Report on final project
Optimizing optimal transport
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Optimal transport is a problem for finding

$$\mathcal{W}(p,q) := \min_{\pi \in U(p,q)} \langle C, \pi \rangle$$

where p and q are probability distributions,

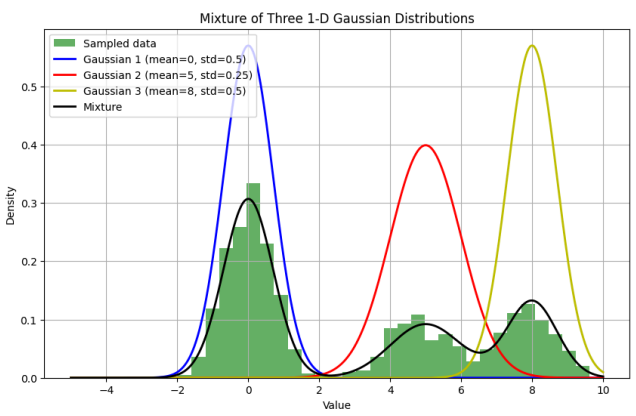
$$U(p,q) \triangleq \{\pi \in \mathbb{R}_{+}^{n \times n} : \pi \mathbf{1}_n = p, \pi^T \mathbf{1}_n = q\}.$$

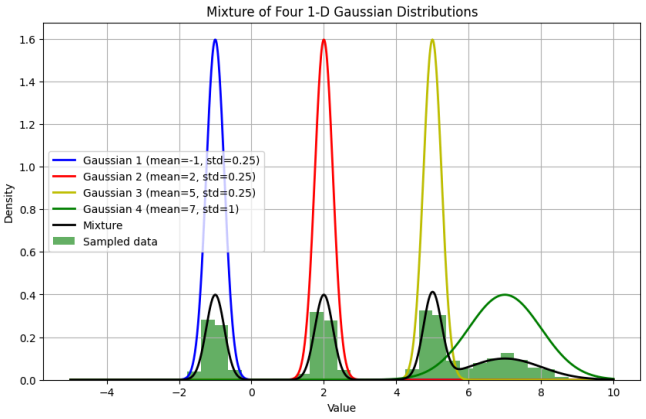
and C is a cost matrix.

There are lot's of approaches to solve this task. To make it M-convex let's regularize by adding regularizing term:

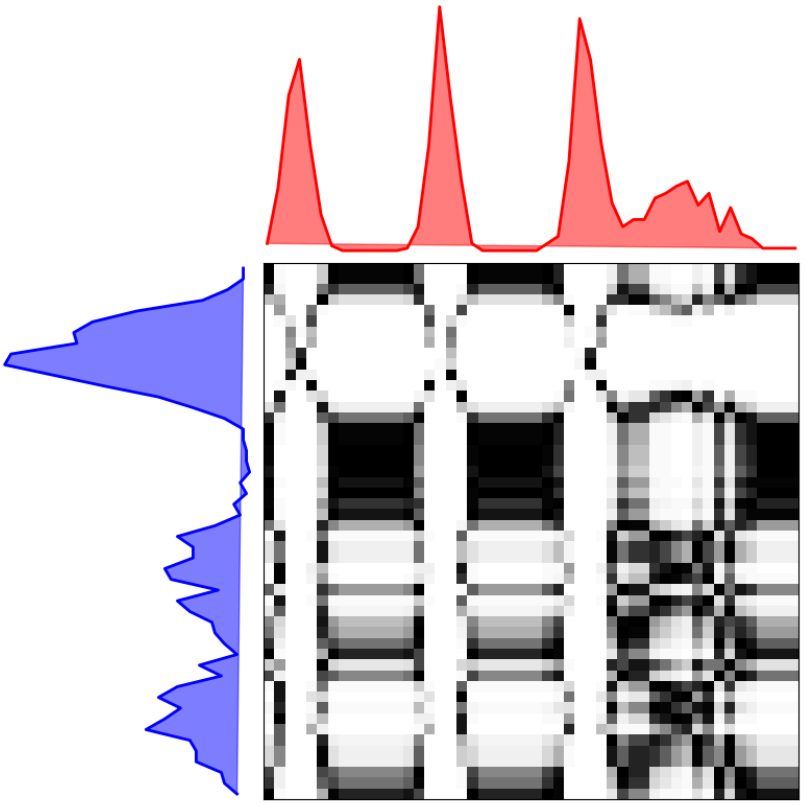
$$\mathcal{W}_\gamma(p,q) := \min_{\pi \in U(p,q)} \{g(\pi) = \langle \pi, C \rangle + \gamma H(\pi)\}$$

I considered the problem of transporting optimally between these two highly entropic distributions (marked black)



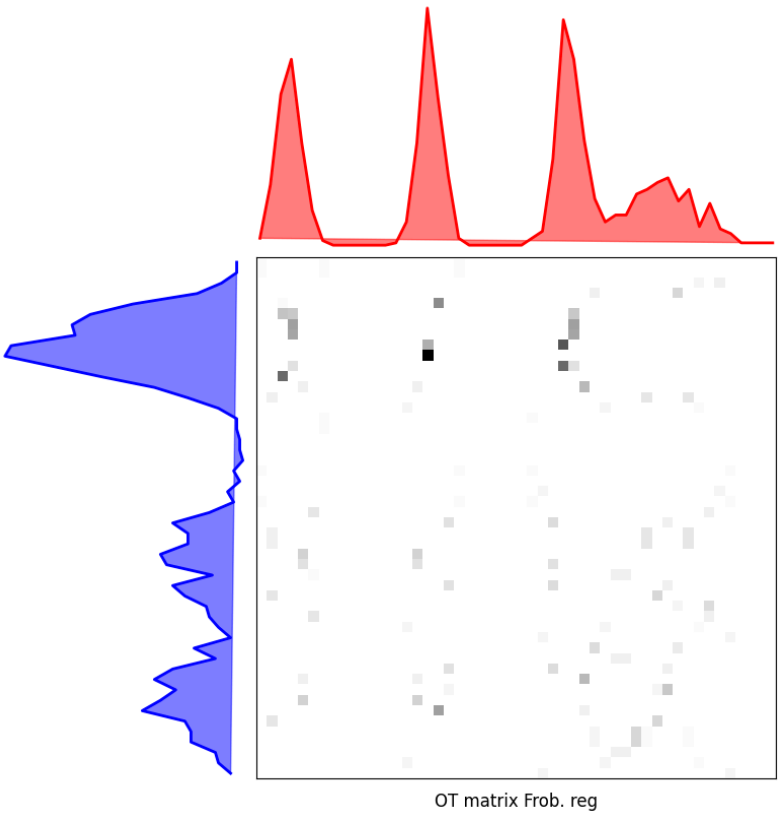


The asymptotically best algorithm is Sinkhorn's algorithm (see [1]). It give us the following transportation matrix π.



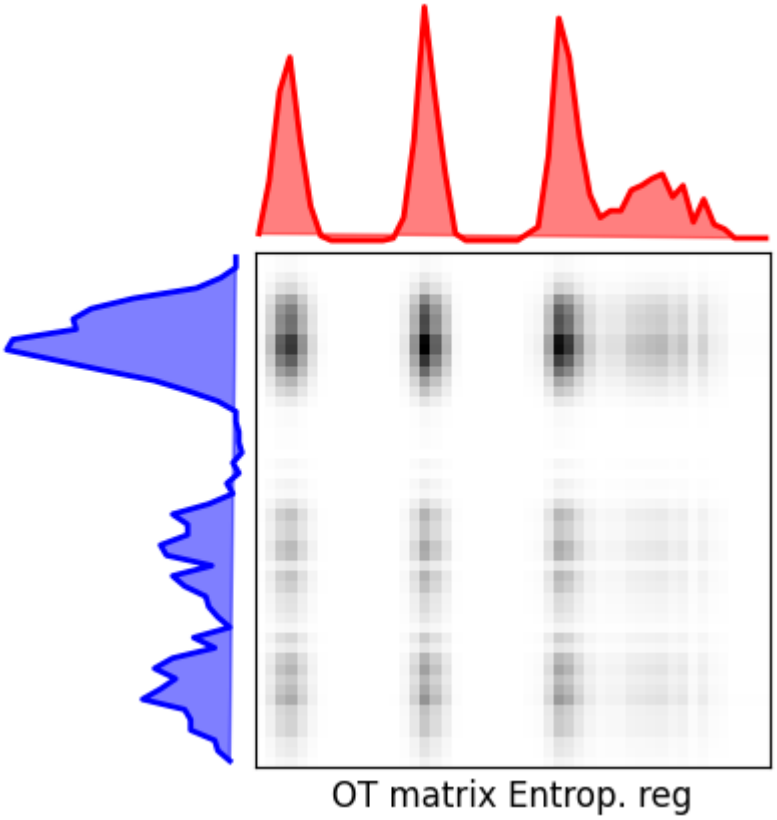
OT matrix Sinkhorn

But Frank-Wolfe algorithm (see [2]) gives this:
with quadratic regularization:



OT matrix Frob. reg

and with entropic regularization:



The last one looks much cleaner and appropriate (however, worked 2 times longer)

Literature:

[1] https://arxiv.org/pdf/2210.11368

[2] https://arxiv.org/pdf/1307.5551