Worksheet 08 - Bases and Coordinates

1 Definitions

Definition 1.1. Let $\vec{u}_1, \vec{u}_2, \in \mathbb{R}^2$. We say that the ordered pair $(\vec{u}_1, \vec{u}_2)$ is a *basis* for $\mathbb{R}^2$ whenever $Span(\vec{u}_1, \vec{u}_2) = \mathbb{R}^2$.

Definition 1.2. Let $\alpha = (\vec{u}_1, \vec{u}_2)$ be a basis for $\mathbb{R}^2$. We define the *coordinate function with respect to* α,

$$Coord_\alpha : \mathbb{R}^2 \to \mathbb{R}^2$$

in the following way: For any $\vec{v}$ in the domain $\mathbb{R}^2$, we set

$$Coord_\alpha(\vec{v}) = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

where $c_1, c_2 \in \mathbb{R}$ are the unique scalars such that

$$\vec{v} = c_1 \cdot \vec{u}_1 + c_2 \cdot \vec{u}_2$$

2 Applications

Exercise 1. If you are given that $\alpha = (\vec{u}_1, \vec{u}_2)$ a basis for $\mathbb{R}^2$, and that

$$\vec{u}_1 = \begin{pmatrix} a \\ c \end{pmatrix} \text{ and } \vec{u}_2 = \begin{pmatrix} b \\ d \end{pmatrix}$$

then you should be able to describe any vector in $\mathbb{R}^2$ as a linear combination of $\vec{u}_1$ and $\vec{u}_2$. That is, for any arbitrary vector $\vec{v} = \begin{pmatrix} j \\ k \end{pmatrix}$ we should be able to find real numbers, r_1 and r_2, so that

$$\begin{pmatrix} j \\ k \end{pmatrix} = r_1 \cdot \begin{pmatrix} a \\ c \end{pmatrix} + r_2 \cdot \begin{pmatrix} b \\ d \end{pmatrix}$$

Find expressions for r_1 and r_2 in terms of a, b, c, d, j, and k. (*Hint: Your notes from Worksheet 05 may be useful.*)

Exercise 2. Using your observations (and assumptions) from Exercise 1, propose a specific formula for

$$Coord_\alpha : \mathbb{R}^2 \to \mathbb{R}^2$$

Exercise 3. Pick three distinct bases for $\mathbb{R}^2$ and demonstrate how to apply your formula from Exercise 2 to express $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ as linear combination in terms of each of those bases.

For next time

- Complete this worksheet
- Review section 2.3
- Read Section 2.4, through the proof of Proposition 2.4.3.