Mathematical Induction

Well ordering of natural numbers N.

Let $S \subseteq N$ that have the following two properties.

(1) $1 \in S$

(2) For all $k \in N$, if $k \in S$, then $k+1 \in S$.

$S = N$.

$S \neq N$

$N \setminus S \neq \emptyset$.

$1 \in S$, $m > 1$ $\Rightarrow m+1 \in N$.

$m-1 < m$ & $m-1 \in S$.

If $k = m-1$ the $k+1$ also belongs to S

but m does not belong to S thus we got a contradiction.

$\Rightarrow$ For each $n \in N$, $P(n) =$ statement about n.

(1) $P(1)$ is true.

(2) It is true for every $k \in N$ is $P(k)$ is true, then $P(k+1)$ is also true.

Example:

$$1+2+3 \cdots n = \frac{n(n+1)}{2}$$

case (i) $n = 1$

$$1 = \frac{1(1+1)}{2} = 1.$$

Case 2 (ii)

$n = k$

$1+2+3 \ldots k = \frac{k(k+1)}{2}$ $\quad n = k+1 \Rightarrow \frac{(k+1)(k}{2}$

Case 3 (iii)

$n = k+1$

$\frac{(k+1)(k+2)}{2} \times \frac{\cancel{k}}{\cancel{k}} = \frac{n(n+1)}{2}$

$1^2 + 2^2 + 3^2 \; ---- = \frac{n}{6}(n+1)(2n+1)$

Case (i)

$n = 1$

$1 = \frac{1}{6}(2)(3) = 1.$

Case (ii)

$n = k$

$1^2 + 2^2 + 3^2 --- + k^2 = \frac{k}{6}(k+1)(2k+1)$

Case (iii)

$n = k+1$

$1^2 + 2^2 + 3^2 \ldots + k^2 + (k+1)^2$

$\Rightarrow \frac{k}{6}(k+1)(2k+1) + (k+1)^2$

$\Rightarrow \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$

$$\Rightarrow \frac{(k+1)\left[k(2k+1)+6(k+1)\right]}{6}$$

$$\Rightarrow \frac{(k+1)\left[2k^2+k+6k+6\right]}{6}$$

$$\Rightarrow \frac{(k+1)(k+2)(2k+3)}{6} \quad --(i)$$

if $n = k+1$

then $1^2 + 2^2 \; --- + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

thus

$$1^2 + 2^2 \; --- + n^2 = \frac{(n+1)(n)(2n+1)}{6}$$

$2^n \leq (n+1)!$

$\Rightarrow$ case (i) $n = 1$

$2^1 \leq (1+1)! \Rightarrow 2 \leq 2$.

case (ii) $n = k$

$2^k \leq (k+1)!$

case (iii) $n = k+1$

$2^{k+1} \Rightarrow 2^k \cdot 2 \; \cancel{\leq} \; (k+1)! \cdot 2 \leq (k+1)! (k+2)$

$2^{k+1} \leq (k+2)!$.

$\Rightarrow$ $\underset{\Omega^k}{1+\Omega+\Omega^2 \dots} = \dfrac{1-\Omega^{k+1}}{1-\Omega}$.

case(i)

$k=1$

$1+\gamma+\gamma^2 \dots = \dfrac{1-\gamma^2}{1-\gamma} = 1+\gamma$.

Case (ii)

$k=k$.

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$\rightarrow$ a) The empty set is the set ϕ having 0 elements

b) If $n \in N$, a set S is said to have n elements is there exists a bijection from the set N onto S.

$= \{1, 2, 3, \dots n\}$.

No. of elements in the sets is called as cardinality. denoted by $|A|$ for a set A.

Unique ness theorem

If the set S is a finite set, then the number of elements in S is a unique number belonging to the set of natural numbers $\in N$.

Theorem:

(a) If A is a set with m elements & B is another set with n elements, & if $A \cap B = \emptyset$, then $A \cup B$ must have $m+n$ elements.

(b) If A is a set with $m \in \mathbb{N}$ elements & $C \subseteq A$, with only 1 element, then $A \setminus C$ has $m-1$ elements.

(c) If C is an infinite set & B is a finite set, then $C \setminus B$ is an infinite set.

suppose S & T, $T \subseteq S$

(a) If ~~S is~~ S is finite set, then T is also finite

b) If T is infinite, the S is also infinite.

~~Le~~ Denumerable (~~or~~ or countable infinite set) ~~set~~.

a) → A set S is said to be denumerable/countably infinite if there exists a bijection from N onto S.

(b) A set is countable if it is either denumer-able / finite.

The set which contains all the subsets of a given set it called as power set.

→ Power set also contains null set.

→ Can consider the set $E = \{\{x\} : x \in A\}$, the set of all single-element subsets of A

→ Suppose we have a map

$F : A \to P(A)$ is surjective

$f(a)$ is a subset of A either it belongs to $B = \{a \in A . a \notin f(a)\}$ can not

$B = f(a_0)$ $a_0 \in B$ $a_0 \notin B$.

$B = f(a_0) \to a_0 \in B$.

Let $x, y, z \in F$ then

a) If $x+y = x+z$ then $y = z$

(b) If $x+y = x$ then $y = 0$

(c) If $x+y = 0$ then $x = -y$

(d) $-(-x) = x$

Proof

(c) Take $z = 0 - x$ in (a)

$$x + y = x + (-x)$$
$$y = -x$$

(d)

Let $-(-x) = A$

$$A + -x = 0$$
$$A + x - x = 0 + x$$
$$A + 0 = x$$
$$A = x$$

Thus $-(-x) = x$.

⇒ Let x, y & $z \in F$

a) If $x \neq 0$ & $xy = xz$ then $y = z$.

(b) If $x \neq 0$ & $xy = x$ then $y = 1$.

(c) If $x \neq 0$ & $xy = 1$ the $y = 1/x$

(d) If $x \neq 0$ the $\frac{1}{1/x} = x$.

i) If $x>0$ then $-x<0$ & vice versa

(ii) If $x>0$ & $y<z$ then $xy<xz$

(iii) If $x<0$ & $y<z$ then $xy>xz$.

(iv) If $x\neq 0$ then $x^2>0$ in particular, $1>0$.

(v) If $0<x<y$ then $0<\frac{1}{y}<\frac{1}{x}$.

Proof

(i) If $x>0$ then $0=-x+x>-x+0$. So $-x<0$

If $x<0$ then $0=-x+x<-x+0$, So $-x>0$

(ii) Since $z>y$ we have $z-y>y-y=0$.

which means that $z-y>0$. Also $x>0$.

Therefore $x(z-y)>0$.

(iii) Since $y<z \Rightarrow -y+y$