Experiment 7, Radioactive Decay

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Abstract

In this experiment our aim is to find the half-life of ^{220}Rn. The Radon gas is one of the elements produced in the radioactive decay chain starting from ^{232}Th(which comes as a Thorium salt). We first calculated the decay constant, λ, of the ^{220}Rn. Then found the half-life of it and compared our result with the accepted value of the half-life of the ^{220}Rn.

Theoretical Motivation

Radioactive decay occurs in unstable atomic nuclei – that is, ones that don't have enough binding energy to hold the nucleus together due to an excess of either protons or neutrons. It comes in three main types – named alpha, beta and gamma for the first three letters of the Greek alphabet. Radioactive decay is determined by quantum mechanics – which is inherently probabilistic. So it's impossible to work out when any particular atom will decay, but we can make predictions based on the statistical behaviour of large numbers of atoms.

However, unstable isotopes may decay through various processes but they all follow the same decay law:

$$N(t) = N_0 e^{-\lambda t} \tag{1}$$

where λ is the decay constant and N_0 is the initial number of the unstable isotope nuclei.

By taking the time derivative of the both sides

$$\frac{dN(t)}{dt} = -\lambda N_0 e^{-\lambda t} \tag{2}$$

$$\frac{dN(t)}{N(t)} = -\lambda dt \tag{3}$$

Since the amount of charge required before the Wulf's electroscope discharges is fixed, the current is proportional to the inverse of the time elapsed to accumulate the full charge, assuming the charging is linear with respect to the time:

$$Q = Is \rightarrow I = \frac{Q}{s} \rightarrow I \propto \frac{1}{s} \propto \frac{dN}{dt} \propto e^{-\lambda t} \tag{4}$$

where s is the time elapsed to reach the full charge. I is related to the number of particles that have been decayed.

$$T_i = \frac{t_i + t_{i+1}}{2} \tag{5}$$

$$s_i = t_{i+1} - t_i \tag{6}$$

Then, we get

$$\lambda t = lns + constant \tag{7}$$

where the constant is lns_0.

Now we define the half-life of a radioactive isotope as the time after which, on average, half of the original material will have decayed.

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}} \tag{8}$$

$$ln(2) = \lambda t_{1/2} \tag{9}$$

Also, we need for the later calculations;

$$\lambda_{weighted\ average} = \frac{\sum_i \omega_i \lambda_i}{\sum_i \omega_i} \tag{10}$$

where $w_i = \frac{1}{\sigma_i^2}$.

$$\sigma_y^2 = \sum_i^m \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_i^2 + \ldots \tag{11}$$

Apparatus, Experimental Procedure, Data

Our apparatuses are

Wulf's Electroscope; One of its electrode connected to the ground. It discharges after reaching full charge. We observed the discharging times and recorded them.

Thorium Salt; It produces Radon gas in its radioactive decay chain.

Ionization Chamber; There is ionized air inside the ionization chamber and this produces current.

HV Power Supply (0-5 kV); We give voltage to the ionization chamber.
Stopwatch; We record with it the time laps between the neutralizations of the Wulf's electroscope.

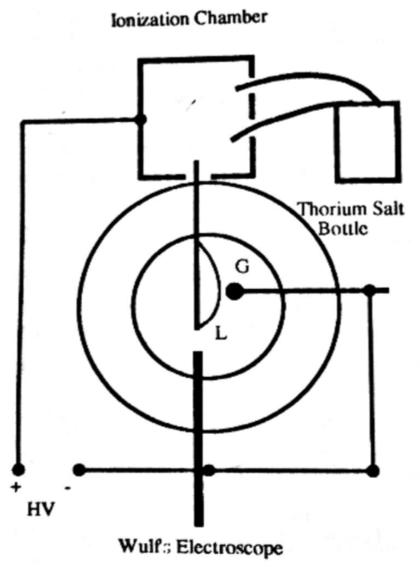

Figure 1: Schematic diagram of the Wulf's electroscope and the ionization chamber

Procedure

- We set the set-up as above illustration.
- We set the HV power supply to 2500V. We squeeze the Thorium salt bottle 5 times quickly. Then, we start the stopwatch.
- We record the time when the electroscope discharges each time. We continue to do it until approximately 3 minutes past.
- We do the the same process for 10 and 15 squeezes.
- Then, we set the HV power supply to 3000V, 3500V, 4000V and 4500V and repeat the above steps for these voltages.

Data

$t_i(sec.)$	5 squeezes	10 squeezes	15 squeezes
t_1	11.46	9.03	8.38
t_2	27.89	22.81	22.55
t_3	47.94	39.44	39.71
t_4	77.14	60.54	61.42
t_5	122.12	89.69	90.89
t_6	227.67	137.69	138.69

Figure 2: The Recorded Discharge Moments for Different Squeezes and for 2500V. Each t_i's σ is 0.10sec.

$t_i(sec.)$	5 squeezes	10 squeezes	15 squeezes
t_1	1.85	1.51	5.23
t_2	16.54	13.10	17.84
t_3	35.02	27.78	33.86
t_4	60.53	45.14	53.30
t_5	97.62	66.20	81.39
t_6	166.86	95.03	122.68
t_7	-	143.04	208.89

Figure 3: The Recorded Discharge Moments for Different Squeezes and for 3000V. Each t_i's σ is 0.10sec.

$t_i(sec.)$	5 squeezes	10 squeezes	15 squeezes
t_1	5.83	10.48	6.28
t_2	20.62	24.45	25.82
t_3	38.93	41.18	52.94
t_4	62.45	62.80	95.58
t_5	97.21	92.48	174.24
t_6	158.92	139.24	-

Figure 4: The Recorded Discharge Moments for Different Squeezes and for 3500V. Each t_i's σ is 0.10sec.

$t_i(sec.)$	5 squeezes	10 squeezes	15 squeezes
t_1	8.55	18.28	10.77
t_2	23.13	45.05	24.05
t_3	40.32	87.10	41.91
t_4	62.01	180.22	65.55
t_5	92.46	-	101.06
t_6	141.94	-	169.98

Figure 5: The Recorded Discharge Moments for Different Squeezes and for 4000V. Each t_i's σ is 0.10sec.

$t_i(sec.)$	5 squeezes	10 squeezes	15 squeezes
t_1	8.61	2.89	13.41
t_2	22.21	18.86	29.91
t_3	40.75	39.80	50.07
t_4	65.14	62.95	77.15
t_5	99.25	98.29	117.72
t_6	157.75	164.00	204.43

Figure 6: The Recorded Discharge Moments for Different Squeezes and for 4500V. Each t_i's σ is 0.10sec.

Analysis

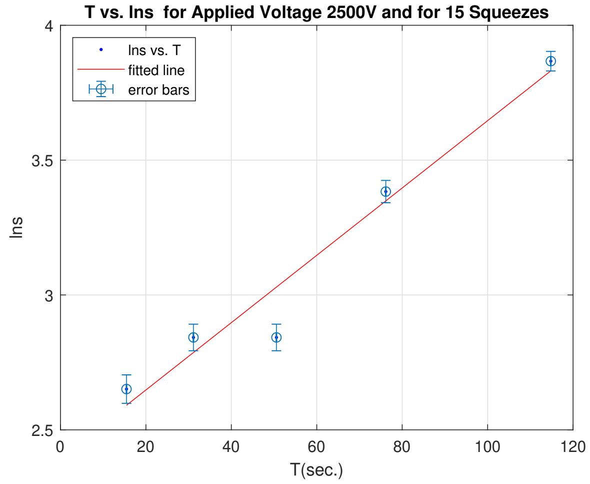

Figure 7: Goodness of fit: SSE: 8.074e-05, R-square: 0.9999, Adjusted R-square: 0.9999, RMSE: 0.005188

Because the errors of the T's are so small, it's hard to see them. Here I use the R-square instead of the χ^2 because I use MATLAB and I couldn't a way to see χ^2 in MATLAB(it's quite complicated). R-square always gives value between 0 and 1. Closer to 1 the R-square, the better the model fits data. Therefore, the fitted line to the data is quite good.
The slope of the line gives the λ from eq.(7).

$$\lambda_3 = 0.01221 \pm 0.00010 s^{-1}$$

Also, we need to know the other λ's but their graphs are not here;

$Given\ Voltage(V)$	$5\ squeezes$	$10\ squeezes$	$15\ squeezes$
2500	$\lambda_1 = 0.01204 \pm 0.00105$	$\lambda_2 = 0.01278 \pm 0.00095$	$\lambda_3 = 0.01221 \pm 0.00010$
3000	$\lambda_4 = 0.01255 \pm 0.00054$	$\lambda_5 = 0.01605 \pm 0.00172$	$\lambda_6 = 0.01235 \pm 0.00049$
3500	$\lambda_7 = 0.01247 \pm 0.00023$	$\lambda_8 = 0.01234 \pm 0.00022$	$\lambda_9 = 0.01163 \pm 0.00139$
4000	$\lambda_{10} = 0.01219 \pm 0.00031$	$\lambda_{11} = 0.01219 \pm 0.00031$	$\lambda_{12} = 0.01369 \pm 0.00072$
4500	$\lambda_{13} = 0.01254 \pm 0.00098$	$\lambda_{14} = 0.01162 \pm 0.00112$	$\lambda_{15} = 0.01196 \pm 0.00012$

Figure 8: The Slopes of the all Lines in Unit of 1/sec.

Then, we find $\lambda_{w.ave.} = 0.0122 \pm 0.0001(1/sec.)$.

Now we can find the $t_{1/2}$ from the eq.(9).

$$t_{1/2} = 56.8 \pm 0.5(sec.)$$

Discussion & Conclusion

We found our $t_{1/2}$ 2.4σ far from the accepted value of the half-life of the ^{220}Rn(55.6sec.). If we consider the all errors, actually it is not that much. These errors may caused by squeezing the Thorium salt bottle not equally each time and our accuracy to detect and record the discharging moments of the electroscope were not well. We gave our accuracy error as 0.10sec. but this value is just the best response of a human to something. Therefore, we couldn't response to the discharging moments at best because we may not able to do it each time. Also, we need to clear the ionization chamber after every turn but we may not did it well. Other errors may caused by the set-up. These are may caused by power supply because it may not work well. Also, the electroscope may caused error because it is a sensitive device it may effected by the radiation coming from the space. We may have found closer value to 55.6sec by taking more data. Also, we saw that λs are independent of the values of the voltages and the number of squeezes. Because we know that λ is the characteristic value of the isotopes, it is not affected by outer changes.

References

http://www.iop.org/resources/topic/archive/radioactivity/
Advanced Physics Experiments - Gulmez, Prof. Dr. Erhan

Link

https://github.com/yarenaksel/Radioactive-Decay.git