Geotechnical Analysis

CENG6202: Advanced Computational Methods in Geotechnical Engineering

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Fall Semester 2019

Contents

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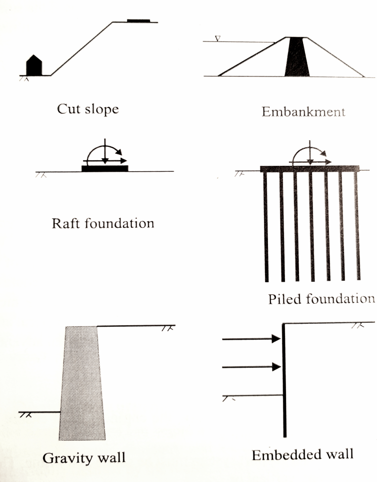
Construction of almost all civil engineering structures in one way or another involve the ground.



Examples of geotechnical structures, after Potts and Zdravkovic 1999.

Contents

- Loads imposed on the soil and structural members must be assessed under working and ultimate loading conditions.
- The relative movements of the soil and structure must be evaluated.
- Simplified analyses and empirical approaches are commonly used in geotechnical design.
- The use of computational/numerical analysis and design methods in geotechnical engineering practice is still limited.

Analysis and Design Requirements

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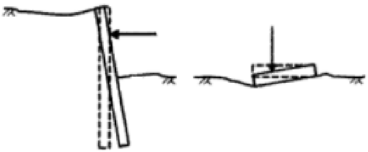
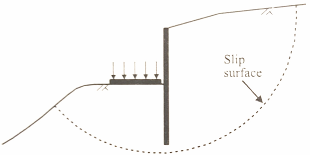


Local stability



Global stability

Assert that there is no danger of translational and rotational failure! Geotechnical Engineer's Responsibility

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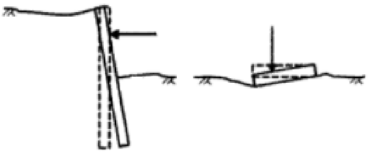

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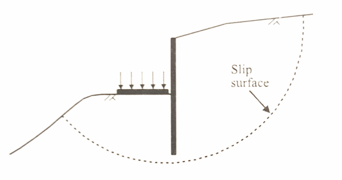

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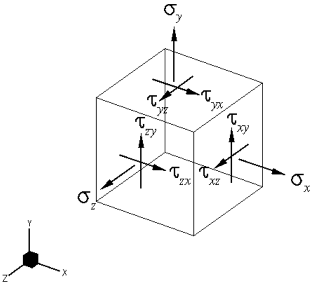
Theoretical Considerations

A theoretical solution is required to satisfy

1. Equilibrium requirements
2. Compatibility requirements
3. Constitutive material behavior
4. Boundary conditions

Equilibrium equations set the external applied loads to be equal to the the sum of all the internal forces.



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Equilibrium equations for a complete system:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$
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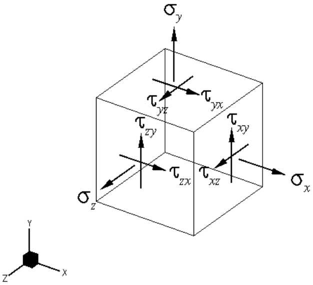

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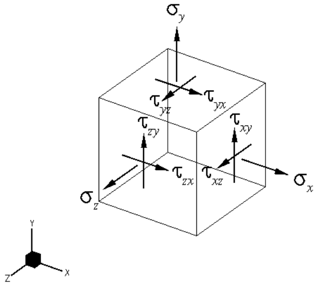

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Mathematical compatibility: Physical compatibility expressed in terms of equations.

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In incremental form, a constitutive relationship is written as

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Compact form:

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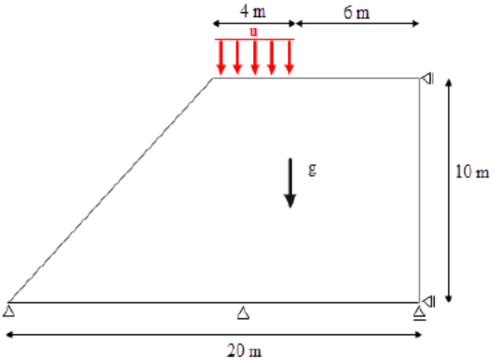
A theoretical solution should also satisfy the different boundary conditions of the problem.



Different types of boundary conditions will be discussed in a later chapter.

Contents

Idealized Computational Domains

Real geotechnical problems are usually simplified by idealizing the geometry and/or boundary conditions of the problem.

Full three-dimensional simulations are usually demanding and most problems are solved by reducing the problem to equivalent two-dimensional models.

The most common geometric idealizations are:

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2. Axi-symmetry

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Idealized Computational Domains

↪Plane Strain

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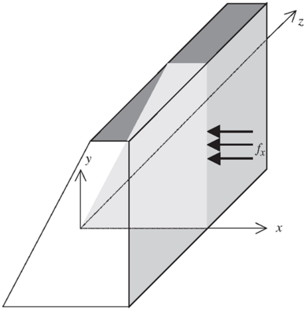

The displacement in the z-dimension is zero ($w = 0$) and the displacements u and v are independent of the z coordinate. Thus,

$$\varepsilon_z = \frac{\partial w}{\partial z} = 0$$

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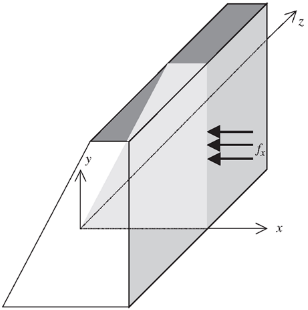

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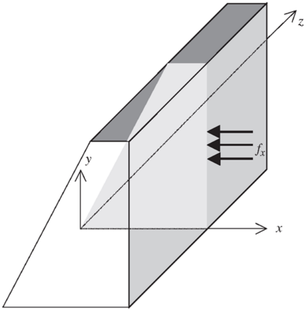

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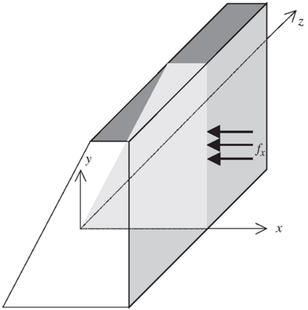

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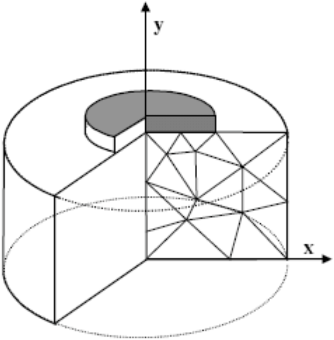
Some problems have a domain with a rotational axis of symmetry where the model could be idealized using axi-symmetric models



Cylindrical coordinates are used for axi-symmetric models

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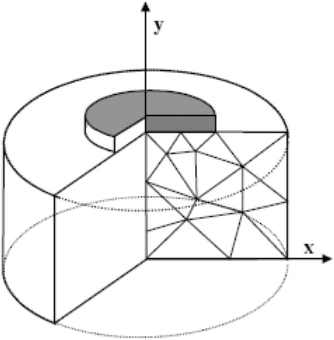

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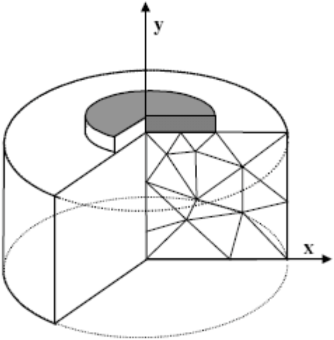

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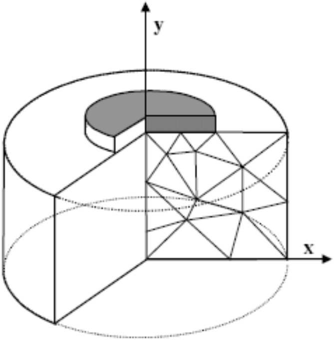

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The different analysis methods in geomechanics may be divided into three groups:

1. Analytical Methods
2. Simple Methods
3. Numerical Methods

Applicable when the governing equation(s) of the problem can be solved such that an analytical expression for the solution can be found.

Example: One-dimensional consolidation

Governing equation:

$$\frac{\partial u}{\partial t} - c_{\text{v}} \frac{\partial^2 u}{\partial z^2} = 0$$

Solution for excess pore pressure:

$$\frac{u(t,z)}{u_0} = \frac{4}{\pi} \sum_{i=1}^{\infty} \frac{(-1)^{i-1}}{2i-1} \exp\left[-(2i-1)^2 \frac{\pi^2 t_{\text{s}}}{4}\right] \cos\left[(2i-1)\frac{\pi z}{2h}\right]$$

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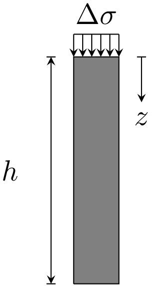

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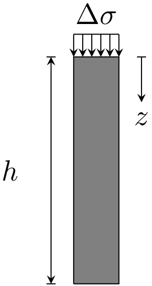

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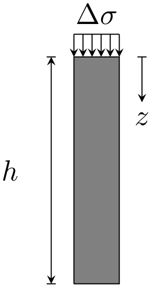

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Three types of simple methods in geotechnical engineering:

1. Limit Equilibrium Method
2. Stress Field Method
3. Limit Analysis Method

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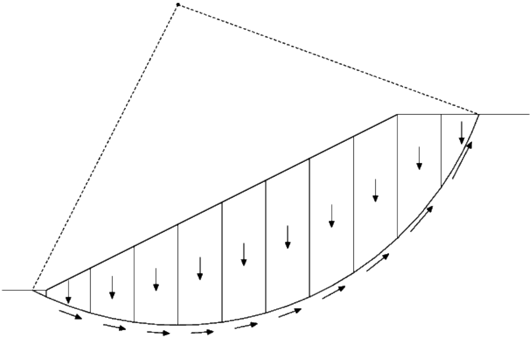
Limit Equilibrium:

A failure criterion is established and an arbitrary failure mechanism is assumed.

Equilibrium equations are written for the global system.

Example: Slope stability analysis

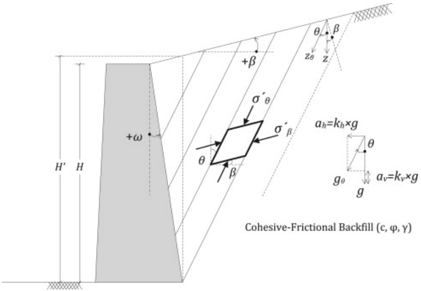


Stress Field:

A failure criterion is established and the soil is assumed to be at the point of failure everywhere.

Equilibrium equations are combined with the failure criterion to solve the problem.

Example: Retaining structures/Earth pressure calculations



Limit Analysis:
This method is based on the following assumptions:

- The soil behavior can be represented by perfect plasticity.
- The plastic strains can be derived from the yield function.
- Changes in the geometry of the soil mass that occur at failure are insignificant.

Failure condition may be established based on two approaches:

1. *Upper bound theorem*: The failure load determined is unsafe or close to the true failure load.
2. *Lower bound theorem*: The estimated load is safe or close to the true failure load.

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- The plastic strains can be derived from the yield function.
- Changes in the geometry of the soil mass that occur at failure are insignificant.

Failure condition may be established based on two approaches:

1. *Upper bound theorem*: The failure load determined is unsafe or close to the true failure load.
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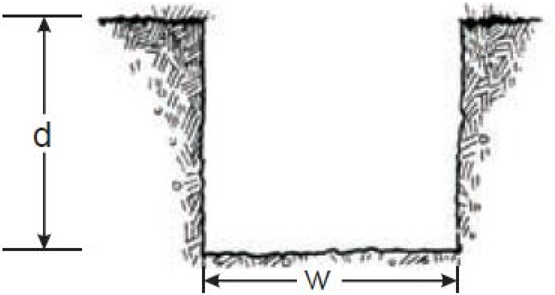
Example: Critical height of a vertical cut in cohesive soil



Upper and lower bound solutions can be calculated.

In a broad sense, the numerical methods used in geotechnical engineering may be classified into two groups:

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Beam-Spring Methods:

It is especially used for soil-structure interaction problems.

The soil behavior is approximated using two approaches:

1. A set of unconnected vertical and horizontal springs
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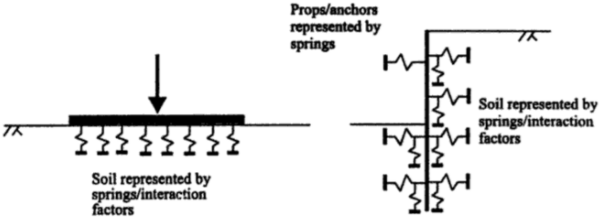
Example: Raft foundations, Laterally loaded piles



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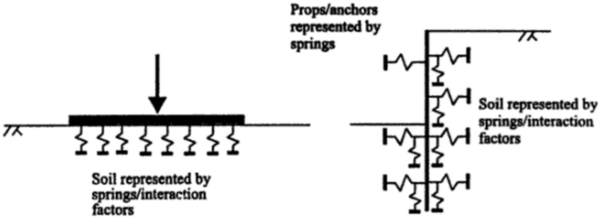

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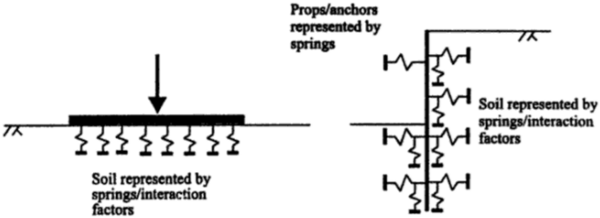

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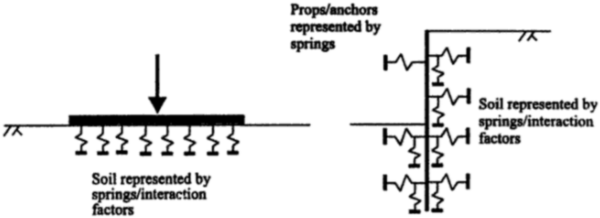

Computational Methods:

Attempt to satisfy all theoretical requirements using realistic soil models and boundary conditions.

Complex in nature and numerical solution usually requires the use of a computer.

Unlike simple methods, a failure mechanisms is not required to be assumed but is rather predicted by the analysis.

Virtually applicable to almost all types of geotechnical problems.

Examples of common computational methods:

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Limit Equilibrium	✓	×	Rigid with failure criterion	✓	×
Stress Field	✓	×	Rigid with failure criterion	✓	×
Limit Analysis (LB)	✓	×	Ideal plasticity	✓	×
Limit Analysis (UB)	×	✓	Ideal plasticity	×	✓
Beam-Spring	✓	✓	Springs or interaction factors	✓	✓
Computational	✓	✓	Any	✓	✓

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