Problem Set 18: Due Wednesday, April 18

Problem 1: Determine whether

$$\left(\begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ -4 \\ 14 \end{pmatrix} \right)$$

is a linearly independent sequence in $\mathbb{R}^3$.

Problem 2: By setting up a system and using Gaussian Eliminations, find one specific example of nontrivial linear combination of

$$\left(\begin{pmatrix} 0 \\ 1 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} -8 \\ 2 \\ -2 \\ 2 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \\ 9 \\ 5 \end{pmatrix} \right)$$

giving $\vec{0}$.

Problem 3: Consider the following three functions in the vector space $\mathcal{P}_2$:

- $f_1(x) = 9x^2 - x + 3$.
- $f_2(x) = 3x^2 - 2x + 5$.
- $f_3(x) = -5x^2 + x + 1$.

Is (f_1, f_2, f_3) linearly independent? Explain.

Problem 4: Consider the following three functions in the vector space $\mathcal{F}$:

- $f_1(x) = 2^x$.
- $f_2(x) = x^2$.
- $f_3(x) = x - 2$.

Is (f_1, f_2, f_3) linearly independent? Explain.

Problem 5: Find a sequence $(\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4)$ of vectors in $\mathbb{R}^3$ such that whenever we omit a vector, the resulting 3 are linearly independent. You should justify why your sequence has this property.

Problem 6: Let $\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n \in \mathbb{R}^n$ (notice the same n). Explain why $\text{Span}(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n) = \mathbb{R}^n$ if and only if $(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n)$ is linearly independent.