Q2

a) Assume CFLs closed under intersections then

$$A = \{a^m b^n c^n \mid m,n \geq 0\} \quad B = \{a^n b^n c^m \mid m,n \geq 0\} \longrightarrow A \cap B = \{a^n b^n c^n \mid n \geq 0\}$$

We know that $A \cap B$ is not CFL therefore we reach to a contradiction. Our assumption was wrong. □

b) Assume CFLs closed under complementation. Then $\overline{A}$ and $\overline{B}$ are CFLs then we can write

$$\underbrace{\overline{(\underbrace{\overline{A} \cup \overline{B}}_{\text{CFL}})}}_{\text{should be}}$$

CFL since we assumed CFLs are closed under complement

$$\overline{(\overline{A} \cup \overline{B})} = A \cap B$$ which is not CFL, contradiction.

Q3)

a) Pick a word $w \in B$ s.t. $|w| \geq p$ where p is the pumping length $w = \underbrace{0^p 1^{2p} 0^p}_{4p > p} \in B$ then

we can split w as $uvxyz$ where $|xy| > 0$ and $|vxy| \leq p$ then we have 5 cases

* vxy contains 0s from left then
uv^2xy^2z contains more 0s than 1s

* vxy contains 0s and 1s from left then
uv^2xy^2z contains $00\ldots\underbrace{00\ldots11\ldots00\ldots}1111 0\ldots0$

0s follow 1s on the left and not a palindrome

* vxy contains 1s only then
  uv^2xy^2z contains more 1s than 0s
* vxy contains 1s and 0s then
  similar to case in *

* vxy contains 0s from right then
  uv^2xy^2z contains more 0s than 1s

b) Similarly let p be P.L and pick $w \in 1^p3^p2^p4^p$ then we can split w as $uvxyz$ where $|vy|>0$ $|vxy| \leq p$

* vxy contains only one of the digits (e.g 1)
  then uv^2xy^2z contains more 1s than 2s.
* vxy contains 2 of the digits (e.g 2 and 4)
  then uv^0xy^0z contains less 2s and 4s than 1s and 3s

c) Let p be P.L s be $1^{2p}1^p0^p1^{2p} \in A$ $|w| \geq p$ where $\tilde{6}p$
then we can split s as $uvxyz$ where we have $|vy|>0$ $|vxy| \leq p$ $s = \underbrace{1^{2p}}_{w}\underbrace{1^p0^p}_{t}\underbrace{1^{2p}}_{w^R}$

* vxy contains only 1s from w then uv^2xy^2z
  $|w| > 2p$ but only has 1s
  w^R should also have cardinality $2p$ but can only do it by borrowing 0s from t meaning $(w^R)^R \neq w$
* vxy contains 1s from w and t then uv^2xy^2z will have $|w|>2p$ and similarly $|w^R| > 2p$ can only be achieved by borrowing 0s from t.
* vxy containing 1s, 1s and 0s or 0s only from t then uv^2xy^2z will make it impossible to obtain $(w^R)^R = w$ by breaking the balance of 1s and 0s.

... 1s from w^R

and 0s.

* vxy contains 0s from t and 1s from w^R

** If amount of 0s in vxy is more than 1s then w^R will borrow 0s from t and $(w^R)^R \neq w$

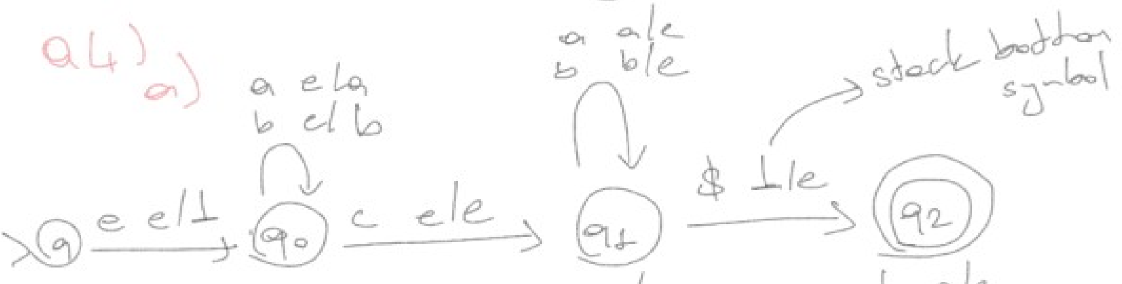
** If amount of 0s in vxy is equal to amount of 1s in vxy

then in uv^2xy^2z we will have $|w^R| = |t| \neq |w|$

** If amount of 1s in vxy is larger than amount of 0s in vxy then in uv^0xy^0z we will have w^R with 0s but w won't have any 0s.

* vxy contains only 1s from w^R then in uv^0xy^0z we will have w^R with 0s but w won't have any 0s.

Q4) a)



b)