PHYS 600: Homework 2

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Problem 1 Friedmann Equation II

We wish to derive

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \tag{1}$$

from the equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{R_0^2 a^2} \tag{2}$$

$$0 = \dot{\rho} + 3H(\rho + P) \tag{3}$$

Begin by taking the time derivative on both sides of Eq. 2:

$$\begin{aligned}
2H\frac{a\ddot{a} - \dot{a}^2}{a^2} &= \frac{8\pi G}{3}\dot{\rho} - 2H\frac{\kappa}{R_0^2 a^2} && \left(H = \frac{\dot{a}}{a}\right)\\
2H\frac{\ddot{a}}{a} - 2H^3 &= -3H\frac{8\pi G}{3}(\rho + P) - 2H\frac{\kappa}{R_0^2 a^2} && \textit{(Eq. 3)}\\
\frac{\ddot{a}}{a} - H^2 &= -\frac{8\pi G}{3}\rho - \frac{4\pi G}{3}\rho - 3\frac{4\pi G}{3}P - \frac{\kappa}{R_0^2 a^2} &&\\
\frac{\ddot{a}}{a} - \cancel{H^2} &= -\cancel{H^2} - \frac{4\pi G}{3}\rho - 3\frac{4\pi G}{3}P && \textit{(Eq. 2)}\\
\frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3P) &&
\end{aligned}$$

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Problem 2 Cosmological Dimming

We saw in class that the angular diameter distance goes as $(1+z)^{-1}$, which implies that angular size goes as $(1+z)$. Additionally, we note that the bolometric luminosity L scales as $(1+z)^{-2}$, where the two factors of $(1+z)$ are due to cosmological redshift and hubble drag, respectively.

By definition, the bolometric surface brightness of an object, I_{e} is given by

$$I_{\text{e}} = \frac{L}{4\pi r^2},$$

where L is the intrinsic bolometric luminosity and $r$is the radius of the object. Using the scalings above, we then get for the observed surface brightness:

$$\begin{aligned} I_{\text{o}} &= \frac{L(1+z)^{-2}}{4\pi\left(r(1+z)\right)^2} \\ &= I_{\text{e}}(1+z)^{-4} \end{aligned}$$

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