# 5_lab

April 20, 2021

# 1 Раздел 5. Уравнения с частными производными гиперболического типа.

## 1.1 Лабораторная рабода №5. Методы решения уравнения переноса.

### 1.1.1 Вариант 3, задание 9.

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## 1.2 Аналитическое решение

Дифференциальная задача:

$$\frac{\partial u}{\partial t} - 4\frac{\partial u}{\partial x} = x; \quad 0 < t \leq 1,\ 0 \leq x \leq 1$$

$$u(x,0) = \sin x - 0.125x^2;\ \ u(1,t) = \sin{(1+4t)} - 0.125$$

Аналитическое решение:

$$u(x,t) = \sin{(4t+x)} - \frac{x^2}{8}$$

## 1.3 Численное решение

Разностная схема:

$$D_h = \left\{(x_l, t^n) : x_l = hl, hL = 1, l = \overline{0,L};\ t^n = n\tau, \tau N = 1, n = \overline{0,N}\right\}$$

$$u_l^{n+1} = u_l^n + \frac{2\tau}{3h}\left(2u_{l+3}^n - 9u_{l+2}^n + 18u_{l+1}^n - 11u_l^n\right) + \frac{8\tau^2}{h^2}\left(-u_{l+3}^n + 4u_{l+2}^n - 5u_{l+1}^n + 2u_l^n\right) + \frac{32\tau^3}{3h^3}\left(u_{l+3}^n - 3u_{l+2}^n + 3u_{l+1}^n\right.$$

$$u_l^0 = \sin x_l - 0.125x_l^2 = \varphi_l,\ l = \overline{0,L};\ \ u_L^n = \sin{(1+4t^n)} - 0.125 = \psi^n,\ n = \overline{1,N}$$

$$u_{L-1}^n = ?,\ n = \overline{1,N}; \quad u_{L-2}^n = ?,\ n = \overline{1,N}.$$

Поиск $u_{L-1}^n$, $u_{L-2}^n$:

$$[u]^n_{L-1} = [u]^n_L - [u'_x]^n_L h + [u''_{xx}]^n_L \frac{h^2}{2} - [u'''_{xxx}]^n_L \frac{h^3}{6}$$

$$[u]^n_{L-2} = [u]^n_L - [u'_x]^n_L (2h) + [u''_{xx}]^n_L \frac{(2h)^2}{2} - [u'''_{xxx}]^n_L \frac{(2h)^3}{6}$$

Выразим производные по координате через производные по времени:

$$[u'_x]^n_L = \frac{1}{a^n_L}\left(-(\dot{\psi}_t)^n + b^n_L\right), \quad [u''_x]^n_L = \frac{1}{(a^n_L)^2}\left\{(\ddot{\psi}_{tt})^n + a^n_L (b'_x)^n_L\right\}, \quad [u'''_{xxx}]^n_L = -\frac{1}{(a^n_L)^3}(\dddot{\psi}_{ttt})^n,$$

где $a^n_L = -4, \;\; b^n_L = x_L$

Условие спектральной устойчивости:

$$\tau \leq \frac{2h}{a}$$

В нашем случае отношение $K = \tau/h$

$$K = \tau/h \leq 0.5$$

Другое определение сходимости через число Куранта

$$K = a\tau/2h$$

[361]:
```
import numpy as np
from matplotlib import pyplot as plt
import pandas as pd
from mpl_toolkits.mplot3d import Axes3D
```

[362]:
```
def anal(x,t):
    return np.sin(4*t + x) - x**2 / 8
```

[363]:
```
def U_anal(N, L, x, t):
    t_a = []
    for i in range(N+1):
        for j in range(L+1):
            t_a.append(t[i])
    t_a = np.array(t_a)

    x_a = np.array([])
    for i in range(N+1):
        x_a = np.concatenate((x_a,x))

    U_an = np.vectorize(anal)(x_a,t_a).reshape((N+1,L+1))

    return U_an
```

[364]:
```python
def U_num(N, L, x, t):

    h = 1/L
    tau = 1/N

    psi = np.sin(1+4*t) - 0.125
    psi_der1 = 4*np.cos(1+4*t)
    psi_der2 = -16*np.sin(1+4*t)
    psi_der3 = -64*np.cos(1+4*t)
    b = x
    b_der1 = 1
    phi = np.sin(x)-0.125*x**2

    u_der1 =  1/a * (-psi_der1 + b[L])
    u_der2 = 1/a**2 * (psi_der2 + a)
    u_der3 = -1/a**3 * psi_der3


    U = np.zeros((N+1,L+1))
    U[0] = phi
    U.reshape((1,(N+1)*(L+1)))[0][L::L+1] = psi

    h_arr = [h*i for i in range(0,3)]
    for j in range(1,3):
        k = L-j
        for i in range(1,N+1):

            U[i][k] = U[i][L] - u_der1[i] * h_arr[j] + u_der2[i] * h_arr[j]**2 /␣
↪2 - u_der3[i] * h_arr[j]**3 / 6


    for n in range(N):
        for l in range(L-2):
            U[n+1][l] = U[n][l] + (2*tau/(3*h)*(2*U[n][l+3] - 9*U[n][l+2] +␣
↪18*U[n][l+1] - 11*U[n][l]) +
                                   8*(tau**2)/(h**2)*(-U[n][l+3] +4*U[n][l+2]␣
↪-5*U[n][l+1] +2*U[n][l]) +
                                   32*(tau**3)/(3*h**3)*(U[n][l+3] - 3*U[n][l+2] +␣
↪3*U[n][l+1] - U[n][l]) +
                                   tau*x[l] + 2*tau**2)

    return U
```

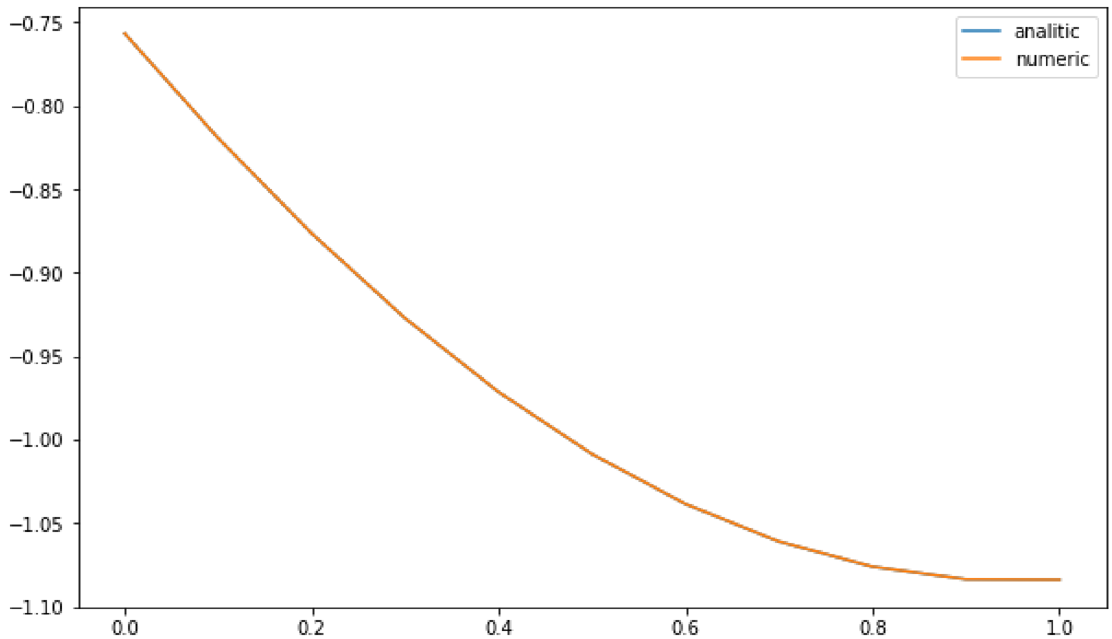
[365]:
```python
def result(L):

    K = 0.25
    N = int(L/K)
```

```
    h = 1/L
    tau = 1/N
    a = -4

    x = np.arange(0,1+1/(100*L),1/L)
    t = np.arange(0,1+1/(100*N),1/N)

    pd.set_option('display.float_format', lambda x: '{:.5E}'.format(x))

    value = abs(U_anal(N,L, x, t)[N]-U_num(N,L,x,t)[N])
    data = np.concatenate((x,U_anal(N,L,x,t)[N],U_num(N,L,x,t)[N], value, [␣
↪max(value) for i in range(L+1) ]))
    data = data.reshape((5,L+1))
    table = pd.DataFrame(data.T, columns = ['x', 'Analitic', 'Numeric', 'Diff',␣
↪'Max Diff'])

    plt.figure(figsize = (10,6))
    plt.plot(x,np.vectorize(anal)(x,[t[N] for i in range(L+1)]))
    plt.plot(x,U_num(N,L,x,t)[N])
    plt.legend(['analitic','numeric'])
    plt.show()

    return table
```

[366]:
```
def result_double_grid(num):
    pd.set_option('display.float_format', lambda x: '{:.5E}'.format(x))
    L = np.array([10*2**i for i in range(num+1)])
    K = 0.25
    N = np.array(L/K, dtype = 'int')
    h = 1/L
    tau = 1/N
    a = -4
    diff = []
    for i in range(num+1):
        x = np.arange(0,1+1/(100*L[i]),1/L[i])
        t = np.arange(0,1+1/(100*N[i]),1/N[i])

        diff.append(max(abs(U_anal(N[i],L[i], x,␣
↪t)[N[i]]-U_num(N[i],L[i],x,t)[N[i]])))
    data = np.concatenate((L, diff))
    data = data.reshape((2,num+1))
    table = pd.DataFrame(data.T, columns = ['L', 'Max Diff'])

    return table
```

### 1.4 Результат на фиксированной сетке

[345]:
```
result(10)
```




[345]:

| | x | Analitic | Numeric | Diff | Max Diff |
|---|---|---|---|---|---|
| 0 | 0.00000E+00 | -7.56802E-01 | -7.56746E-01 | 5.67215E-05 | 6.65556E-05 |
| 1 | 1.00000E-01 | -8.19527E-01 | -8.19467E-01 | 5.99286E-05 | 6.65556E-05 |
| 2 | 2.00000E-01 | -8.76576E-01 | -8.76513E-01 | 6.25368E-05 | 6.65556E-05 |
| 3 | 3.00000E-01 | -9.27416E-01 | -9.27351E-01 | 6.45203E-05 | 6.65556E-05 |
| 4 | 4.00000E-01 | -9.71602E-01 | -9.71536E-01 | 6.58590E-05 | 6.65556E-05 |
| 5 | 5.00000E-01 | -1.00878E+00 | -1.00871E+00 | 6.65397E-05 | 6.65556E-05 |
| 6 | 6.00000E-01 | -1.03869E+00 | -1.03862E+00 | 6.65556E-05 | 6.65556E-05 |
| 7 | 7.00000E-01 | -1.06117E+00 | -1.06111E+00 | 6.59065E-05 | 6.65556E-05 |
| 8 | 8.00000E-01 | -1.07616E+00 | -1.07610E+00 | 6.45988E-05 | 6.65556E-05 |
| 9 | 9.00000E-01 | -1.08370E+00 | -1.08370E+00 | 4.01782E-06 | 6.65556E-05 |
| 10 | 1.00000E+00 | -1.08392E+00 | -1.08392E+00 | 0.00000E+00 | 6.65556E-05 |

### 1.5 Результат на последовательно удваиваемых сетках

[360]:
```
result_double_grid(5)
```

[360]:

| | L | Max Diff |
|---|---|---|
| 0 | 1.00000E+01 | 6.65556E-05 |
| 1 | 2.00000E+01 | 4.16547E-06 |
| 2 | 4.00000E+01 | 2.60407E-07 |
| 3 | 8.00000E+01 | 1.62757E-08 |

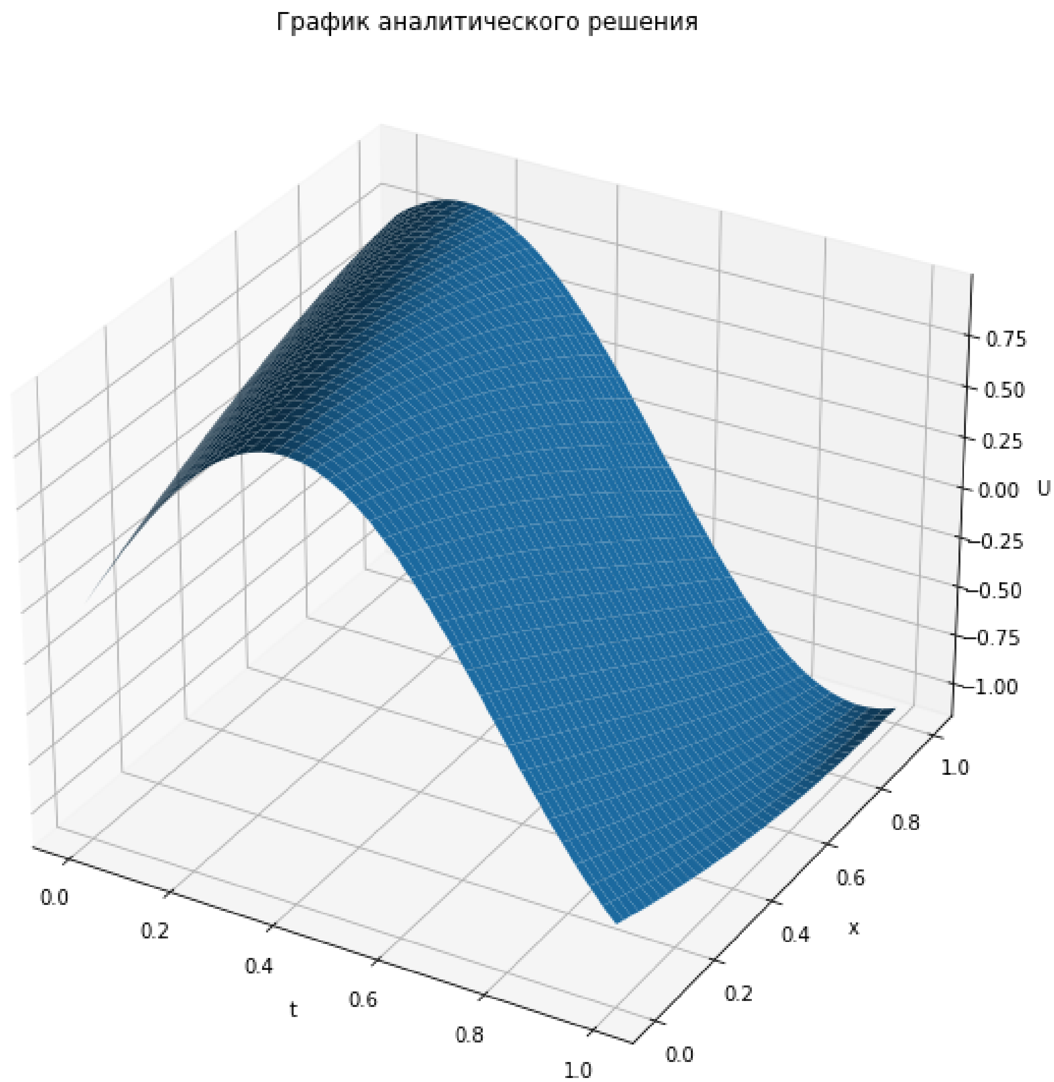
```
4 1.60000E+02 1.01725E-09
5 3.20000E+02 6.35814E-11
```

### 1.6 3-х мерный график поверхности

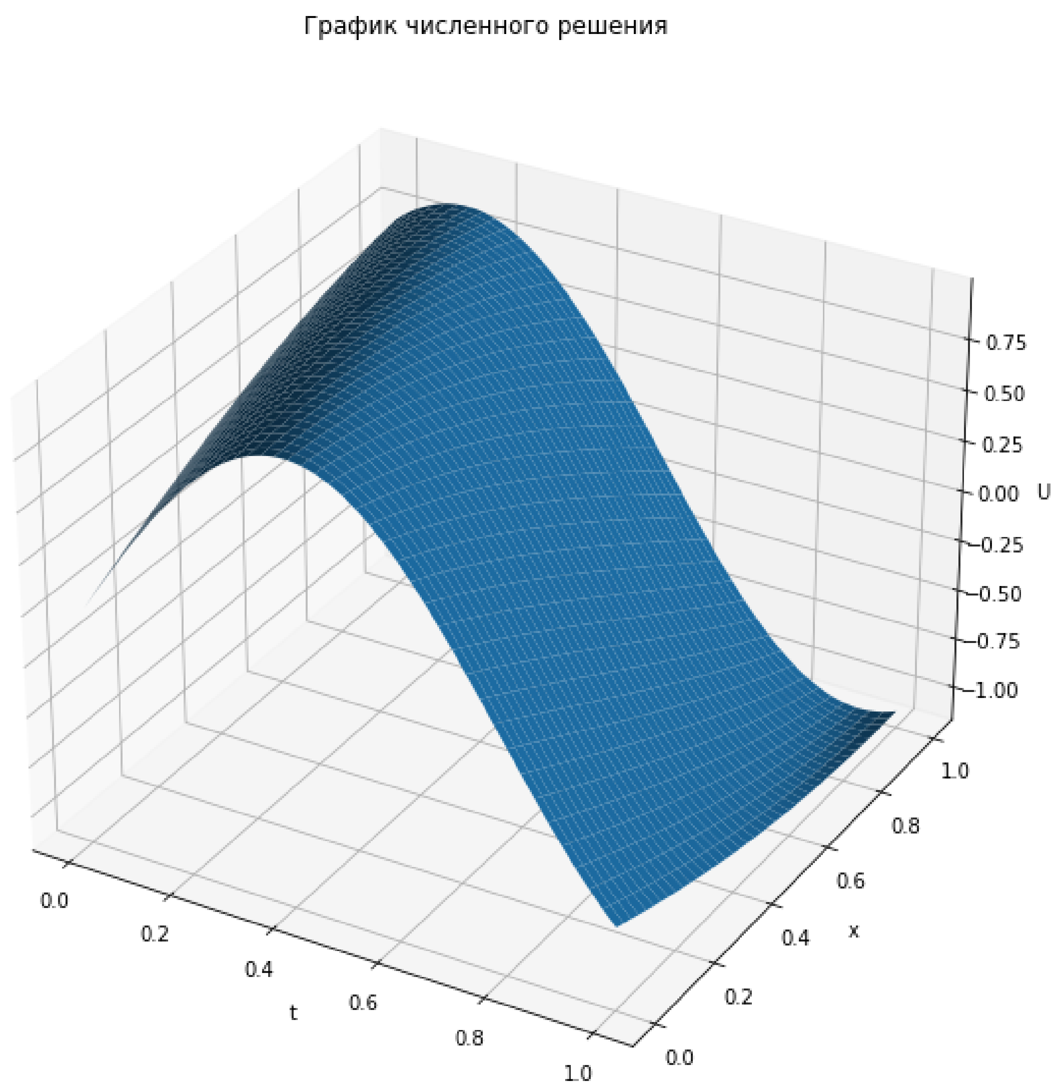
[357]:
```python
L = 40
K = 0.25
N = int(L/K)

x = np.arange(0,1+1/(10*L),1/L)
t = np.arange(0,1+1/(10*N),1/N)

t_a = []
for i in range(N+1):
    for j in range(N+1):
        t_a.append(t[i])
t_a = np.array(t_a)

x_a = np.array([])
for i in range(N+1):
    x_a = np.concatenate((x_a,x))


fig = plt.figure(figsize = (15,10))
ax = fig.add_subplot(111, projection='3d')
xgrid, tgrid = np.meshgrid(x, t)
ax.plot_surface(tgrid, xgrid, anal(xgrid,tgrid))
ax.set_zlabel('U')
ax.set_ylabel('x')
ax.set_xlabel('t')
ax.set_title('График аналитического решения')
plt.savefig('surface.png', dpi=400, quality=100)
plt.show()

fig = plt.figure(figsize = (15,10))
ax = fig.add_subplot(111, projection='3d')
xgrid, tgrid = np.meshgrid(x, t)
ax.plot_surface(tgrid, xgrid, U_num(N, L, x, t))
ax.set_zlabel('U')
ax.set_ylabel('x')
ax.set_xlabel('t')
ax.set_title('График численного решения')
plt.savefig('surface_2.png', dpi=400, quality=100)
plt.show()
```

```
<ipython-input-357-d9d073539a93>:27: MatplotlibDeprecationWarning: savefig() got
unexpected keyword argument "quality" which is no longer supported as of 3.3 and
```

```
will become an error two minor releases later
  plt.savefig('surface.png', dpi=400, quality=100)
```




```
<ipython-input-357-d9d073539a93>:38: MatplotlibDeprecationWarning: savefig() got
unexpected keyword argument "quality" which is no longer supported as of 3.3 and
will become an error two minor releases later
  plt.savefig('surface_2.png', dpi=400, quality=100)
```




[281]:
```
# L = 10
# k1 = 0.5
# h = 1/L
# N = int(abs(a/(2*k1*h)))
# tau = 1/N
# N
```

[203]:
```
# condition = abs(2*h/a)
# def check():
#     if tau <= condition:
#         return 'Устойчиво'
#     else:
#         return 'Не устойчиво'
```

```
# check()
```

[ ]:

```
```