

1b) prove by induction that the sum of all elements in the  $m^{\text{th}}$  row of the triangle is  $2^m$ .

First, we will define  $a_{m,n}$  term

such  $a_{m,n} = a_{m-1,n} + a_{m-1,n-1}$

where  $m$  is the row and  $n$  the column

prove by Induction:

$m=0$

$2^0 = 1$  and the sum of line 0 is indeed 1

Induction step:

Assuming that  $a_{m,0} + a_{m,1} + \dots + a_{m,n} = \sum_{k=0}^n a_{m,k} = 2^m$  is correct,

we need to prove  $a_{m+1,0} + a_{m+1,1} + \dots + a_{m+1,n} + a_{m+1,n+1} = \sum_{k=0}^{n+1} a_{m+1,k} = 2^{m+1}$

proof:

$$\sum_{k=0}^{n+1} a_{m+1,k} = a_{m+1,0} + a_{m+1,1} + a_{m+1,2} + \dots + a_{m+1,n} + a_{m+1,n+1}$$

using def of pascal triangle

$$= a_{m,0} + a_{m-1} + a_{m,1} + a_{m-1} + a_{m,2} + a_{m-1} + \dots + a_{m,n} + a_{m-1} + a_{m,n+1} + a_{m-1}$$

simplify:  $= a_{m-1} + 2a_{m,0} + 2a_{m,1} + \dots + 2a_{m,n} + a_{m,n+1}$

simplify  $= 0 + 2(a_{m,0} + a_{m,1} + \dots + a_{m,n}) + 0$

$$= 2 \cdot \sum_{k=0}^n a_{m,k}$$

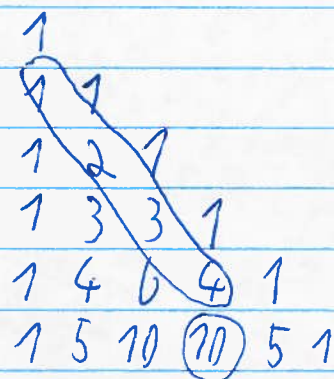
using the induction hypo  $= 2 \cdot 2^m = 2^{m+1} \quad \square$

● 1c) Prove that:

$$C(i,0) + C(i+1,1) + \dots + C(i+j,j) = C(i+j+1,j) \quad \forall i, j \in \mathbb{N}$$

example  $i=1, j=3$

$$\begin{aligned} & C(1,0) + C(1+1,1) + C(1+2,2) + C(1+3,3) = \\ & = C(1,0) + C(2,1) + C(3,2) + C(4,3) = \\ & = C(1,0) + C(2,1) + C(3,2) + C(4,3) = \\ & = 1 + 2 + 3 + 4 = 10 \end{aligned}$$



Proof by induction: ( $j$  is the variable while  $i$  is constant)

base case:  $j=0$

$C(i,0)=1$  for any  $i$ , and also

$$C(i+j+1,j) = C(i+0+1,0) = C(i+1,0) = 1 \quad \text{for any } i. \quad \textcircled{a}$$

Induction step: Assuming  $\sum_{k=0}^j C(i+k,k) = C(i+j+1,j)$  is true  
 need to prove  $\sum_{k=0}^{j+1} C(i+k,k) = C(i+j+2,j+1)$

proof:  $\sum_{k=0}^{j+1} C(i+k,k) = C(i,0) + C(i+1,1) + \dots + C(i+j,j) + C(i+j+1,j+1) =$

using induction =  $C(i+j+1,j) + C(i+j+1,j+1)$

using definition =  $C(i+j+2,j+1) \quad \square$

OE pascal triangle

$$a_{m,n} = a_{m-1,n} + a_{m-1,n-1}$$