Final exam

Focus on material before 5.2.16

$T: P_5 \to P_5$ by $T(f) = f''$

$\alpha = (1, x, x^2, x^3, x^4, x^5)$

$$c_0 \cdot 1 + c_1 \cdot x + c_2 \cdot x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5$$

$$\xrightarrow{T} 0 \cdot 0 + 0 \cdot 0 + c_2 \cdot (2) + c_3 6x + c_4 12x^2 + c_5 20 x^3$$

$$\begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{pmatrix} \to \begin{pmatrix} c_2 \cdot 2 \\ c_3 \cdot 6 \\ c_4 \cdot 12 \\ c_5\, 20 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \to \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ 6 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix}0\\0\\0\\0\\1\\0\end{pmatrix} \to \begin{pmatrix}0\\0\\12\\0\\0\\0\end{pmatrix} \qquad \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix} \to \begin{pmatrix}0\\0\\0\\20\\0\\0\end{pmatrix}$$

$$[T]_\alpha^\alpha = \begin{pmatrix}0&0&2&0&0&0\\0&0&0&6&0&0\\0&0&0&0&12&0\\0&0&0&0&0&20\\0&0&0&0&0&0\\0&0&0&0&0&0\end{pmatrix}\begin{pmatrix}d_1\\d_2\\d_3\\d_4\\d_5\\d_6\end{pmatrix}$$

$$\begin{bmatrix}3&0&0\\0&4&0\\0&0&5\end{bmatrix}\begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix} \to c_1\begin{pmatrix}3\\0\\0\end{pmatrix} + c_2\begin{pmatrix}0\\4\\0\end{pmatrix} + c_3\begin{pmatrix}0\\0\\5\end{pmatrix}$$

Range(T) = Span(columns of T)

$$\begin{bmatrix}3&0&0&1\\0&4&0&1\\0&0&5&1\end{bmatrix} \qquad \mathbb{R}^4 \to \mathbb{R}^3$$

$$\begin{bmatrix}3&0&0&1&2\\0&4&0&1&1\\0&0&5&1&3\end{bmatrix} \qquad \mathbb{R}^5 \to \mathbb{R}^3$$

$$\begin{bmatrix}3&0&0\\0&4&0\\0&0&5\end{bmatrix}\begin{pmatrix}1\\2\\1\end{pmatrix} = \begin{pmatrix}3\\8\\5\end{pmatrix}$$

$$\begin{bmatrix}3&0&0\\0&4&0\\0&0&5\end{bmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}3\\0\\5\end{pmatrix}$$

$$\Rightarrow \begin{bmatrix}3&0&0\\0&4&0\\0&0&5\end{bmatrix}\begin{bmatrix}1&1\\2&0\\1&1\end{bmatrix} = \begin{pmatrix}3&3\\8&0\\5&5\end{pmatrix}$$

Vector spaces

- some vector spaces have bases

  what does a basis give us?

  - encode <u>every</u> element of the space as a <u>unique</u> linear combination of the basis elements

- Once we have a basis then we get a Coordinate function.

$$\mathrm{Coord}_{\alpha} : V \longrightarrow \mathbb{R}^n$$

- So we can use coordinate functions to express any linear transformation as a matrix.

$$\begin{array}{c} \\ n\downarrow \end{array}\begin{bmatrix} & \\ & \\ & \xrightarrow{m} \end{bmatrix}_{n\times m}\begin{pmatrix} \\ \\ \\ \end{pmatrix}_{m\times 1}\downarrow m = \begin{pmatrix} \\ \\ \\ \end{pmatrix}_{n\times 1}$$

$$\begin{bmatrix} & \\ & \\ & \end{bmatrix}_{n\times m}\begin{bmatrix} \\ \\ \\ \end{bmatrix}_{m\times 2} = \begin{bmatrix} & \\ & \\ & \end{bmatrix}_{n\times 2}$$

$\mathbb{R}^m \to \mathbb{R}^n$ $\qquad$ $\mathbb{R}^2 \to \mathbb{R}^m$