Boundary Element Method and Hybrid Methods

CENG6202: Advanced Computational Methods in Geotechnical Engineering

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Contents

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Some examples include

- Boundary Element Method (BEM)
- Discrete Element Method (DEM)
- Material Point Method (MPM)
- IsoGeometric Analysis (IGA)

A combination of different computational methods to simulate problems results in what's referred to as a hybrid method. An example is coupled BEM-FEM.

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Main Idea of BEM

The solution to a PDE can be approximated by looking at the solution of the PDE on the boundary and using this information to find the solution inside the domain.

- BEM is useful on very large domains where a FEM approximation would have too many elements to be practical.
- An example is a modeling a series of wells in an infinite reservior. FEM would require an extremely large mesh and is thus computationally demanding. BEM may be applied with discretization only over the boundary of the domain.

Boundary Element Method

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The main steps in the BEM are

- Formulation of the mathematical model of the problem
- Derivation of representation formulas
- Derivation of boundary integral equations
- Discretization of boundary into boundary elements
- Formulation of discrete equations based on discretization
- Solution of the final equation system
- Interpretation of results

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Consider the equation of steady-state flow in 2D:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

The fundamental solution satisfies the equation

$$\frac{d^2 w}{dx^2} + \frac{d^2 w}{dy^2} + \delta(\xi - x, \eta - y) = 0$$

The Dirac-Delta function δ is a generalized function which is equal to zero everywhere except for zero and its integral over the entire real line is equal to one.

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The solution is expected to be symmetric about the point (ξ, η) since $\delta(\xi - x, \eta - y)$ is symmetric about this point.

A local polar coordinate system is adopted about the point (ξ, η) and we define

$$r = \sqrt{(\xi - x)^2 + (\eta - y)^2}$$

We can now write

$$\nabla^2 w = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} = 0$$

For $r > 0$, $\delta(\xi - x, \eta - y) = 0$ and $\frac{\partial^2 w}{\partial \theta^2} = 0$ from symmetry. Thus,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = 0$$

The solution to this takes the form

$$w = A \ln r + B$$

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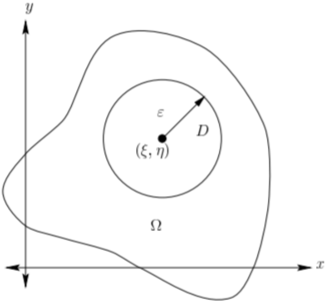
The integral property of the Dirac-Delta function is used to find the constants A and B.



For a domain D containing $r = 0$, we have

$$\int_D \nabla^2 w dD = -\int_D \delta dD = -1$$

The BEM solution for $\nabla^2 h = 0$ can now be developed for a domain Ω. Multiplying the PDE by a weighting function w gives

$$\nabla^2 h = 0 \Rightarrow w\left[\nabla^2 h = 0\right] \Rightarrow \int_\Omega \left[\nabla^2 h w\right] d\Omega = 0$$

Applying Green's theorem and performing integration by parts gives

$$\int_\Omega \left[\nabla^2 h w\right] d\Omega = \int_{\partial\Omega} \frac{\partial h}{\partial n} w d\Gamma - \int_{\partial\Omega} h \frac{\partial w}{\partial n} d\Gamma + \int_\Omega h \nabla^2 w d\Omega = 0$$

In BEM, the last term is chosen as

$$\int_\Omega h \nabla^2 w d\Omega = -\int_\Omega h \delta(\xi - x, \eta - y) d\Omega = -h(\xi, \eta)$$

assuming that $(\xi, \eta) \in \Omega$ and not on the boundary.

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Thus, the boundary integral equation becomes

$$-\int_{\partial\Omega} \frac{\partial h}{\partial n} w \, d\Gamma + \int_{\partial\Omega} h \frac{\partial w}{\partial n} d\Gamma + h(\xi, \eta) = 0 \quad \text{for} \quad (\xi, \eta) \in \Omega$$

This implies that we can find h at any arbitrary point $(\xi, \eta) \in \Omega$ by looking at h and w only on the boundary.

It doesn't help if we don't know h or $\frac{\partial h}{\partial n}$ on the boundary!

If $(\xi, \eta) \notin \Omega$, the property of δ implies

$$\int_{\Omega} h \nabla^2 w \, d\Omega = -\int_{\Omega} h \delta(\xi - x, \eta - y) d\Omega = 0$$

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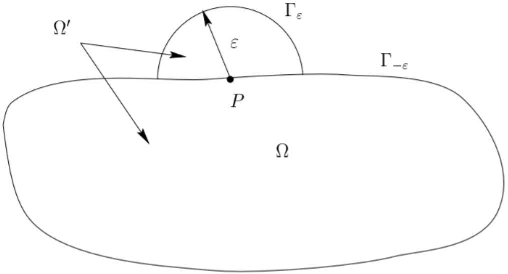
Special consideration is required when the point $P = (\xi, \eta)$ is on the boundary.



The integral in this case is evaluated by defining a disc with radius ϵ such that we have two subdomains Γ_ϵ and $\Gamma_{-\epsilon}$. The limit of the boundary integral equation is taken for both as $\epsilon \to 0$.

Evaluating the limits of the integrals as $\epsilon \to 0$ gives

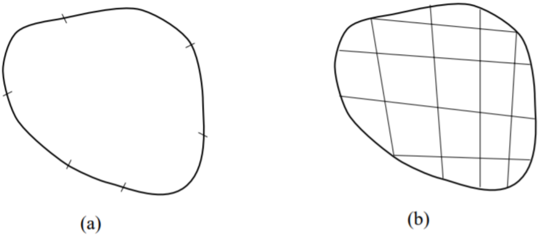
$$-\int_{\partial\Omega} \frac{\partial h}{\partial n} w d\Gamma + \int_{\partial\Omega} h \frac{\partial w}{\partial n} d\Gamma + c(P)h(P) = 0$$

where

$$c(P) = \begin{cases} 1 & P \in \Omega \\ \frac{1}{2} & P \in \partial\Omega, \text{smooth boundary} \\ 1 - \frac{\alpha}{2\pi} & P \in \partial\Omega, \text{non-smooth boundary} \\ 0 & P \notin \Omega \end{cases}$$

and α is the interior angle of the corner at P for a non-smooth boundary.

The boundary Γ is divided into a desired number (say N) of boundary elements Γ_i such that $\Gamma = \bigcup_{i=1}^{N} \Gamma_i$.



a) BEM Mesh and b) FEM Mesh

- The values of h and $q = \frac{\partial h}{\partial n}$ will be known from boundary conditions on some Γ_i. These are used to find the values of h on other boundary elements.
- The simplest boundary element has one node at its center. As in FEM, the number of nodes per element may be increased.
- The basis functions for h and q are usually the same but different order functions may also be used.

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For N boundary elements, the boundary integral equation may be written as

$$-\sum_{i=1}^{N}\int_{\Gamma_i}\frac{\partial h}{\partial n}w d\Gamma + \sum_{i=1}^{N}\int_{\Gamma_i} h\frac{\partial w}{\partial n}d\Gamma + c(P)h(P) = 0$$

Standard basis functions are introduced to approximate the unknowns in terms of the nodal values.

Formulation of the boundary element equations and assembly results in an equation system of the form

$$\boldsymbol{A}\boldsymbol{h} = \boldsymbol{B}\boldsymbol{q}$$

which is similar to the FEM equation $\boldsymbol{K}\boldsymbol{h} = \boldsymbol{F}$. The vectors $\boldsymbol{h}$ and $\boldsymbol{q}$ are vectors of the nodal values of h and q.

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FEM	**BEM**
Discretization of whole domain	Discretization of only boundary
Solution obtained over the entire domain	Solution first obtained over the boundary
Element integrals are easy to evaluate	Integrals are more difficult to evaluate
Best for finite domains	Best for infinite or semi-infinite domains
Large and sparse matrix in final equation system $\boldsymbol{Kh} = \boldsymbol{F}$	Small and filled-in matrix in final equation system $\boldsymbol{Ah} = \boldsymbol{Bq}$
Requires no prior knowledge of solution	Requires a fundamental solution of the PDE
Applicable to most linear second-order PDEs	Difficult to apply to inhomogeneous or nonlinear problems

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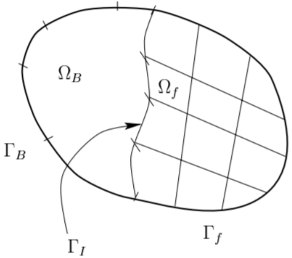
Coupled FEM-BEM

There are various cases where some region of the computational domain favors BEM and another region FEM.



A combined FEM-BEM model has: Ω_B - BEM Region, Ω_F - FEM Region, Γ_B - BEM Boundary, Γ_F - FEM Boundary and Γ_I - Interface boundary.

Coupled FEM-BEM

Two possible way of coupling BEM and FEM regions:

1. Consider the BEM region as a finite element and combine with FEM
2. Consider the FEM region as a boundary element and combine with BEM

The final equation systems for the BEM and FEM regions are

$$\boldsymbol{A}\boldsymbol{h} = \boldsymbol{B}\boldsymbol{q} \quad \text{and} \quad \boldsymbol{K}\boldsymbol{h} = \boldsymbol{F}$$

To apply method 1, we write the first system as

$$\boldsymbol{B}^{-1}\boldsymbol{A}\boldsymbol{h} = \boldsymbol{q}$$

Convert $\boldsymbol{q}$ into an equivalent load vector by weighting the nodal values of $\boldsymbol{q}$ with the appropriate basis functions producing matrix $\boldsymbol{M}$, i.e. $\boldsymbol{f}_B = \boldsymbol{M}\boldsymbol{q}$.

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Thus, we get

$$\boldsymbol{M}(\boldsymbol{B}^{-1}\boldsymbol{A})\boldsymbol{h} = \boldsymbol{M}\boldsymbol{q} = \boldsymbol{f}_B$$

which may be written as

$$\boldsymbol{K}_B\boldsymbol{h} = \boldsymbol{f}_B$$

$\boldsymbol{K}_B = \boldsymbol{M}(\boldsymbol{B}^{-1}\boldsymbol{A})$ is thus an equivalent stiffness matrix obtained from the BEM region and may be assembled together with the FEM equations.

The second method can be applied by following a similar procedure. The FEM equation is converted into an equivalent BEM equation i.e.

$$\boldsymbol{K}\boldsymbol{h} = \boldsymbol{F} \Rightarrow \boldsymbol{K}\boldsymbol{h} = \boldsymbol{M}\boldsymbol{q}$$

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