Problem Set # 6

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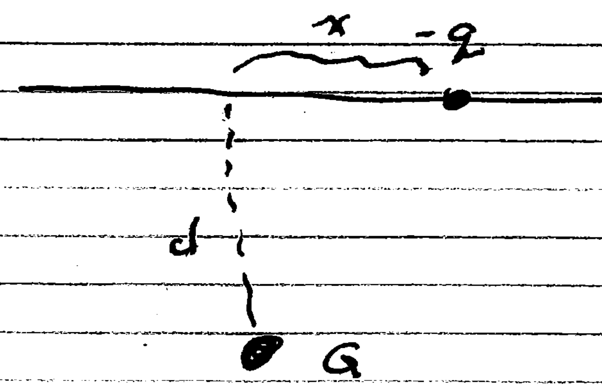


k of E & M.

a)

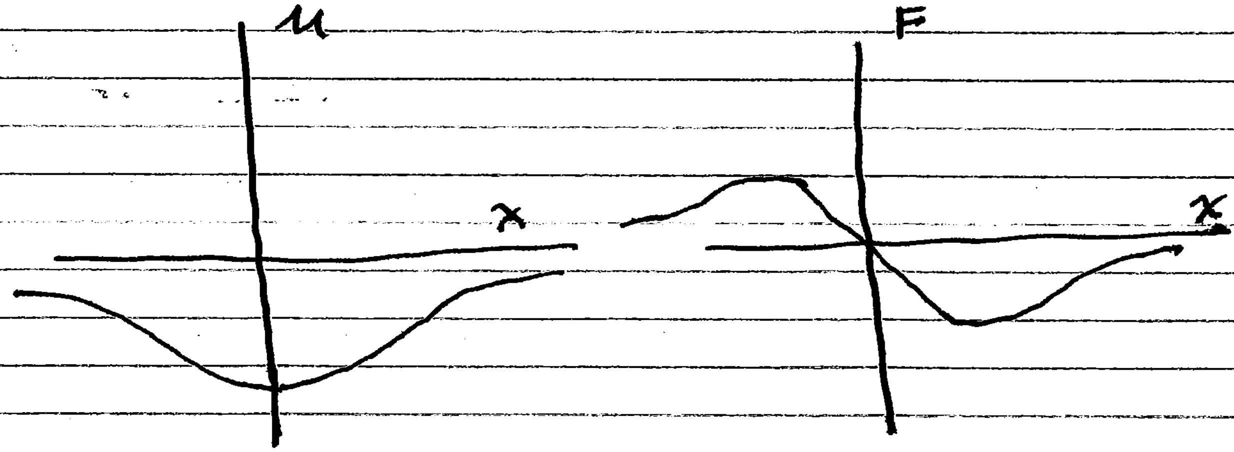
$$\mathcal{U} = -\frac{k_0 G q}{r} = -\frac{k_0 G q}{\sqrt{d^2 + x^2}}$$

b)

$$F = -\frac{d\mathcal{U}}{dx} = -\frac{k_0 G q}{2}\left(d^2 + x^2\right)^{-3/2}(2x)$$

$$= -\frac{k_0 G q}{\cancel{2}}\left(d^2 + x^2\right)^{-3/2} x$$

c)



d)

Well for $x << d$ (small)

$$F \cong \underbrace{-\frac{k_o Q q}{d^3}}\, x$$

So this is the effective k

$$\omega_o = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_o Q q}{d^3 m}}$$

e)

$$\omega_o = \sqrt{\frac{9\times10^{9} \times (1.6\times10^{-19})^2}{(2\times10^{-10})^3\, 9\times10^{-31}}} = 5.65\times10^{15}$$

$$\nu = \frac{\omega_o}{2\pi} = 9\times10^{14} \text{ Hz}$$

$$\lambda = \frac{c}{\nu} = 3.3\times10^{-7} \text{ m}$$

ultraviolet ~~visible~~ light.

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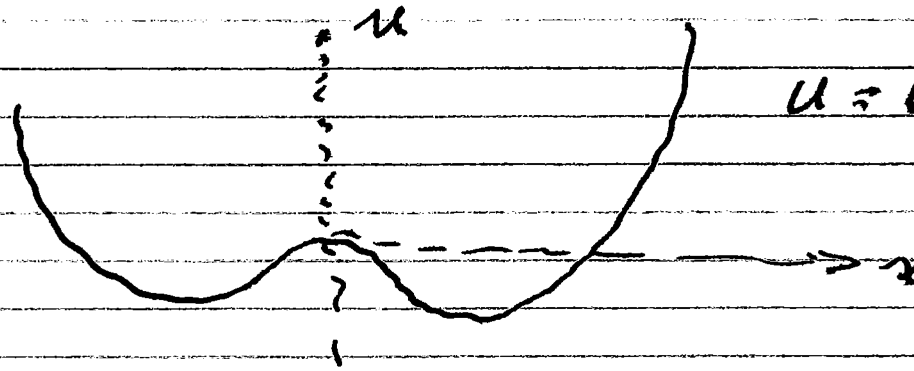
a)



$$\mathcal{U} = (x^4 - \lambda^2 x^2)\frac{\mathcal{U}_0}{\lambda^4}$$

b) ~~Stable~~ Equilibrium

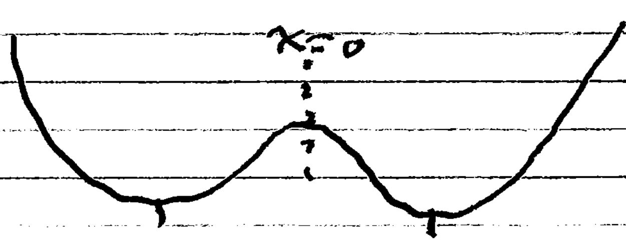
$$F = -\frac{d\mathcal{U}}{dx} = -(4x^3 - 2\lambda^2 x)\frac{\mathcal{U}_0}{\lambda^4}$$

This $= 0$ when $4x^3 = 2\lambda^2 x$

$x = 0$ is one solu.

another is $2x^2 = \lambda^2$

$$x = \pm\frac{\lambda}{\sqrt{2}}$$



$x = -\frac{\lambda}{\sqrt{2}}$ $\quad$ $x = \frac{\lambda}{\sqrt{2}}$

These 2 are stable.

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For the k about $x = \pm \frac{\lambda}{\sqrt{2}}$

Take $\left.\frac{d^2 U}{dx^2}\right|_{x=\pm\frac{\lambda}{\sqrt{2}}}$

$$\frac{d^2 U}{dx^2} = \left(12x^2 - 2\lambda^2\right)\frac{U_0}{\lambda^4}$$

$$\left.\frac{d^2 U}{dx^2}\right|_{x=\pm\frac{\lambda}{\sqrt{2}}} = \left(12\left(\frac{\lambda^2}{2}\right) - 2\lambda^2\right)\frac{U_0}{\lambda^4}$$

$$= 4\lambda^2 \frac{U_0}{\lambda^4} = \frac{4U_0}{\lambda^2} = k$$

So $\omega_0^2 = \frac{k}{m} = \frac{4U_0}{m\lambda^2}$

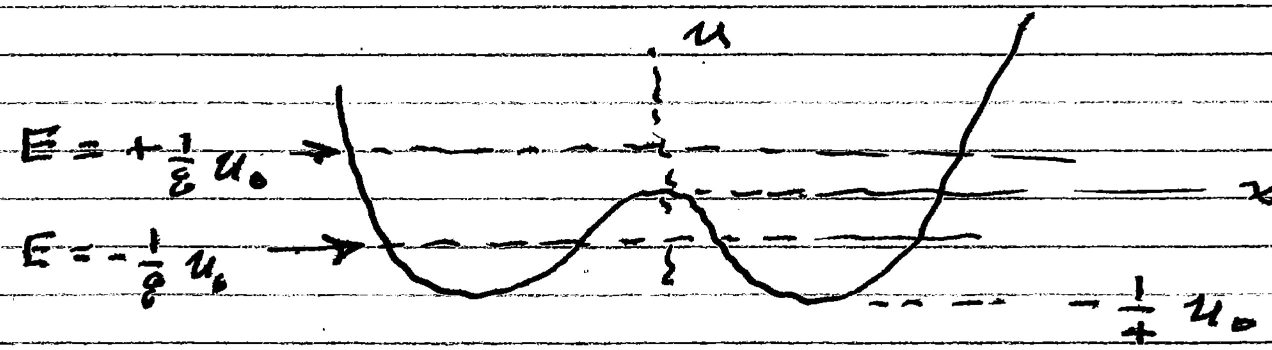
$$\omega_0 = \frac{2}{\lambda}\sqrt{\frac{U_0}{m}}$$

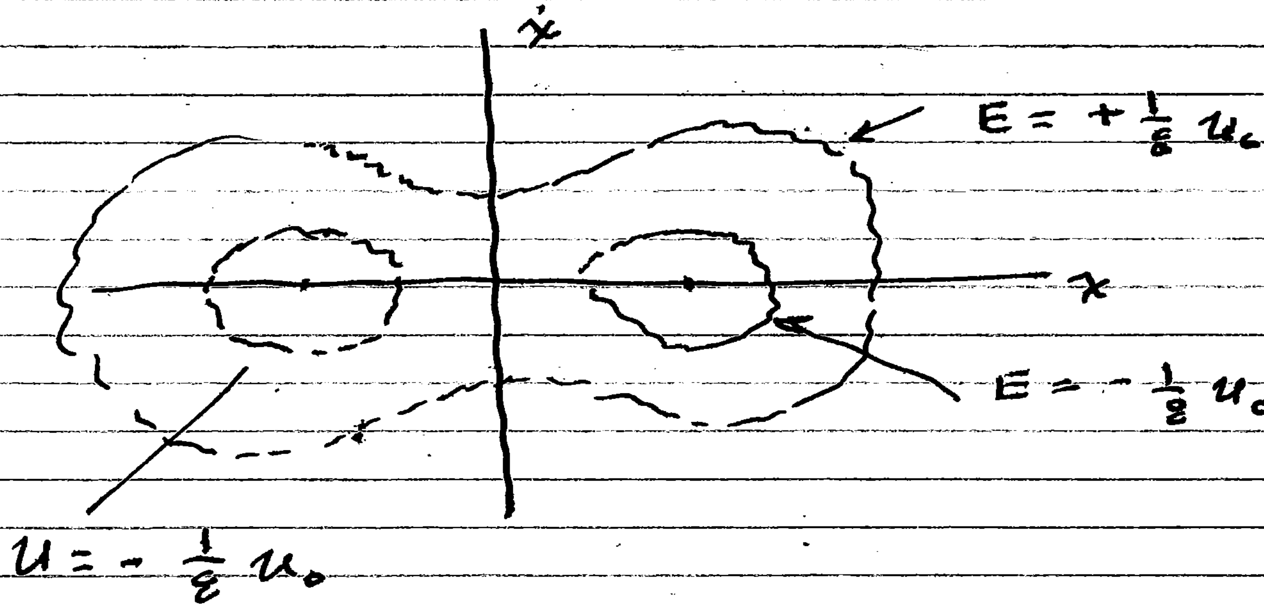
c) Here we would like to know the U's for the valleys & hills -

a $x = 0$ $U = 0$. at $x = \pm \frac{\lambda}{\sqrt{2}}$

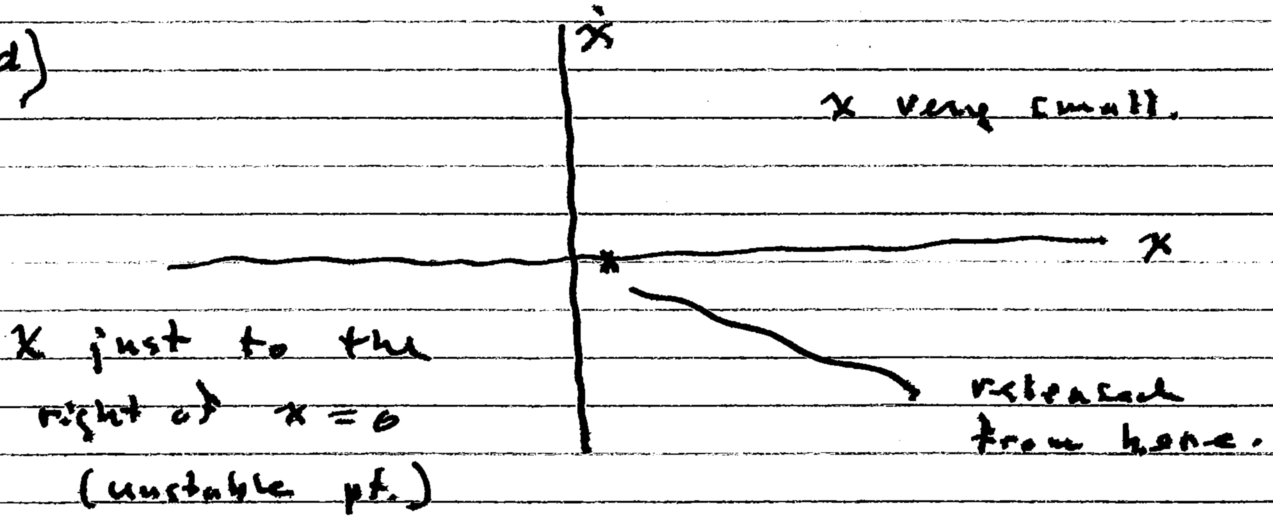
$$U = +\left(\frac{\lambda^4}{4} - \lambda^2\frac{\lambda^2}{2}\right)\frac{U_0}{\lambda^4}$$

$U = -\frac{1}{4} U_0$


U
$E = +\frac{1}{8} U_0$
$E = -\frac{1}{8} U_0$
x
$-\frac{1}{4} U_0$


$\dot{x}$
$E = +\frac{1}{8} U_0$
x
$E = -\frac{1}{8} U_0$
$U = -\frac{1}{8} U_0$

d)


$\dot{x}$
x very small.
x
x just to the right of $x = 0$ (unstable pt.)
released from here.

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$$F = -\frac{dU}{dx} = -(4x^3 - 2\lambda^2 x)\frac{U_0}{\lambda^4}$$

for x very small

$$m\ddot{x} = F = 2\lambda^2 \frac{U_0}{\lambda^4} x$$

$$\ddot{x} = \frac{2U_0}{m\lambda^2} x$$

Here we might try ~~$x = Ae$~~

$$x = Ae^{\beta t}$$

Sub.

$$\beta^2 A e^{\beta t} = \frac{2U_0}{m\lambda^2} A e^{\beta t}$$

$$\beta = \pm\sqrt{\frac{2U_0}{m\lambda^2}}$$

$$x = Ae^{+\sqrt{}\, t} + Be^{-\sqrt{}\, t}$$

want $\dot{x} = 0$ at $t=0$ $\Rightarrow$ $\dot{x}(0) = \sqrt{}A - \sqrt{}B$

or $A = B$

and

$$x(0) = 2A \Rightarrow A = \frac{1}{2} 10^{-4} \lambda$$

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$$x = 10^{-4} \lambda \cosh\left(\sqrt{\frac{2U_0}{m\lambda^2}}\, t\right)$$

~~at $5 (m\lambda^2/U_0)^{1/2}$~~

$$= 10^{-4} \lambda \cosh\left(\sqrt{\frac{2U_0}{m\lambda^2}}\, 5 \left(\frac{m\lambda^2}{U_0}\right)^{1/2}\right)$$

$$= 10^{-4} \lambda \cosh(5) \simeq 10^{-4} \lambda e^{5}$$

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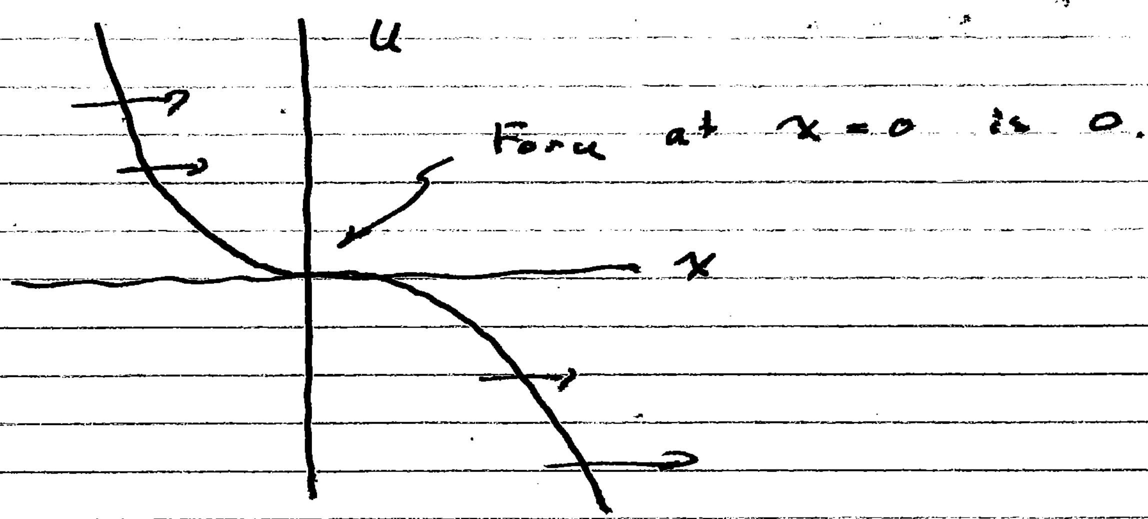
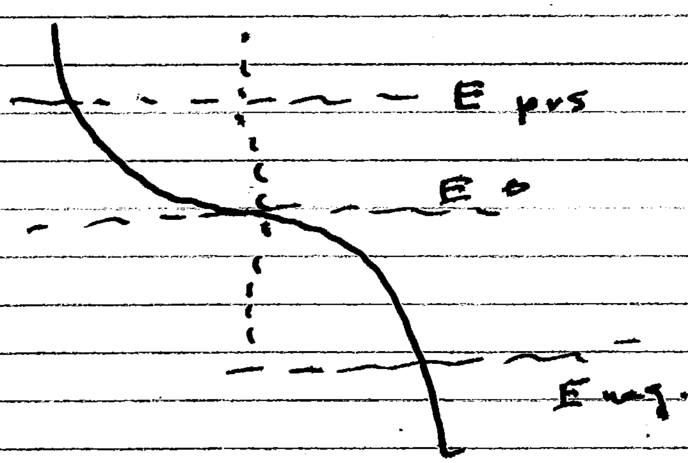
$$U = -cx^3$$





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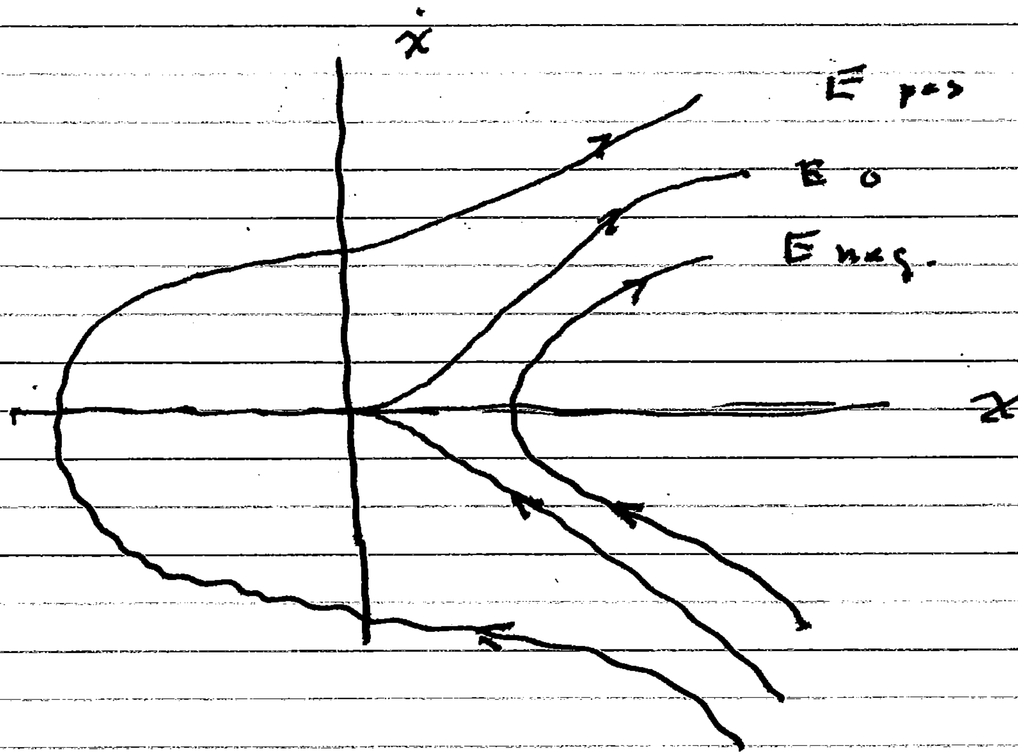

For $E = 0$ an incoming trajectory does not go into an out going — It take ∞ time to reach $x = 0$

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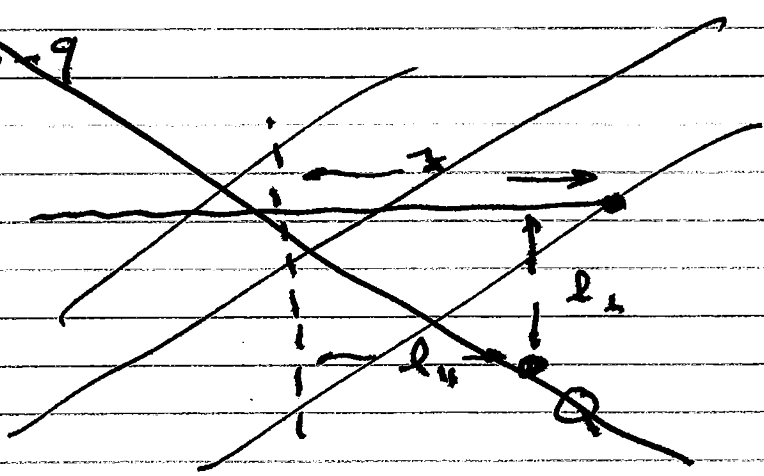

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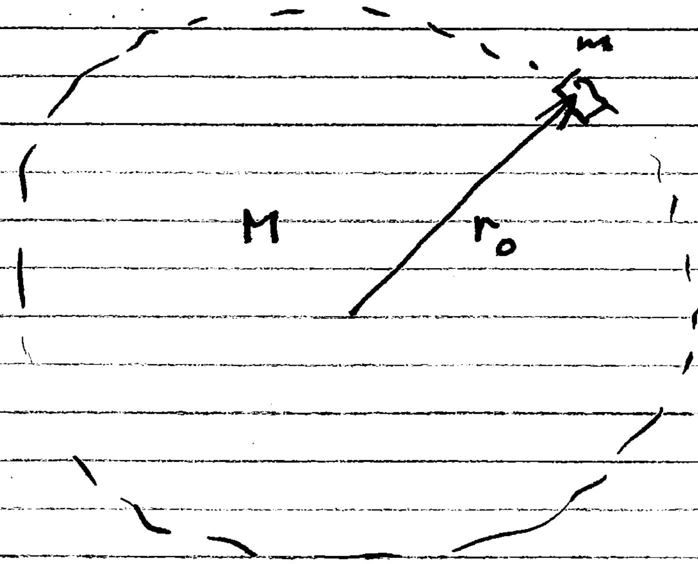

Consider a mass at the edge. How long does it take to reach the center?

I am assuming the outer mass do not pass inner masses. The force is always from the whole of mass M. Thus

$$U = -\frac{GMm}{r}$$

$$E = \frac{1}{2}m\dot{r}^2 + U$$

$$E = -\frac{GMm}{r_0}$$ (starts from rest.)

$$-\frac{GMm}{r_0} = \frac{1}{2}m\dot{r}^2 - \frac{GMm}{r}$$

$$GMm\left(\frac{1}{r} - \frac{1}{r_0}\right) = \frac{1}{2}m\dot{r}^2$$

$$\dot{r}^2 = 2GM\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

$$\dot{r} = -\sqrt{2GM}\left(\frac{1}{r} - \frac{1}{r_0}\right)^{1/2}$$

motion in inward

$$\frac{-\frac{dr}{dt}}{\sqrt{2GM}\sqrt{\frac{1}{r} - \frac{1}{r_0}}} = 1$$

$$-\frac{1}{\sqrt{2GM}}\int_{r_0}^{0}\frac{dr}{\sqrt{\frac{1}{r} - \frac{1}{r_0}}} = \int_0^t dt' = t$$

$$\frac{1}{\sqrt{2GM}}\int_0^{r_0}\frac{dr}{\sqrt{\frac{1}{r} - \frac{1}{r_0}}} = t$$

Change var. – (class recommendation)

$$r = r_0 \cos^2 \theta$$

$$dr = -r_0\, 2 \cos\theta \sin\theta \, d\theta$$

Note:

$$\sqrt{\frac{1}{r} - \frac{1}{r_0}} = \sqrt{\frac{1}{r_0 \cos^2\theta} - \frac{1}{r_0}}$$

$$= \frac{1}{\sqrt{r_0}\cos\theta} \sqrt{1 - \cos\theta} = \frac{\sin\theta}{\sqrt{r_0}\cos\theta}$$

so

$$\frac{-1}{\sqrt{2GM}} \int_{\pi/2}^{0} \frac{2 r_0 \cos\theta \sin\theta \, d\theta}{\frac{\sin\theta}{\sqrt{r_0}\cos\theta}} = t$$

$$- \frac{\sqrt{r_0}\, 2 r_0}{\sqrt{2GM}} \int_{\pi/2}^{0} \cos^2\theta \, d\theta = t$$

$$r_0^{3/2} \sqrt{\frac{2}{GM}} \int_{0}^{\pi/2} \cos^2\theta \, d\theta = t$$

$$t = \sqrt{\frac{2r_0^3}{GM}} \; \frac{1}{2}\left(\frac{\pi}{2}\right)$$

$$t = \frac{\pi}{4}\sqrt{\frac{2r_0^3}{GM}}$$

express in terms of orig. density

$$M = \rho_0 \frac{4}{3}\pi r_0^3$$

$$t = \frac{\pi}{4}\sqrt{\frac{2r_0^3}{G\rho_0 \frac{4}{3}\pi r_0^3}} = \frac{\pi}{4}\sqrt{\frac{3}{G\rho_0 2\pi}}$$

$$t = \sqrt{\frac{3\pi}{32 G\rho_0}}$$