Homework Assignment 2

PHYSICS 314 - THERMODYNAMICS & STATISTICAL PHYSICS (Spring 2018)

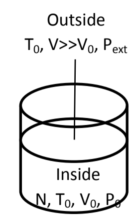
Due Friday, February 9th, by noon, Noyce 1135

I cannot award full credit for work that I am unable to read or follow. For my benefit and for yours, please:

- *Write neatly*
- *Show and EXPLAIN all steps*
- *Make diagrams large and clearly-labeled*

You are welcome to collaborate with others on this assignment. However, the work you turn in should be your own. Please cite collaborators and outside sources. See the syllabus for details.

Regardless of the number of parts, all homework problems are weighted equally. Regardless of the number of questions, all homework assignments are weighted equally.



1) A cylindrical container with cross-sectional area *S* and closed off at its top by a piston of mass *m*, contains n moles of an ideal gas. The walls of the cylinder are *diathermal*, which means they conduct heat readily. It is placed in a room where the pressure is P_{ext}, and the temperature is T_0. In its initial state, the temperature of the gas is also T_0, its volume is V_0, and its pressure is P_0. The piston is attached to the ceiling with a string. You cut the string and wait for the gas to attain its final state.
   a) Determine the final state of the gas by finding:
      i) P_f, (*Hint: When does the piston stop moving? Think Physics 131.*)
      ii) T_f, and
      iii) V_f.
   b) Calculate the variation in the total energy of the gas (ΔU).
   c) The amount of heat that flows into the room in the process is measured to be Q_0. What is the work done on the gas by the piston? (*Be careful about the sign!*)

2) A cylinder with adiabatic (non-thermally conducting) walls is divided into two compartments, A and B. Each one contains one mole of a monatomic ideal gas. The separating wall between A and B is a non-permeable piston that is free to move. Initially, in compartment A, a small heater (of negligible heat capacity) slowly heats the gas until a *long time* later, t_f, when a thermometer is placed in compartment B, indicating its final temperature, T_B = 321 K.
   Assume that $C_V = 3/2\ R$ for both compartments, and assume that initially for both compartments T_0 = 273 K and P_0 = 105 Pa. (*Note: This is not a typo: 105 Pa, NOT 10^5 Pa.*) Calculate the initial and final temperatures, pressures, and volumes in each of the two compartments, as well as the energy provided by the heater in the case of:
   a) a diathermal piston.
   b) an adiabatic piston.
      In both cases, present your results in a table.
      Hint: For the energy from the heater in b), consider the First Law for each compartment individually.

3) You wish to keep your dorm room at a constant temperature of T_1 = 22° C while the outside temperature in Grinnell is 0° C. To evaluate the thermal losses, you turn off the heat. After two hours, the temperature in your dorm room has dropped to 17° C. We assume that during an interval of time, dt, the amount of heat lost is given by:

$$Q = -a(T - T_{ext})dt$$

a) Derive a relationship T(t) between the temperature T (in K) and time *t* (in seconds), in terms of *a* and the heat capacity of the dorm room, *C*.
b) Calculate the numerical value for *a*, given that $C = 10^7$ J K^{-1}.
c) Calculate the power that must be exerted by your heater to maintain a constant temperature of 22° C in your dorm room when the temperature outside is 0° C.

4) Mount Ogden in Weber County, Utah, is 9,579 feet above sea level. (For contrast, the highest point in Iowa is Hawkeye Point in the northwest part of the state, which is 1,670 feet above sea level.) A 60-kg hiker wishes to climb to the summit of Mt. Ogden, an ascent of 5,000 vertical feet (≈ 1,500 m) above the surrounding terrain.
   a) Assuming that she is 25% efficient at converting chemical energy from food into mechanical work, and that essentially all of the mechanical work is used to climb vertically, roughly how many bowls of corn flakes (standard serving size 1 oz., 100 kcal) should the hiker eat before setting out? (1 kcal = 4,184 J)
   b) As the hiker climbs the mountain, 75% of the energy from the corn flakes is converted to thermal energy. If there were no way to dissipate this energy, and this energy were equivalent to the heat that flows into her body, by how many degrees would her body temperature increase? (The specific heat capacity of the human body is roughly 3,500 J K^{-1} kg^{-1}.)
   c) Thankfully, the extra energy does not warm the hiker's body significantly. (The temperature increase you found in part b) would probably be fatal.) Instead, it goes (mostly) into evaporating water from her skin. If the temperature is 25° C, how many liters of water should she drink during the hike to replace fluids lost due to the thermal energy gained? (At 25° C, the latent heat of vaporization of water is 580 cal / g. Recall that 1 L of water has a mass of about 1 kg.)

5) Consider the combustion of one mole of methane (CH_4) gas. Assume the system is at standard temperature (298 K) and pressure (10^5 Pa = 1 bar) both before and after the reaction.

$$CH_4 \text{ (gas)} + 2\, O_2 \text{ (gas)} \rightarrow CO_2 \text{ (gas)} + 2\, H_2O \text{ (gas)}$$

   a) First imagine the process of converting a mole of methane into its elemental constituents (graphite and hydrogen gas). Use the reference data at the back of Schroeder to find ΔH for this process. *Careful: You cannot quite just read the value from the chart on page 404.*
   b) Now imagine forming a mole of CO_2 and two moles of water vapor from their elemental constituents. Determine ΔH for each of these processes, and find the total ΔH.
   c) What is the ΔH for the actual reaction in which methane and oxygen form carbon dioxide and water vapor directly? You probably will want to use your answers from a) and b). However, the processes described in part a) and b) are not actually how methane forms. Explain why it is still ok to use a) and b).
   d) How much heat is given off during this reaction, assuming that no "other" (non-compression-expansion) forms of work are done?
   e) What is the change in the system's energy during this reaction?
   f) What would the change in enthalpy be if the H_2O ended up as liquid water instead of vapor?
   g) What would the change in energy be if the H_2O ended up as liquid water instead of vapor? Assume the volume of the liquid is negligible. *Hint: In the Ideal Gas Law, n = moles of gas.*
   h) The Sun has a mass of 2×10^{30} kg and gives off energy at a rate of 3.9×10^{26} watts. Suppose the Sun were made of methane and oxygen (in the proper proportions to react all of each). Suppose further that the source of the Sun's energy were ordinary combustion this chemical fuel. About how long would the Sun last?

6) I flip a two-sided coin five times. My results, in order, were T T H H H. (H = heads and T = tails)
   a) If you flip your own coin five times, what is the probability that you get this same sequence?

b) Try it with your own coin. What was your sequence? Did you match mine?

c) There are 15 people in this class. Assume they all do part b). What is the probability that not a single person out of the 15 matched my sequence? *Hint: What is the probability for any particular person to NOT match?*

7)

a) When rolling three six-sided dice, what is the probability that their sum is six or lower?

b) Compare the results of a) to the Einstein solids results from Schroeder (2.8) and (2.9). Explain qualitatively how the two situations are analogous. What is the difference that explains why $\Omega(3) = 1$ for dice and $\Omega(3) = 10$ for oscillators?

8) When rolling a single six-sided die (with numbers 1-6) N times:

a) What is the probability that not a single one of the rolls yields a one?

b) What is the probability of not rolling a single one in the first (N-1) rolls, and then rolling a one on the N^{th} try?

c) What is the mean number of rolls it takes to roll a one?

Hint: For $|x| < 1, \sum_{n=1}^{\infty} nx^n = \frac{x}{(x-1)^2}$.

9) List *three* main ideas from this homework assignment. For example, you could write a few-sentence explanation of a concept, or list an equation and explain the variables and in what circumstances the equation applies.

The goal is for you to review and to reflect on the big picture. Think about what you might want to remember when you look back at this homework before the test. I hope that this will be useful for your studying. I am not looking for anything specific here; you will be graded on effort and completion.