

# Geotechnical Analysis

## Advanced Computational Methods in Geotechnical Engineering

MSc Course - Addis Ababa Science and Technology University

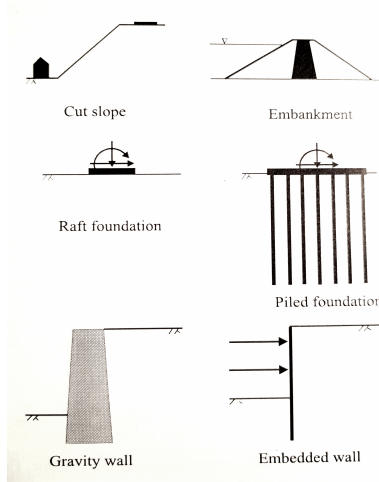
Yared W. Bekele, PhD

- 1 Introduction
- 2 Analysis and Design Requirements
- 3 Theoretical Considerations
- 4 Idealized Computational Domains
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# Introduction

Construction of almost all civil engineering structures in one way or another involve the ground.



Examples of geotechnical structures, after Potts and Zdravkovic 1999.

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# Analysis and Design Requirements

- Loads imposed on the soil and structural members must be assessed under working and ultimate loading conditions.
- The relative movements of the soil and structure must be evaluated.
- Simplified analyses and empirical approaches are commonly used in geotechnical design.
- The use of computational/numerical analysis and design methods in geotechnical engineering practice is still limited.

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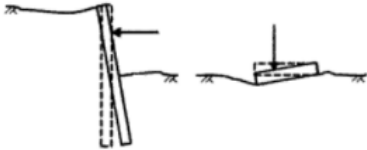


# Analysis and Design Requirements

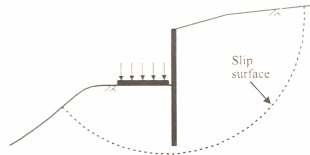
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# Analysis and Design Requirements

*Stability* is one of the main requirements in the design of geotechnical structures. Two aspects: *local stability* and *global stability*



Local stability

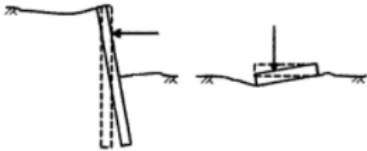


Global stability

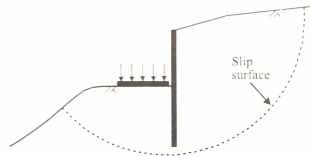
Assert that there is no danger of translational and rotational failure! **Geotechnical Engineer's Responsibility**

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Local stability



Global stability

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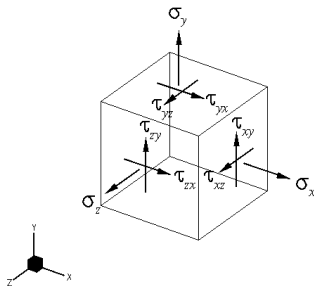
A theoretical solution is required to satisfy

- ① Equilibrium requirements
- ② Compatibility requirements
- ③ Constitutive material behavior
- ④ Boundary conditions

# Theoretical Considerations

↪ Equilibrium

Equilibrium equations set the external applied loads to be equal to the the sum of all the internal forces.



$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + b = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0$$

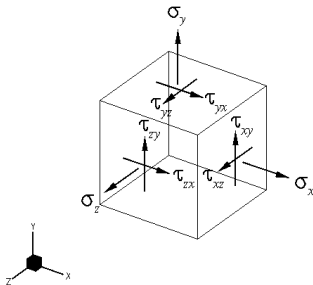
Equilibrium equations for a complete system:

$$\begin{aligned} \sum F_x &= 0 & \sum F_y &= 0 & \sum F_z &= 0 \\ \sum M_x &= 0 & \sum M_y &= 0 & \sum M_z &= 0 \end{aligned}$$

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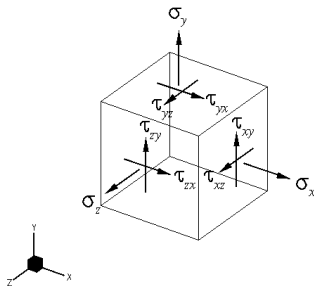
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↪ Compatibility

*Physical compatibility:* Deformations should not cause unrealistic overlapping of materials or generation of holes.

*Mathematical compatibility:* Physical compatibility expressed in terms of equations.

$$\begin{aligned}\varepsilon_x &= \frac{\partial u}{\partial x} & \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\ \varepsilon_y &= \frac{\partial v}{\partial y} & \gamma_{yz} &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \\ \varepsilon_z &= \frac{\partial w}{\partial z} & \gamma_{xz} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\end{aligned}$$

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# Theoretical Considerations

## ↔ Constitutive Equations

Constitutive equations are stress-strain relationships that govern the behavior of the material.

In incremental form, a constitutive relationship is written as

$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\sigma_z \\ \Delta\tau_{xy} \\ \Delta\tau_{xz} \\ \Delta\tau_{zy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{45} & D_{46} \\ D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\varepsilon_z \\ \Delta\gamma_{xy} \\ \Delta\gamma_{xz} \\ \Delta\gamma_{zy} \end{Bmatrix}$$

Compact form:

$$\Delta\sigma = D\Delta\varepsilon$$

In terms of effective stresses:

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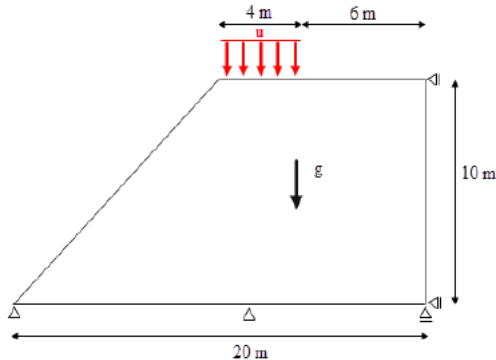
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# Theoretical Considerations

## ↪ Boundary Conditions

A theoretical solution should also satisfy the different boundary conditions of the problem.



Different types of boundary conditions will be discussed in a later chapter.



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# Idealized Computational Domains

Real geotechnical problems are usually simplified by idealizing the geometry and/or boundary conditions of the problem.

Full three-dimensional simulations are usually demanding and most problems are solved by reducing the problem to equivalent two-dimensional models.

The most common geometric idealizations are:

- 1 Plane strain
- 2 Axi-symmetry

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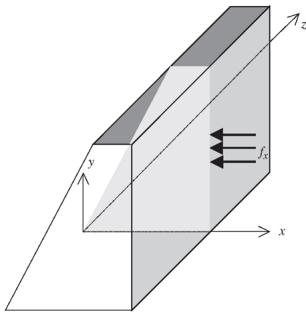
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↪ Plane Strain

Applied when the  $z$ -dimension of the problem is large and it can be assumed that the state existing in the  $x$ - $y$  plane holds for all planes parallel to it.



The displacement in the  $z$ -dimension is zero ( $w = 0$ ) and the displacements  $u$  and  $v$  are independent of the  $z$  coordinate. Thus,

$$\epsilon_z = \frac{\partial w}{\partial z} = 0$$

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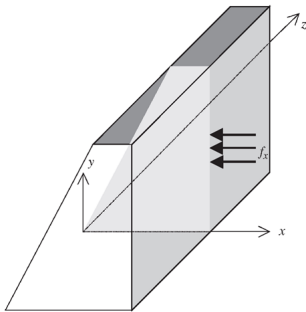
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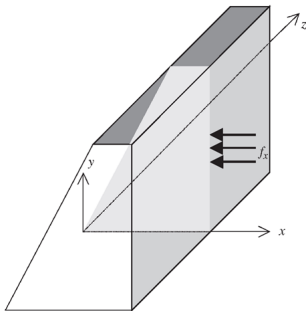
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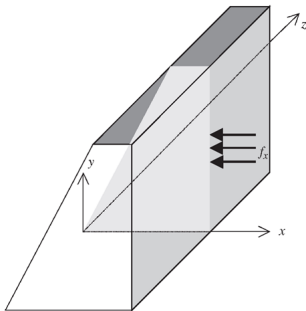
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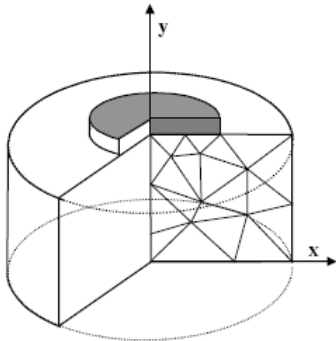
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# Idealized Computational Domains

↪ Axi-symmetry

Some problems have a domain with a rotational axis of symmetry where the model could be idealized using axi-symmetric models



Cylindrical coordinates are used for axi-symmetric models

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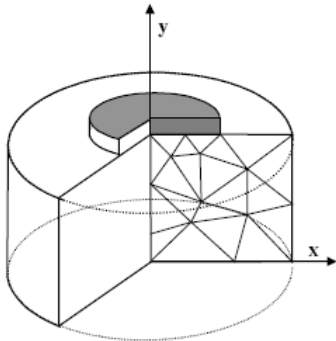
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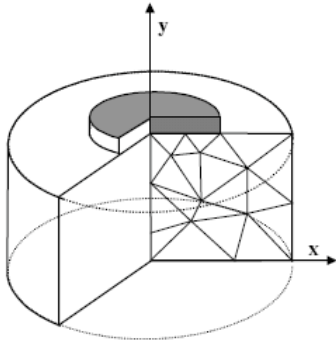
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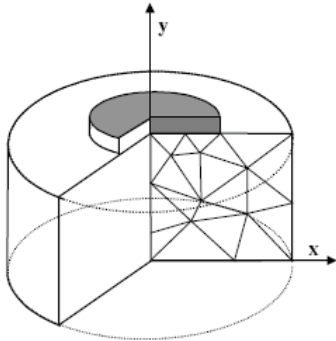
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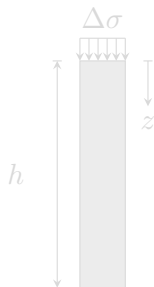
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The different analysis methods in geomechanics may be divided into three groups:

- ① Analytical Methods
- ② Simple Methods
- ③ Numerical Methods

Applicable when the governing equation(s) of the problem can be solved such that an analytical expression for the solution can be found.

Example: One-dimensional consolidation



Governing equation:

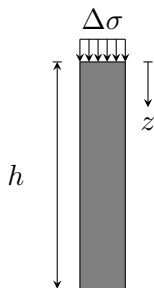
$$\frac{\partial u}{\partial t} - c_v \frac{\partial^2 u}{\partial z^2} = 0$$

Solution for excess pore pressure:

$$\frac{u(t, z)}{u_0} = \frac{4}{\pi} \sum_{i=1}^{\infty} \frac{(-1)^{i-1}}{2i-1} \exp \left[ -(2i-1)^2 \frac{\pi^2 t_s}{4} \right] \cos \left[ (2i-1) \frac{\pi z}{2h} \right]$$

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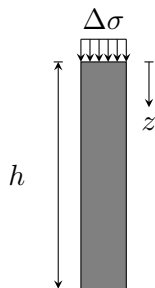
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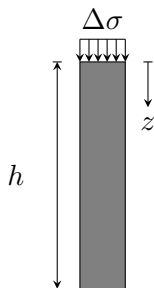
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Three types of simple methods in geotechnical engineering:

- ① Limit Equilibrium Method
- ② Stress Field Method
- ③ Limit Analysis Method

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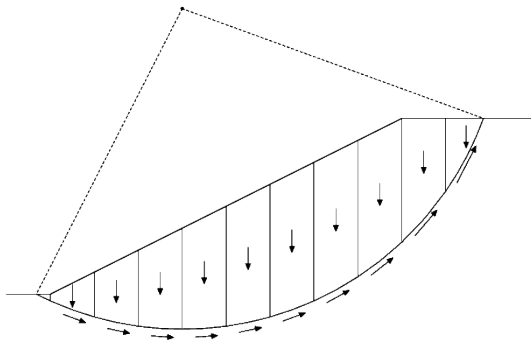
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## Limit Equilibrium:

A failure criterion is established and an arbitrary failure mechanism is assumed.

Equilibrium equations are written for the global system.

Example: Slope stability analysis

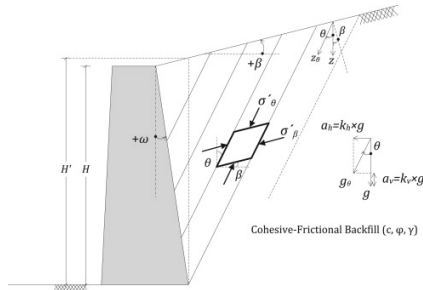


## Stress Field:

A failure criterion is established and the soil is assumed to be at the point of failure everywhere.

Equilibrium equations are combined with the failure criterion to solve the problem.

Example: Retaining structures/Earth pressure calculations



## Limit Analysis:

This method is based on the following assumptions:

- The soil behavior can be represented by perfect plasticity.
- The plastic strains can be derived from the yield function.
- Changes in the geometry of the soil mass that occur at failure are insignificant.

Failure condition may be established based on two approaches:

- ① *Upper bound theorem:* The failure load determined is unsafe or close to the true failure load.
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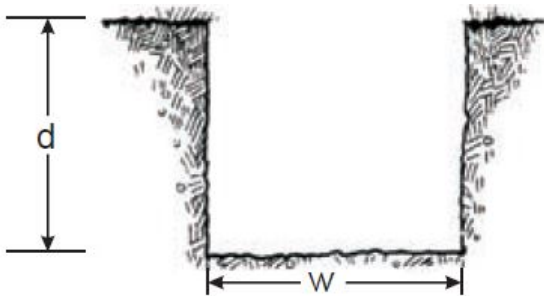
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## Limit Analysis:

Example: Critical height of a vertical cut in cohesive soil



Upper and lower bound solutions can be calculated.

In a broad sense, the numerical methods used in geotechnical engineering may be classified into two groups:

- 1 Beam-Spring Methods
- 2 Full Computational Methods

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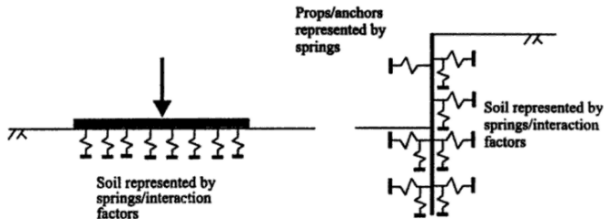
## Beam-Spring Methods:

It is especially used for soil-structure interaction problems.

The soil behavior is approximated using two approaches:

- ① A set of unconnected vertical and horizontal springs
- ② Linear elastic interaction factors

Example: Raft foundations, Laterally loaded piles



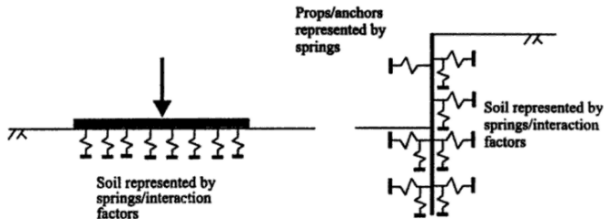
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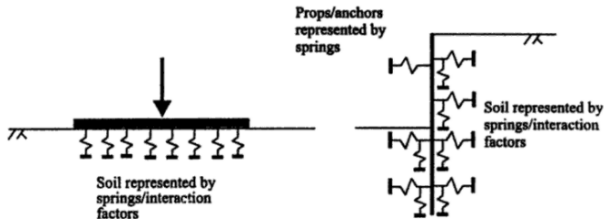
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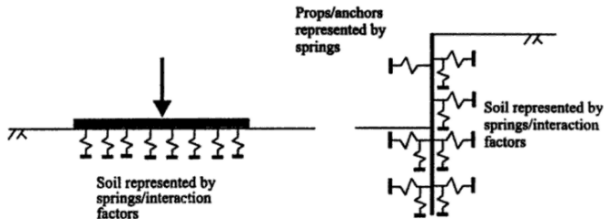
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## Computational Methods:

Attempt to satisfy all theoretical requirements using realistic soil models and boundary conditions.

Complex in nature and numerical solution usually requires the use of a computer.

Unlike simple methods, a failure mechanisms is not required to be assumed but is rather predicted by the analysis.

Virtually applicable to almost all types of geotechnical problems.

Examples of common computational methods:

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# Analysis Methods

↪ Comparison of Methods

Analysis Method	Solution Requirements					
	Equ.	Com.	Constitutive Behavior	Soil	Force BCs	Disp. BCs
Analytical	✓	✓	Linear elastic		✓	✓
Limit Equilibrium	✓	×	Rigid with failure criterion		✓	×
Stress Field	✓	×	Rigid with failure criterion		✓	×
Limit Analysis (LB)	✓	×	Ideal plasticity		✓	×
Limit Analysis (UB)	×	✓	Ideal plasticity		×	✓
Beam-Spring	✓	✓	Springs or interaction factors		✓	✓
Computational	✓	✓	Any		✓	✓

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