Experiment 5, Poisson Statistics

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Abstract

In this experiment we observe the distribution of radiation emitted by two different source of ^{137}Cs with 10UC and 1.0UC. Using a Geiger counter, we count the number of gamma rays emitted from the radiation source by setting the counter 1s and 10s interval. From this we can make histogram of the counts per interval vs. distribution of counts. From these histograms we can decide on which distribution (Poisson or Gaussian) is fitting with our data.

Theoritical Motivation

Radioactive decay is determined by quantum mechanics which is inherently probabilistic. So it is impossible to work out when any particular atom will decay, but we can make predictions based on the statistical behaviour of large numbers of atoms. Therefore, a ^{137}Cs source is an excellent, predictable gamma ray source. It randomly releases radiation at a predictable average rate. Because the radiation releases are independent events, we should be able to model radioactive decay of ^{137}Cs with a Poisson distribution. If we are able to do this, we can make predictions about the spread of radiation over time from such a radioactive source if we can determine the mean rate of emitted radiation.

The Poisson distribution is defined by the equation;

$$P(x;\mu) = \frac{\lambda^x e^{-\lambda}}{x!} \tag{1}$$

We know that the Poisson distribution comes from Binomial distribution. Let's see how it is. Binomial distribution equation is defined as;

$$Pr(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \tag{2}$$

The binomial distribution works when we have a fixed number of events n, each with a constant probability of success p. If we don't know the number of trials that will happen. Instead, we only know the average number of successes per time period. Then, we take μ, the mean rate of events, to equal pn, we can then evaluate $Pr(x)$ as n goes to infinity:

$$\lim_{x\to\infty} \frac{n!}{x!(n-x)!} \left(\frac{\lambda}{n}\right)^x (1-\frac{\lambda}{n})^{n-x} \tag{3}$$

$$\lim_{x\to\infty} \underbrace{\frac{n!}{n^x(n-x)!}}_{\approx 1} \frac{\lambda^x}{x!} \underbrace{(1-\frac{\lambda}{n})^n}_{\approx e^{-\lambda}} \underbrace{(1-\frac{\lambda}{n})^{-x}}_{\approx 1} \tag{4}$$

$$\approx \frac{\lambda^x e^{-\lambda}}{x!} \tag{5}$$

Also, there are properties of the Poisson distribution such as; mean is equal to λ and $\sigma^2 = \mu$. I won't show the former explicitly. Let's show the letter;

$< n >= \mu$ & $\sigma^2 =< n^2 > - < n >^2$

$$< n^2 >= \sum_{n=0}^{\infty} n^2 \frac{\lambda^n e^{-\lambda}}{n!} = e^{-\lambda} \sum_{n=1}^{\infty} (n\,(n-1)+n)\,\frac{\lambda^n}{n!} \tag{6}$$

$$= e^{-\lambda} \sum_{n=2}^{\infty} (n\,(n-1))\,\frac{\lambda^n}{n!} + e^{-\lambda} \sum_{n=1}^{\infty} \frac{\lambda^n}{(n-1)!} \tag{7}$$

$$e^{-\lambda}\lambda^2 \sum_{n=2}^{\infty} \frac{\lambda^{n-2}}{(n-2)!} + e^{-\lambda}\lambda \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = \lambda^2 + \lambda \tag{8}$$

$$\tag{9}$$

Also we can derive Gaussian distribution from Poisson distribution. Using Stirling's formula for $n!$;

$$x! \to \sqrt{2\pi x}\, e^{-x} x^x \quad \text{as} \quad x \to \infty$$

$$\begin{aligned} p(x) &= \frac{\lambda^{\lambda(1+\delta)} e^{-\lambda}}{\sqrt{2\pi} e^{-\lambda(1+\delta)} [\lambda(1+\delta)]^{\lambda(1+\delta)+1/2}} \\ &= \frac{e^{\lambda\delta}(1+\delta)^{-\lambda(1+\delta)-1/2}}{\sqrt{2\pi\lambda}} \\ &= \frac{e^{-\lambda\delta^2/2}}{\sqrt{2\pi\lambda}} \end{aligned}$$

Substituting back for x, with $\delta = \frac{(x-\lambda)}{\lambda}$, yields

$$p(x) = \frac{e^{-(x-\lambda)^2/(2\lambda)}}{\sqrt{2\pi\lambda}}$$

Hence, it is not so easy to distinguish between the Poisson distribution and the Gaussian distribution as μ becomes large. However, the Poisson processes show a different behavior when it comes to the distribution of successive events.
The probability of observing n counts during a time interval t is

$$P(\mu, n) = P(\alpha, t, n) = \frac{(\alpha t)^n e^{-\alpha t}}{n!} \tag{10}$$

where $\alpha = \frac{\mu}{t}$. Then, the probability of having one event in a time interval dt is

$$P(\alpha, dt, 1) = \frac{(\alpha dt) e^{-\alpha dt}}{1!} \tag{11}$$

Hence, the probability of having n events in a t interval followed by another event within a dt time is (assuming $e^{\alpha dt} \approx 1$

$$P_q(n+1, t)dt = P(\alpha, t, n)P(\alpha, dt, 1) = \frac{(\alpha t)^n e^{-\alpha dt}}{n!} \frac{(\alpha dt) e^{-\alpha dt}}{1!} \tag{12}$$

$$\approx \frac{(\alpha t)^n e^{-\alpha t} \alpha dt}{n!} \tag{13}$$

$$P_q(n+1, t) = \frac{(\alpha t)^n e^{-\alpha t} \alpha}{n!} \tag{14}$$

Again, as n becomes large, the distribution, $P_q(n+1, t)$ approaches the Gaussian distribution. Here n will be limited to 0 and 1.
Also, to analyse our graphs we need to know χ^2 of the fitted functions

$$\chi_{poisson}{}^2 = \sum_{i=n} \frac{\left(y(n) - W\left(\frac{\lambda^n e^{-\lambda}}{n!}\right)\right)^2}{\sigma_{y_i}^2} \tag{15}$$

$$\chi_{gaussian}{}^2 = \sum_{i=n} \frac{\left(y(n) - W\frac{1}{\sigma\sqrt{2\pi}} exp[\frac{-1}{2}\left(\frac{n-\lambda}{\sigma}\right)^2]\right)^2}{\sigma_{y_i}^2} \tag{16}$$

where W is bin width.

Apparatus, Experimental Procedure, Data

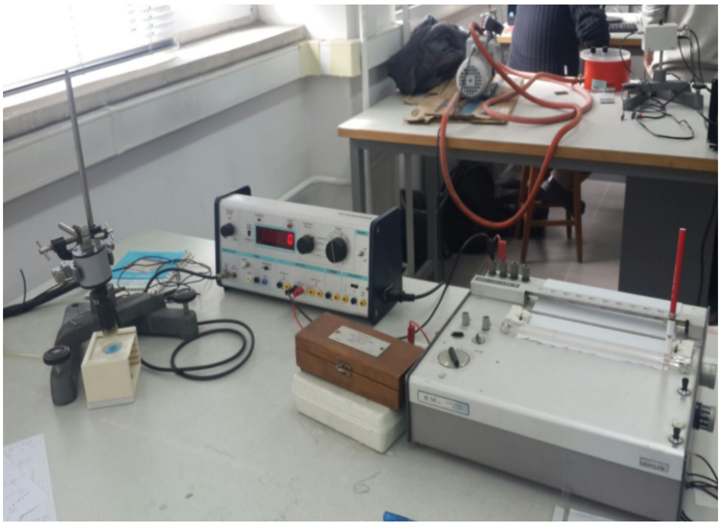

Figure 1: Set-up

Our apparatuses are

Geiger counter with a scalar; A Geiger-Mueller (GM) tube is a gas-filled radiation detector. The fill gas is generally argon at a pressure of less than 0.1 atm plus a small quantity of a quenching vapor. The fill gas is generally argon at a pressure of less than 0.1 atm plus a small quantity of a quenching vapor. If a gamma ray interacts with the GM tube (by either the Photoelectric Effect or Compton scattering) it will produce an energetic electron that may pass through the interior of the tube.

Sample holder; We put our samples on it.

Various gamma-ray sources; We used ^{137}Cs with 10UC and 1.0UC as gamma ray sources.

Lead absorbers; Absorbs the radiation.

Chart recorder; Draws the peaks which caused by gamma rays.

Procedure

First, we put ^{137}Cs with 10UC on the sample holder in top three tray position, below the GM tube. Then, to take good data we need to find the operating voltage of GM tube. For this we set the counter to radioactivity, 100s and single mode. Then, we gave voltage to the GM tube between 300-500V and we did that by first giving 300V and then increasing it by 20V after every 100s until we reach 500V. Then, we see a plateau and we chose 460V as operating voltage. After finding the operating voltage, we didn't change our source but we set Geiger counter to 10s and countinuous mode. At this mode we took 100-data. After that we set Geiger counter to 1s continuous mode and again we took 100-data. Then, we changed our source to ^{137}Cs with 1.0UC at top 1 tray position. We did the same, at 10s and 1s continuous mode 100-data taking, procedure for ^{137}Cs with 10UC. At the end, we set our last source as it gives 1 count in 1s. For this we put the source at the bottom of the tray. Then, we run the chart recorder and gave as a drawing. After seeing aprroximately 100 peaks, we stopped the device.

Data

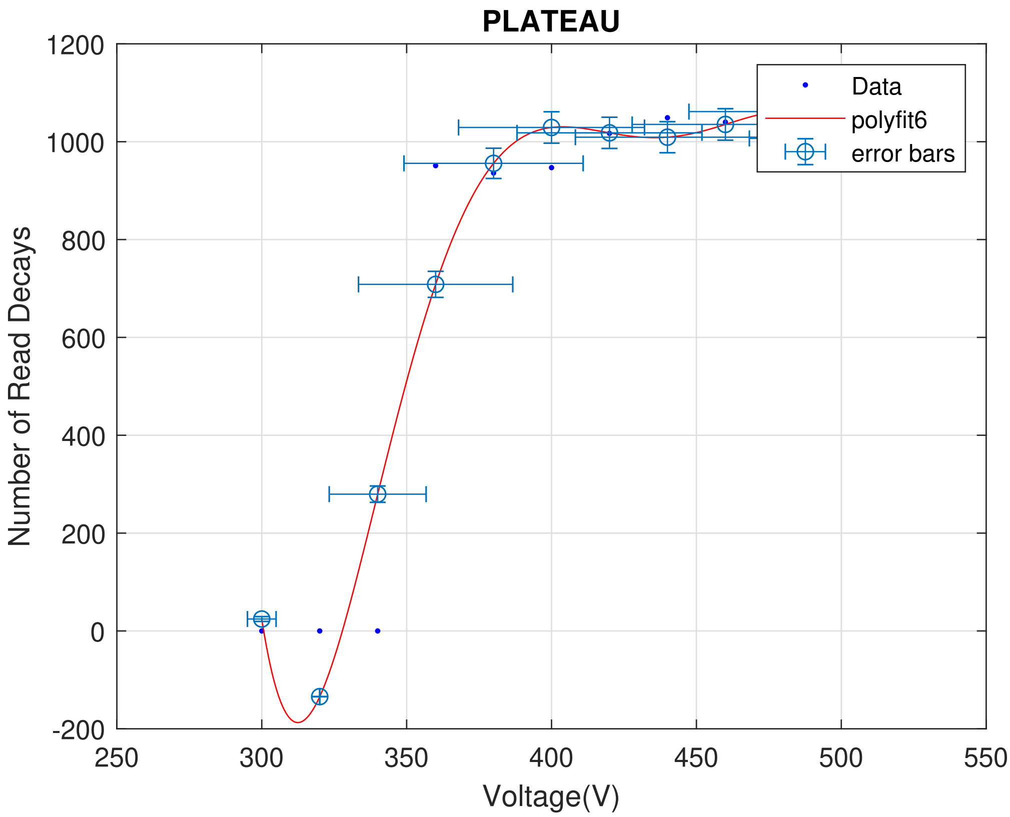

Figure 2: We take 460V from the plateau

The other datas can be shown from the analysis part of the report as histograms. I think that there is no need for showing the same thing twice.

Analysis

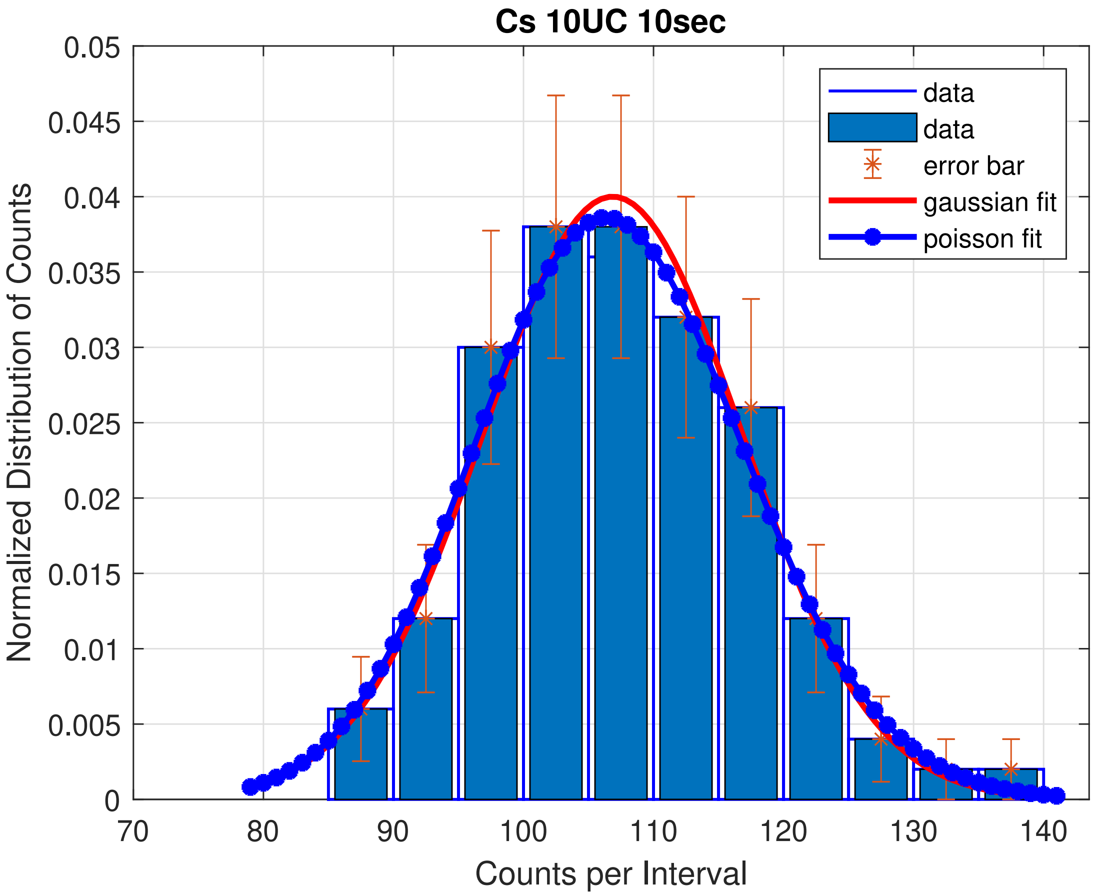

Figure 3: Histogram of ^{137}Cs with 10UC for 10s count

μ of the Gaussşian distribution is $\mu = 106.86 \pm 0.99$
σ of the Gaussşian distribution is $\sigma = 9.97271 \pm 0.60830$
λ of the Poisson distribution is $\lambda = 106.86 \pm 1.01$
${\chi_{gaussian}}^2 = 11.6763$ therefore ${\chi_{gaussian}}^2/ndf = 1.29737$
${\chi_{poisson}}^2 = 11.2223$ therefore ${\chi_{poisson}}^2/ndf = 1.12223$

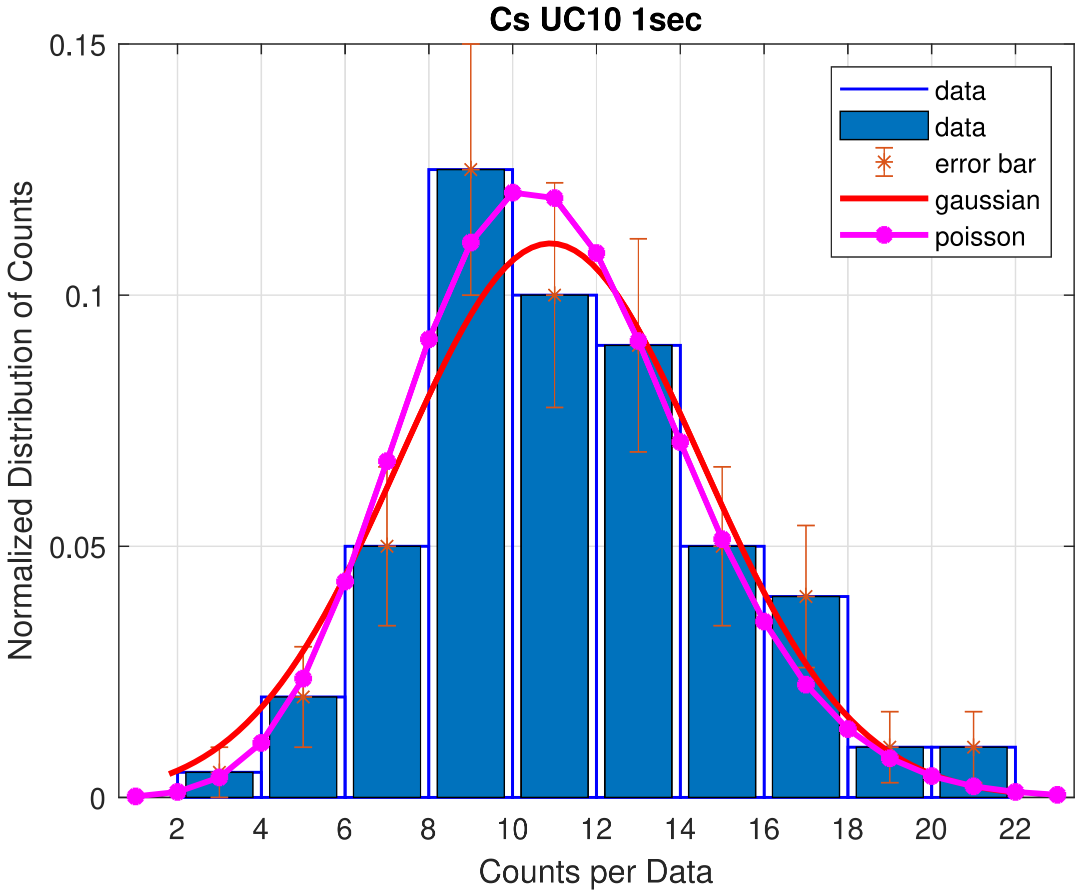

Figure 4: Histogram of ^{137}Cs with 10UC for 1s count

μ of the Gaussşian distribution is $\mu = 10.9 \pm 0.4$
σ of the Gaussşian distribution is $\sigma = 3.61674 \pm 0.22061$
λ of the Poisson distribution is $\lambda = 10.9 \pm 0.3$
${\chi_{gaussian}}^2 = 2.1036$ therefore ${\chi_{gaussian}}^2/ndf = 0.26295$
${\chi_{poisson}}^2 = 2.3335$ therefore ${\chi_{poisson}}^2/ndf = 0.25928$

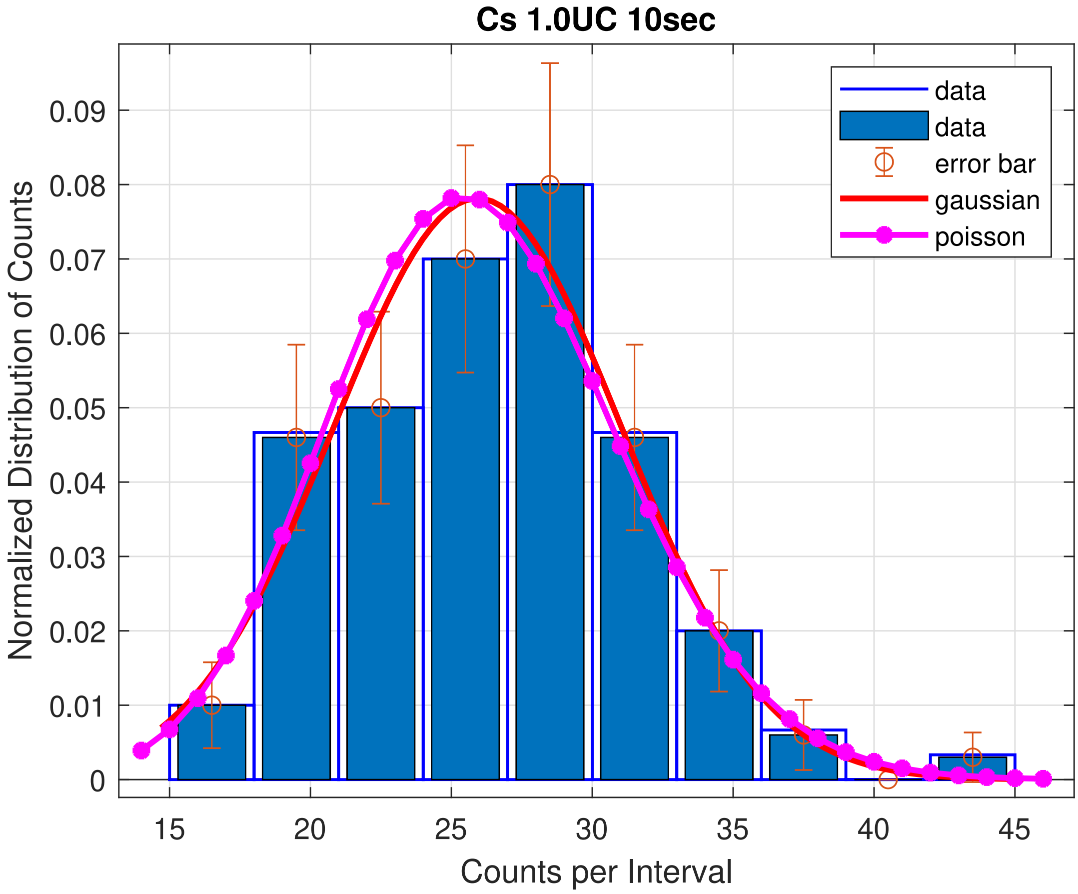

Figure 5: Histogram of ^{137}Cs with 1,0UC for 10s count

μ of the Gaussşian distribution is $\mu = 25.93 \pm 0.51$
σ of the Gaussşian distribution is $\sigma = 5.10546 \pm 0.31142$
λ of the Poisson distribution is $\lambda = 25.93 \pm 0.50$
${\chi_{gaussian}}^2 = 5.4334$ therefore ${\chi_{gaussian}}^2/ndf = 0.67918$
${\chi_{poisson}}^2 = 5.4963$ therefore ${\chi_{poisson}}^2/ndf = 0.61070$

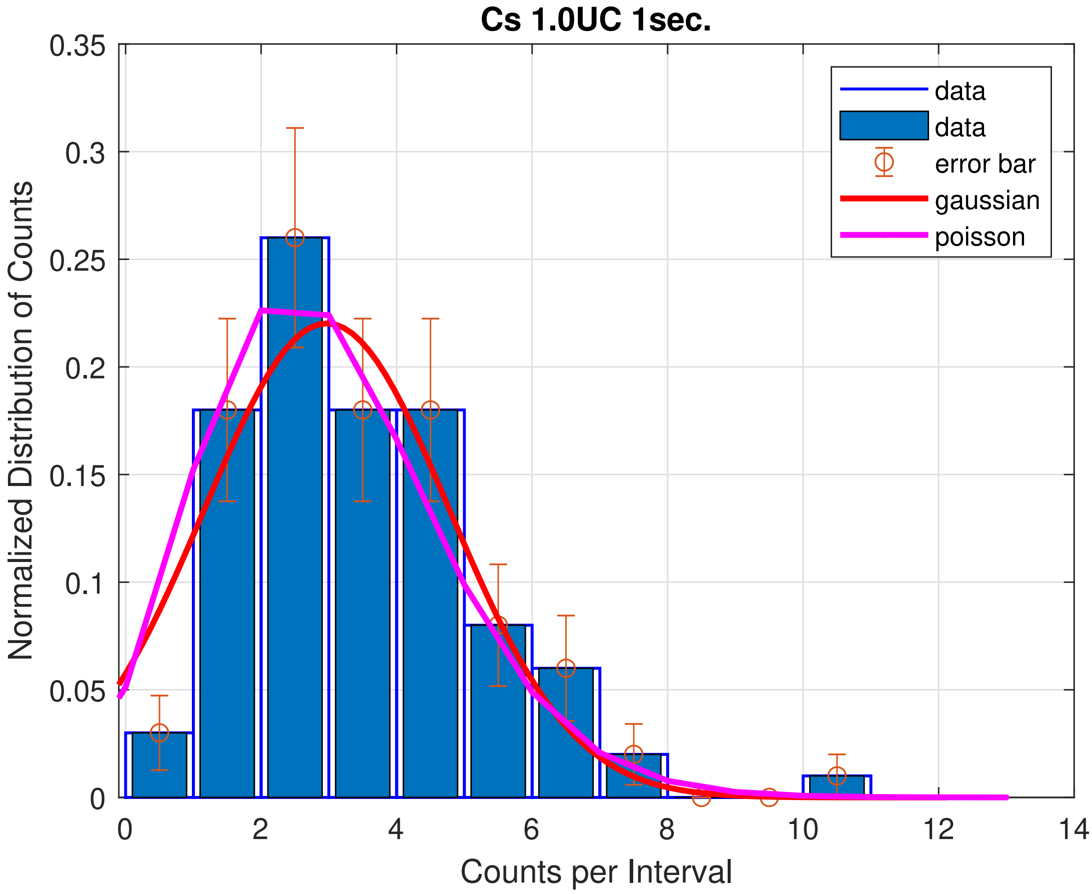

Figure 6: Histogram of ^{137}Cs with 1,0UC for 1s count

μ of the Gaussşian distribution is $\mu = 2.97 \pm 0.18$
σ of the Gaussşian distribution is $\sigma = 1.8116 \pm 0.1105$
λ of the Poisson distribution is $\lambda = 2.97 \pm 0.17$
${\chi_{gaussian}}^2 = 0.3274$ therefore ${\chi_{gaussian}}^2/ndf = 0.0364$
${\chi_{poisson}}^2 = 0.1734$ therefore ${\chi_{poisson}}^2/ndf = 0.01734$

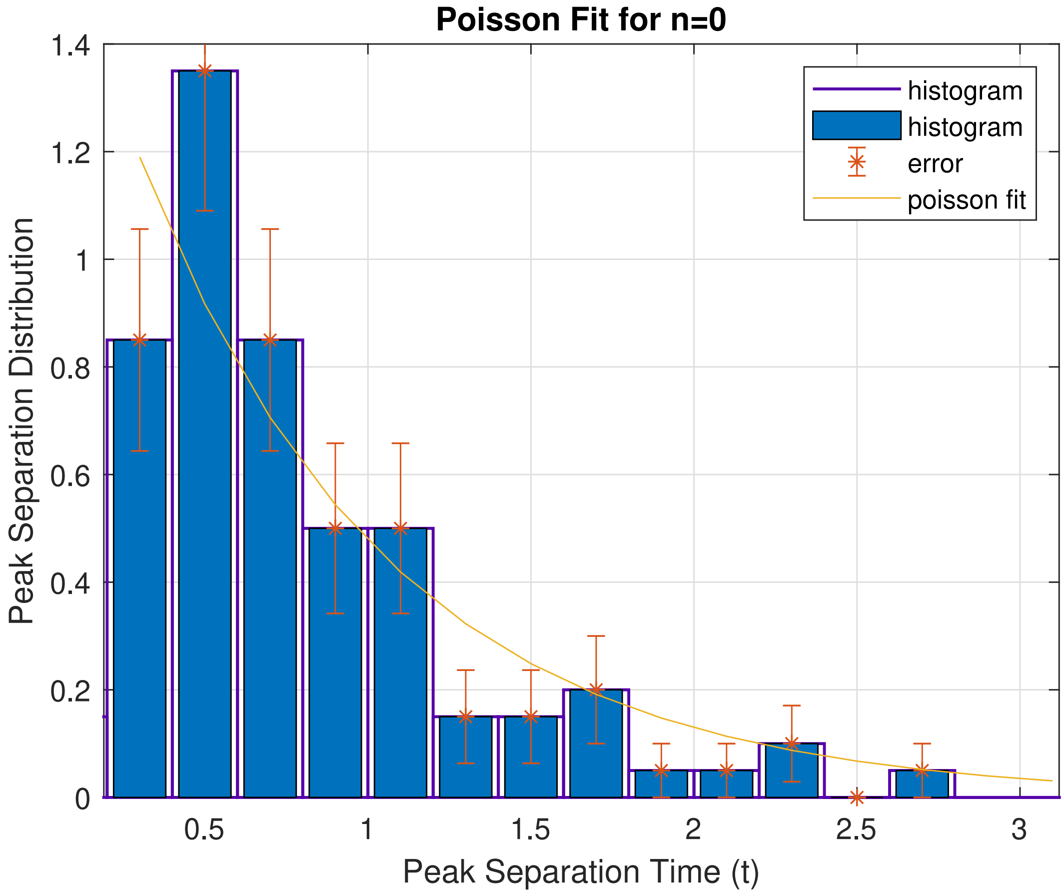

Figure 7: Histogram and Poisson fit for n=0

We fit $P(1,t) = \frac{(\alpha t)^0 e^{-\alpha t}\alpha}{0!}$ with some normalization constant to the histogram. n=0 corresponds to the situation where there is no peak between the peak points. That means, the distance between two consecutive peaks.

$b \cdot e^{-\alpha t}$ is the Poisson fit for the n=0.
$\alpha = 1.305 \pm 0.269$
$SSE = 0.393$
$R - square = 0.8316$
$AdjustedR - square : 0.8187$
$RMSE : 0.1739$

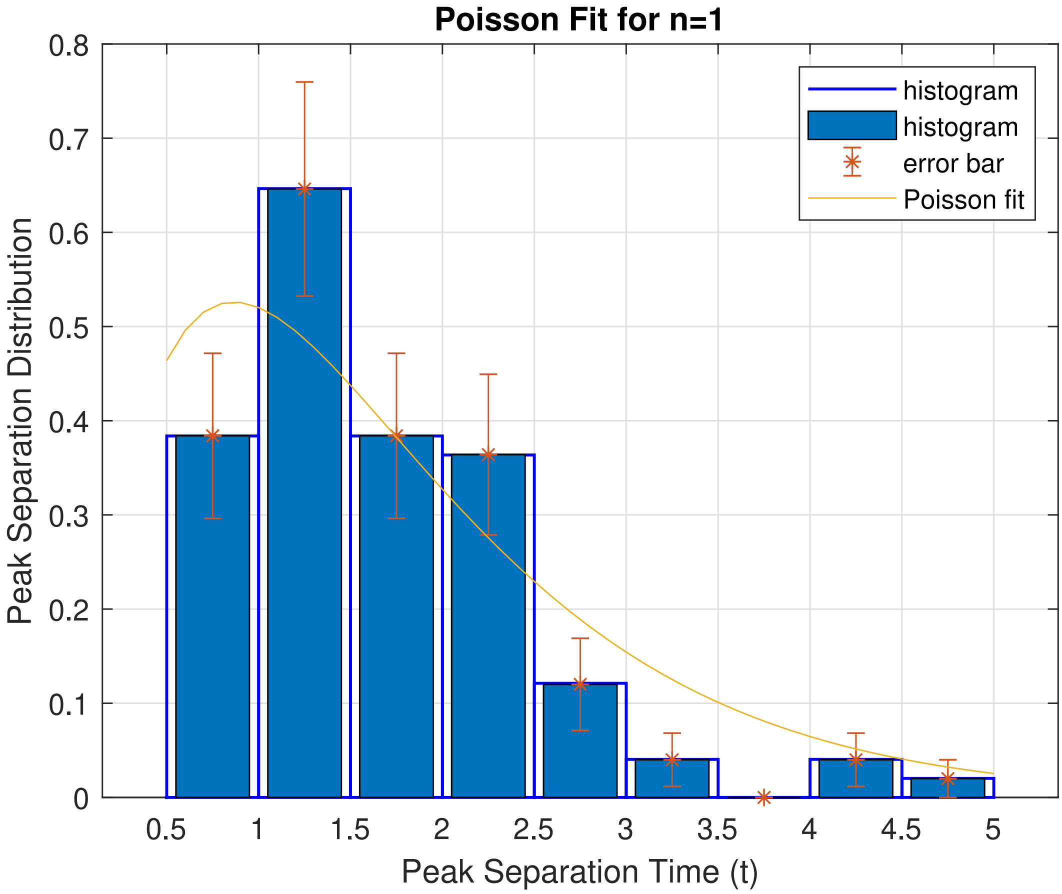

Figure 8: Histogram and Poisson fit for n=1

We fit $P_q(2,t) = \frac{(\alpha t)^1 e^{-\alpha t}\alpha}{1!}$ with some normalization constant to the histogram. n=1 corresponds to the situation where there is one peak between two peak points. Therefore we must measure distance between peaks by this mean.

$b \cdot \alpha^2 \cdot t \cdot e^{-\alpha t}$ is the Poisson fit for the n=0.
$\alpha = 1.157 \pm 0.399$
$SSE = 0.07073$
$R - square = 0.8312$
$AdjustedR - square : 0.8071$
$RMSE : 0.1005$
R-square always gives value between 0 and 1. Closer to 1 the R-square, the better the model fits data.

Discussion & Conclusion

We compared the Gaussian and Poisson fits for radioactive decaying. Also, we examine which one is behaving better in different situations. We see from the histograms that Poisson distribution gives better value of χ^2 than Gaussian distribution with small μ. Also, their value of χ^2 is not close. With bigger value of μ, we see that Poisson distribution approaches to Gaussian distribution and their χ^2's are very close(percentage wise). In the last part of the experiment, we see that Poisson distribution is fitted very well to the histograms, we understand this from the closeness of the R-square's to 1 and from the independence of gama decaying. Therefore, we see that radioactive decaying behaves like Poisson distribution.

References

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Advanced Physics Experiments - Gulmez, Prof. Dr. Erhan

Link

https://github.com/yarenaksel/442poisson.git