Homework Assignment 10

PHYSICS 314 - THERMODYNAMICS & STATISTICAL PHYSICS (Spring 2018)

Due Friday, May 4th, by noon, Noyce 1135

I cannot award full credit for work that I am unable to read or follow. For my benefit and for yours, please:

- *Write neatly*
- *Show and EXPLAIN all steps*
- *Make diagrams large and clearly-labeled*

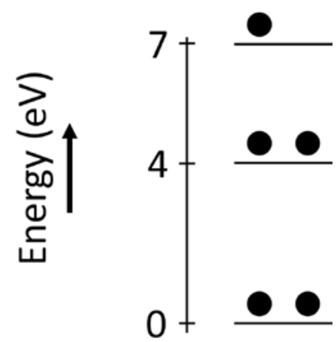
You are welcome to collaborate with others on this assignment. However, the work you turn in should be your own. Please cite collaborators and outside sources. See the syllabus for details.

Regardless of the number of parts, all homework problems are weighted equally. Regardless of the number of questions, all homework assignments are weighted equally.

---

1) H_2 excited state probabilities (See Schroeder page 373 for information on the hydrogen atom.)
   a) For a hydrogen atom at room temperature, estimate the probability of being in the first excited state. *Hint: You may approximate an infinite sum by only considering the first term in the sum, but you must justify why doing so is a reasonable approximation.*
   b) Repeat the calculation for hydrogen at $9500\ K$, as in the atmosphere of the star γ Uma.

2) In previous physics and math classes, you have probably encountered the concept of *standard deviations*, a measure of fluctuations around an average value. It is one way of quantifying uncertainty in experimental results. For this problem, consider a simplified system that has five atoms, each of which can occupy one of three possible states. Let the energies of the states be as shown, and suppose the atoms are distributed in the states as represented by the dots.



   a) For each of the five atoms, calculate the *deviation* of the energy from the average energy of the five atoms, $\Delta E_i = E_i - \bar{E}$.
   b) Compute the *standard deviation*, $\sigma_E \equiv \sqrt{\overline{(\Delta E_i)^2}}$. This quantity is also called the *root-mean-square (rms) deviation* because it is the square **root** of the **mean** (average) of the **square** of the deviation.
   c) For this system, perform explicit calculations to show that the following relationship is true.

$$\sigma_E^2 = \overline{E^2} - (\bar{E})^2$$

   d) Prove that the relationship in part c) is true in general. (This relationship is often an easier way to calculate the standard deviation.)

3) Cyanogen (CN) is a molecule often found in cold interstellar molecular clouds. The first rotational excited states of CN have an energy that is greater than that of the ground state by $4.7 \times 10^{-4}\ eV$. There are three excited states that share this same energy. From studies of absorption spectra of starlight that passes through the molecular clouds, one can learn about the about the relative population of the excited states of CN. Measurements from 1941 showed that for every ten CN molecules in the ground state, there are roughly three molecules in the excited states - on average, one in each of the three states. To account for these relative populations, astronomers suggested that the molecules could be in thermal equilibrium with some thermal reservoir with a well-defined temperature, T_{res}. Find T_{res}. (The thermal reservoir is

now known to be a gas of photons that fills the entire observable universe. It is known as the *cosmic background radiation*, and its discoverers won a Nobel Prize.)

4) Consider a system that is in equilibrium with a reservoir at temperature T. Prove the following equation for the average value of energy in such a system. (We have been using this equation in lecture without proof.) Z is the partition function, and $\beta = {}^{1}/_{kT}$. Do not forget to explain both equalities.

$$\bar{E} = -\frac{\partial}{\partial\beta}\ln Z = -\frac{1}{Z}\frac{\partial Z}{\partial\beta}$$

5) Consider the (quantized) rotation of the diatomic molecule carbon monoxide, CO. For CO, the rotational quantum ϵ has been measured to be approximately $0.00024\ eV$.
   a) This value can be spectroscopically measured by exposing the molecules to varying frequencies of electromagnetic radiation.
      i) What is the frequency of a photon that excites CO from the rotational ground state ($j = 0$) to the first excited rotational state ($j = 1$)?
      ii) What is the frequency of a photon that excites CO from the first excited state ($j = 1$) to the second excited state ($j = 2$)?
      iii) In what region of the electromagnetic spectrum do these photons lie?
   b) Calculate the rotational partition function for CO at $300\ K$ using the high-temperature approximation derived in class.
   c) Now examine the exact formula for the same rotational partition function, Schroeder equation (6.30). The sum is infinite, but it can be approximated by a finite sum from zero to some $j = j_f$. Use a computer program of your choice to calculate the approximate finite sum.

$$Z_{rot}(j_f) \approx \sum_{j=0}^{j_f}(2j+1)e^{-j(j+1)/kT}$$

   Plot Z_{rot} as a function of j_f. Start from $j_f = 0$, and continue at to higher j_f until Z_{rot} seems to be converging (at least until it changes by less than 0.1 when you add the next term to the sum). Plot your approximate answer from part b) (a constant value) on the same axes. Does the high temperature approximation seem to be higher or lower than the actual value?

6) Gaussian integrals appear frequently in physics. Recall that we used the result of a Gaussian integral in class.

$$\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$$

I highly recommend that you read the two or so pages of Appendix B.1 about Gaussian integrals. The proof of the formula above is very clever. From this one Gaussian integral result, one can evaluate many different integrals. This problem looks at a slightly different integrands of the form $x^n e^{-ax^2}$, where n is odd, and a is a constant.
   a) Evaluate the following <u>*definite*</u> integral entirely by reasoning. Do not actually integrate.

$$\int_{-\infty}^{\infty} xe^{-ax^2}dx$$

   b) Evaluate the following <u>*indefinite*</u> integral using u substitution.

$$\int xe^{-ax^2}dx$$

c) Evaluate the following <u>*definite*</u> integral.

$$\int_0^\infty xe^{-ax^2}dx$$

d) Evaluate the following <u>*definite*</u> integral. *Hint: Use part c) and the method used from Schroeder equation (B.6) to (B.8).*

$$\int_0^\infty x^3e^{-ax^2}dx$$

<u>NOTICE!!! THIS PROBLEM IS TIME SENSITIVE!</u>

7) You have two choices for the reflection problem this week. You may choose to do either Option I or Option II. *I strongly encourage you to do Option I if you are able,* but both options are worth the same number of points.

Option I

Attend the second special *Squire Lecture* Physics seminar on Tuesday, May 1st, at noon, <u>OR</u> the general public seminar at 4 pm that afternoon.

The speaker is Professor Andrew Davis (a 1971 Grinnell graduate!) from the University of Chicago. The noon presentation is entitled, *Making CHILI, a new instrument for exploring the origin of our solar system*, and the afternoon seminar is entitled, *Nature's ultimate sample return mission: seeing into stars by studying stardust in the laboratory*. Professor Davis studies geochemistry and cosmochemistry, which are related to the geophysics we have been discussing recently.

After attending the lecture, write a half-page reflection on the presentation.

- Summarize the main points of the presentation.
- Discuss the connections between the science discussed in the presentation and the material covered in this course.
- Fulfilling the above requirements will earn you a $^{3.25}/_{4}$. The rest of the points will be awarded based on the depth and quality of your reflection.

Option II

Read (or listen to) the following material discussing recent topics from class.

Lab-grown diamonds come into their own

https://www.npr.org/2016/12/01/502330818/lab-grown-diamonds-come-into-their-own

Big diamonds bring scientists a message from superdeep Earth

https://www.npr.org/sections/thetwo-way/2016/12/15/505386423/big-diamonds-bring-scientists-a-message-from-superdeep-earth

Write a short response to the two articles. One paragraph per article is sufficient.

- Summarize the main points of each article in a few sentences.
- For each article, discuss in a few sentences the connections between the science discussed in the article and the material covered in this course.
- Which article did you feel better conveyed the relevant science? Why? (Keep in mind the intended audience of each piece.)

- Fulfilling the above requirements will earn you a $^{3.25}/_{4}$. The rest of the points will be awarded based on the depth and quality of your explanations of the connections.

8) PLEASE DO #8 ON PIECES OF PAPER THAT ARE SEPARATE FROM THE REST OF YOUR HOMEWORK.

On the first piece of paper, write or type your problem without the solution; this will be copied for your peers. On a separate piece of paper (that is also separate from the rest of your homework), write or type the solution.

In preparation for the end of the semester, continue thinking about questions that could be on the final. For this week's assignment, look at my feedback from your proposed questions from HW9 problem 8. Choose one of your problems to write-up more formally. Make any revisions necessary to the problem, and then write up a detailed solution. As a model for the level of detail, use the solutions you are provided for regular homework. Be sure to not only show any mathematics clearly, but *also to clearly explain your reasoning for each step.*

You will be graded on the appropriateness, originality, and clarity of your questions and on the thoroughness and clarity of the solution.

For next week's homework, I will have you exchange problems and solve a problem created by one of your classmates.

Remember, if any of the problems are particularly good, I may use them on the final! Wouldn't it be nice to see a problem you wrote on the final?