Writing Assignment 01 - General Feedback

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1 Citing Previous Results

Results in mathematics are built on top of one another. It is important to keep track of which results we rely upon, to avoid circular reasoning. So, you must keep track of the definitions and results we prove and cite them whenever you use them in an argument. See Table 1 for examples of claims that require justification or citations for your argument to be completely valid. Especially at this early stage referencing specific results and definitions may seem tedious, but we need to practice good habits now before our results become more entangled.

As the semester progresses you should always cite, at least, any result used that is found in the chapter containing the material relevant to that assignment. You should cite every definition that has been formally stated in this course when you use it. If you use a definition or result more than once in a given argument, use your judgment to decide whether you need to cite it again.

Table 1: Some assertions that can be formally justified.

Claim or assertion	**With formal justification**
Let a be an even integer. There is an integer k with $a = 2k$.	Let a be an even integer. *By the definition of even* there is an integer k with $a = 2k$.
Every integer is either even or odd.	*By Fact 1.4.6*, every integer is either even or odd.
Every integer is exactly one of Type 0, Type 1, Type 2, or Type 3.	*By the fact given to us in the problem statement*, every integer is exactly one of Type 0, Type 1, Type 2, or Type 3.

2 What is Arbitrary?

Consider the first problem:

> Show that for all $a \in \mathbb{Z}$ we have that a^2 has type 0 or type 1.

A possible starting point for proof might be:

> Let $a \in \mathbb{Z}$ be an arbitrary type 0 integer. By definition, we can fix $n \in \mathbb{Z}$ with $a = 4n$.

Here, the integer, a, we start with is *arbitrary*, but the n is *fixed*. One way to think about the connection is as follows:

- You have a bag containing all of the type 0 integers.
- You reach in, grab one at random, and put it in a sealed envelope without looking at it.
- You mark the envelope with the letter a.

- Now, because we know that it came from the bag of type 0 integers, we know that there exists an associated n with $a = 4n$. However, n is not arbitrary, it is just guaranteed to exist and is completely dependent on a.

It is important that we declare where our variables are from, first, before using definitions or results to get our hands on additional objects or properties.

3 Assuming the Result

Consider the following arguments for Problem 2C:

A valid argument

Let $a \in \mathbb{Z}$ be an arbitrary type 0 integer. By definition of type 0, we may fix $n \in \mathbb{Z}$ such that $a = 4n$. Now, suppose we pick $b \in \mathbb{Z}$ and $c \in \mathbb{Z}$ by setting $b = n + 1$ and $c = n - 1$. We observe that

$$\begin{aligned} b^2 - c^2 &= (n+1)^2 - (n-1)^2 && \text{by our choice of } b \text{ and } c \\ &= n^2 + 2n + 1 - n^2 + 2n - 1 \\ &= 4n \\ &= a && \text{by our choice of } n \end{aligned}$$

Thus, we have established the existence of integers, b and c, such that $a = b^2 - c^2$. As our choice of a was arbitrary, the result holds for all type 0 integers.

An invalid argument

Let $a \in \mathbb{Z}$ be an arbitrary type 0 integer. By definition of type 0, we may fix $n \in \mathbb{Z}$ such that $a = 4n$. Now, suppose we pick $b \in \mathbb{Z}$ and $c \in \mathbb{Z}$ by setting $b = n + 1$ and $c = n - 1$. We observe that

$$\begin{aligned} a &= b^2 - c^2 \\ 4n &= (n+1)^2 - (n-1)^2 && \text{by our choice of } b \text{ and } c \\ 4n &= n^2 + 2n + 1 - n^2 + 2n - 1 \\ 4n &= 4n \\ &= a && \text{by our choice of } n \end{aligned}$$

Thus, we have established the existence of integers, b and c, such that $a = b^2 - c^2$. As our choice of a was arbitrary, the result holds for all type 0 integers.

The problem in the second version begins with the first line of the chain of equations. Our goal is to conclude that our choice of b and c are valid. By including the a on the left side of the equation, we are assuming our conclusion before we have proved it.

This is a very common error, when we start to prove statements of this nature. The work we did to discover the pattern feels like the work we want to do to verify the pattern. You want to show that that you chose the correct b and c. So, you start with them by themselves, and manipulate them until you arrive at a.