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Problem Set # 1

1-3 a)

1.1-5
$$\frac{dv}{v_t^2 - v^2} = \frac{g}{v_t^2}\,dt$$

Now
$$v = \frac{ds}{dt} \quad \text{so} \quad v\,dt = \frac{ds}{dt}dt = ds$$

~~[illegible]~~
$$dt = \frac{ds}{v}$$

Sub
$$\frac{dv}{v_t^2 - v^2} = \frac{g}{v_t^2}\,\frac{ds}{v}$$

or
$$\frac{v\,dv}{v_t^2 - v^2} = \frac{g}{v_t^2}\,ds$$

$$\frac{1}{2}\,\frac{d\,v^2}{v_t^2 - v^2} = \frac{g}{v_t^2}\,ds$$

Integrate
$$\int_{v_0}^{v_f} \frac{d\,v^2}{v_t^2 - v^2} = \frac{2g}{v_t^2}\int_0^{s_f} ds$$

$$-\left[\ln\left(v_t^2 - v_f^2\right) - \ln\left(v_t^2 - v_0^2\right)\right] = \frac{2g}{v_t^2}\,s_f$$

$$\ln\left(\frac{v_t^2 - v_f^2}{v_t^2 - v_0^2}\right) = -\frac{2g}{v_t^2} s_f$$

Solve for v_f (use $e^{\ln y} = y$)

$$\frac{v_t^2 - v_f^2}{v_t^2 - v_0^2} = e^{-\frac{2g s_f}{v_t^2}}$$

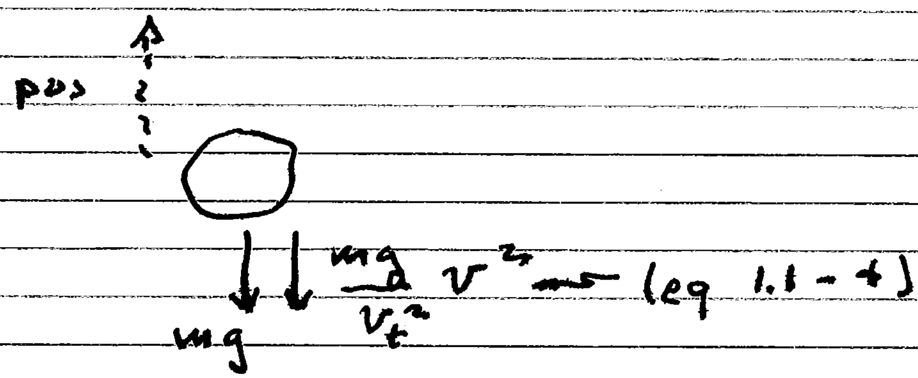
$$v_t^2 - v_f^2 = (v_t^2 - v_0^2)\, e^{-\frac{2g s_f}{v_t^2}}$$

$$\boxed{v_t^2\left(1 - e^{-\frac{2g s_f}{v_t^2}}\right) + v_0^2\, e^{-\frac{2g s_f}{v_t^2}} = v_f^2}$$

Clearly if $s_f \to 0$ $\quad v_f^2 = v_0^2$

if $s_f \to \infty$ $\quad v_f^2 \to v_t^2$

b)



$$F = ma$$

$$m\frac{dv}{dt} = -mg - \frac{mg}{v_t^2}v^2 = -mg\left(1 + \frac{v^2}{v_t^2}\right)$$

$$-\frac{dv}{\cancel{g}\left(1+\frac{v^2}{v_t^2}\right)} = g\,dt$$

From part a) $\quad dt = \frac{ds}{v}$

$$\frac{v\,dv}{v_t^2 + v^2} = -\frac{g\,ds}{v_t^2}$$

$$\frac{1}{2}\,\frac{dv^2}{v_t^2 + v^2} = -\frac{g\,ds}{v_t^2}$$

Integrate $\displaystyle\int_{v_0}^{0 \leftarrow \text{final } v} \frac{dv^2}{v_t^2+v^2} = -\frac{2g}{v_t^2}\int_0^{s_f} ds$

$$\ln\left(v_t^2 + v^2\right)\Big|_{v_0}^{0} = -\frac{2gs_f}{v_t^2}$$

$$\ln(v_t^2) - \ln\left(v_t^2 + v_0^2\right) = -\frac{2gs_f}{v_t^2}$$

$$\cancel{\ln\left(\frac{v_t^2+v_0^2}{v_t^2}\right) = \frac{2gs_f}{v_t^2}}$$

A/

$$\boxed{\frac{v_t^2}{2g} \ln\left[\frac{v_t^2 + v_0^2}{v_t^2}\right] = S_f}$$

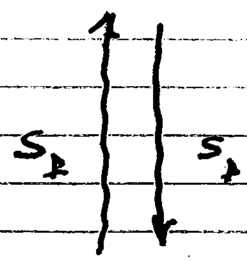
$$\boxed{\frac{v_t^2}{2g} \ln\left[1 + v_0^2/v_t^2\right] = S_f}$$

Note: for ε small $\ln(1+\varepsilon) \simeq \varepsilon$

so for $v_t \to \infty$

$$S_f \simeq \frac{v_t^2}{2g}\left(\frac{v_0^2}{v_t^2}\right) = \frac{v_0^2}{2g} \checkmark$$

Combining a) & b)



Sub. b) result in a.

What is $e^{-\frac{2gS_f}{v_t^2}}$?

$$\frac{2gS_f}{v_t^2} = \frac{2g}{v_t^2}\left[\frac{v_t^2}{2g}\ln\left(1 + \frac{v_0^2}{v_t^2}\right)\right]$$

$$= \ln\left(1 + \frac{v_0^2}{v_t^2}\right)$$

5/

Sub into a) result - with $v_0^2 = 0$

$$v_f^2 = v_t^2 \left(1 - e^{-2g s_p / v_t^2}\right)$$

$$= v_t^2 \left(1 - \frac{1}{e^{\ln(1 + v_0^2/v_t^2)}}\right)$$

$$v_f^2 = v_t^2 \left(1 - \frac{1}{1 + v_0^2/v_t^2}\right)$$

$$v_f^2 = v_t^2 \left[\frac{1 + \frac{v_0^2}{v_t^2} - 1}{1 + \frac{v_0^2}{v_t^2}}\right]$$

$$v_f^2 = \frac{v_0^2}{1 + v_0^2/v_t^2}$$

So $$\frac{v_f^2}{v_0^2} = \frac{v_t^2}{v_t^2 + v_0^2}$$

for $v_t \to \infty$ (low damping) $\frac{v_f^2}{v_0^2} \to 1$

$v_t \to 0$ $\frac{v_f^2}{v_0^2} \to \frac{v_t^2}{v_0^2}$ ✓

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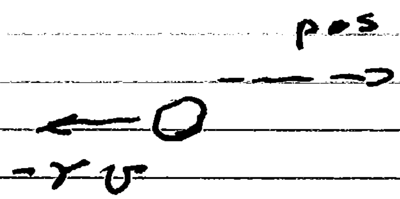


$$F = ma$$

$$-rv = m\frac{dv}{dt}$$

$$\frac{dv}{dt} = -\frac{r}{m}v$$

$$\frac{dv}{v} = -\frac{r}{m}dt$$

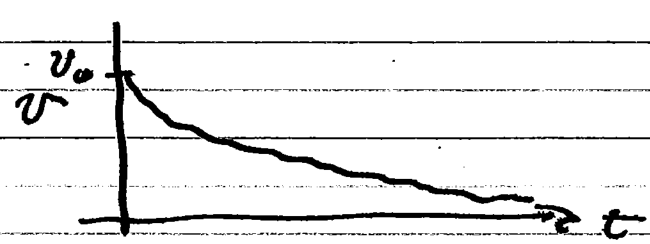
$$\int_{v_0}^{v_f} \frac{dv}{v} = -\frac{r}{m}\int_0^{t_f} dt = -\frac{r\,t_f}{m}$$

$$\ln(v_f) - \ln(v_0) = -\frac{r\,t_f}{m}$$

$$\ln\left(\frac{v_f}{v_0}\right) = -\frac{r\,t_f}{m}$$

$$\frac{v_f}{v_0} = e^{-\frac{r\,t_f}{m}}$$

$$v_f = v_0\, e^{-\frac{r\,t_f}{m}}$$



$$\frac{ds}{dt} = v_0 e^{-\frac{rt}{m}}$$

$$ds = v_0 e^{-rt/m} dt$$

$$s = -\frac{v_0 m}{r} e^{-rt/m} \Big|_0^{t_f}$$

$$s = -\frac{v_0 m}{r} \left[e^{-\frac{rt_f}{m}} - 1 \right]$$

$$s = \frac{m v_0}{r} \left(1 - e^{-\frac{rt_f}{m}} \right)$$