Phys 600: HW 3

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In [2]:
```
# Install packages

from astropy.cosmology import FlatLambdaCDM
from astropy import units as u
import numpy as np
import matplotlib.pyplot as plt
```

In [3]:
```
plt.rcParams.update({
    "text.usetex": True,
    "font.family": "Helvetica"
})
```

Problem 1

In [4]:
```
# Set up an Astropy cosmology to check our answers

H0 = 70.0
Om0 = 0.3

cosmo = FlatLambdaCDM(H0=H0, Om0=Om0)

Ode0 = cosmo.Ode0
```

In [5]:
```
Ode0
```

Out[5]:
```
0.7
```

Density Parameters

Check our work for $\Omega_{\mathrm{x}}(z)$ using Astropy

In [6]:
```
z = 0.5

cosmo.Om(z), cosmo.Ode(z)
```

Out[6]:
```
(0.5912408759124087, 0.4087591240875912)
```

Agrees!

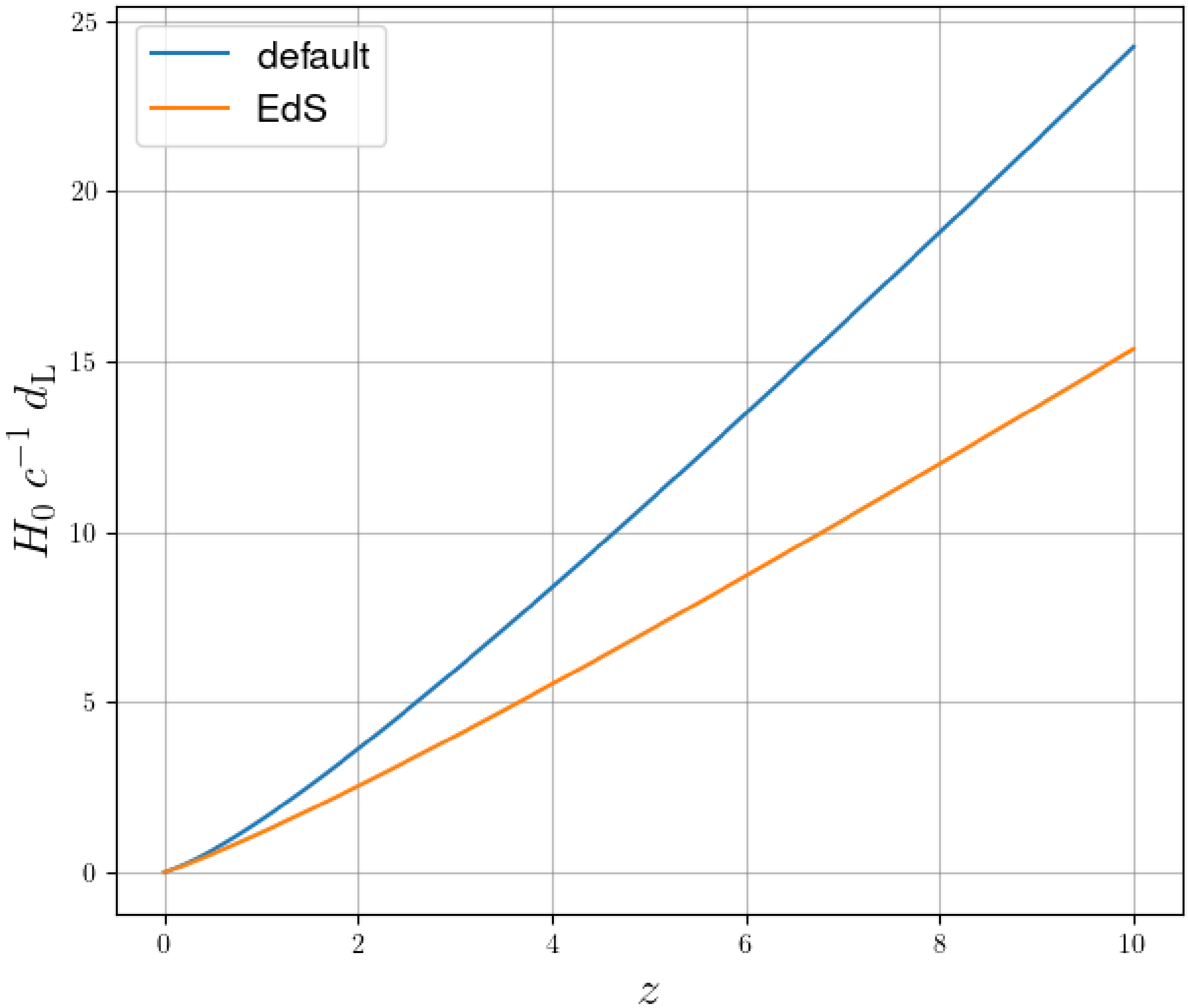
Luminosity and Angular Diameter Distances

In [7]:
```
res = 100 # Resolution of plot
z_axis = np.linspace(0, 10, res)

d_l = np.zeros(len(z_axis))
d_a = np.zeros(len(z_axis))

d_l_eds = np.zeros(len(z_axis))
d_a_eds = np.zeros(len(z_axis))
```

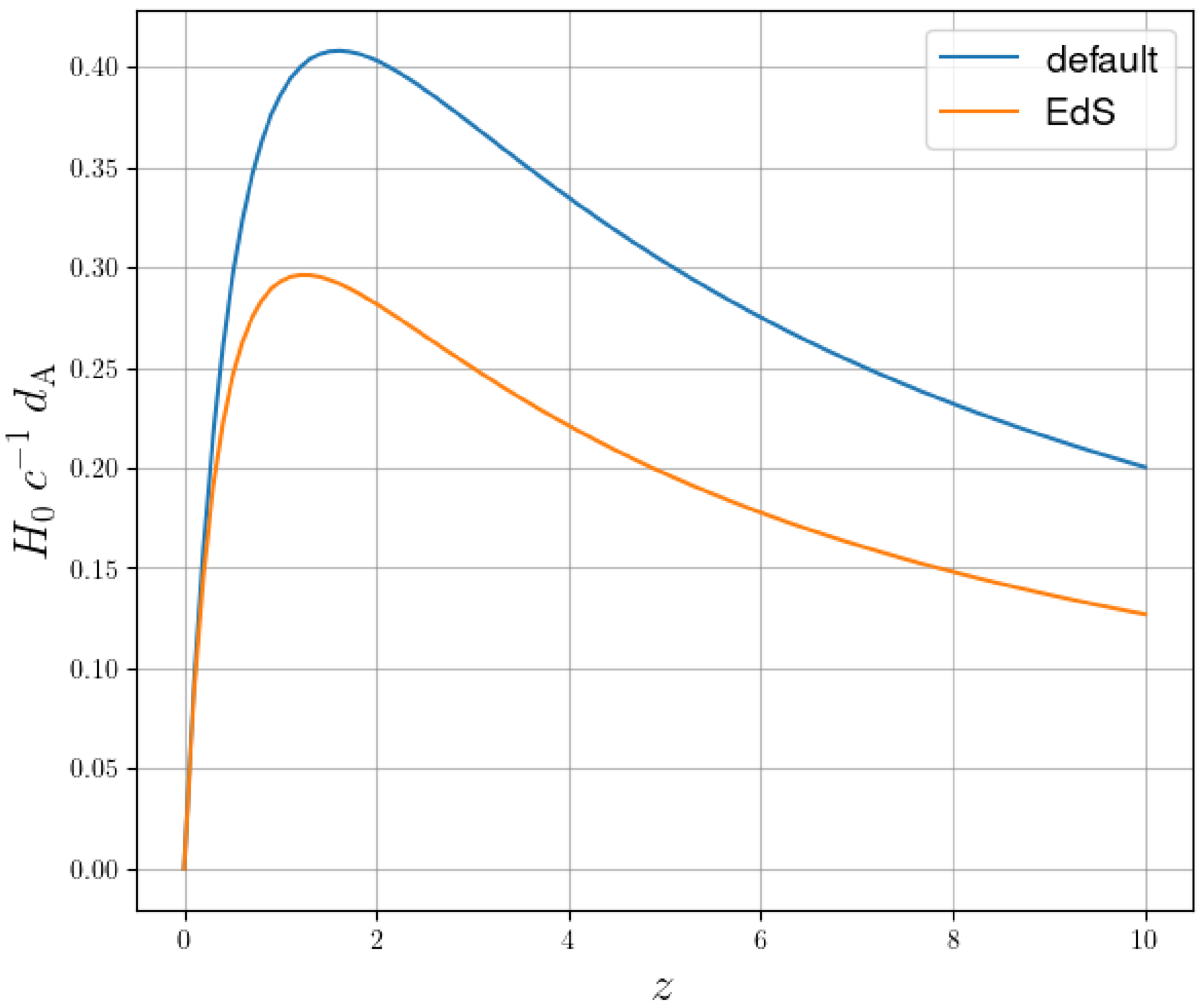
In [8]:
```
# Calculate d_l and d_a for the default cosmology

for i in range(res):
    z = z_axis[i]
    Ode0 = 0.7

    # Calculate comoving distance from z=0 to z
    z_array = np.linspace(0, z, 1000)
    integrand = 1 / ( np.sqrt(Om0 * (1 + z_array)**3 + Ode0) )
    d_m = np.trapz(integrand, x=z_array)

    # Convert to d_l and d_a
    d_l[i] = (1 + z) * d_m
    d_a[i] = d_m / (1 + z)
```

In [9]:
```
# Calculate d_l and d_a for a matter only universe (EdS)

for i in range(res):
    z = z_axis[i]

    # Calculate comoving distance from z=0 to z
    z_array = np.linspace(0, z, 1000)
    integrand = 1 / ( np.sqrt((1 + z_array)**3) ) # Om0=1, Omde0=0
    d_m = np.trapz(integrand, x=z_array)

    # Convert to d_l and d_a
    d_l_eds[i] = (1 + z) * d_m
    d_a_eds[i] = d_m / (1 + z)
```

In [10]:
```
fig, ax = plt.subplots(1, 1, figsize=(7,6))

ax.plot(z_axis, d_l, label='default')
ax.plot(z_axis, d_l_eds, label='EdS')

ax.set_axisbelow(True)
ax.xaxis.grid(color='gray', alpha=0.5, linestyle='-')
ax.yaxis.grid(color='gray', alpha=0.5, linestyle='-')

ax.set_xlabel('$z$', fontsize=18)
ax.set_ylabel(r'$ H_0\ c^{-1}\ d_\mathrm{L}$', fontsize=18)

ax.legend(fontsize=14)

fig.savefig('/Users/yaronetokayer/Yale Drive/Classes/PHYS 600/phys600 hw/phy
           dpi=300, bbox_inches='tight')
```



In [11]:
```
fig, ax = plt.subplots(1, 1, figsize=(7,6))

ax.plot(z_axis, d_a, label='default')
ax.plot(z_axis, d_a_eds, label='EdS')

ax.set_axisbelow(True)
ax.xaxis.grid(color='gray', alpha=0.5, linestyle='-')
ax.yaxis.grid(color='gray', alpha=0.5, linestyle='-')

ax.set_xlabel('$z$', fontsize=18)
ax.set_ylabel(r'$ H_0\ c^{-1}\ d_\mathrm{A}$', fontsize=18)

ax.legend(fontsize=14)

fig.savefig('/Users/yaronetokayer/Yale Drive/Classes/PHYS 600/phys600 hw/phy
           dpi=300, bbox_inches='tight')
```



Looking Back

In [12]:
```
def t_l(z, h, Om0, Ode0):
    '''
    Function to compute the lookback time as a function of z

    Inputs:
    z - redshift
    h - H0/(100 km/s/Mpc)
    Om0 - Fractional matter density at z=0
    Ode0 - Fractional dark energy density at z=0

    Returns:
    t_l - lookback time in years
    '''

    H0 = h * 100 * u.km / u.s / u.Mpc

    z_array = np.linspace(0, z, 1000)

    integrand = 1 / H0 / ( 1 + z_array ) / np.sqrt(Om0 * ( 1 + z_array )**3
    return np.trapz(integrand, x=z_array).to(u.year)
```

In [13]:
```python
def t_l_to_z(t_l_target, h, Om0, Ode0, tolerance=1e-6):
    '''
    Function to find redshift (z) given the lookback time (t_l) using a bise

    Inputs:
    t_l_target - Lookback time (astropy units expected)
    h - H0/(100 km/s/Mpc)
    Om0 - Fractional matter density at z=0
    Ode0 - Fractional dark energy density at z=0
    tolerance - Tolerance for the bisection method convergence

    Returns:
    z - Redshift corresponding to the given lookback time
    '''

    # Initialize the search interval for z
    z_low, z_high = 0.0, 1e3

    # Perform the bisection search
    while z_high - z_low > tolerance:
        z_mid = (z_low + z_high) / 2
        t_mid = t_l(z_mid, h, Om0, Ode0)
        if t_mid > t_l_target:
            z_high = z_mid
        else:
            z_low = z_mid

    # The bisection method has converged; return the redshift
    return z_mid
```

In [14]:
```python
# Compute z for t_l = 1e10 years

h = 0.7
Om0 = 0.3
Ode0 = 0.7
t_l_target = 1e10 * u.year

t_l_to_z(t_l_target, h, Om0, Ode0)
```

Out[14]: 1.8555903807282448

Check our work for t_t using Astropy

In [15]:
```python
z=1.8555903807282448
cosmo.lookback_time(z)
```

Out[15]: 9.9999956 Gyr

We are correct to 3 significant figures, as required.

Problem 3

To what redshift is our expansion accurate to 10%? To answer this, we create two functions: one that computes the exact numerical integral $\chi(z)$, and another that computes the series expansion to third order. Then we use a bisection method to find the z at which the expansion differs by 10%. We also define a function to compute the second order approximation.

Define functions

In [16]:
```
def chi(z, Om0, Ode0):
    '''
    Function to compute the dimensionless comoving distance as a function of

    Inputs:
    z - redshift
    Om0 - Fractional matter density at z=0
    Ode0 - Fractional dark energy density at z=0

    Returns:
    chi - dimensionless comoving distance (H_0 \chi * c^-1)
    '''

    z_array = np.linspace(0, z, 1000)

    integrand = 1 / np.sqrt( Om0 * ( 1 + z_array )**3 + Ode0 + (1 - Om0 - Oc
    return np.trapz(integrand, x=z_array)
```

In [22]:
```
def chi_thirdorder(z, Om0, Ode0):
    '''
    Function to compute the approximate dimensionless comoving distance to t
    as a function of z

    Inputs:
    z - redshift
    Om0 - Fractional matter density at z=0
    Ode0 - Fractional dark energy density at z=0

    Returns:
    chi - dimensionless comoving distance (H_0 \chi * c^-1)
    '''

    z_array = np.linspace(0, z, 1000)

    integrand = (
        1
        + ( ( -2 + 2*Ode0 - Om0 ) / 2 ) * z_array
        + ( ( 8 - 20*Ode0 + 12*Ode0**2 + 4*Om0 + 3*Om0**2 - 12*Ode0*Om0 ) /
    )

    return np.trapz(integrand, x=z_array)

def chi_secondorder(z, Om0, Ode0):
    '''
```

```
    Function to compute the approximate dimensionless comoving distance to t
    as a function of z

    Inputs:
    z - redshift
    Om0 - Fractional matter density at z=0
    Ode0 - Fractional dark energy density at z=0

    Returns:
    chi - dimensionless comoving distance (H_0 \chi * c^-1)
    '''

    z_array = np.linspace(0, z, 1000)

    integrand = (
        1
        + ( ( -2 + 2*Ode0 - Om0 ) / 2 ) * z_array
    )

    return np.trapz(integrand, x=z_array)
```

Find the z at which the error exceed 10%

For the third order approximation:

In [18]:

```
# Initialize the search interval for z
z_low, z_high = 0.0, 2

tolerance = 1e-6 # Precision of z
target = 0.1 # Looking for delta to be within 10%

# Perform the bisection search
while z_high - z_low > tolerance:
    z_mid = (z_low + z_high) / 2
    delta_mid = np.abs(( chi_thirdorder(z_mid, Om0, Ode0) - chi(z_mid, Om0,
    if delta_mid > target:
        z_high = z_mid
    else:
        z_low = z_mid

z_mid
```

Out[18]: 1.2042932510375977

For the second order approximation:

In [63]:

```
# Initialize the search interval for z
z_low, z_high = 0.0, 2.5

tolerance = 1e-6 # Precision of z
target = 0.1 # Looking for delta to be within 10%

# Perform the bisection search
while z_high - z_low > tolerance:
```

```
    z_mid = (z_low + z_high) / 2
    delta_mid = np.abs(( chi_secondorder(z_mid, Om0, Ode0) - chi(z_mid, Om0,
    if delta_mid > target:
        z_high = z_mid
    else:
        z_low = z_mid

z_mid
```

Out[63]: 2.0541709661483765

For the first oder approximation:

This will help us see at what redshift we begin to probe the cosmological density parameters.

In [59]:
```
# Initialize the search interval for z
z_low, z_high = 0.0, 2

tolerance = 1e-6 # Precision of z
target = 0.1 # Looking for delta to be within 10%

# Perform the bisection search
while z_high - z_low > tolerance:
    z_mid = (z_low + z_high) / 2
    delta_mid = np.abs(( z_mid - chi(z_mid, Om0, Ode0) ) / chi(z_mid, Om0, O
    if delta_mid > target:
        z_high = z_mid
    else:
        z_low = z_mid

z_mid
```

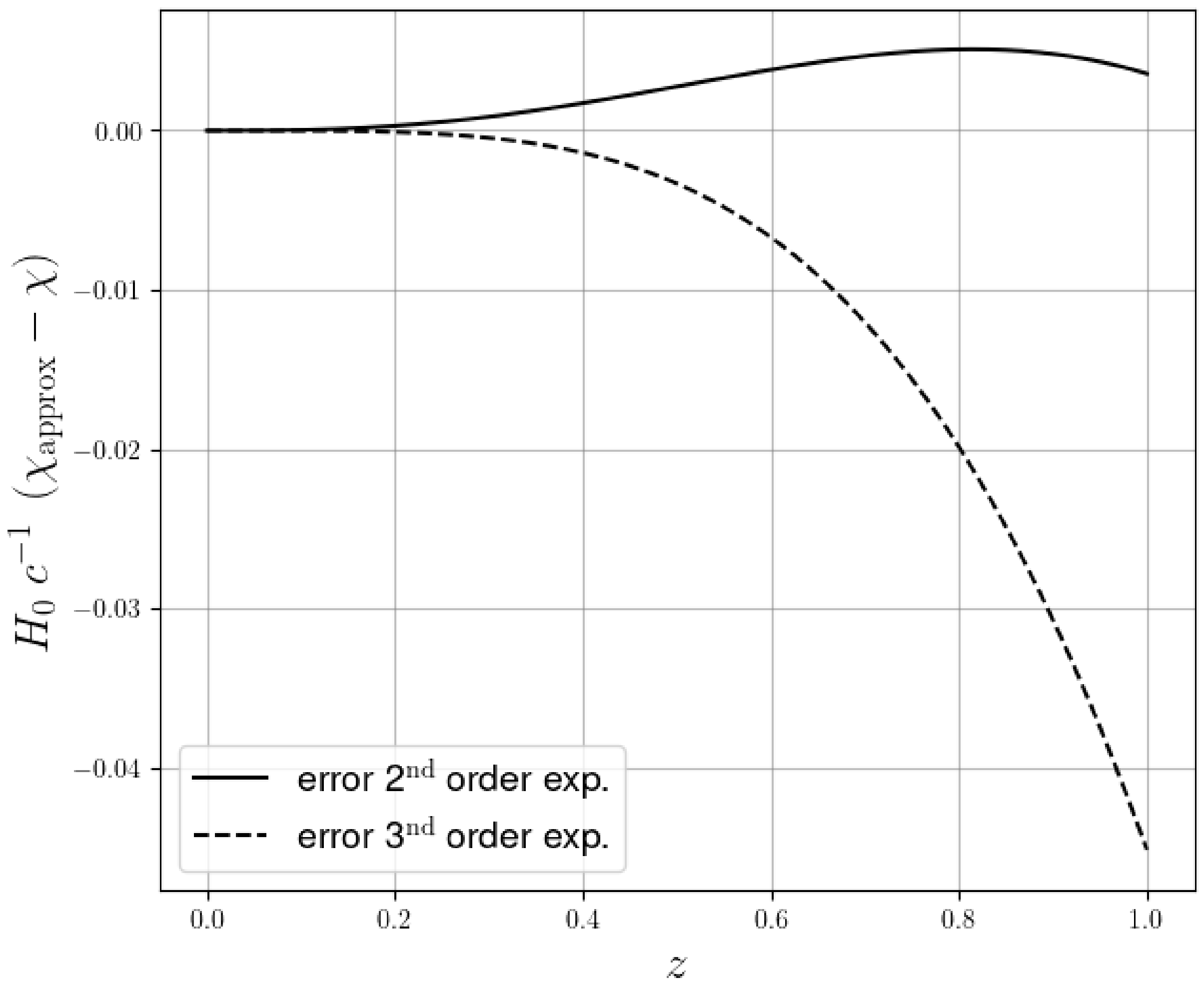
Out[59]: 0.3857755661010742

Surprisingly, we find that the third order approximation diverges at a smaller redshift than the second order approximation. Let's plot each term of the expansion relative to the exact value

In [61]:
```
z = np.linspace(0,1,1000)
exact = chi(z, Om0, Ode0)
third = chi_thirdorder(z, Om0, Ode0)
second = chi_secondorder(z, Om0, Ode0)
first = z

error2 = second - exact
error3 = third - exact
```

In [64]:
```
fig, ax = plt.subplots(1, 1, figsize=(7,6))

# ax.plot(z, exact, ls='-', color='black', label='exact')
# ax.plot(z, first, ls='--', color='black', label='one term exp.')
# ax.plot(z, second, ls=':', color='black', label='two term exp.')
```

```python
# ax.plot(z, third, ls='-.', color='black', label='three term exp.')

ax.plot(z, error2, ls='-', color='black', label='error 2$^\mathrm{nd}$ order
ax.plot(z, error3, ls='--', color='black', label='error 3$^\mathrm{nd}$ orde

ax.set_axisbelow(True)
ax.xaxis.grid(color='gray', alpha=0.5, linestyle='-')
ax.yaxis.grid(color='gray', alpha=0.5, linestyle='-')

ax.set_xlabel('$z$', fontsize=18)
ax.set_ylabel(r'$ H_0\ c^{-1}\ \left(\chi_\mathrm{approx} - \chi\right)$', f

ax.legend(fontsize=14)

fig.savefig('/Users/yaronetokayer/Yale Drive/Classes/PHYS 600/phys600 hw/phy
            dpi=300, bbox_inches='tight')
```




```
In [ ]:
```