

## Problem Set 16: Due Wednesday, April 11

**Problem 1:** Use Gaussian Elimination to solve the following system:

$$\begin{array}{rrcr} x & & - & z & = & 0 \\ 3x & + & y & & = & 1 \\ -x & + & y & + & z & = & 4. \end{array}$$

**Problem 2:** Find the coefficients  $a$ ,  $b$ , and  $c$  so that the graph of  $f(x) = ax^2 + bx + c$  passes through the points  $(1, 2)$ ,  $(-1, 6)$ , and  $(2, 3)$ .

**Problem 3:** Is

$$\begin{pmatrix} 20 \\ 0 \\ 5 \\ 10 \end{pmatrix} \in \text{Span} \left( \begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right)?$$

Explain.

**Problem 4:** Give a parametric description of the solution set of the following system:

$$\begin{array}{rrrrcr} x & + & 2y & - & z & & = & 3 \\ 2x & + & y & & & + & w & = & 4 \\ x & - & y & + & z & + & w & = & 1. \end{array}$$

**Problem 5:** Use Gaussian Elimination to classify for which values of  $h, k \in \mathbb{R}$  the system

$$\begin{array}{rrcr} x & + & hy & = & 2 \\ 4x & + & 8y & = & k \end{array}$$

has each of the following: (i) no solution, (ii) one solution, and (iii) infinitely many solutions.

**Problem 6:** Determine exact conditions (that is, conditions that are both necessary and sufficient) on  $a, b, c, d \in \mathbb{R}$  such that

$$\begin{array}{rrcr} x & - & 3y & = & a \\ 3x & + & y & = & b \\ x & + & 7y & = & c \\ 2x & + & 4y & = & d \end{array}$$

has a solution.