Homework Assignment 5

PHYSICS 314 - THERMODYNAMICS & STATISTICAL PHYSICS (Spring 2018)

Due Friday, March 9th, by noon, Noyce 1135

I cannot award full credit for work that I am unable to read or follow. For my benefit and for yours, please:

- *Write neatly*
- *Show and EXPLAIN all steps*
- *Make diagrams large and clearly-labeled*

You are welcome to collaborate with others on this assignment. However, the work you turn in should be your own. Please cite collaborators and outside sources. See the syllabus for details.

Regardless of the number of parts, all homework problems are weighted equally. Regardless of the number of questions, all homework assignments are weighted equally.

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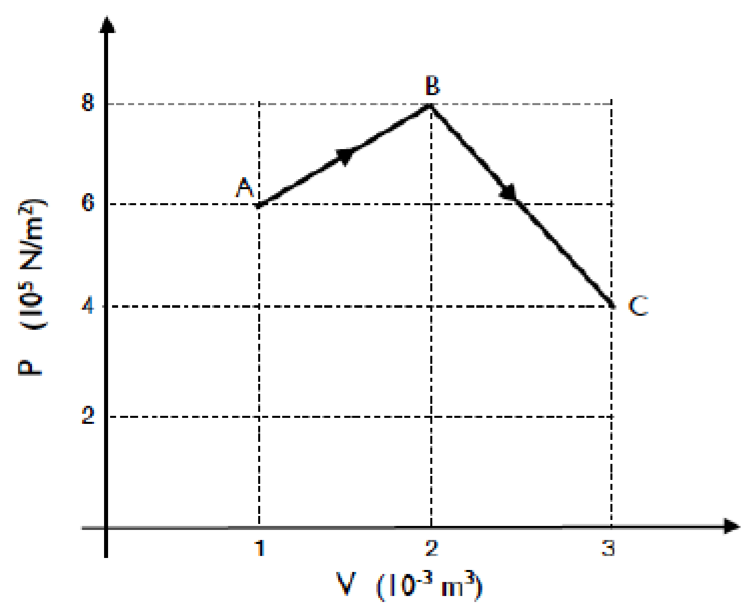
1) Two identical blocks of copper *A* and *B* initially at temperatures T_A and T_B, respectively, are placed in thermal contact with one another. Each block has mass *m* and *specific* heat capacity *c*. Assume that the combined system of the two copper blocks is isolated.
   a) Derive an expression for the total change in energy of the system, ΔU, as the system goes from its initial state to equilibrium. Express your answer in terms of *c*, *m*, T_A, T_B, and T_f, the equilibrium temperature of the system.
   b) Use your expression from a) to find the final temperature of the system. *I know you probably know the answer without doing any calculations. I want you to follow this method anyway.*
   c) Find the change in entropy of each block (ΔS_A and ΔS_B) and the total variation in entropy of the system (ΔS). Express your answer in terms of *c*, *m*, T_A, and T_B.
   d) Show that the second law of thermodynamics holds for the system.

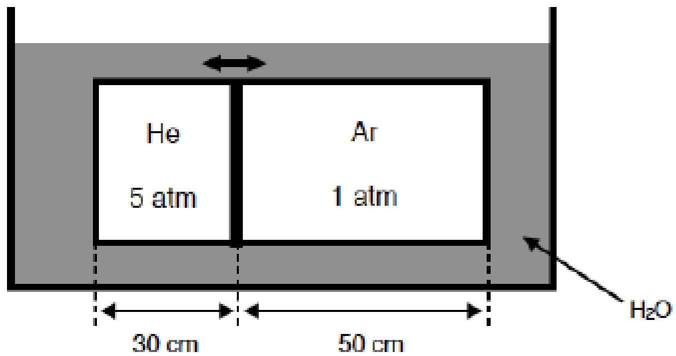
2) Assume there is one mole of an ideal diatomic gas. This gas is taken quasistatically first from state **A** to state **B**, and then from state **B** to state **C** along the straight-line paths shown in the pressure vs. volume diagram.



   a) What is the heat capacity at constant volume of the gas, C_V?
   b) What is the work done on the gas in the process **A** → **B** → **C**?

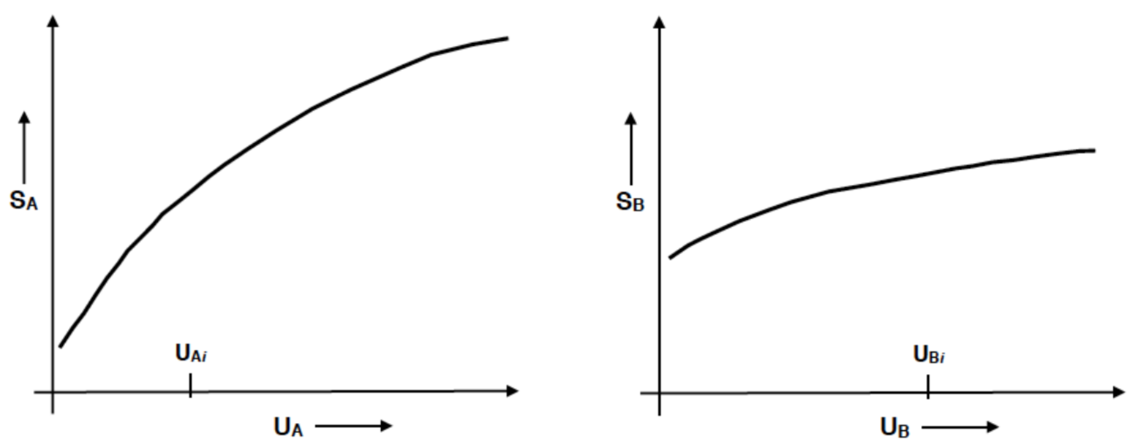
c) What is the heat absorbed by the gas in this process?
d) What is the change of entropy in this process?

3) An 80-cm-long cylindrical container is separated into two compartments by a thin piston. The piston is originally clamped in position 30 cm from the left end. The left compartment is filled with one mole of helium gas at a pressure of 5 atmospheres. The right compartment is filled with argon gas at 1 atmosphere of pressure. Consider the gases to be ideal. The cylinder is submerged in 1 liter of water, and the entire system is initially at a uniform temperature of 25° C. The heat capacities of the cylinder and of the piston may be neglected. When the piston is unclamped, it moves until a new equilibrium situation is ultimately reached. Assume the system as a whole is isolated, that is, assume that the water can exchange energy with the helium and the argon, but not with anything else.



a) What is the increase in temperature of the water?
b) How far from the left end of the cylinder will the piston come to rest? *Hint: First find the number of moles of argon.*
c) What is the increase in total entropy of the system?

4) Entropy versus energy plots for two objects, *A* and *B*, are shown below.



Both graphs use the same horizontal and vertical scales. The initial energies of the two objects are U_{Ai} and U_{Bi} at the values shown.

a) Explain what happens initially when objects *A* and *B* are brought into thermal contact, and explain *why*. **You may NOT use the word "temperature" in your answer.**
b) Now you can use "temperature" again. Which object was initially at a higher temperature? How do you know?
c) Sketch the two plots above and indicate on each plot the equilibrium energy of the object. *Explain.*

5) Assume your kitchen is 25°C. A 30.0-g ice cube at 0°C is left in the sink, where it eventually melts.
   a) Calculate the change in entropy for the phase change of the ice cube as it melts into water at 0°C. Ignore the small change in volume that happens with the phase change. *Hint: You will probably have to look up a value to complete this problem.*
   b) Calculate the change in entropy of the resulting water as its temperature rises from 0°C to 25°C. *You can assume that the heat capacity of water is constant over this range, which is a pretty good assumption. Hint: You will probably need to look up a value again.*
   c) Calculate the change in entropy *of the kitchen* as it gives up heat to the melt ice and warm the water to room temperature.
   d) Calculate the net change in entropy of the universe during the process. Comment on the meaning of the sign of your answer.

6) Fill in some of the gaps from our discussion of paramagnetism.
   a) Start with the expression for the multiplicity of a two-state paramagnet with N dipoles. Assume N is large. Derive an expression for the entropy in terms of $N_\uparrow$ and N. Manipulate your expression so that it does not have any factorials, fractions, or exponents. You can use the form of Sterling's approximation that ignores large terms.
$$\Omega = \frac{N!}{N_\uparrow!\,N_\downarrow!}$$
   b) Use your expression from part a) to calculate $\frac{\partial S}{\partial N_\uparrow}$, the partial derivative of entropy with respect to the number of spin-up dipoles.
   c) In class, we found an expression for energy as a function of spin-up dipoles. Differentiate to find $\frac{\partial U}{\partial N_\uparrow}$.
$$U = \mu B(N_\downarrow - N_\uparrow)$$
   d) Combine your previous results to find an expression for temperature as a function of $N_\uparrow$. *Hint: Use the chain rule and the fact that* $\frac{\partial N_\uparrow}{\partial U} = \left(\frac{\partial U}{\partial N_\uparrow}\right)^{-1}$.

7) There are three partial derivative of *entropy* formulas in Schroeder Table 3.3: one for temperature, one for pressure, and one for chemical potential. Use the thermodynamic identity to derive three similar formulas for temperature, pressure, and chemical potential that involve only partial derivatives of *energy*.
   Hint: This is fairly straightforward. I just want you to spend a little time thinking about all of the information contained in this one formula.
$$dU = TdS - PdV + \mu dN$$

8) List <u>*three*</u> main ideas from this homework assignment. For example, you could write a few-sentence explanation of a concept, or list an equation and explain the variables and in what circumstances the equation applies.
   The goal is for you to review and to reflect on the big picture. Think about what you might want to remember when you look back at this homework before the test. I hope that this will be useful for your studying. I am not looking for anything specific here; you will be graded on effort and completion.