Assignment2

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3/11/2018

Matrix problems

1. Suppose

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

(a) Check that $A^3 = \mathbf{0}$
(b) Replace the third column of A by the sum of the second and third columns

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First, produce A

```
A <- matrix(c(1,1,3,5,2,6,-2,-1,-3), nrow = 3, byrow = TRUE)
A%%A%%A
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0    0    0
## [3,]    0    0    0
```

Then, add the columns 2 and 3 and assign the sum to the third column

```
A[,3] <- A[,2] + A[,3]

A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    4
## [2,]    5    2    8
## [3,]   -2   -1   -4
```

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2. Create the following matrix B with 15 rows

$$B = \begin{bmatrix} 10 & -10 & 10 \\ 10 & -10 & 10 \\ \cdots & \cdots & \cdots \\ 10 & -10 & 10 \end{bmatrix}$$

Calculate the 3x3 matrix $B^T B$. You can make this calculation with the function crossprod(). See the documentaion.

```
B<-matrix(rep(c(10,-10,10),15), nrow=15,ncol=3,byrow=TRUE)

crossprod(B)
```

```
##       [,1]  [,2]  [,3]
## [1,]  1500 -1500  1500
## [2,] -1500  1500 -1500
## [3,]  1500 -1500  1500
```

3. Create a 6 x 6 matrix matE with every element equal to 0. check what the functions row() and col() return when applied to matE.

Now, create the 6 x 6 matix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Here is matE, a 6x6 matrix of 0's followed by row(matE) and col(matE)

```
matE <- matrix(0, nrow = 6, ncol=6)
```

```
# With a little experimentation you would see
# that the specified pattern is in the |1|'s
row(matE)-col(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0   -1   -2   -3   -4   -5
## [2,]    1    0   -1   -2   -3   -4
## [3,]    2    1    0   -1   -2   -3
## [4,]    3    2    1    0   -1   -2
## [5,]    4    3    2    1    0   -1
## [6,]    5    4    3    2    1    0
```

```r
# so you use the locations of the 1's to modify matE
matE[abs(row(matE)-col(matE))==1] <- 1
matE
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    1    0    0    0    0
## [2,]    1    0    1    0    0    0
## [3,]    0    1    0    1    0    0
## [4,]    0    0    1    0    1    0
## [5,]    0    0    0    1    0    1
## [6,]    0    0    0    0    1    0
```

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4. Look at the help for the function outer(). Now, create the following patterned matrix:

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```r
a <- 0:4
A <- outer(a,a,"+")
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    5
## [3,]    2    3    4    5    6
## [4,]    3    4    5    6    7
## [5,]    4    5    6    7    8
```

Use outer() a little more to make sure you get it.

```r
B <- outer(a,a, "*")
B
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    0    0    0    0
## [2,]    0    1    2    3    4
## [3,]    0    2    4    6    8
## [4,]    0    3    6    9   12
## [5,]    0    4    8   12   16
```

```r
# and
b <- 5:10
C <- outer(a,b,"+")
C
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    5    6    7    8    9   10
## [2,]    6    7    8    9   10   11
## [3,]    7    8    9   10   11   12
## [4,]    8    9   10   11   12   13
## [5,]    9   10   11   12   13   14
```

```r
# and finally  --  make sure you check the values.
D <- outer(b,a, "%%")
D
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   NA    0    1    2    1
## [2,]   NA    0    0    0    2
## [3,]   NA    0    1    1    3
## [4,]   NA    0    0    2    0
## [5,]   NA    0    1    0    1
## [6,]   NA    0    0    1    2
```

5. Create the following patterned matrices. Your solutions should be generalizable to enable creating larger matrices with the same structure.

(a)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \\ 2 & 3 & 4 & 0 & 1 \\ 3 & 4 & 0 & 1 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$$

```r
f<-0:4
F<-outer(f,f,"+")%%5
F
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    0
## [3,]    2    3    4    0    1
## [4,]    3    4    0    1    2
## [5,]    4    0    1    2    3
```

(b)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 8 & 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```r
m<-0:9
M<-outer(m,m,"+")%%10
M
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]    0    1    2    3    4    5    6    7    8     9
##  [2,]    1    2    3    4    5    6    7    8    9     0
##  [3,]    2    3    4    5    6    7    8    9    0     1
##  [4,]    3    4    5    6    7    8    9    0    1     2
##  [5,]    4    5    6    7    8    9    0    1    2     3
##  [6,]    5    6    7    8    9    0    1    2    3     4
##  [7,]    6    7    8    9    0    1    2    3    4     5
##  [8,]    7    8    9    0    1    2    3    4    5     6
##  [9,]    8    9    0    1    2    3    4    5    6     7
```

```
## [10,]    9    0    1    2    3    4    5    6    7     8
```

(c)

$$\begin{bmatrix} 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 \\ 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 \\ 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 \\ 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 \\ 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

```
o<-0:8
O<-outer(o,o,"-")%%9
O
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##  [1,]    0    8    7    6    5    4    3    2    1
##  [2,]    1    0    8    7    6    5    4    3    2
##  [3,]    2    1    0    8    7    6    5    4    3
##  [4,]    3    2    1    0    8    7    6    5    4
##  [5,]    4    3    2    1    0    8    7    6    5
##  [6,]    5    4    3    2    1    0    8    7    6
##  [7,]    6    5    4    3    2    1    0    8    7
##  [8,]    7    6    5    4    3    2    1    0    8
##  [9,]    8    7    6    5    4    3    2    1    0
```

6. Solve the following system of linear equations by setting up and solving the matrix equation $\mathbf{Ax} = \mathbf{y}$.

$x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 7$
$2x_1 + x_2 + 2x_3 + 3x_4 + 4x_5 = -1$
$3x_1 + 2x_2 + x_3 + 2x_4 + 3x_5 = -3$
$4x_1 + 3x_2 + 2x_3 + x_4 + 2x_5 = 5$
$5x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 = 17$

```
y<-c(7,-1,-3,5,17)
A<-matrix(0, nr=5,nc=5)
A<-abs(col(A)-row(A))+1
x<-solve(A,y)
x
```

```
## [1] -2  3  5  2 -4
```

```
A%*%x
```

```
##      [,1]
## [1,]    7
## [2,]   -1
## [3,]   -3
## [4,]    5
## [5,]   17
```

7. Create a 6 x 10 matrix of random integers chosen from 1,2,. . . ,10 by executing the following two lines of code:

```
set.seed(75)
aMat <- matrix(sample(10, size=60, replace=TRUE), nr=6)
aMat
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## [1,]    3    6    7    7    2    4    3    7    1     4
## [2,]    1    9    8    7    2    6   10    9    5     2
## [3,]    7   10    8    4   10    5    4    8    4     4
## [4,]    4    3    1    1    3    3    9    7    4     2
## [5,]    1    8    1    9    9    8    1    3    7     7
## [6,]    2    6    7    5    6   10    4    6   10     1
```

Use the matrix you have created to answer these questions:

(a) Find the number of entries in each row which are greater than 4.

```
apply(aMat,1,function(n){sum(n>4)})
```

```
## [1] 4 7 6 2 6 7
```

(b) Which rows contain exactly two occurrences of the number seven?

```
aMat
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## [1,]    3    6    7    7    2    4    3    7    1     4
## [2,]    1    9    8    7    2    6   10    9    5     2
## [3,]    7   10    8    4   10    5    4    8    4     4
## [4,]    4    3    1    1    3    3    9    7    4     2
## [5,]    1    8    1    9    9    8    1    3    7     7
## [6,]    2    6    7    5    6   10    4    6   10     1
```

```
which(apply(aMat,1, function(x){sum(x==7)==2}))
```

```
## [1] 5
```

(c) Find those pairs of columns whose total (over both columns) is greater tnan 75. The answer should be a matrix with two columns; so, for example, the row (1,2) in the output matrix means that the sum of columns 1 and 2 in the original matrix is greater than 75. Repeating a column is permitted; so, for exalmple, the final output matrix could contain the rows (1,2), (2,1), and (2,2).

What if repetitions are not permitted? Then only (1,2) from (1,2),(2,1) and (2,2) would be permitted.

```
aSums<-colSums(aMat)
N<-outer(aSums,aSums,"+")>75
which(N, arr.ind = TRUE)
```

```
##      row col
## [1,]   2   2
## [2,]   6   2
## [3,]   8   2
## [4,]   2   6
## [5,]   8   6
## [6,]   2   8
## [7,]   6   8
## [8,]   8   8
```

when not permitted

```
aSums<-colSums(aMat)
N<-outer(aSums,aSums,"+")>75
N[lower.tri(N, diag=TRUE)]<-FALSE
which(N,arr.ind=TRUE)
```

```
##      row col
## [1,]   2   6
## [2,]   2   8
## [3,]   6   8
```

8. Calculate

(a) $\sum_{i=1}^{20}\sum_{j=1}^{5}\frac{i^4}{(3+j)}$

```
sum((1:20)^4) * sum(1/(3+(1:5)))
```

```
## [1] 639215.3
```

```
# or

sum(outer((1:20)^4, (3+(1:5)), "/"))
```

```
## [1] 639215.3
```

(b) $\sum_{i=1}^{20}\sum_{j=1}^{5}\frac{i^4}{(3+ij)}$

```
sum((1:20)^4/(3+outer(1:20,1:5,"*")))
```

```
## [1] 89912.02
```

(c) $\sum_{i=1}^{10}\sum_{j=1}^{i}\frac{i^4}{(3+ij)}$

```
sum(outer(1:10,1:10,function(i,j){(i>=j)*i^4/(3+i*j) }) )
```

```
## [1] 6944.743
```