Weightage.

Midsem - 25%

End Sem - 35%

Quiz - 25%

Assignments - 15%

Set Theory:

Theorem:

a) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.

(b) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.

$x \in A \setminus B$

$x \in A \setminus C$.

$x \quad (A \setminus B) \cap (A \setminus C)$

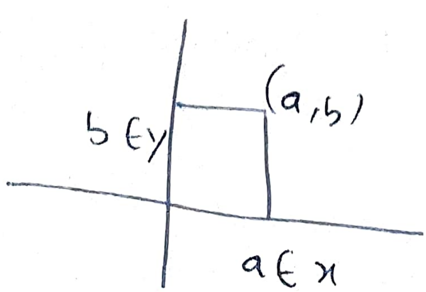
Definition => Cartesian product

A and $B \neq \phi$

$A = \{1, 2, 3\}$

$B = \{6, 7\}$.

$A \times B = \{(1,6); (1,7); (2,6); (2,7); (3,6); (3,7)\}$.

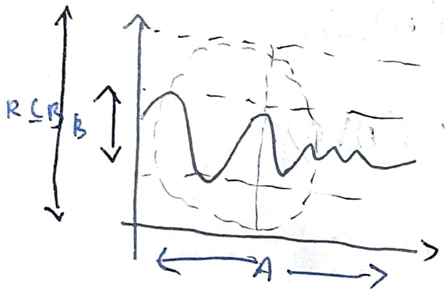


Definition function.

Let A and B are two sets, then a function from A to B is a set of ordered pairs in $A \times B$ such that each $a \in A$ $\exists$ a unique $b \in B$ with $(a, b) \in f$.

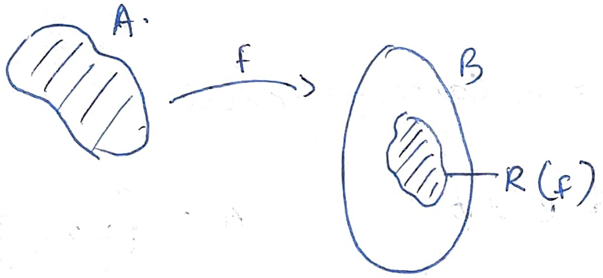
domain of f $\rightarrow D(f) = A$.

Range of f $R(f) \subseteq B$.



$D(f) = A$.

1-08-2022



Direct Image:

If E is a subset of A, then the direct image of E under f is the subset $f(E)$

$$f(E) = \{f(x) : x \in E\}.$$

Inverse image :

If $H \subseteq B$ then inverse image of H under the function f is the subset $f^{-1}(H)$ of the input set A which is defined as

$$f^{-1}(H) = \{x \in A : f(x) = H\}.$$

$\lambda_i \Rightarrow$ Vector V_i

$\sum \lambda_i V_i V_i^{*} \rightarrow$ Spectral representation

$A A^{*} - A^{*} A = 0$ [Normal matrix]

$\hookrightarrow [A^{*} = \overline{(A^T)}]$

$f: R \rightarrow R \qquad f(x) = x^2.$

$$E = \{x \mid 0 \le x \le 2\}$$

$$f(E) = \{y \mid 0 \le y \le 4\} = G.$$

$$f^{-1}(G) = \{x \mid -2 \le x \le 2\} \neq E.$$

$$f^{-1}(f(E)) \neq E$$

If we restrict our self to only positive real numbers our assumption would be correct.

$$f^{-1}(G \cap H) \subseteq f^{-1}(G) \cap f^{-1}(H).$$

$x \in f^{-1}(G \cap H)$

$f(x) \in G \cap H. \Rightarrow f(x) \in G$ &

$f(x) \in H.$

$x \in f^{-1}(G)$

$x \in f^{-1}(H).$

$x \in f^{-1}(G) \cap f^{-1}(H).$

Need to prove $x \in f^{-1}(G \cap H).$

$[f^{-1}(\cancel{G}) \cancel{\cap f^{-1}(H)}]$

Injective function:

The function f is said to be injective if when ever $x_1 \neq x_2$ then $f(x_1) \neq f(x_2).$

one-one mapping.

$\Rightarrow \quad f(x) = \dfrac{2x}{x-1}$

Prove $f(x)$ is injective or not.

$x_1 \neq x_2$

but

$$\frac{2x_1}{x_1-1} = \frac{2x_2}{x_2-1}$$

$$\Rightarrow 2x_1(x_2-1) = 2x_2(x_1-1)$$

$$\Rightarrow \cancel{2x_1x_2} - 2x_1 = \cancel{2x_1x_2} - 2x_2$$

$$x_1 = x_2$$

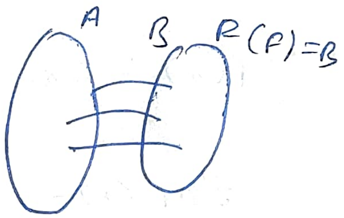
which is a contradiction which says that

$f(x_1) \neq f(x_2)$ when $x_1 \neq x_2$.

Surjective function:

The function f is said to be surjective, if

$F(A) = B \Rightarrow R(F) = B$.



If a function is both injective & bijective then it is called as bijective.

Inverse function:

If $f : A \to B$. is a bijection of A on B. then

$$g : \{ (b, a) \in B \times A . (a, b) \in f \}$$

$g = f^{-1}$.

$$f(x) = \frac{2x}{x-1}$$

$$y = \frac{2x}{x-1} \Rightarrow x = \frac{y}{y-2}.$$

⇒ **Composition of functions:**

$g o f \neq f o g$

$f(x) = 2x$, $g(x) = 3x^2 - 1$.

$g o f(x) = 3(2x)^2 - 1 = 12x^2 - 1$.

$f o g(x) = 2(3x^2 - 1) = 6x^2 - 2$.

Theorem: Let $f : A \to B$ & $g : B \to C$. & $H \subseteq C$. Then we have.

$$(g o f)^{-1}(H) = f^{-1}(g^{-1}(H)).$$