# Experiment 6, Scattering in 2D

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**Abstract**

The aim of the experiment to find the radius of the vertically placed cylindirical 2d target by finding its cross section. We shot a number of steel balls towards the target and counted the number of them scattered at a specific solid angle as a function of the scattering angle. We use two methods to find cross section length and compare them with the measured value of the cross section length by the vernier caliper.

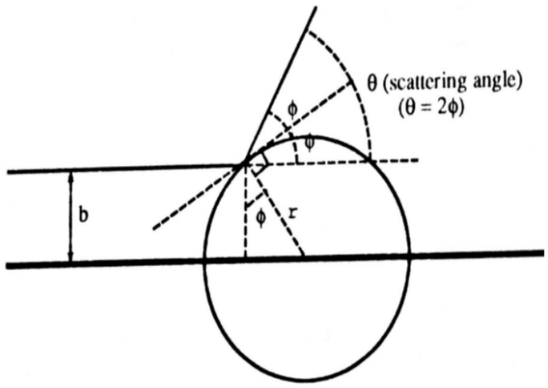
**Theoretical Motivation**

In nuclear or subatomic particle physics, probability that a given atomic nucleus or subatomic particle will collide or scatter is determined by their scattering parameters. Cross sections come in many varieties. They can help describe what happens when a particle hits a nucleus. In elastic reactions, particles bounce off one another but maintain their identities. In inelastic reactions, one or more particle shatters apart. In a resonance state, short-lived virtual particles appear.

In this experiment we assume that our steel balls as point particle. The differential cross section is defined simply as the ratio of the number of scattered particles observed in a region normalized by corresponding solid angle to the total number of particles shot towards the target.

$$\frac{d\sigma}{d\Omega} = \frac{Y}{Id\Omega} \tag{1}$$

where Y is the yield in $d\Omega$, $d\Omega$ is the solid angle and I is the incident flux (shots/m).




Assuming that the distance between the target and the detector (pressure sensitive paper around the rim of the scattering tray) is larger, the scattering angle $\theta$ given by

$$\frac{b}{r} = cos\frac{\theta}{2} \tag{2}$$

The number of particles scattered at a given angle is

$$dN = -Idb = \frac{Ir}{2}sin\frac{\theta}{2}d\theta \tag{3}$$

there is a minus sign in front of the $Idb$ because when we take derivative of the $\cos(\theta/2)$ we get a minus sign and count can't be negative. As a result there is minus sign as a sign correction. Hence, the differential cross section is simply

$$\frac{d\sigma}{d\theta} = \frac{dN}{Id\theta} = \frac{r}{2}sin\frac{\theta}{2}. \tag{4}$$

Because our experiment is at 2d, solid angle is $\theta$. Also because of that cross section is equal to $2r$. We can see this result from integrating the differential cross section over the full angular range, from 0 to $2\pi$.

$$\frac{d\sigma}{d\theta} = \frac{dN}{Id\theta} \tag{5}$$

$$\int \frac{d\sigma}{d\theta} = \int \frac{dN}{Id\theta} \tag{6}$$

$$\int_C d\sigma = \int_0^{2\pi} \frac{r}{2}\sin\left(\frac{\theta}{2}\right) d\theta \tag{7}$$

$$\sigma = 2r \tag{8}$$

Also,

$$2r = \sigma = \sum_{\theta} \frac{dN}{I} \tag{9}$$

Also, we need to find error propagation of the $sin(\frac{\theta}{2})$.

$$\sigma_y^2 = \sum_{i}^{m} \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_i^2 + \dots \tag{10}$$

$$\sigma_{sin\frac{\theta}{2}}^2 = \left(\frac{1}{2}\cos\left(\frac{\theta_i}{2}\right)\right)^2 .\sigma_{\theta_i}^2 \tag{11}$$

& for $f = a\frac{x}{y}$ or $f = axy$ ;

$$\frac{\sigma_f^2}{f^2} = \frac{\sigma_x^2}{x^2} + \frac{\sigma_y^2}{y^2} \tag{12}$$

**Apparatus, Experimental Procedure, Data**

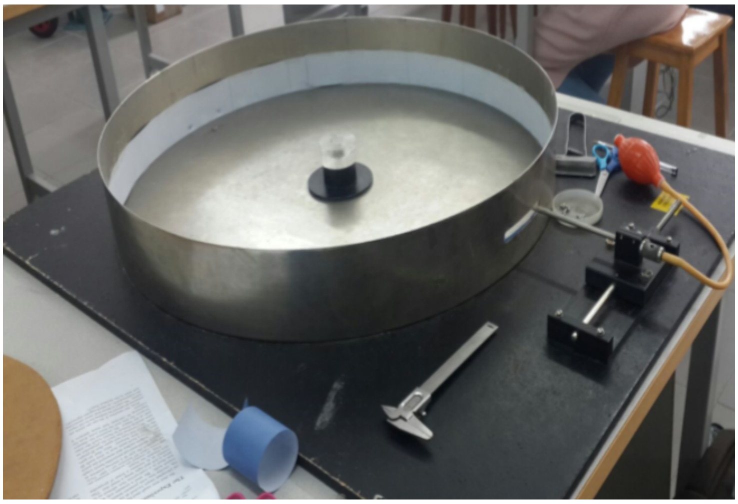


Figure 1: Set-up

Our apparatuses are
**Scattering Tray**; It is consist of the gun, circular tray, cylindrical plexiglass target and adjustable screw.
**Pressure Sensitive Paper Tape** ; It gives a mark when there is a touch to the paper and we covered the wall of the scattering tray with this. Therefore, we can read the marks of the balls. Therefore, sensitive

paper tape is like detector.
**20 Steel Balls**; We use them as point particles and we shoot them at the target. Then, we see their marks on the tape as scattering angle.
**Ruler**; We measure distance between the initial and final position of the gun.
**Vernier Caliper**; We measure the diameter of the cylindirical target.

### Procedure

- We cover the rim with the pressure sensitive paper tape.
- We shoot 20 balls from the gun to the target with the same pressure at the every position of the gun.
- After shooting 20 balls we move the gun by turning the screw one turn and shoot 20 ball again. We repeat the procedure until there is no scattering from the cylindirical target.
- We record the number of turns of the screw.
- We measure the distance between the initial and the final position of the gun.
- We mark angles on the tape(10 30 50... 350). Then, we count the number of marks on the tape in each interval.
- We measure the diameter of the cylindirical target with vernier caliper.

### Data

| d$\theta$ | dN | Error of dN | d$\theta$ | dN | Error of dN |
|---|---|---|---|---|---|
| 20 | 45 | 6.7 | 200 | 41 | 6.4 |
| 40 | 21 | 4.6 | 220 | 20 | 4.5 |
| 60 | 27 | 5.2 | 240 | 29 | 5.4 |
| 80 | 36 | 6 | 260 | 44 | 6.6 |
| 100 | 45 | 6.7 | 280 | 48 | 6.9 |
| 120 | 59 | 7.7 | 300 | 33 | 5.7 |
| 140 | 45 | 6.7 | 320 | 9 | 3 |
| 160 | 63 | 7.9 | 340 | 14 | 3.7 |
| 180 | 62 | 7.9 | - | - | - |

Figure 2: The Raw Data of $d\theta$, dN and error of dN

| $d\theta$ | dN | Error of dN |
|---|---|---|
| 20 | 59 | 7.7 |
| 40 | 30 | 5.5 |
| 60 | 60 | 7.7 |
| 80 | 84 | 9.2 |
| 100 | 89 | 9.4 |
| 120 | 88 | 9.4 |
| 140 | 65 | 8.1 |
| 160 | 104 | 10.2 |
| 180 | 124 | 11.1 |

Figure 3: The Cumulative Raw Data of $d\theta$, dN and error of dN

- The number of turns of the screw: 48.
- Distance between the initial and the final position of the gun (let's say $\boldsymbol{D}$): $6.8 \times 10^{-2} \pm 0.1 \times 10^{-2}$m.
- Diameter of the cylindirical target(let's say $\boldsymbol{2r_0}$):$5.77 \times 10^{-2} \pm 0.01 \times 10^{-2}$m.

**Analysis**

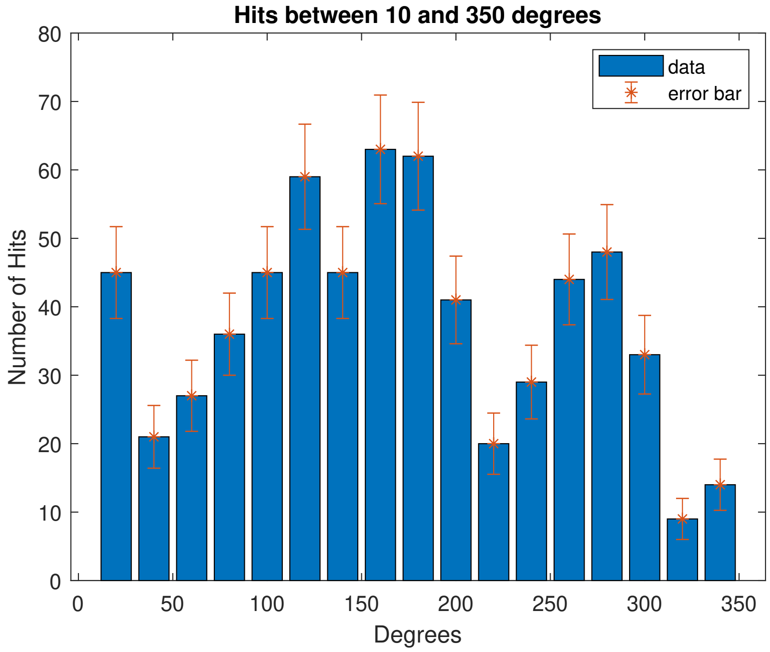



Figure 4: Histogram of d$\theta$, dN and error of dN

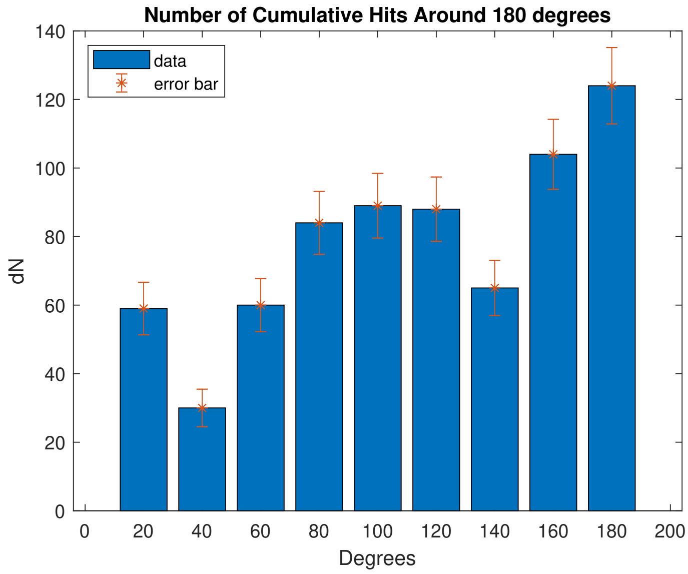



Figure 5: Histogram of cumulative d$\theta$, dN and error of dN

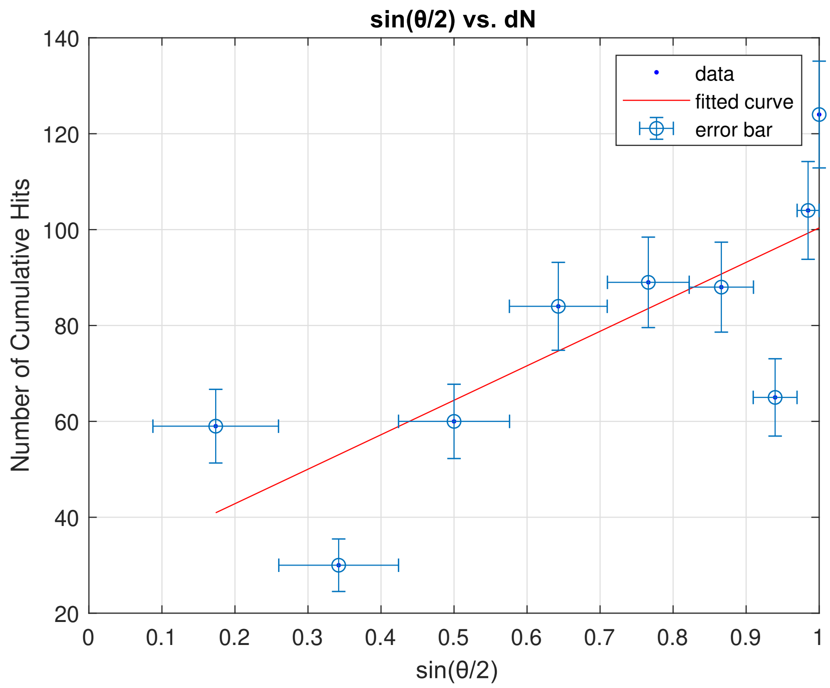



Figure 6: $\sin(\theta/2)$ vs. dN line fitting with SSE: 2545, R-square: 0.59, Adjusted R-square: 0.5315, RMSE: 19.07

Firstly, I will calculate the I.

$I = \frac{number\ of\ shots\ per\ turn}{D/48} = \frac{20}{6.8 \times 10^{-2}/48} = 14 \times 10^3 \pm 2.0 \times 10^2 (shots/m)$

Then, we can calculate the radius by using equation (3) and slope of the fitted line gives us dN/sin($\theta$/2)

Slope of the line p1= 71.91 $\pm$ 26.78

$$p1 = \frac{Ir}{2} d\theta \text{ }, where\ d\theta = \frac{\pi}{9} \pm \frac{\pi}{18} (rad.) \tag{13}$$

$$2r_1 = 0.058 \pm 0.036 (m)$$

Secondly, by equation (9)

$$2r_2 = \frac{total\ number\ of\ flux}{I} = \frac{703}{14 \times 10^3} = 0.050 \pm 0.0020 (m)$$

$Error_{\sigma_{r_1}} = \frac{|2r_0 - 2r_1|}{\sigma_{2r_1}} = 0.0083$

$Error_{\sigma_{r_2}} = \frac{|2r_0 - 2r_2|}{\sigma_{2r_2}} = 3.8$

**Conclusion & Discussion**

We used two methods to find diameter of the target and we compared our results with the measured value of the taget by vernier caliper. First method gave better result because the result is 0.0083$\sigma$ away from the measured value. Second method gave much worse result because the result is 3.8$\sigma$ away from the measured value. This difference between the first and the second method values may be caused by the number of marks on the tape(703) because the number of marks are very less than the expected(960). To fix this we may put an error to the number of shots per turn. The same type of error may be caused by some data points because they look too away from the line. Also, errors may be caused by us because we did many things by our hands, eyes, etc. We may not have given the gun same pressure each time, we may not have shot 20 balls each turn, we may have left our marks on the tape, we may have measured the distance between the initial and the final position of the gun wrongly. And also, errors may be caused by the set-up. The rim, the cylindrical target, the balls may not be perfectly circular. In order to minimize the error we may make our set-up more human independent especially the gun. Also, we could be more awake while performing the experiment.

### References

https://www.symmetrymagazine.org/article/speak-physics-what-is-a-cross-section
https://www.britannica.com/science/cross-section-physics
Advanced Physics Experiments - Gulmez, Prof. Dr. Erhan

### Link

https://github.com/yarenaksel/PHYS442-2d-Scattering.git