

Assignment 1 – Written Answers - Yariv Edry and Shai Nosatzki

1.

$$Ax = b$$

$$\underbrace{\begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix}}_{\text{Transformer}} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} u' \\ v' \\ w' \end{pmatrix}$$

$$\textcircled{1} \quad \frac{h_{00}u + h_{01}v + h_{02}}{h_{20}u + h_{21}v + h_{22}} = \frac{u'}{1}$$

$$h_{00}u + h_{01}v + h_{02} - u'(h_{20}u + h_{21}v + h_{22}) = 0$$

$$\textcircled{2} \quad \frac{h_{10}u + h_{11}v + h_{12}}{h_{20}u + h_{21}v + h_{22}} = \frac{v'}{1}$$

$$h_{10}u + h_{11}v + h_{12} - v'(h_{20}u + h_{21}v + h_{22}) = 0$$

$$\Downarrow$$

$$\underbrace{\begin{pmatrix} u & v & 1 & 0 & 0 & 0 & -u'u & -u'v & -u'w \\ 0 & 0 & 0 & u & v & 1 & -v'u & -v'v & -v'w \end{pmatrix}}_{2 \times 9} \underbrace{\begin{pmatrix} h_{00} \\ h_{01} \\ \vdots \\ h_{22} \end{pmatrix}}_{9 \times 1} = \underbrace{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{2 \times 1}$$

$$Ah = 0$$

add constraint:

$$\|h\| = 1, \min \|Ah\|$$

$$\Downarrow$$

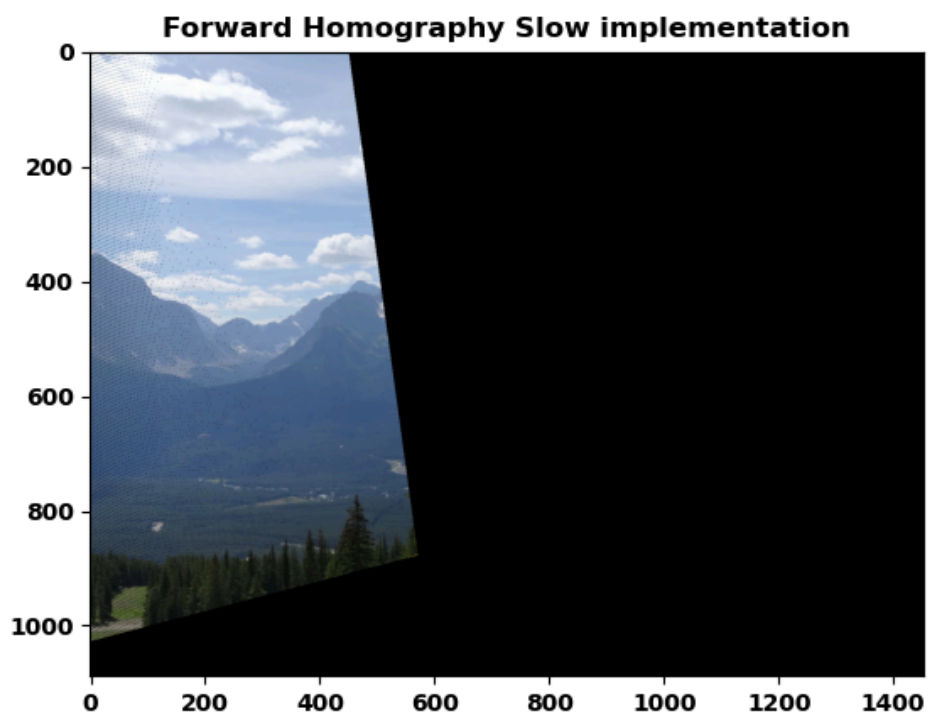
eigenvectors of $A^T A$ (equals to the smallest eigen value)

2. In code.

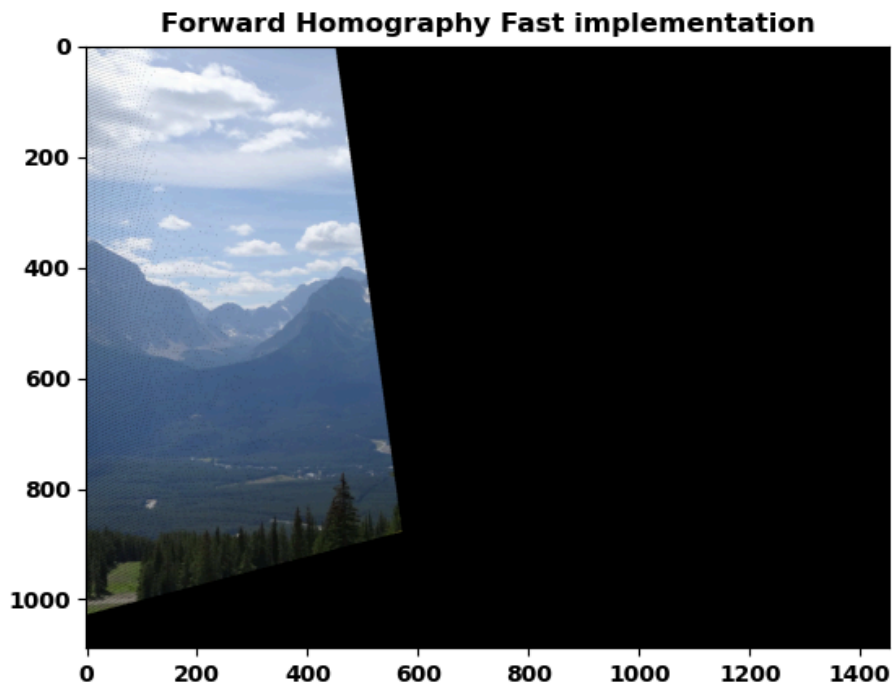
3.

	0	1	2
0	1.4345721408666705	0.21044323172569873	-1277.1867901768135
1	0.013426515371006021	1.3470612260043795	-16.045587306327313
2	0.0003792792982078017	5.565231459622085e-05	1.0

4. Slow Forward Homography Image:

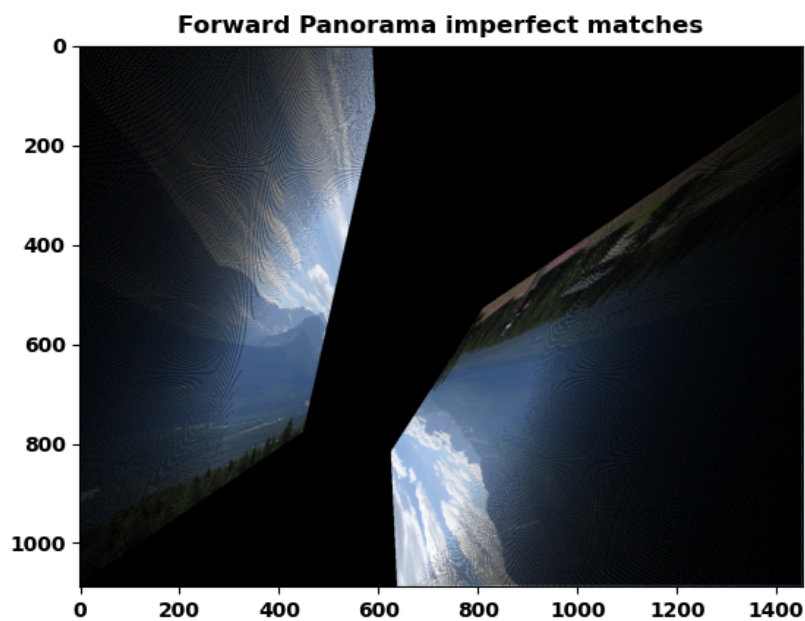


5. Fast Forward Homography Image:



6. In a forward mapping, there can be unfilled pixels (blank\black spots in the image) or overridden pixels (several pixels from the source point to the same pixel in the destination). In addition, the forward mapping can map a source pixel to a non integer pixel in the destination (can result in artifacts due to an estimation of color\ intensity based on the non integer location).

7.



As the points are mismatched, the resulting orientation of the output is obviously wrong.

8. In code.

9. In code.

10.

$$k = \frac{\log(1-p)}{\log(1-w^k)}$$

For $p=0.9$ (90% confidence):

$$k = \left\lceil \frac{\log(1-0.9)}{\log(1-(0.8)^4)} \right\rceil = 5$$

For $p=0.99$ (99% confidence):

$$k = \left\lceil \frac{\log(1-0.99)}{\log(1-(0.8)^4)} \right\rceil = 9$$

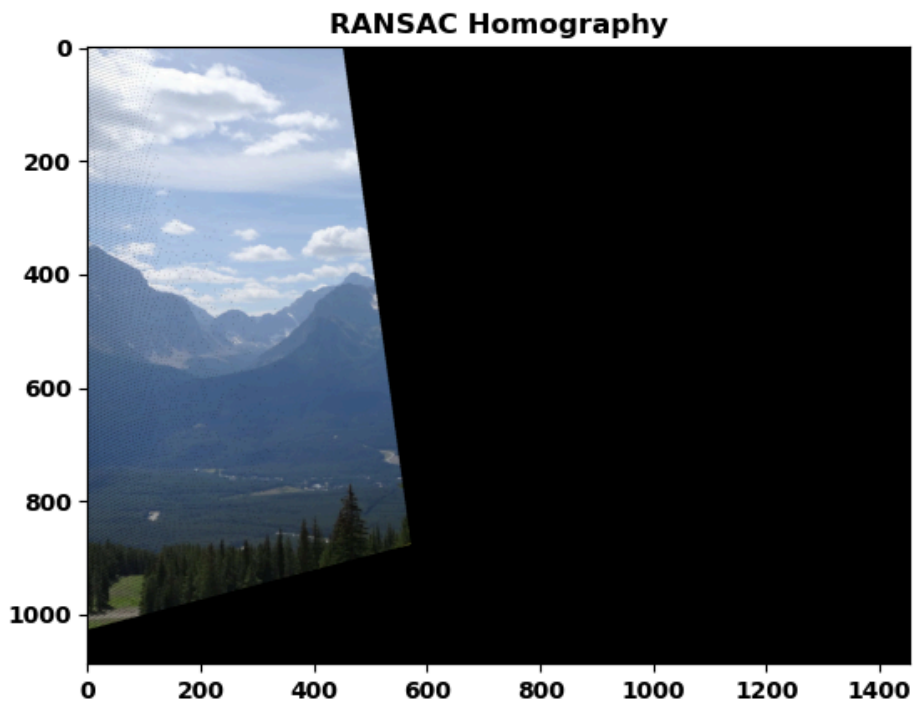
To cover all options:

$$\binom{30}{4} = \frac{30!}{4!(30-4)!} = 27,405 \text{ iterations}$$

11. In code.

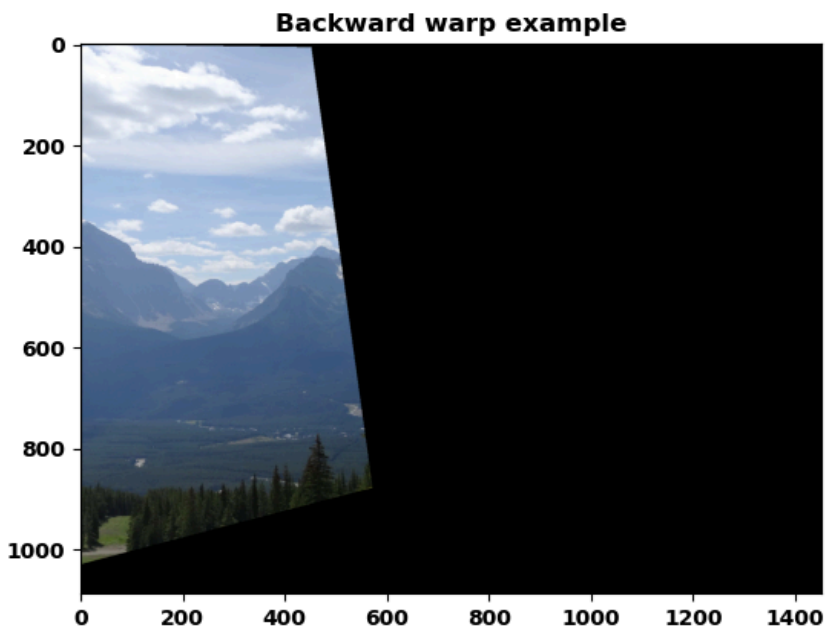
12.

```
RANSAC Homography 0.0020 sec
[[ 1.12313781e-03  1.64757662e-04 -9.99919585e-01]
 [ 1.05117245e-05  1.05462483e-03 -1.25622165e-02]
 [ 2.96940746e-07  4.35706350e-08  7.82907867e-04]]
```



The result looks similar to the forward homography used in Section 5. The orientation is fine (not like in section 7).

13.

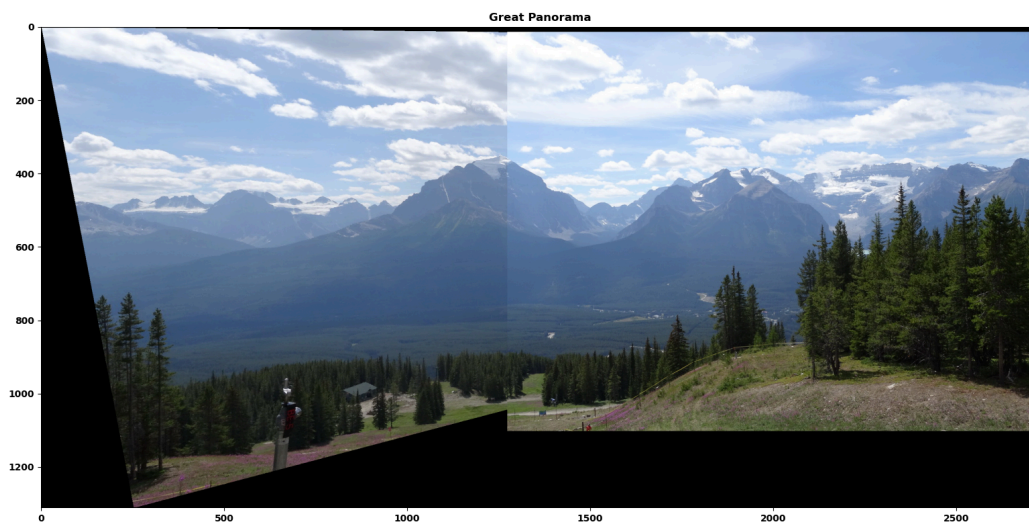


The image looks pretty the same but with less artifacts.

14. In code.

15. In code.

16.



17.

