Measurement Error (2)

1. $\underline{Z} \sim MVN(\underline{0}, \Sigma_Z)$

2. $\Sigma_Z = \begin{bmatrix} \sigma_A^2 & 0 \\ 0 & \sigma_B^2 \end{bmatrix}$

3. $\sigma_A^2, \sigma_B^2 \sim \Gamma(0.001, 1000)$

Source Isotope Values (15)

4. $\underline{X_s} \sim MVN(\underline{\mu_s}, \Sigma_s + \Sigma_{disc} + \Sigma_Z)$

5. $\underline{\mu_s}^\top = (\mu_A, \mu_B)$

6. $\mu_A, \mu_B \sim N(0, 1000)$

7. $\Sigma_s = \begin{bmatrix} \sigma_A^2 & \rho\sigma_A\sigma_B \\ \rho\sigma_A\sigma_B & \sigma_B^2 \end{bmatrix}$

8. $\sigma_A^2, \sigma_B^2 \sim \Gamma(0.001, 1000)$

9. $\rho \sim Unif(-1, 1)$

Source Concentrations (12)

10. $\underline{D_s} \sim MVN(\epsilon_s \underline{\mu_{D,s}}, \Sigma_{D,s})$

11. $\underline{\mu_{D,s}}^\top = (\mu_A, \mu_B)$

12. $\mu_A, \mu_B \sim N(0, 1000)$

13. $\Sigma_{D,s} = \begin{bmatrix} \sigma_A^2 & 0 \\ 0 & \sigma_B^2 \end{bmatrix}$

14. $\sigma_A^2, \sigma_B^2 \sim \Gamma(0.001, 1000)$

Mixtures (46)

15. $\underline{M_j} \sim MVN(\underline{\mu_j}, \Sigma_j + \Sigma_{res} + \Sigma_Z)$

16. $\Sigma_j = \sum_s Iso_{j,s}^2 \Sigma_s$

17. $\underline{\mu_j} = \sum_s Iso_{j,s} \underline{\mu_s}$

18. $Iso_{j,s} = \frac{\mu_{D,s} i_{j,s}}{\sum_s \mu_{D,s} i_{j,s}}$

19. $\underline{i_{j,s}} \sim CLR(\underline{f_s}, \Sigma_i)$

20. $\underline{f_s} \sim CLR(\underline{\mu}, \Sigma_f)$

21. $\underline{\mu}^\top = (\mu_A, \mu_B)$

22. $\mu_A, \mu_B \sim N(0, 1000)$

23. $\Sigma_f = \Sigma_i = \begin{bmatrix} \sigma_A^2 & 0 \\ 0 & \sigma_B^2 \end{bmatrix}$

24. $\sigma_A^2, \sigma_B^2 \sim \Gamma(0.001, 1000)$

25. $\Sigma_{res} = \begin{bmatrix} \sigma_A^2 & 0 \\ 0 & \sigma_B^2 \end{bmatrix}$

26. $\sigma_A^2, \sigma_B^2 \sim \Gamma(0.001, 1000)$

1. Measurement error (Z) follows a multivariate normal distribution. Mass spectrometer calibration runs (i.e., isotope standards) are centralized and used to estimate the variance (Σ_z) of the distribution.
2. The variance terms ($\sigma_{A,B}^2$) makeup the covariance matrix (Σ_z).
3. Priors on the variances ($\sigma_{A,B}^2$) are assumed to follow a gamma distribution.

4. Each source (X_s) follows a multivariate normal distribution. Source error (Σ_s), measurement error (Σ_z), and discrimination error (Σ_{disc}) are incorporated into the source distributions.
5. Source isotope values (μ_s) are composed of means for each isotope ($\mu_{A,B}$).
6. Priors on source istope value means ($\mu_{A,B}$) are assumed to follow a normal distribution.
7. The variance terms ($\sigma_{A,B}^2$) make up the source covariance matrix (Σ_s).
8. Priors on source isotope variances ($\sigma_{A,B}^2$) are assumed to follow a gamma distribution.
9. Priors on source isotope correlation values (ρ) are assumed to follow a uniform distribution.

10. Elemental concentrations values (D_s) follow a multivariate normal distribution.
11. Elemental concentration values ($\mu_{D,s}$) are composed of means for each isotope ($\mu_{A,B}$) and are rescaled by the digestibility (ϵ_s).
12. Priors on concentration means ($\mu_{A,B}$) are assumed to follow normal distributions
13. The variance terms ($\sigma_{A,B}^2$) makeup the concentration covariance matrix ($\Sigma_{D,s}$).
14. Priors on the concentration variances ($\sigma_{A,B}^2$) are assumed to follow a gamma distribution.

15. Each individual in the mixture data (M_j) follows a multivariate normal mixture distribution with multiple sources of error. These error sources include mixture error (Σ_j), residual error (Σ_{res}) and measurement error (Σ_Z). The mixture distribution (M_j) is a weighted sum of the source isotope distributions (X_s). Weights are the fraction of an assimilated isotope for a given source and individual ($Iso_{j,s}$).
16. The covariance matrix for the mixture data (Σ_j) is the weighted sum of all source covariance matrices (Σ_S).
17. Mixture data for each individual (μ_j) are composed of mixture means for each isotope ($\mu_{A,B}$). These mixture means ($\mu_{A,B}$) are a weighted sum of all source means (μ_s).
18. The contribution of a particular source to an individual ($i_{j,s}$) is calculated using the Phillips and Koch (2002) concentration dependence model.
19. Each individual's food source contribution ($i_{j,s}$) is distributed using the CLR transform of the normal distribution as described in Semmens et al. (2009).
20. Population level food source contributions (f_s) are distributed using the CLR transform of the normal distribution.
21. The population level contribution (μ) is composed of mean values for each isotope ($\mu_{A,B}$).
22. Priors on the population level ($\mu_{A,B}$) are assumed to follow a normal distribution.
23. Each individual's covariance matrix (Σ_i) and the population covariance matrix (Σ_f) are made up of variance terms for each isotope ($\sigma_{A,B}^2$).
24. Priors on the mixture proportion variances ($\sigma_{A,B}^2$) are assumed to follow a gamma distribution.
25. The residual error term (Σ_{res}) is composed of variance terms for each isotope ($\sigma_{A,B}^2$).
26. Priors on residual error variances ($\sigma_{A,B}^2$) are assumed to follow a gamma distribution.