Geotechnical Analysis

CENG6202: Advanced Computational Methods in Geotechnical Engineering

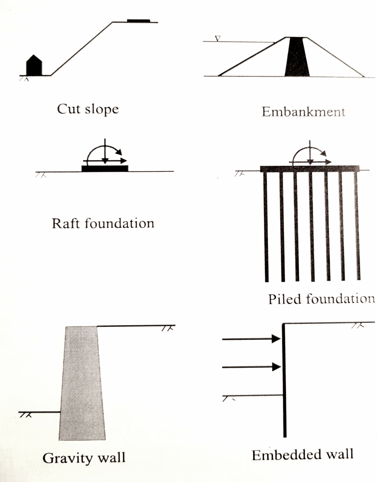
Yared Worku, PhD

Fall Semester 2019

Contents

Contents

Construction of almost all civil engineering structures in one way or another involve the ground.



Examples of geotechnical structures, after Potts and Zdravkovic 1999.

Contents

Analysis and Design Requirements

- Loads imposed on the soil and structural members must be assessed under working and ultimate loading conditions.
- The relative movements of the soil and structure must be evaluated.
- Simplified analyses and empirical approaches are commonly used in geotechnical design.
- The use of computational/numerical analysis and design methods in geotechnical engineering practice is still limited.

Analysis and Design Requirements

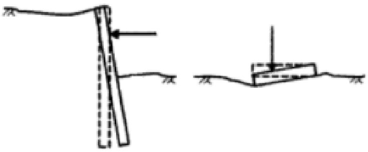
- Loads imposed on the soil and structural members must be assessed under working and ultimate loading conditions.
- The relative movements of the soil and structure must be evaluated.
- Simplified analyses and empirical approaches are commonly used in geotechnical design.
- The use of computational/numerical analysis and design methods in geotechnical engineering practice is still limited.

- Loads imposed on the soil and structural members must be assessed under working and ultimate loading conditions.
- The relative movements of the soil and structure must be evaluated.
- Simplified analyses and empirical approaches are commonly used in geotechnical design.
- The use of computational/numerical analysis and design methods in geotechnical engineering practice is still limited.

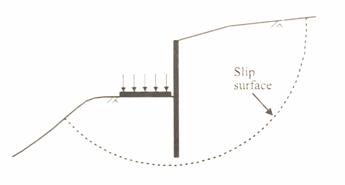
Analysis and Design Requirements

- Loads imposed on the soil and structural members must be assessed under working and ultimate loading conditions.
- The relative movements of the soil and structure must be evaluated.
- Simplified analyses and empirical approaches are commonly used in geotechnical design.
- The use of computational/numerical analysis and design methods in geotechnical engineering practice is still limited.

Stability is one of the main requirements in the design of geotechnical structures. Two aspects: *local stability* and *global stability*



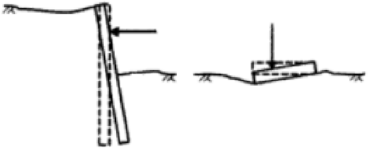
Local stability



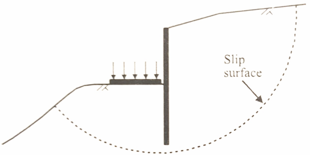
Global stability

Assert that there is no danger of translational and rotational failure! Geotechnical Engineer's Responsibility

Stability is one of the main requirements in the design of geotechnical structures. Two aspects: *local stability* and *global stability*



Local stability



Global stability

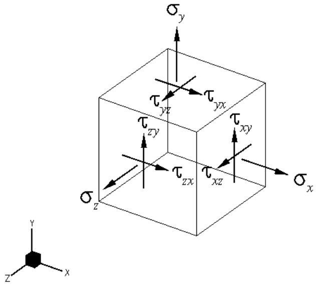
Assert that there is no danger of translational and rotational failure! Geotechnical Engineer's Responsibility

Contents

A theoretical solution is required to satisfy

1. Equilibrium requirements
2. Compatibility requirements
3. Constitutive material behavior
4. Boundary conditions

Equilibrium equations set the external applied loads to be equal to the the sum of all the internal forces.



$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + b = 0$$

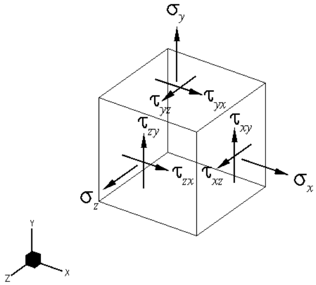
$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0$$

Equilibrium equations for a complete system:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

Equilibrium equations set the external applied loads to be equal to the the sum of all the internal forces.

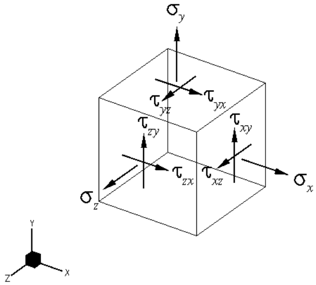


$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0$$
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + b = 0$$
$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0$$

Equilibrium equations for a complete system:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$
$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

Equilibrium equations set the external applied loads to be equal to the the sum of all the internal forces.



$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0$$
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + b = 0$$
$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0$$

Equilibrium equations for a complete system:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$
$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

Physical compatibility: Deformations should not cause unrealistic overlapping of materials or generation of holes.

Mathematical compatibility: Physical compatibility expressed in terms of equations.

$$
\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} & \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\
\varepsilon_y &= \frac{\partial v}{\partial y} & \gamma_{yz} &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \\
\varepsilon_z &= \frac{\partial w}{\partial z} & \gamma_{xz} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial yz}
\end{aligned}
$$

Physical compatibility: Deformations should not cause unrealistic overlapping of materials or generation of holes.

Mathematical compatibility: Physical compatibility expressed in terms of equations.

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \qquad \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial yz}$$

Physical compatibility: Deformations should not cause unrealistic overlapping of materials or generation of holes.

Mathematical compatibility: Physical compatibility expressed in terms of equations.

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$
$$\varepsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$
$$\varepsilon_z = \frac{\partial w}{\partial z} \qquad \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial yz}$$

Physical compatibility: Deformations should not cause unrealistic overlapping of materials or generation of holes.

Mathematical compatibility: Physical compatibility expressed in terms of equations.

$$
\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} & \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\
\varepsilon_y &= \frac{\partial v}{\partial y} & \gamma_{yz} &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \\
\varepsilon_z &= \frac{\partial w}{\partial z} & \gamma_{xz} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial yz}
\end{aligned}
$$

Constitutive equations are stress-strain relationships that govern the behavior of the material.

In incremental form, a constitutive relationship is written as

$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\sigma_z \\ \Delta\tau_{xy} \\ \Delta\tau_{xz} \\ \Delta\tau_{zy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{45} & D_{46} \\ D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\varepsilon_z \\ \Delta\gamma_{xy} \\ \Delta\gamma_{xz} \\ \Delta\gamma_{zy} \end{Bmatrix}$$

Compact form:

$$\Delta\boldsymbol{\sigma} = \boldsymbol{D}\Delta\varepsilon$$

In terms of effective stresses:

$$\Delta\boldsymbol{\sigma}' = \boldsymbol{D}'\Delta\varepsilon$$

Constitutive equations are stress-strain relationships that govern the behavior of the material.

In incremental form, a constitutive relationship is written as

$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\sigma_z \\ \Delta\tau_{xy} \\ \Delta\tau_{xz} \\ \Delta\tau_{zy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{45} & D_{46} \\ D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\varepsilon_z \\ \Delta\gamma_{xy} \\ \Delta\gamma_{xz} \\ \Delta\gamma_{zy} \end{Bmatrix}$$

Compact form:

$$\Delta\boldsymbol{\sigma} = \boldsymbol{D}\Delta\boldsymbol{\varepsilon}$$

In terms of effective stresses:

$$\Delta\boldsymbol{\sigma}' = \boldsymbol{D}'\Delta\boldsymbol{\varepsilon}$$

Constitutive equations are stress-strain relationships that govern the behavior of the material.

In incremental form, a constitutive relationship is written as

$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\sigma_z \\ \Delta\tau_{xy} \\ \Delta\tau_{xz} \\ \Delta\tau_{zy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{45} & D_{46} \\ D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\varepsilon_z \\ \Delta\gamma_{xy} \\ \Delta\gamma_{xz} \\ \Delta\gamma_{zy} \end{Bmatrix}$$

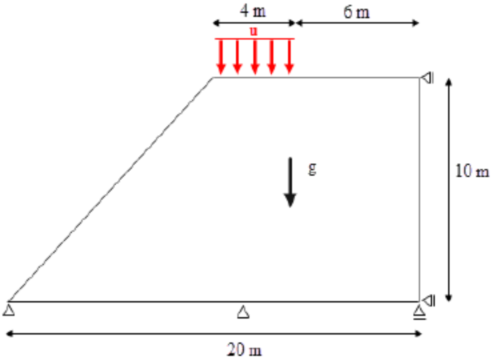
Compact form:

$$\Delta\boldsymbol{\sigma} = \boldsymbol{D}\Delta\boldsymbol{\varepsilon}$$

In terms of effective stresses:

$$\Delta\boldsymbol{\sigma}' = \boldsymbol{D}'\Delta\boldsymbol{\varepsilon}$$

A theoretical solution should also satisfy the different boundary conditions of the problem.



Different types of boundary conditions will be discussed in a later chapter.

Contents

Idealized Computational Domains

Real geotechnical problems are usually simplified by idealizing the geometry and/or boundary conditions of the problem.

Full three-dimensional simulations are usually demanding and most problems are solved by reducing the problem to equivalent two-dimensional models.

The most common geometric idealizations are:

1. Plane strain
2. Axi-symmetry

Idealized Computational Domains

Real geotechnical problems are usually simplified by idealizing the geometry and/or boundary conditions of the problem.

Full three-dimensional simulations are usually demanding and most problems are solved by reducing the problem to equivalent two-dimensional models.

The most common geometric idealizations are:

1. Plane strain
2. Axi-symmetry

Idealized Computational Domains

Real geotechnical problems are usually simplified by idealizing the geometry and/or boundary conditions of the problem.

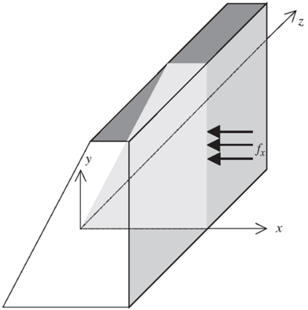
Full three-dimensional simulations are usually demanding and most problems are solved by reducing the problem to equivalent two-dimensional models.

The most common geometric idealizations are:

1. Plane strain
2. Axi-symmetry

Idealized Computational Domains
↪Plane Strain

Applied when the z-dimension of the problem is large and it can be assumed that the state existing in the x-y plane holds for all planes parallel to it.

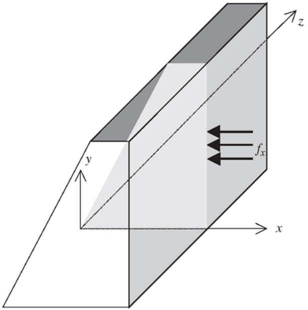


The displacement in the z-dimension is zero ($w = 0$) and the displacements u and v are independent of the z coordinate. Thus,

$$\varepsilon_z = \frac{\partial w}{\partial z} = 0$$
$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 0$$
$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial yz} = 0$$

Equilibrium, compatibility and constitutive equations are reduced with respect to the stresses and strains relevant for the problem.

Applied when the z-dimension of the problem is large and it can be assumed that the state existing in the x-y plane holds for all planes parallel to it.

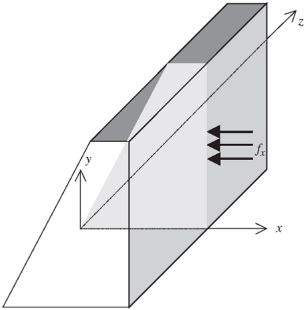


The displacement in the z-dimension is zero ($w = 0$) and the displacements u and v are independent of the z coordinate. Thus,

$$\varepsilon_z = \frac{\partial w}{\partial z} = 0$$
$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 0$$
$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial yz} = 0$$

Equilibrium, compatibility and constitutive equations are reduced with respect to the stresses and strains relevant for the problem.

Applied when the z-dimension of the problem is large and it can be assumed that the state existing in the x-y plane holds for all planes parallel to it.

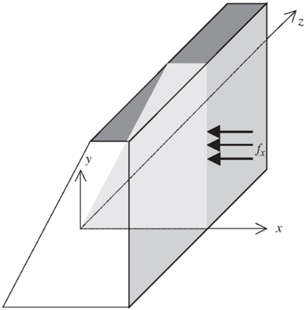


The displacement in the z-dimension is zero ($w = 0$) and the displacements u and v are independent of the z coordinate. Thus,

$$\varepsilon_z = \frac{\partial w}{\partial z} = 0$$
$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 0$$
$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial yz} = 0$$

Equilibrium, compatibility and constitutive equations are reduced with respect to the stresses and strains relevant for the problem.

Applied when the z-dimension of the problem is large and it can be assumed that the state existing in the x-y plane holds for all planes parallel to it.



The displacement in the z-dimension is zero ($w = 0$) and the displacements u and v are independent of the z coordinate. Thus,

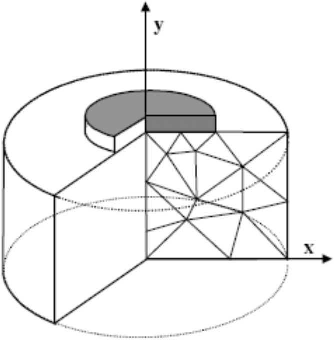
$$\varepsilon_z = \frac{\partial w}{\partial z} = 0$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 0$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial yz} = 0$$

Equilibrium, compatibility and constitutive equations are reduced with respect to the stresses and strains relevant for the problem.

Some problems have a domain with a rotational axis of symmetry where the model could be idealized using axi-symmetric models

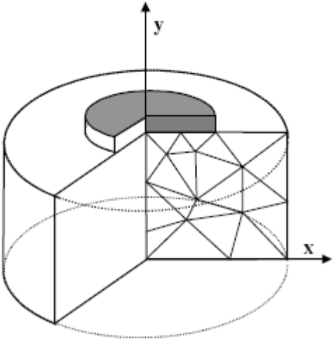


Cylindrical coordinates are used for axi-symmetric models

$$\varepsilon_r = \frac{\partial u}{\partial r}$$
$$\varepsilon_z = \frac{\partial v}{\partial z}$$
$$\varepsilon_\theta = \frac{u}{r}$$
$$\gamma_{rz} = \frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}$$
$$\gamma_{r\theta} = \gamma_{z\theta} = 0$$

Equilibrium, compatibility and constitutive equations are written in radial coordinates considering the relevant stresses and strains.

Some problems have a domain with a rotational axis of symmetry where the model could be idealized using axi-symmetric models

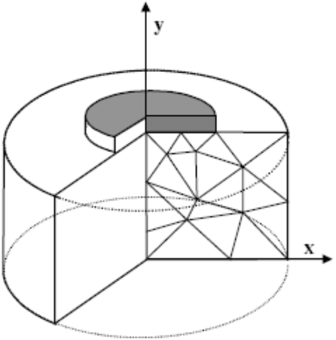


Cylindrical coordinates are used for axi-symmetric models

$$\varepsilon_r = \frac{\partial u}{\partial r}$$
$$\varepsilon_z = \frac{\partial v}{\partial z}$$
$$\varepsilon_\theta = \frac{u}{r}$$
$$\gamma_{rz} = \frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}$$
$$\gamma_{r\theta} = \gamma_{z\theta} = 0$$

Equilibrium, compatibility and constitutive equations are written in radial coordinates considering the relevant stresses and strains.

Some problems have a domain with a rotational axis of symmetry where the model could be idealized using axi-symmetric models

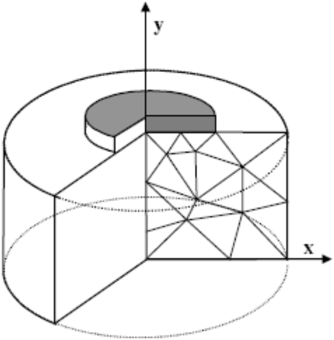


Cylindrical coordinates are used for axi-symmetric models

$$\varepsilon_r = \frac{\partial u}{\partial r}$$
$$\varepsilon_z = \frac{\partial v}{\partial z}$$
$$\varepsilon_\theta = \frac{u}{r}$$
$$\gamma_{rz} = \frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}$$
$$\gamma_{r\theta} = \gamma_{z\theta} = 0$$

Equilibrium, compatibility and constitutive equations are written in radial coordinates considering the relevant stresses and strains.

Some problems have a domain with a rotational axis of symmetry where the model could be idealized using axi-symmetric models



Cylindrical coordinates are used for axi-symmetric models

$$\varepsilon_r = \frac{\partial u}{\partial r}$$
$$\varepsilon_z = \frac{\partial v}{\partial z}$$
$$\varepsilon_\theta = \frac{u}{r}$$
$$\gamma_{rz} = \frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}$$
$$\gamma_{r\theta} = \gamma_{z\theta} = 0$$

Equilibrium, compatibility and constitutive equations are written in radial coordinates considering the relevant stresses and strains.

Contents

Analysis Methods

The different analysis methods in geomechanics may be divided into three groups:

1. Analytical Methods
2. Simple Methods
3. Numerical Methods

Applicable when the governing equation(s) of the problem can be solved such that an analytical expression for the solution can be found.

Example: One-dimensional consolidation

Governing equation:

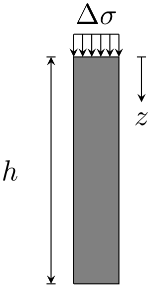
$$\frac{\partial u}{\partial t} - c_\mathrm{v}\frac{\partial^2 u}{\partial z^2} = 0$$

Solution for excess pore pressure:

$$\frac{u(t,z)}{u_0} = \frac{4}{\pi}\sum_{i=1}^{\infty}\frac{(-1)^{i-1}}{2i-1}\exp\left[-(2i-1)^2\frac{\pi^2 t_\mathrm{s}}{4}\right]\cos\left[(2i-1)\frac{\pi z}{2h}\right]$$

Applicable when the governing equation(s) of the problem can be solved such that an analytical expression for the solution can be found.

Example: One-dimensional consolidation



Governing equation:

$$\frac{\partial u}{\partial t} - c_{\mathrm{v}}\frac{\partial^2 u}{\partial z^2} = 0$$

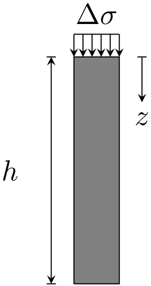
Solution for excess pore pressure:

$$\frac{u(t,z)}{u_0} = \frac{4}{\pi}\sum_{i=1}^{\infty}\frac{(-1)^{i-1}}{2i-1}\exp\left[-(2i-1)^2\frac{\pi^2 t_{\mathrm{s}}}{4}\right]\cos\left[(2i-1)\frac{\pi z}{2h}\right]$$

Analysis Methods
↪Analytical Methods

Applicable when the governing equation(s) of the problem can be solved such that an analytical expression for the solution can be found.

Example: One-dimensional consolidation



Governing equation:

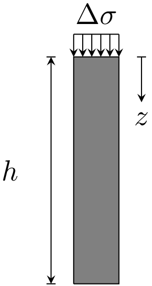
$$\frac{\partial u}{\partial t} - c_{\mathrm{v}} \frac{\partial^2 u}{\partial z^2} = 0$$

Solution for excess pore pressure:

$$\frac{u(t,z)}{u_0} = \frac{4}{\pi} \sum_{i=1}^{\infty} \frac{(-1)^{i-1}}{2i-1} \exp\left[-(2i-1)^2 \frac{\pi^2 t_{\mathrm{s}}}{4}\right] \cos\left[(2i-1)\frac{\pi z}{2h}\right]$$

Applicable when the governing equation(s) of the problem can be solved such that an analytical expression for the solution can be found.

Example: One-dimensional consolidation



Governing equation:

$$\frac{\partial u}{\partial t} - c_{\mathrm{v}} \frac{\partial^2 u}{\partial z^2} = 0$$

Solution for excess pore pressure:

$$\frac{u(t,z)}{u_0} = \frac{4}{\pi} \sum_{i=1}^{\infty} \frac{(-1)^{i-1}}{2i-1} \exp\left[-(2i-1)^2 \frac{\pi^2 t_{\mathrm{s}}}{4}\right] \cos\left[(2i-1)\frac{\pi z}{2h}\right]$$

Three types of simple methods in geotechnical engineering:

1. Limit Equilibrium Method
2. Stress Field Method
3. Limit Analysis Method

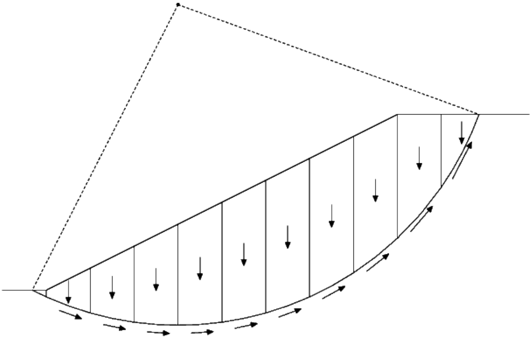
Three types of simple methods in geotechnical engineering:

1. Limit Equilibrium Method
2. Stress Field Method
3. Limit Analysis Method

Limit Equilibrium:
A failure criterion is established and an arbitrary failure mechanism is assumed.

Equilibrium equations are written for the global system.

Example: Slope stability analysis

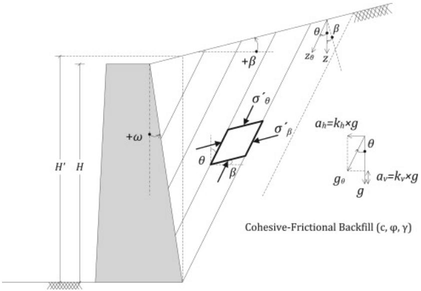


Stress Field:

A failure criterion is established and the soil is assumed to be at the point of failure everywhere.

Equilibrium equations are combined with the failure criterion to solve the problem.

Example: Retaining structures/Earth pressure calculations



Limit Analysis:

This method is based on the following assumptions:

- The soil behavior can be represented by perfect plasticity.
- The plastic strains can be derived from the yield function.
- Changes in the geometry of the soil mass that occur at failure are insignificant.

Failure condition may be established based on two approaches:

1. *Upper bound theorem*: The failure load determined is unsafe or close to the true failure load.
2. *Lower bound theorem*: The estimated load is safe or close to the true failure load.

Limit Analysis:

This method is based on the following assumptions:

- The soil behavior can be represented by perfect plasticity.
- The plastic strains can be derived from the yield function.
- Changes in the geometry of the soil mass that occur at failure are insignificant.

Failure condition may be established based on two approaches:

1. *Upper bound theorem*: The failure load determined is unsafe or close to the true failure load.
2. *Lower bound theorem*: The estimated load is safe or close to the true failure load.

Limit Analysis:

This method is based on the following assumptions:

- The soil behavior can be represented by perfect plasticity.
- The plastic strains can be derived from the yield function.
- Changes in the geometry of the soil mass that occur at failure are insignificant.

Failure condition may be established based on two approaches:

1. *Upper bound theorem*: The failure load determined is unsafe or close to the true failure load.
2. *Lower bound theorem*: The estimated load is safe or close to the true failure load.

Limit Analysis:

This method is based on the following assumptions:

- The soil behavior can be represented by perfect plasticity.
- The plastic strains can be derived from the yield function.
- Changes in the geometry of the soil mass that occur at failure are insignificant.

Failure condition may be established based on two approaches:

1. *Upper bound theorem*: The failure load determined is unsafe or close to the true failure load.
2. *Lower bound theorem*: The estimated load is safe or close to the true failure load.

Limit Analysis:
This method is based on the following assumptions:

- The soil behavior can be represented by perfect plasticity.
- The plastic strains can be derived from the yield function.
- Changes in the geometry of the soil mass that occur at failure are insignificant.

Failure condition may be established based on two approaches:

1. *Upper bound theorem*: The failure load determined is unsafe or close to the true failure load.
2. *Lower bound theorem*: The estimated load is safe or close to the true failure load.

Limit Analysis:
This method is based on the following assumptions:

- The soil behavior can be represented by perfect plasticity.
- The plastic strains can be derived from the yield function.
- Changes in the geometry of the soil mass that occur at failure are insignificant.

Failure condition may be established based on two approaches:

1. *Upper bound theorem*: The failure load determined is unsafe or close to the true failure load.
2. *Lower bound theorem*: The estimated load is safe or close to the true failure load.

Limit Analysis:

This method is based on the following assumptions:

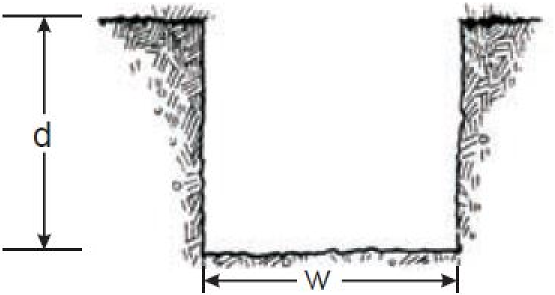
- The soil behavior can be represented by perfect plasticity.
- The plastic strains can be derived from the yield function.
- Changes in the geometry of the soil mass that occur at failure are insignificant.

Failure condition may be established based on two approaches:

1. *Upper bound theorem*: The failure load determined is unsafe or close to the true failure load.
2. *Lower bound theorem*: The estimated load is safe or close to the true failure load.

Limit Analysis:

Example: Critical height of a vertical cut in cohesive soil



Upper and lower bound solutions can be calculated.

In a broad sense, the numerical methods used in geotechnical engineering may be classified into two groups:

1. Beam-Spring Methods
2. Full Computational Methods

In a broad sense, the numerical methods used in geotechnical engineering may be classified into two groups:

1. Beam-Spring Methods
2. Full Computational Methods

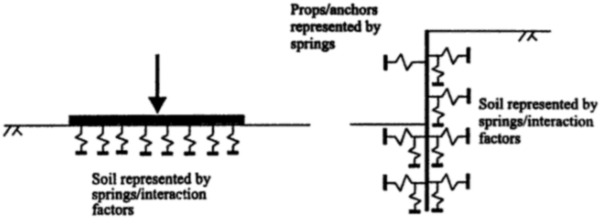
Beam-Spring Methods:

It is especially used for soil-structure interaction problems.

The soil behavior is approximated using two approaches:

1. A set of unconnected vertical and horizontal springs
2. Linear elastic interaction factors

Example: Raft foundations, Laterally loaded piles

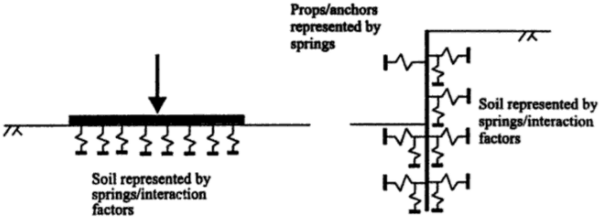


Beam-Spring Methods:
It is especially used for soil-structure interaction problems.

The soil behavior is approximated using two approaches:

1. A set of unconnected vertical and horizontal springs
2. Linear elastic interaction factors

Example: Raft foundations, Laterally loaded piles

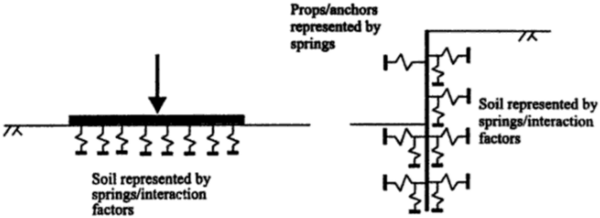


Beam-Spring Methods:
It is especially used for soil-structure interaction problems.

The soil behavior is approximated using two approaches:

1. A set of unconnected vertical and horizontal springs
2. Linear elastic interaction factors

Example: Raft foundations, Laterally loaded piles



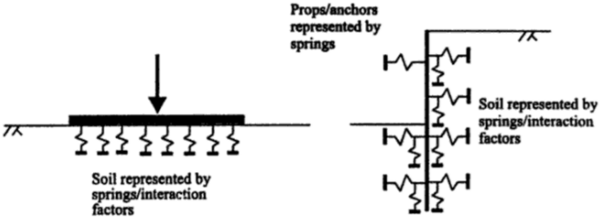
Beam-Spring Methods:

It is especially used for soil-structure interaction problems.

The soil behavior is approximated using two approaches:

1. A set of unconnected vertical and horizontal springs
2. Linear elastic interaction factors

Example: Raft foundations, Laterally loaded piles



Computational Methods:

Attempt to satisfy all theoretical requirements using realistic soil models and boundary conditions.

Complex in nature and numerical solution usually requires the use of a computer.

Unlike simple methods, a failure mechanisms is not required to be assumed but is rather predicted by the analysis.

Virtually applicable to almost all types of geotechnical problems.

Examples of common computational methods:

- Finite Difference Method (FDM)
- Finite Element Method (FEM)
- Boundary Element Method (BEM)
- Discrete Element Method (DEM)

Computational Methods:
Attempt to satisfy all theoretical requirements using realistic soil models and boundary conditions.

Complex in nature and numerical solution usually requires the use of a computer.

Unlike simple methods, a failure mechanisms is not required to be assumed but is rather predicted by the analysis.

Virtually applicable to almost all types of geotechnical problems.

Examples of common computational methods:

- Finite Difference Method (FDM)
- Finite Element Method (FEM)
- Boundary Element Method (BEM)
- Discrete Element Method (DEM)

Computational Methods:
Attempt to satisfy all theoretical requirements using realistic soil models and boundary conditions.

Complex in nature and numerical solution usually requires the use of a computer.

Unlike simple methods, a failure mechanisms is not required to be assumed but is rather predicted by the analysis.

Virtually applicable to almost all types of geotechnical problems.

Examples of common computational methods:

- Finite Difference Method (FDM)
- Finite Element Method (FEM)
- Boundary Element Method (BEM)
- Discrete Element Method (DEM)

Computational Methods:

Attempt to satisfy all theoretical requirements using realistic soil models and boundary conditions.

Complex in nature and numerical solution usually requires the use of a computer.

Unlike simple methods, a failure mechanisms is not required to be assumed but is rather predicted by the analysis.

Virtually applicable to almost all types of geotechnical problems.

Examples of common computational methods:

- Finite Difference Method (FDM)
- Finite Element Method (FEM)
- Boundary Element Method (BEM)
- Discrete Element Method (DEM)

Computational Methods:

Attempt to satisfy all theoretical requirements using realistic soil models and boundary conditions.

Complex in nature and numerical solution usually requires the use of a computer.

Unlike simple methods, a failure mechanisms is not required to be assumed but is rather predicted by the analysis.

Virtually applicable to almost all types of geotechnical problems.

Examples of common computational methods:

- Finite Difference Method (FDM)
- Finite Element Method (FEM)
- Boundary Element Method (BEM)
- Discrete Element Method (DEM)

Computational Methods:

Attempt to satisfy all theoretical requirements using realistic soil models and boundary conditions.

Complex in nature and numerical solution usually requires the use of a computer.

Unlike simple methods, a failure mechanisms is not required to be assumed but is rather predicted by the analysis.

Virtually applicable to almost all types of geotechnical problems.

Examples of common computational methods:

- Finite Difference Method (FDM)
- Finite Element Method (FEM)
- Boundary Element Method (BEM)
- Discrete Element Method (DEM)

Analysis Method	Solution Requirements				
	Equ.	Com.	Constitutive Soil Behavior	Force BCs	Disp. BCs
Analytical	✓	✓	Linear elastic	✓	✓
Limit Equilibrium	✓	×	Rigid with failure criterion	✓	×
Stress Field	✓	×	Rigid with failure criterion	✓	×
Limit Analysis (LB)	✓	×	Ideal plasticity	✓	×
Limit Analysis (UB)	×	✓	Ideal plasticity	×	✓
Beam-Spring	✓	✓	Springs or interaction factors	✓	✓
Computational	✓	✓	Any	✓	✓

Analysis Method	Solution Requirements				
	Equ.	Com.	Constitutive Soil Behavior	Force BCs	Disp. BCs
Analytical	✓	✓	Linear elastic	✓	✓
Limit Equilibrium	✓	×	Rigid with failure criterion	✓	×
Stress Field	✓	×	Rigid with failure criterion	✓	×
Limit Analysis (LB)	✓	×	Ideal plasticity	✓	×
Limit Analysis (UB)	×	✓	Ideal plasticity	×	✓
Beam-Spring	✓	✓	Springs or interaction factors	✓	✓
Computational	✓	✓	Any	✓	✓

Analysis Method	Solution Requirements				
	Equ.	**Com.**	**Constitutive Soil Behavior**	**Force BCs**	**Disp. BCs**
Analytical	✓	✓	Linear elastic	✓	✓
Limit Equilibrium	✓	×	Rigid with failure criterion	✓	×
Stress Field	✓	×	Rigid with failure criterion	✓	×
Limit Analysis (LB)	✓	×	Ideal plasticity	✓	×
Limit Analysis (UB)	×	✓	Ideal plasticity	×	✓
Beam-Spring	✓	✓	Springs or interaction factors	✓	✓
Computational	✓	✓	Any	✓	✓

Analysis Method	Solution Requirements				
	Equ.	**Com.**	**Constitutive Soil Behavior**	**Force BCs**	**Disp. BCs**
Analytical	✓	✓	Linear elastic	✓	✓
Limit Equilibrium	✓	×	Rigid with failure criterion	✓	×
Stress Field	✓	×	Rigid with failure criterion	✓	×
Limit Analysis (LB)	✓	×	Ideal plasticity	✓	×
Limit Analysis (UB)	×	✓	Ideal plasticity	×	✓
Beam-Spring	✓	✓	Springs or interaction factors	✓	✓
Computational	✓	✓	Any	✓	✓

Analysis Method	Solution Requirements				
	Equ.	**Com.**	**Constitutive Soil Behavior**	**Force BCs**	**Disp. BCs**
Analytical	✓	✓	Linear elastic	✓	✓
Limit Equilibrium	✓	×	Rigid with failure criterion	✓	×
Stress Field	✓	×	Rigid with failure criterion	✓	×
Limit Analysis (LB)	✓	×	Ideal plasticity	✓	×
Limit Analysis (UB)	×	✓	Ideal plasticity	×	✓
Beam-Spring	✓	✓	Springs or interaction factors	✓	✓
Computational	✓	✓	Any	✓	✓

Analysis Method	Solution Requirements				
	Equ.	Com.	Constitutive Soil Behavior	Force BCs	Disp. BCs
Analytical	✓	✓	Linear elastic	✓	✓
Limit Equilibrium	✓	×	Rigid with failure criterion	✓	×
Stress Field	✓	×	Rigid with failure criterion	✓	×
Limit Analysis (LB)	✓	×	Ideal plasticity	✓	×
Limit Analysis (UB)	×	✓	Ideal plasticity	×	✓
Beam-Spring	✓	✓	Springs or interaction factors	✓	✓
Computational	✓	✓	Any	✓	✓

Analysis Method	Solution Requirements				
	Equ.	Com.	Constitutive Soil Behavior	Force BCs	Disp. BCs
Analytical	✓	✓	Linear elastic	✓	✓
Limit Equilibrium	✓	×	Rigid with failure criterion	✓	×
Stress Field	✓	×	Rigid with failure criterion	✓	×
Limit Analysis (LB)	✓	×	Ideal plasticity	✓	×
Limit Analysis (UB)	×	✓	Ideal plasticity	×	✓
Beam-Spring	✓	✓	Springs or interaction factors	✓	✓
Computational	✓	✓	Any	✓	✓

Analysis Method	Solution Requirements				
	Equ.	**Com.**	**Constitutive Soil Behavior**	**Force BCs**	**Disp. BCs**
Analytical	✓	✓	Linear elastic	✓	✓
Limit Equilibrium	✓	×	Rigid with failure criterion	✓	×
Stress Field	✓	×	Rigid with failure criterion	✓	×
Limit Analysis (LB)	✓	×	Ideal plasticity	✓	×
Limit Analysis (UB)	×	✓	Ideal plasticity	×	✓
Beam-Spring	✓	✓	Springs or interaction factors	✓	✓
Computational	✓	✓	Any	✓	✓

Analysis Method	Solution Requirements				
	Equ.	**Com.**	**Constitutive Soil Behavior**	**Force BCs**	**Disp. BCs**
Analytical	✓	✓	Linear elastic	✓	✓
Limit Equilibrium	✓	×	Rigid with failure criterion	✓	×
Stress Field	✓	×	Rigid with failure criterion	✓	×
Limit Analysis (LB)	✓	×	Ideal plasticity	✓	×
Limit Analysis (UB)	×	✓	Ideal plasticity	×	✓
Beam-Spring	✓	✓	Springs or interaction factors	✓	✓
Computational	✓	✓	Any	✓	✓