

## Problem Set 1: Due Monday, January 29

**Problem 1:** Let  $P$  be the plane in  $\mathbb{R}^3$  containing the origin and both of the following two vectors:

$$\vec{u} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \quad \text{and} \quad \vec{w} = \begin{pmatrix} -7 \\ 1 \\ 4 \end{pmatrix}$$

In the notes, we discussed one way to parametrize  $P$ , and we will discuss this in more detail later. Now find an equation of the form  $ax + by + cz = d$  for  $P$ . Explain your process using a sentence or two.

**Problem 2:** Let  $L$  be the line in  $\mathbb{R}^3$  that is the intersection of the two planes  $3x + 4y - z = 2$  and  $x - 2y + z = 4$ .

- Using the equations of the planes, determine if the points  $(1, 0, 1)$  and  $(1, 1, 5)$  are on  $L$ .
- Find a parametric description of  $L$ . Explain your process using a sentence or two.
- Use the parametric description of  $L$  to determine if  $(5, 2, 3)$  is a point on  $L$ . Explain.

*Note:* Given a point, it seems easier to determine if it is on  $L$  using the equations of the planes rather than the parametric description. In contrast, if you want to *generate* points on  $L$ , it is easier to use the parametric description (just plug in values for the parameter) than the plane equations.

**Problem 3:**

- Do the planes with equations  $2x - 3y + z = 7$  and  $-4x + 9y - 2z = 3$  intersect? Explain your reasoning.
- Do the lines described by the two parametric equations

$$\begin{array}{rclcl} x & = & -4 & + & 6t \\ y & = & 2 & + & t \\ z & = & 1 & + & 3t \end{array} \qquad \begin{array}{rclcl} x & = & 4 & + & 4t \\ y & = & 5 & - & t \\ z & = & 9 & - & 2t \end{array}$$

intersect? Explain your reasoning.

**Problem 4:** Determine if the following are true or false. Justify your answers using complete sentences in each case.

- There exists  $x \in \mathbb{R}$  with  $\sin x = \cos x$ .
- There exists  $x \in \mathbb{R}$  with  $\sin x = \cos x + 2$ .
- There exists  $m, n \in \mathbb{N}$  with  $9m + 15n = 3$ .
- There exists  $m, n \in \mathbb{Z}$  with  $9m + 15n = 3$ .
- For all  $t \in \mathbb{R}$ , we have

$$2 \cos^4(3t) + 2 \cos^2(3t) \cdot \sin^2(3t) - \cos(6t) = 1.$$

- For all  $a \in \mathbb{R}$ , we have  $a^2 + 6a + 10 > 0$ . (Do not rely on a picture.)