Homework Assignment 1

PHYSICS 314 - THERMODYNAMICS & STATISTICAL PHYSICS (Spring 2018)

Due Friday, February 2nd, by noon, Noyce 1135

I cannot award full credit for work that I am unable to read or follow. For my benefit and for yours, please:

- *Write neatly*
- *Show and EXPLAIN all steps*
- *Make diagrams large and clearly-labeled*

You are welcome to collaborate with others on this assignment. However, the work you turn in should be your own. Please cite collaborators and outside sources. See the syllabus for details.

Regardless of the number of parts, all homework problems are weighted equally. Regardless of the number of questions, all homework assignments are weighted equally.

---

1) In the Fahrenheit scale, water freezes at a temperature of 32° F and boils at a temperature of 212° F.
   a) Derive an equation to convert from Fahrenheit to Celsius and one to convert back in the opposite direction. The two scales are linear.
   b) The normal body temperature of a human is 98.6° F. What is this temperature in Celsius? In kelvin?

2) See equation 1.40 in Schroeder (pg. 26). For an ideal gas undergoing adiabatic compression, determine the relationships between the following parameters. Each answer should be in terms of the adiabatic exponent, γ, and a constant.
   a) V and T
   b) P and T

3) Thermal Expansion
   a) For a liquid, the fractional increase in volume per unit change in temperature (with fixed pressure) is called the *thermal expansion coefficient*, β.

$$\beta \equiv \frac{\Delta V / V}{\Delta T}$$

   V is the volume. T is the temperature. Δ is a (technically *infinitesimal)* change.

   When the temperature of liquid mercury increases by one kelvin, its volume increase by one part in 5500. Thus, $\beta_{Hg} = {}^{1}/_{5500K}$. (This value actually changes a small amount with temperature, but the variation is less than a percent over a couple hundred degrees in the temperature range of interest, so it can be ignored.)

   A mercury thermometer uses this thermal expansion to measure temperature. Suppose such a thermometer has a bulb at the bottom that is full of mercury. Most of the mercury is contained in the bulb, but as this mercury expands, a very small amount travels up a narrow tube that is marked with temperatures.

   Suppose that the bulb at the bottom contains 60 mm^3. Also suppose that a 1 degree ***Celsius*** temperature change changes the height of the mercury in the cylindrical tube by 8.5 mm. What is the radius of the narrow inner tube?

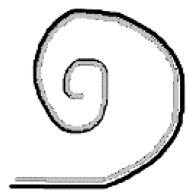
b) Suppose that now that we have two new thermometers. Their dimensions are unknown. However, they are identical, except that one is filled with mercury and one is filled with the mercury-alternative toluene. For a temperature change of 0.5 degrees Celsius, the height of the mercury changes by 4 mm. By how much does the height of the toluene change?

$$\beta_{toluene} = 0.001K^{-1}$$

c) For a solid, the *linear thermal expansion coefficient*, α, is the fractional increase in length per degree change in temperature.

$$\alpha \equiv \frac{\Delta L / L}{\Delta T}$$

A dial thermometer uses a coiled metal strip made out of two different metals laminated together. Qualitatively explain in a few sentences how this could be used as a thermometer.



d) Show that the volume thermal expansion coefficient of a solid is *approximately* equal to the sum of its linear expansion coefficients in three directions.

$$\beta = \alpha_x + \alpha_y + \alpha_z$$

Hint: Consider a rectangular prism. Choose one of the (at least) two ways to do this. (1) Consider a differential form of the definition of the thermal expansion coefficients. (2) Calculate the change in volume from the prism expanding in each of the three dimensions, ignoring very small contributions.

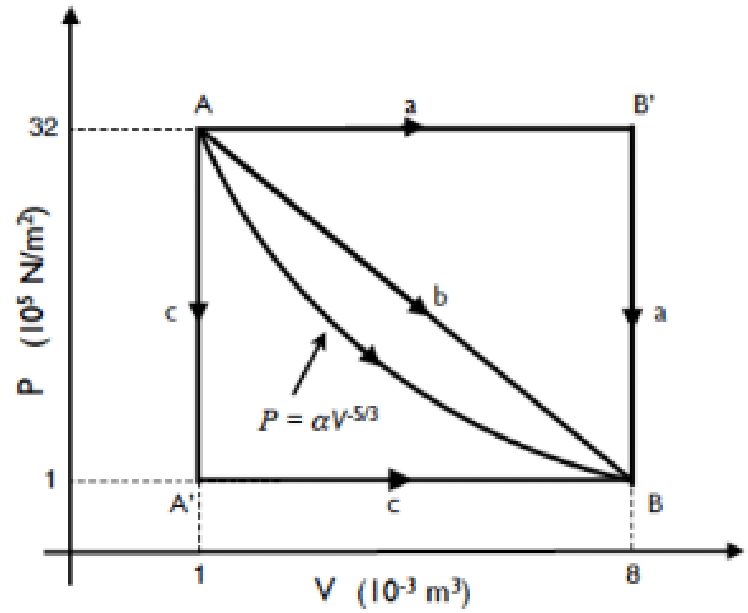
4) Consider one mole of an ideal gas.
   a) Depict each of the following processes on a separate *P–V* diagram.
      i) An *isobaric* process (one that takes place at a constant pressure) carrying the gas from an initial state *A* to a final state *B*.
      ii) An *isochoric* process (one that takes place at a constant volume) carrying the gas from an initial state *A* to a final state *C*.
      iii) An *isothermal* process (one that takes place at a constant temperature) carrying the gas from an initial state *A* to a final state *D.*
   b) Depict these same three processes (isobaric, isochoric, and isothermal) on separate *P–T* diagrams.

5) A horizontal cylinder, closed off by a piston, contains a monatomic ideal gas (adiabatic exponent $\gamma = 5/3$) with pressure P_0. Its initial volume is V_0 and its initial temperature is T_0. You wish to reduce the volume of the gas by a factor of 2, and you consider a few different ways of doing so.
   a) Calculate the work done on the system if you reduce the volume by a factor of 2 with a process that is:
      i) Isothermal
      ii) Isobaric
      iii) Adiabatic
   b) Rank the three different processes in order of increasing work done on the system.
   c) Sketch each of these three processes on a single P-V diagram, and comment on how this illustrates the result of part b).

6) In a process A → B in which no heat is exchanged with the environment, the pressure *P* of a certain amount of gas is found to change with its volume *V* according to the relation:

$$P = \alpha V^{-5/3}$$

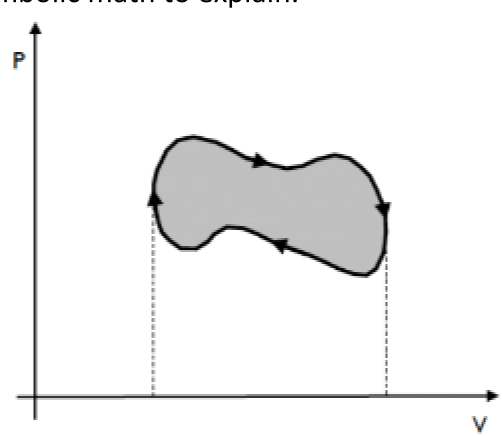
where α is a constant. This process, along with three other processes (a, b, and c) that take the gas from state A to state B, is shown in the diagram below.



Now consider the other three processes shown. Find the work done on this system ***and*** the net heat absorbed by this system in each case. *Note the labels on the diagram; give numerical answers.*

a) The system is expanded from its original to its final volume, with heat being added to maintain a constant pressure. The volume is then kept constant, and heat is extracted to reduce the pressure to 10^5 N m^{-2}.
b) The volume is increased, and heat is supplied to cause the pressure to decrease linearly with volume.
c) The two steps of process a) are performed in the opposite order.

7) A system undergoes the process that appears in the P-V diagram below. Such a process is called 'cyclic' since the system ends up in a final state that is identical to its initial state; it is a closed loop. Show that the work done ***by*** the system is given by the area contained within the closed curve. Use a combination of labeled sketches, words, and symbolic math to explain.



8) List *three* main ideas from this homework assignment. For example, you could write a few-sentence explanation of a concept, or list an equation and explain the variables and in what circumstances the equation applies.
The goal is for you to review and to reflect on the big picture. Think about what you might want to remember when you look back at this homework before the test. I hope that this will be useful for your studying. I am not looking for anything specific here; you will be graded on effort and completion.