1 Solution Sets and Equations

Everything we do in this course boils down to studying properties of sets and functions. When we consider the fact that our function notation is shorthand for elements of a set with special properties, then everything boils down to sets. We continue this translation of familiar concepts into the language of set theory with equations.

Definition 1.1. Suppose that we have two functions, $f : A \to B$ and $g : A \to B$. We define the *solution set* of the equation $f(x) = g(x)$ to be $\{a \in A : f(a) = g(a)\}$.

Exercise 1. Suppose that we have two functions, $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 - 1$, and $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = x + 1$. Find the solution set of the equation $f(x) = g(x)$.

Exercise 2. Suppose that we have two functions, $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = ax - by$, and $g : \mathbb{R}^2 \to \mathbb{R}$ defined by $g(x, y) = c$. Find the solution set of the equation $f(x, y) = g(x, y)$. What is the geometric interpretation of the solutions set?

Exercise 3. Suppose that we have functions

- $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = x - 4y$
- $g : \mathbb{R}^2 \to \mathbb{R}$ defined by $g(x, y) = -2$
- $s : \mathbb{R}^2 \to \mathbb{R}$ defined by $s(x, y) = 2x - 5y$
- $t : \mathbb{R}^2 \to \mathbb{R}$ defined by $t(x, y) = 4$

a) Find the solution set of the equation $f(x, y) = g(x, y)$.

b) Find the solution set of the equation $s(x, y) = t(x, y)$.

c) Find the intersection of the solution sets. What is the geometric interpretation of this intersection?

Exercise 4. Suppose that we have functions

- $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = ax + by$
- $g : \mathbb{R}^2 \to \mathbb{R}$ defined by $g(x, y) = j$
- $s : \mathbb{R}^2 \to \mathbb{R}$ defined by $s(x, y) = cx + dy$
- $t : \mathbb{R}^2 \to \mathbb{R}$ defined by $t(x, y) = k$

a) Find the solution set of the equation $f(x, y) = g(x, y)$.

b) Find the solution set of the equation $s(x, y) = t(x, y)$.

c) Find the intersection of the solution sets. What is the geometric interpretation of this intersection?

Prepare for 2/7

- Finish this worksheet
- Read Sections 1.7 and 2.1.