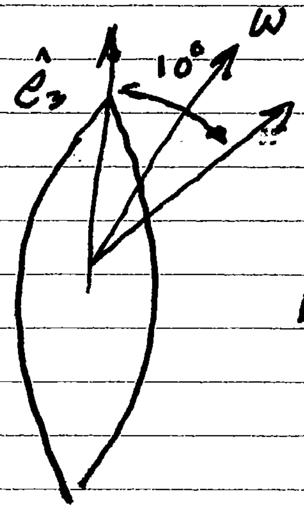
1/

Problem Set 11

7-11



$\hat{L}$ ← fixed in space

Note: 10° is $\measuredangle$ between L & e_3

also $2\pi(6)$ is the wobble freq. ω_w [Note it is not ω_3]

In class we saw

$$\omega_w = \frac{|L|}{I_x} \Rightarrow |L| = I_x \omega_w = 12\pi I_x$$

$$L = \begin{pmatrix} I_x & & \\ & I_x & \\ & & .6 I_x \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = I_x \begin{pmatrix} \omega_1 \\ \omega_2 \\ .6\omega_3 \end{pmatrix} \qquad (1)$$

Component of L in $\hat{e}_3$ dir $\quad I_x(.6)\omega_3$

Thus $\quad |L| \cos(10^\circ) = I_x(.6)\omega_3$

2

or $$12\pi \cos(10°) I_x = I_x (.6) \omega_3$$

$$\omega_3 = \frac{12\pi \cos(10°)}{.6} = 19.7\pi$$

From (1)

$$L^2 = I_x^2 \left(\omega_1^2 + \omega_2^2 + (.6)^2 \omega_3^2 \right)$$

$$(12\pi)^2 I_x^2 = I_x^2 \left(\omega_1^2 + \omega_2^2 + (.6)^2 (19.7\pi)^2 \right)$$

$$(12\pi)^2 - (.6)^2 (19.7)^2 \pi^2 = \omega_1^2 + \omega_2^2$$

$$4.29\,\pi^2 = \omega_1^2 + \omega_2^2 \equiv \omega_\perp^2$$

$$2.1\,\pi = \omega_\perp$$

b) In class, just after the intro. of Euler eq. we showed

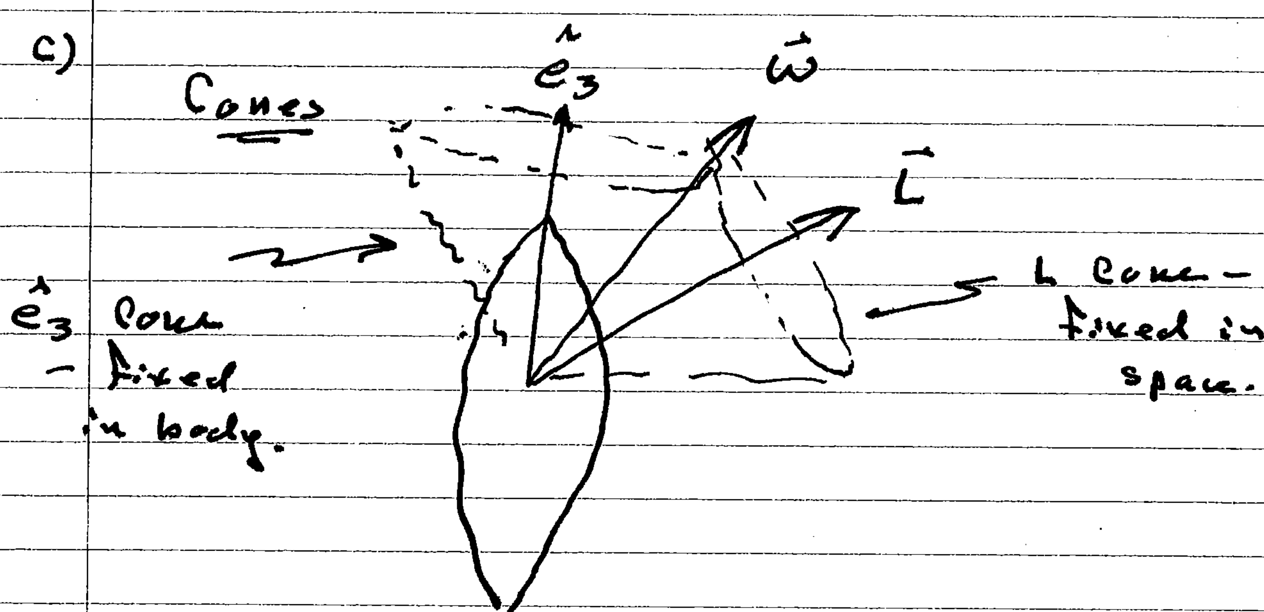
$$\ddot{\omega}_1 = -\Omega^2 \omega_1$$

where $$\Omega = \frac{(I_3 - I_x)}{I_x} \omega_3$$

$$= (.6 - 1.)\, \omega_3$$

$$|\Omega| = .4\,(19.7\,\pi) = 7.9\,\pi$$

c)



The $\hat{e}_3$ cone rolls without slipping around the $\vec{L}$ cone. For disc the $\hat{e}_3$ cone was inside the $\vec{L}$ cone. Here it is outside since $I_3 < I_1$. Seen from the $\hat{e}_3$ cone $\vec{\omega}$ (pt. of contact with L cone) goes the other way around from ω.

7-24

$$I_1 \dot{w}_1 = w_2 w_3 (I_2 - I_3)$$

$$I_2 \dot{w}_2 = w_3 w_1 (I_3 - I_1)$$

$$I_3 \dot{w}_3 = w_1 w_2 (I_1 - I_2)$$

$$\dot{w}_1 = w_2 w_3 \frac{(I_2 - I_3)}{I_1}$$

$$\dot{w}_2 = w_3 w_1 \frac{(I_3 - I_1)}{I_2}$$

$$\dot{w}_3 = w_1 w_2 \frac{(I_1 - I_2)}{I_3}$$

$I_3 > I_2 > I_1$ $\qquad \frac{I_2 - I_3}{I_1} = -\alpha_1$

$$\frac{I_3 - I_1}{I_2} = \alpha_2$$

$$\frac{I_1 - I_2}{I_3} = -\alpha_3$$

when all α's are pos.

$$\dot{w}_1 = -w_2 w_3 \alpha_1$$

$$\dot{w}_2 = w_3 w_1 \alpha_2$$

$$\dot{w}_3 = -w_1 w_2 \alpha_3$$

1st ~~let~~ ω close to $\hat{e}_1$ => ω_1

large seek evolution of ω_2 & ω_3

$$\dot{\omega}_2 = \omega_1 \alpha_2 \omega_3$$

$$\dot{\omega}_3 = -\omega_1 \alpha_3 \omega_2$$

Take $\omega_1 \simeq$ Const

$$\ddot{\omega}_2 = \omega_1 \alpha_2 \dot{\omega}_3 = -\underbrace{\omega_1 \alpha_2 \omega_1 \alpha_3}_{\text{pos.}} \omega_3$$

So harmonic oscillator —

2nd ω close to $\hat{e}_2$ => ω_2 large

& ~ const

See evolution of ω_1 & ω_3

$$\dot{\omega}_1 = -\omega_2 \alpha_1 \omega_3$$

$$\dot{\omega}_3 = -\omega_2 \alpha_3 \omega_1$$

$$\ddot{\omega}_1 = -\omega_2 \alpha_1 \dot{\omega}_3 = +\underbrace{\omega_2^2 \alpha_1 \alpha_3}_{\text{pos.}} \omega_1$$

HO with the wrong sign

— unstable.