Problem Set # 5

2-16

$$I\ddot{\theta} = -k'\theta - r'\dot{\theta}$$

$$I = 4.4 \times 10^{-8}$$

$$k' = .1 \qquad r' = 1.4 \times 10^{-10}$$

$$\omega_0^2 = \frac{k'}{I} = \frac{.1}{4.4\times10^{-8}} = 2.27\times10^{6}$$

$$\omega_0 = 1500$$

$$\tau = \frac{I}{r'} = 314$$

$$Q = \omega_0\tau = 4.7\times10^{5}$$

$$FWHM = \frac{\omega_0}{Q} = .0032$$

$$A = \frac{G}{\sqrt{(\omega_0^2-\omega^2)^2 + (\eta\omega)^2}}$$

For $\omega \cong \omega_0$

$$A = \frac{G}{(\eta\omega)\sqrt{1+\frac{(\omega_0^2-\omega^2)^2}{(\eta\omega)^2}}} \simeq \frac{G}{(\eta\omega)\left(1+\frac{1}{2}\frac{(\omega_0^2-\omega^2)^2}{(\eta\omega)^2}\right)}$$

$$= \frac{G}{(\eta\omega)\left(1+\frac{1}{2}\frac{(\omega_0-\omega)^3(\omega_0+\omega)^2}{(\eta\omega)^2}\right)}$$

$$\omega = \omega_0 + \Delta$$

So $$\frac{1}{2}\frac{(\omega_0-\omega)^2(\omega_0+\omega)^2}{(\eta\omega)^2}$$

$$= \frac{1}{2}\frac{\Delta^2(\omega_0+\omega_0+\Delta)^2}{\eta^2(\omega_0+\Delta)^2} \simeq \frac{2\Delta^2\omega_0^2}{\eta^2\omega_0^2} = \frac{2\Delta^2}{\eta^2}$$

$$A = \frac{G}{\eta(\omega_0+\Delta)\left(1+\frac{2\Delta^2}{\eta^2}\right)} = \frac{G}{\eta\omega_0\left(1+\frac{\Delta}{\omega_0}\right)\left(1+\frac{2\Delta^2}{\eta^2}\right)}$$

use $$\frac{1}{1+\varepsilon} \simeq 1-\varepsilon$$

$$A \simeq \frac{G}{\eta\omega_0}\left(1-\frac{\Delta}{\omega_0}\right)\left(1-\frac{2\Delta^2}{\eta^2}\right)$$

$$\simeq \frac{G}{\eta\omega_0}\left(1-\frac{\Delta}{\omega_0}-\frac{2\Delta^2}{\eta^2}\right)$$

neglect Δ^3

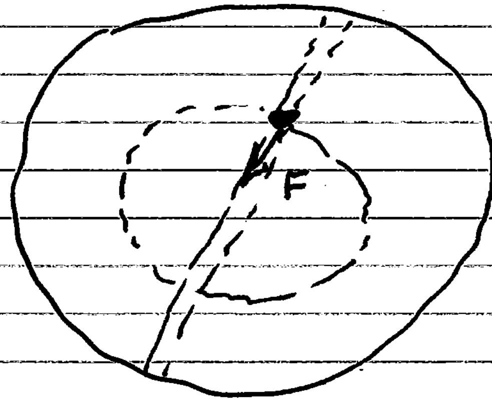
max $$\frac{d}{d\Delta} \rightarrow -\frac{1}{\omega_0} - \frac{4\Delta}{\eta^2} = 0$$

$$\Rightarrow \Delta = -\frac{\eta^2}{4\omega_0}$$ then $-\frac{2\Delta^2}{\eta}$ factor

Alice in Wonderland Problem.

Consider the earth with a hole through the diameter. How long would it take a mass (a person) to fall through the hole and arrive at the other side. Assume the mass of the earth, 6×10^{24} kg, is distributed uniformly within the earth's radius 6.4×10^{6} m. If you enjoyed that, do the same for a cord. Neglect friction.

Alice in Wonderland Problem.



$$F = \frac{G M m}{R^2}$$

where R is distance to center & M

Density $\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3}$

$$M = \frac{4}{3}\pi R^3 \rho = \frac{\frac{4}{3}\pi R^3 M_E}{\frac{4}{3}\pi R_E^3} = \frac{R^3}{R_E^3} M_E$$

$$F = \frac{G R^3 m M_E}{R_E^3 R^2} = \frac{G M_E m}{R_E^3} R$$

So $m\ddot{R} = -\frac{G M_E m}{R_E^3} R$

$$\ddot{R} = -\frac{G M_E}{R_E^3} R$$

Force is inward

$$\omega_0 = \sqrt{\frac{G M_E}{R_E^3}} = \sqrt{\frac{6.7\times10^{-11} \times 6\times10^{24}}{(6.4\times10^6)^3}}$$

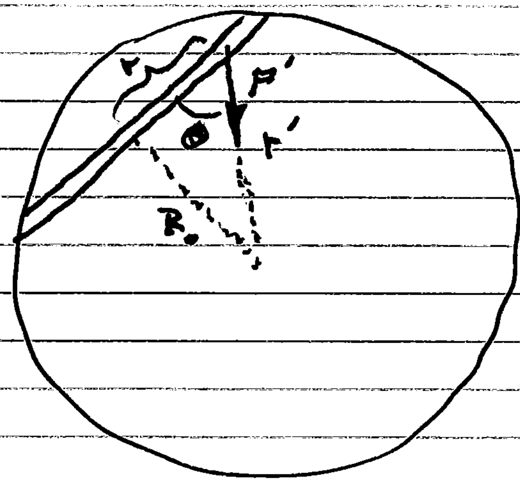
$$\omega_0 = .0012 = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{.0012} = 5070 \text{ sec}$$

Time to reach other side $= T/2$

$= 2540$ sec

or 42 min.



$$F = -\cos\theta \; F'$$

$$F' = \frac{G M_E m}{R_E^3} r' \qquad \text{from part 1}$$

$$\cos\theta = \frac{r}{r'}$$

$$F' = -\frac{G M_E m}{R_E^3} r$$

$$\ddot{r} = -\frac{G M_E}{R_E^3} r$$

$$\omega_0 = \sqrt{\frac{G M_E}{R_E^3}}$$

Same answer as in first part.

42 min.