Homework Assignment 9

PHYSICS 314 - THERMODYNAMICS & STATISTICAL PHYSICS (Spring 2018)

Due Friday, April 27th, by noon, Noyce 1135

I cannot award full credit for work that I am unable to read or follow. For my benefit and for yours, please:

- *Write neatly*
- *Show and EXPLAIN all steps*
- *Make diagrams large and clearly-labeled*

You are welcome to collaborate with others on this assignment. However, the work you turn in should be your own. Please cite collaborators and outside sources. See the syllabus for details.

Regardless of the number of parts, all homework problems are weighted equally. Regardless of the number of questions, all homework assignments are weighted equally.

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1) Calcite and aragonite are two common crystalline forms of calcium carbonate, $CaCO_3$. Use information from the table in the back of Schroeder to answer the following questions.
   a) Which of the two forms is the more stable phase at $298\ K$ and $1\ bar$? *Explain.*
   b) At room temperature, what is the pressure at which the other phase becomes more stable? *Hint: Assume the solids are incompressible.*
   c) Explain why it would be significantly more difficult to do a similar stability analysis for CO_2 at a temperature between its triple point temperature and its critical point temperature.

2) Gibbs Free Energy of H_2O
   a) Sketch plots of Gibbs free energy as a function of temperature for the three phases of H_2O (ice, water, and steam) at $P = 1\ atm$. Plot the three phases on the same axes, including all three phases, not just the most stable phase at each temperature. The sketch should be qualitatively accurate. (I would like more detail here than on HW8 #6.) Label 0°C and 100°C.
   b) Repeat part a), but at $P = 1\ mbar$. Discuss the differences between this plot and that from part a).

3) Clausius-Clapeyron and $\mathrm{H_2O}$
   a) Use the Clausius-Clapeyron relation to explain why the slope of the water-ice phase boundary on a $P-T$ phase diagram has the opposite sign of the slope of most liquid-solid phase boundaries.
   b) The melting point of water is 0°C at atmospheric pressure, and the latent heat of water is $334\ kJ/kg$. The density of ice is $917\ kg/m^3$ and the density of water is $1000\ kg/m^3$. Use these facts to determine approximately by how much and in which direction the pressure must be changed to get ice to melt at just one degree lower, -1°C.
   c) What is the pressure z meters below the surface of a glacier with ice of density ρ?
   d) Find the depth in meters below the surface of a glacier at which ice melts at -1°C.

4) The Clausius-Clapeyron relation is a differential equation. In principle, it can be solved to find the shape of the entire phase-boundary curve. This can be difficult, though, as the latent heat, L, and the change in volume, ΔV, generally change with pressure and temperature. However, over a reasonably small section of the boundary, the latent heat can be approximated as constant. Also, if the phase transformation is from either a liquid or a solid to a gas, the volume of the initial phase can be ignored because the gas phase is so much larger.

   Make all the assumptions mentioned above. Solve the differential equation and find an expression for pressure as a function of temperature. *Hint: In addition to the given variables, your expression will include an unknown constant.*

5) Suppose there is a particle in equilibrium with a reservoir at $300\ K$. The particle has three possible energy states; the particle can either be in state 1 with energy $-0.05\ eV$, in state 2 with energy $0.00\ eV$, or in state 3 with energy $+0.05\ eV$.
   a) Calculate the partition function, Z, for this particle.
   b) Calculate the probability of the particle being in each of the three states.
   c) Suppose that the energy scale is now shifted so that the ground state is considered $0.00\ eV$. Recalculate part a) and part b) with this new scale.
   d) Comment on the significance of what does and what does not change with the new scale.

6) Consider now energy *levels* made up of n degenerate energy states. Let the multiplicity of the level be equal to the number of degenerate states for that level. Show that the probability of finding an atom in a particular energy level with energy E is given by the expression shown, in which F is the Helmholtz free energy of the level.

$$\mathcal{P}(E) = \frac{1}{Z} e^{-F/kT}$$

<u>NOTICE!!! THIS PROBLEM IS TIME SENSITIVE!</u>

7) You have two choices for the reflection problem this week. You may choose to do either Option I or Option II. *I strongly encourage you to do Option I if you are able,* but both options are worth the same number of points.

   Option I

   Attend the special *Squire Lecture* Physics seminar on Tuesday, April 24th, at noon.
   The speaker is Professor Meredith Hughes from Wesleyan University. The presentation is entitled, "STEM Equity and Inclusion: A Female Astronomer's View". It is an important and often-overlooked topic in the field, and I believe the presentation will be useful for all of us.

   After attending the lecture, write a half-page reflection on the presentation.
   - Summarize the main points of the presentation.
   - Respond to the presentation. If you wish, you could consider some of the following questions.
     - Was there any information in the presentation that surprised you?
     - What was the most important conclusion that you drew from the presentation?
     - Was there anything in this presentation that reminded you of your or your peers' experiences in STEM fields? *There is no need to respond to this potentially personal question if you do not feel comfortable doing so. Feel free to skip this question or to respond in very general terms.*
     - Were there any additional related topics that you would have liked to seen discussed?

- Fulfilling the above requirements will earn you a $^{3.25}/_{4}$. The rest of the points will be awarded based on the depth and quality of your reflection.

Option II

Read the following material discussing recent topics from class.

Kamerlingh Onnes and the discovery of superconductivity

https://aapt.scitation.org/doi/pdf/10.1119/1.17669

Strange phenomena in matter's flatlands (Nobel Prize in Physics 2016)

https://www.nobelprize.org/nobel_prizes/physics/laureates/2016/popular-physicsprize2016.pdf

Write a short response to the two articles. One paragraph per article is sufficient.

- Summarize the main points of each article in a few sentences.
- For each article, discuss in a few sentences the connections between the science discussed in the article and the material covered in this course.
- Which article did you feel better conveyed the relevant science? Why? (Keep in mind the intended audience of each piece.)
- Fulfilling the above requirements will earn you a $^{3.25}/_{4}$. The rest of the points will be awarded based on the depth and quality of your explanations of the connections. (For example, a response that discusses the energy flow of a negative temperature system will earn more points than a response that simply states that negative temperature is something we saw in class.)

NOTICE!!! THE FINAL PROBLEM HAS CHANGED!

8) In preparation for the end of the semester, start thinking about questions that could be on the final.

a) Create two problems on material from after Test 2 that could be on the final. These problems should be the level of difficulty that could be on a test. They should not be too easy, such as a problem that just requires plugging numbers into a formula. They should also not be so difficult that they could not be solved in the amount of time available on a test with many questions.

b) For each problem, provide a very brief description of ***how*** the problem could be solved. You do NOT need to solve the problem completely… yet. Just outline the method that could be used to solve the problem.

You will be graded on the appropriateness and originality of your questions and on the outlines of the solutions you provide.

I will provide feedback on your problems, and for the next week's homework, I will ask you to write up full, formal solutions to one problem. For the last homework of the semester, I will have you exchange problems and solve a problem created by one of your classmates. *If any of the problems are particularly good, I may use them on the final. Hopefully that is an incentive to write good problems!*