Homework Assignment 10
PHYSICS 314 - THERMODYNAMICS & STATISTICAL PHYSICS (Spring 2018)
Due Friday, May 4th, by noon, Noyce 1135

I cannot award full credit for work that I am unable to read or follow. For my benefit and for yours, please:

- o   *Write neatly*
- o   *Show and EXPLAIN all steps*
- o   *Make diagrams large and clearly-labeled*

You are welcome to collaborate with others on this assignment. However, the work you turn in should be your own. Please cite collaborators and outside sources. See the syllabus for details.

Regardless of the number of parts, all homework problems are weighted equally. Regardless of the number of questions, all homework assignments are weighted equally.

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1) H_2 excited state probabilities (See Schroeder page 373 for information on the hydrogen atom.)

    Based on SCHROEDER 6.6

a) For a hydrogen atom at room temperature, estimate the probability of being in the first excited state. *Hint: You may approximate an infinite sum by only considering the first term in the sum, but you must justify why doing so is a reasonable approximation.*

    The probability of being in the first excited state, s_2, is given by the Boltzmann factor over the partition function multiplied by the degeneracy, n.

    $$P(s_2) = n_2 \times \frac{1}{Z} e^{-E(s_2)/kT}$$

    The partition function is an infinite sum over all energy states' Boltzmann functions.

    $$Z = \sum_s e^{-E(s)/kT}$$

    $$Z = e^{-E(s_1)/kT} + e^{-E(s_2)/kT} + e^{-E(s_3)/kT} \dots$$

    Examine the relative size of these terms by plugging in values, including energies from Appendix A.

    $$Z = e^{13.6\ eV/(8.62\times10^{-5}eV/K)(300\ K)} + e^{3.4\ eV/(8.62\times10^{-5}eV/K)(300\ K)} + e^{1.5\ eV/(8.62\times10^{-5}eV/K)(300\ K)} \dots$$

    $$Z = 3 \times 10^{228} + 1 \times 10^{57} + 2 \times 10^{25} \dots$$

    The first term dominates this sum, and thus we can approximate the sum as the first term.

    $$Z \approx e^{-E(s_1)/kT} \approx 3 \times 10^{228}$$

    From Appendix A, the first excited state has a degeneracy of 4. Plug values into the probability.

    $$P(s_2) = n_2 \times \frac{1}{Z} e^{-E(s_2)/kT} \approx n_2 \times \frac{e^{-E(s_2)/kT}}{e^{-E(s_1)/kT}}$$

$$\mathcal{P}(s_2) \approx 4 \times \frac{1 \times 10^{57}}{3 \times 10^{228}}$$

$$\mathcal{P}(s_2) \approx 2 \times 10^{-171}$$

This is extremely small!

b) Repeat the calculation for hydrogen at $9500\ K$, as in the atmosphere of the star γ Uma.

Even at this high temperature, the same approximation for the partition function can be used.

$$Z = e^{-E(s_1)/kT} + e^{-E(s_2)/kT} + e^{-E(s_3)/kT} \dots$$

$$Z = e^{13.6\ eV/(8.62\times10^{-5}eV/K)(9500\ K)} + e^{3.4\ eV/(8.62\times10^{-5}eV/K)(9500\ K)} + e^{1.5\ eV/(8.62\times10^{-5}eV/K)(9500\ K)} \dots$$

$$Z = 1.6 \times 10^7 + 64 + 6.2 \dots$$

$$Z \approx 1.6 \times 10^7$$

The first term still dominates.

$$\mathcal{P}(s_2) \approx n_2 \times \frac{e^{-E(s_2)/kT}}{e^{-E(s_1)/kT}}$$

$$\mathcal{P}(s_2) \approx 4 \times \frac{64}{1.6 \times 10^7}$$

$$\mathcal{P}(s_2) \approx 1.6 \times 10^{-5}$$

Even at this high temperature, the excited population is very low.

2) In previous physics and math classes, you have probably encountered the concept of *standard deviations*, a measure of fluctuations around an average value. It is one way of quantifying uncertainty in experimental results. For this problem, consider a simplified system that has five atoms, each of which can occupy one of three possible states. Let the energies of the states be as shown, and suppose the atoms are distributed in the states as represented by the dots.
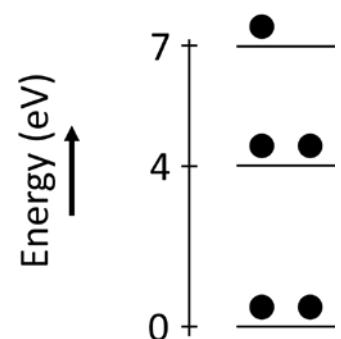
Based on SCHROEDER 6.17

a) For each of the five atoms, calculate the *deviation* of the energy from the average energy of the five atoms, $\Delta E_i = E_i - \bar{E}$.

First calculate the average energy.

$$\bar{E} = \frac{\sum_i E_i}{N_{atoms}} = \frac{7\ eV + 4\ eV + 4\ eV + 0\ eV + 0\ eV}{5} = \frac{15\ eV}{5} = 3\ eV$$

Now find the deviations.

$$\Delta E_i = E_i - \bar{E}$$

$$\Delta E_i = E_i - 3 \; eV$$

$$\Delta E_i = 0 \; eV - 3 \; eV = -3 \; eV$$

$$\Delta E_i = 0 \; eV - 3 \; eV = -3 \; eV$$

$$\Delta E_i = 4 \; eV - 3 \; eV = 1 \; eV$$

$$\Delta E_i = 4 \; eV - 3 \; eV = 1 \; eV$$

$$\Delta E_i = 7 \; eV - 3 \; eV = 4 \; eV$$

b) Compute the *standard deviation*, $\sigma_E \equiv \sqrt{\overline{(\Delta E_i)^2}}$. This quantity is also called the *root-mean-square (rms) deviation* because it is the square **root** of the **mean** (average) of the **square** of the deviation.

First calculate the squares of the deviations.

$$(\Delta E_i)^2 = (-3 \; eV)^2 = 9 \; eV^2$$

$$(\Delta E_i)^2 = (-3 \; eV)^2 = 9 \; eV^2$$

$$(\Delta E_i)^2 = (1 \; eV)^2 = 1 \; eV^2$$

$$(\Delta E_i)^2 = (1 \; eV)^2 = 1 \; eV^2$$

$$(\Delta E_i)^2 = (4 \; eV)^2 = 16 \; eV^2$$

Now calculate the mean.

$$\overline{(\Delta E_i)^2} = \frac{\sum_i (\Delta E_i)^2}{N_{atoms}} = \frac{9 \; eV^2 + 9 \; eV^2 + 1 \; eV^2 + 1 \; eV^2 + 16 \; eV^2}{5} = \frac{36 \; eV^2}{5} = 7.2 \; eV^2$$

$$\sigma_E^2 = 7.2 \; eV^2$$

Finally take the square root to get the standard deviation.

$$\sigma_E \equiv \sqrt{\overline{(\Delta E_i)^2}} = \sqrt{7.2 \; eV^2} \approx 2.7 \; eV$$

Comparing the standard deviation to the individual deviations calculated in part a), it seems to be a reasonable approximation of the deviations, as the magnitudes of the deviations range from $1 \; eV$ to $4 \; eV$.

c) For this system, perform explicit calculations to show that the following relationship is true.

$$\sigma_E^2 = \overline{E^2} - (\bar{E})^2$$

Calculate the square of the energies.

$$E^2 = (7 \ eV)^2 = 49 \ eV^2$$

$$E^2 = (4 \ eV)^2 = 16 \ eV^2$$

$$E^2 = (4 \ eV)^2 = 16 \ eV^2$$

$$E^2 = (0 \ eV)^2 = 0 \ eV^2$$

$$E^2 = (0 \ eV)^2 = 0 \ eV^2$$

Now calculate the mean.

$$\overline{E^2} = \frac{\Sigma_i (E_i)^2}{N_{atoms}} = \frac{49 \ eV^2 + 16 \ eV^2 + 16 \ eV^2 + 0 \ eV^2 + 0 \ eV^2}{5} = \frac{81 \ eV^2}{5} = 16.2 \ eV^2$$

Plug in this value and the standard deviation and average energy values from above.

$$\sigma_E^2 = \overline{E^2} - (\bar{E})^2$$

$$7.2 \ eV^2 = 16.2 \ eV^2 - (3 \ eV)^2$$

$$7.2 \ eV^2 = 16.2 \ eV^2 - 9 \ eV^2$$

$$7.2 \ eV^2 = 7.2 \ eV^2$$

d) Prove that the relationship in part c) is true in general. (This relationship is often an easier way to calculate the standard deviation.)

Start with the definition of standard deviation.

$$\sigma_E \equiv \sqrt{\overline{(\Delta E_i)^2}}$$

Square both sides.

$$\sigma_E^2 = \overline{(\Delta E_i)^2}$$

Substitute out the second term for the expression used to calculate it above.

$$\sigma_E^2 = \frac{\Sigma_i (\Delta E_i)^2}{N_{atoms}}$$

Drop the subscript for clarity.

$$\sigma_E^2 = \frac{\sum_i (\Delta E_i)^2}{N}$$

Now use the definition of deviation.

$$\sigma_E^2 = \frac{\sum_i (E_i - \bar{E})^2}{N}$$

Expand.

$$\sigma_E^2 = \frac{\sum_i (E_i^2 - 2\bar{E}E_i + \bar{E}^2)}{N}$$

Separate out into three separate sums.

$$\sigma_E^2 = \frac{\sum_i E_i^2}{N} - \frac{\sum_i 2\bar{E}E_i}{N} + \frac{\sum_i \bar{E}^2}{N}$$

Pull the constant average value out of the sums.

$$\sigma_E^2 = \frac{\sum_i E_i^2}{N} - 2\bar{E}\frac{\sum_i E_i}{N} + \bar{E}^2\frac{\sum_i (1)}{N}$$

The first term is the average of the energy squared. The second term is the average energy. The third term has a sum that is just the number of particles.

$$\sigma_E^2 = \overline{E_i^2} - 2\bar{E}\bar{E} + \bar{E}^2\frac{N}{N}$$

$$\sigma_E^2 = \overline{E_i^2} - 2\bar{E}^2 + \bar{E}^2$$

$$\sigma_E^2 = \overline{E_i^2} - \bar{E}^2$$

3) Cyanogen (CN) is a molecule often found in cold interstellar molecular clouds. The first rotational excited states of CN have an energy that is greater than that of the ground state by $4.7 \times 10^{-4} \ eV$. There are three excited states that share this same energy. From studies of absorption spectra of starlight that passes through the molecular clouds, one can learn about the about the relative population of the excited states of CN. Measurements from 1941 showed that for every ten CN molecules in the ground state, there are roughly three molecules in the excited states - on average, one in each of the three states. To account for these relative populations, astronomers suggested that the molecules could be in thermal equilibrium with some thermal reservoir with a well-defined temperature, T_{res}. Find T_{res}. (The thermal reservoir is now known to be a gas of photons that fills the entire observable universe. It is known as the *cosmic background radiation*, and its discoverers won a Nobel prize.)

Based on SCHROEDER 6.12

Examine the ratio of the probabilities of being in any one of the excited states (s_1, $E = E_e$) compared to being in the ground state (s_0, $E = E_g$). For convenience, call this ratio $\mathcal{P}_{ratio}$. Note that since we are only considering a particular one of the three excited states, there is no need to consider multiplicity.

$$\mathcal{P}_{ratio} \equiv \frac{\mathcal{P}(s_1)}{\mathcal{P}(s_0)} = \frac{\frac{1}{Z}e^{-E_e/kT}}{\frac{1}{Z}e^{-E_g/kT}}$$

The common partition functions cancel.

$$\mathcal{P}_{ratio} = \frac{e^{-E_e/kT}}{e^{-E_g/kT}}$$

Combine the exponents.

$$\mathcal{P}_{ratio} = e^{-(E_e-E_g)/kT}$$

Solve for temperature.

$$\ln \mathcal{P}_{ratio} = -\frac{(E_e - E_g)}{kT}$$

$$kT \ln \mathcal{P}_{ratio} = -(E_e - E_g)$$

$$kT \ln \mathcal{P}_{ratio} = -(E_e - E_g)$$

$$T = \frac{-(E_e - E_g)}{k \ln \mathcal{P}_{ratio}}$$

Plug in the values. Note that the observed ratio was one to ten, and the energy difference was given.

$$T = \frac{-(4.7 \times 10^{-4} \ eV)}{(8.62 \times 10^{-5} eV/K) \ln \left(\frac{1}{10}\right)}$$

Note that there is no need to worry about the negative sign. The negative sign cancels with the negative sign from the log term.

$$T \approx 2.4 \ K$$

The actual cosmic microwave background temperature is closer to $2.7 \ K$.

4) Consider a system that is in equilibrium with a reservoir at temperature T. Prove the following equation for the average value of energy in such a system. (We have been using this equation in lecture without proof.) Z is the partition function, and $\beta = 1/kT$. Do not forget to explain both equalities.

$$\bar{E} = -\frac{\partial}{\partial \beta} \ln Z = -\frac{1}{Z} \frac{\partial Z}{\partial \beta}$$

Based on SCHROEDER 6.16

First of all, note that the second equality comes from the chain rule.

$$\frac{\partial}{\partial \beta} \ln Z = \frac{\partial}{\partial \beta} (\ln Z) \frac{\partial Z}{\partial \beta} = \frac{1}{Z} \frac{\partial Z}{\partial \beta}$$

Now examine the partial derivative with the explicit expression of the partition function.

$$\frac{\partial Z}{\partial \beta} = \frac{\partial}{\partial \beta} \left(\sum_s e^{-E(s)\beta} \right)$$

Differentiate each term of the sum. (Differentiation is linear.)

$$\frac{\partial Z}{\partial \beta} = \sum_s \frac{\partial}{\partial \beta} e^{-E(s)\beta}$$

$$\frac{\partial Z}{\partial \beta} = \sum_s (-E(s)) e^{-E(s)\beta}$$

Plug this in to the far-right-hand side of the original expression.

$$-\frac{1}{Z} \frac{\partial Z}{\partial \beta} = -\frac{1}{Z} \sum_s (-E(s)) e^{-E(s)\beta}$$

$$-\frac{1}{Z} \frac{\partial Z}{\partial \beta} = \frac{1}{Z} \sum_s E(s) e^{-E(s)\beta}$$

The partition function is constant, so it can be moved inside the sum.

$$-\frac{1}{Z} \frac{\partial Z}{\partial \beta} = \sum_s \frac{E(s) e^{-E(s)\beta}}{Z}$$

$$-\frac{1}{Z} \frac{\partial Z}{\partial \beta} = \sum_s E(s) \frac{e^{-E(s)\beta}}{Z}$$

Note that the fraction is the Boltzmann factor over the partition function. This is the probability of the state.

$$-\frac{1}{Z} \frac{\partial Z}{\partial \beta} = \sum_s E(s) \mathcal{P}(s)$$

This is now our expression for calculating average energy, Schroeder (6.16).

$$-\frac{1}{Z} \frac{\partial Z}{\partial \beta} = \bar{E}$$

(Note that we could have notice this a few lines ago using Schroeder (6.17).)

$$\frac{1}{Z}\sum_s E(s)\,e^{-E(s)\beta} = \bar{E}$$

5) Consider the (quantized) rotation of the diatomic molecule carbon monoxide, CO. For CO, the rotational quantum ϵ has been measured to be approximately $0.00024\ eV$.

Based on SCHROEDER 6.23 and SCHROEDER A.22

a) This value can be spectroscopically measured by exposing the molecules to varying frequencies of electromagnetic radiation.
   i) What is the frequency of a photon that excites CO from the rotational ground state ($j = 0$) to the first excited rotational state ($j = 1$)?

   The frequency of the photon is related to the energy through Planck's constant.

   $$f = \frac{E_{photon}}{h}$$

   To excite the molecule, the energy of the photon must correspond to the energy difference between the two states.

   $$f = \frac{E_{j=1} - E_{j=0}}{h}$$

   Calculate the energy difference using the expression for energy in terms of rotational quantum number j.

   $$E_j = j(j+1)\epsilon$$

   $$f = \frac{[1(1+1)\epsilon] - [0(0+1)\epsilon]}{h}$$

   $$f = \frac{[2\epsilon] - [0\epsilon]}{h}$$

   $$f = \frac{2\epsilon}{h}$$
   $$f \approx \frac{2(0.00024\ eV)}{4.14 \times 10^{-15}\ eV\ s}$$

   $$f \approx 1.16 \times 10^{11}\ Hz = 116\ GHz$$

   ii) What is the frequency of a photon that excites CO from the first excited state ($j = 1$) to the second excited state ($j = 2$)?

   The method is the same as part i).

$$f = \frac{E_{j=2} - E_{j=1}}{h}$$

$$f = \frac{[2(2+1)\epsilon] - [1(1+1)\epsilon]}{h}$$

$$f = \frac{[6\epsilon] - [2\epsilon]}{h}$$

$$f = \frac{4\epsilon}{h}$$

Instead of plugging in values, note that this is just twice the previous result.

$$f \approx 2 \times (1.16 \times 10^{11} \ Hz) = 2 \times 116 \ GHz$$

$$f \approx 232 \ GHz$$

iii) In what region of the electromagnetic spectrum do these photons lie?

Electromagnetic radiation with frequency of a couple hundred gigahertz is in the microwave region of the spectrum.

b) Calculate the rotational partition function for CO at $300 \ K$ using the high-temperature approximation derived in class.

From class, the rotational partition function can be approximated at high temperatures with a fairly simple expression.

$$Z_{rot} = \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT} \approx \frac{kT}{\epsilon}$$

Plug in values given.

$$Z_{rot} \approx \frac{kT}{\epsilon} \approx \frac{(8.62 \times 10^{-5} \ eV/K)(300 \ K)}{0.00024 \ eV}$$

$$Z_{rot} \approx 107.8$$

c) Now examine the exact formula for the same rotational partition function, Schroeder equation (6.30). The sum is infinite, but it can be approximated by a finite sum from zero to some $j = j_f$. Use a computer program of your choice to calculate the approximate finite sum.

$$Z_{rot}(j_f) \approx \sum_{j=0}^{j_f} (2j+1) e^{-j(j+1)\epsilon/kT}$$

Plot Z_{rot} as a function of j_f. Start from $j_f = 0$, and continue to higher j_f until Z_{rot} seems to be converging (at least until it changes by less than 0.01 when you add the next term to the sum). Plot

your approximate answer from part b) (a constant value) on the same axes. Does the high temperature approximation seem to be higher or lower than the actual value?

I chose to use Mathematica, but this could be done equally well in other programs. The code and plot are below. The code is recopied here for easier reading.

$T = 300;$
$k = 8.62 * 10^{-5};$
$\epsilon = 0.00024;$

$\text{Plot}[\{\text{Sum}[(2j + 1)\text{Exp}[-j(j + 1)\,\epsilon/(k * T)], \{j, 0, j_f\}], k * T/\epsilon\}, \{j_f, 0,30\}, \text{PlotLegends}$
$\rightarrow \text{"Expressions"}, \text{LabelStyle} \rightarrow \text{Large}, \text{ImageSize} \rightarrow \text{Large}, \text{AxesLabel} \rightarrow \{j_f, Z_{\text{rot}}\}]$
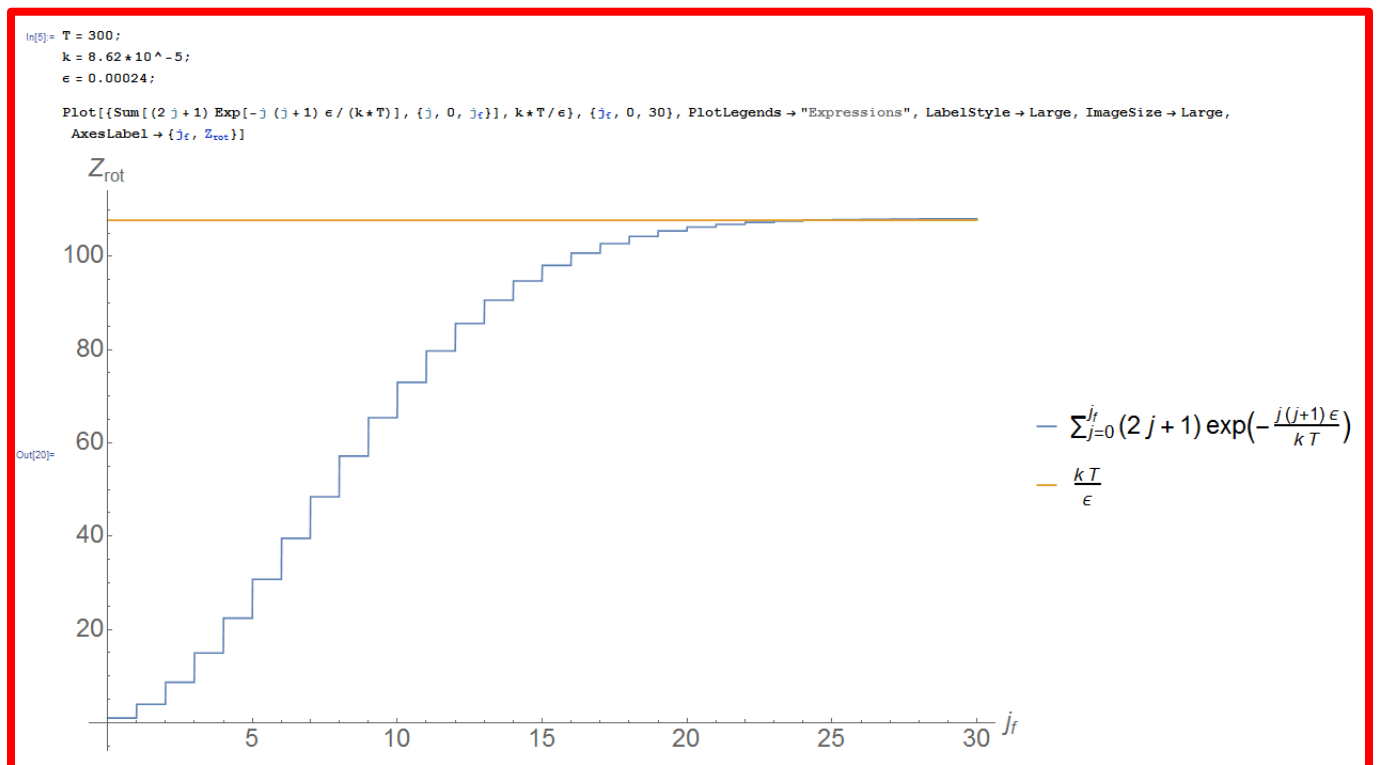
A zoom in the y direction allows for a better comparison between the sums and the high-temperature approximation.
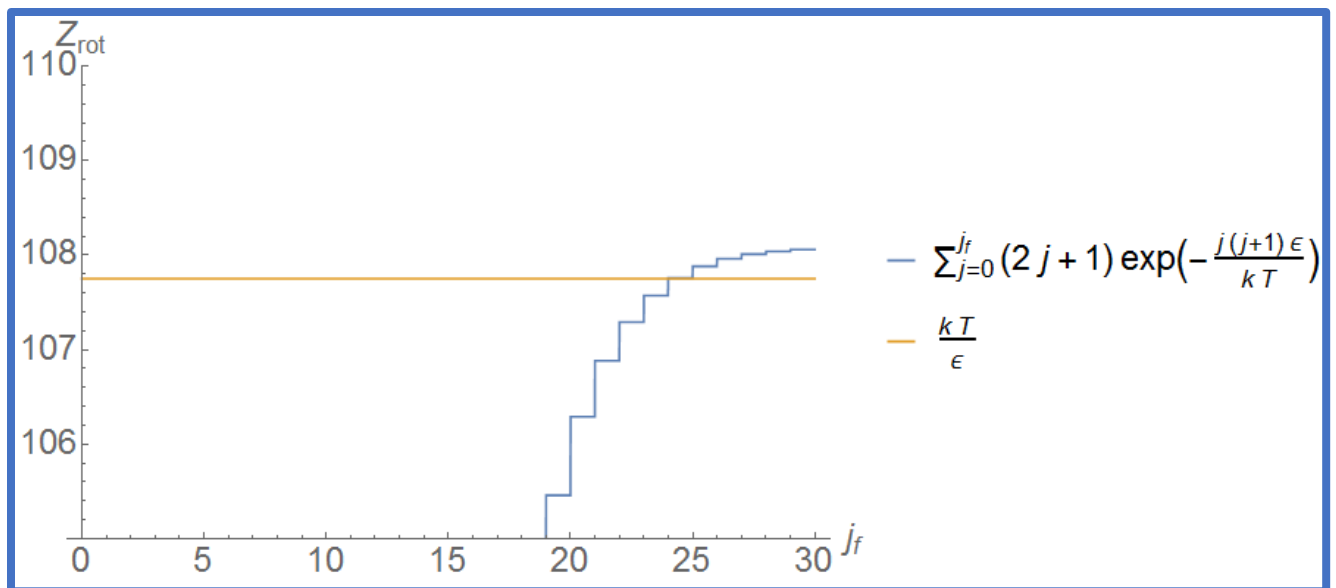
The high-temperature estimate for the partition function appears to be a little too low.

Note that I choose to go to $j_f = 30$ mostly for aesthetic reasons. However, the sum does meet the criteria at this point; the value changes by much less than 0.01 with the addition of a new term.

```
Sum[(2 j + 1) Exp[-j (j + 1) e / (k * T)], {j, 0, 29}]

108.059

Sum[(2 j + 1) Exp[-j (j + 1) e / (k * T)], {j, 0, 30}]

108.07
```

6) Gaussian integrals appear frequently in physics. Recall that we used the result of a Gaussian integral in class.

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

I highly recommend that you read the two or so pages of Appendix B.1 about Gaussian integrals. The proof of the formula above is very clever. From this one Gaussian integral result, one can evaluate many different integrals. This problem looks at a slightly different integrands of the form $x^n e^{-ax^2}$, where n is odd, and a is a constant.

Based on SCHROEDER B.3

a) Evaluate the following _definite_ integral entirely by reasoning. Do not actually integrate.

$$\int_{-\infty}^{\infty} xe^{-ax^2} dx$$

Note that the integrand is an odd function of x; its value for any x is the opposite of its value for $(-x)$.

$$f(x) = -f(-x) \quad odd \; function$$

$$xe^{-ax^2} = (-x)e^{-a(-x)^2}$$

$$xe^{-ax^2} = (-x)e^{-a(-x)^2}$$

$$xe^{-ax^2} = -\left(xe^{-ax^2}\right)$$

The integral of any odd function from negative infinity to infinity must equal zero. For every contribution from x values greater than zero, there is an equal and opposite contribution from $-x$ to cancel it. Thus, the integral for this integrand (and every odd integrand) must be zero.

$$\int_{-\infty}^{\infty} xe^{-ax^2} dx = 0$$

b) Evaluate the following *indefinite* integral using u substitution.

$$\int xe^{-ax^2} dx$$

As did for an integral in class, set u equal to the exponent.

$$u = -ax^2$$

Differentiate to find du.

$$du = -2ax \; dx$$

Solve for $x \; dx$, which appears in the integrand.

$$-\frac{du}{2a} = x \; dx$$

Substitute in u and du.

$$\int xe^{-ax^2} dx = \int e^{-ax^2} x \; dx$$

$$\int xe^{-ax^2} dx = \int e^u \left(-\frac{du}{2a}\right)$$

$$\int xe^{-ax^2} dx = -\frac{1}{2a} \int e^u du$$

The indefinite integral of this exponential is just the exponential itself.

$$\int xe^{-ax^2}\,dx = -\frac{1}{2a}e^u$$

$$\int xe^{-ax^2}\,dx = -\frac{1}{2a}e^{-ax^2}$$

We could also have an additive constant at the end.

c) Evaluate the following *definite* integral.

$$\int_0^\infty xe^{-ax^2}\,dx$$

Use the result from part b) for the antiderivative.

$$\int_0^\infty xe^{-ax^2}\,dx = -\frac{1}{2a}e^{-ax^2}\Big|_{x=0}^{x=\infty}$$

$$\int_0^\infty xe^{-ax^2}\,dx = -\frac{1}{2a}e^{-a\infty^2} - \left(-\frac{1}{2a}e^{-a0^2}\right)$$

$$\int_0^\infty xe^{-ax^2}\,dx = -\frac{1}{2a}(0) + \frac{1}{2a}(1)$$

$$\int_0^\infty xe^{-ax^2}\,dx = \frac{1}{2a}$$

d) Evaluate the following *definite* integral. *Hint: Use part c) and the method used from Schroeder equation (B.6) to (B.8).*

$$\int_0^\infty x^3 e^{-ax^2}\,dx$$

As mentioned in the hint, use the differentiation trick used by Schroeder. Differentiate both sides of the answer from part c) with respect to a.

$$\frac{\partial}{\partial a}\left(\int_0^\infty xe^{-ax^2}\,dx\right) = \frac{\partial}{\partial a}\left(\frac{1}{2a}\right)$$

Differentiate the right side.

$$\frac{\partial}{\partial a}\left(\int_0^\infty xe^{-ax^2}\,dx\right) = -\frac{1}{2a^2}$$

As in Schroeder, the function is well enough behaved that the derivative can be brought inside the integral.

$$\int_0^\infty \frac{\partial}{\partial a}\left(xe^{-ax^2}\right)\,dx = -\frac{1}{2a^2}$$

Differentiate with respect to a.

$$\int_0^\infty x(-x^2)e^{-ax^2}\,dx = -\frac{1}{2a^2}$$

$$-\int_0^\infty x^3 e^{-ax^2}\,dx = -\frac{1}{2a^2}$$

Cancel the negative signs. Note that this is the integral that the problem asks about.

$$\int_0^\infty x^3 e^{-ax^2}\,dx = \frac{1}{2a^2}$$

NOTICE!!! THIS PROBLEM IS TIME SENSITIVE!

7) You have two choices for the reflection problem this week. You may choose to do either Option I or Option II. *I strongly encourage you to do Option I if you are able,* but both options are worth the same number of points.

Option I

Attend the second special *Squire Lecture* Physics seminar on Tuesday, May 1st, at noon, OR the general public seminar at 4 pm that afternoon.
The speaker is Professor Andrew Davis (a 1971 Grinnell graduate!) from the University of Chicago. The noon presentation is entitled, *Making CHILI, a new instrument for exploring the origin of our solar system*, and the afternoon seminar is entitled, *Nature's ultimate sample return mission: seeing into stars by studying stardust in the laboratory*. Professor Davis studies geochemistry and cosmochemistry, which are related to the geophysics we have been discussing recently.

After attending the lecture, write a half-page reflection on the presentation.
- Summarize the main points of the presentation.
- Discuss the connections between the science discussed in the presentation and the material covered in this course.
- Fulfilling the above requirements will earn you a $3.25/4$. The rest of the points will be awarded based on the depth and quality of your reflection.

Option II

Read (or listen to) the following material discussing recent topics from class.

Lab-grown diamonds come into their own
    https://www.npr.org/2016/12/01/502330818/lab-grown-diamonds-come-into-their-own
Big diamonds bring scientists a message from superdeep Earth
    https://www.npr.org/sections/thetwo-way/2016/12/15/505386423/big-diamonds-bring-scientists-a-message-from-superdeep-earth

Write a short response to the two articles. One paragraph per article is sufficient.
- Summarize the main points of each article in a few sentences.

- For each article, discuss in a few sentences the connections between the science discussed in the article and the material covered in this course.
- Which article did you feel better conveyed the relevant science? Why? (Keep in mind the intended audience of each piece.)
- Fulfilling the above requirements will earn you a $3.25/4$. The rest of the points will be awarded based on the depth and quality of your explanations of the connections.

8) **PLEASE DO #8 ON PIECES OF PAPER THAT ARE SEPARATE FROM THE REST OF YOUR HOMEWORK.**

On the first piece of paper, write or type your problem without the solution; this will be copied for your peers. On a separate piece of paper (that is also separate from the rest of your homework), write or type the solution.

In preparation for the end of the semester, continue thinking about questions that could be on the final. For this week's assignment, look at my feedback from your proposed questions from HW9 problem 8. Choose <u>one</u> of your problems to write-up more formally. Make any revisions necessary to the problem, and then write up a detailed solution. As a model for the level of detail, use the solutions you are provided for regular homework. Be sure to not only show any mathematics clearly, but *also to clearly explain your reasoning for each step.*

You will be graded on the appropriateness, originality, and clarity of your questions and on the thoroughness and clarity of the solution.

For next week's homework, I will have you exchange problems and solve a problem created by one of your classmates.

Remember, if any of the problems are particularly good, I may use them on the final! Wouldn't it be nice to see a problem you wrote on the final?