

# CENG 280

## Formal Languages and Abstract Machines

Spring 2023-2024

### Homework 3

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#### Question 1

Draw a PDA for each of the following languages.

- $\{a^m b^n \mid m, n \geq 0 \text{ and } n \leq m \leq 2n\}$
- $\{w \mid w \in \{a, b\}^* \text{ and } w \text{ has twice as many } b's \text{ as } a's\}$
- $\{a^n b^m c^m d^n \mid n, m \geq 0 \text{ and either } n \text{ or } m \text{ is odd but not both.}\}$

#### Question 2

- Use the languages  $A = \{a^m b^n c^n \mid m, n \geq 0\}$  and  $B = \{a^n b^n c^m \mid m, n \geq 0\}$  to show that the class of context-free languages is not closed under intersection.
- Use the first part and DeMorgan's Law to show that the class of context-free languages is not closed under complementation. Hint: you can use proof by contradiction.

#### Question 3

For the following languages prove that each one of those languages are not context-free.

- Let  $B$  be the language of all palindromes over  $\{0, 1\}$  containing an equal number of 0s and 1s.
- Let  $\Sigma = \{1, 2, 3, 4\}$  and  $C = \{w \in \Sigma^* \mid \text{in } w \text{ number of 1s equals number of 2s, and number of 3s equals number of 4s}\}$ .
- Let  $A = \{wtw^R \mid w, t \in \{0, 1\}^* \text{ and } |w| = |t|\}$ .

#### Question 4

For the following languages prove that each one of those languages are deterministic context-free.

- $\{wcw^R \mid w \in \{a, b\}^*\}$
- $\{ca^m b^n \mid m \neq n\} \cup \{da^m b^{2m} \mid m \geq 0\}$