Collisional generation of runaway electron seed distributions leading to sub-criticality, avalanche, or fast transfer

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Presented at the IAEA Fusion Energy Conference,
Kyoto, October 2016

Outline

- Brief History and Motivation
  - Recent developments address critical issues with seminal papers
  - Need for nonlinear collisional treatment to understand seed generation
- New code with FEM treatment of electron f with background ions including impurities
  - Nonlinear e-e collisions, linear e-i collisions, f dependent and independent sourcing
  - Backward Euler implicit timestepping
  - Initial electric field drives current to steady state – triggers impurity injection
  - Electric field found from current density constraint after impurities injected
- Interface to runaway probability calculation used to calculate seed population
- Initial results indicate the fast transfer difficult to achieve under realistic timescales
- Concluding Remarks

- The SCREAM Collaboration
  - Goals and Scope
  - Who, Where and What
- Concluding Remarks

Understanding of runaway electron kinetic physics started with seminal studies

- First analysis of runaway phenomena carried by [Dreicer 1958, 1959]
- Relativistic case studied by [Connor&Hastie 1975]
- First well-known and most-cited work on secondary runaway electron generation by [Rosenbluth & Putvinski 1997]
  - Through large angle collision, high energy RE can transfer large fraction of energy and momentum to low energy electron and knock it into runaway regime ➔ avalanche
  - Critical electric field for avalanche is Connor-Hastie Ec
  - The growth rate is (almost) a linear function of E/Ec-1

$$\frac{\partial f}{\partial t} + E\{f\} + C\{f\} + R\{f\} = S$$

E: Electric field drive
C: Collision operator
R: Synchrotron radiation back-reaction force
S: Secondary runaway electron generation

Limitations of Rosenbluth-Putvinski model prevent quantitative experimental analyses

- A simplified source term for the secondary generation

$$S = \frac{n_r}{4\pi \ln\Lambda}\delta(\xi - \xi_2)\frac{1}{p^2}\frac{\partial}{\partial p}\left(\frac{1}{1-\sqrt{1+p^2}}\right) \qquad \xi_2 = \frac{\sqrt{1+p^2}-1}{p}$$

  - All RE assumed to have infinite momentum and zero pitch angle
  - Secondary RE can have larger momentum than the seed one! (unphysical)
  - Pitch angle distribution is singular
  - Change of momentum and pitch angle of seed electron after collision not considered – violate conservation law
- Missing kinetic effects
  - Radiation reaction force (synchrotron and bremsstrahlung) important for high energy electron
  - Other effects (magnetic perturbations, kinetic instabilities)

Recent progress has accelerated - quantitative understanding of experiments advancing

Multiple groups have accelerated effort in the last two years (2015-16 papers)

- IFS, IPP and ITER: Monte Carlo methods and rigorous marginal E analytics
  - P. Aleynikov, B. Breizman, Phys. Rev. Lett., **114**, 155001 (2015).
- Columbia: Theoretical analyses of runaway dynamics
  - A. Boozer, Phys. Plasmas **22**, 032504 (2015).
- PPPL: Adjoint Fokker-Planck probability, nonlinear continuum seed and avalanche, Monte Carlo
  - C. Liu, D.P. Brennan, A. Bhattacharjee, and A.H. Boozer, Phys. Plasmas **23**, 010702 (2016).
- GA: MHD Simulations with relativistic tracer particles
  - V.A. Izzo, D.A. Humphreys, and M. Kornbluth, Plasma Phys. Control. Fusion 54, 095002 (2012).
- ORNL: Full-orbit effects in toroidal geometry, impurity transport, thermal anisotropy, Monte Carlo, UQ
  - D. del-Castillo-Negrete and L. Carbajal, 58th Annual Meeting of the APS Division of Plasma Physics GP10.00095 (2016); to be submitted to Physics of Plasmas.
  - L. Carbajal, D. del-Castillo-Negrete, D. Spong, S.Seal and L. Baylor. 58th Annual Meeting of the APS Division of Plasma Physics. GP10.00097 (2016).
  - G. Zhang, C. Webster, M. Gunzburger and J. Burkardt, *SIAM Review*, **58**, 517 (2016).
- LANL: Phase space structure and runaway transport processes, Vlasov Fokker-Planck
  - Z.Guo, C. Mcdevitt and X. Tang, Proceedings of the International Sherwood Fusion Theory Conference, Madison, Wisconsin, April 2016.
- European Groups: Complementary continuum solvers, Monte Carlo, and time dependent simulations
  - Stahl, Hirvijoki, Decker, Embréus and Fülöp, PRL **114**, 115002 (2015).
  - Hirvijoki, Pusztai, Decker, Embréus, Stahl and Fülöp, **JPP** (2015).
  - E. Nilsson, J. Decker, Y. Peysson, R.S. Granetz, F. Saint-Laurent and M. Vlainic, Plasma Phys. Control. Fusion **57**, 095006 (2015).

Motivation: initial seed distribution study including nonlinear e,e and linear e,i collisions

Important question: how many seed electrons available for runaway and avalanche

High energy tail can be lost in two ways

- Collisional drag on cold electrons
- Lost to walls due to destroyed surfaces

Leads to three possible outcomes

- Fast transfer : eneough seed electrons to immediately take up Ip
- Avalanche : some seed electrons, generate enough to take ~ Ip
- Sub-Critical : not enough seed electrons

Fokker-Planck solver with background Maxwellian ions and sources/sinks

Kinetic equation includes electric field E, e,e and e,i collisions, with L a source dependent on f (eg. for parallel loss) and s an independent source (eg. for cold e source)

$$\frac{\partial f_a}{\partial t} + \frac{e_a \boldsymbol{E}}{m_a} \cdot \frac{\partial f_a}{\partial \boldsymbol{v}} = C_{aa}[f_a, f_a] + C_{ab}[f_a, f_b] + L[f_a] + s,$$

Both e and i distributions are represented in Rosenbluth potentials

$$\phi(\boldsymbol{v}) = -\frac{1}{4\pi}\int f(\boldsymbol{v}')\frac{1}{|\,\boldsymbol{v}-\boldsymbol{v}'\,|}d\boldsymbol{v}', \qquad \psi(\boldsymbol{v}) = -\frac{1}{8\pi}\int f(\boldsymbol{v}')\,|\,\boldsymbol{v}-\boldsymbol{v}'\,|\,d\boldsymbol{v}',$$

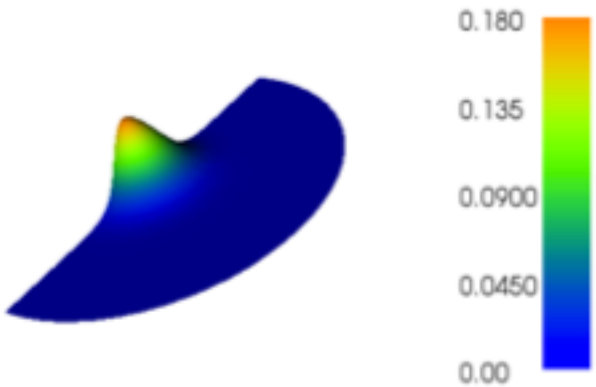
$$C_{ab}[f_a, f_b] = \left(\frac{e_a^2 e_b^2 \ln\Lambda_{ab}}{m_a^2 \varepsilon_0^2}\right)\frac{\partial}{\partial \boldsymbol{v}} \cdot \left(\frac{m_a}{m_b}\frac{\partial \phi_b}{\partial \boldsymbol{v}} f_a - \frac{\partial^2 \psi_b}{\partial \boldsymbol{v}\partial \boldsymbol{v}} \cdot \frac{\partial f_a}{\partial \boldsymbol{v}}\right).$$

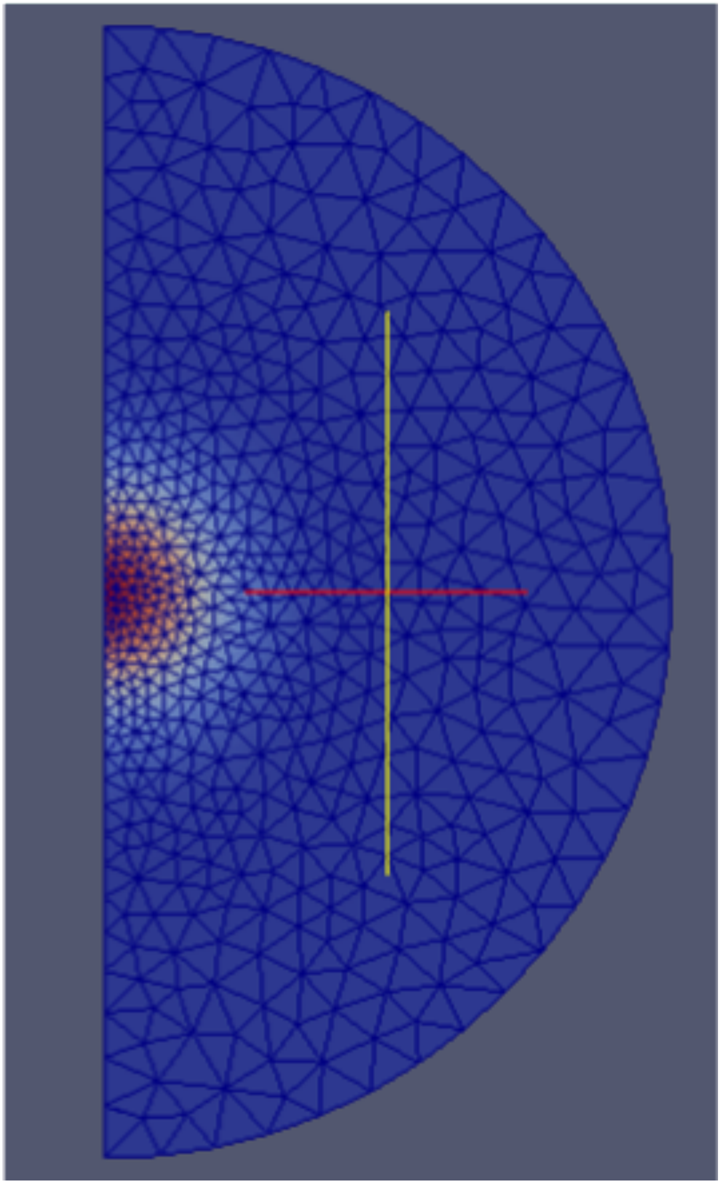
Can impose a current constraint $\boldsymbol{E} \cdot \int \boldsymbol{v}\frac{\partial f_a}{\partial t} d\boldsymbol{v} = 0,$

Finite Element unstructured mesh representation allows for flexibility

Developed quickly using Fenics library
Fenicsproject.org

Density conserved, energy conservation an issue (losses O(10^-4) per collision time, working on it. Note: Hirvijoki talk)



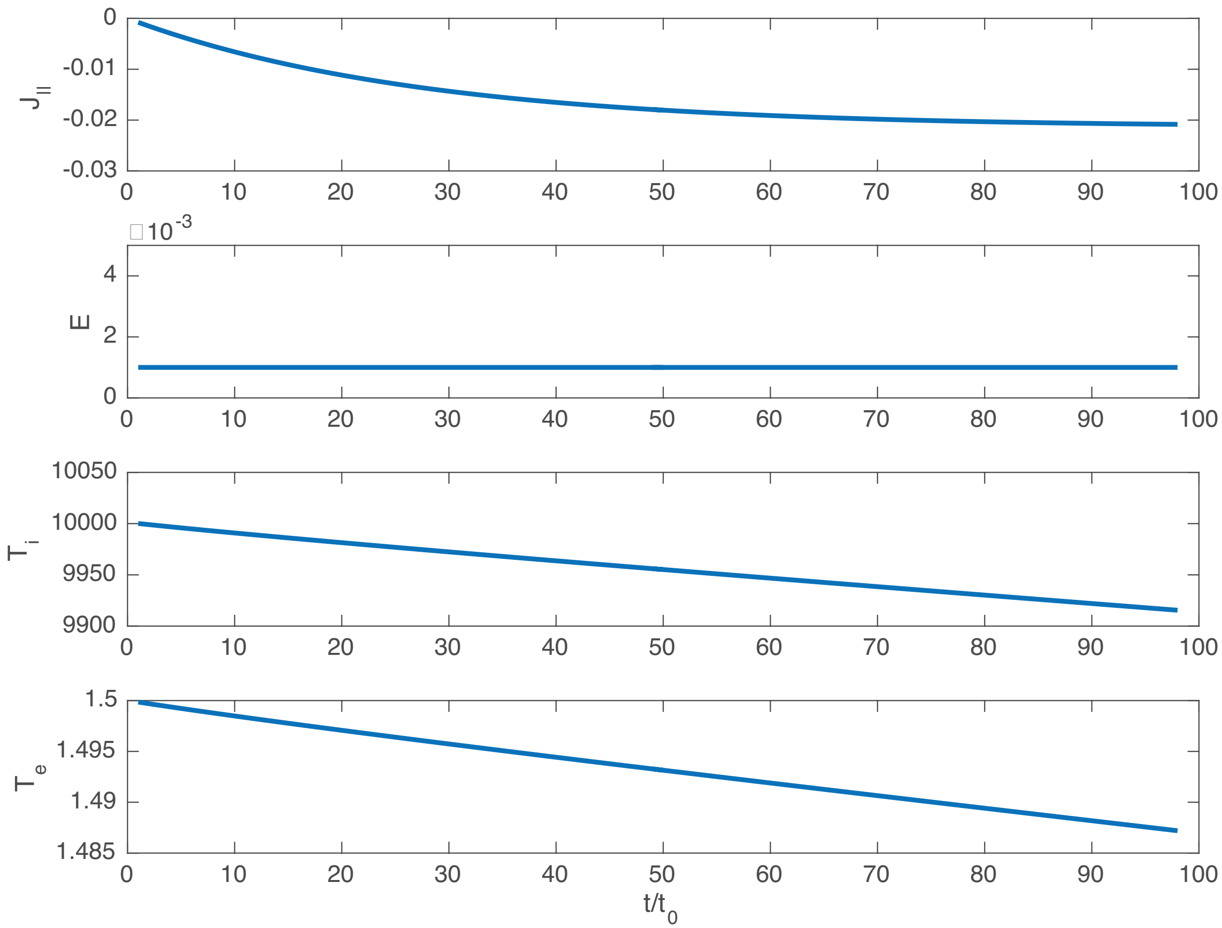


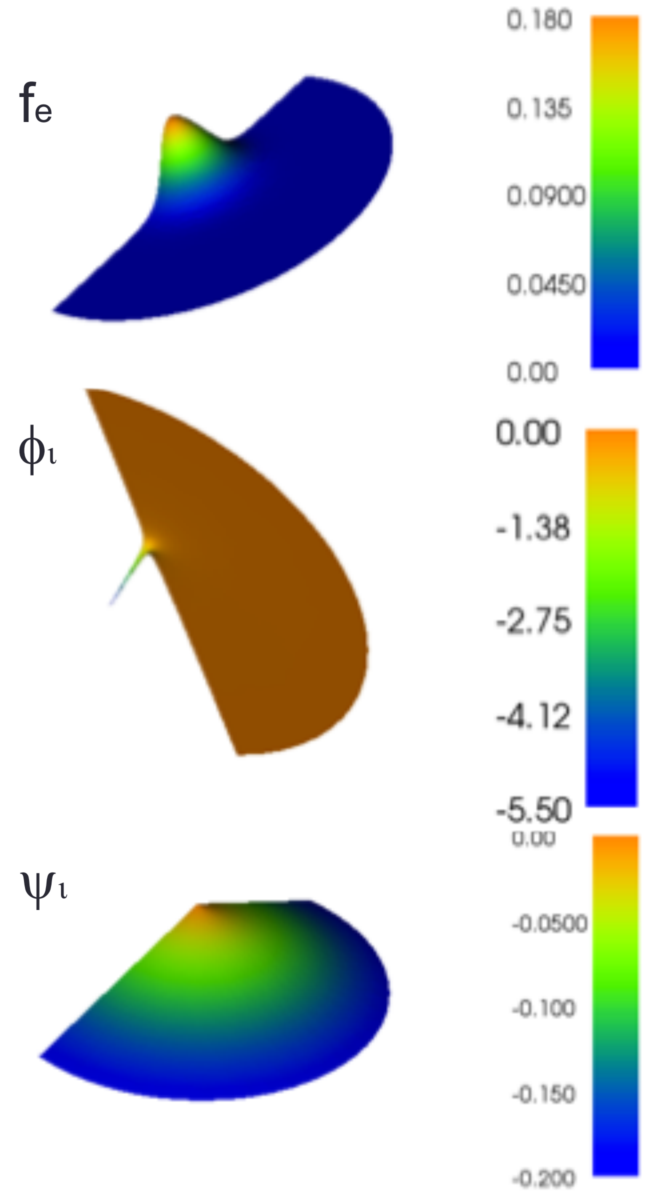
Initial case simply draw out current against E

Density conserved

Te dropping slowly on collisional drag with ions

Ti adjusted to track Te





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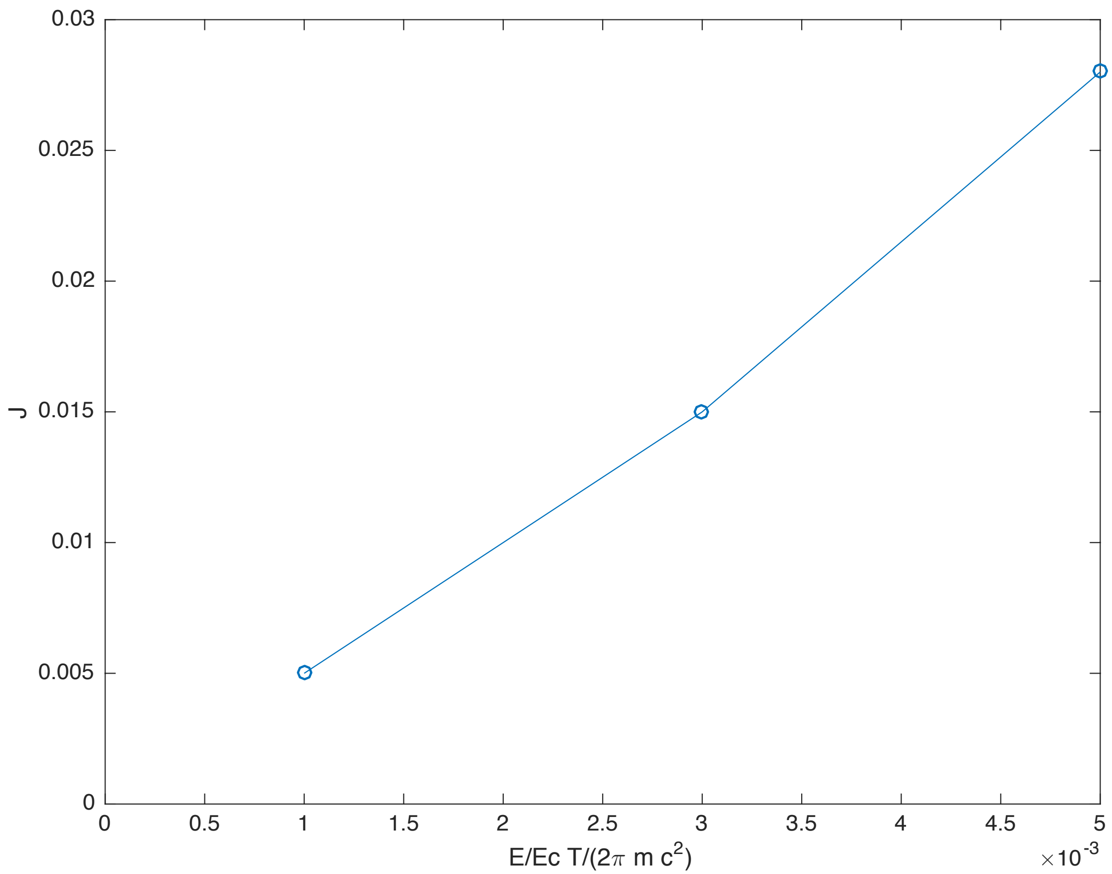


Saturated current linear with E (will check against Spitzer)
Heating causes deviation at higher E.

Plan: Use the Adjoint method and the Runaway Probability Function to get the seed population

F is the Green's function of the Fokker-Planck operator *L*.

$$\frac{dF}{dt}=\frac{\partial F}{\partial t}-\hat{L}[F]=\delta(p-p_0)$$
$$F(p_1)=0, F(p_2)=0.$$

P is solution of adjoint Fokker-Planck equation.

$$\hat{L}^{\dagger}[P]=a(p)\frac{\partial P}{\partial p}+D(p)\frac{\partial^2 P}{\partial p^2}=0$$
$$P(p_1)=0, P(p_2)=1$$

$$\int_{p_1}^{p_2}\hat{L}[F]P\,dp=\left[PU+D\frac{\partial P}{\partial p}F\right]_{p1}^{p2}+\int_{p_1}^{p_2}F\hat{L}^{\dagger}[P]dp$$

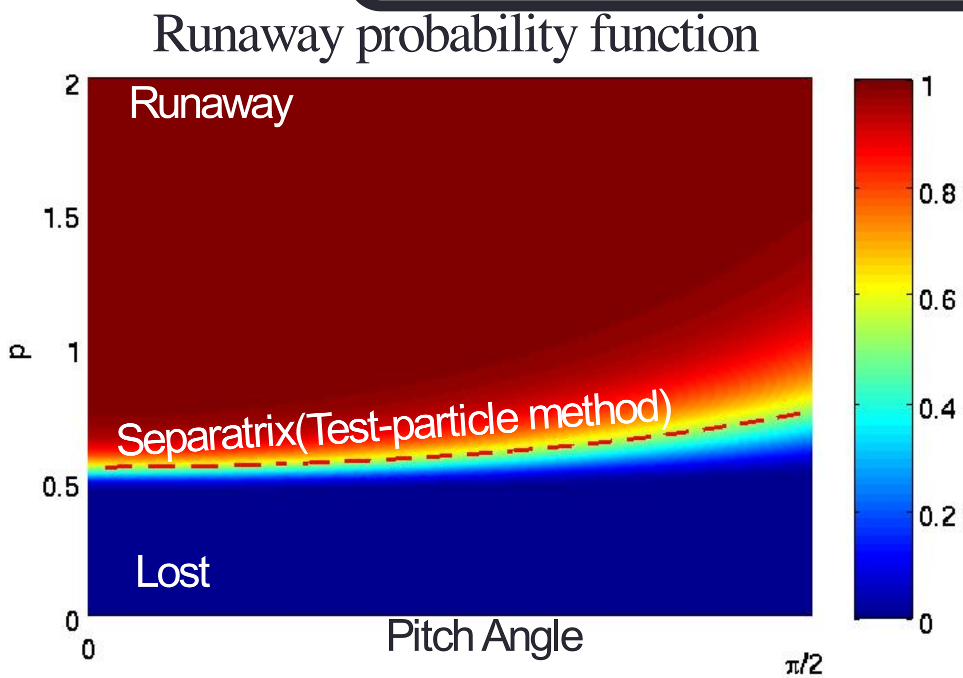
$$P(p=p_0)=U\big|_{p=p_2}$$

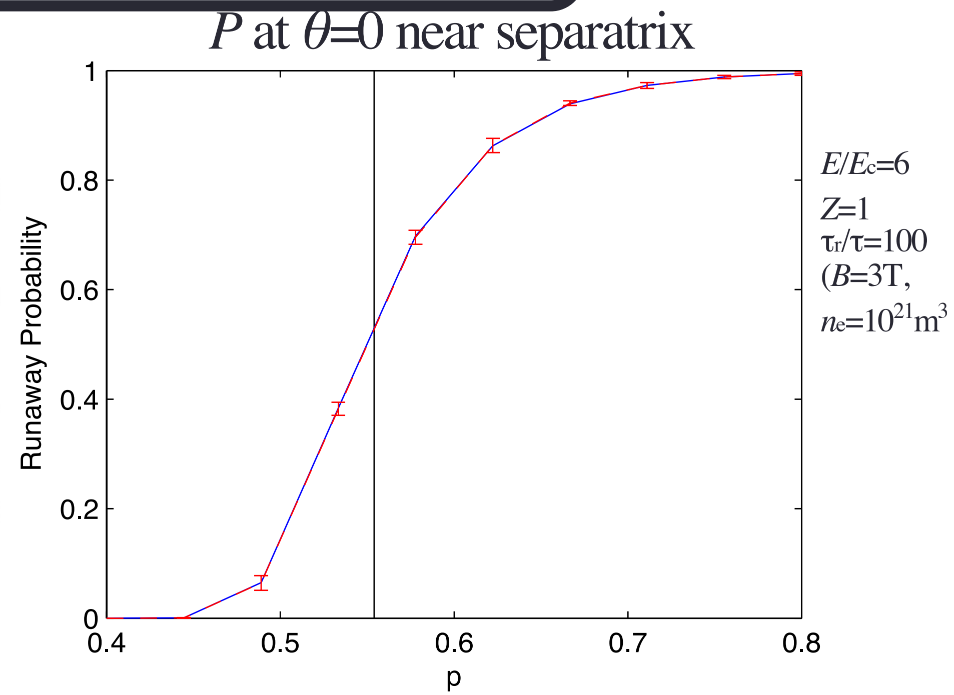
P characterize the probability for electron to eventually reach boundary p=p_2.

C.F.F. Karney and N.J. Fisch, Phys. Fluids 29, 180 (1986).

Runaway Probability Function for Z=1

$$a(p)\frac{dP(p)}{dx}+D(p)\frac{d^2P(p)}{dp^2}=0 \quad P\big|_{p_1}=0 \quad P\big|_{p_2}=1$$





- P gives probability for electron to reach high momentum boundary
- Result of P shows smooth transition near separatrix
  - The test-particle method (relying on truncation of pitch angel scattering) only gives a line of separatrix, equivalent to a Heaviside P function.
- Results agree well with Monte-Carlo Simulation

C. Liu, D.P. Brennan, A. Bhattacharjee, and A.H. Boozer, Phys. Plasmas **23**, 010702 (2016).
J.R. Martín-Solís, R. Sánchez, and B. Esposito, Phys. Rev. Lett. **105**, 185002 (2010).

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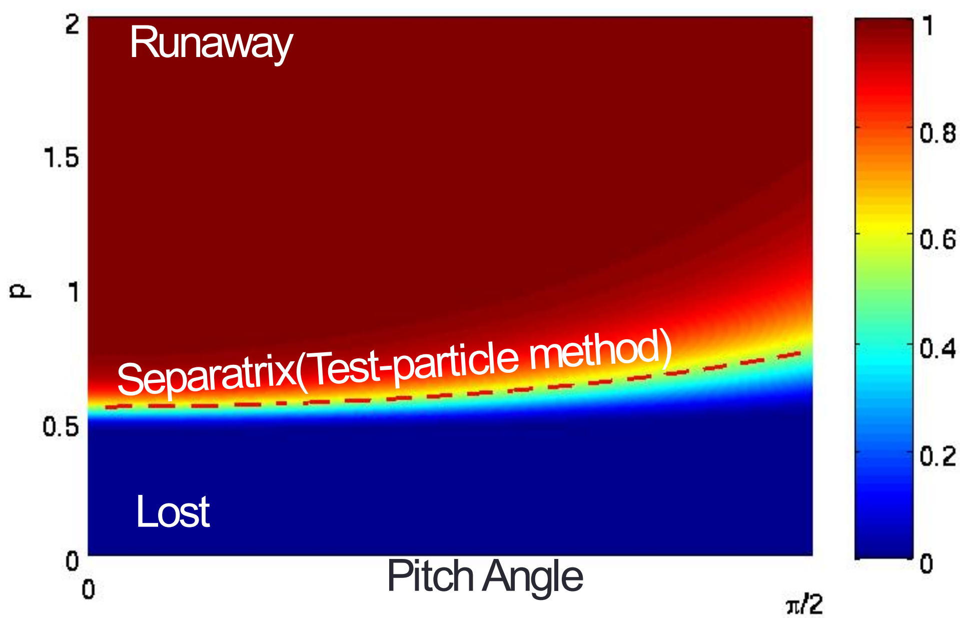
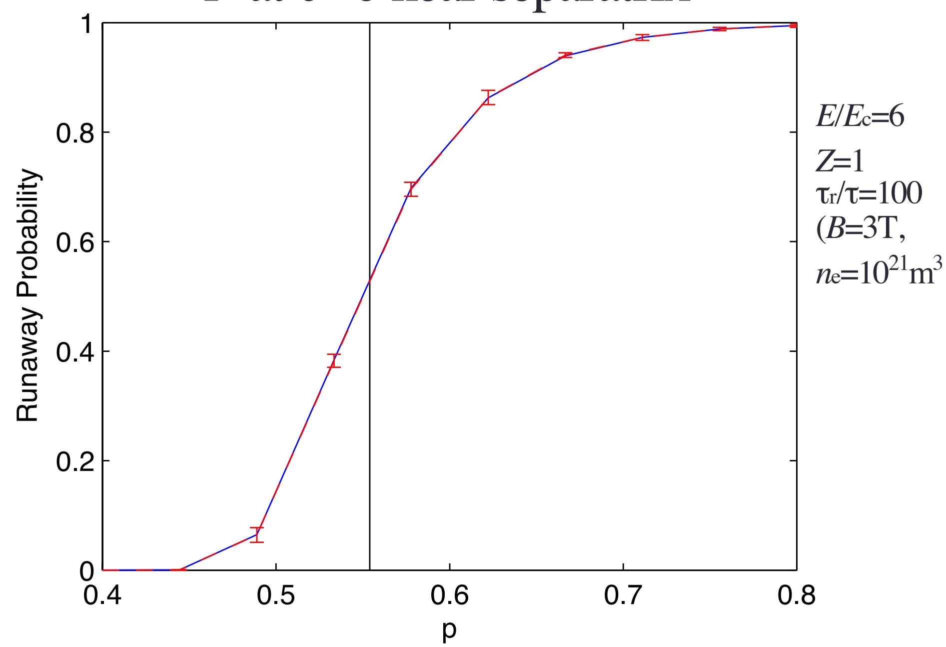


P at θ=0 near separatrix



P can be used to estimate the number of seed RE in thermal quench.

Interface coded, cases being explored

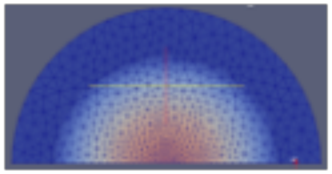
$$n_{se}=\int d^3v\, f\cdot P$$

C. Liu, D.P. Brennan, A. Bhattacharjee, and A.H. Boozer, Phys. Plasmas **23**, 010702 (2016).
J.R. Martín-Solís, R. Sánchez, and B. Esposito, Phys. Rev. Lett. **105**, 185002 (2010).

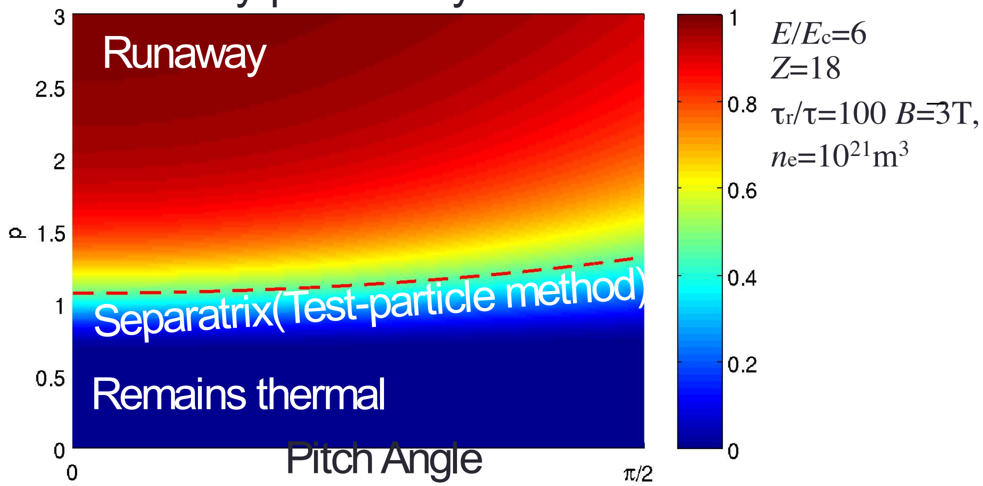
Runaway Probability Function applied to RE seed calculation

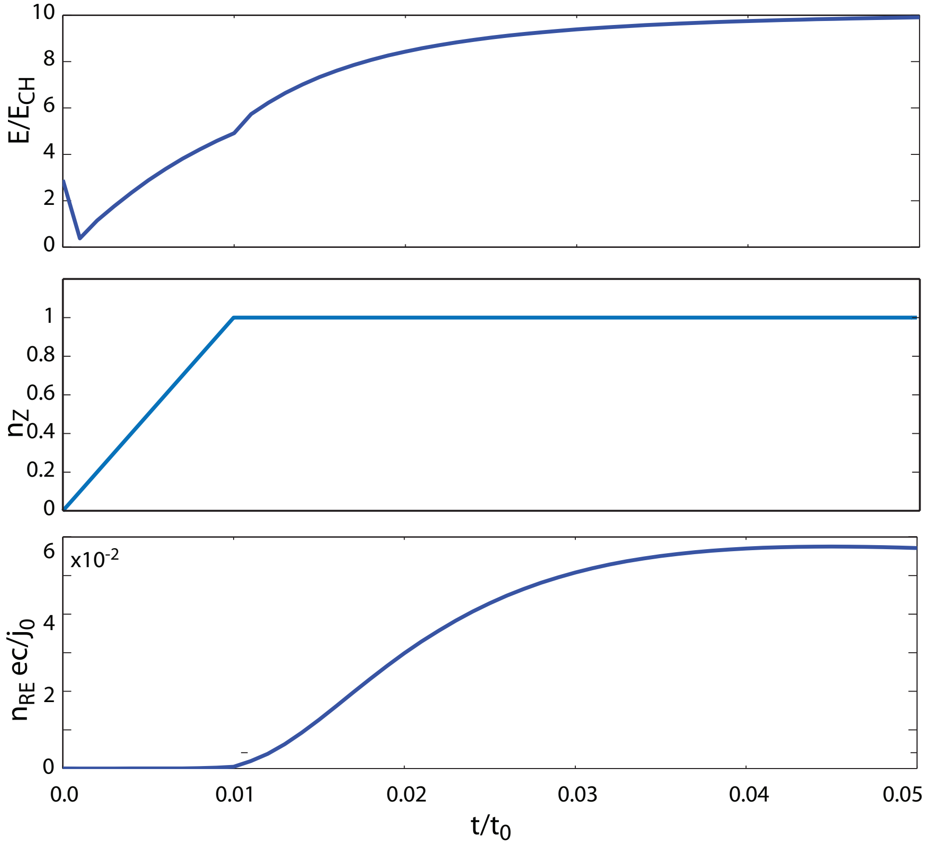
$$\frac{\partial f_a}{\partial t} + \frac{e_a \boldsymbol{E}}{m_a} \cdot \frac{\partial f_a}{\partial \boldsymbol{v}} = C_{aa}[f_a, f_a] + C_{ab}[f_a, f_b] + L[f_a] + s,$$

f(p,θ,t) solved for various temperature collapse and impurity injection scenarios



Runaway probability function





- *f and P* used to estimate the number of seed RE in thermal quench $n_{se} = \int d^3v\, f \cdot P$
- Result: Fast transfer difficult to achieve, avalanche dominant

C. Liu, D.P. Brennan, A. Bhattacharjee, and A.H. Boozer, Phys. Plasmas **23**, 010702 (2016).
D.P. Brennan, E. Hirvijoki, C. Liu, A.H. Boozer and A. Bhattacharjee, Proceedings IAEA FEC, TH/P1-35, Kyoto 2016

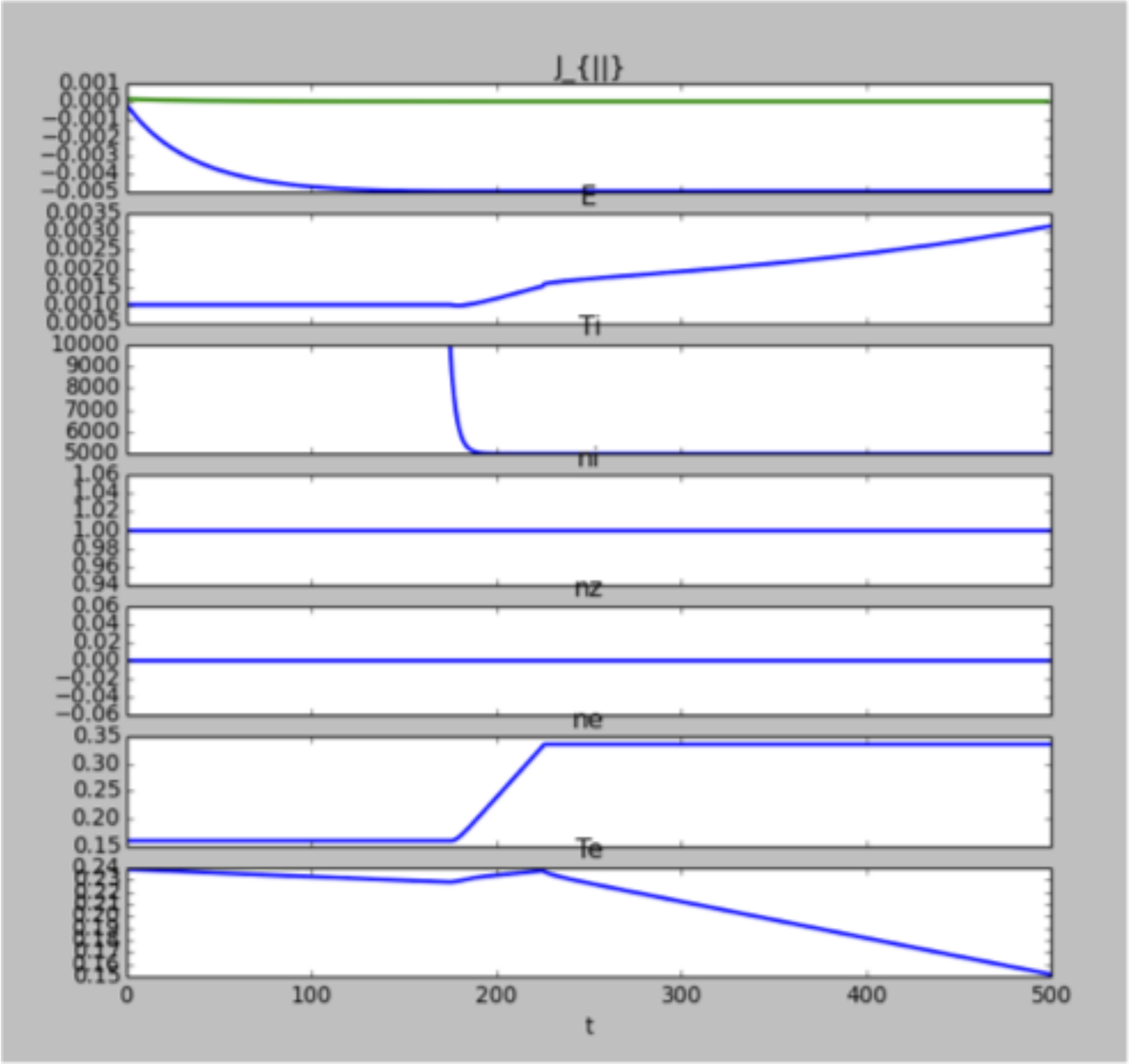
Exploring impurity deposition scenarios

Argon (Z=18) put in as independent species from background ions

(ignore nz plot .

Ion impurity density made consistent with deposited electron source of cold electrons (fully stripped impurity).

Imposed J constraint at onset of impurity deposition.



For fast deposition expect largest population of REs, but also depend on initial E and current

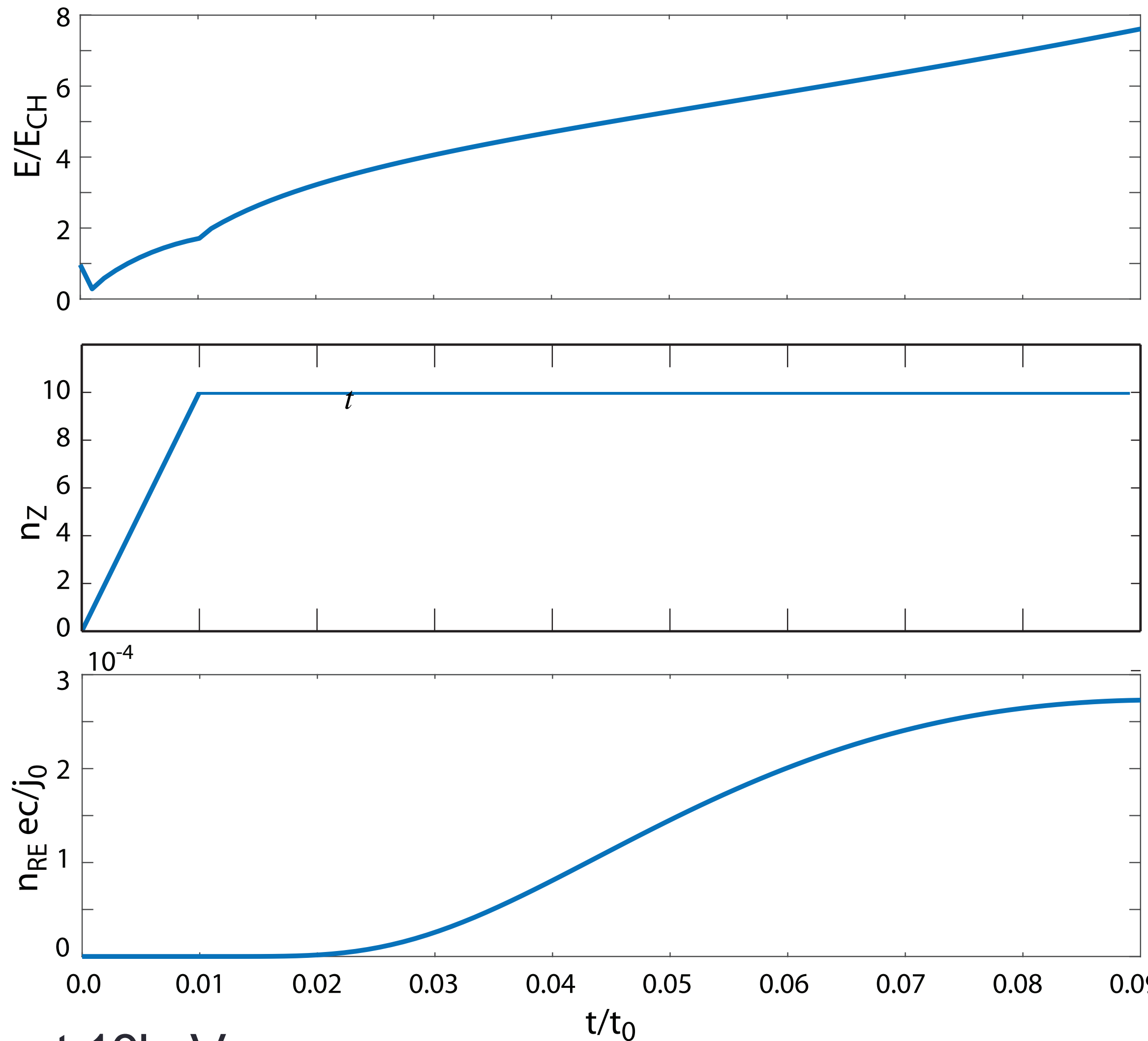
Initia $E=E_{ch}$

Fast deposition ~3us

Initial transient before collisional diffusion

Electric field eventually rises as electrons cool

Few RE's driven, subcritical



$t_0 = 1/\nu_{ee} \approx 0.25ms$ at 10keV

Higher initial electric field increases initial J and eventual E/Ech: More RE's -> avalanche

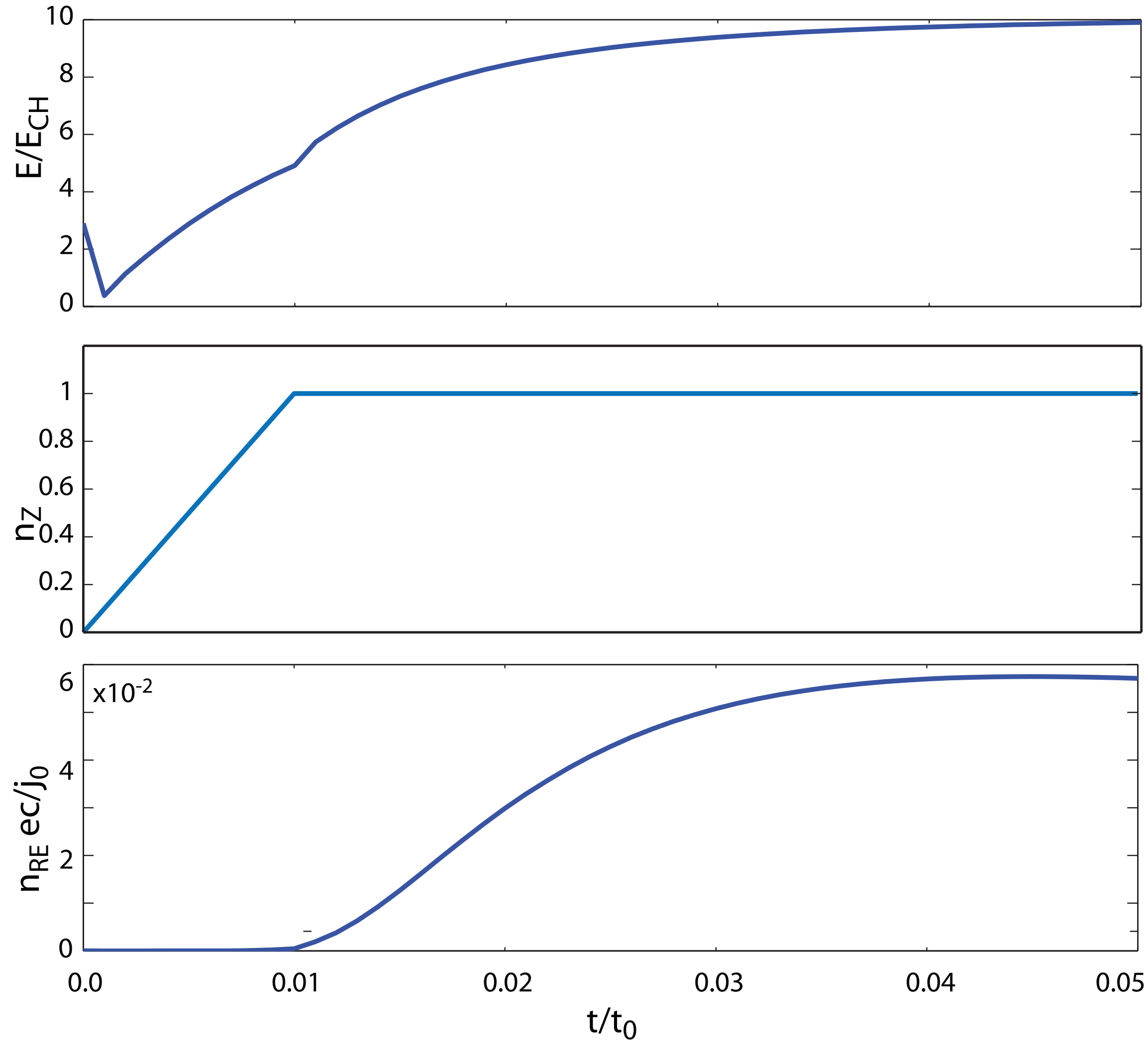
Initia E=3Ech

Fast deposition ~3us

Initial transient before collisional diffusion

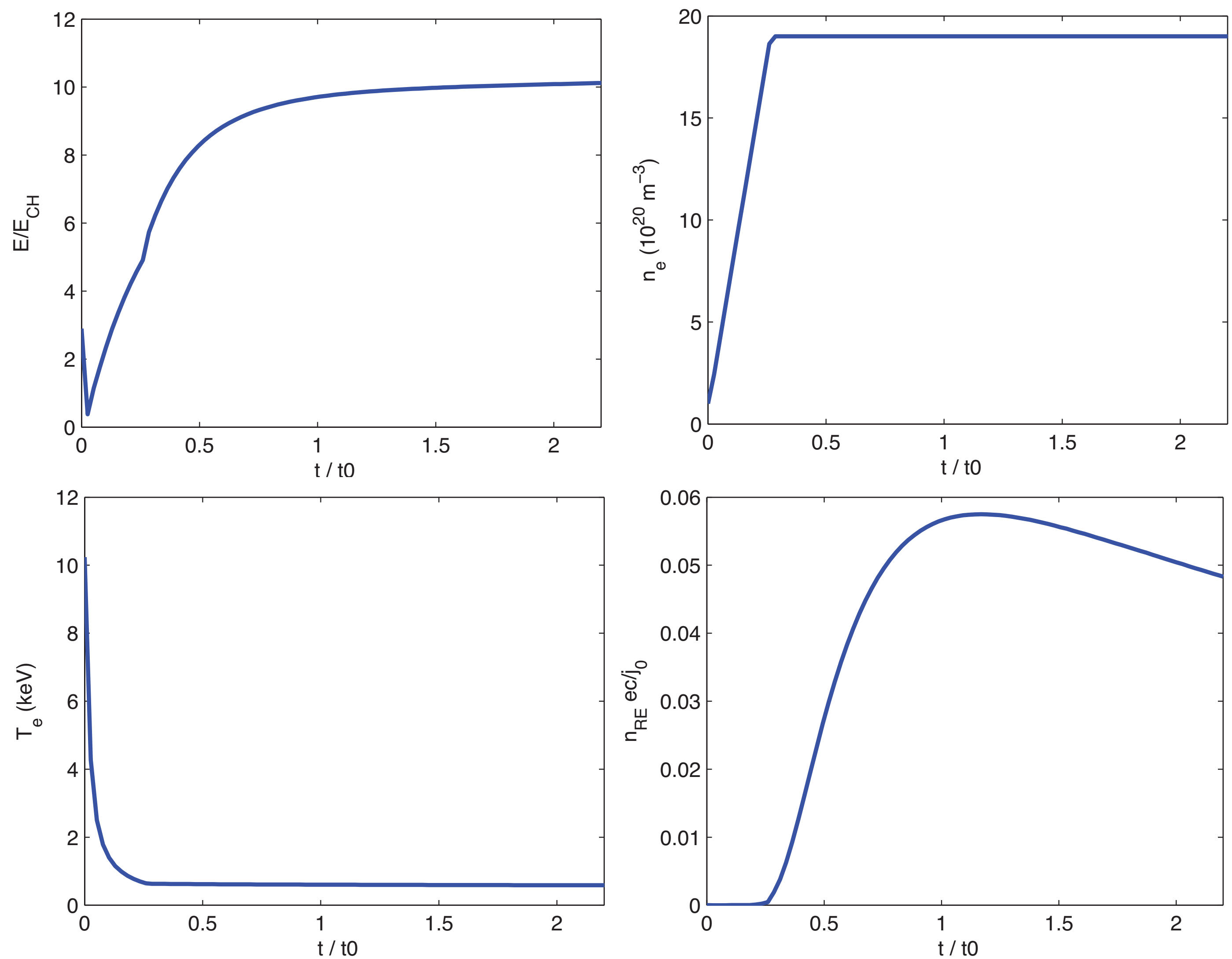
Electric field rises faster as electrons cool

Enough electrons for avalanche, but weak



Longer deposition timescales show little change in seed: difficult to get fast transfer

Slower deposition approaching 100us shows little change in the avalanche seed.



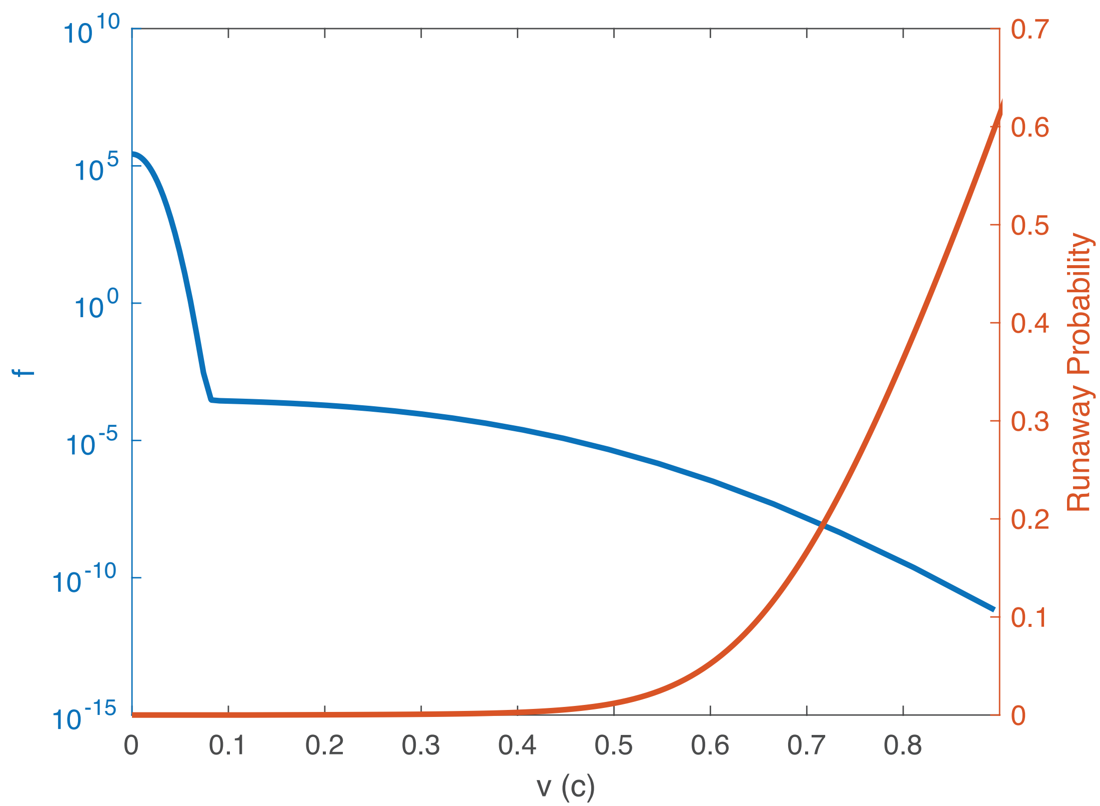
Cold electrons cool fast population but electric field does not increase fast enough for REs

Why?

Deposition of impurity electrons at low temperature

High energy tail quickly collisionally cools before electric field increases large enough.

Collsional cooling of low temperature electrons due to radiation is necessary component. To get high electric field.



Concluding Remarks

- Motivation: Include fully nonlinear collisonality in kinetic modeling of runaway seed generation
- New code with FEM treatment of electron f with background ions including impurities
  - Nonlinear e-e collisions, linear e-i collisions, f dependent and independent sourcing
  - Backward Euler implicit timestepping
  - Initial electric field drives current to steady state – triggers impurity injection
  - Electric field found from current density constraint after impurities injected
- Interface to runaway probability calculation used to calculate seed population
  - Fast and robust
  - Avalanche growth, and slowing down time also available
  - Because of large electric field, synchrotron unimportant
- Initial results indicate the fast transfer difficult to achieve under realistic timescales
  - Initial electric field and current must be large to get Avalanche regime
  - Ion cooling may be necessary to model radiative cooling of low temperature electron and increase Ohmic electric field in early response

Important issues with RE dynamics remain to be addressed to be predictive about the DMS

- RE interaction with High-Z impurities
- Seed distribution (hot tail) effects in thermal quench events
- Spatial / configuration space dependence
- Kinetic instability
  - Whistler wave scattering
  - Bump on tail
- Magnetic fluctuations
- MHD instability coupling
- RE termination (magnetic energy conversion), RE-wall interaction

Open questions remain as to the best technical methods for coupling the runaway electron, impurity transport, and MHD simulation codes, managing and visualizing the large volumes of data, and determining its uncertainty, both in experiment and in simulation.

Recent Workshops and Exascale Review Highlighted Need for a Center

- Need for progress in runaway electron physics was clearly made in both the 2015 Integrated Simulations and Transients workshops, and simulation of disruptions has been a recent focus of the 2016 Exascale Requirements Review, all three involving partnerships between FES and ASCR.
  - C. Greenfield and R. Nazikian. Report on scientific challenges and research opportunities in transient research, 2015.
  - P. Bonoli and L.C. McInnes. Report of the workshop on integrated simulations for magnetic fusion energy sciences, 2015.

- An eventual reliable design tool for runaway mitigation requires almost the full functionality of the whole device modeling (WDM) of a tokamak, the proposed physics studies will naturally lead to a runaway physics module for WDM.

Collaboration needed between theory, simulation and algorithmic development

Center assembles experts in runaway electron physics, tokamak disruptions, magnetohydrodynamics (MHD) simulations, and advanced computing.

Theory: Analytic plasma theory, or employing light weight code for analysis

Simulation: Production code development/improvement and large simulations

Algorithms: Designing, implementing and testing innovative algorithms and performance enhancements for runaways

Collaborative center needed to address best technical methods for coupling the runaway electron, impurity transport, and MHD simulation codes, managing and visualizing the large volumes of data, and determining its uncertainty, both in experiment and in simulation.

Center needed to address physics questions

SCREAM is a FES/ASCR Collaboration between 12 Principal Investigators at 9 Institutions

Team Includes 9 Institutions with 12 PI's
8 Associated with FES
4 Associated with ASCR
$4.9M / 2yrs

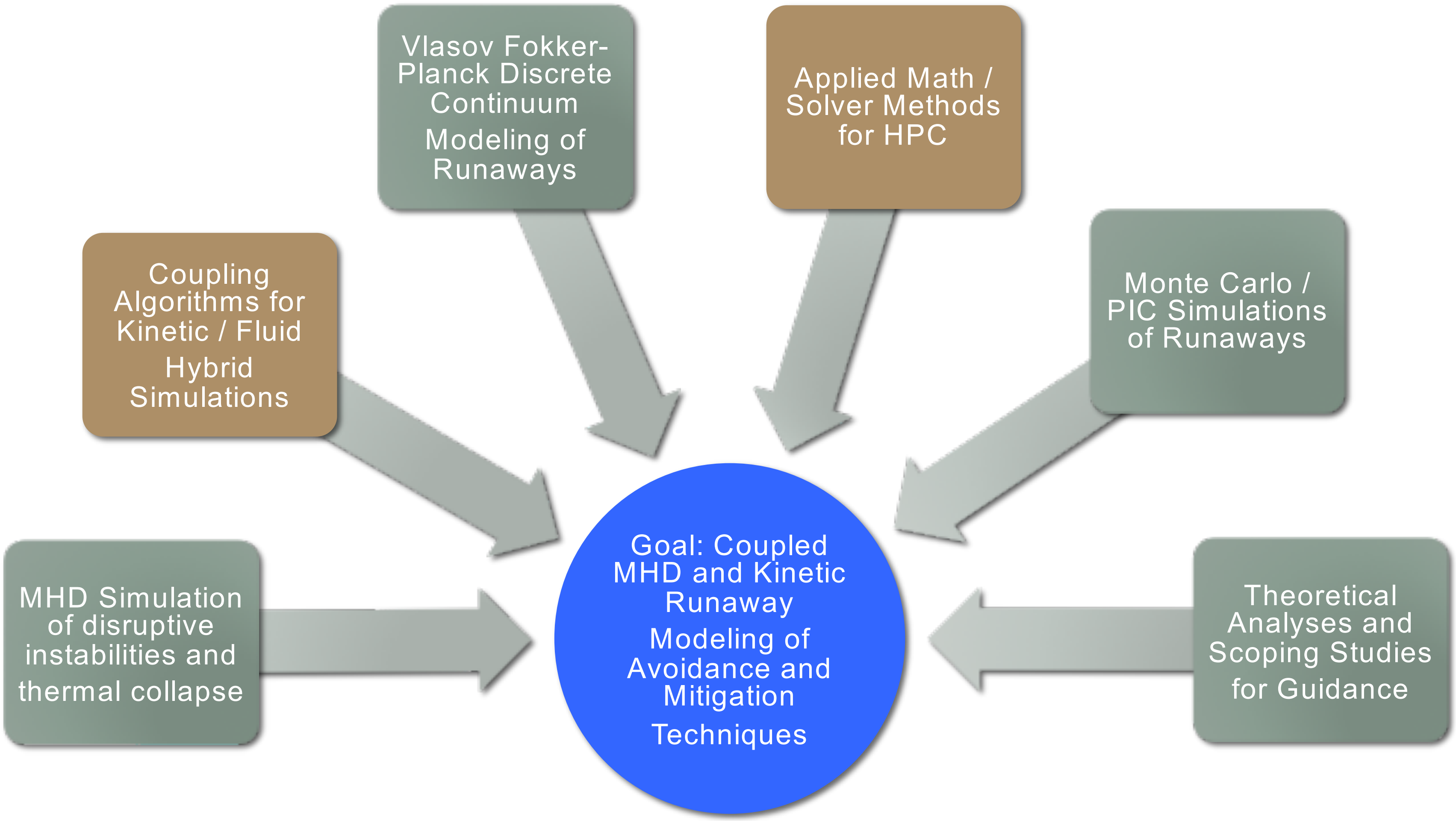
Mission: combine theoretical models with advanced simulation and analysis facilitated by direct participation of ASCR SciDAC institutes to focus on the runaway risk for ITER and tokamaks in general.

Principal Investigators:

- FES:
  - Dylan Brennan (Princeton)
    - Lead PI - Universities
  - Xianzhu Tang (LANL)
    - Lead PI - Labs
  - Amitava Bhattacharjee (PPPL)
  - Allen Boozer (Columbia)
  - Boris Breizman (UT, Austin)
  - Diego Del-Castillo-Negrete (ORNL)
  - Valerie Izzo (UCSD)
  - Lang Lao (GA)
- ASCR
  - Mark Adams (LBNL)
  - Luis Chacon (LANL)
  - Irene Gamba (UT, Austin)
  - Guannan Zhang (ORNL)

Collaborations underway between several groups

Effort combines FES theory and modeling of runaway physics with numerical simulation facilitated by ASCR applied mathematics



Collaboration aims to advance understanding and quantitative prediction of runaway physics

Overall Goals
- Establish the physical basis for generation and evolution
- Explore scenarios for avoidance
- Investigate the leading candidates for mitigation

Initial Scope
- Theoretical investigation of runaway physics and mitigation
- Scoping studies of runaway electron generation with reduced modeling
- Relativistic Vlasov-Fokker-Planck simulations of runaway electrons using phase-space discretization
- Modeling of Disruptions and Runaway Electrons with NIMROD
- Simulating of Runaway Seed Current Generation with XGC1
- Monte Carlo simulations of runaway electrons including full-orbit, spatial/configuration space with KORC

Computational Methods
- Relativistic Fokker-Planck solvers using grid discretization in phase space
- Self-consistent particle-in-cell
- Particle-based Monte-Carlo
- MHD-particle hybrid

Cross-check between these different methods will provide an additional means for verification and will further bolster the fidelity of physics predictions.

Concluding Remarks about SCREAM

SCREAM will serve as a US collaborative effort on simulation and theory of runaway electrons and directly contribute to ITPA.

Collaborations between groups forming : Multiple groups now in quantitative consensus on several radiative effects on runaway dynamics. Much progress in fundamental theory over past few years. Formulations of advanced algorithms for RE modeling coupled to background plasma advance currently under development.

Advanced Computing Needed : Theory community addressing physics and validation against experiment, but open questions remain, some best addressed through development in advanced computing.

SCREAM will help community address questions accessible through combining theory developments with advanced computing, such as interaction with magnetic fluctuations, to be quantitatively predictive on avoidance and mitigation.