is $100\ m^3/s$. How much will the temperature of the river increase due to this waste heat? *Hint: You will have to look up some properties of water for this problem.*
   c) To avoid this *thermal pollution* of the river, the plant could instead be cooled by evaporation of river water. (This is more expensive, but it is environmentally preferable in some areas.) At what rate must the water evaporate? What fraction of the river must be evaporated? Assume the water is initially at 25°C.

2) **Rectangular Heat Engine:** Look back to Schroeder 1.34 (pg. 23), a problem we did in class earlier in the semester. The problem looks at an ideal diatomic gas taken around a rectangular cycle on a P-V diagram. Suppose that this system is used as a heat engine to convert the heat added into mechanical work.
   a) Evaluate the efficiency of this engine for the case that $V_2 = 3V_1$ and $P_2 = 2P_1$.
   b) Compare your answer from part a) to the efficiency of an ideal Carnot engine operating between the same temperature extremes.

3) **Refrigeration Cycle:** Suppose we have a device that takes n moles of air through the following cycle.

   $\boldsymbol{A \rightarrow B}$**:** Adiabatic compression where the air is taken from the temperature of the cold reservoir, T_c, and pressure P_2 to temperature T_1 and pressure P_1.

   $\boldsymbol{B \rightarrow C}$**:** Isobaric cooling to the temperature of the hot reservoir, T_h.

   $\boldsymbol{C \rightarrow D}$**:** Adiabatic expansion to pressure P_2 to temperature T_2.

   $\boldsymbol{D \rightarrow A}$**:** Return to the initial state by isobaric heating to the temperature of the temperature of the cold reservoir, T_c.

   Assume that air is an ideal gas with (constant) heat capacity at constant pressure C_P and adiabatic exponent γ. Also assume that the ratio of pressures is $\frac{P_1}{P_2} \equiv a > 1$.

   a) Sketch the cycle on a P-V diagram.

b) Calculate the total work exchanged by the system with the exterior over the course of one cycle, as a function of the temperatures $n, R, f,\ T_c,\ T_h,\ T_1$, and T_2.
c) Calculate the coefficient of performance of this cycle in terms of the four temperatures.
d) Calculate T_1 and T_2 in terms of $T_c,\ T_h,\ a$, and γ.
e) Calculate the coefficient of performance for this refrigerator in terms of <u>only</u> the adiabatic exponent γ and the ratio of pressures a.

4) In order to more evenly cool your living room, you consider putting your "window" air conditioning unit in the middle of the room rather than in the window. Make a thermodynamic argument for or against this idea.

5) **Laser Cooling:** Rubidium (Rb) atoms are often used in atomic physics experiments. There are several reasons for this. Rb is easily vaporized, and thus can be studied in a gaseous state. It is one of the alkali metals, a group of elements with relatively simple electronic configurations that have only one electron in their outer shell. Rb also has atomic transition frequencies that are easily accessible with common lasers. *Our book does not discuss laser cooling much. Here is another resource that might be of some help.*
http://hyperphysics.phy-astr.gsu.edu/hbase/optmod/lascool.html
a) Two commonly interrogated atomic transitions in ^{87}Rb, the D_1 and D_2 lines, correspond to wavelengths of 795 nm (D_1) and 780 nm (D_2). Calculate the magnitude of the momentum of a single photon at each of those wavelengths.
b) Suppose an initially stationary ^{87}Rb atom absorbs a single 780 nm photon that was traveling in the positive x direction. What is the velocity of the atom after it absorbs the photon?
c) At some point after absorbing a 780-nm photon, the atom will reemit a photon at the same wavelength. Suppose the photon is emitted in the positive z direction. What is the change in momentum of the atom when this happens?
d) In general, the reemission process happens in a random direction. Assume that the absorption and reemission process happens many, many times and that all of the photons come from a laser that emits light in the positive x direction.
   i) What is the net change in momentum <u>*due to emission only*</u>?
   ii) What is the direction of the <u>*overall*</u> change in momentum (due to both absorption and emission)?
e) The previous parts of the problem refer to how to impart a force on an atom. However, this does not necessarily cool (that is, slow down) the atom. If the atom is moving away from the laser, it would actually heat (speed up) the atom. To cool an atom, the force needs to be velocity dependent, so that the photons only impart a force on atoms that are moving towards the laser. Considering that atoms only absorb particular wavelengths of light, explain how the Doppler shift can be exploited to create a velocity dependent force to slow atoms *in three dimensions*.

6) Read (or listen to) the following two stories discussing recent topics from class.

Negative temperatures that are hotter than the sun
https://www.npr.org/2013/01/04/168624854/negative-temperatures-that-are-hotter-than-the-sun
How strong can a hurricane get?
https://physicstoday.scitation.org/do/10.1063/PT.6.1.20170908a/full/

Write a short response to the two articles. One paragraph per article is sufficient.

- Summarize the main points of each article in a sentence or two.
- For each article, discuss in a few sentences the connections between the science discussed in the article and the material covered in this course.

- Which article did you feel better conveyed the relevant science? Why? (Note that unlike the *Physics Today* article, the *NPR* piece is intended for a broader audience that does not necessarily have a background in physics.)
- Fulfilling the above requirements will earn you a $^{3.25}/_{4}$. The rest of the points will be awarded based on the depth and quality of your explanations of the connections. (For example, a response that discusses the energy flow of a negative temperature system will earn more points than a response that simply states that negative temperature is something we saw in class.)

7) List *three* main ideas from this homework assignment. For example, you could write a few-sentence explanation of a concept, or list an equation and explain the variables and in what circumstances the equation applies.
The goal is for you to review and to reflect on the big picture. Think about what you might want to remember when you look back at this homework before the test. I hope that this will be useful for your studying. I am not looking for anything specific here; you will be graded on effort and completion.