Problem Set 17: Due Monday, April 16

Problem 1: Does

$$\text{Span}\left(\begin{pmatrix}2\\0\\1\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right) = \mathbb{R}^3?$$

Explain.

Problem 2: Given $b_1, b_2, b_3 \in \mathbb{R}$, determine necessary and sufficient conditions so that

$$\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix} \in \text{Span}\left(\begin{pmatrix}0\\1\\5\end{pmatrix}, \begin{pmatrix}3\\1\\-1\end{pmatrix}, \begin{pmatrix}1\\1\\3\end{pmatrix}\right)$$

is true.

Problem 3: Working in $\mathcal{P}_3$, consider the following functions:

- $f_1(x) = x^3 + 2x^2 + x$.
- $f_2(x) = -3x^3 - 5x^2 + x + 2$.
- $f_3(x) = x^2 - x + 1$.
- $g(x) = x^3 + 8x^2 + 7$.

Is $g \in \text{Span}(f_1, f_2, f_3)$? Explain.

Problem 4: Let V be the vector space of all 2×2 matrices. Does

$$\text{Span}\left(\begin{pmatrix}1 & 1\\2 & 0\end{pmatrix}, \begin{pmatrix}2 & 3\\7 & 2\end{pmatrix}, \begin{pmatrix}0 & 1\\2 & 6\end{pmatrix}\right) = V?$$

Explain.

Problem 5: Consider the vector space $\mathbb{R}$, under the usual addition and scalar multiplication (so $\vec{0} = 0$ here). Show that the only subspaces of $\mathbb{R}$ are $\{0\}$ and $\mathbb{R}$.
Hint: Let W be an arbitrary subspace of $\mathbb{R}$ with $W \neq \{0\}$. We know that $0 \in W$, so we can fix some $a \in W$ with $a \neq 0$. Now explain why every element of $\mathbb{R}$ is in W.

Problem 6: In Problem 5 on Problem Set 14, you showed that

$$W = \left\{\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} \in \mathbb{R}^3 : a_1 + a_2 + a_3 = 0\right\}$$

was a subspace of $\mathbb{R}^3$. Show that

$$W = \text{Span}\left(\begin{pmatrix}1\\-1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\-1\end{pmatrix}\right)$$

by giving a double containment proof.
Aside: Using this result, we can instead apply Proposition 4.1.16 to conclude that W is a subspace of $\mathbb{R}^3$.