Auto Covariance

$$C_{xx}(t_1, t_2) = E\Big[\underbrace{(x(t_1) - \eta_x(t_1))}_{\tilde{x}(t_1)}\underbrace{(x(t_2) - \eta_x(t_2))}_{\tilde{x}(t_2)}\Big] = R_{xx}(t_1, t_2) - \eta_x(t_1)\eta_x(t_2)$$

* for zero mean R.P.

$$R_{xx}(t_1, t_2) = C_{xx}(t_1, t_2)$$

Correlation-coefficient

$$\rho_{xx}^{(t_1, t_2)} = \frac{C_{xx}(t_1, t_2)}{\sqrt{C_{xx}(t_1, t_1) \cdot C_{xx}(t_2, t_2)}} \quad \text{as} \quad C_{xx}(t_1, t_1) = \mathrm{Var}(X(t_1)).$$

Note if $X(t_1)$ and $X(t_2)$ are independent, then

$$R_{xx}(t_1, t_2) = E[X(t_1) \cdot X(t_2)] = \eta_x(t_1) \cdot \eta_x(t_2).$$

and thus,

$$C_{xx}(t_1, t_2) = R_{xx}(t_1, t_2) - \eta_x(t_1) \cdot \eta_x(t_2) = 0.$$

[if covariance is zero $\Rightarrow$ R.V. are un-correlated]

Let $x(t) = \underset{\text{R.V.}}{r_0} \cos(\omega t + \underset{U[-\pi, \pi)}{\phi})$

$$R_{xx}(t_1, t_2) = E[x(t_1) \cdot x(t_2)]$$

$$= E[r^2 \cos(\omega t_1 + \phi) \cdot \cos(\omega t_2 + \phi)]$$

$$= E[r^2] \cdot E[\cos(\omega t_1 + \phi) \cdot \cos(\omega t_2 + \phi)]$$

$$= \frac{E[r^2]}{2\pi} \cdot \int_{-\pi}^{\pi} \cos(\omega t_1 + \phi) \cdot \cos(\omega t_2 + \phi) \, d\phi$$

$$\frac{E[r^2]}{2\pi}\int\left(\cos\omega t_1\cos\phi - \sin\omega t_1\sin\phi\right)\left(\cos\omega t_2\cos\phi - \sin\omega t_2\sin\phi\right)d\phi$$

$$R_{xx}(t_1,t_2) = \frac{E[r^2]}{2}\cos(\omega(t_1-t_2))$$

we see that autocorrelation does not depend on actual values of t_1, t_2 (only upon the difference $t_1 - t_2$).

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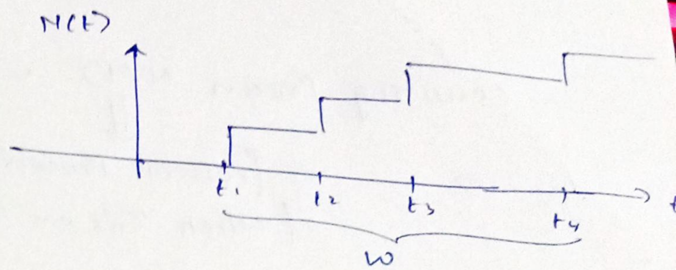
let $\omega \in \Omega$

and $\omega = (t_1, t_2, t_3, t_4)$.

then $N(t)$ for this ω is

then $N(t;\omega)$ is R.P. function



$N(t)$ is R.P.

let $T(n)$ represent the arrival time of nth customer.

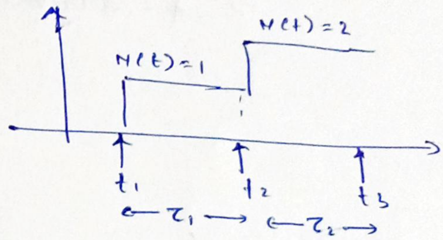
let $\tau(n) = T(n+1) - T(n)$

we can easily show that

$\Rightarrow$

$$T(n) = \sum_{i=1}^{n-1}\tau[i]$$

{sum of iid exp(λ)}. — iid of exp(λ) (say)



$\Rightarrow$ $T(n) \Rightarrow$ conv. of τ_is

we will get $f_T(\alpha, n) = \dfrac{(\lambda\alpha)^{n-1}}{(n-1)!}\exp(-\lambda\alpha)$; $\alpha > 0$

Erlang's Distribution.

[Sum of iid exp(λ) is Erlang's Dist]

$$f_\tau(t) = \lambda\exp(-\lambda t) \qquad t \geq 0$$

$\mathbb{P}[N(t) = n] = \mathbb{P}[T(n) \le t \text{ and } T(n+1) > t]$.

Now since $\tau(n) = T(n+1) - T(n) \Rightarrow T(n+1) = \tau(n) + T(n)$

$\mathbb{P}[N(t) = n] = \mathbb{P}[T(n) \le t ; \tau(n) > t - T(n)]$ $\xrightarrow{} \text{let} = \alpha$

$= \cancel{\mathbb{P}[T(n) \le t] \cdot \mathbb{P}[\tau(n) > t - T(n)]}$. (we will avg over whole $T(n)$ later)

$$\mathbb{P}[N(t) = n] = \int_0^t f_T(\alpha, n) \cdot \int_{t-\alpha}^{\alpha} f_\tau(\beta)\, d\beta\, d\alpha = \underbrace{\frac{(\lambda t)^n \exp(-\lambda t)}{n!}}$$

Counting Process $N(t)$ ~ $\underline{\underline{\text{Poisson}(\lambda t)}}$

$\Downarrow$

$\Rightarrow$ [Poisson Process]

[when τ_n's are iid & $\exp(\lambda)$]

- $E[N(t)] = \lambda t$.

$\Rightarrow \lambda$ = arrival rate of customers.