BHM_example

February 17, 2022

NRT Lectures - Statistical Modeling

1 Bayesian Hierarchical Model

1.0.1 Rat Tumor Example

[1]:
```
import math
import random
import numpy as np
import pandas as pd
# import graphviz
# from pymc3 import model_to_graphviz
import pymc3 as pm
from pymc3 import Model, sample, Beta, Binomial, Exponential, Uniform, summary,
    ↪plot_posterior, model_to_graphviz, Deterministic
import matplotlib.pyplot as plt
# import os
# os.environ["PATH"] += os.pathsep + 'C:\Program
    ↪Files\Python37\Lib\site-packages\graphviz\dot.py'
```

[2]:
```
d =  pd.read_table("rattumor.txt", sep = " ")
d = d.iloc[:,:2]
d
```

[2]:
```
     y   N
0    0  20
1    0  20
2    0  20
3    0  20
4    0  20
..  ..  ..
66  16  52
67  15  46
68  15  47
69   9  24
70   4  14

[71 rows x 2 columns]
```

[3]:
```python
d.describe()
```

[3]:

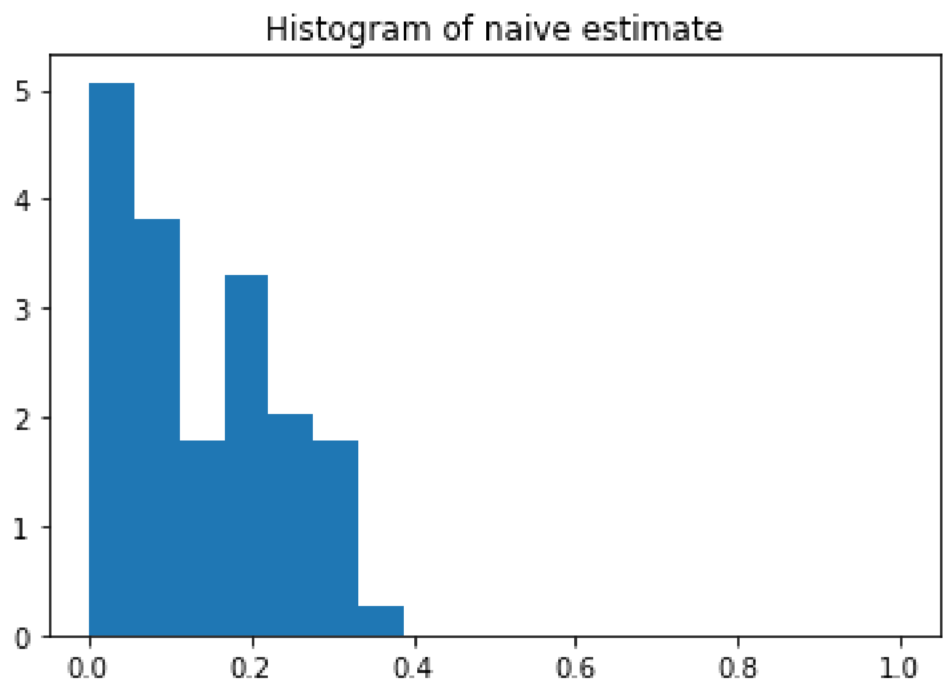
	y	N
count	71.000000	71.000000
mean	3.760563	24.492958
std	3.811504	10.973830
min	0.000000	10.000000
25%	1.000000	19.000000
50%	3.000000	20.000000
75%	5.000000	22.500000
max	16.000000	52.000000

A naive estimate of θ_j is $\hat{\theta}_j = y_j/n_j$

Histogram of $\hat{\theta}_j$

[4]:
```python
plt.hist(d.y/d.N, range = (0,1), bins = 18, density=True)
plt.title("Histogram of naive estimate")
plt.show
```

[4]: <function matplotlib.pyplot.show(close=None, block=None)>



[5]:
```python
with Model() as model1:

    # Priors
```

```python
    alpha = Exponential('alpha', 0.001)
    beta = Exponential('beta', 0.001)

    theta = Beta('theta', alpha=alpha, beta=beta, shape=71)

    # Data likelihood
    y_like = Binomial('y_like', n=d.N, p=theta, observed=d.y)
```

[6]:

```python
random.seed(100)
with model1:
    trace1 = sample(100, tune=100)
```

```
/tmp/ipykernel_9952/1044724547.py:3: FutureWarning: In v4.0, pm.sample will
return an `arviz.InferenceData` object instead of a `MultiTrace` by default. You
can pass return_inferencedata=True or return_inferencedata=False to be safe and
silence this warning.
  trace1 = sample(100, tune=100)
Only 100 samples in chain.
Auto-assigning NUTS sampler…
Initializing NUTS using jitter+adapt_diag…
Multiprocess sampling (2 chains in 2 jobs)
NUTS: [theta, beta, alpha]
```

```
<IPython.core.display.HTML object>
```

```
<IPython.core.display.HTML object>
```

```
Sampling 2 chains for 100 tune and 100 draw iterations (200 + 200 draws total)
took 3 seconds.
The acceptance probability does not match the target. It is 0.9763936443680605,
but should be close to 0.8. Try to increase the number of tuning steps.
The acceptance probability does not match the target. It is 0.920349624272922,
but should be close to 0.8. Try to increase the number of tuning steps.
The rhat statistic is larger than 1.05 for some parameters. This indicates
slight problems during sampling.
The number of effective samples is smaller than 10% for some parameters.
```

[7]:

```python
summary(trace1)
```

```
/home/ooblack/miniconda3/envs/viz/lib/python3.10/site-
packages/arviz/data/io_pymc3.py:96: FutureWarning: Using `from_pymc3` without
the model will be deprecated in a future release. Not using the model will
return less accurate and less useful results. Make sure you use the model
argument or call from_pymc3 within a model context.
  warnings.warn(
```

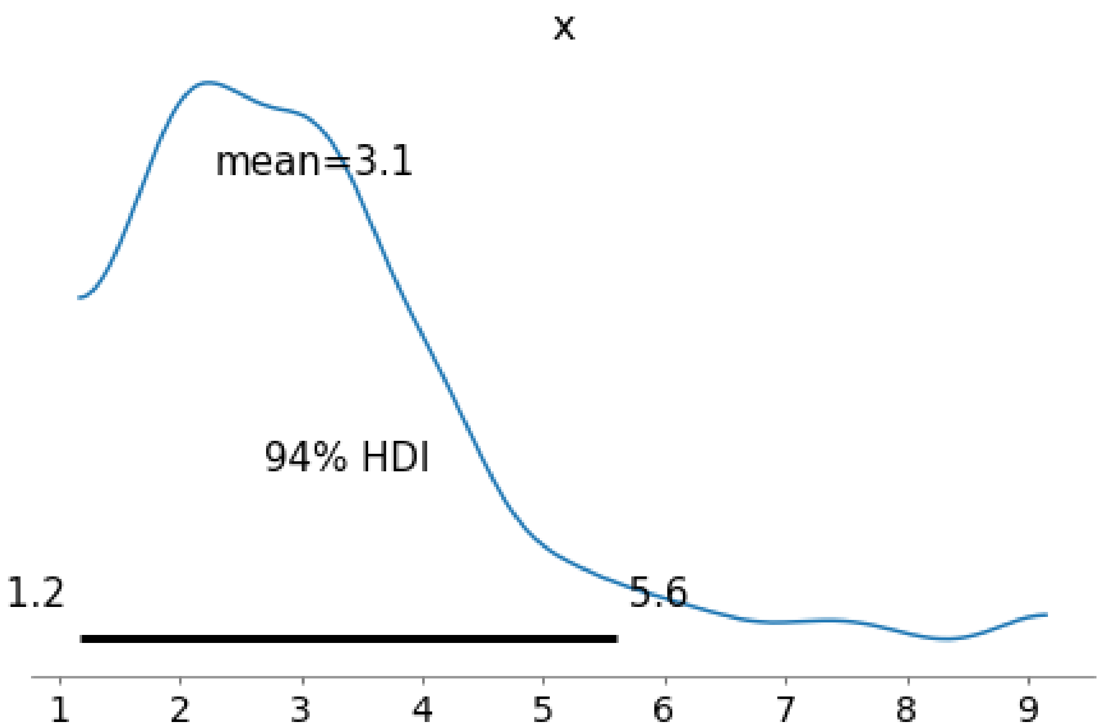
[7]:

```
              mean     sd  hdi_3%  hdi_97%  mcse_mean  mcse_sd  ess_bulk  \
alpha        3.119  1.399   1.164    5.611      0.368    0.272      17.0
beta        18.891  7.971   6.632   34.316      1.771    1.271      22.0
theta[0]     0.075  0.045   0.001    0.163      0.005    0.004      58.0
```

```
theta[1]     0.071  0.038   0.003    0.135      0.005    0.003      56.0
theta[2]     0.069  0.045   0.004    0.150      0.004    0.003     119.0
…              …      …       …        …          …        …
theta[66]    0.259  0.056   0.160    0.357      0.003    0.002     349.0
theta[67]    0.274  0.058   0.170    0.384      0.003    0.003     419.0
theta[68]    0.265  0.057   0.151    0.371      0.003    0.003     304.0
theta[69]    0.267  0.079   0.151    0.444      0.005    0.004     288.0
theta[70]    0.201  0.066   0.094    0.324      0.003    0.002     285.0

           ess_tail  r_hat
alpha          21.0   1.09
beta           33.0   1.07
theta[0]       63.0   1.03
theta[1]       73.0   1.03
theta[2]       87.0   1.01
…                …     …
theta[66]     121.0   1.01
theta[67]     157.0   1.05
theta[68]     141.0   1.02
theta[69]     118.0   1.08
theta[70]     198.0   1.00

[73 rows x 9 columns]
```

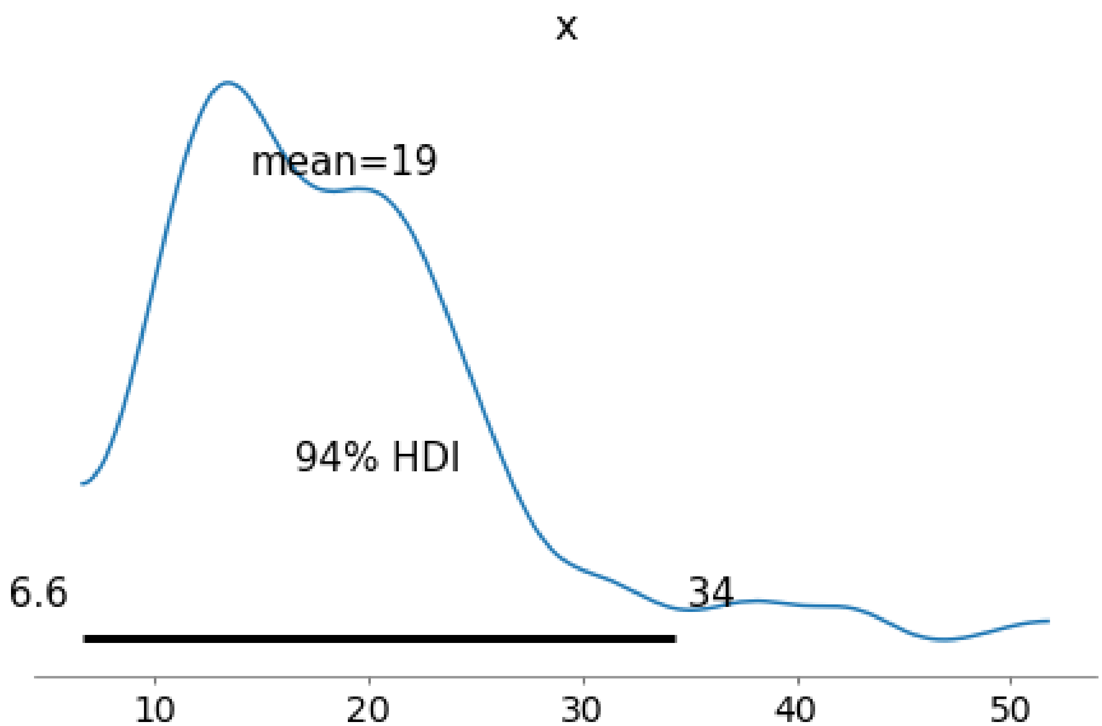
[8]:
```
plot_posterior(trace1['alpha'])
```

[8]:
```
<AxesSubplot:title={'center':'x'}>
```

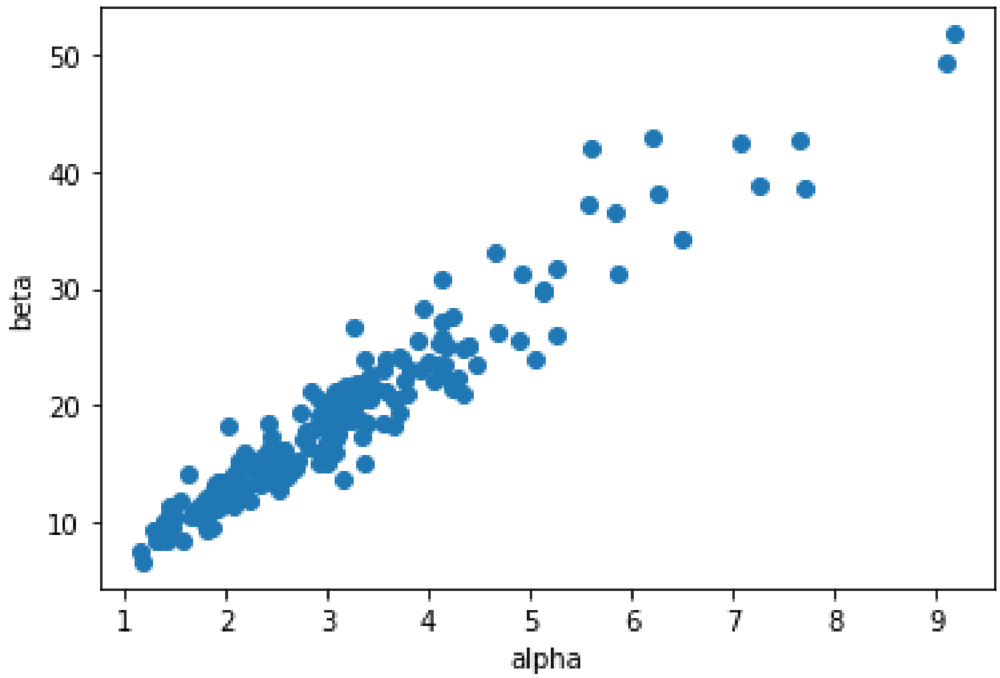



```
[9]: plot_posterior(trace1['beta'])
```

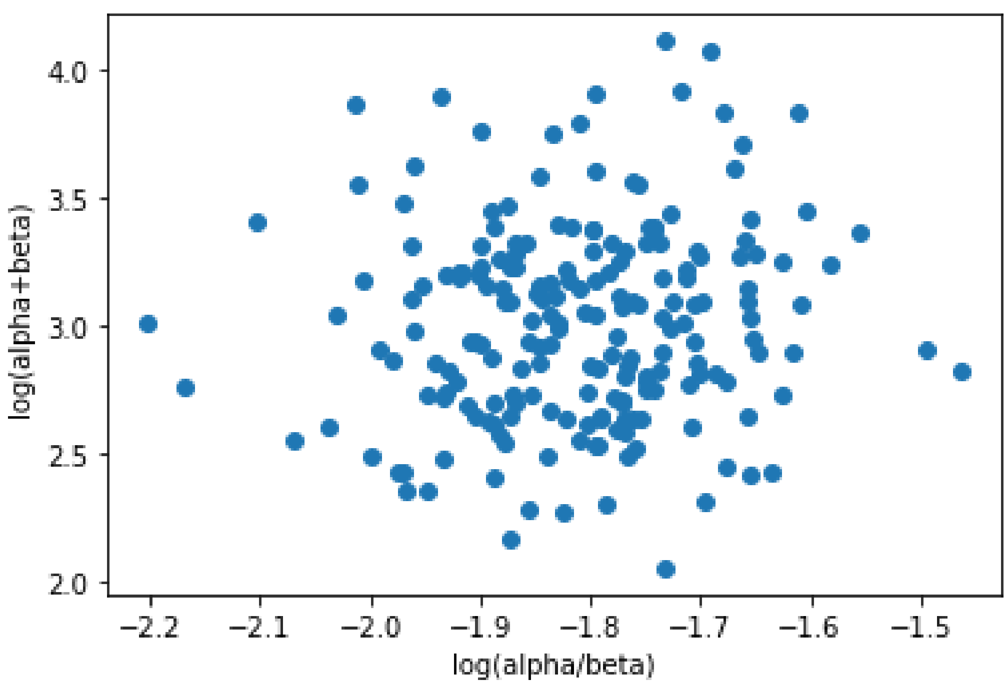
```
[9]: <AxesSubplot:title={'center':'x'}>
```




```
[10]: alpha = trace1.get_values(varname='alpha')
beta = trace1.get_values(varname='beta')
plt.scatter(alpha, beta)
plt.xlabel('alpha')
plt.ylabel('beta')
plt.show()
```




```
[11]: plt.scatter(np.log(alpha/beta), np.log(alpha+beta))
      plt.xlabel('log(alpha/beta)')
      plt.ylabel('log(alpha+beta)')
      plt.show()
```



1.0.2 Try another prior

[12]:
```
with Model() as model2:

    phi1 = Uniform('phi1', lower=0, upper=1)
    phi2 = Uniform('phi2', lower=0, upper=1000)

    alpha = Deterministic('alpha', phi1 / (phi2**2))
    beta = Deterministic('beta', (1-phi1) / phi2**2)

    theta = Beta('theta', alpha=alpha, beta=beta, shape=71)

    # Data likelihood
    y_like = Binomial('y_like', n=d.N, p=theta, observed=d.y)
```

[13]:
```
random.seed(100)
with model2:
    trace2 = sample(100, tune=100)
```

```
/tmp/ipykernel_9952/2992495953.py:3: FutureWarning: In v4.0, pm.sample will
return an `arviz.InferenceData` object instead of a `MultiTrace` by default. You
can pass return_inferencedata=True or return_inferencedata=False to be safe and
silence this warning.
  trace2 = sample(100, tune=100)
Only 100 samples in chain.
Auto-assigning NUTS sampler…
Initializing NUTS using jitter+adapt_diag…
Multiprocess sampling (2 chains in 2 jobs)
NUTS: [theta, phi2, phi1]
```

```
<IPython.core.display.HTML object>
```

```
<IPython.core.display.HTML object>
```

```
Sampling 2 chains for 100 tune and 100 draw iterations (200 + 200 draws total)
took 5 seconds.
The acceptance probability does not match the target. It is 0.9346941703130048,
but should be close to 0.8. Try to increase the number of tuning steps.
The acceptance probability does not match the target. It is 0.9318999435860623,
but should be close to 0.8. Try to increase the number of tuning steps.
The rhat statistic is larger than 1.05 for some parameters. This indicates
slight problems during sampling.
The number of effective samples is smaller than 25% for some parameters.
```

[14]:
```
summary(trace2)
```

```
/home/ooblack/miniconda3/envs/viz/lib/python3.10/site-
```

```
packages/arviz/data/io_pymc3.py:96: FutureWarning: Using `from_pymc3` without
the model will be deprecated in a future release. Not using the model will
return less accurate and less useful results. Make sure you use the model
argument or call from_pymc3 within a model context.
  warnings.warn(
```

[14]:

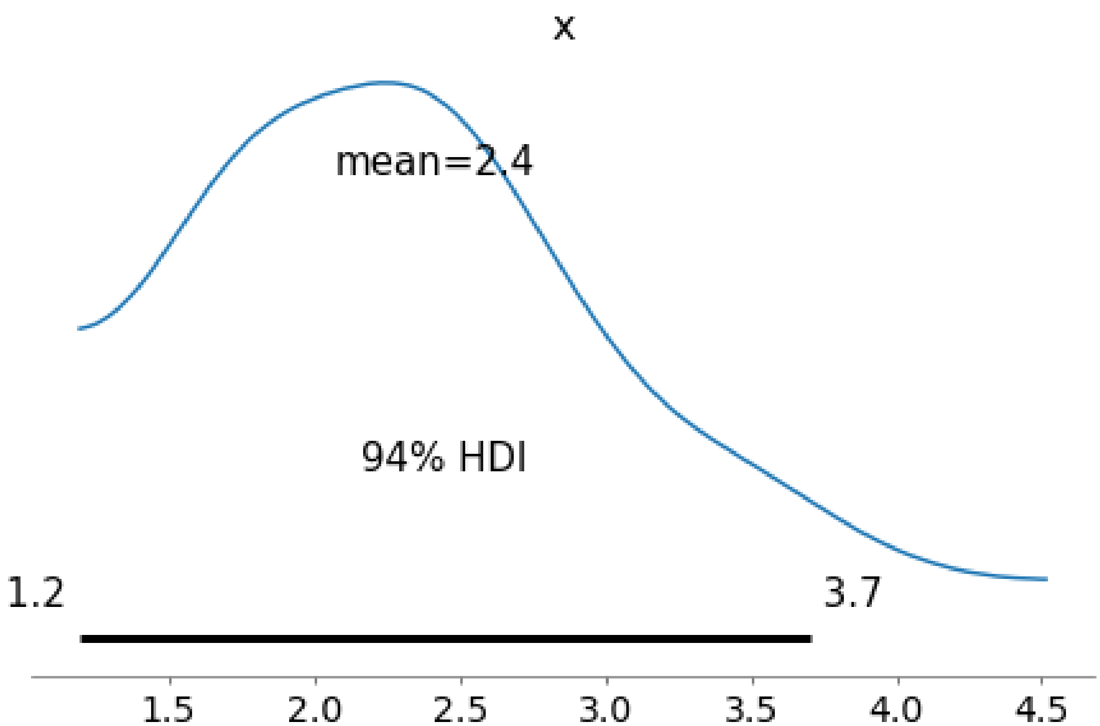
```
               mean     sd  hdi_3%  hdi_97%  mcse_mean  mcse_sd  ess_bulk  \
phi1          0.144  0.013   0.119    0.167      0.001    0.001     182.0
phi2          0.252  0.038   0.184    0.320      0.006    0.004      43.0
alpha         2.414  0.730   1.190    3.703      0.120    0.086      35.0
beta         14.438  4.277   6.428   21.429      0.605    0.430      50.0
theta[0]      0.061  0.041   0.006    0.148      0.003    0.002     159.0
…               …      …       …        …          …        …
theta[66]     0.266  0.059   0.166    0.363      0.003    0.002     442.0
theta[67]     0.278  0.055   0.171    0.381      0.003    0.002     460.0
theta[68]     0.275  0.061   0.175    0.381      0.003    0.002     384.0
theta[69]     0.282  0.065   0.179    0.411      0.003    0.002     371.0
theta[70]     0.215  0.074   0.081    0.345      0.004    0.003     296.0

           ess_tail  r_hat
phi1          119.0   1.00
phi2          217.0   1.04
alpha         149.0   1.05
beta          217.0   1.04
theta[0]      132.0   1.00
…               …      …
theta[66]     138.0   1.00
theta[67]     162.0   1.02
theta[68]     230.0   0.99
theta[69]     124.0   1.00
theta[70]     181.0   1.00

[75 rows x 9 columns]
```

[15]:

```python
plot_posterior(trace2['alpha'])
```

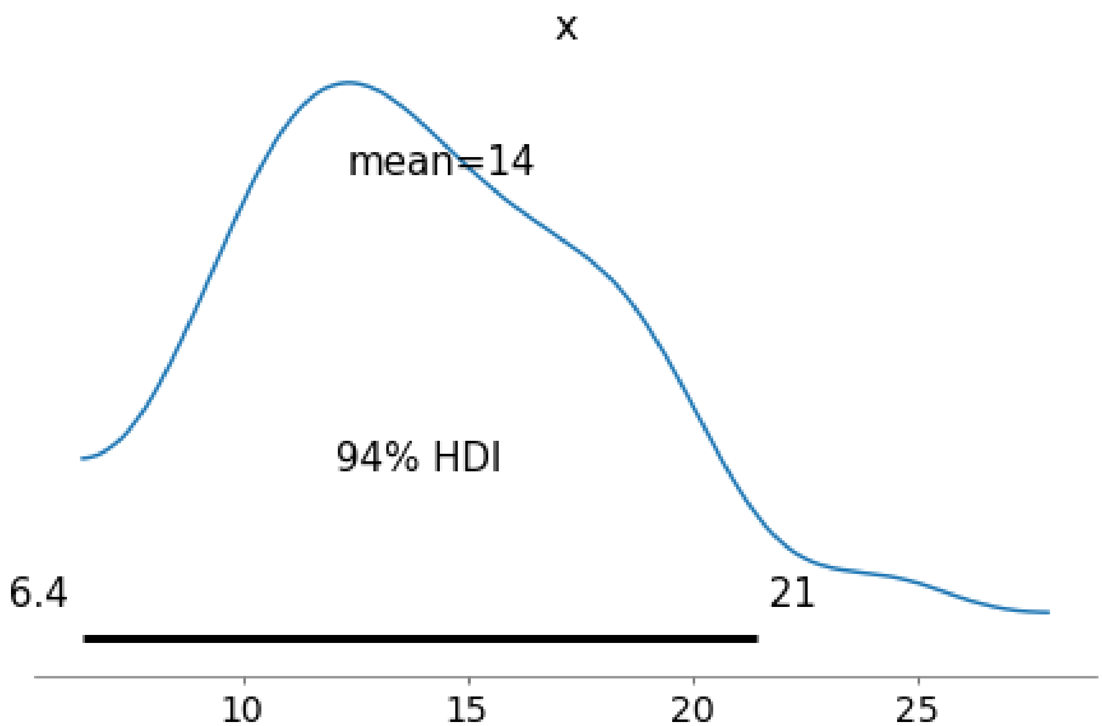
[15]:

```
<AxesSubplot:title={'center':'x'}>
```

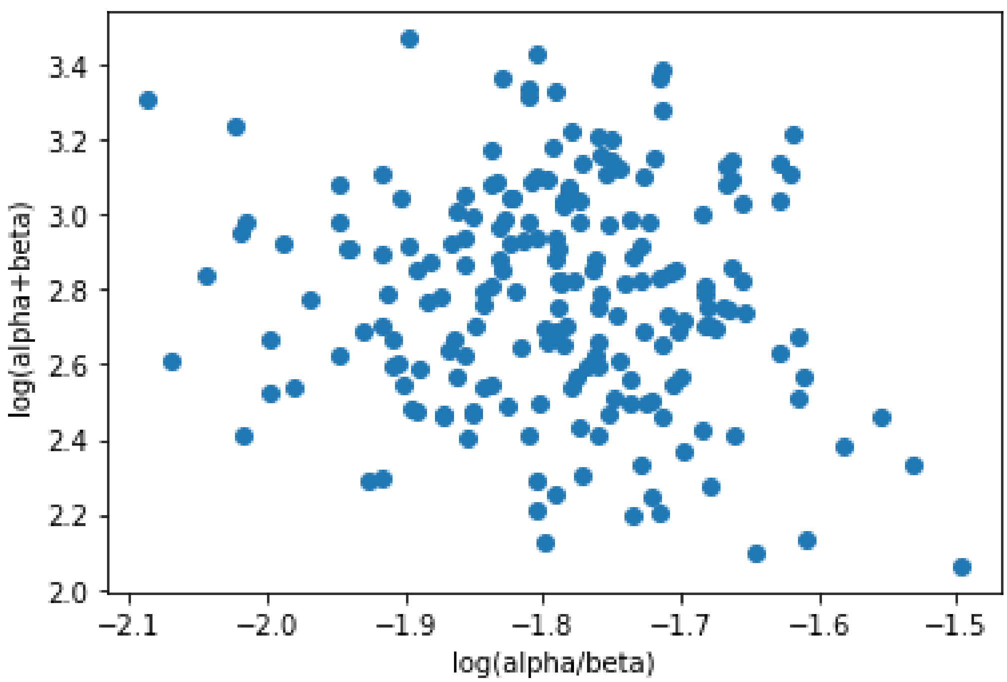


[16]:
```
plot_posterior(trace2['beta'])
```

[16]:
```
<AxesSubplot:title={'center':'x'}>
```



[17]:
```
alpha2 = trace2.get_values(varname='alpha')
beta2 = trace2.get_values(varname='beta')
plt.scatter(np.log(alpha2/beta2), np.log(alpha2+beta2))
plt.xlabel('log(alpha/beta)')
plt.ylabel('log(alpha+beta)')
plt.show()
```


3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
log(alpha+beta)
−2.1
−2.0
−1.9
−1.8
−1.7
−1.6
−1.5
log(alpha/beta)