

## 1 Review

**Exercise 1.** Give a parametric description of the solution set from Worksheet 5, Exercise 2.

**Exercise 2.** In Exercise 4 on Worksheet 5, what restrictions must you place on the constants,  $a, b, c$ , and  $d$ , to be sure that the solution sets make sense?

## 2 The Vector Space $\mathbb{R}^2$

**Definition 2.1.** The *vectors* in  $\mathbb{R}^2$  are the elements of the set

$$\mathbb{R}^2 = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} : a, b \in \mathbb{R} \right\}$$

We define the *sum of vectors* in  $\mathbb{R}^2$  by

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a + c \\ b + d \end{pmatrix}$$

We define *scalar multiplication* of a vector by

$$d \cdot \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} d \cdot a \\ d \cdot b \end{pmatrix}$$

for all  $d \in \mathbb{R}$ , where “ $\cdot$ ” denotes standard real number multiplication.

**Notation.** We use the notation  $\vec{w}$  to indicate that  $w$  is a vector. We use the notation  $\vec{0}$  to denote the vector whose entries are all zero.

**Definition 2.2.** Let  $\vec{u}$  be a vector in  $\mathbb{R}^2$ . We define the *span of  $\vec{u}$*  to be the following subset of  $\mathbb{R}^2$ :

$$Span(\vec{u}) = \{c \cdot \vec{u} : c \in \mathbb{R}\}$$

**Exercise 3.** Let  $\vec{u} \in \mathbb{R}^2$  be arbitrary, and let  $S = Span(\vec{u})$ . Prove that  $\vec{0} \in S$ .

**Exercise 4.** Let  $\vec{u} \in \mathbb{R}^2$  be arbitrary, and let  $S = Span(\vec{u})$ . Prove that for all  $\vec{v}_1, \vec{v}_2 \in S$  we have that  $\vec{v}_1 + \vec{v}_2 \in S$

**Exercise 5.** Let  $\vec{u} \in \mathbb{R}^2$  be arbitrary, and let  $S = Span(\vec{u})$ . Prove that for all  $\vec{v} \in S$ , and all  $d \in \mathbb{R}$ , we have that  $d \cdot \vec{v} \in S$

**STOP**

**Exercise 6.** Compare and contrast your set from Exercise 1 to the definition of  $Span(\vec{u})$ .

**Exercise 7.** Which properties from Exercises 3, 4, and 5 does the set from Exercise 1 exhibit? Prove, or provide a counterexample for each.

## For Next Time

- Complete this Worksheet
- Read 2.2 and 2.3 (up to Proposition 2.3.7)