Homework 1

PHYS 600
Sep 8, 2023

Numerical Integration

In[103]:=

$$f[\Omega\_,\ z\_] := \frac{1}{\left(\Omega\ (1+z)^3 + (1-\Omega)\ (1+z)^{\frac{3}{2}}\right)^{\frac{1}{2}}};$$

In[104]:=

```
integrateFunction[Ω_?NumericQ, z0_?NumericQ] := NIntegrate[f[Ω, z], {z, 0, z0}];
```

In[105]:=

```
ΩValues = {0, 0.3, 0.7, 1};
```

In[106]:=

```
(*Generate a plot for each Ω*)
plots = Table[
   Plot[
    integrateFunction[i, z0], {z0, 0, 1},
    PlotStyle → ColorData[97, "ColorList"][[i]],
    AxesLabel → {"z", "Integration Result"},
    Frame → True,
    FrameLabel → {{"Integration Result", None}, {"z", None}},
    LabelStyle → {FontSize → 14},
    PlotRange → All,
    PlotLegends → Placed[
      {"Ω = " <> ToString[ΩValues[[i]]]}, {0.7, 0.1}
     ]
   ], {i, Length[ΩValues]}
  ];
```

Analytic Integration

In[107]:=

$$\Omega = 0:$$

$$\int_0^z \frac{1}{(1+z')^{\frac{3}{4}}}\,dz' = \int_1^{1+z} u^{-\frac{3}{4}}\,du = \boxed{4\ (1+z)^{\frac{1}{4}} - 4}$$

In[107]:=

$\Omega = 1$:

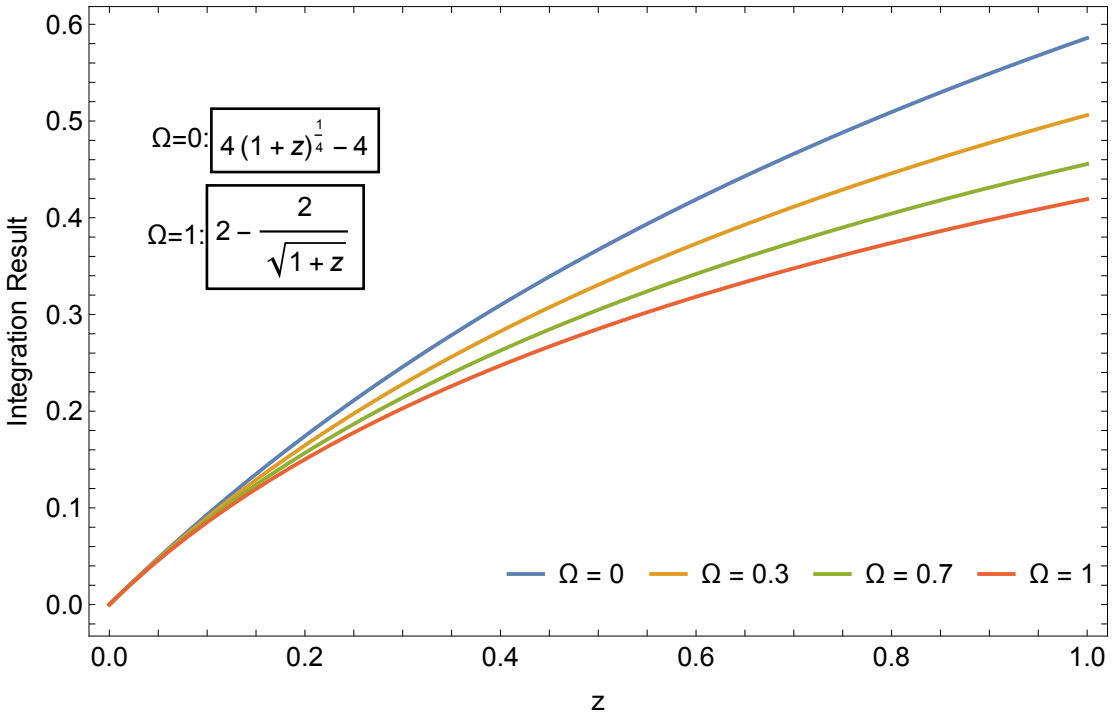
$$\int_0^z \frac{1}{(1+z')^{\frac{3}{2}}}\,dz' = \int_1^{1+z} u^{-\frac{3}{2}}\,du = \boxed{2 - \frac{2}{(1+z)^{\frac{1}{2}}}}$$

Plot

In[107]:=

```
combinedPlot = Show[
  plots,
  PlotRange → All,
  ImageSize → Large,
  Epilog → {Text[Style["Ω=0:" <> 4 (1 + z)^(1/4) - 4, 14], {0.16, 0.48}],
    Text[Style["Ω=1:" <> 2 - 2/Sqrt[1 + z], 14], {0.15, 0.38}]}
 ]
```

Out[107]=



In[108]:=

```
(*Export image to png*)
Export["/Users/yaronetokayer/Yale Drive/Classes/PHYS
    600/phys600 hw/phys600 hw 1/combined_plot.png", combinedPlot];
```

In[109]:=

```
(*Export the notebook as a PDF*)
NotebookSave[];
NotebookPrint[InputNotebook[],
 "/Users/yaronetokayer/Yale Drive/Classes/PHYS 600/phys600
   hw/phys600 hw 1/phys600 hw 1.pdf"]
```